

Stellar evolution and nucleosynthesis

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Nozomu Tominaga

- Astronomy/Astrophysics
- Research interests
 - Relativistic radiation hydrodynamics
 - **Origin of elements**
 - Time-domain astronomy
 - Multi-messenger astronomy
 - Galactic archaeology
- <https://nozomu-tominaga.jp/>

Contents

- Thermonuclear reaction
- Stellar structure
 - Static burning
- Supernova explosion
 - Explosive nucleosynthesis
- Synthesis of heavy elements
- Chemical evolution
- Recent topics

Origin of elements

- Introduction given by Livius Trache-san's lectures.

THE ELEMENTS

Radioactive elements

Photographs show samples of the pure or nearly pure element except for Helium, Al, Be, Fe, Ac, Pa, and Np. Some radioactive elements contain traces of other elements. Pu, Ba, Po, Ra, and Am show artificial objects containing traceable amounts of the element. Technetium shows a 99Mo source. Helium shows a bubble space telescope image of the Eagle Nebula, which is nearly featureless. Se-111 shows the purple or pink color which the element is named. 112-118 had not been obtained yet in 2016.

Poster and photography by Theodore W. Gray and Nick Mann.

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More images and complete technical data can be found at www.periodictable.com.

PERIODICTABLE.COM

Thermonuclear reaction

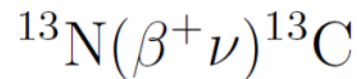
Nuclear reaction equations

$$\frac{dn_A}{dt} = \sum_B \lambda_B^{(A)} n_B + \sum_{C,D} \lambda_{CD}^{(A)} n_C n_D + \sum_{E,F,G} \lambda_{EFG}^{(A)} n_E n_F n_G$$

Number density: n

Reaction rate: r

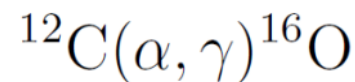
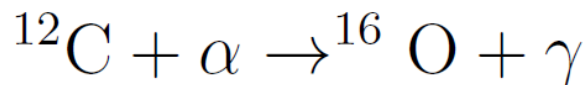
- B -> A + ...



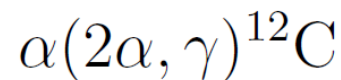
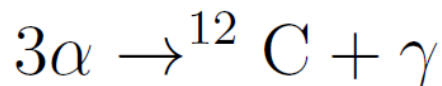
$$\epsilon_n = rQ/\rho$$

Energy generation rate
per reaction Q

- C + D -> A + ...



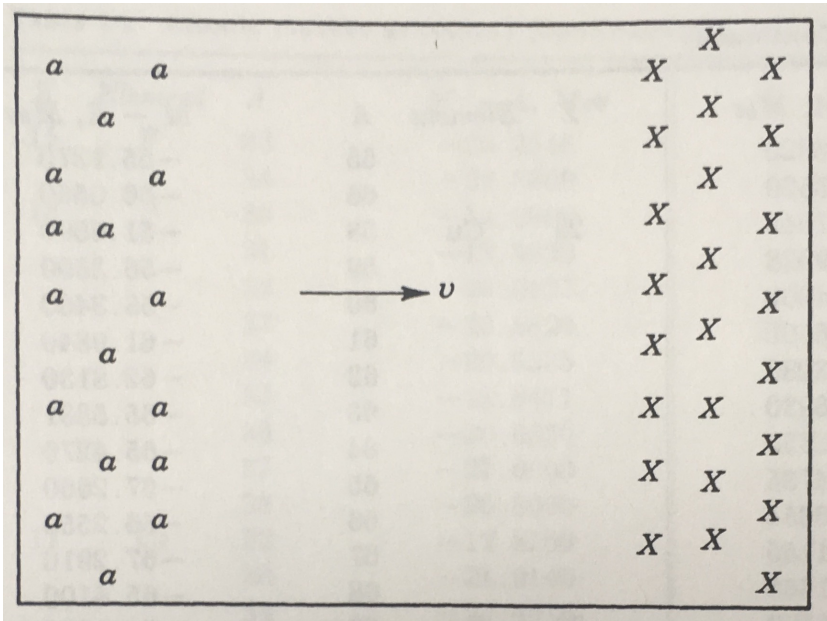
- E + F + G -> A + ...



How are reaction rates evaluated?

Collision of particles

- Reaction: $a + X \rightarrow Y + b$ $X(a,b)Y$
- The reaction occurs when a collides with X.
- a nucleus move to X nucleus with relative velocity of v .



The reaction rate r should be

$$r(v) = \sigma(v)vn_a(\mathbf{v}_a)n_X(\mathbf{v}_X)$$

where $\sigma(v)$ is the cross section for relative velocity of $v = |\mathbf{v}_a - \mathbf{v}_X|$.

Total reaction rate

$$r = \int r(v) d^3 \mathbf{v}_a d^3 \mathbf{v}_X$$

$$r = n_a n_X \langle \sigma(v) v \rangle$$

where $\langle \sigma(v) v \rangle$ is the average of $\sigma(v) v$.

- Maxwell-Boltzmann distribution

$$n(\mathbf{v}) = n \left(\frac{m}{2\pi k_B T} \right)^{\frac{3}{2}} \exp \left(-\frac{mv^2}{2k_B T} \right)$$

- Cross section

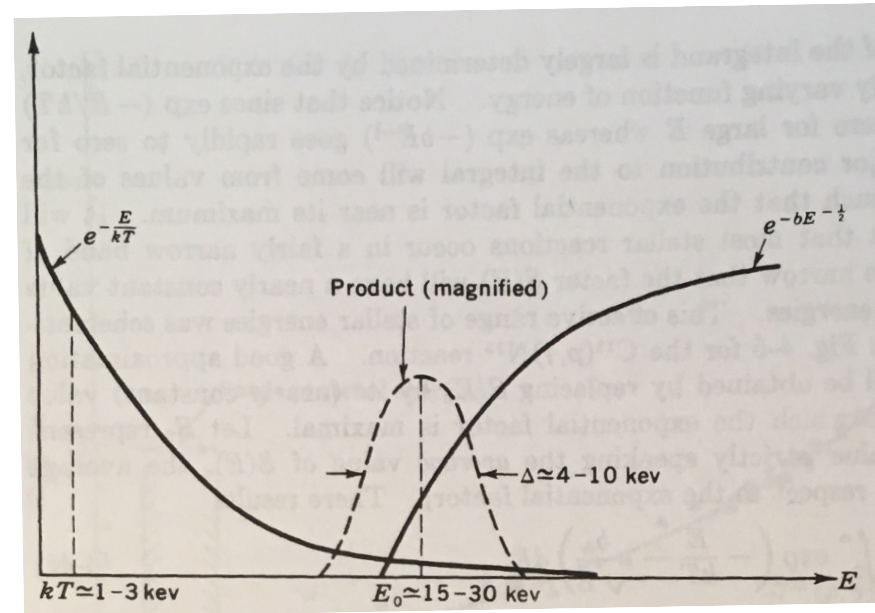
$$\sigma(E) = \frac{S(E)}{E} \exp \left(-\frac{2\pi Z_1 Z_2 e^2}{\hbar v} \right)$$

Value of $\langle \sigma v \rangle$

$$\langle \sigma v \rangle = \frac{\int \sigma(v) v \exp\left(-\frac{\mu v^2}{2k_B T}\right) d^3 v}{\int \exp\left(-\frac{\mu v^2}{2k_B T}\right) d^3 v}$$

- S is almost constant for non-resonant reactions.

$$\langle \sigma v \rangle = \frac{S}{(k_B T)^{3/2}} \frac{4}{(2\pi\mu)^{1/2}} \int \exp\left(-\frac{E}{k_B T} - \left(\frac{2\mu}{E}\right)^{1/2} \frac{\pi Z_1 Z_2 e^2}{\hbar}\right) dE$$



Gamow window

Reaction rate

$$r = n_1 n_2 \frac{\alpha}{T_7^{2/3}} \exp\left(-\frac{\beta}{T_7^{1/3}}\right)$$

where

$$T_7 = T/10^7 K$$

$$\alpha = 2.8 \times 10^{-16} S \left(\frac{Z_1 Z_2 (A_1 + A_2)}{A_1 A_2} \right)^{1/3}$$

$$\beta = 20 \left(\frac{A_1 A_2 Z_1^2 Z_2^2}{A_1 + A_2} \right)^{1/3} \quad [\text{cgs}]$$

S [keV barn]

1 barn = 10^{-24}cm^2

Temperature dependence of reaction rate

$$\varepsilon \propto r \propto \frac{1}{T_7^{2/3}} \exp\left(-\frac{\beta}{T_7^{1/3}}\right)$$

$$\frac{\partial \varepsilon}{\partial T_7} = \frac{1}{T_7^{5/3}} \exp\left(-\frac{\beta}{T_7^{1/3}}\right) \left(\frac{\beta}{3T_7^{1/3}} - \frac{2}{3}\right)$$

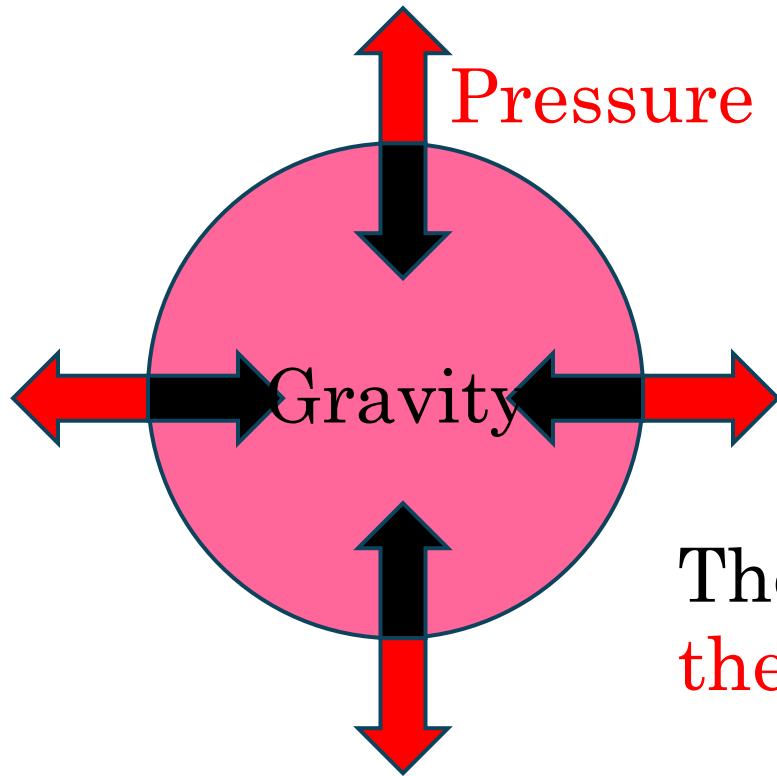
- In stellar core, $\beta \gg T_7^{1/3}$ and thus $\frac{\partial \varepsilon}{\partial T_7} \gg 1$.
- The reaction rate is sensitive to the temperature change. Therefore, the core temperature is almost **fixed** for each burning stage.

Temperature for burning stages

Burning stage	Temperature
H burning	10^7K
He burning	10^8K
C burning	$6 \times 10^8\text{K}$
Ne burning	10^9K
O burning	$2 \times 10^9\text{K}$
Si burning	$3 \times 10^9\text{K}$

Stellar structure

A star is almost static and shines



In order to be static, stars need **pressure** to support their own gravity.

The pressure stems from **thermonuclear fusion reactions**.

Stellar structure

- Mass conservation

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

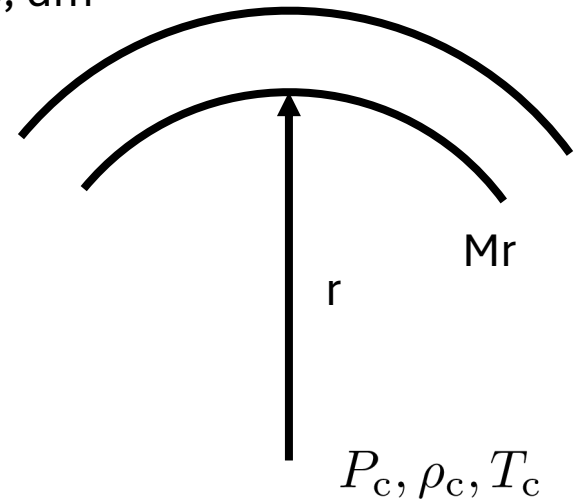
- Equilibrium of force

$$-\frac{GM_r dM_r}{r^2} + 4\pi r^2 P - 4\pi r^2 (P + dP) = 0$$

- Equation of state

$$P = K \rho^{1 + \frac{1}{N}}$$

dr, dp, dm



Lane-Emden equation

$$\frac{d}{dr} \left(-\frac{r^2}{\rho G} \frac{dP}{dr} \right) = 4\pi r^2 \rho$$

Dimensionless parameters

$$\rho = \rho_c \theta(\xi)^N$$

$$\frac{1}{\xi^2} \frac{d}{d\xi} \left(\xi^2 \frac{d\theta}{d\xi} \right) = -\theta^N$$

$$P = P_c \theta(\xi)^{N+1}$$

Boundary condition

$$\theta(0) = 1 \quad \left. \frac{d\theta}{d\xi} \right|_{\xi=0} = 0$$

$$r = \left(\frac{N+1}{4\pi G} \frac{P_c}{\rho_c^2} \right)^{1/2} \xi$$

Solving Lane-Emden eq.,

- Stellar radius: $R = \left(\frac{N + 1}{4\pi G} \frac{P_c}{\rho_c^2} \right)^{1/2} \xi_N$

- Stellar mass: $M = \left(\frac{1}{4\pi G^3} \frac{P_c^4}{\rho_c^3} \right)^{1/2} \varphi_N$

$$\theta(\xi_N) = 0 \qquad \varphi_N = -(N + 1)^{3/2} \left[\xi^2 \frac{d\theta}{d\xi} \right]_{\xi=\xi_N}$$

- Equation of state

- Gas, non-rela. deg. e⁻ gas: N=3/2, $P \propto \rho^{5/3}$

- Radiation, rela. deg. e⁻ gas: N=3, $P \propto \rho^{4/3}$

Degenerate pressure of electrons

- Degenerate pressure at $T=0\text{K}$

- Non-relativistic:
$$P_e^{(0)} = \frac{8\pi}{15h^3 m_e} p_F^5 \propto n_e^{5/3}$$

- Ultrarelativistic:
$$P_e^{(0)} = \frac{2\pi c}{3h^3} p_F^4 \propto n_e^{4/3}$$

- For the ultrarelativistic case,

- Stellar mass

Chandrasekhar's limiting mass
$$M_{\text{ch}} = 1.46 \left(\frac{Y_e}{0.5} \right)^2 M_{\odot}$$

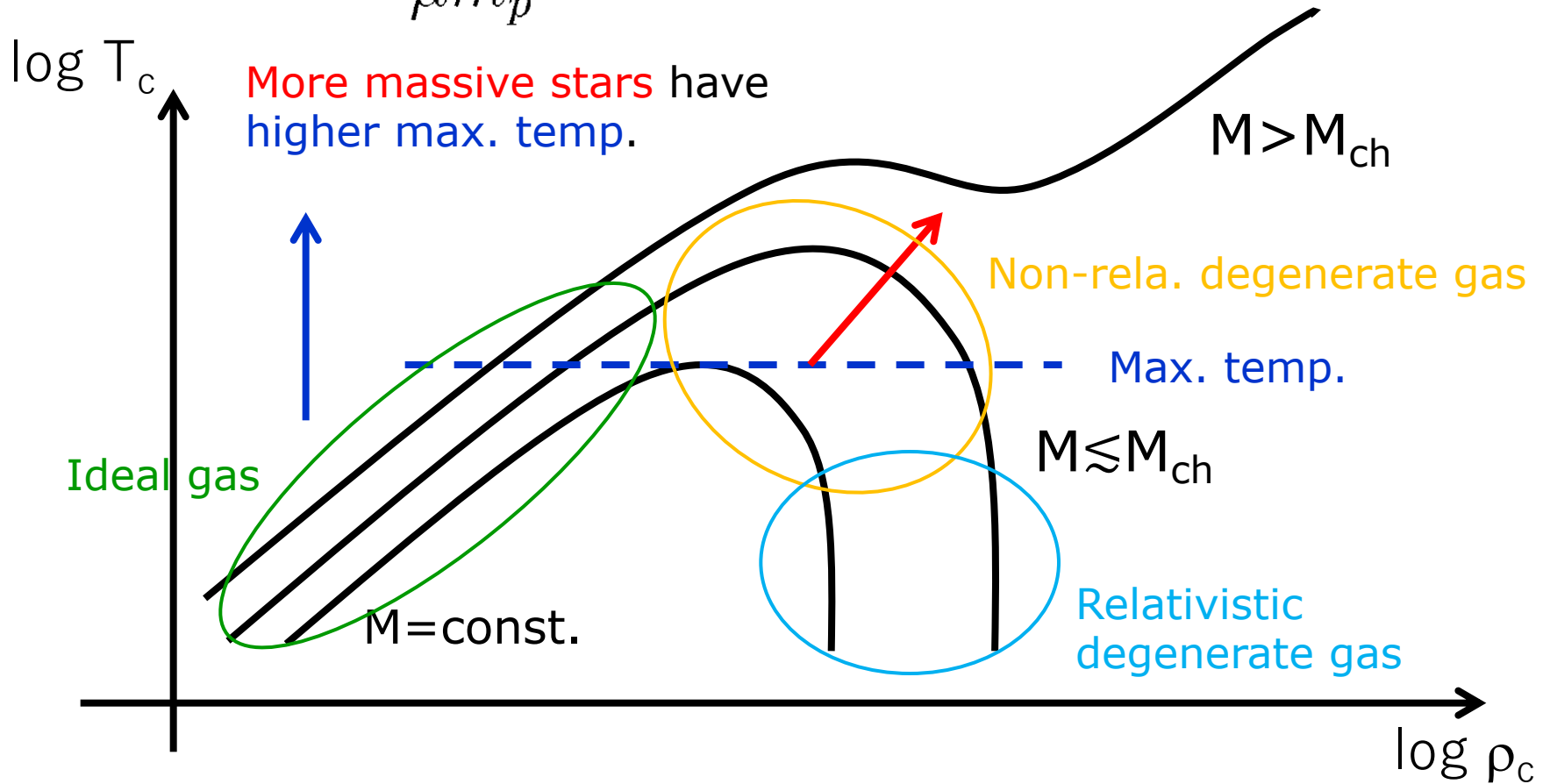
- Stellar radius

$$R \propto Y_e \rho_c^{-1/3} \xrightarrow{\rho_c \rightarrow \infty} 0$$

In reality, e^- -capture takes place and a neutron star will form.

Max. temp. of stars

• EoS: $P = \frac{\rho k T}{\mu m_p} + P_e^{(0)}$ μ : mean molecular weight



Static burning

H burning

- A star mainly consists of H and He.
- However, products of the major two-particle combinations are unstable.
 - $p + p \rightarrow {}^2\text{He} \rightarrow p + p$
 - $p + {}^4\text{He} \rightarrow {}^5\text{Li} \rightarrow p + {}^4\text{He}$
 - ${}^4\text{He} + {}^4\text{He} \rightarrow {}^8\text{Be} \rightarrow {}^4\text{He} + {}^4\text{He}$
- Therefore, bypassing reactions, PP chain and CNO cycle, are needed for H burning.

PP chain - PPI -

Pure H gas

- $p + p \rightarrow d + e^+ + \nu$
- $d + p \rightarrow {}^3\text{He} + \gamma$
- ${}^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + 2p$

Reaction rate

$$r_{pp} = \lambda_{pp} n_p n_p / 2$$

$$r_{pd} = \lambda_{pd} n_p n_d$$

$$r_{33} = \lambda_{33} n({}^3\text{He}) n({}^3\text{He}) / 2$$

$$\frac{dn_p}{dt} = -2\lambda_{pp} \frac{n_p^2}{2} + 2\lambda_{33} \frac{n({}^3\text{He})^2}{2}$$

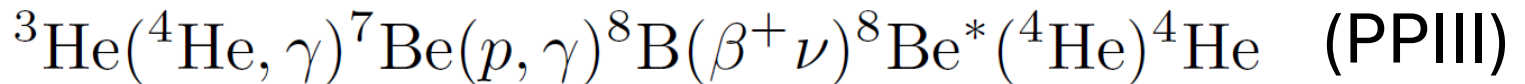
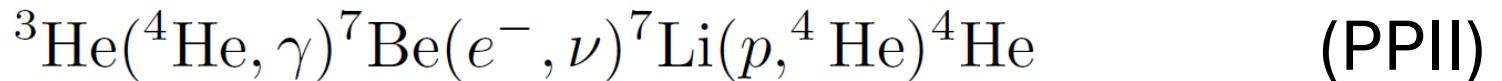
$$\frac{dn_d}{dt} = \lambda_{pp} \frac{n_p^2}{2} - \lambda_{pd} n_p n_d$$

$$\frac{dn({}^3\text{He})}{dt} = \lambda_{pd} n_p n_d - 2\lambda_{33} \frac{n({}^3\text{He})^2}{2}$$

$$\frac{dn({}^4\text{He})}{dt} = \lambda_{33} \frac{n({}^3\text{He})^2}{2}$$

PPII and PPIII chains

- If ${}^4\text{He}$ exists, the following reactions take place



- Since the number of p + p reaction (slow) is once, these could be faster than PPI depending on ρ, T, X .

Table 5-1 Reactions of the PP chains

Reaction	Q value, Mev	Average ν loss, Mev	S_0 , kev barns	$\frac{dS}{dE}$, barns	B	τ_{12} , years†
$\text{H}^1(p, \beta^+ \nu)\text{D}^2$	1.442	0.263	3.78×10^{-22}	4.2×10^{-24}	33.81	7.9×10^9
$\text{D}^2(p, \gamma)\text{He}^3$	5.493		2.5×10^{-4}	7.9×10^{-6}	37.21	4.4×10^{-8}
$\text{He}^3(\text{He}^3, 2p)\text{He}^4$	12.859		5.0×10^3		122.77	2.4×10^5
$\text{He}^3(\alpha, \gamma)\text{Be}^7$	1.586		4.7×10^{-1}	-2.8×10^{-4}	122.28	9.7×10^5
$\text{Be}^7(e^-, \nu)\text{Li}^7$	0.861	0.80				3.9×10^{-1}
$\text{Li}^7(p, \alpha)\text{He}^4$	17.347		1.2×10^2		84.73	1.8×10^{-5}
$\text{Be}^7(p, \gamma)\text{B}^8$	0.135		4.0×10^{-2}		102.65	6.6×10^1
$\text{B}^8(\beta^+ \nu)\text{Be}^8(\alpha)\text{He}^4$	18.074	7.2				3×10^{-8}

† Computed for $X = Y = 0.5$, $\rho = 100$, $T_6 = 15$ (sun).

PP chain

- Solve the following equations

$$\frac{dH}{dt} = -2\lambda_{pp} \frac{H^2}{2} - \lambda_{pd}HD + 2\lambda_{33} \frac{(\text{He}^3)^2}{2} - \lambda_{17}H\text{Be}^7 - \lambda'_{17}H\text{Li}^7$$

$$\frac{dD}{dt} = \lambda_{pp} \frac{H^2}{2} - \lambda_{pd}HD$$

$$\frac{d\text{He}^3}{dt} = \lambda_{pd}HD - 2\lambda_{33} \frac{(\text{He}^3)^2}{2} - \lambda_{34}\text{He}^3\text{He}^4$$

$$\frac{d\text{He}^4}{dt} = \lambda_{33} \frac{(\text{He}^3)^2}{2} - \lambda_{34}\text{He}^3\text{He}^4 + 2\lambda_{17}H\text{Be}^7 + 2\lambda'_{17}H\text{Li}^7$$

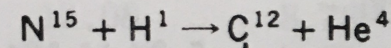
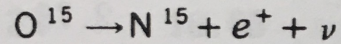
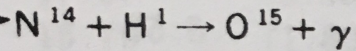
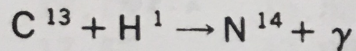
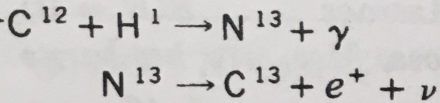
$$\frac{d\text{Be}^7}{dt} = \lambda_{34}\text{He}^3\text{He}^4 - \lambda_{e7}n_e\text{Be}^7 - \lambda_{17}H\text{Be}^7$$

$$\frac{d\text{Li}^7}{dt} = \lambda_{e7}n_e\text{Be}^7 - \lambda'_{17}H\text{Li}^7$$

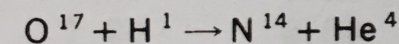
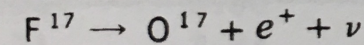
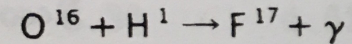
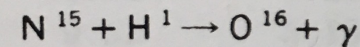
CNO cycle

- If CNO exist, they work as catalysts.

THE CNO BI-CYCLE
($T < 10^8$ °K)



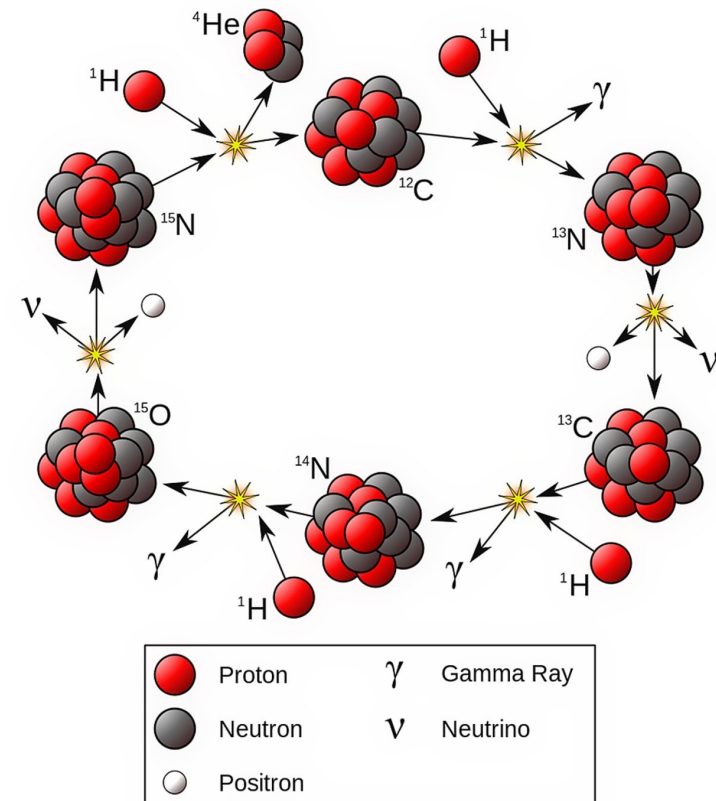
or ($\sim 4 \times 10^{-4}$)



$$\left\{ \begin{array}{l} \tau = 870 \text{ sec} \\ \log \tau (\text{years}) = -4.56 \end{array} \right.$$

$$\left\{ \begin{array}{l} \tau = 178 \text{ sec} \\ \log \tau (\text{years}) = -5.25 \end{array} \right.$$

$$\left\{ \begin{array}{l} \tau = 95 \text{ sec} \\ \log \tau (\text{years}) = -5.52 \end{array} \right.$$



CNO cycle

$$\frac{dC^{12}}{dt} = -\lambda_{p12}HC^{12} + \alpha\lambda_{p15}HN^{15} = -\frac{C^{12}}{\tau_{12}} + \alpha\frac{N^{15}}{\tau_{15}}$$

$$\frac{dN^{13}}{dt} = \frac{C^{12}}{\tau_{12}} - \frac{N^{13}}{\tau_{\beta}(13)} - \frac{N^{13}}{\tau_p(N^{13})} \quad \tau_{\beta}(N^{13}) = 870 \text{ sec}$$

$$\frac{dC^{13}}{dt} = \frac{N^{13}}{\tau_{\beta}(13)} - \frac{C^{13}}{\tau_{13}}$$

$$\frac{dN^{14}}{dt} = \frac{C^{13}}{\tau_{13}} - \frac{N^{14}}{\tau_{14}} + \frac{O^{17}}{\tau_{17}}$$

$$\frac{dO^{15}}{dt} = \frac{N^{14}}{\tau^{14}} - \frac{O^{15}}{\tau_{\beta}(15)} - \frac{O^{15}}{\tau_p(O^{15})} \quad \tau_{\beta}(O^{15}) = 178 \text{ sec}$$

$$\frac{dN^{15}}{dt} = \frac{O^{15}}{\tau_{\beta}(15)} - \frac{N^{15}}{\tau_{15}}$$

$$\frac{dO^{16}}{dt} = \gamma\frac{N^{15}}{\tau_{15}} - \frac{O^{16}}{\tau_{16}}$$

$$\frac{dF^{17}}{dt} = \frac{O^{16}}{\tau_{16}} - \frac{F^{17}}{\tau_{\beta}(17)} - \frac{F^{17}}{\tau_p(F^{17})} \quad \tau_{\beta}(F^{17}) = 95 \text{ sec}$$

$$\frac{dO^{17}}{dt} = \frac{F^{17}}{\tau_{\beta}(17)} - \frac{O^{17}}{\tau_{17}}$$

CNO cycle

- Beta decay is faster than proton capture
- Beta decay and ^{15}N attain equilibrium.
- Branching ratio to ON cycle is small.
- Split CNO cycle to CN and ON cycle.

$$\begin{aligned}\frac{dC^{12}}{dt} &= -\frac{C^{12}}{\tau_{12}} + \frac{N^{14}}{\tau_{14}} \\ \frac{dC^{13}}{dt} &= \frac{C^{12}}{\tau_{12}} - \frac{C^{13}}{\tau_{13}} \\ \frac{dN^{14}}{dt} &= \frac{C^{13}}{\tau_{13}} - \frac{N^{14}}{\tau_{14}}\end{aligned}$$

CN cycle abundance evolution

$$\begin{bmatrix} C^{12} \\ C^{13} \\ N^{14} \end{bmatrix} = N \begin{bmatrix} 0.0122 \\ 0.00305 \\ 0.985 \end{bmatrix} + 0.320N \begin{bmatrix} 1 \\ 0.336 \\ -1.336 \end{bmatrix} \exp(-1.274 \times 10^{-3}t) \\ + \frac{0.222}{240.2} N \begin{bmatrix} 1 \\ 240.5 \\ -241.5 \end{bmatrix} \exp(-4.983 \times 10^{-3}t)$$

- At $t \rightarrow \infty$, most of nucleus are converted to ^{14}N .

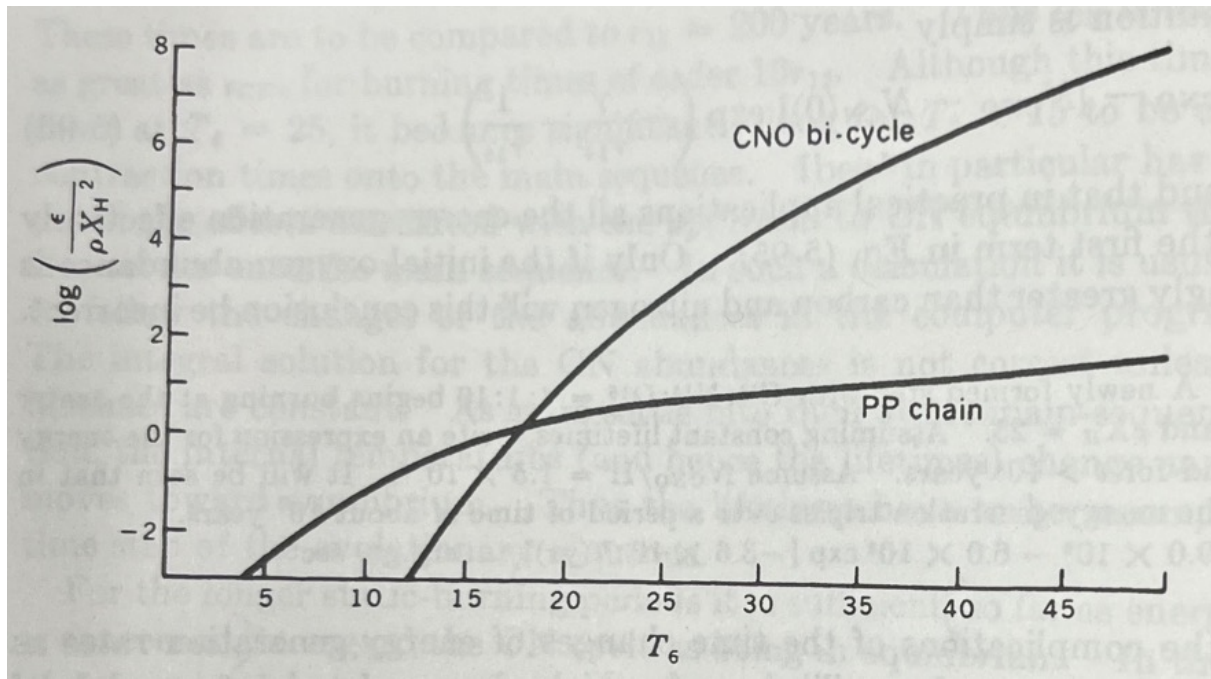
PP chain vs. CNO cycle

$$\epsilon = 2.36 \times 10^6 \rho X_{\text{H}}^2 T_6^{-1} \exp(-33.81 T_6^{-1}) \psi_{\epsilon}(\alpha) (1 + 0.0123 T_6^{\frac{1}{2}} + 0.0109 T_6^{\frac{3}{2}} + 0.00095 T_6) \quad \text{erg g}^{-1} \text{sec}^{-1}$$

where

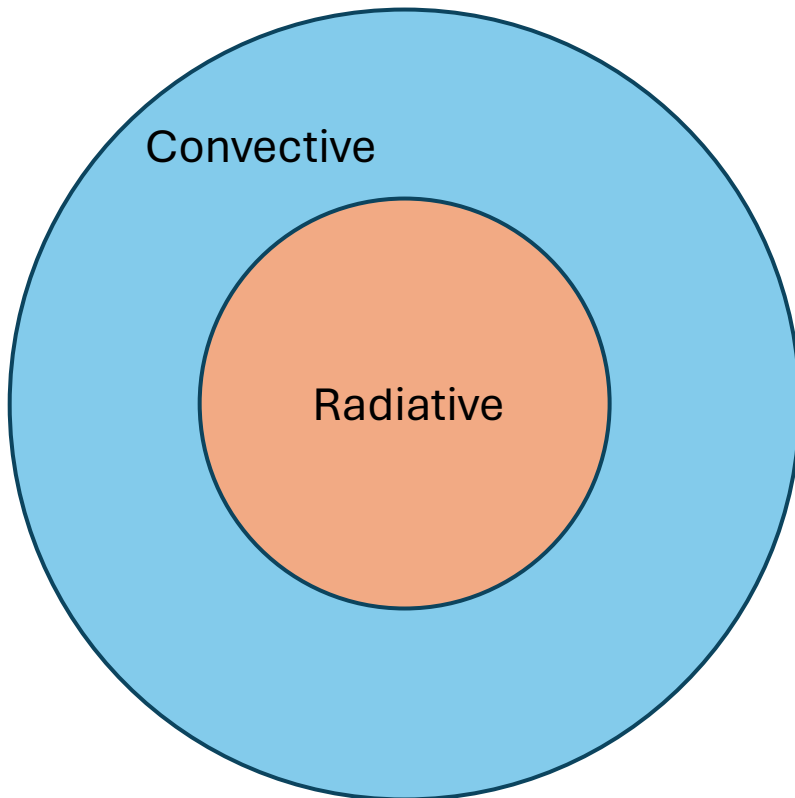
$$\psi_{\epsilon}(\alpha) = \Phi(\alpha) (0.980 F_{\text{PPI}} + 0.960 F_{\text{PPII}} + 0.721 F_{\text{PPIII}})$$

$$\epsilon_{\text{CNO}} \approx 8 \times 10^{27} \rho X_{\text{H}} X_{\text{CN}} f_{\text{N}} T_6^{-\frac{3}{2}} \exp(-152.31 T_6^{-1}) \quad \text{erg g}^{-1} \text{sec}^{-1}$$

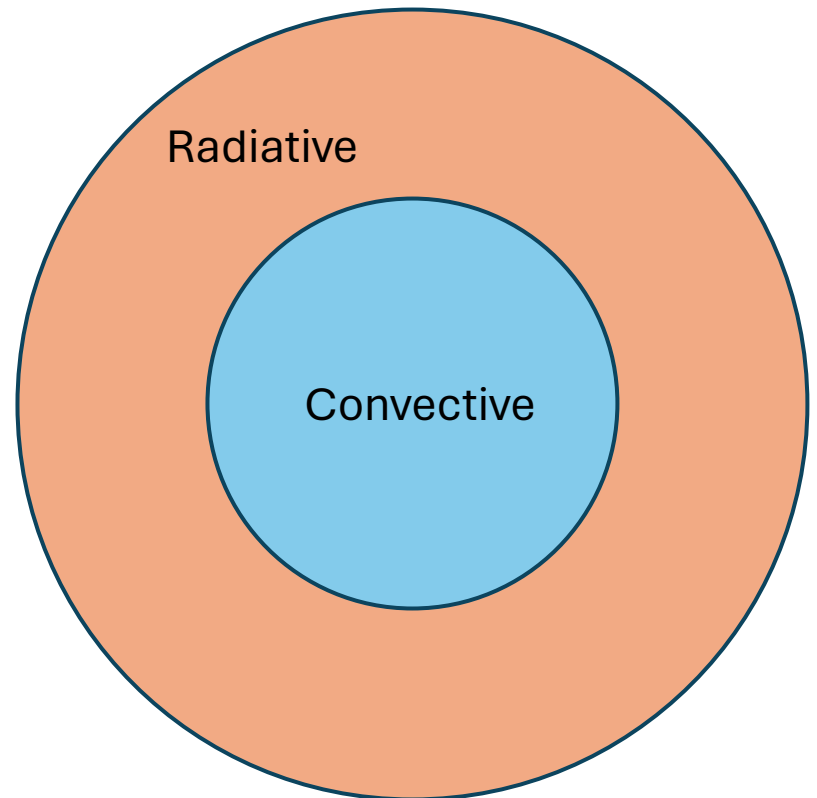


Stellar interior

- PP chain



- CNO cycle



He burning

- ${}^8\text{Be}$ is in equilibrium $\alpha + \alpha \longleftrightarrow {}^8\text{Be}$

$$n({}^8\text{Be}) = n_\alpha^2 \omega f \frac{h^3}{(2\pi\mu k_B T)^{3/2}} \exp\left(-\frac{E_r}{k_B T}\right)$$

- ${}^8\text{Be} + \alpha$ is also close to equilibrium

$$n({}^{12}\text{C}^*) = n({}^8\text{Be})n_\alpha f \frac{h^3}{(2\pi\mu k_B T)^{3/2}} \exp\left(-\frac{E_r}{k_B T}\right)$$

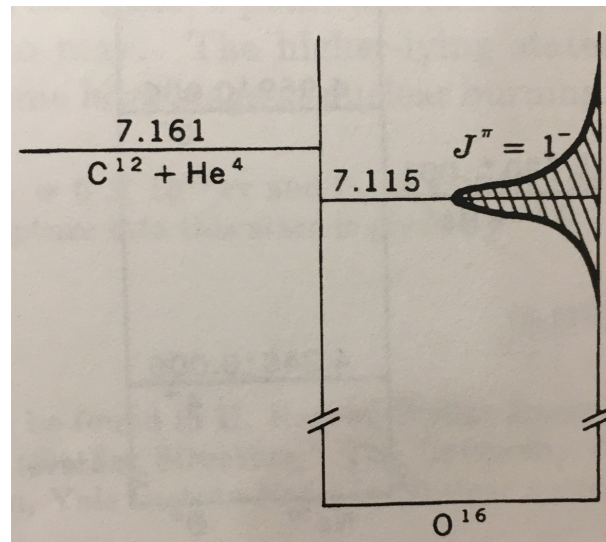
- The decay rate of ${}^{12}\text{C}^* \rightarrow {}^{12}\text{C} + \gamma$ is $\frac{\Gamma_\gamma}{\hbar}$

$$r_{3\alpha \rightarrow {}^{12}\text{C}} = \frac{n({}^{12}\text{C}^*)\Gamma_\gamma}{\hbar} = 9.8 \times 10^{-54} \frac{n_\alpha^3}{T_8^3} f \exp\left(-\frac{42.94}{T_8}\right)$$

$$\varepsilon_{3\alpha} = 3.9 \times 10^{11} \frac{\rho^2 X_\alpha^3}{T_8^3} f \exp\left(-\frac{42.94}{T_8}\right)$$

Nucleosynthesis during He burning

- $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$
 - Important reaction affects subsequent burning stage
- The reaction rate is not well determined.



- And probably $^{16}\text{O}(\alpha, \gamma)^{20}\text{Ne}$

The Sun after 5Gyr

太陽

The white dwarf cools with time.



The envelope will be a planetary nebular and the core will be a white dwarf.

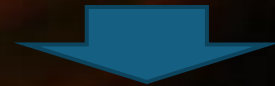
H is exhausted at the center



The Sun expands to be a redgiant star.



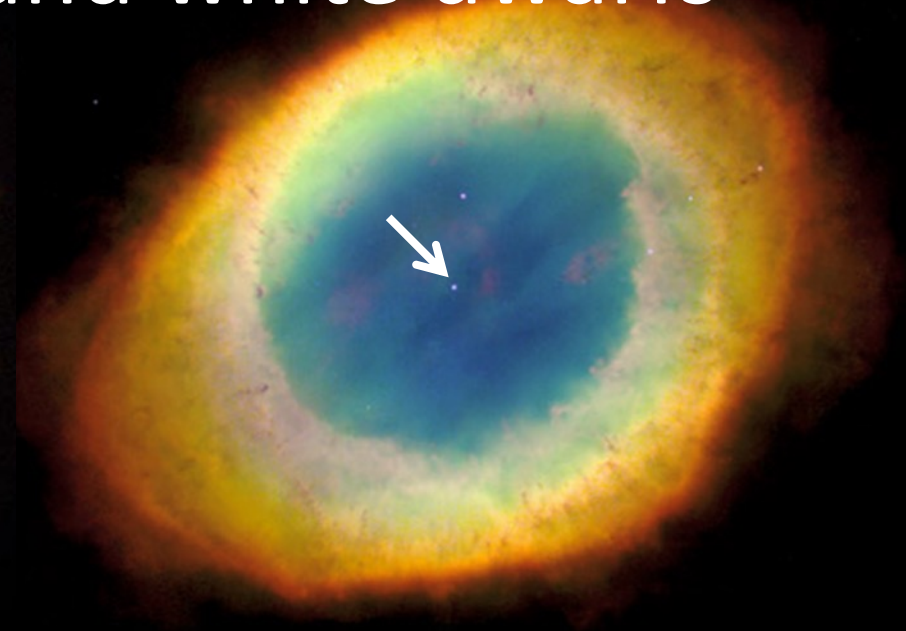
He core burning starts and the Sun becomes a horizontal branch star.



The Sun expands to be an asymptotic redgiant branch star.



Planetary nebular and white dwarfs



Advanced burning stage

- C burning ($\epsilon \sim 2.6 \times 10^{40} \rho X_C^2 \lambda_{C,C}$ erg/g/s)

<i>Reaction Channel</i>	<i>Q, Mev</i>
$C^{12} + C^{12} \rightarrow Mg^{24} + \gamma$	13.930
$\rightarrow Na^{23} + p$	2.238
$\rightarrow Ne^{20} + \alpha$	4.616
$\rightarrow Mg^{23} + n$	-2.605
$\rightarrow O^{16} + 2\alpha$	-0.114

- O burning ($\epsilon \sim 2 \times 10^{40} \rho X_O^2 \lambda_{O,O}$ erg/g/s)

<i>Reaction Channel</i>	<i>Q, Mev</i>
$O^{16} + O^{16} \rightarrow S^{32} + \gamma$	16.539
$\rightarrow P^{31} + p$	7.676
$\rightarrow S^{31} + n$	1.459
$\rightarrow Si^{28} + \alpha$	9.593
$\rightarrow Mg^{24} + 2\alpha$	-0.393

Equilibrium (at high temperature)

- $A + B \rightleftharpoons C + D$
- The reaction releases energy of Q
- Chemical potential of ideal gas

$$\mu = -k_B T \ln \left[\left(\frac{mk_B T}{2\pi\hbar^2} \right)^{3/2} \frac{V}{N} \right] + \phi_r(T)$$

- Equilibrium condition

$$\mu_A + \mu_B + Q = \mu_C + \mu_D$$

Saha equation

$$\frac{n_A n_B}{n_C} = \frac{g_A g_B}{g_C} \left(\frac{k_B T}{2\pi \hbar^2} \frac{m_A m_B}{m_C} \right)^{3/2} \exp \left(-\frac{Q}{k_B T} \right)$$

- After repeated action,

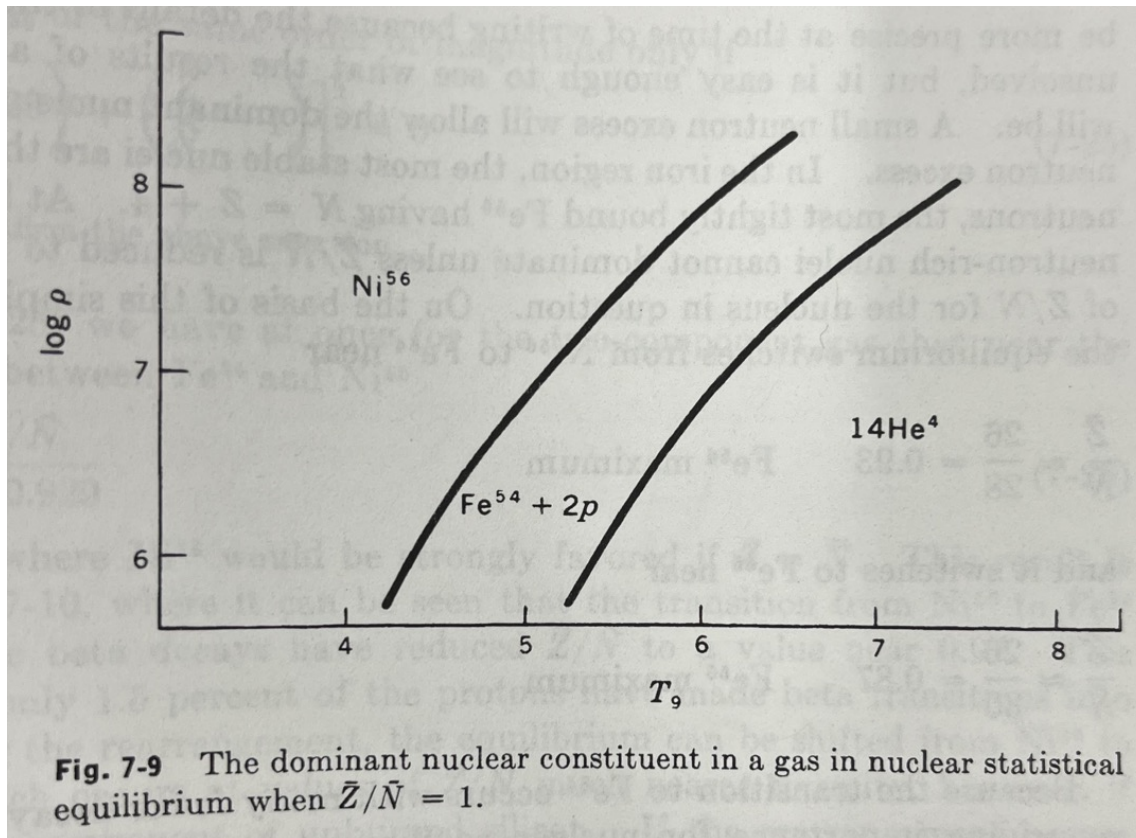
$$\frac{n_p^Z n_n^{(A-Z)}}{n(A, Z)} = \frac{2^A}{g(A, Z)} A^{-3/2} \theta^{A-1} \exp \left(-\frac{Q(A, Z)}{k_B T} \right)$$

$$Q(A, Z) = c^2 [Z m_p + (A - Z) m_n - m(A, Z)] \quad \theta = \left(\frac{2\pi m_p k_B T}{h^2} \right)^{3/2}$$

Here, the change of Y_e is ignored.

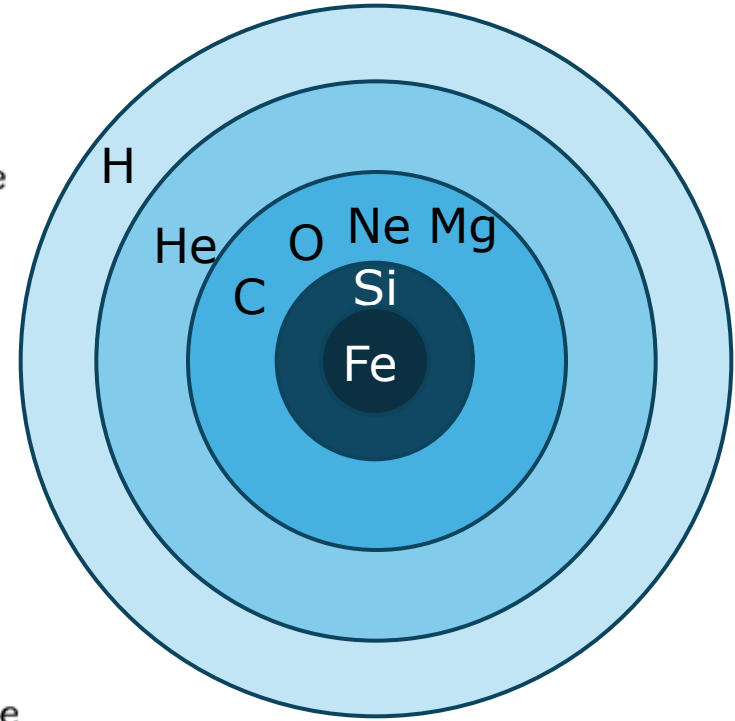
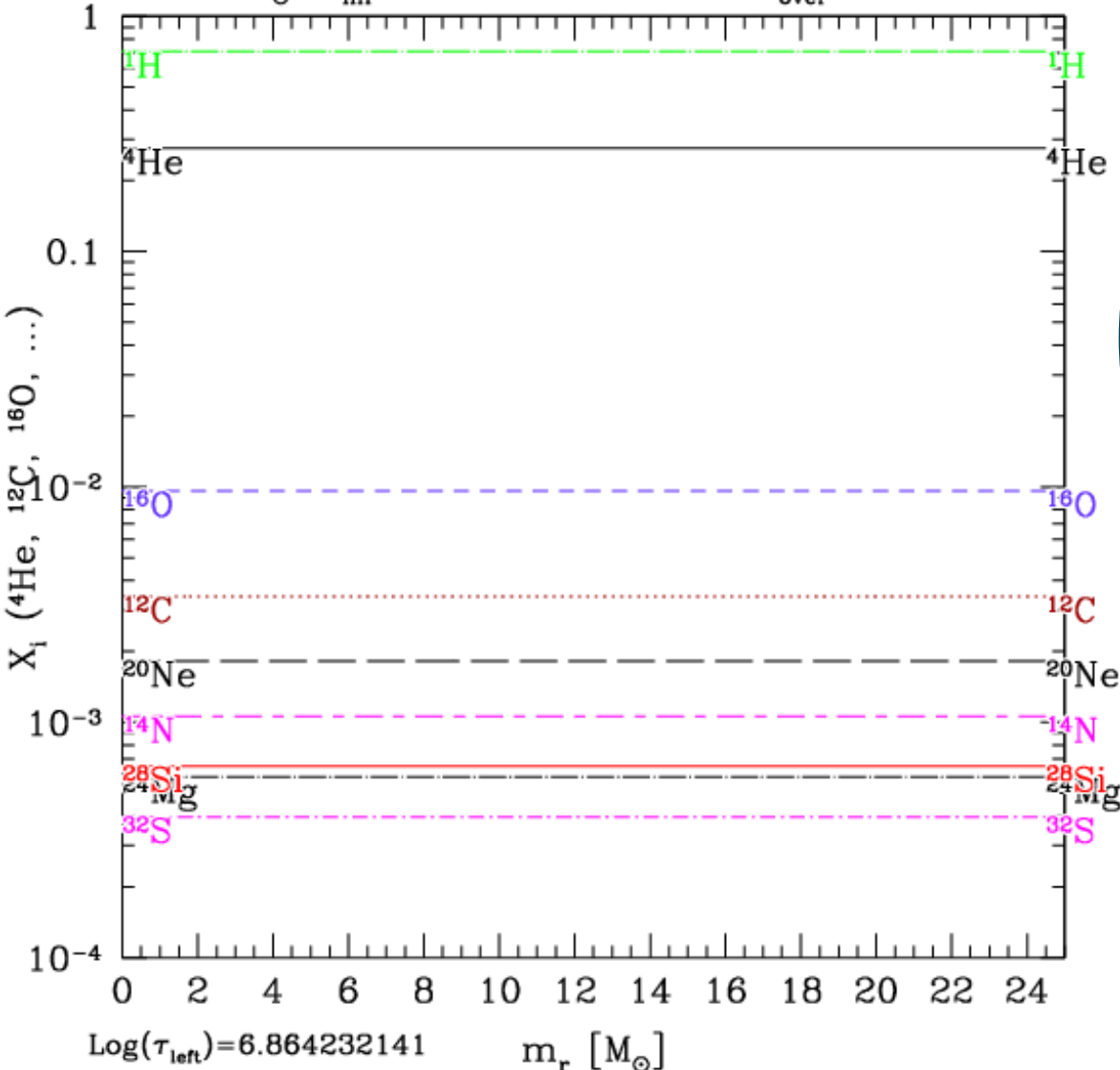
Nuclear Statistical Equilibrium

$$\frac{N_{\alpha}^{14}}{Ni^{56}} = \frac{4^{21}}{56^{\frac{3}{2}}} \theta^{13} \exp \frac{14Q(\alpha) - Q(Ni^{56})}{kT}$$



Stellar evolution

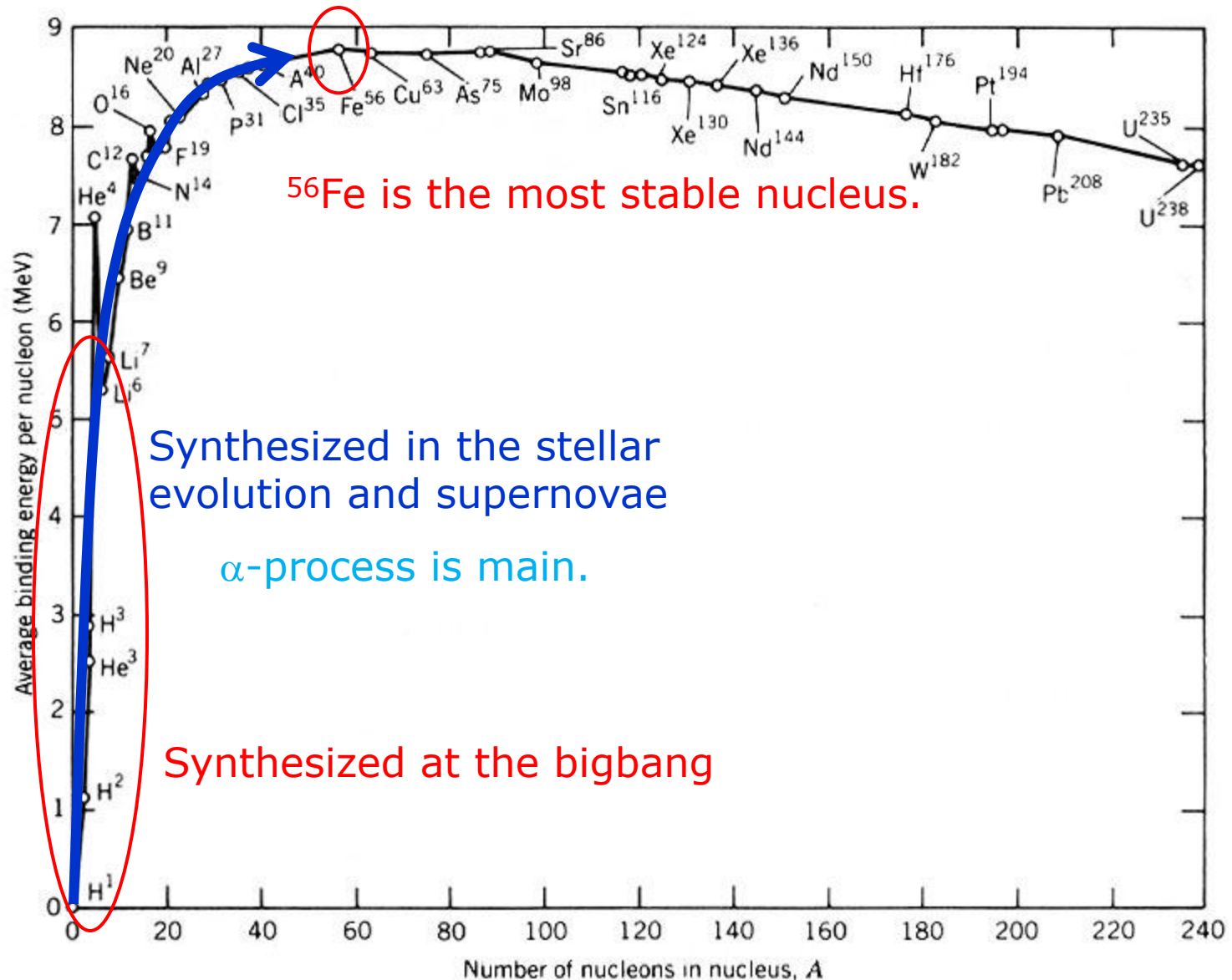
$M=25 M_{\odot}$, $v_{\text{ini}}=0 \text{ km s}^{-1}$, $Z=0.02$, $\alpha_{\text{over}}=0.1$



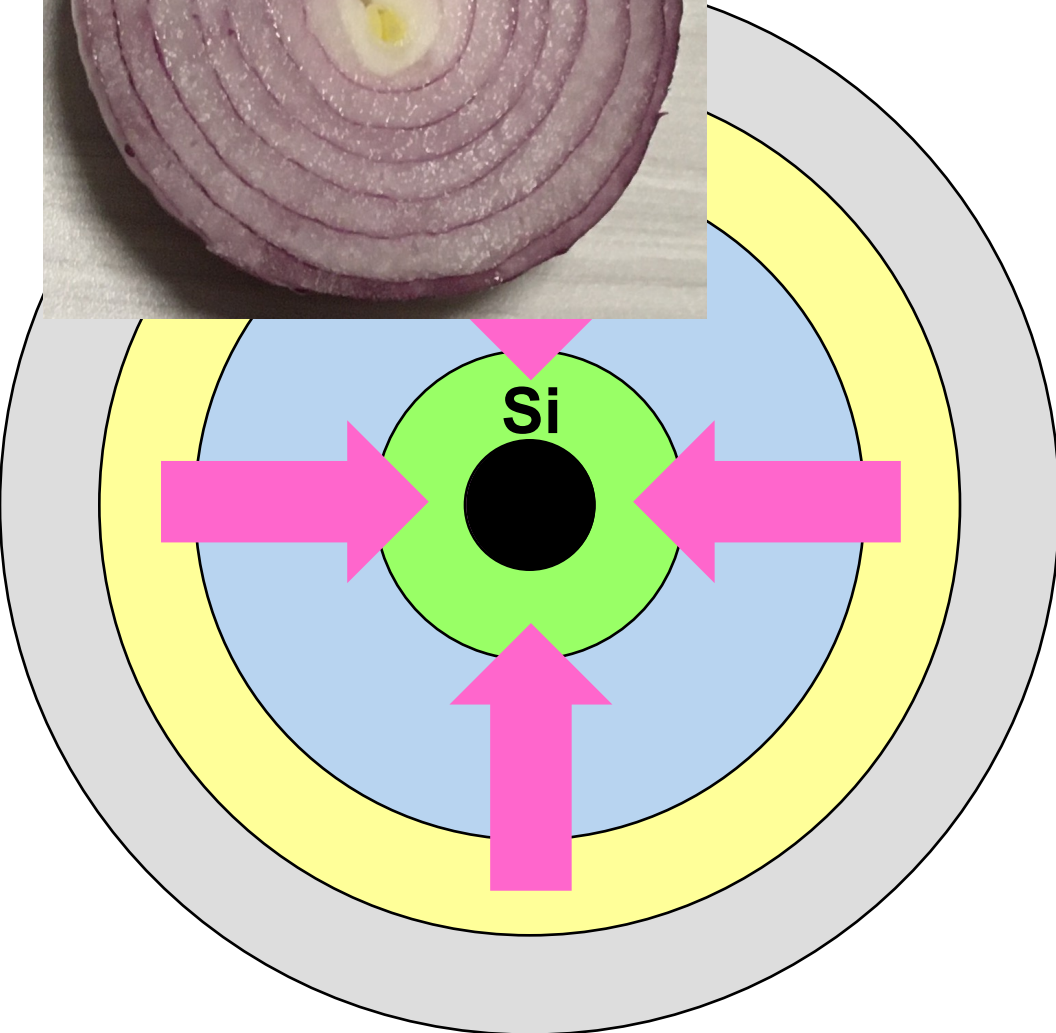
Onion-like structure

(c) Raphael Hirschi

Binding energy of nucleus



Core of massive star



Onion-like structure

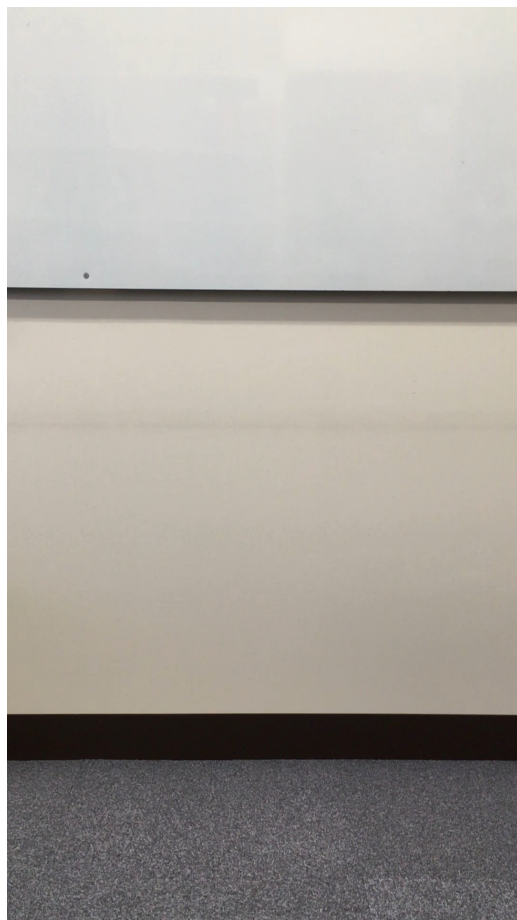
No nuclear energy
can be extracted
from Fe.

Core collapse

Proto neutron star
is formed.

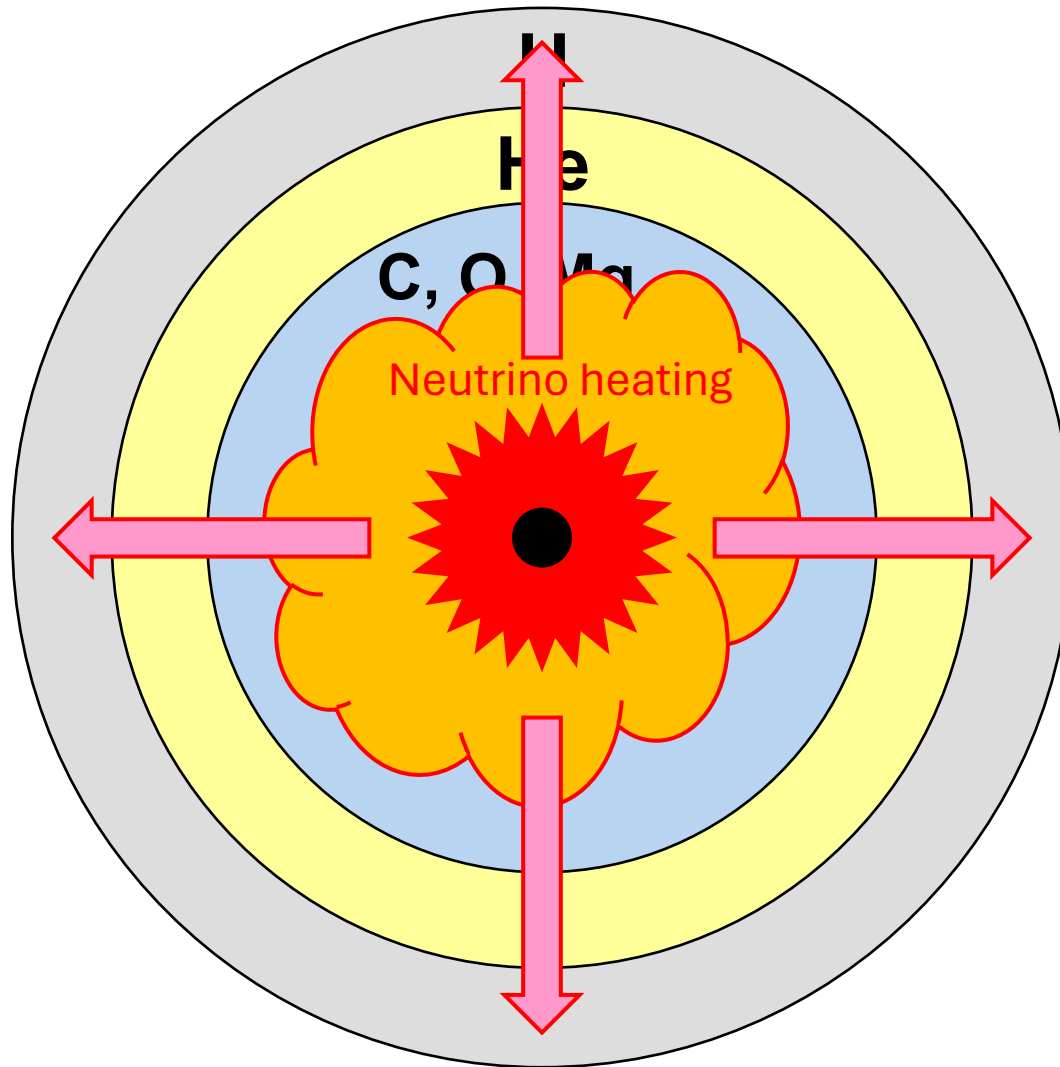
Surrounding
matter continues
falling.

Infalling matter **bounces**.



Supernova explosions

Supernova explosion



The stellar materials are ejected as a supernova.

Supernovae

Very bright

$$L \sim 10^{42} \text{erg/s} \sim 10^9 L_{\odot}$$

Energy source

Shock heating

^{56}Ni - ^{56}Co radioactive decay

Huge energy

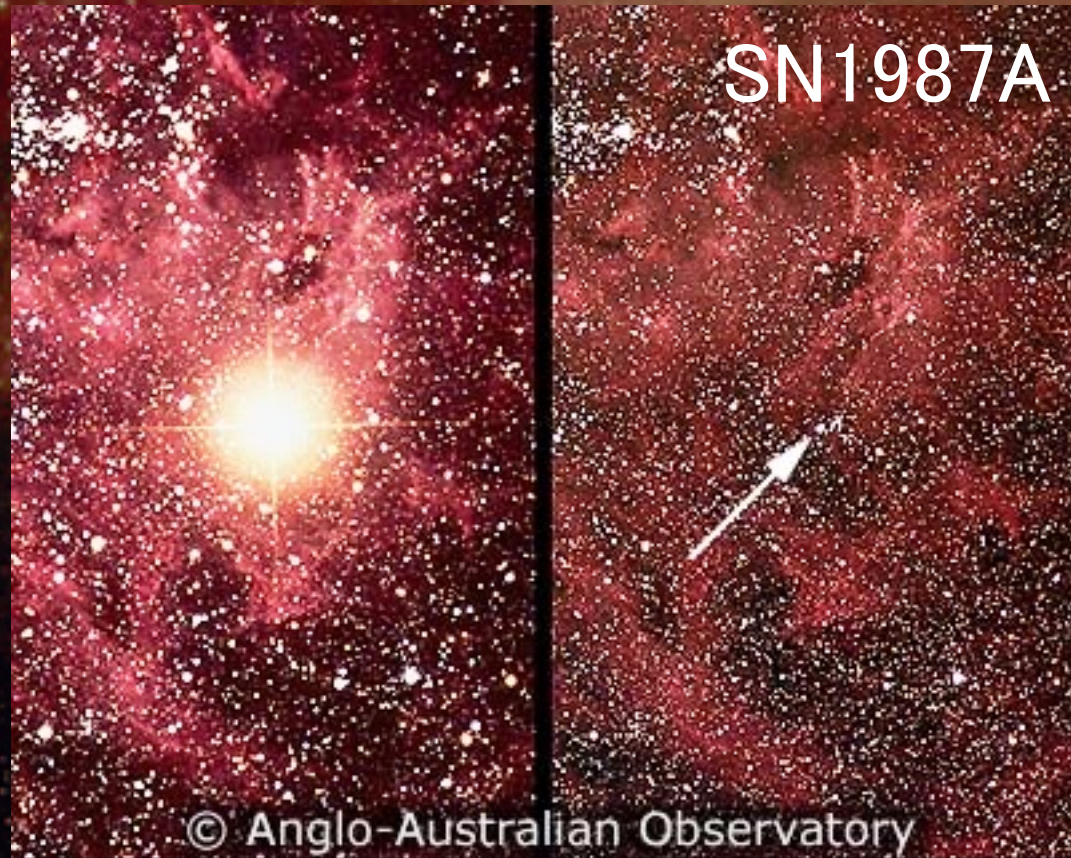
$$E_K \sim 10^{51} \text{erg}$$

Gravitational energy

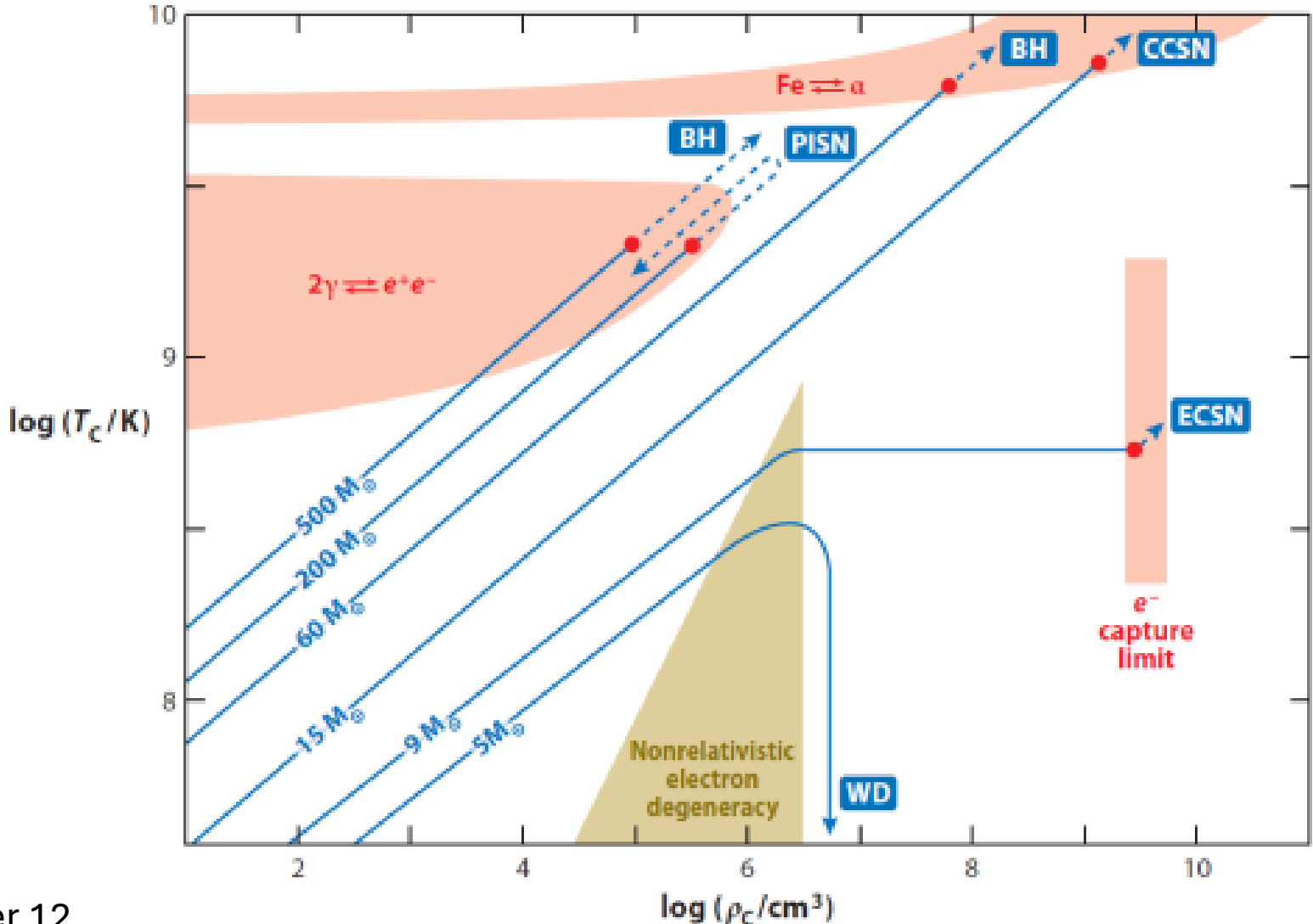
$$GM_{\odot}^2 / R_{\text{NS}} \sim 10^{53} \text{erg}$$

Nuclear energy

$$\Delta(^{12}\text{C} \rightarrow ^{56}\text{Ni}) M_{\odot} \sim 10^{51} \text{erg}$$



Stellar fates



Stellar fates depend on their masses

Type Ia supernova

Brown dwarf

White dwarf

Neutron star or black hole

Pair instability supernova

Mass loss

Supernovae

Core collapse

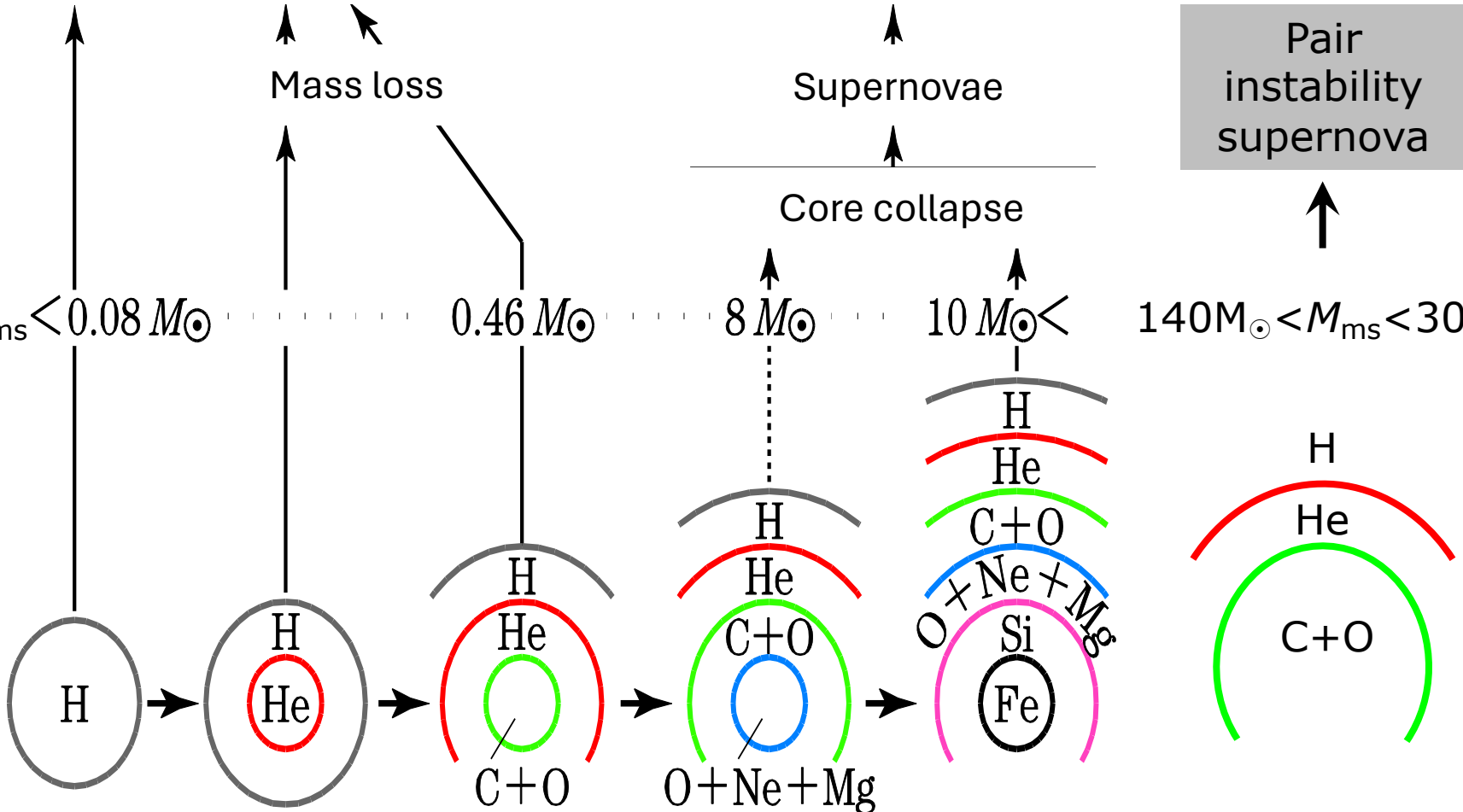
$$M_{\text{ms}} < 0.08 M_{\odot}$$

$$0.46 M_{\odot}$$

$$8 M_{\odot}$$

$$10 M_{\odot} <$$

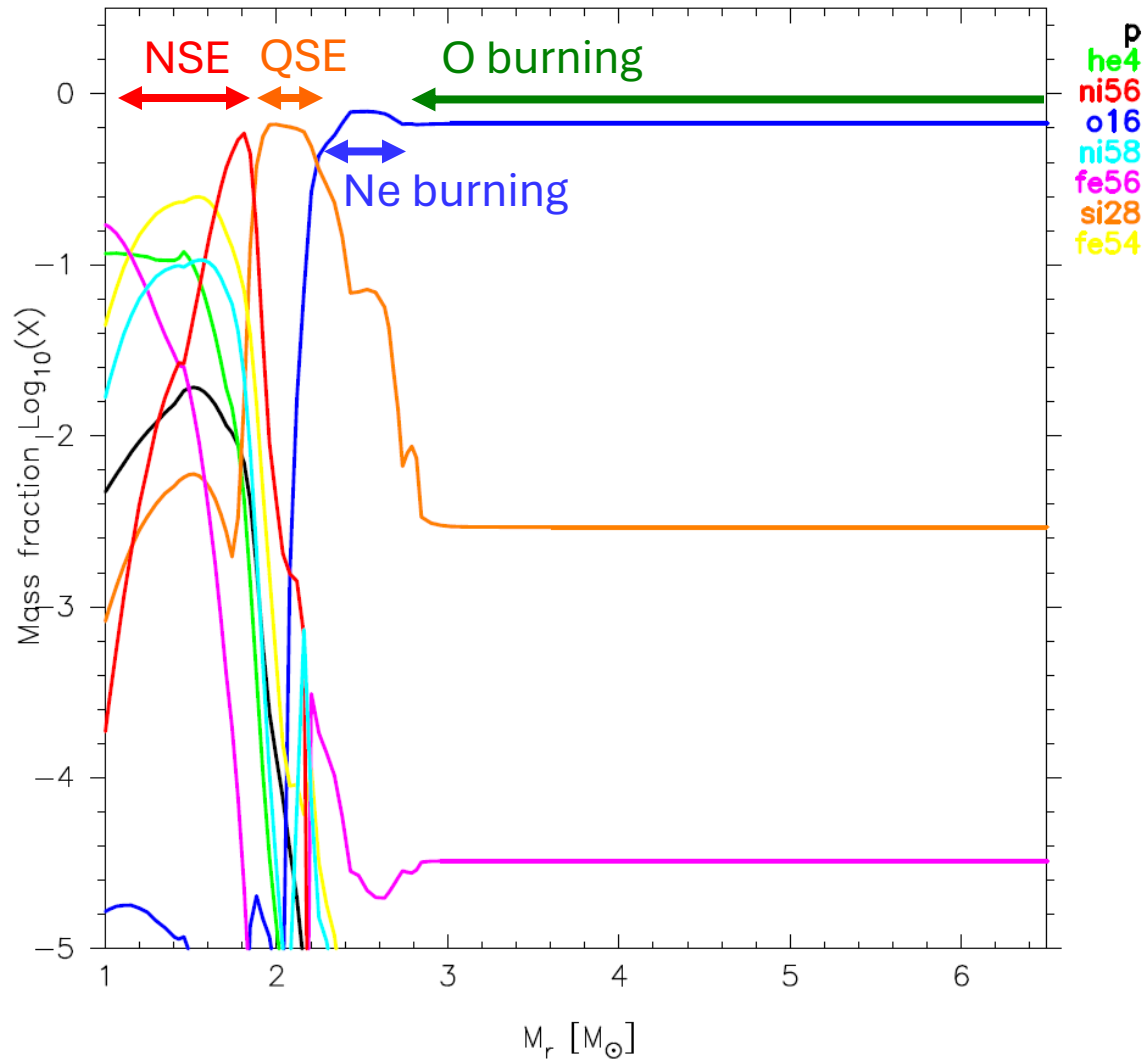
$$140 M_{\odot} < M_{\text{ms}} < 300 M_{\odot}$$



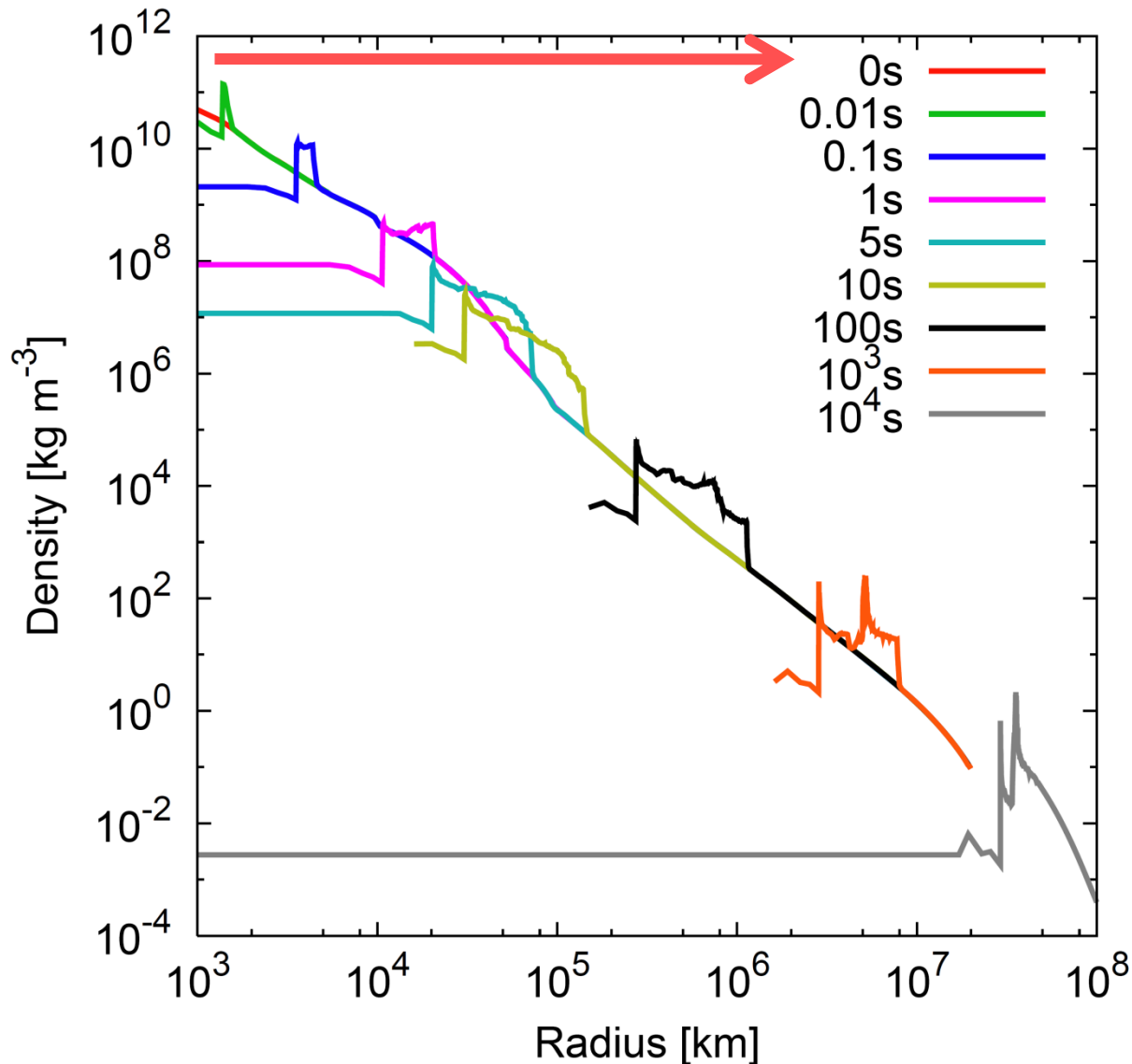
元素はいかにつられたか(野本憲一編)

Explosive nucleosynthesis

Abundance distribution at the presupernova stage

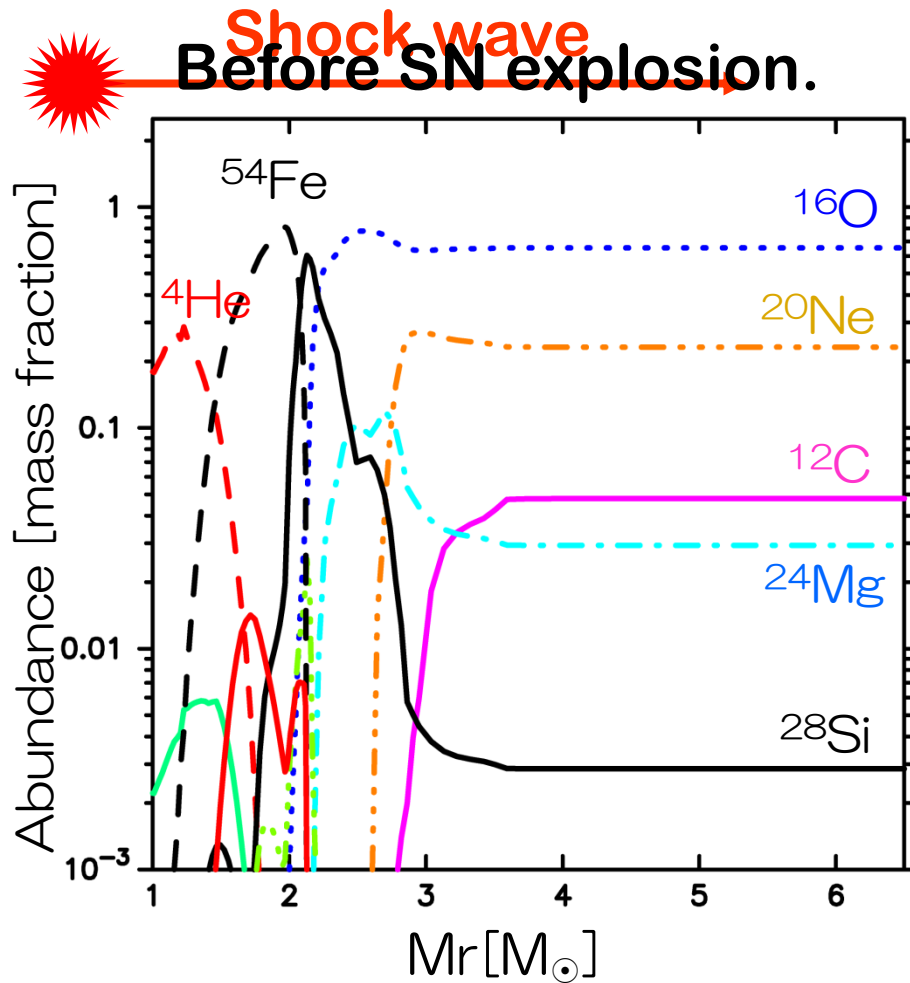


Shock wave in the stellar mantle



Radiation energy is dominant after the shock wave and radiation are fully coupled with matter.

Explosive nucleosynthesis



Explosive nucleosynthesis

- Temp. behind the shock

- radiation dominated

$$E = aT^4 \frac{4\pi R^3}{3}$$

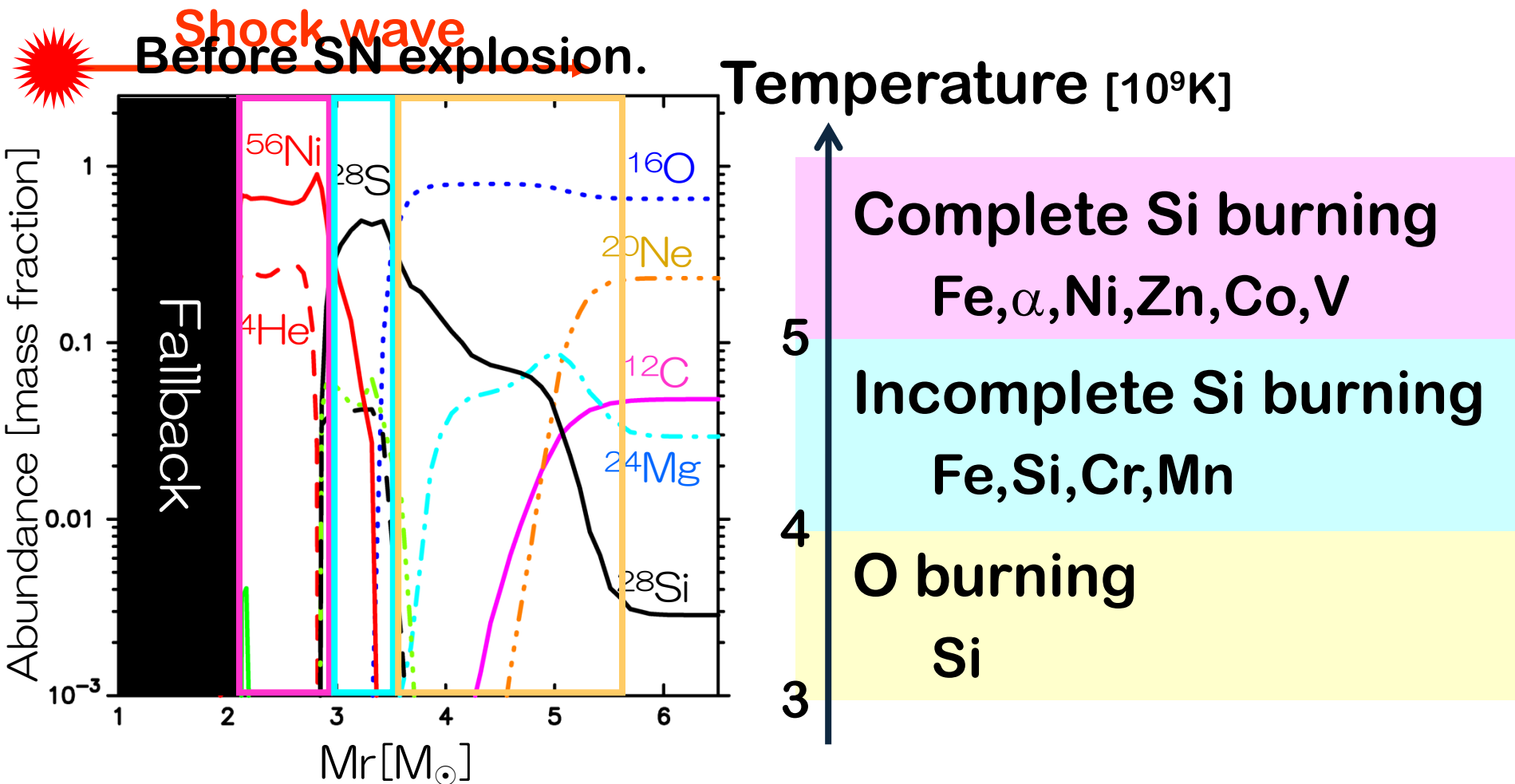
$$\frac{T}{10^9\text{K}} = 13 \left(\frac{E}{10^{51}\text{erg}} \right)^{1/4} \left(\frac{R}{10^3\text{km}} \right)^{-3/4}$$

- Short hydrodynamical timescale
 - Slightly **higher temp.** is required

Burning stage	Temperature
Explosive C & Ne burning	2x10 ⁹ K
Explosive O burning	3-4x10 ⁹ K
Incomplete Si burning (QSE)	4x10 ⁹ K
Complete Si burning (NSE)	5x10 ⁹ K

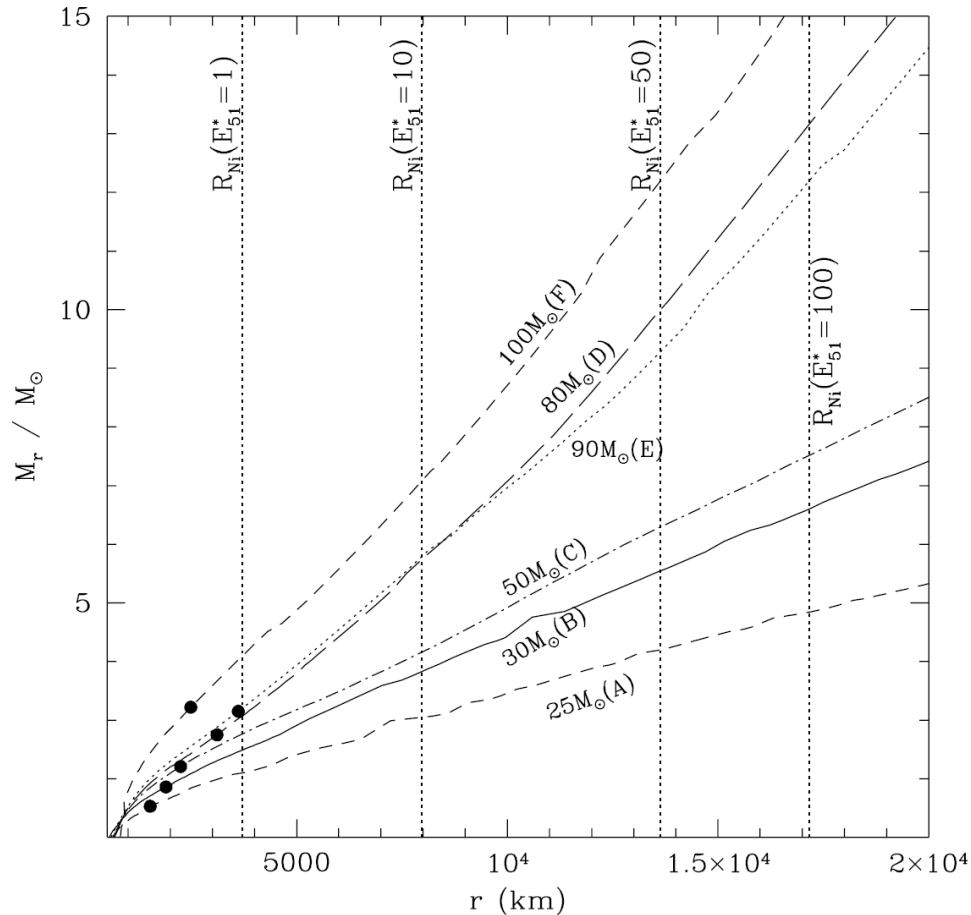
Explosive nucleosynthesis

Heavier elements are synthesized at **higher T** layer (i.e., in the inner layer).

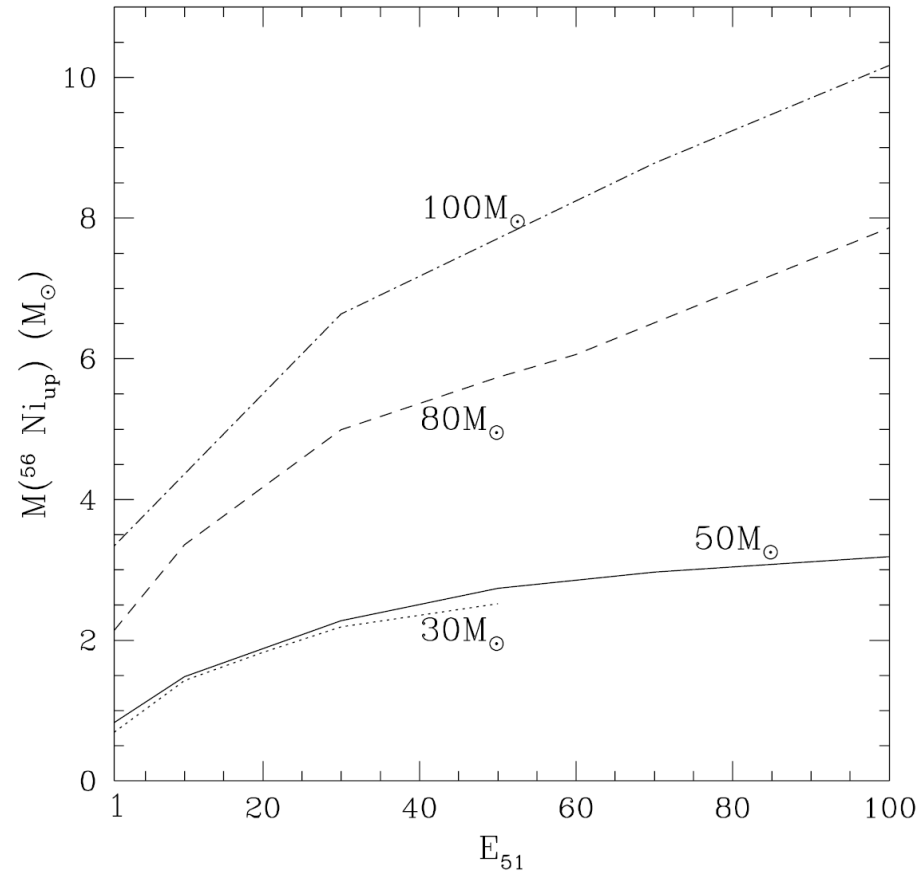


Stellar structure and nucleosynthesis

Radius-mass relation



Max. of synthesized ^{56}Fe mass



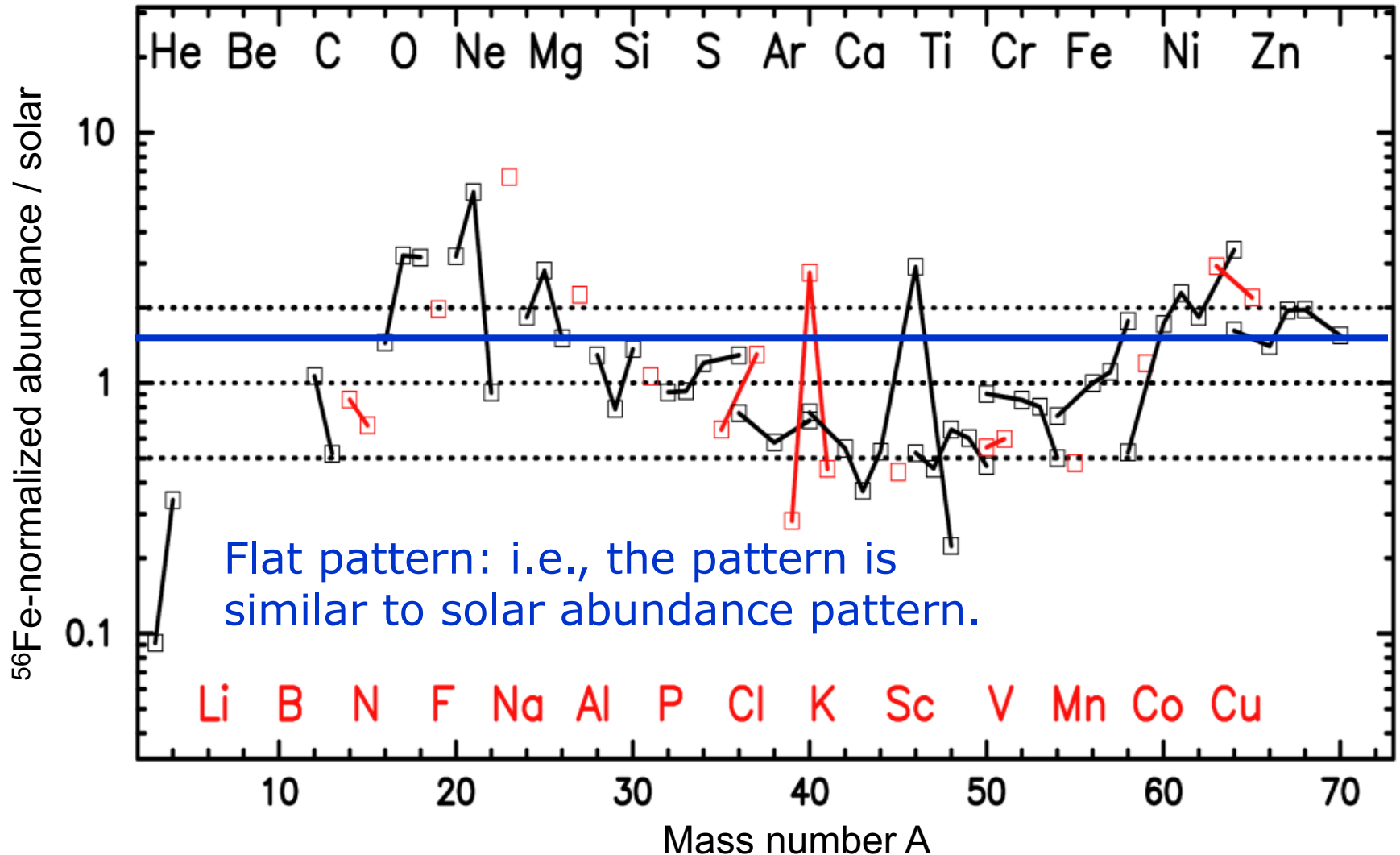
Varieties of supernovae

- Core-collapse SNe (CCSNe)
 - Explosions of massive stars ($M > 8M_{\odot}$)
 - Gravitational energy
- Type Ia SNe (SNe Ia)
 - Explosions of accreting white dwarf
 - Thermonuclear energy
- Pair-instability SNe (PISNe)
 - Explosions of very massive stars ($M \sim 140-300M_{\odot}$)
 - Thermonuclear energy

Nucleosynthesis in CCSNe

- **Explosion of massive stars**
 - $M_{\text{ms}} > 8M_{\odot}$, $R \sim 10^{5-9} \text{km}$
 - $E \sim 10^{51-52} \text{erg}$, $M_{\text{ej}} \sim 1-30M_{\odot}$, $M(\text{Fe}) \sim 0.1M_{\odot}$
- **Gravitational energy**
 - $\sim 1\%$ of released $E_{\text{grav}} = GM_{\odot}^2/R_{\text{NS}} \sim 10^{53} \text{erg}$
- Large amount of **various heavy elements**
- Interval from star formation: **$\sim 2-20 \text{Myr}$**
 - First metal-enrichment in the universe is made by CCSNe.

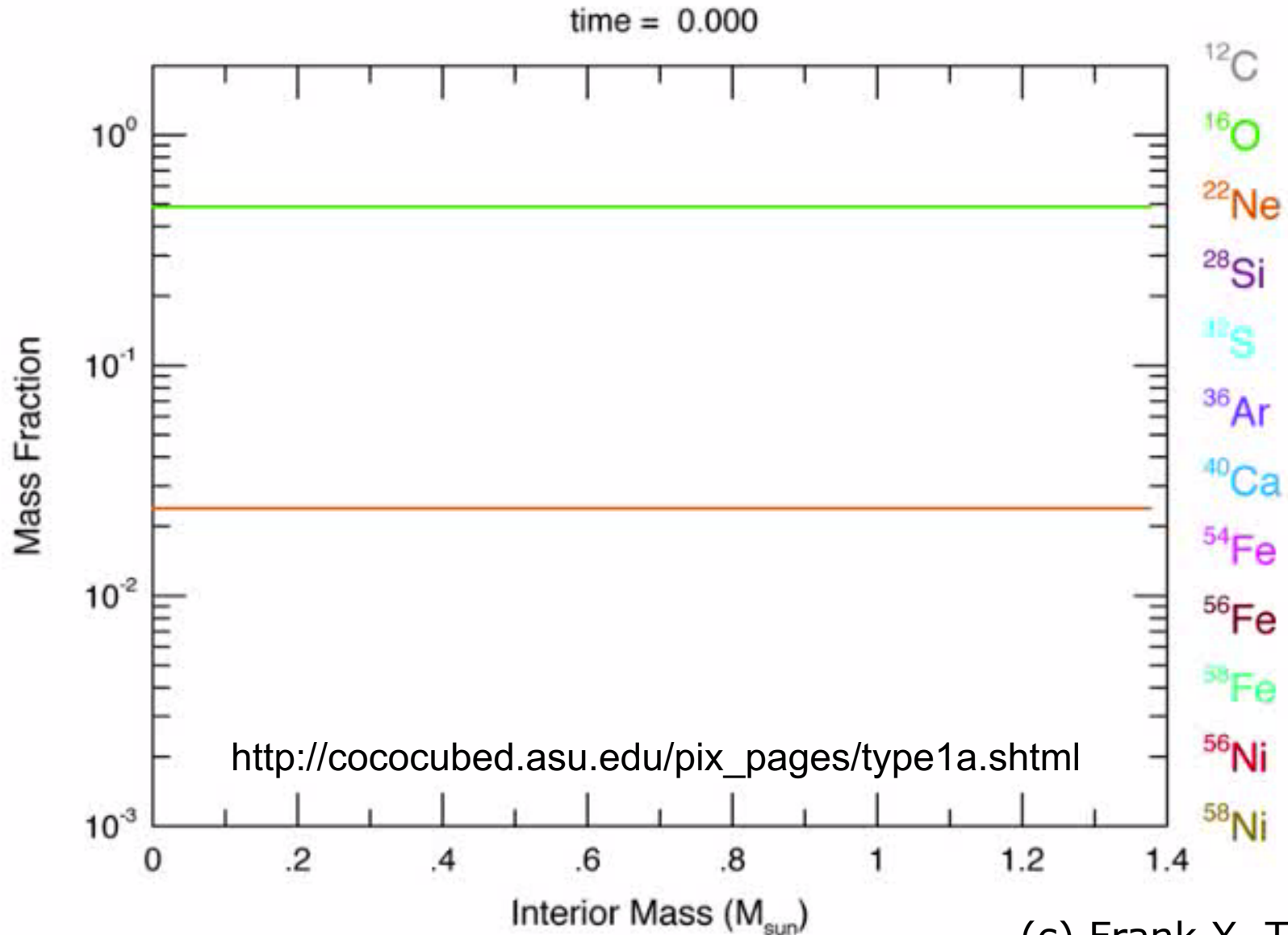
Abundance pattern of CCSN yield



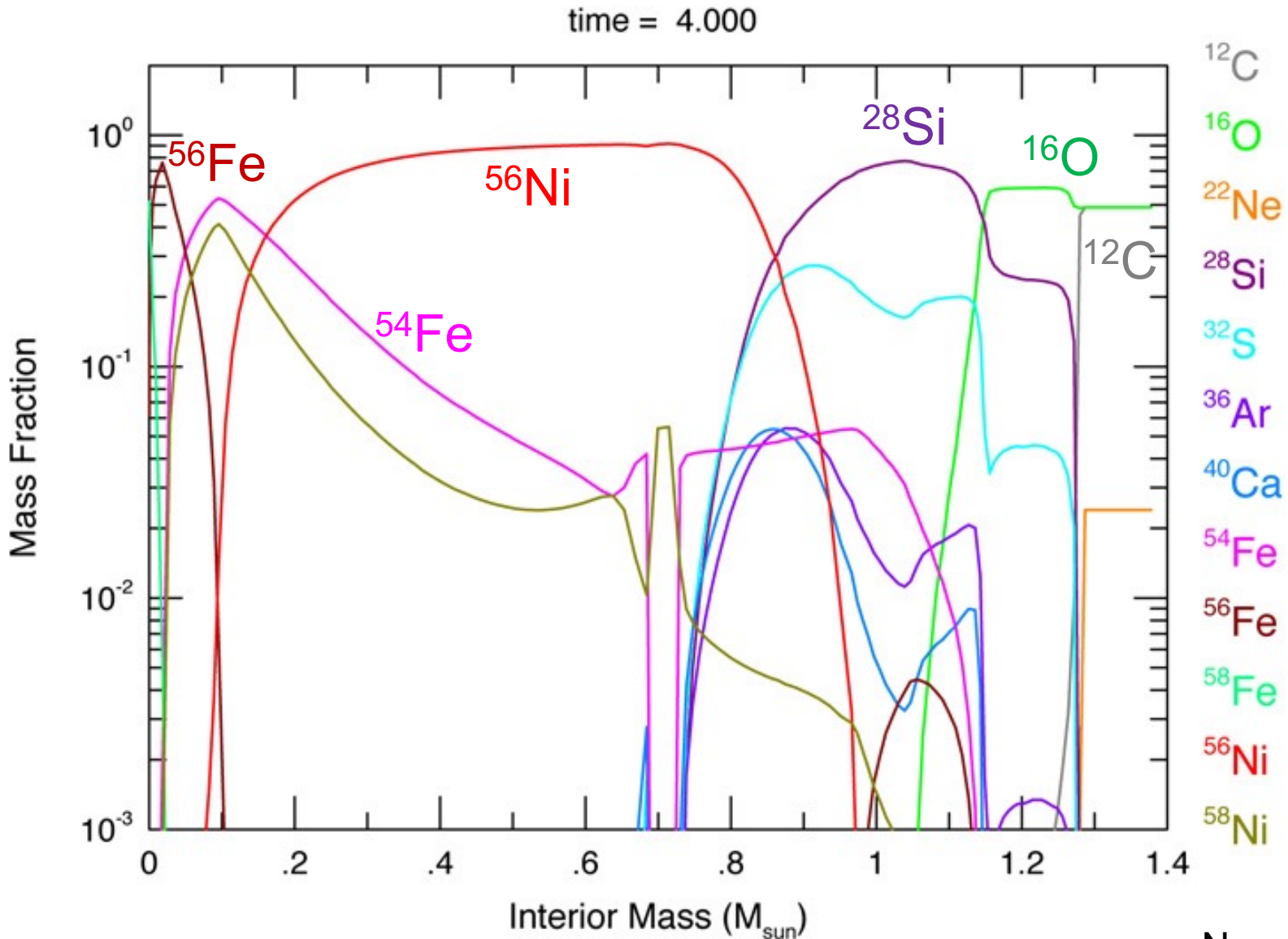
Nucleosynthesis in SNe Ia

- **Explosion of white dwarf**
 - $M_{\text{ms}} < 8M_{\odot}$, Compact ($R \sim 10^3 \text{ km}$)
 - $E = 10^{51} \text{ erg}$, $M_{\text{ej}} = 1.34M_{\odot}$, $M(\text{Fe}) \sim 0.9M_{\odot}$
- **Thermonuclear explosion**
 - $E_{\text{exp}} = E_{\text{nuc}} (+ E_{\text{grav}}) = \Delta(^{12}\text{C} \rightarrow ^{56}\text{Ni})M_{\odot} \sim 10^{51} \text{ erg}$
- Large amount of $^{56}\text{Ni}(\text{Fe})$ and Fe-peak elements
- Interval from star formation: **$\sim \text{Gyr}$**
 - The contribution delays comparing with CCSNe.

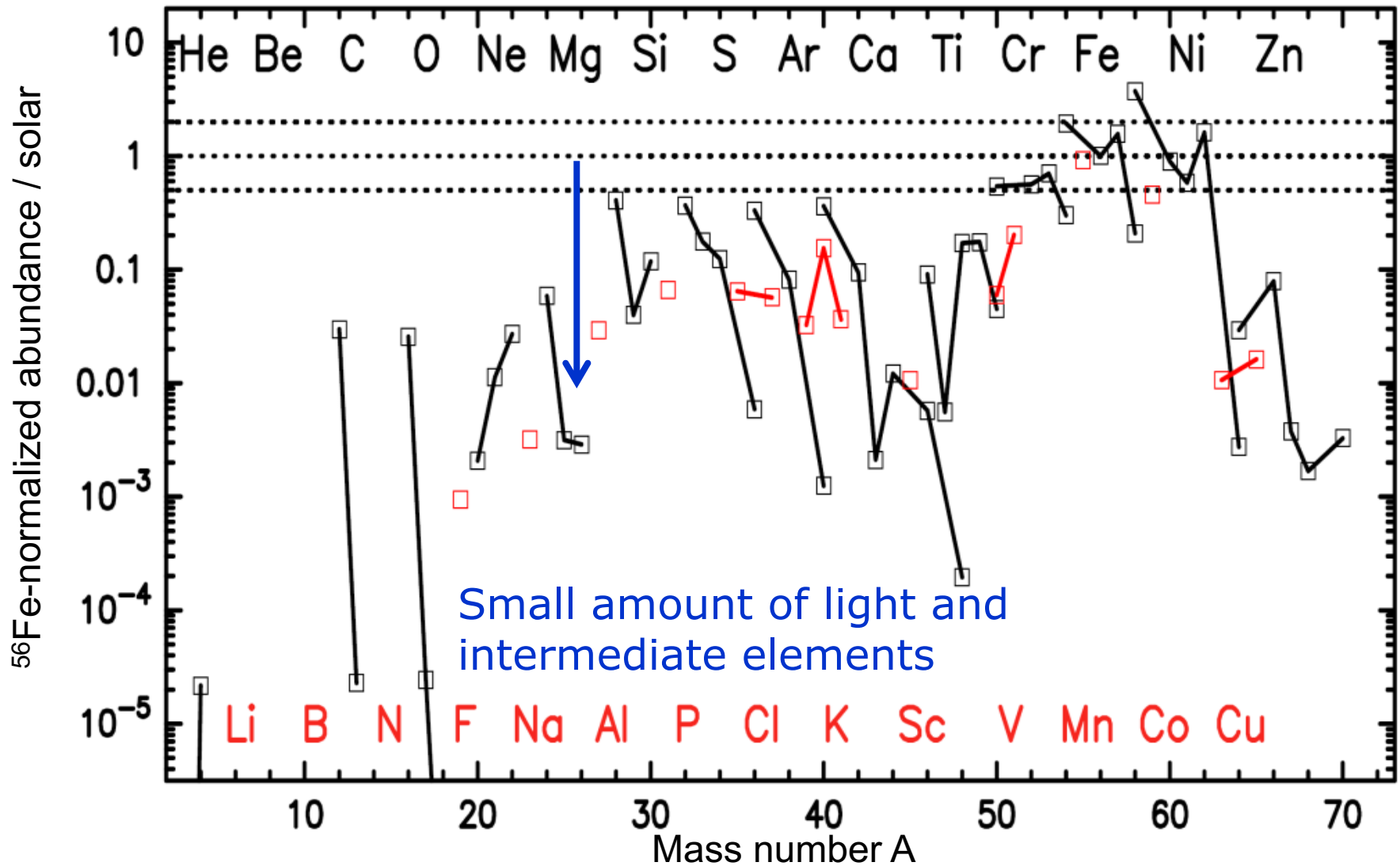
Nucleosynthesis in SNe Ia



Nucleosynthesis in SNe Ia



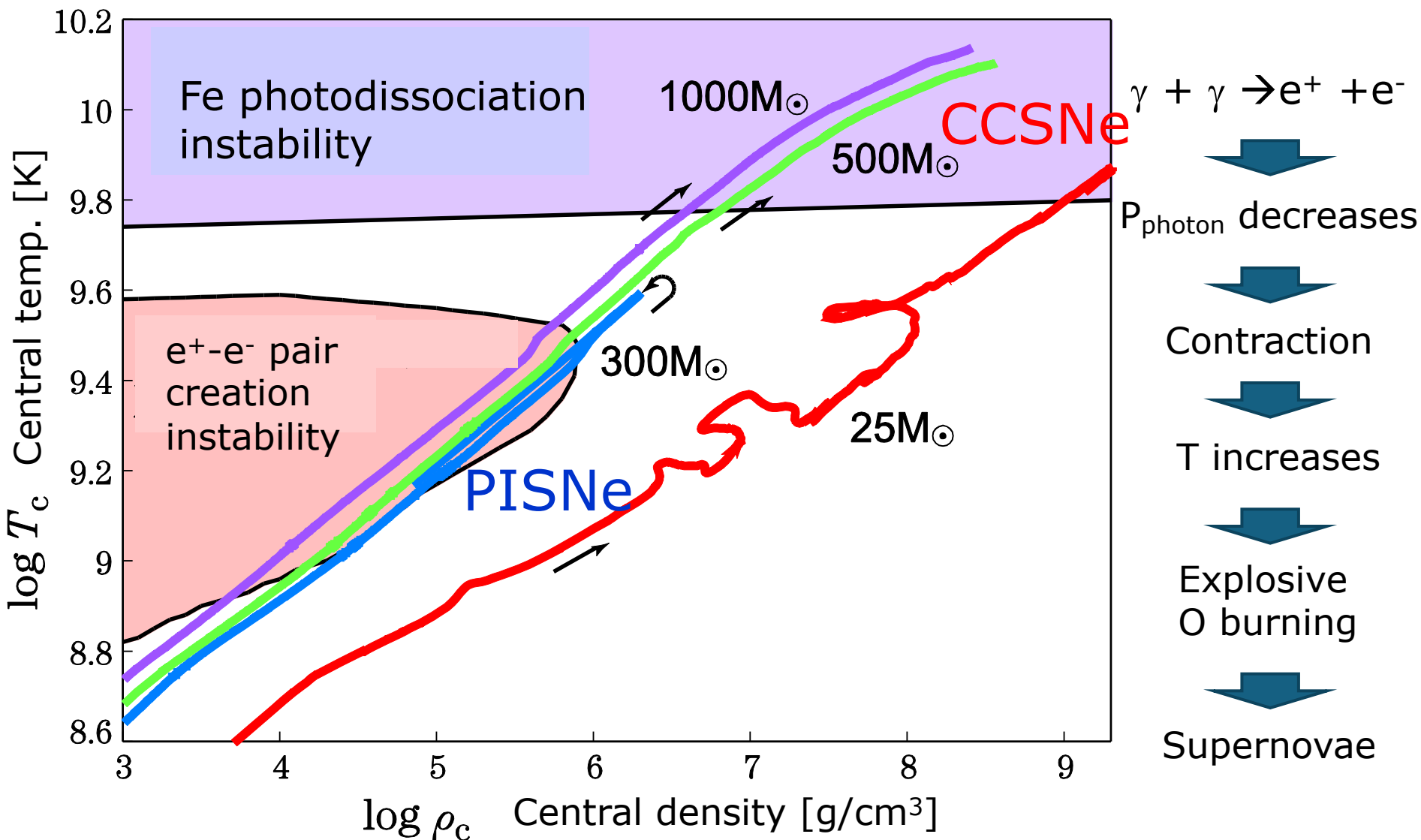
Abundance pattern of SN Ia yield



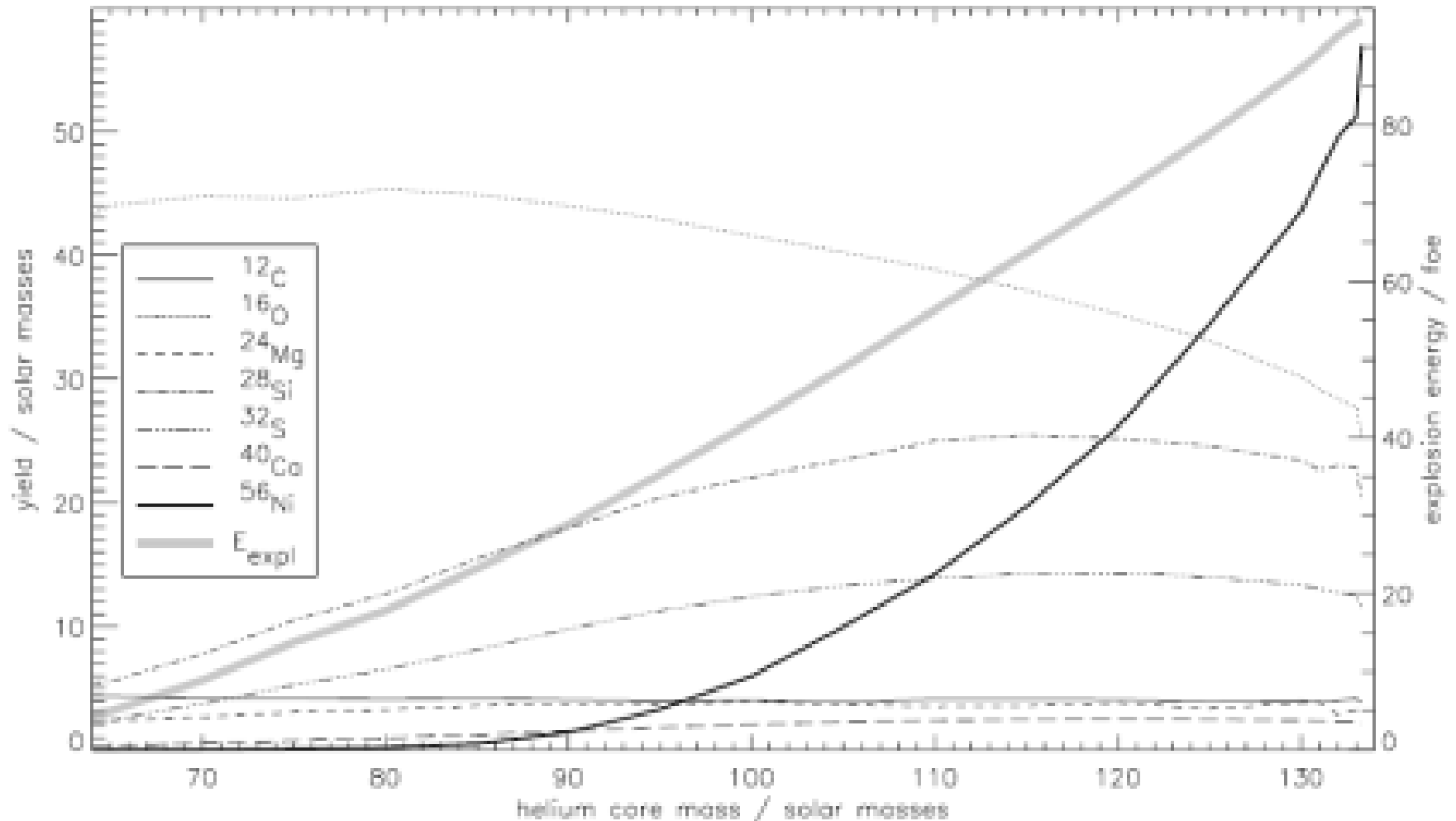
Nucleosynthesis in PISNe

- Explosion of very massive star
 - $M_{\text{ms}} = 140\text{-}300M_{\odot}$, $R \sim 10^9\text{km}$, low entropy
 - Such massive stars can survive only at small Z .
 - $E \sim 10^{51\text{-}53}\text{erg}$, $M_{\text{ej}} \sim 100M_{\odot}$, $M(\text{Fe}) \sim 1\text{-}10M_{\odot}$
- Thermonuclear explosion
 - $E_{\text{exp}} = E_{\text{nuc}} (+ E_{\text{grav}})$
 $= \Delta(^{16}\text{O} \rightarrow ^{28}\text{Si}, ^{56}\text{Ni})(10\text{-}100M_{\odot}) \sim 10^{52\text{-}53}\text{erg}$
- Large amount of $^{56}\text{Ni}(\text{Fe})$ and even- Z elements
- Interval from star formation: $\sim \text{Myr}$

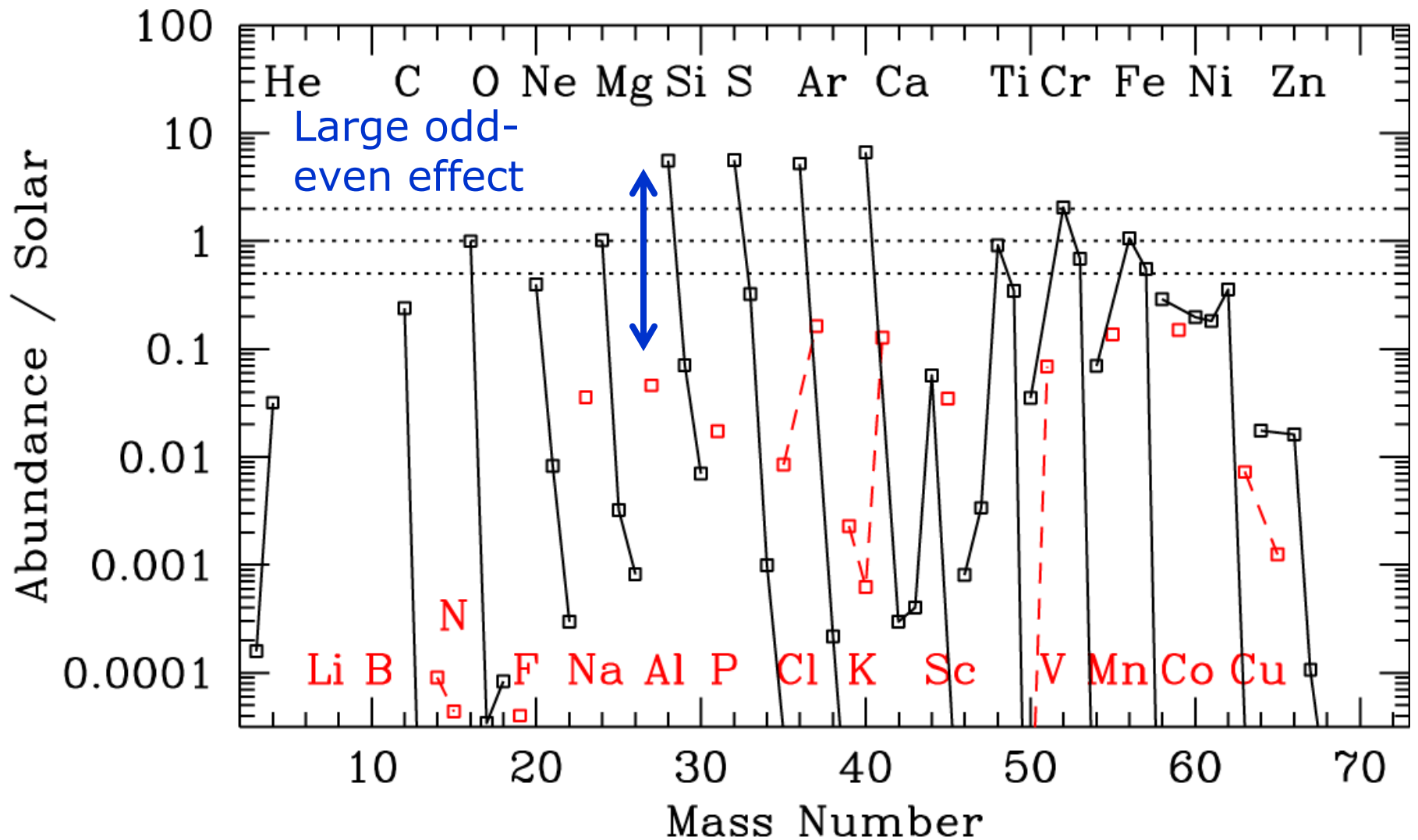
Explosion mechanism of PISNe



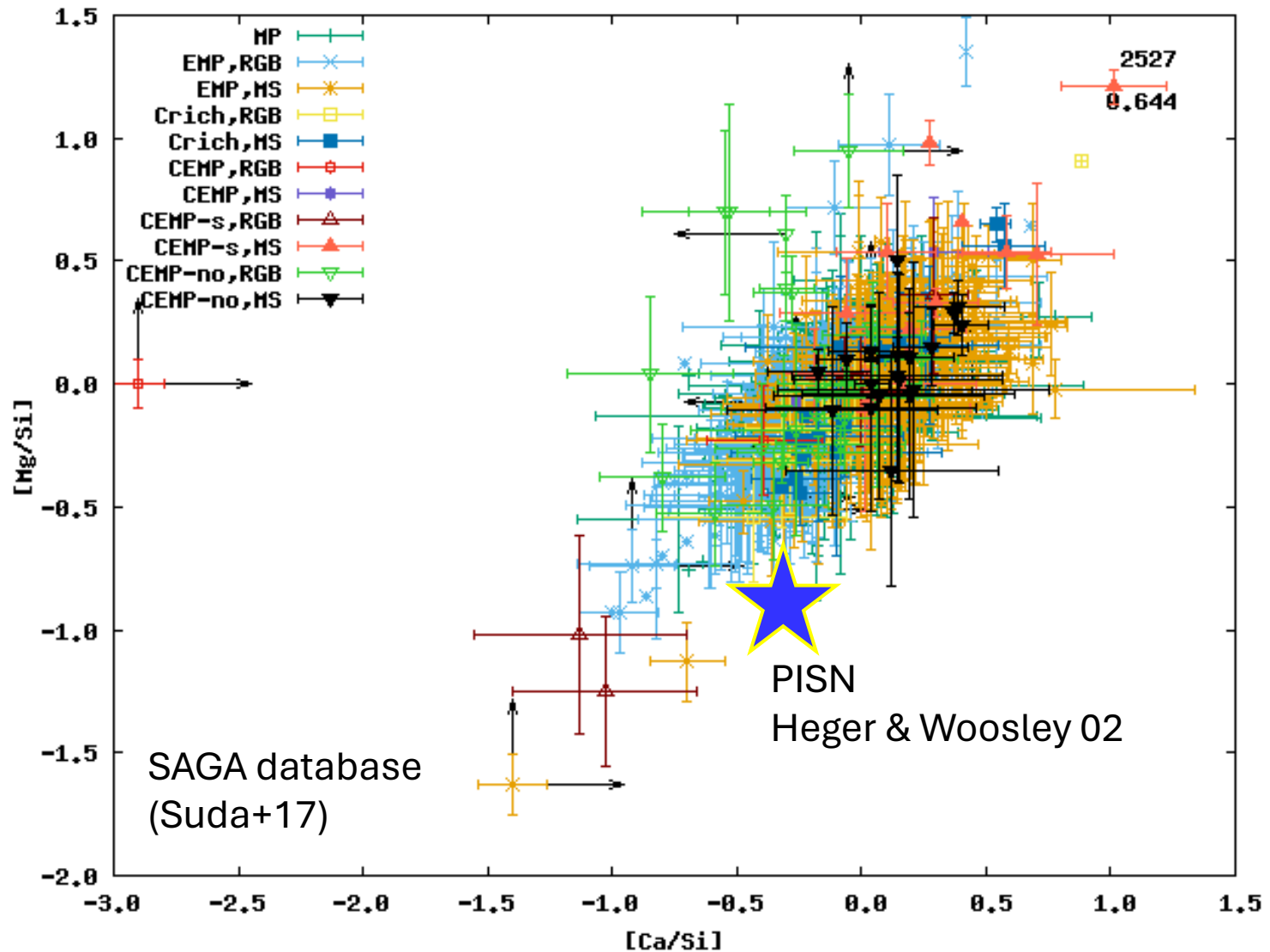
Mass dependence of PISN yields



Abundance pattern of PISNe



PISN: [Ca/Si] vs. [Mg/Si]



Synthesis of heavy elements

How are heavy elements produced?

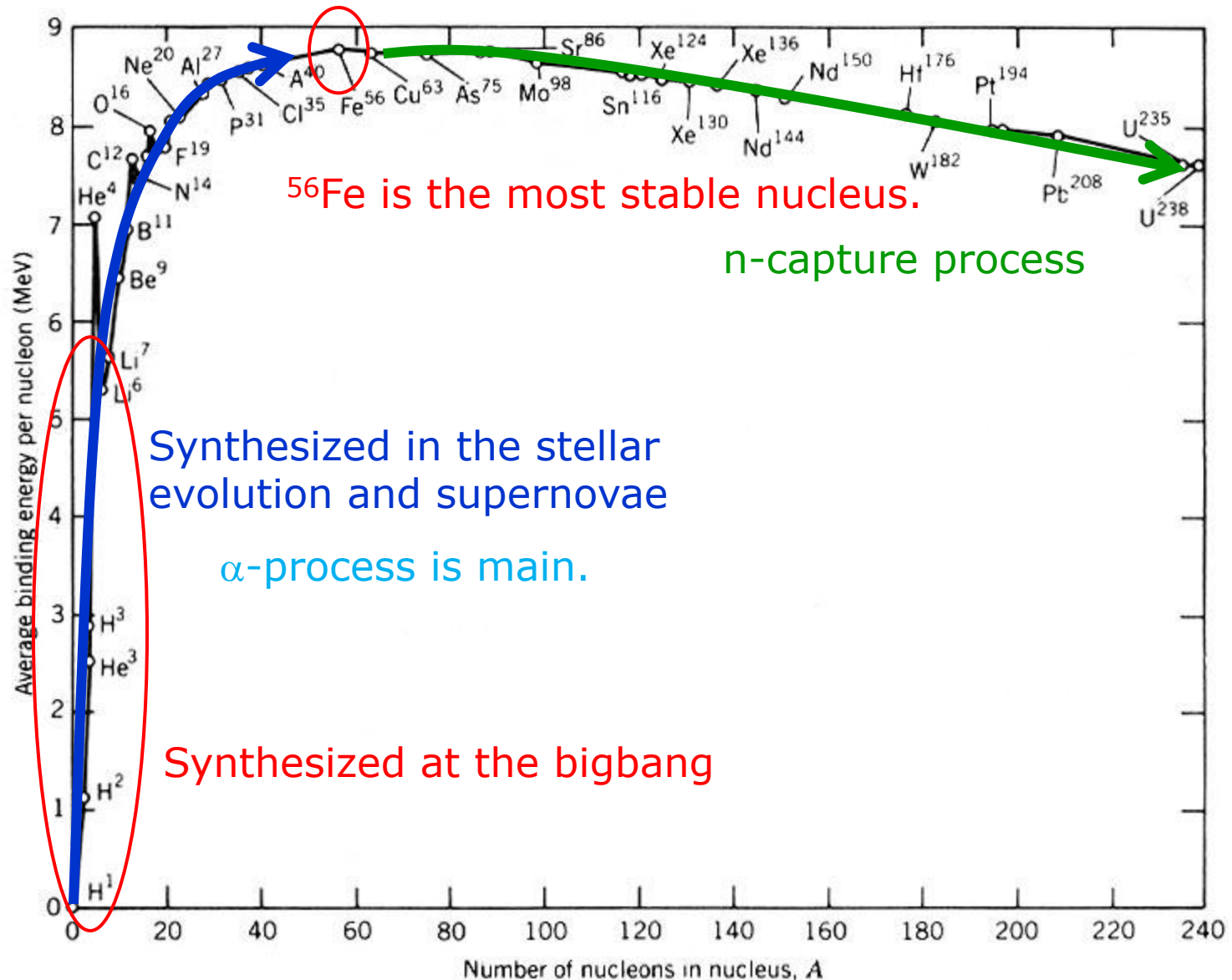
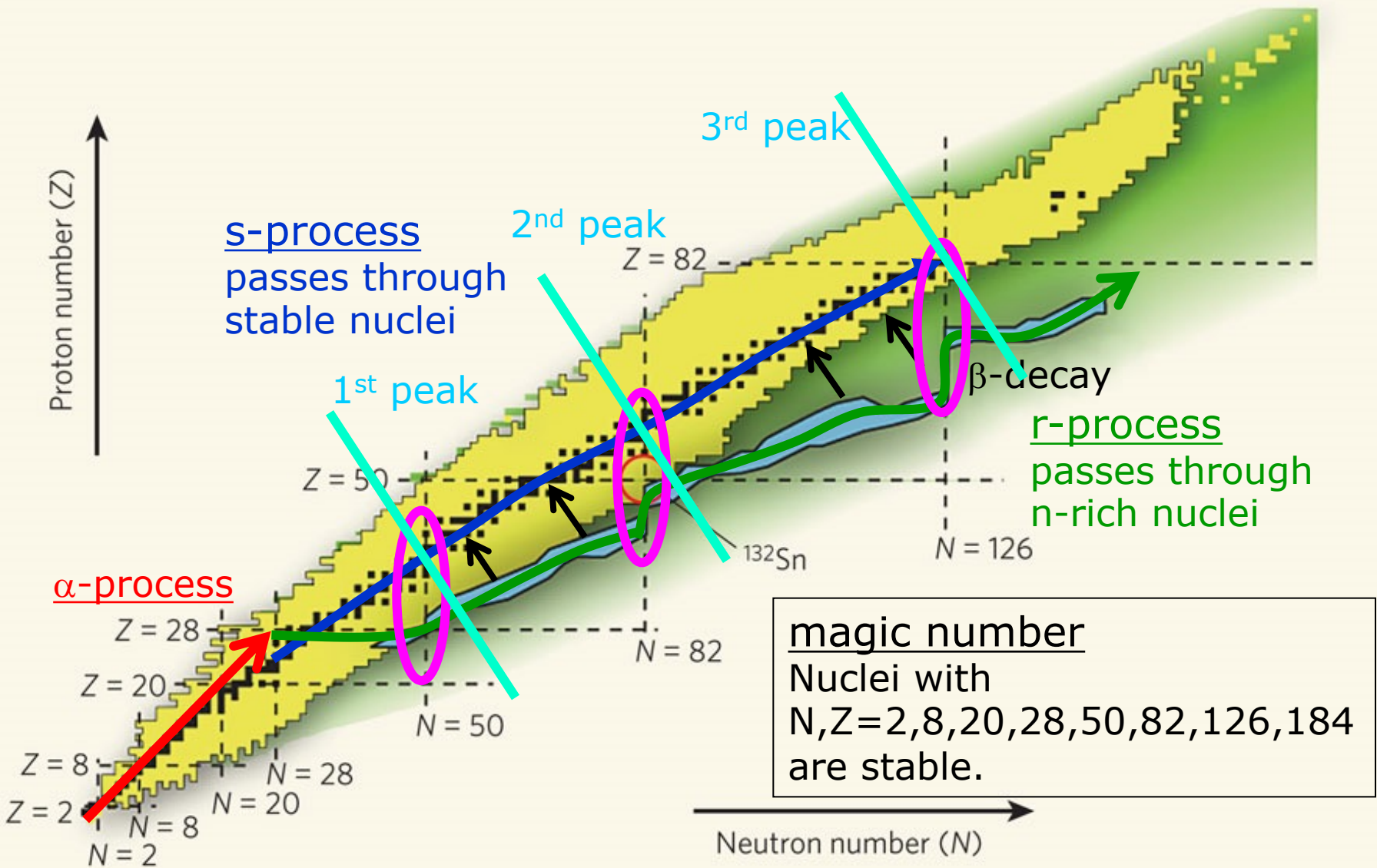
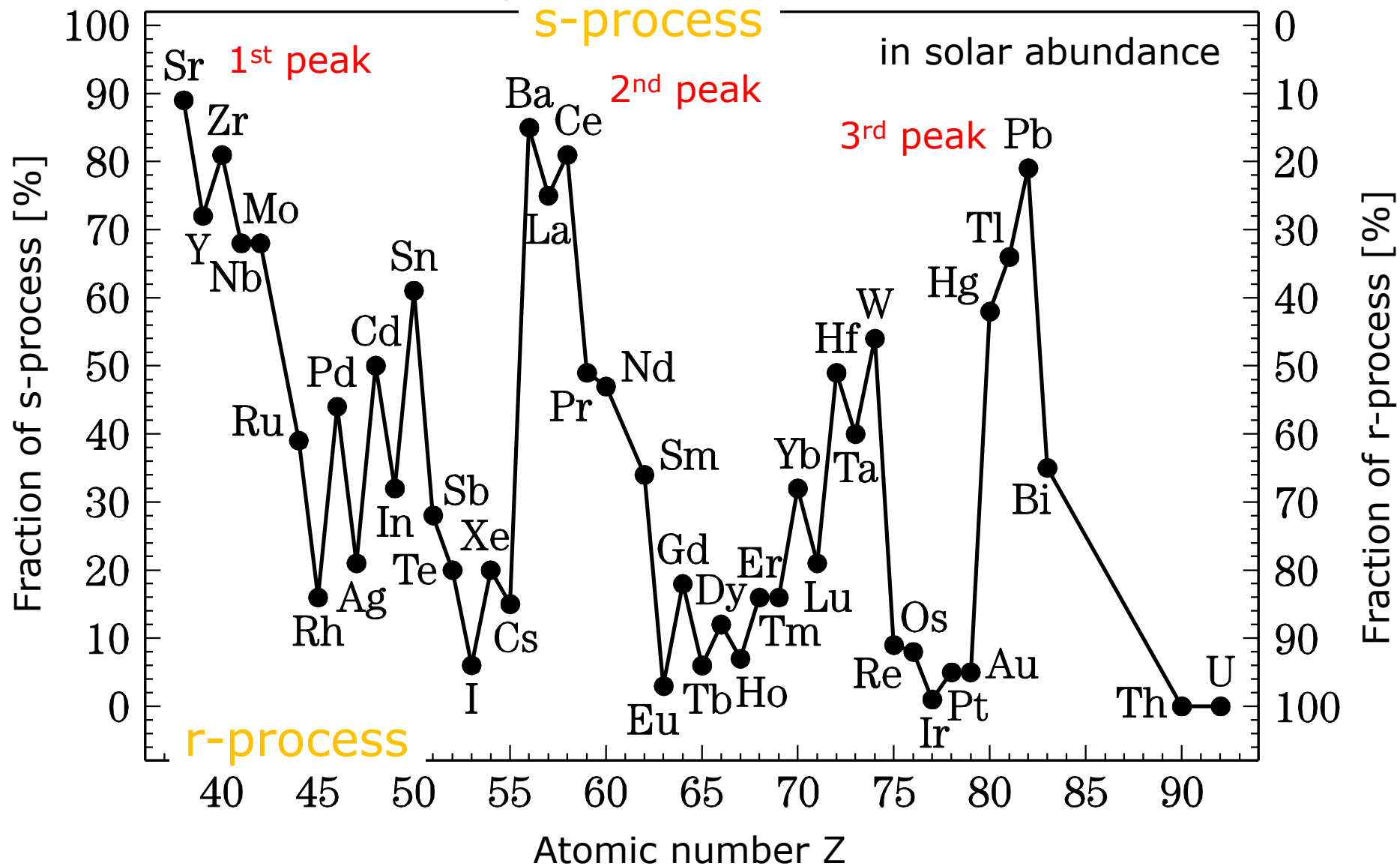


Chart of the nuclides



Production process



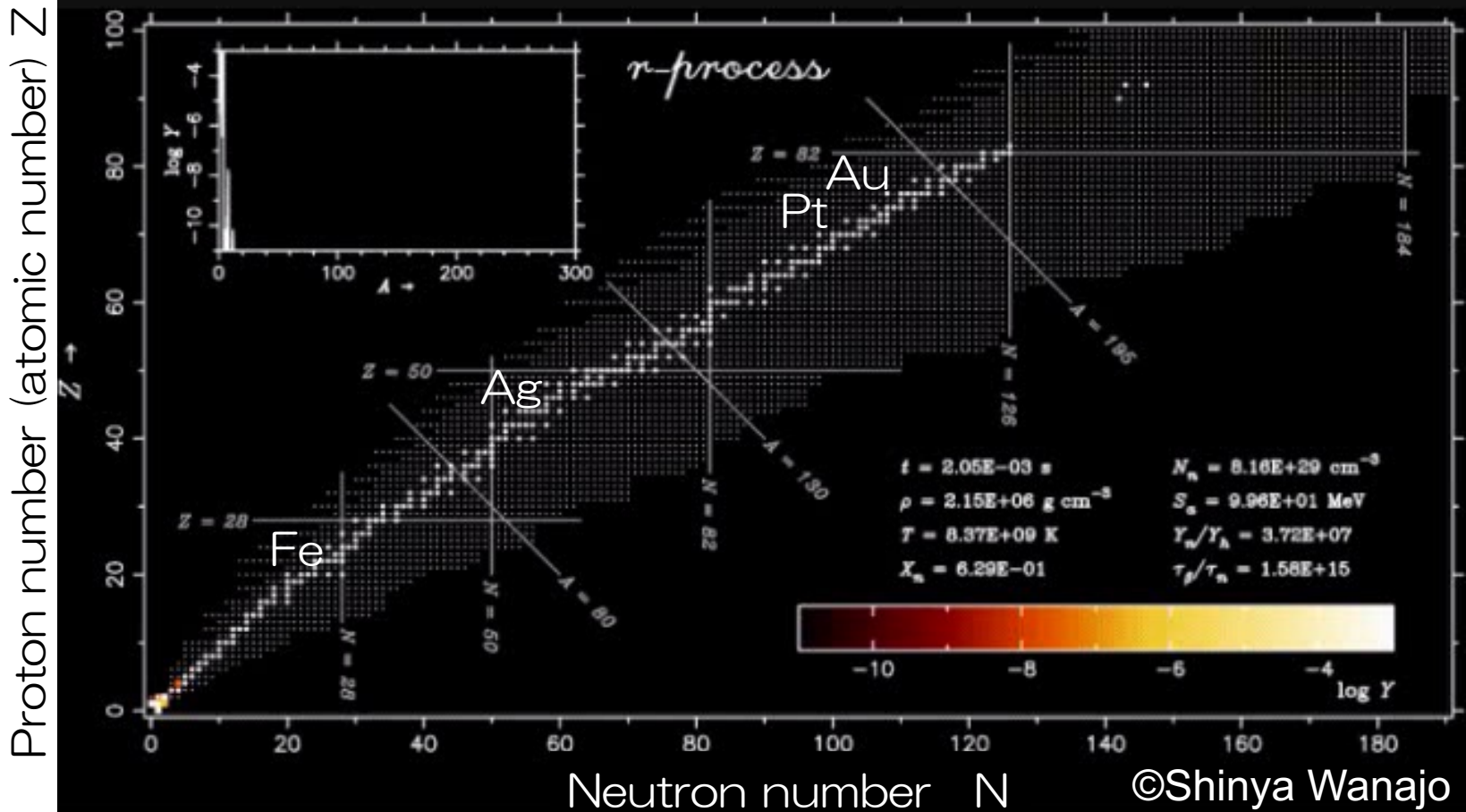
s-process nucleosynthesis

- Secondary process (seed nuclei are required)
 - no s-process in metal-free era
- **Stable** n-cap. elements are synthesized.
 - up to ^{206}Pb , ^{209}Bi (largest Z among stable nuclei).
 - 3 peaks with n magic number
- Halflife of n: 10 min
 - Stable n source is required.
 $^{13}\text{C} + ^4\text{He} \rightarrow ^{16}\text{O} + \text{n}$ (He **shell** burning: $T > 8 \times 10^7 \text{K}$)
 $^{22}\text{Ne} + ^4\text{He} \rightarrow ^{25}\text{Mg} + \text{n}$ (He **core** burning: $T > 2.5 \times 10^8 \text{K}$)

r-process nucleosynthesis

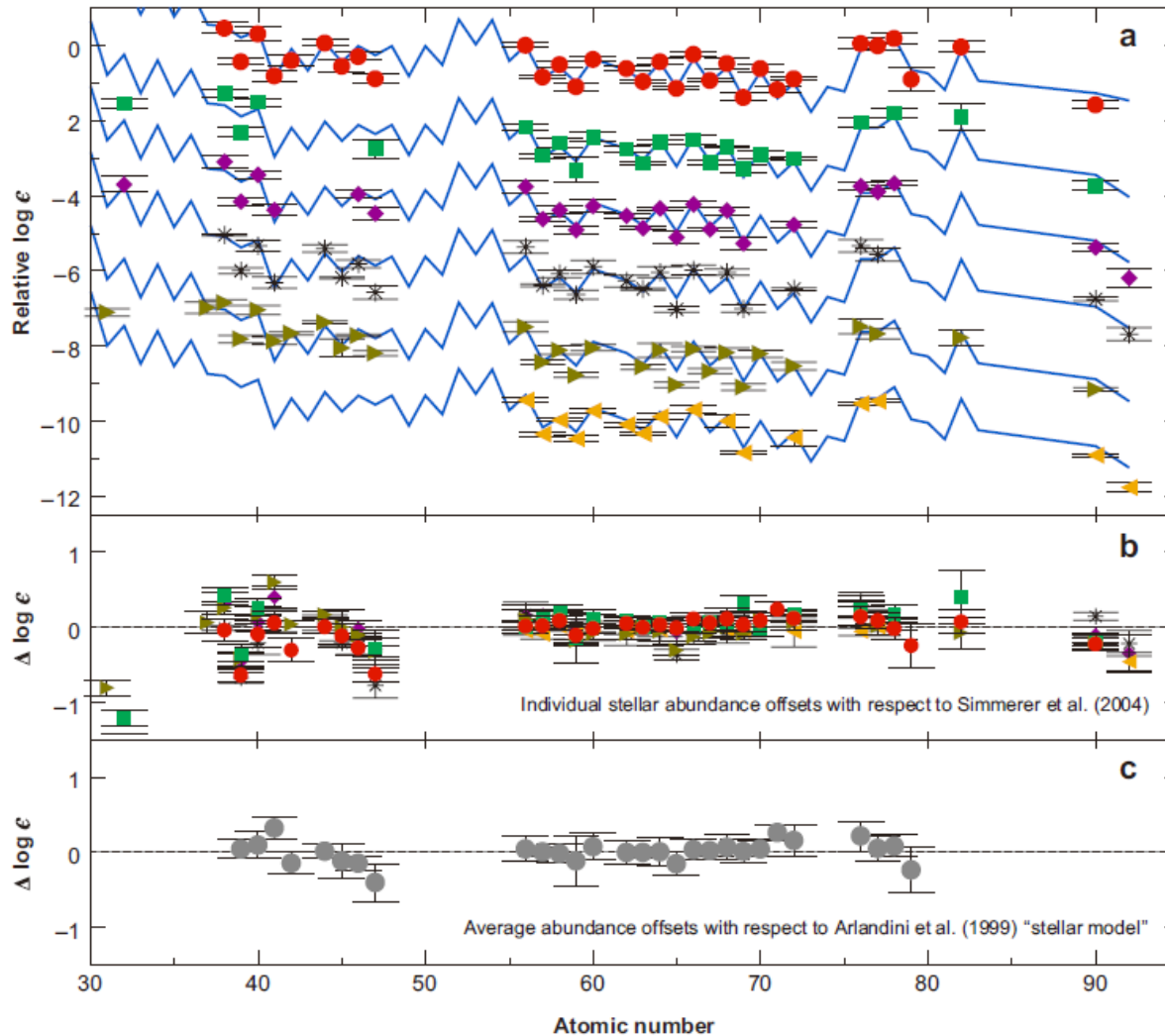
- Primary process
- $n(n) \sim 10^{20-30} \text{cm}^{-3}$, $s/k_B \sim 100$, $\tau_{\text{nuc}} \sim 10^{-2 \sim -6} \text{sec}$
- It passes through **unstable n-rich nuclei** and synthesizes large amount of unstable nuclei with n magic number, that decay to **stable nuclei** via β -decay.
- The 3 peaks shift to **smaller Z** compared to the s-process elements.
- **Products: Ag, Pt, Au, etc.**

r-process nucleosynthesis



r-process elements

- Universality



n-capture sites

- s-process site
 - **Not metal-free** (seed nuclei is required.)
 - AGB stars
 - $n(n) \sim 10^7 \text{ cm}^{-3}$, $\tau_{\text{nuc}} \sim 10^{4-5} \text{ yr}$
 - Rotating massive stars (spinstar)
 - At least, massive stars can produce light s-elements.
- r-process site
 - **High entropy & low electron fraction**
 - Core-collapse supernovae?
 - Neutron star mergers?

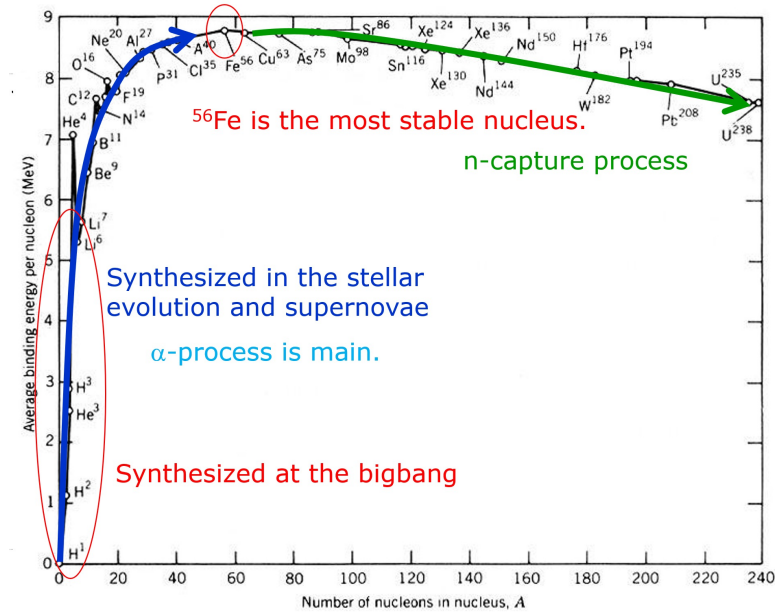
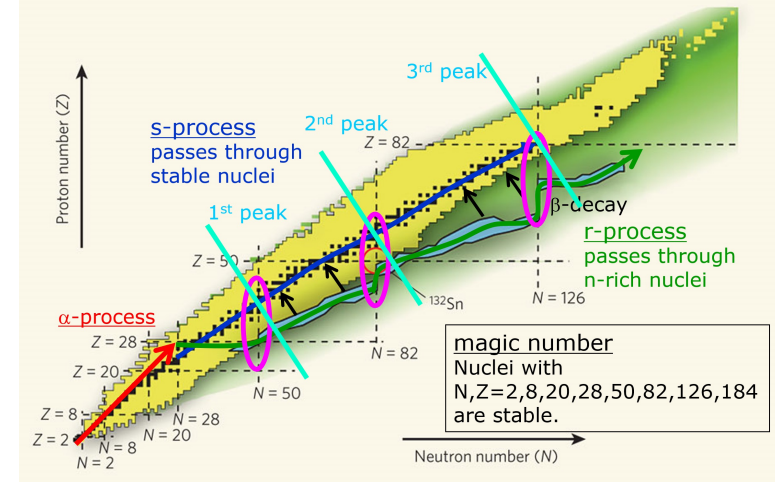
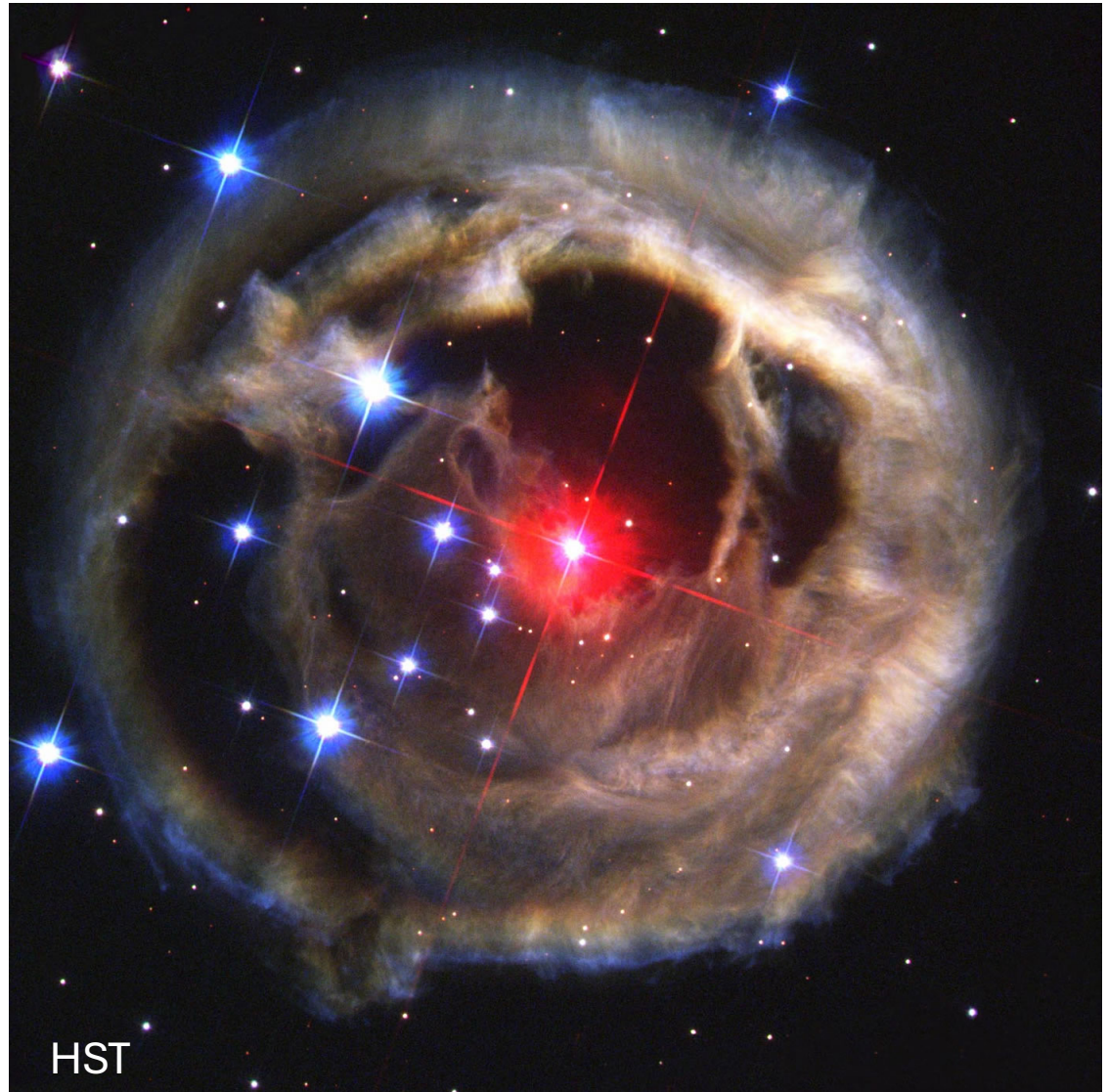


Chart of the nuclides



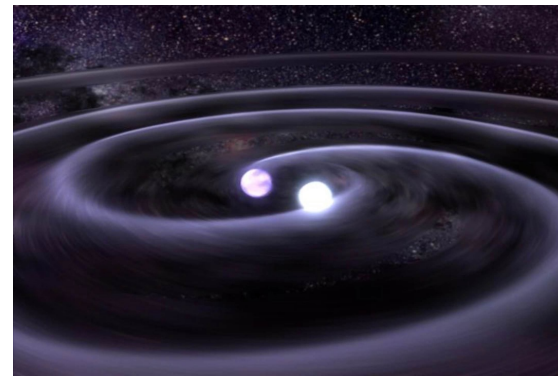
s-process elements

- AGB stars
- $^{13}\text{C}(\alpha, n)^{16}\text{O}$
- $^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}$

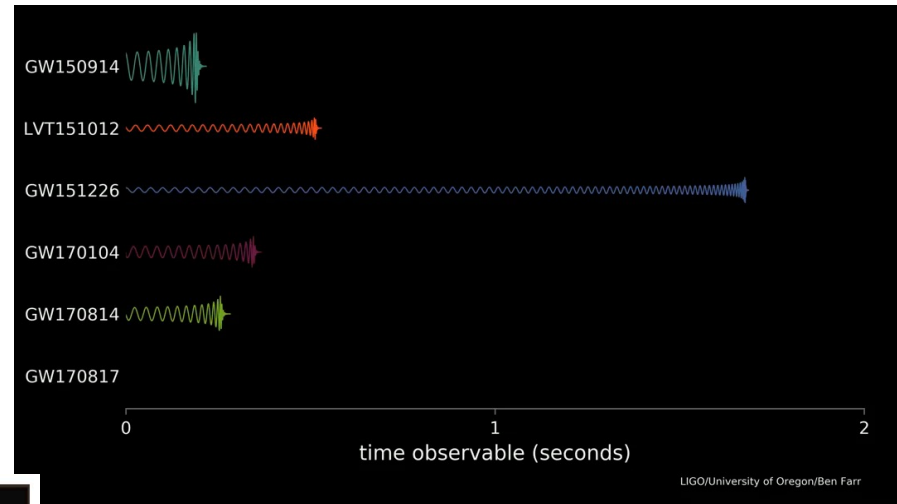


Recent progress r-process site

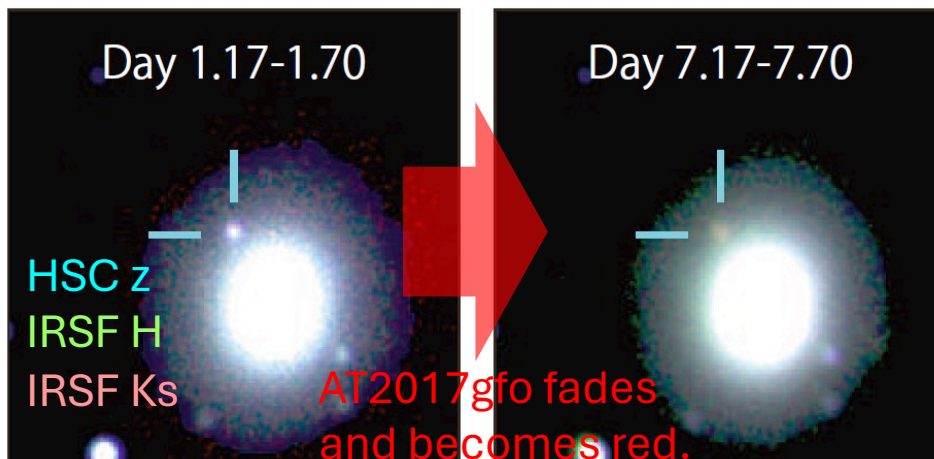
- Neutron star merger
- Gravitational wave
- GW170817



NASA



LIGO



A firm evidence of **synthesis of Lanthanoid elements** has been found.

It is not clear whether the neutron star is the dominant origin of r-process elements or not.

Utsumi, Tanaka, NT+17

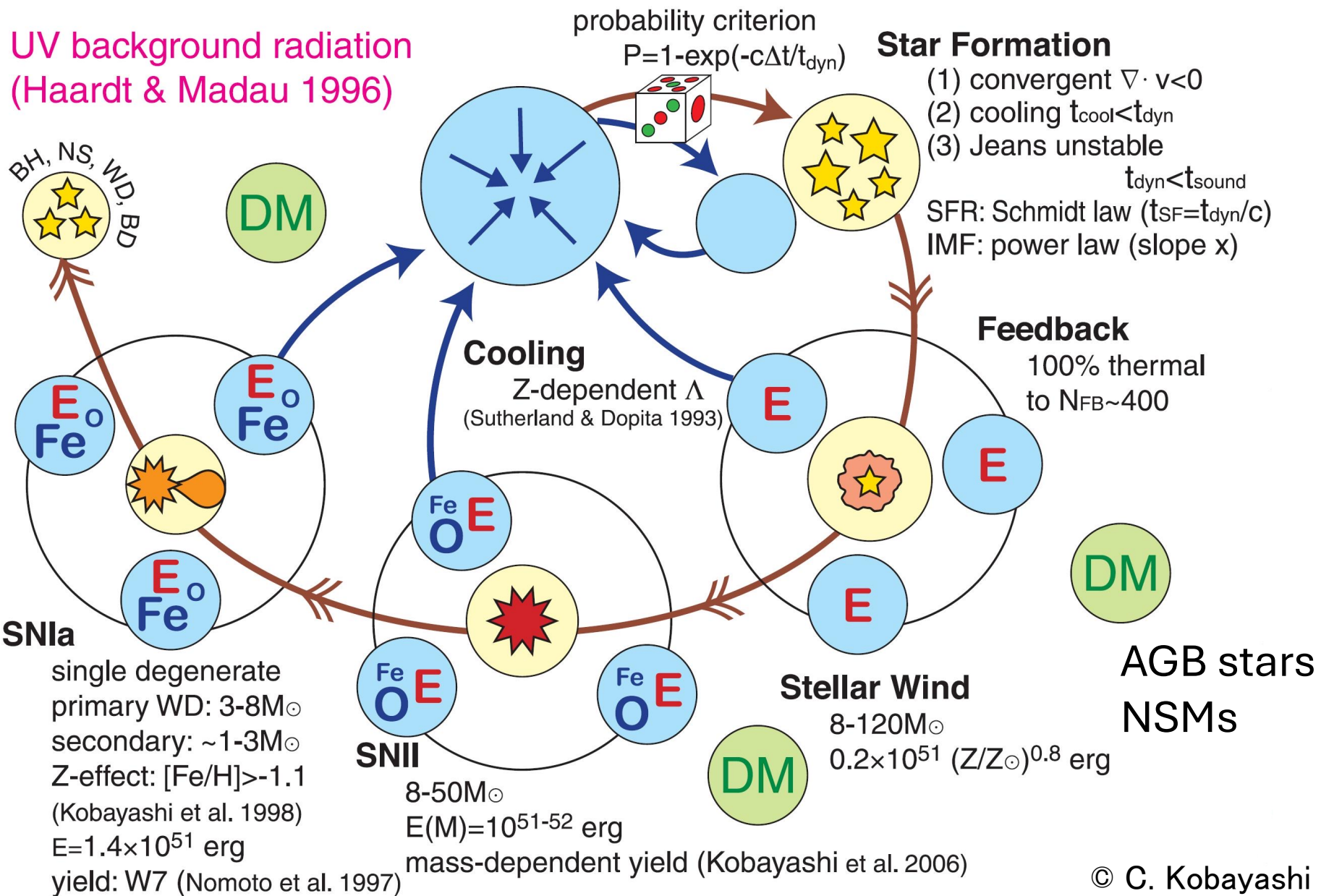
Chemical evolution

Chemical evolution

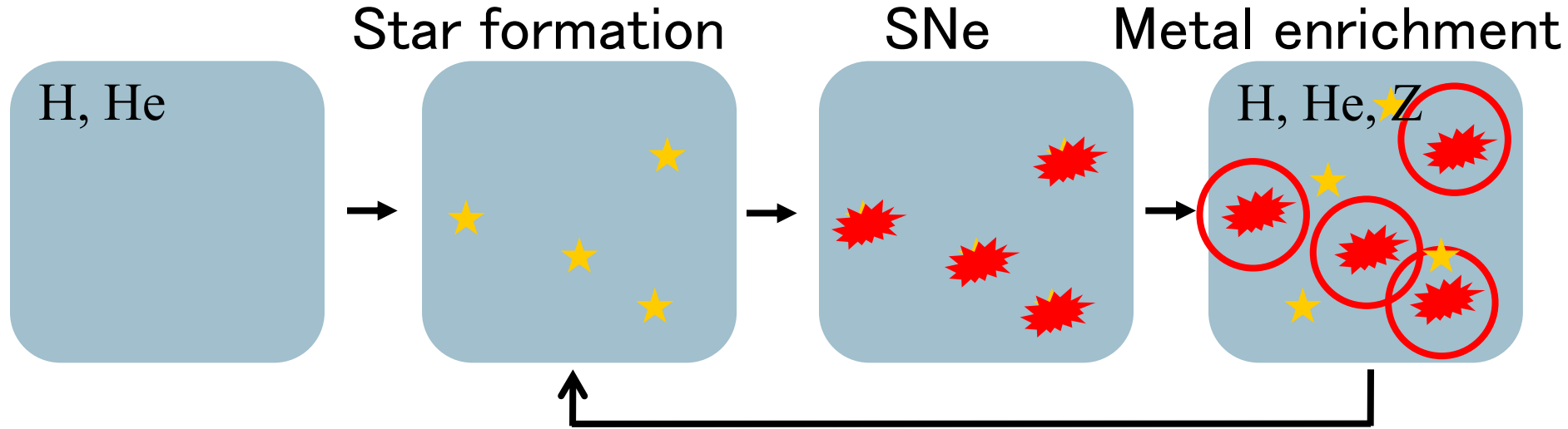
- **Stars and supernovae** evolve universe from metal-free to solar abundance.
- The **chemical evolution** of universe is studied including **star formation, feedback, and metal-enrichment** due to stars and supernovae.
- There are several methods:
 - 1-zone
 - $p(t, [\alpha/\text{Fe}], [\text{Fe}/\text{H}])$
 - Chemodynamical simulation
 - $p(x, v, t, [\alpha/\text{Fe}], [\text{Fe}/\text{H}])$
- Observations
 - SEGUE, APOGEE, LAMOST, GALAH, Kepler, Gaia, ...

Chemical evolution

UV background radiation
(Haardt & Madau 1996)

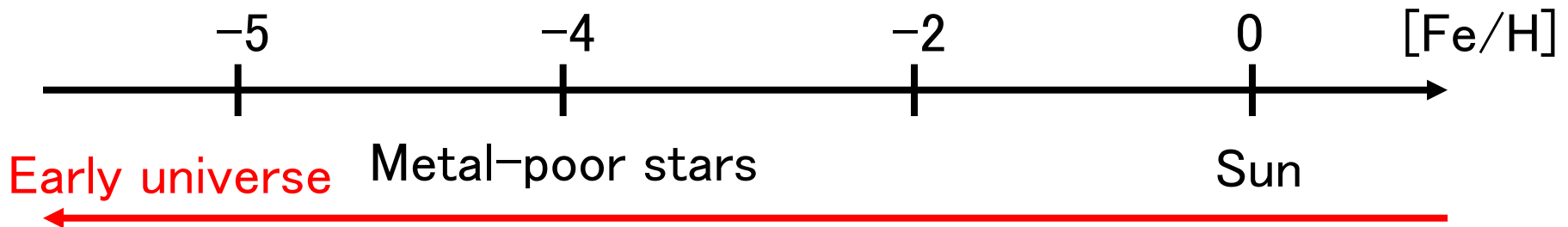


Chemical evolution is recorded in abundance pattern of metal-poor stars.



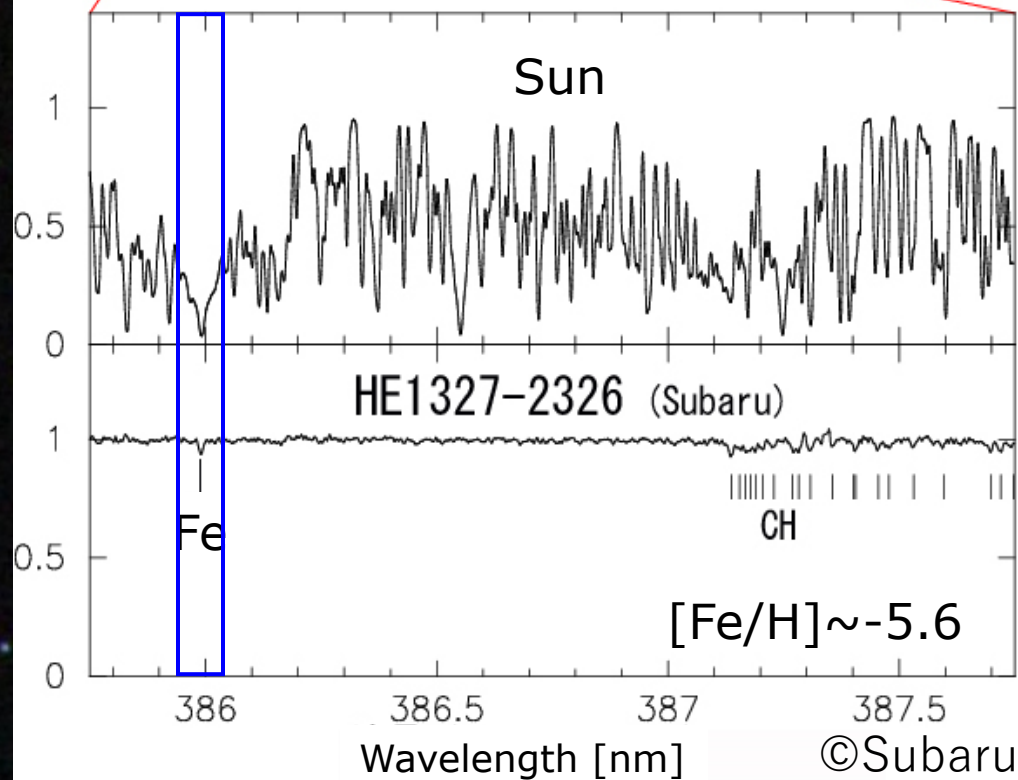
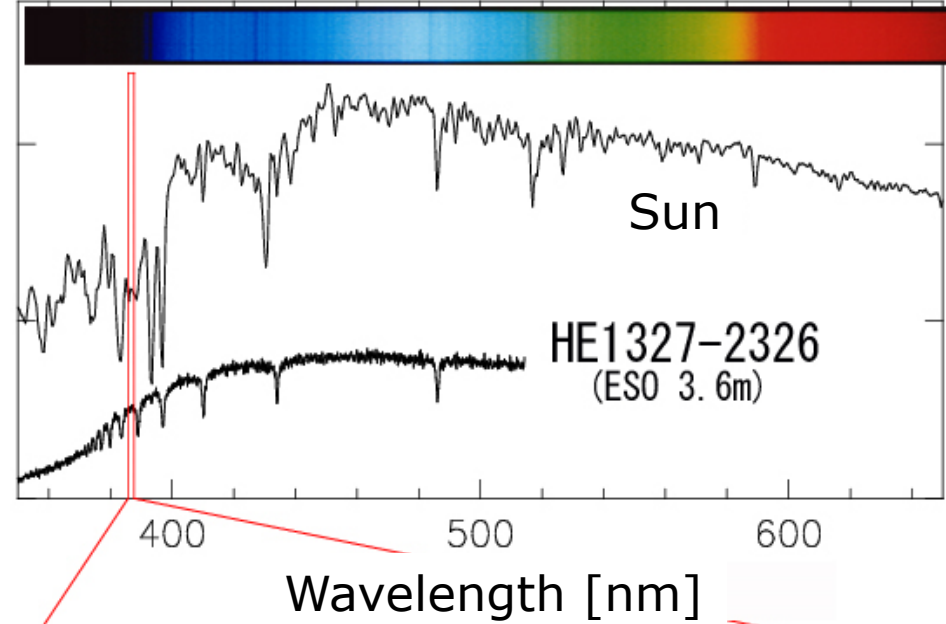
Metallicity increases with time,
thus **metallicity is a time indicator** in the mixed universe.

$$[Fe/H] = \log(Fe/H) - \log(Fe/H)_{\odot}$$

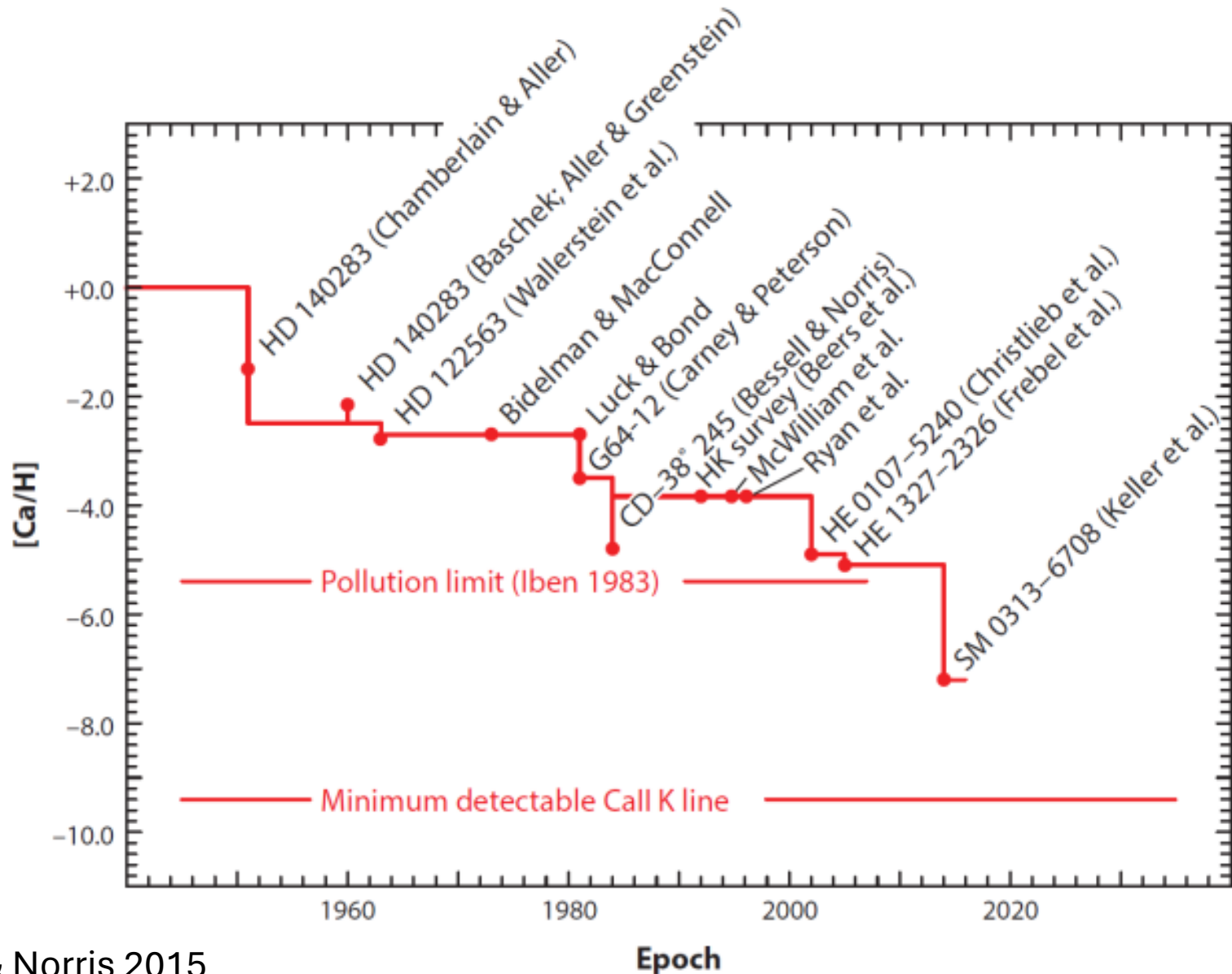


Metal-poor star

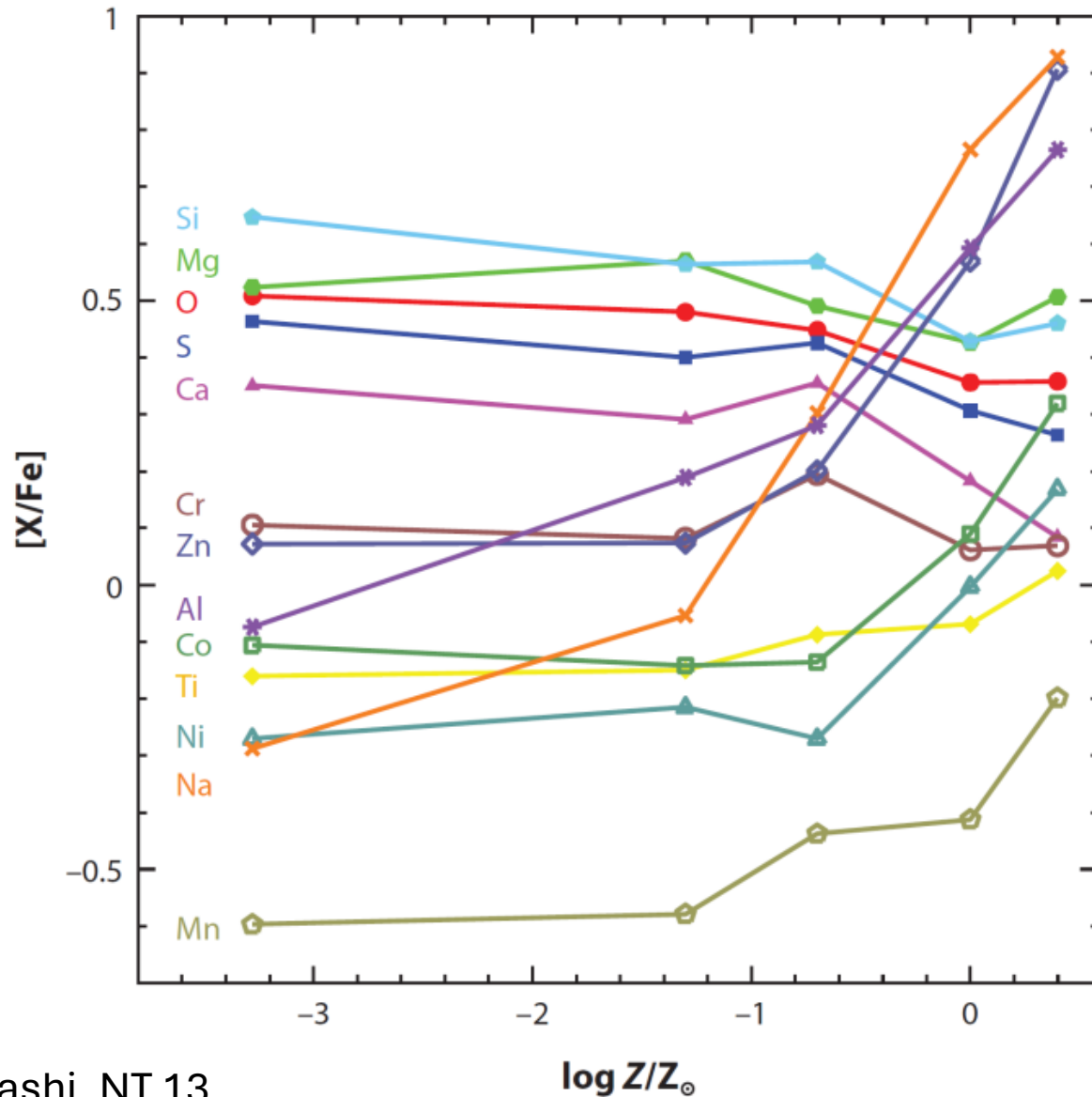
Stars are formed in the early universe and have small amount of metals.



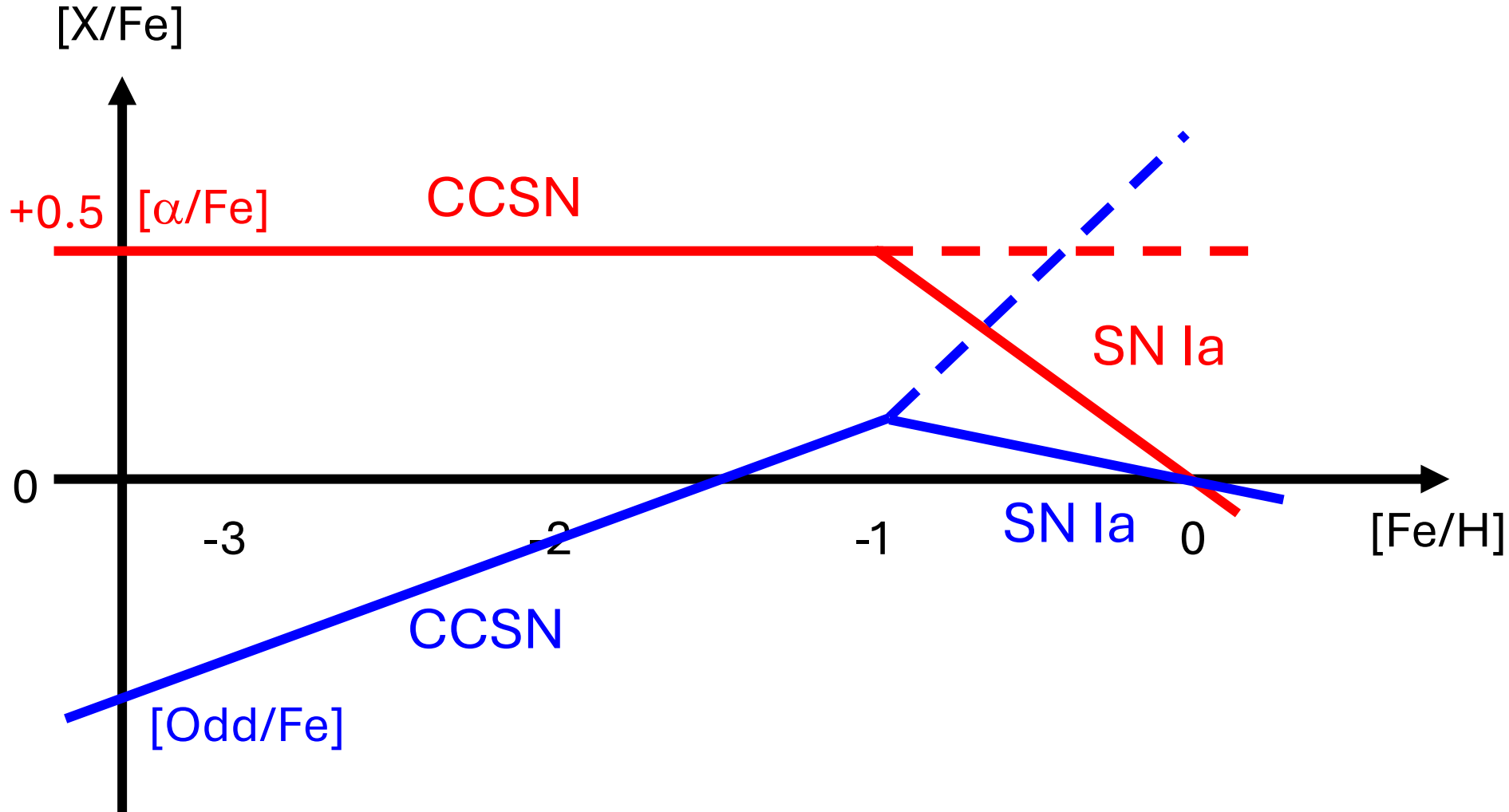
Discoveries of metal-poor stars



Core-collapse supernova yields

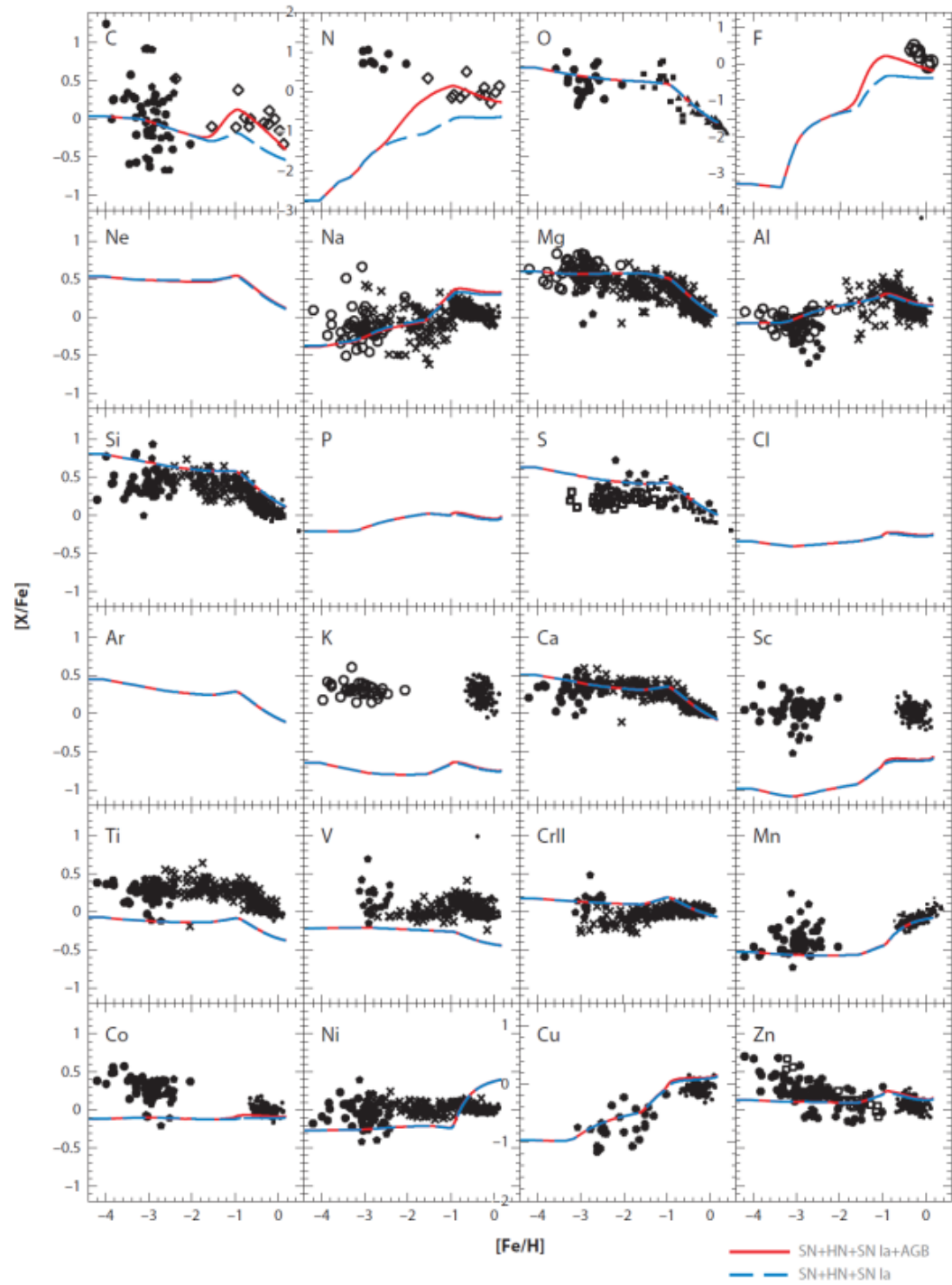
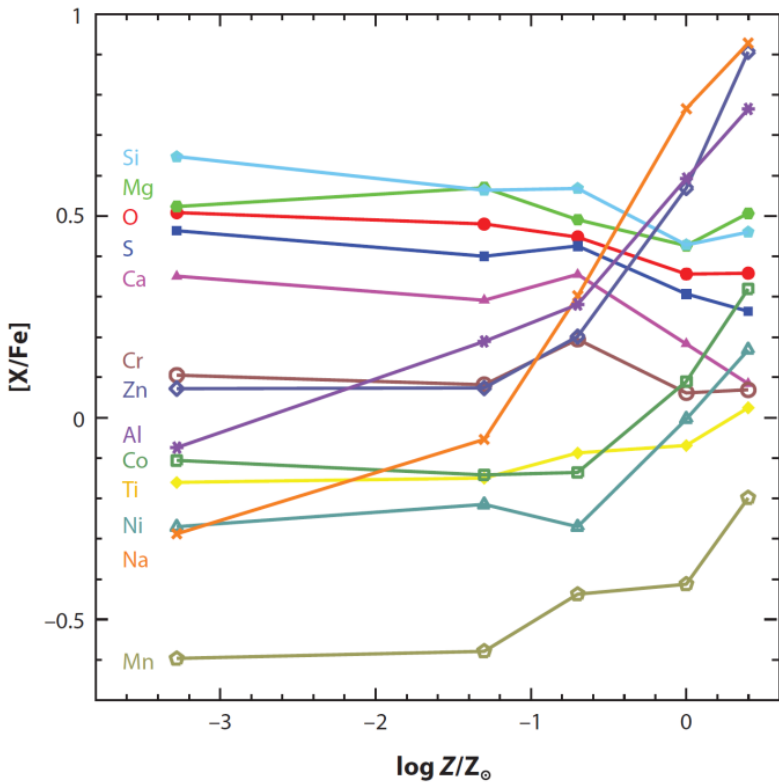


Chemical evolution of elements



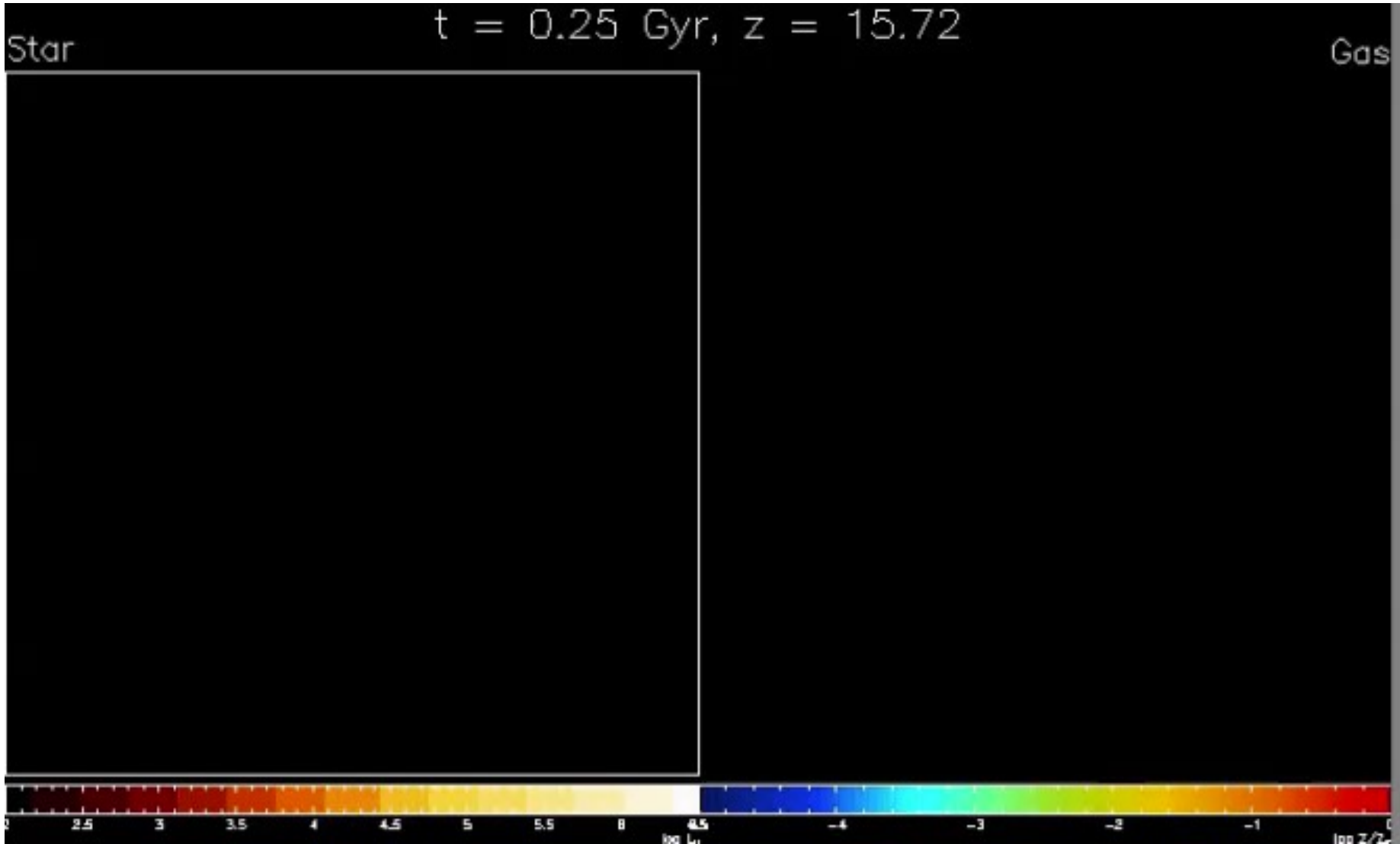
Chemical evolution

Supernova yields

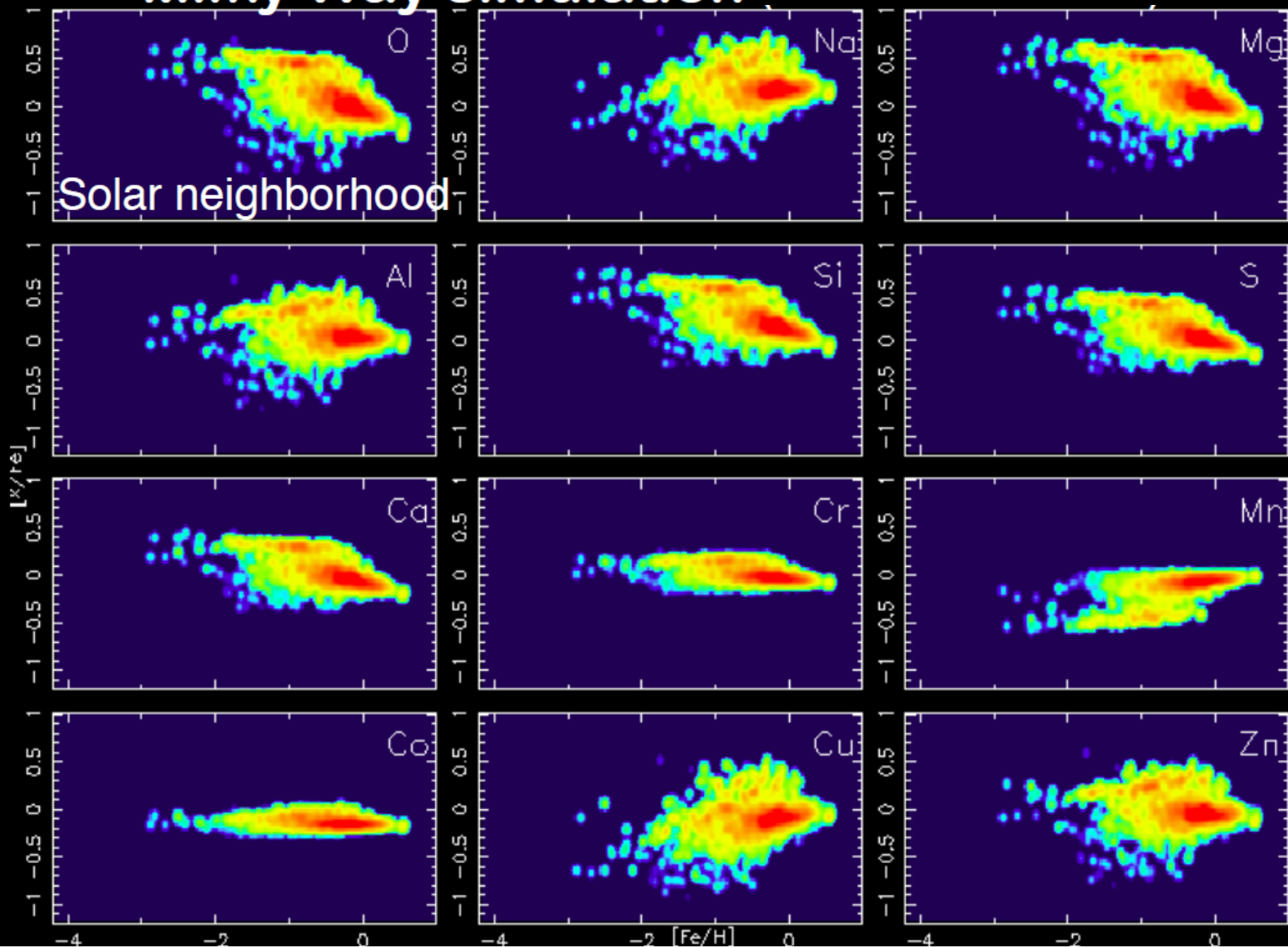


Chrono-chemo-dynamical simulation

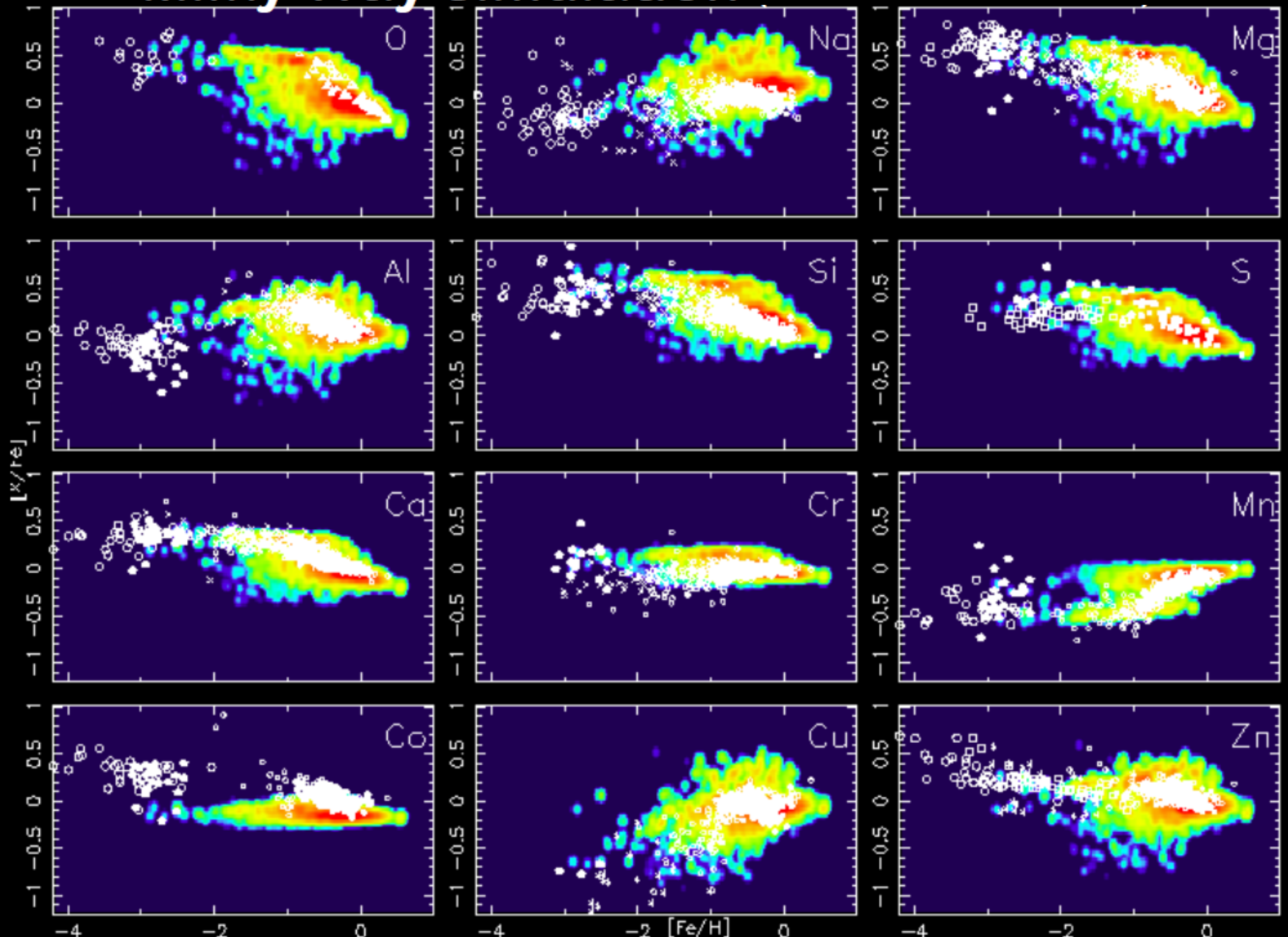
Kobayashi+07



Milky Way simulation Kobayashi & Nakasato 2011



Milky Way simulation Kobayashi & Nakasato 2011



Homework – 1

- Draw several lines of constant masses on the T_c - ρ_c plane for different masses with adopting gas and electron degenerate pressure ($T=0K$), $X(^{12}C)=X(^{16}O)=0.5$, and $\varphi_N = 16.15$.

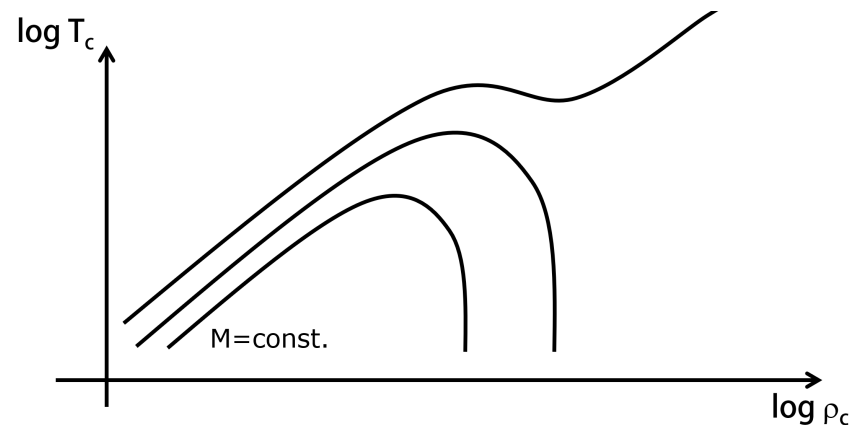
$$P = \frac{\rho k T}{\mu m_p} + P_e^{(0)}$$

$$M = \left(\frac{1}{4\pi G^3} \frac{P_c^4}{\rho_c^3} \right)^{1/2} \varphi_N$$

$$P_e^{(0)} = \frac{\pi}{3} \left(\frac{m_e c}{h} \right)^3 m_e c^2 f(x)$$

$$f(x) = x(2x^2 - 3)(x^2 + 1)^{1/2} + 3 \sinh^{-1}(x)$$

$$x = \frac{p_F}{m_e c}$$



Homework – 2

- Derive the dominant isotopes between ^{56}Ni and alpha in NSE as functions of T and ρ

$$\frac{N_{\alpha}^{14}}{N_{\text{Ni}^{56}}} = \frac{4^{21}}{56^{\frac{3}{2}}} \theta^{13} \exp \frac{14Q(\alpha) - Q(\text{Ni}^{56})}{kT}$$

$$Q(A, Z) = c^2 [Zm_p + (A - Z)m_n - m(A, Z)]$$

$$\theta = \left(\frac{2\pi m_p k_B T}{h^2} \right)^{3/2}$$

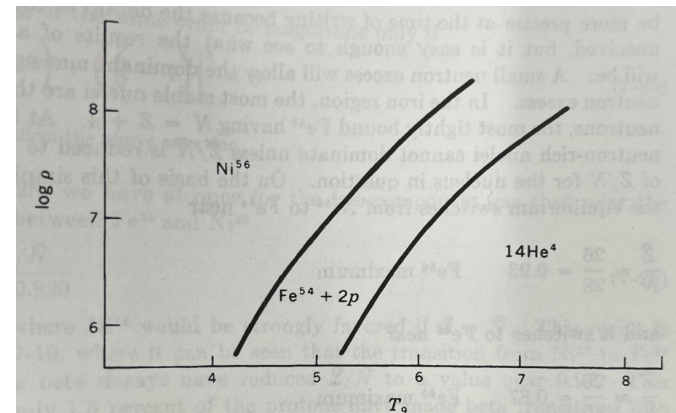
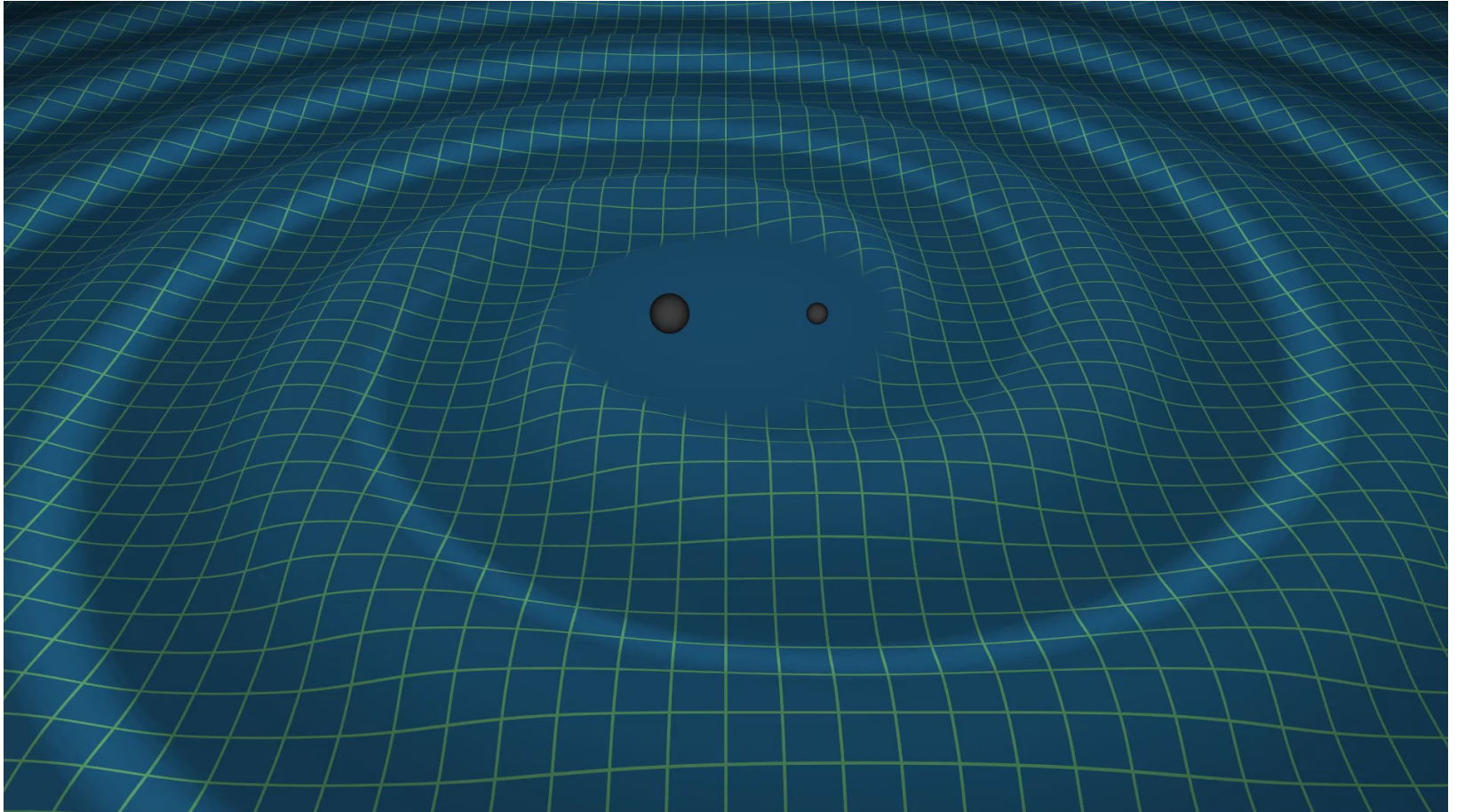


Fig. 7-9 The dominant nuclear constituent in a gas in nuclear statistical equilibrium when $Z/\bar{N} = 1$.

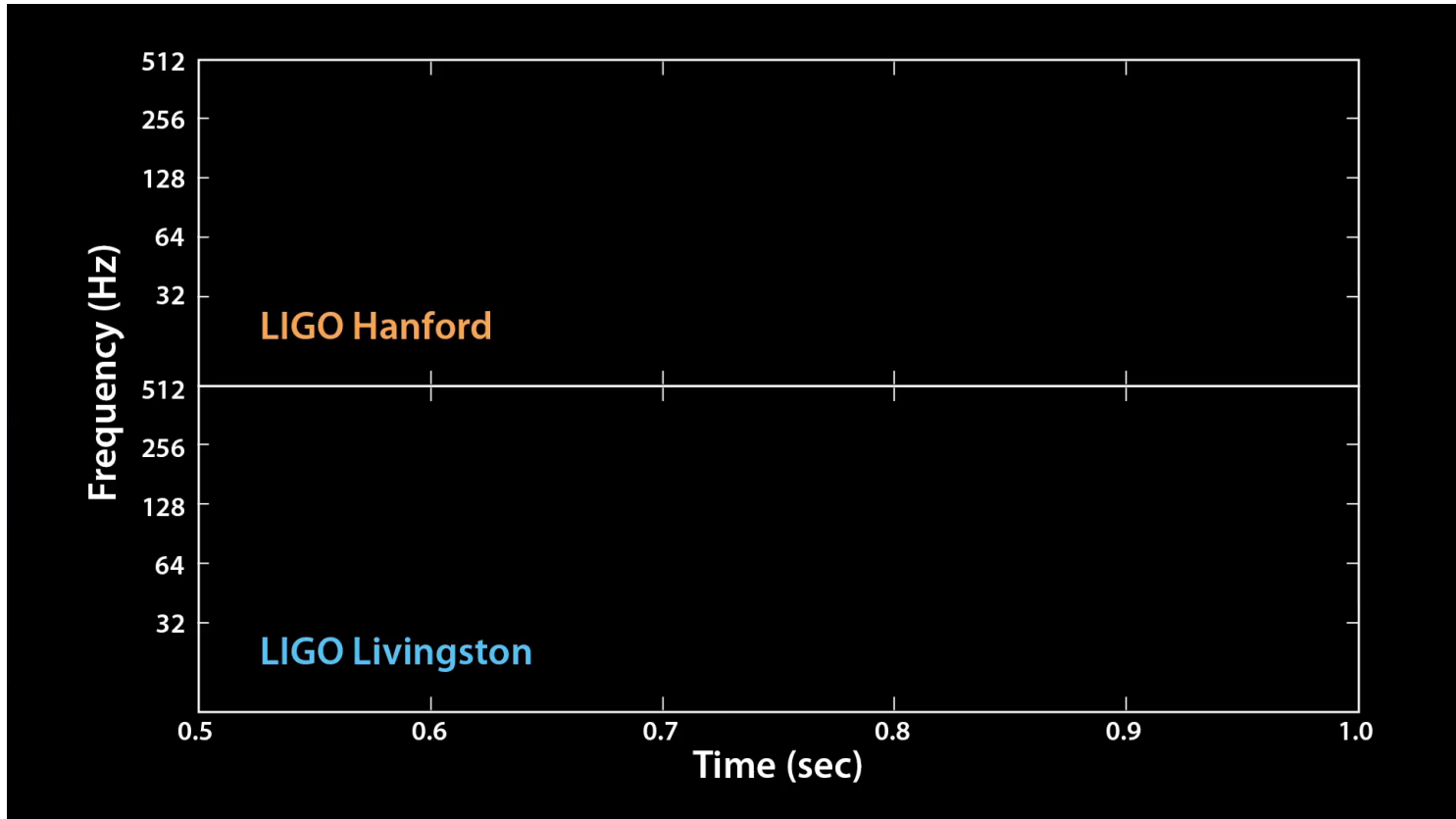
Recent topics

Multi-messenger astronomy

Gravitational wave



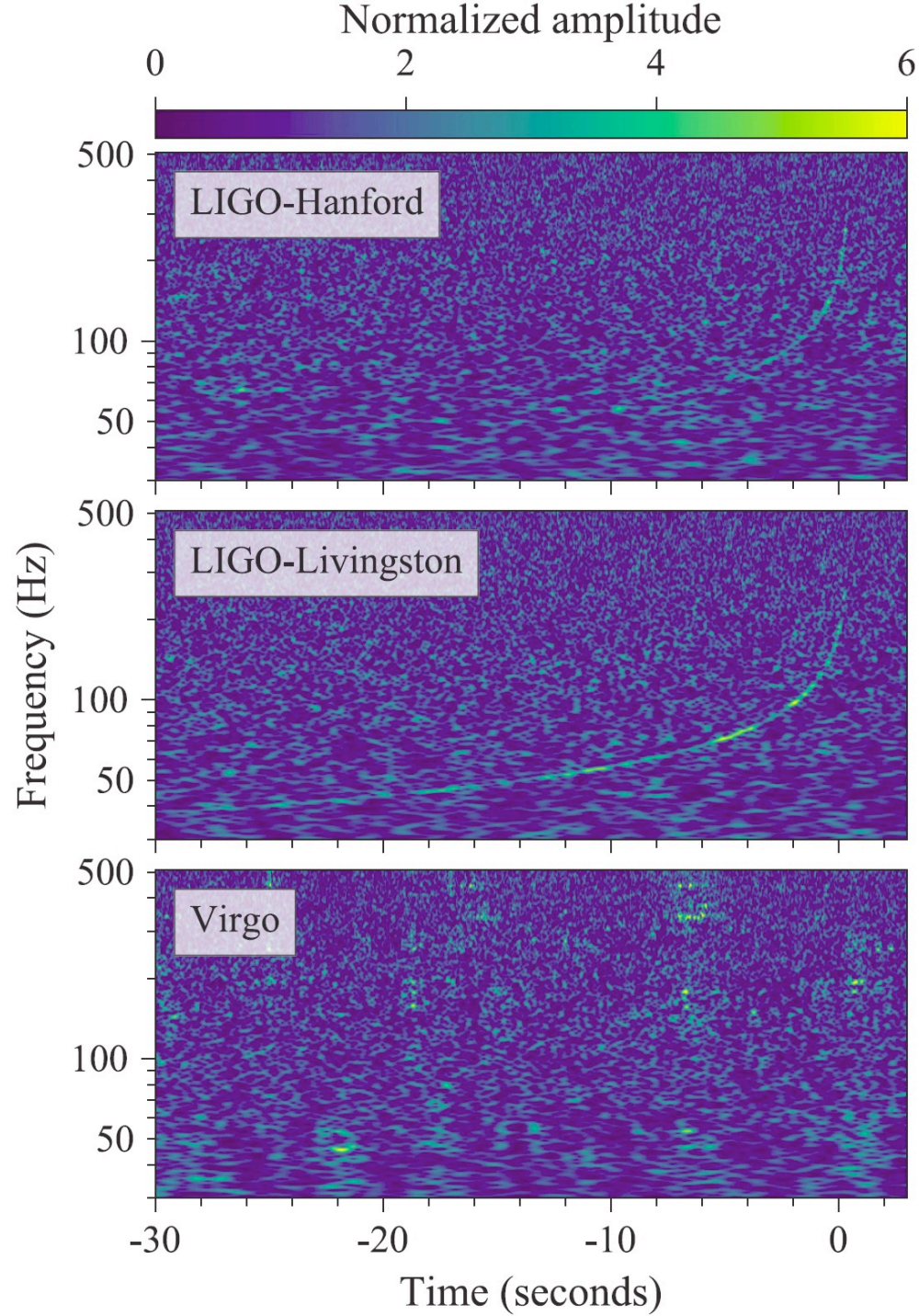
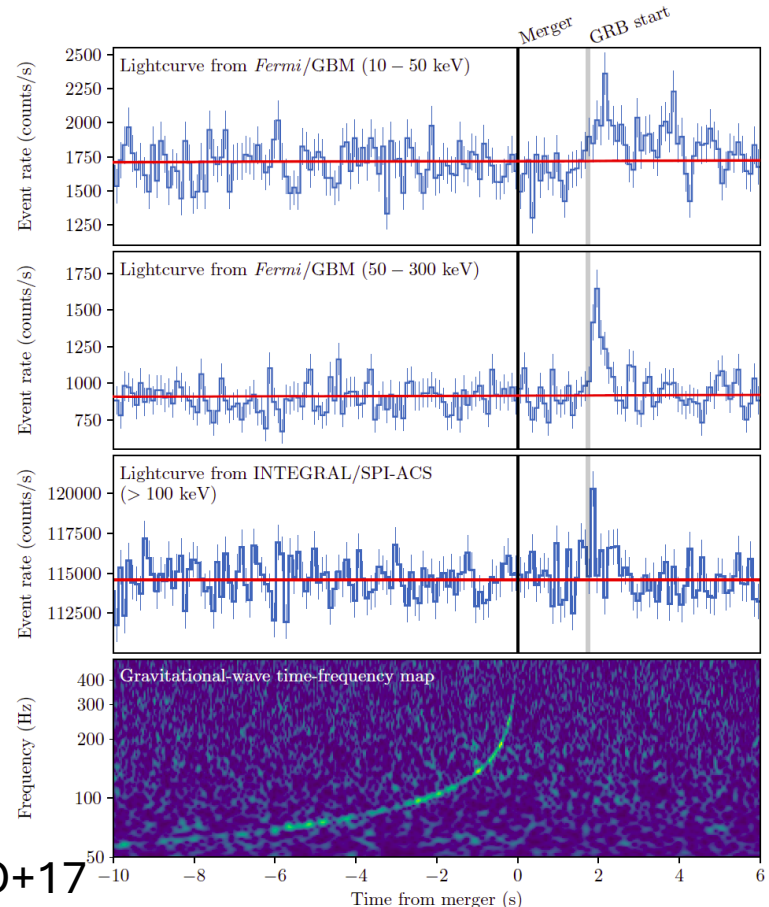
Detection of GW150914



Neutron star merger

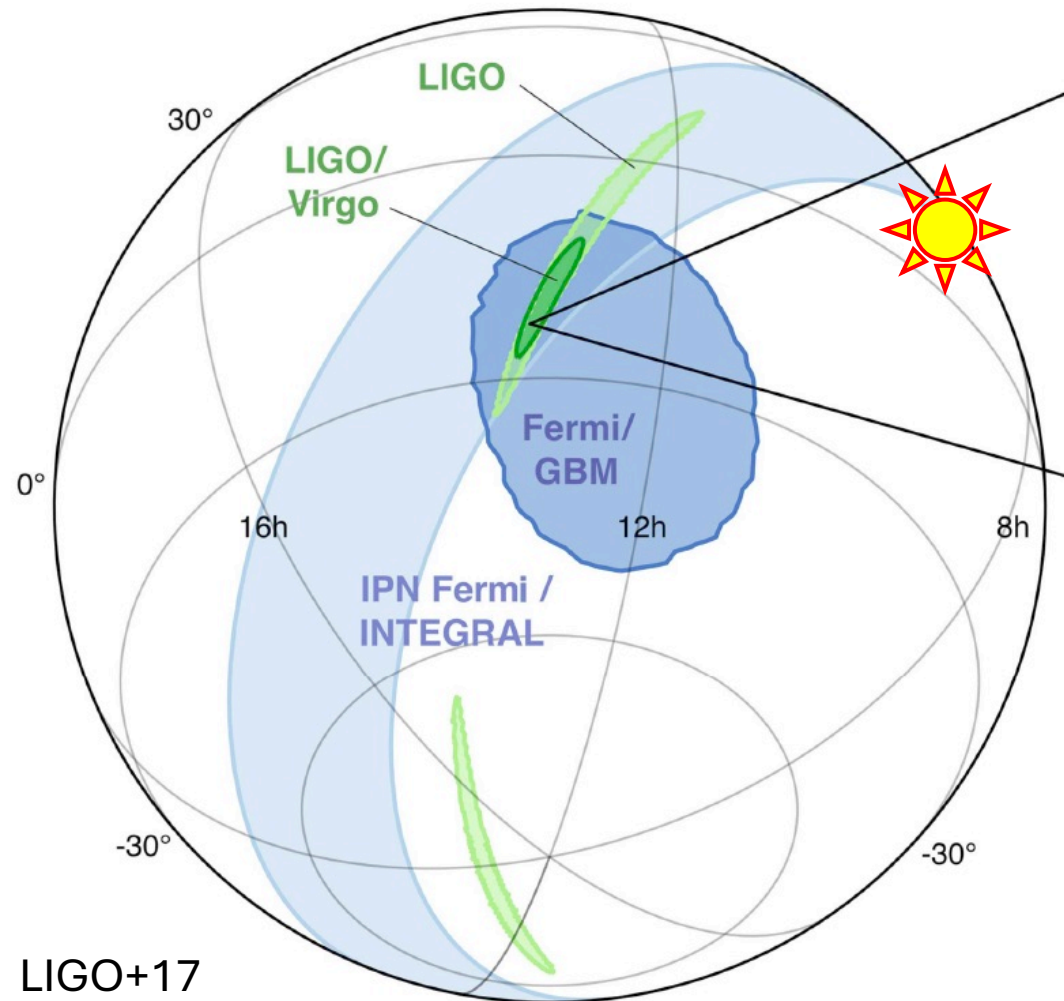
GW170817

The component masses is in the range $1.17\text{--}1.60 M_{\odot}$, with the total mass of the system $2.74^{+0.04}_{-0.01} M_{\odot}$.

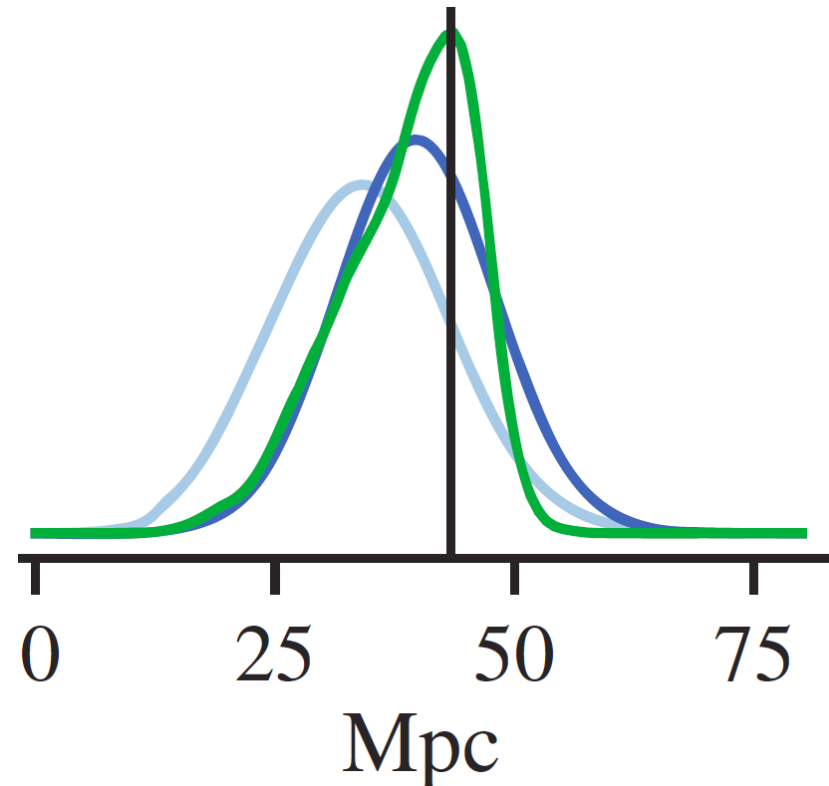


GW170817

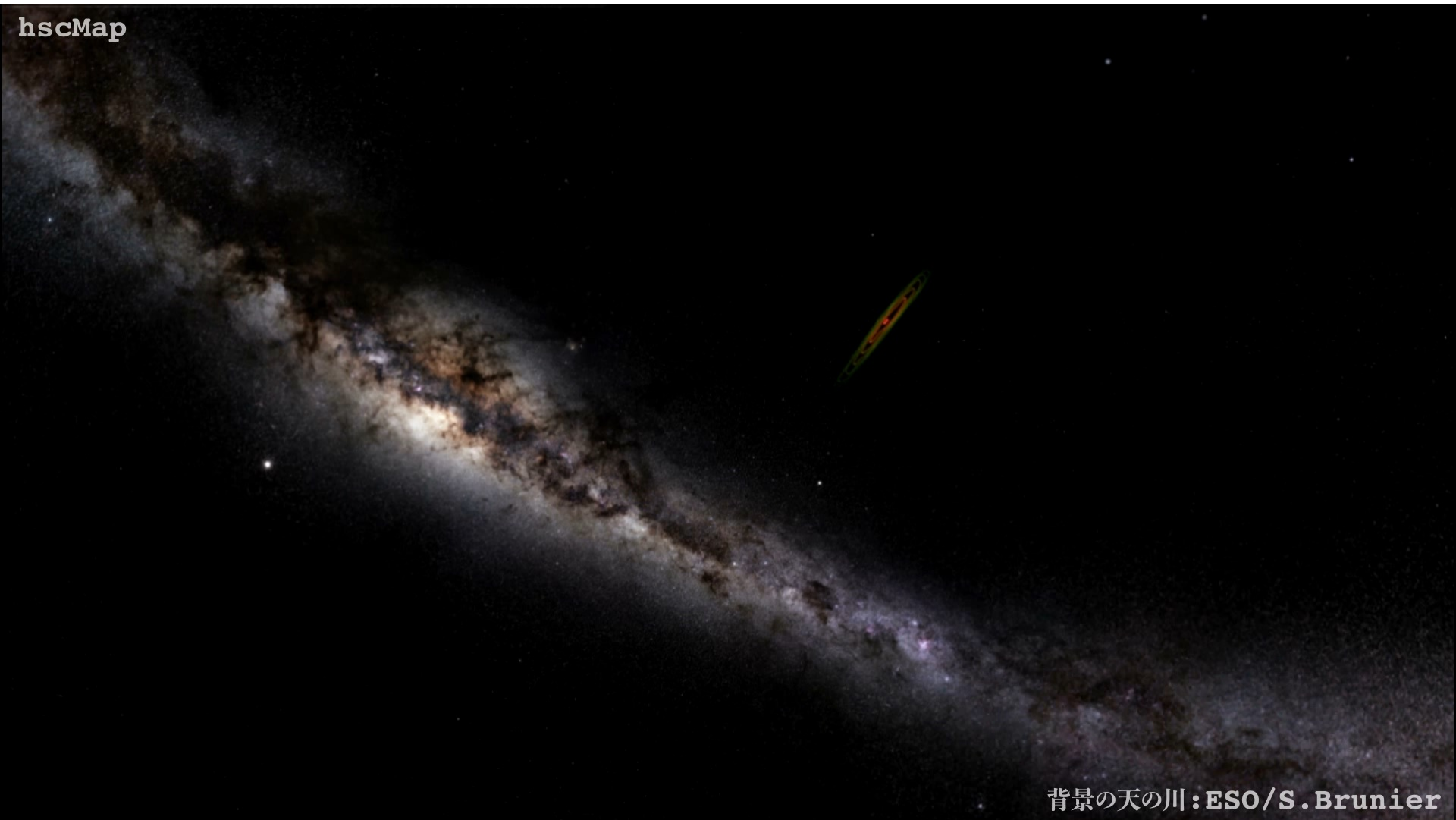
Localization with 3 detectors are as narrow as 28deg^2 for a 90% credible region.



Distance is 40^{+8}_{-14}Mpc .
5 times closer than we prepared.

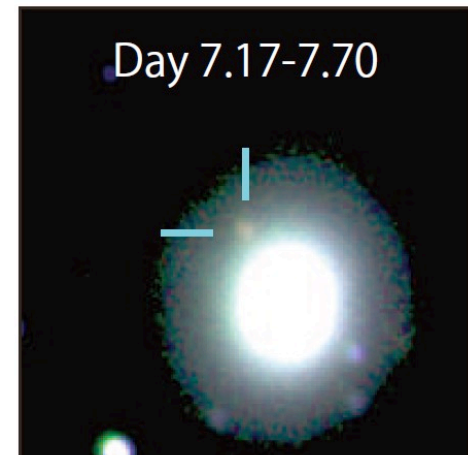
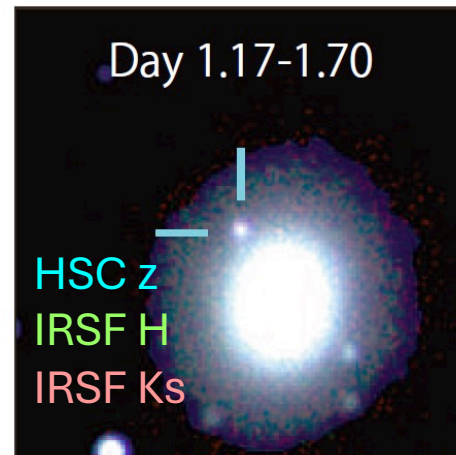
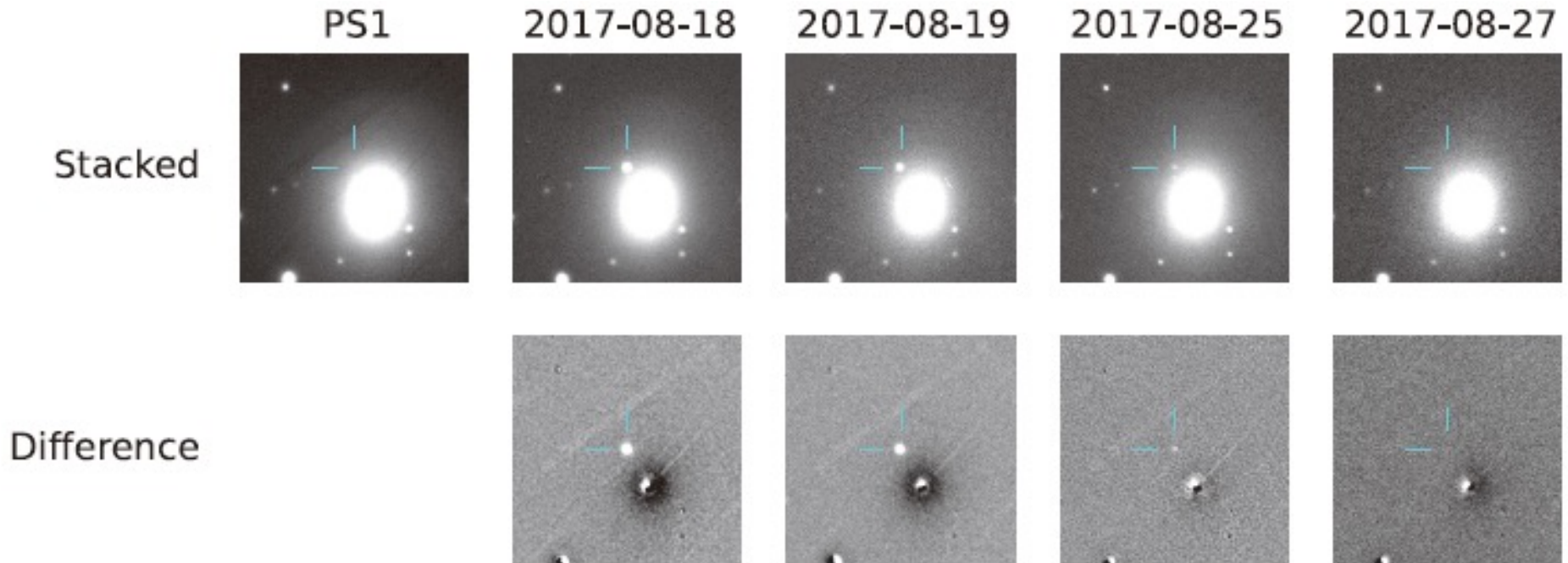


Where is a GW source?

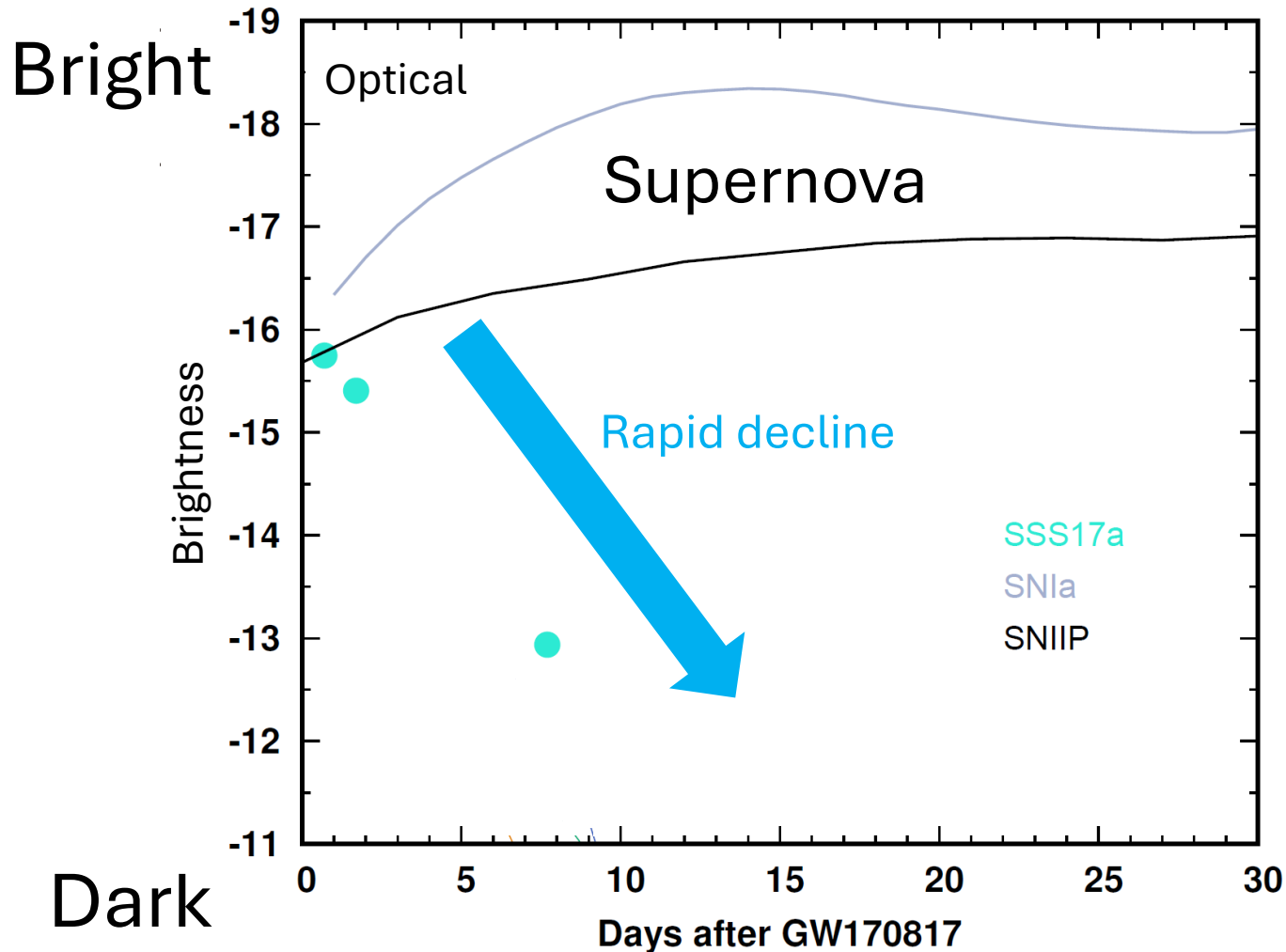


J-GEM17btc (SSS17a/DLT17ck/AT2017gfo)

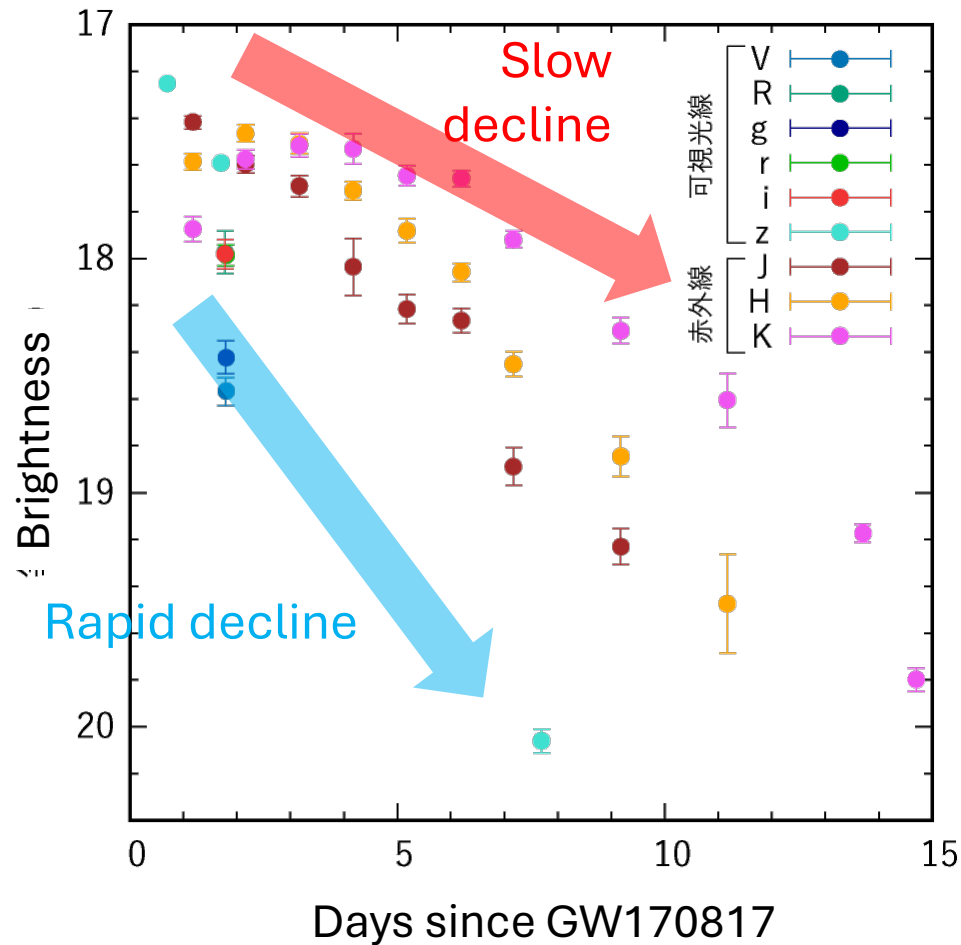
Off-center transient located in NGC4993



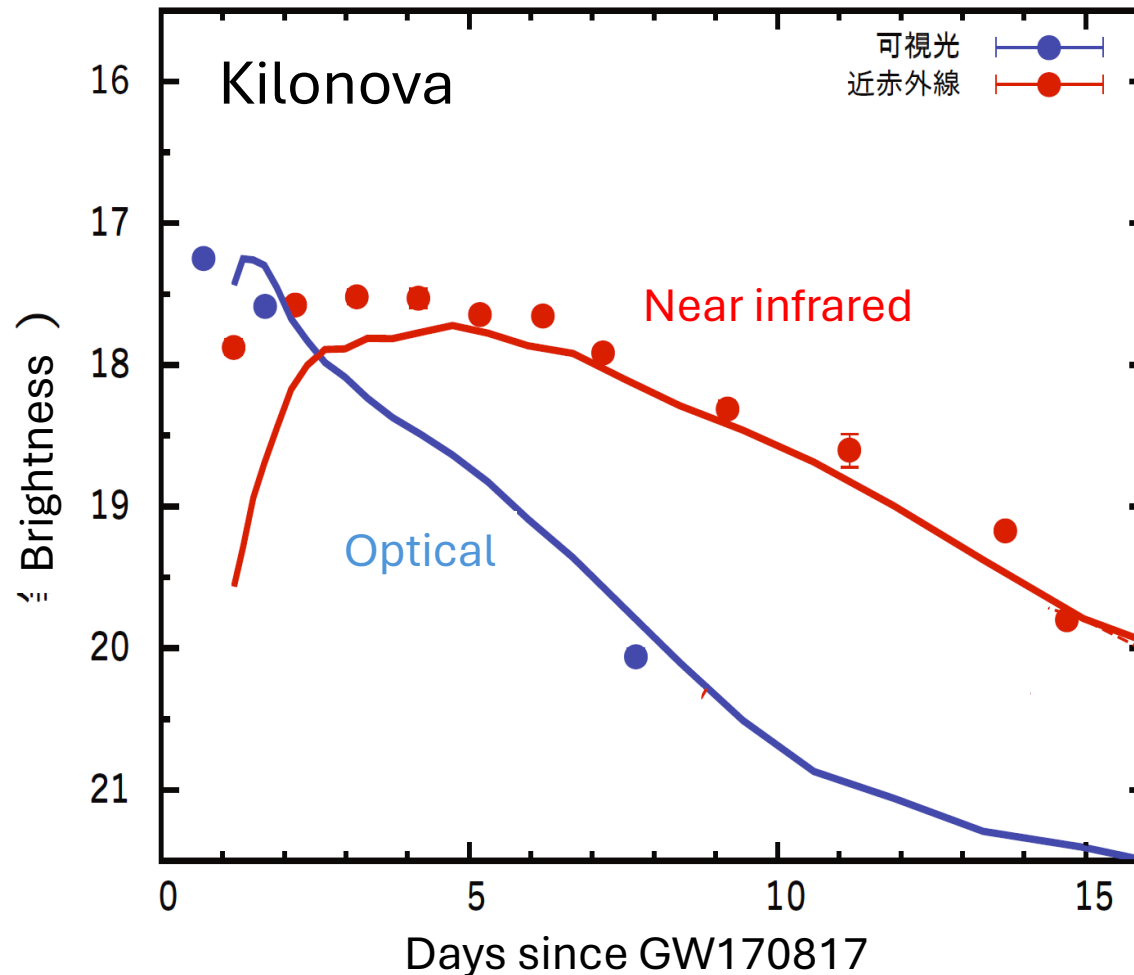
Light curve different from SNe



Decline slowly in red bands



Such an event is predicted



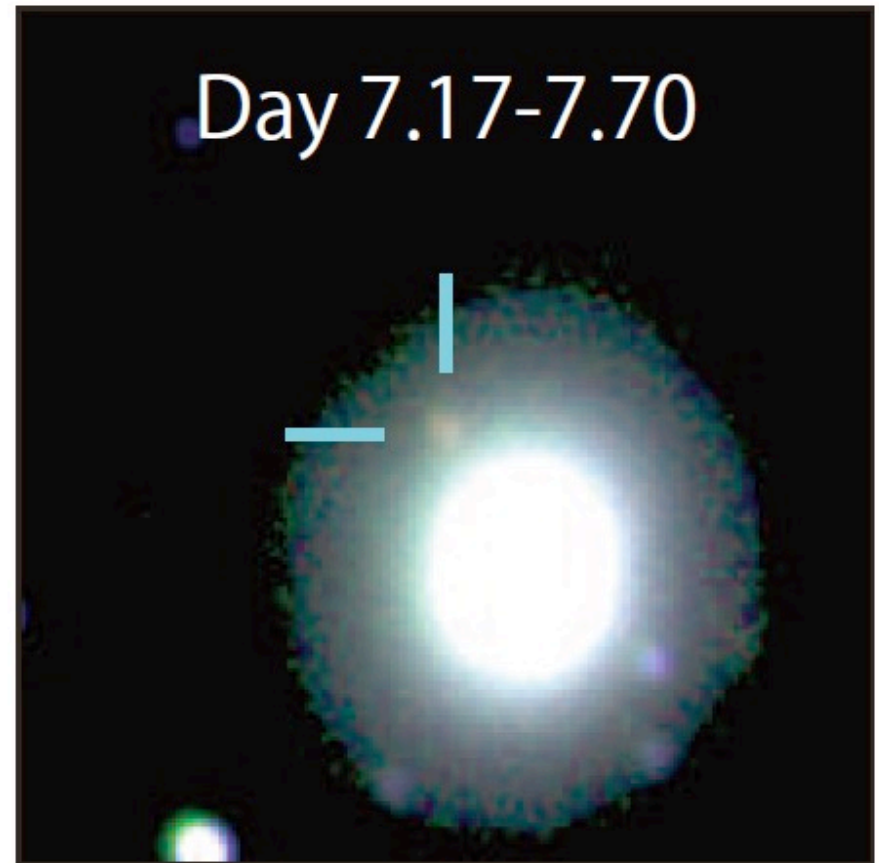
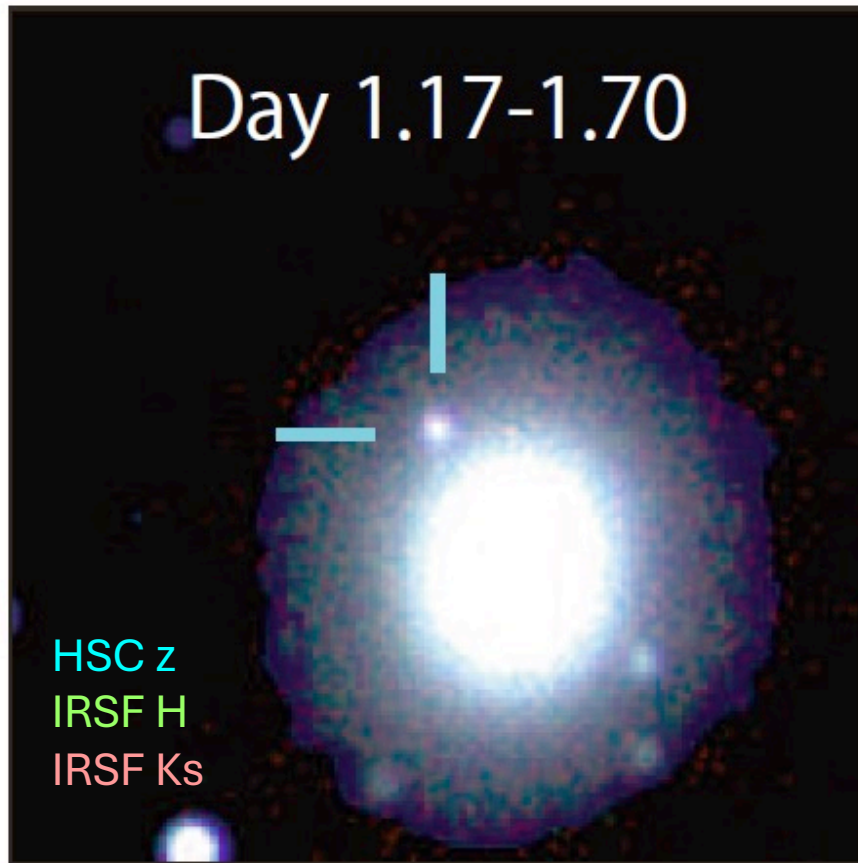
Why is the kilonova red?

- Flame reaction

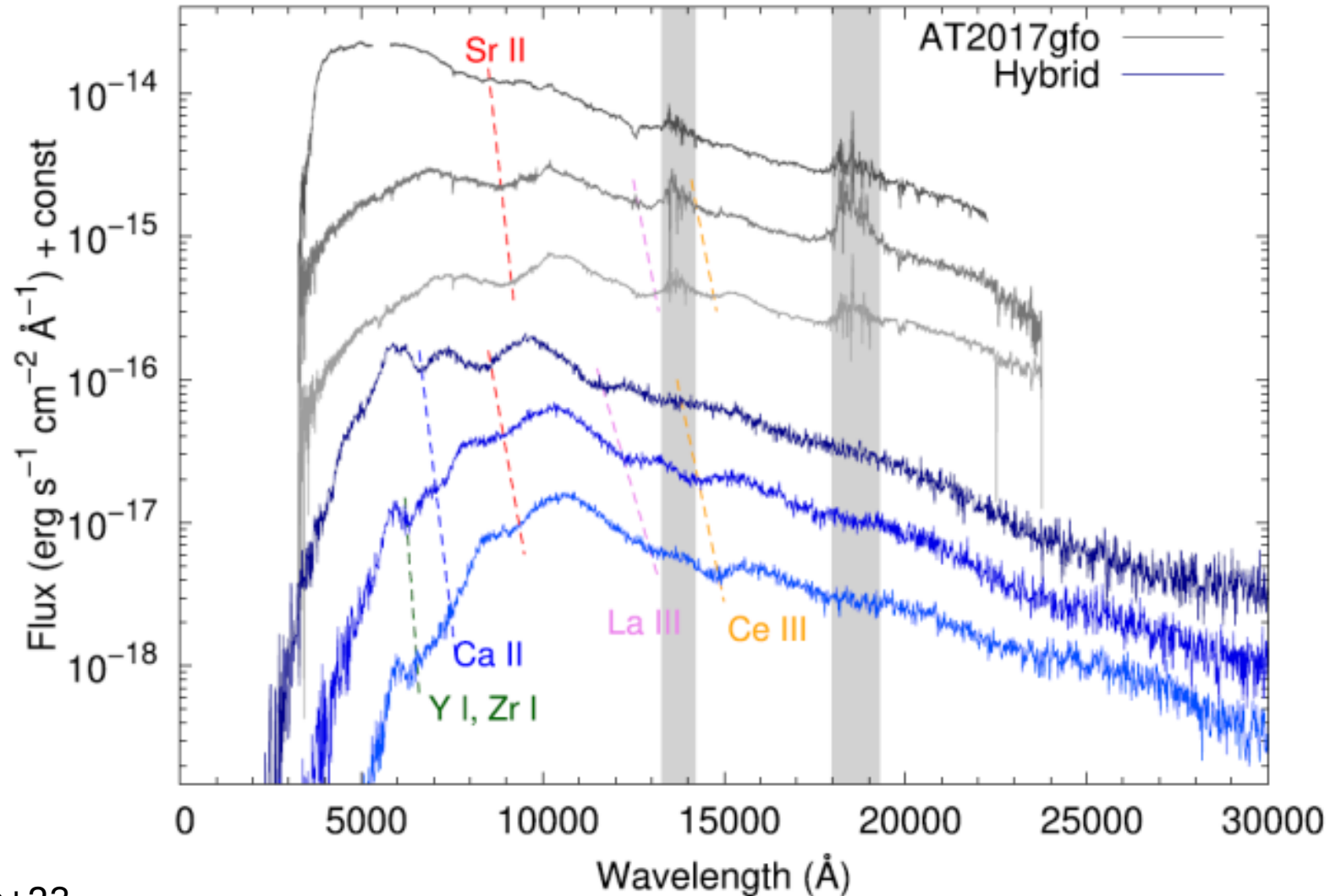


Lanthanoids give **red** colors.

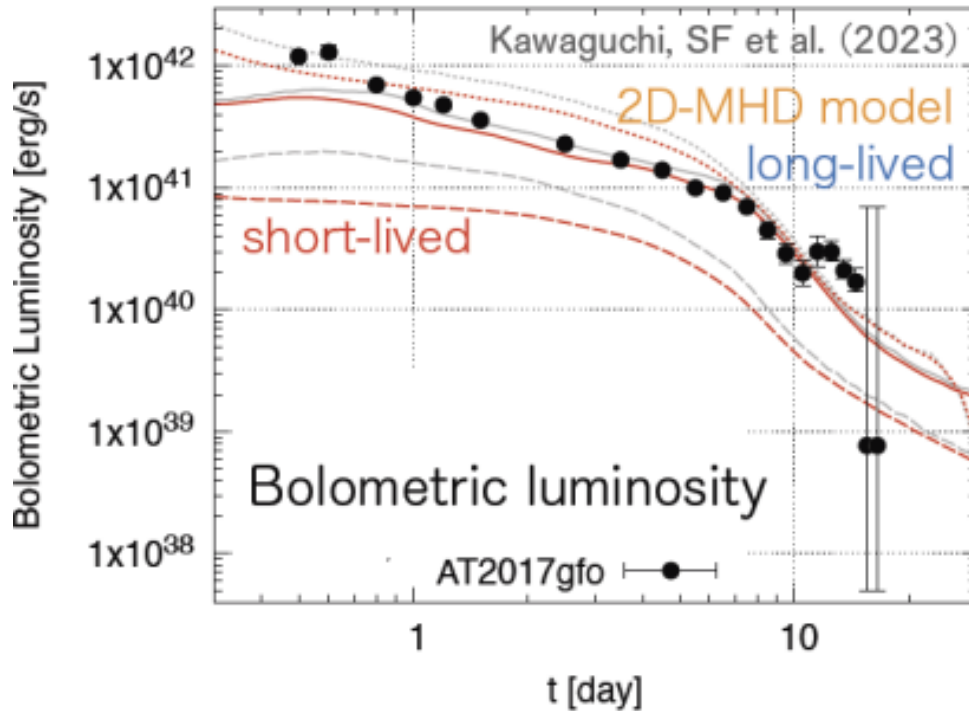
First evidence of synthesis of Lanthanoids in neutron star mergers



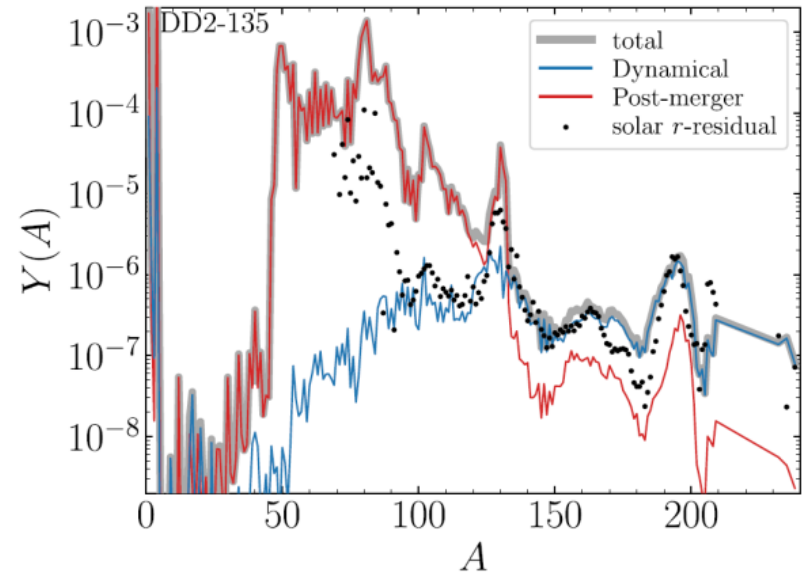
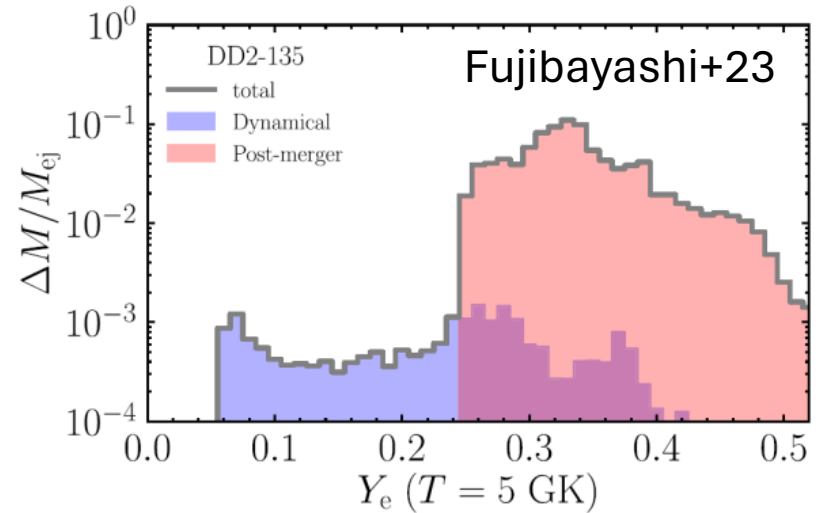
Confirmed elements in kilonova spectra



Abundance pattern of AT2017gfo

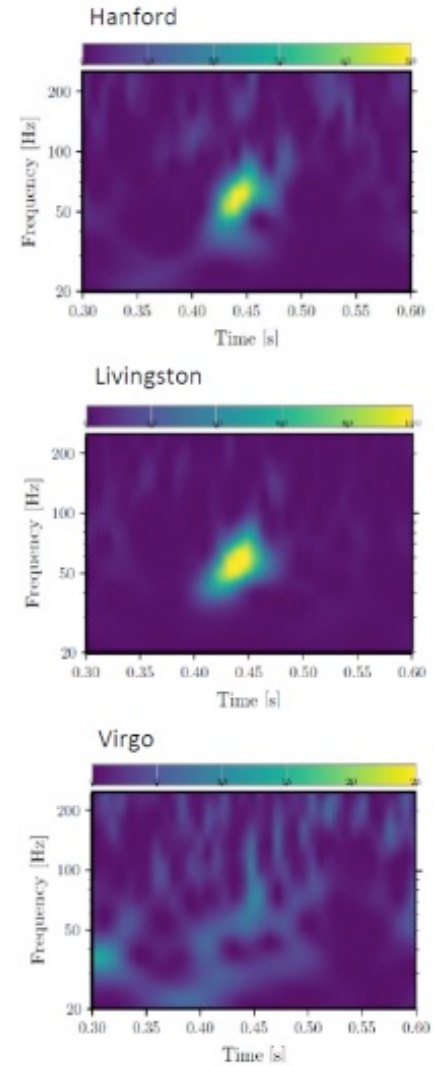
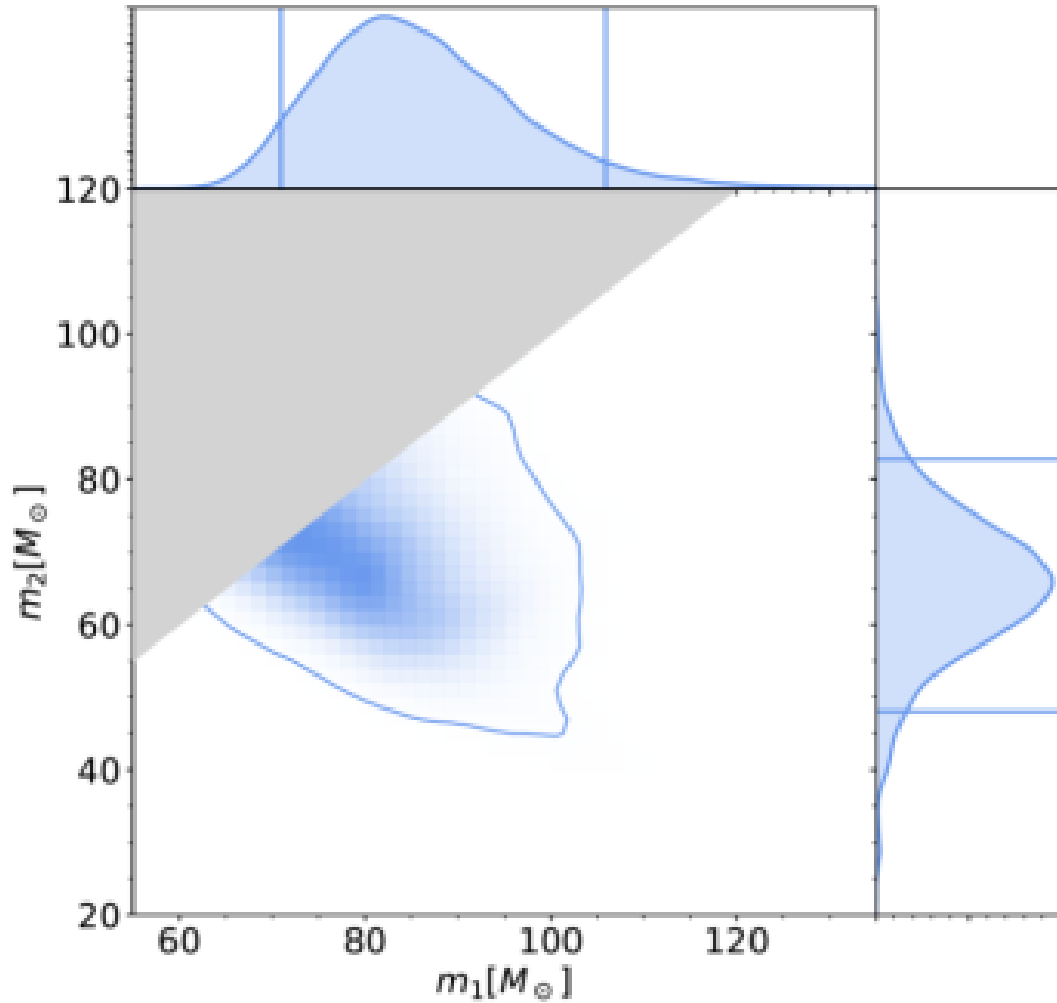


- AT2017gfo is too bright to reproduce solar r-process pattern.



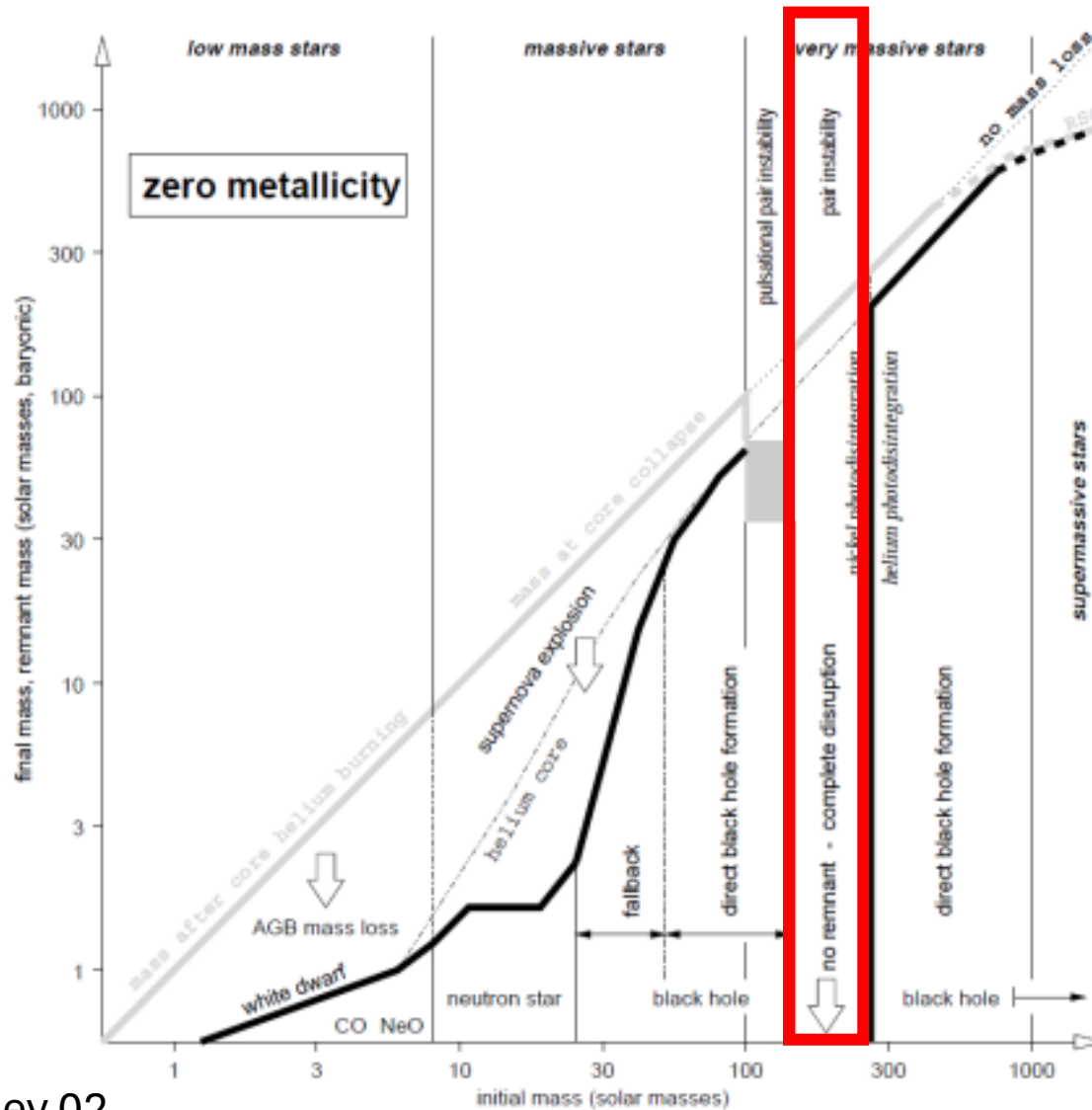
PISN mass gap

GW190521

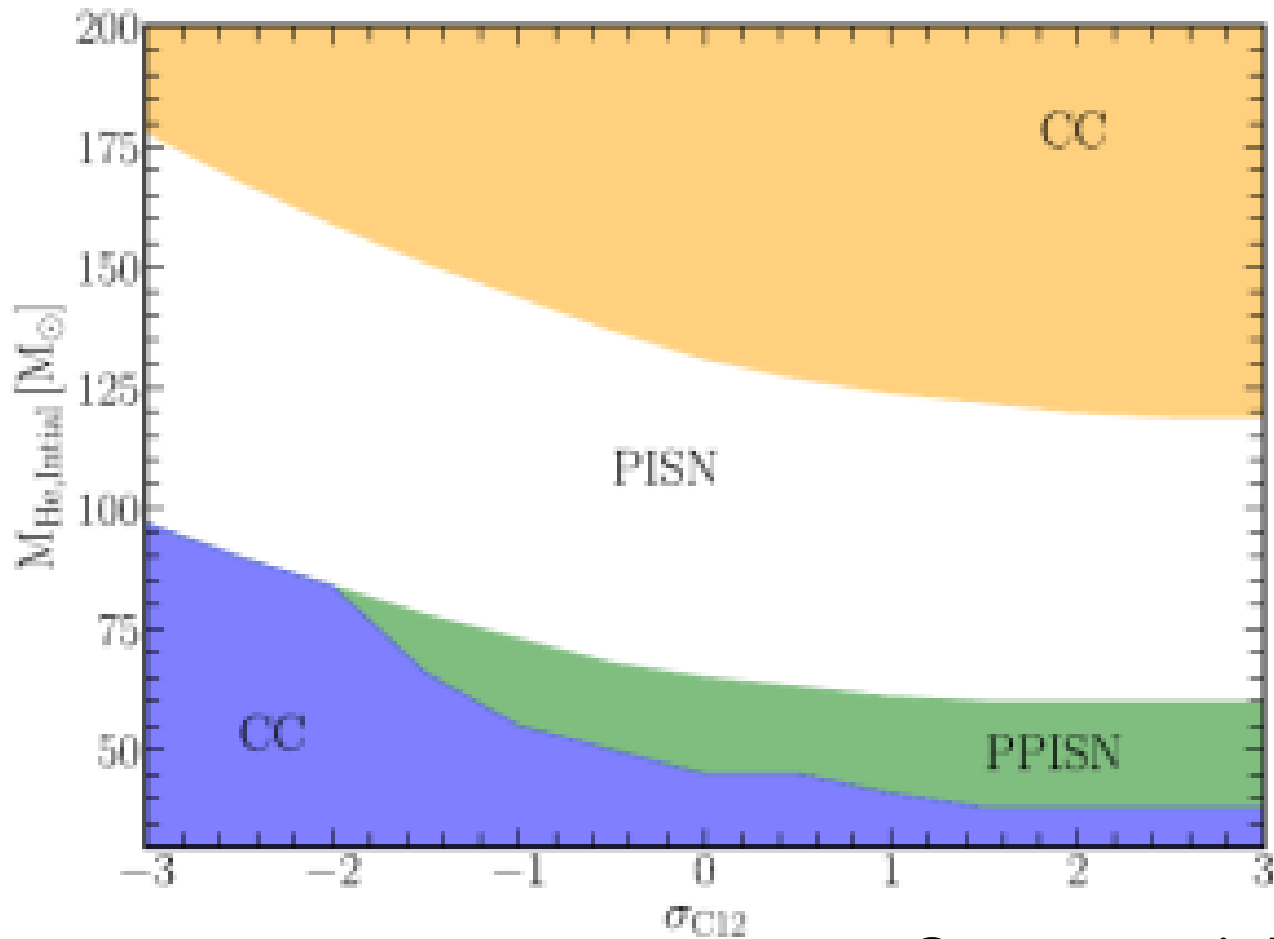


- BH merger with 85Msun+66Msun

PISN mass gap



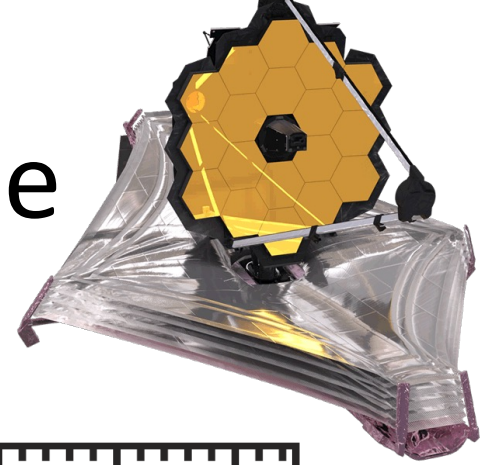
Dependence of $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rate



Or sequential BH mergers?

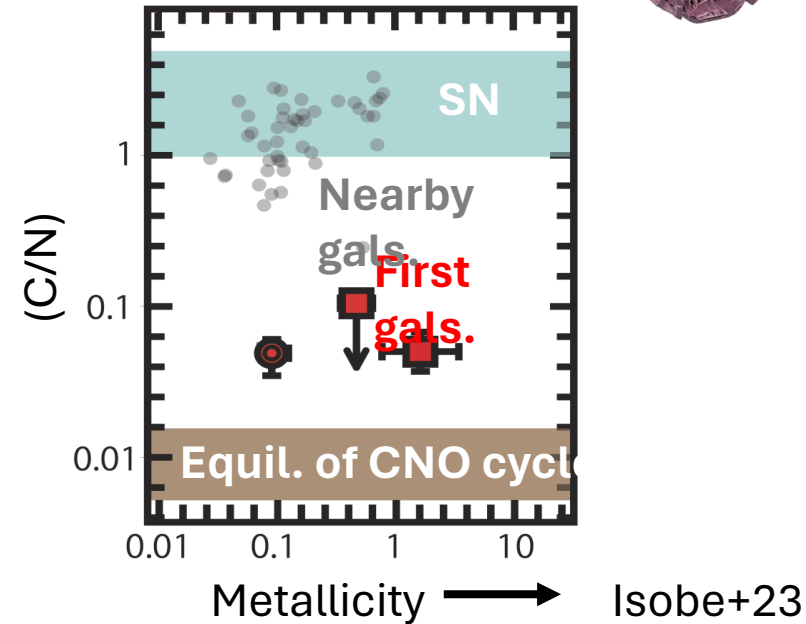
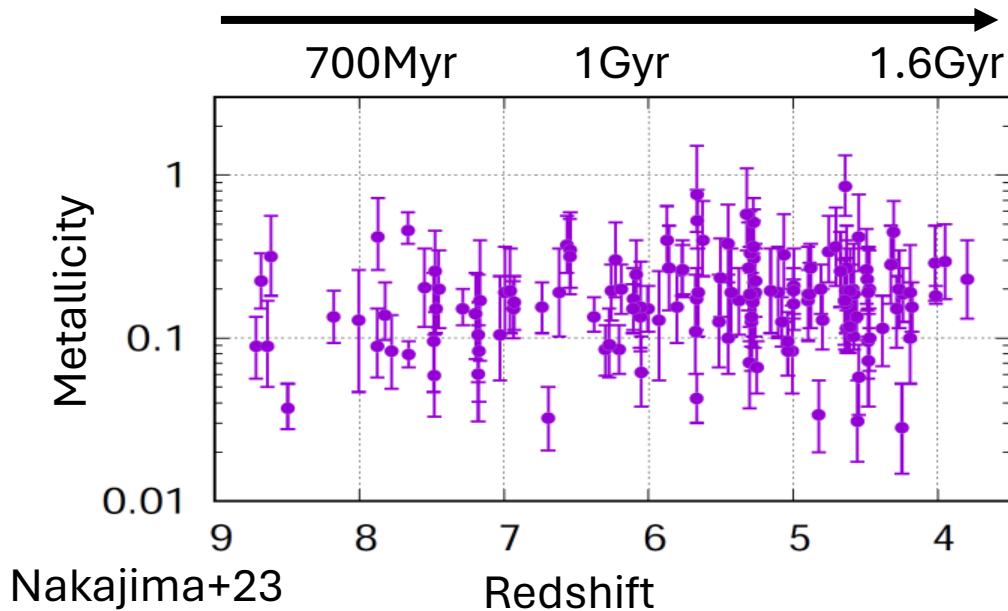
Abundance of first galaxies

James Webb Space Telescope



Scaled with solar

Age of Universe

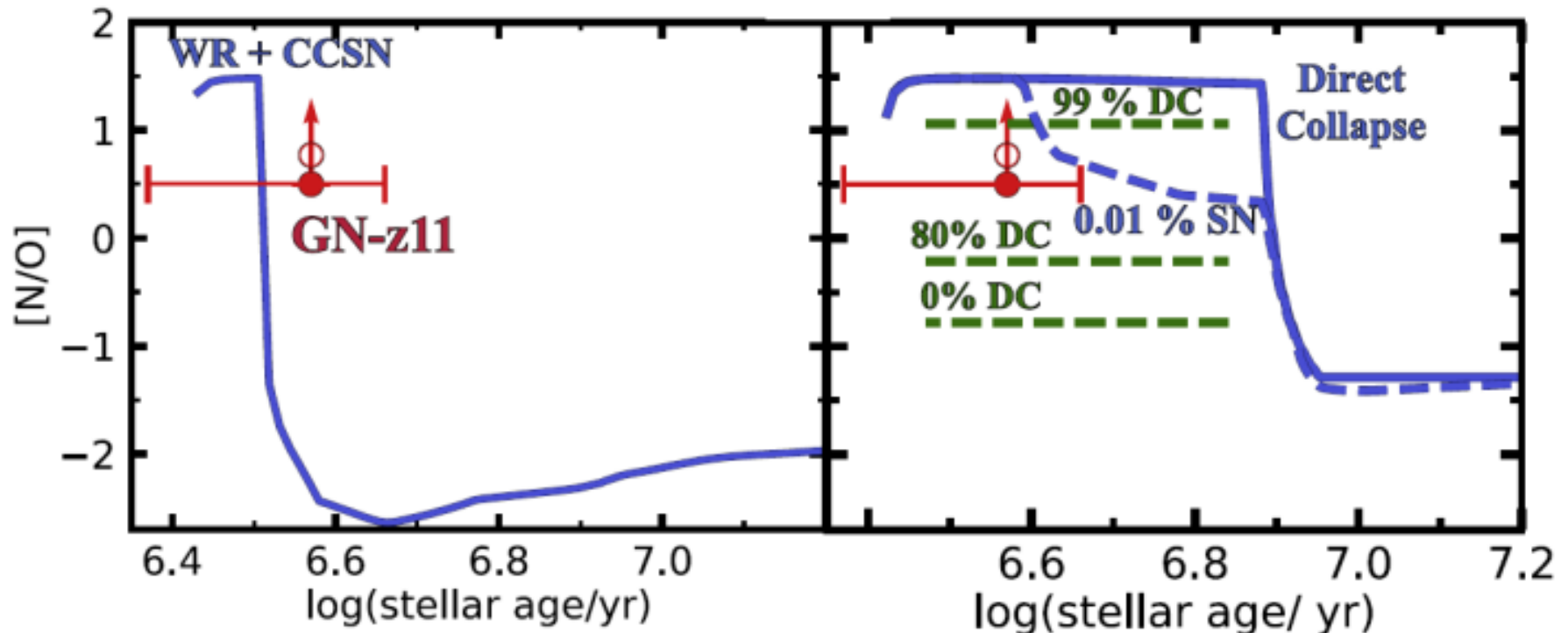


First galaxies at high redshifts have

1. High metallicity of 6-30% of the Sun

2. High N abundance compared to C (and O)

An extreme condition is required to explain the high N abundance.



- Only the winds of rotating Wolf–Rayet stars that end up as a direct collapse can explain the high N/O ratio.

Fluorine detection at $z=4.4$

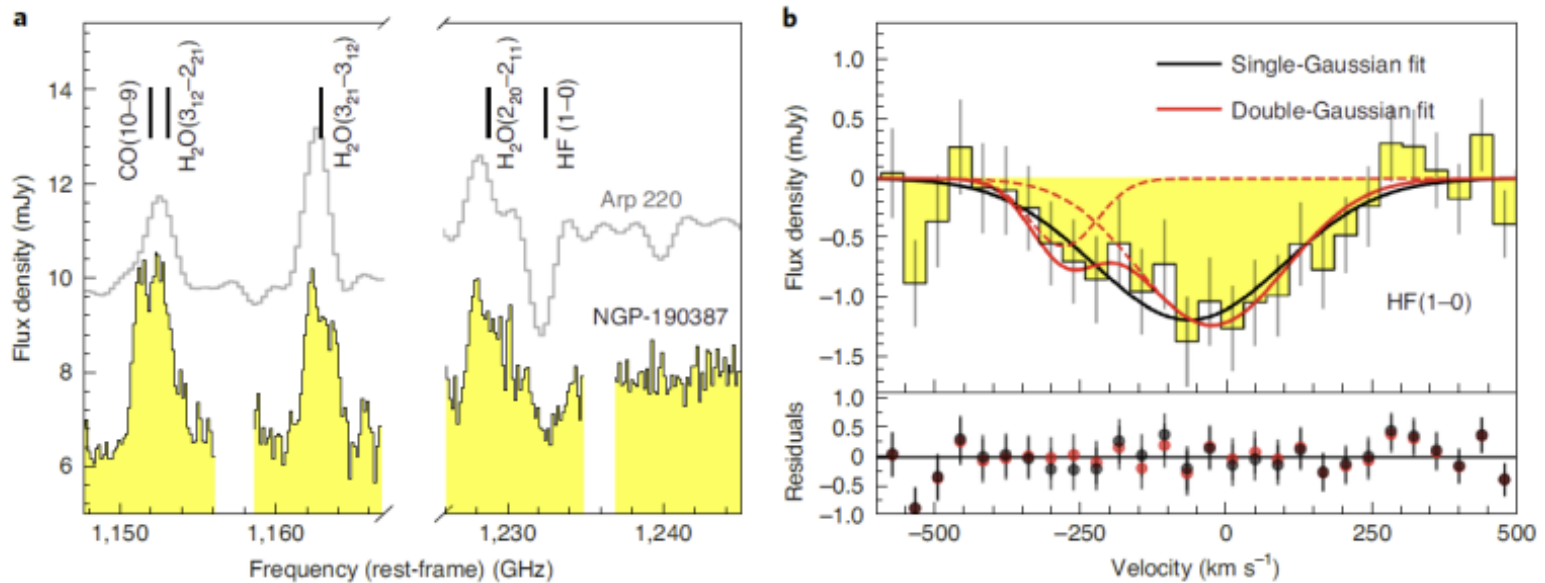
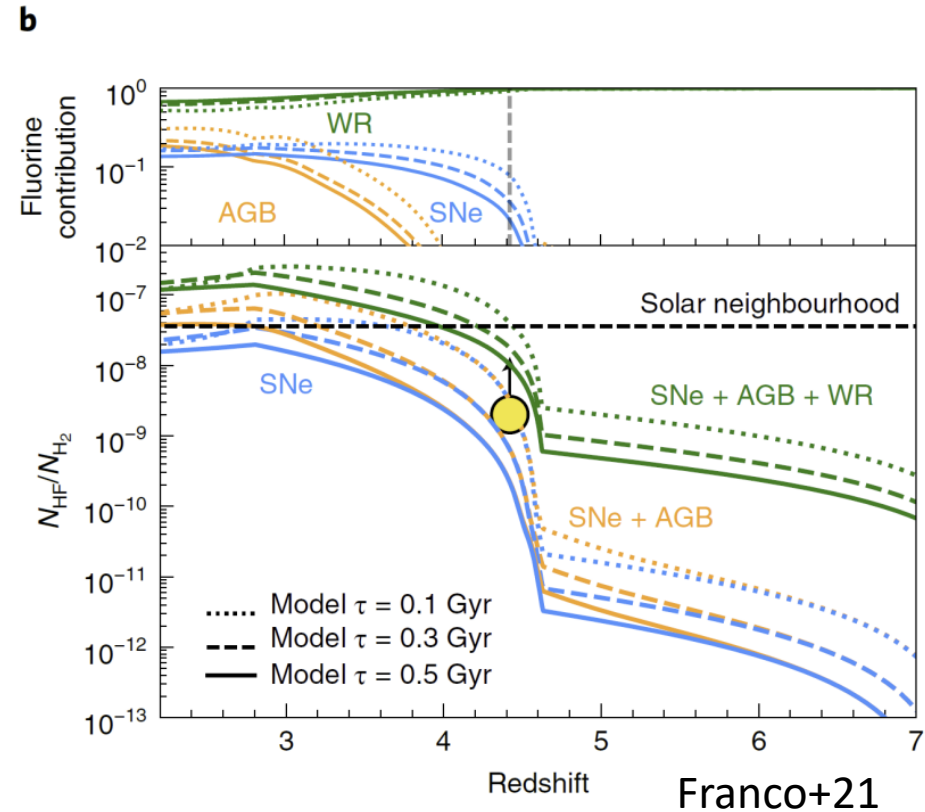
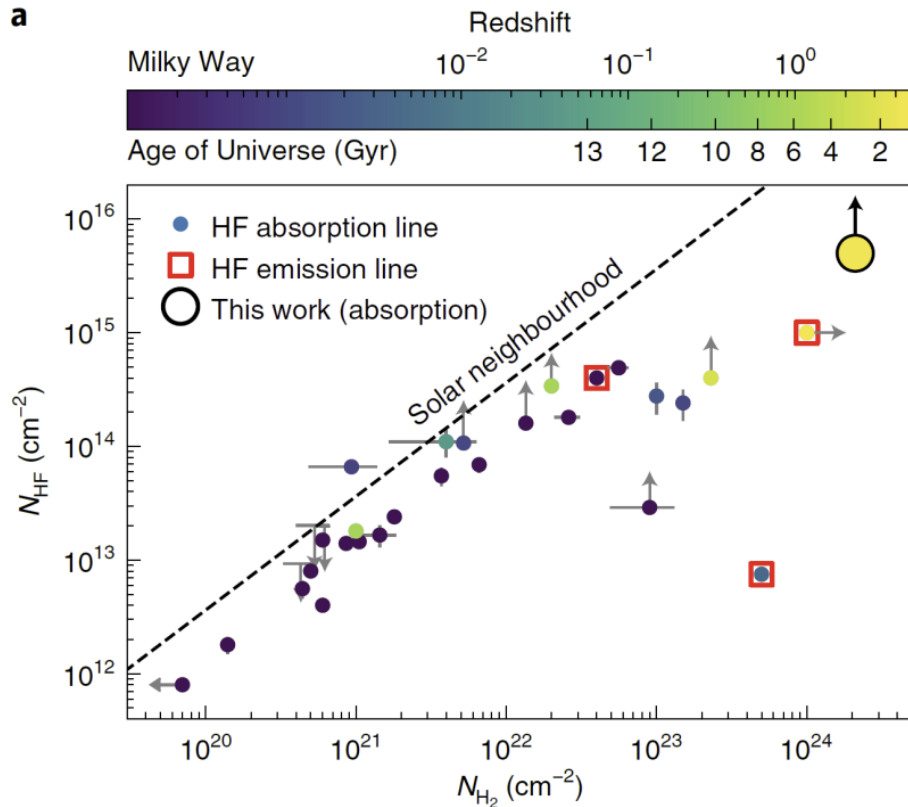


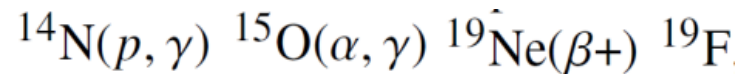
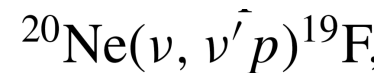
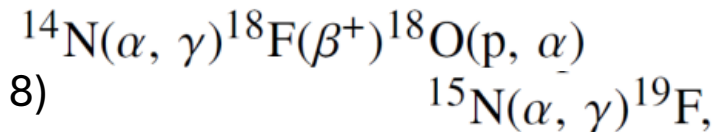
Fig. 2 | ALMA spectrum of NGP-190387. a, Spectrum in band 6 without continuum subtraction. The positions of emission and absorption lines are indicated by labelled vertical lines. For comparison, the Arp 220 spectrum⁷², shifted and scaled by arbitrary units, is also displayed (in grey). The frequency gap is due to the separation of the ALMA spectral windows. By inspection, the plot of the flux density without continuum subtraction shows that the absorption line is not saturated. By integrating the signal-to-noise ratio (S/N) over the absorption line, we obtain an $S/N = 8$. **b**, Observed HF(1-0) line profile ($\nu_{\text{rest}} = 1,232.48$ GHz; ref. ⁷³), continuum subtracted. The spectral resolution is 40 km s^{-1} , with the velocity scale centred on the HF(1-0) transition at $z = 4.420$. The single- and double-component Gaussian fits are displayed, with dashed lines indicating the individual components of the double-Gaussian fit. The error bars show the uncertainties on the flux measurement given by imfit. The bottom panel shows the residuals (data - model) in units of millijansky in black for the single-Gaussian fit and in red for the double-Gaussian fit. In these two panels, the flux density is not corrected by the magnification factor ($\mu \approx 5$).

Fluorine synthesis



- Fluorine can be synthesized via

- He convective shell (Limongi & Chieffi 18)
- Neutrino reaction (Yoshida+08)
- N-rich H layer (Shibata+ in prep.)

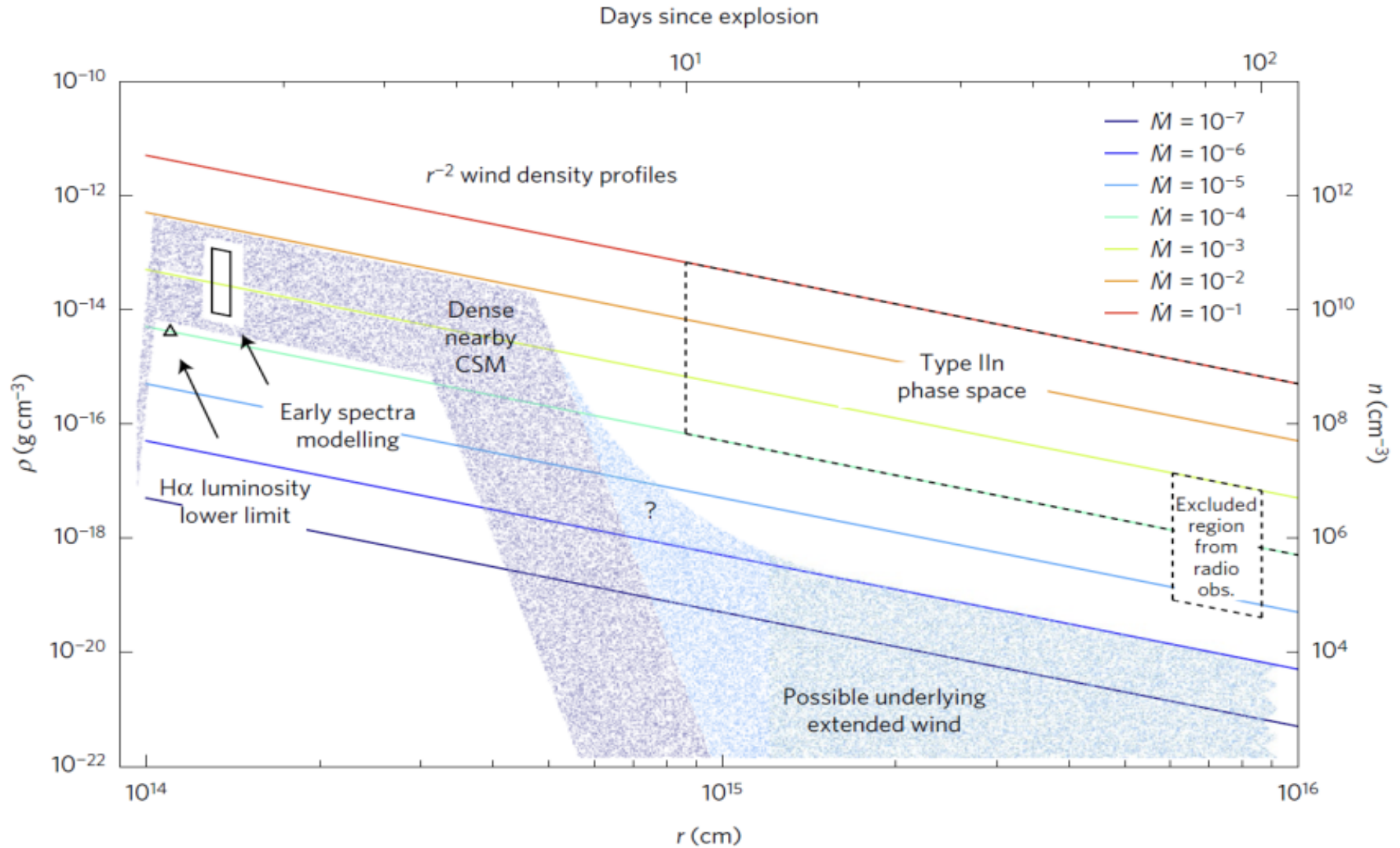


Enhanced mass loss

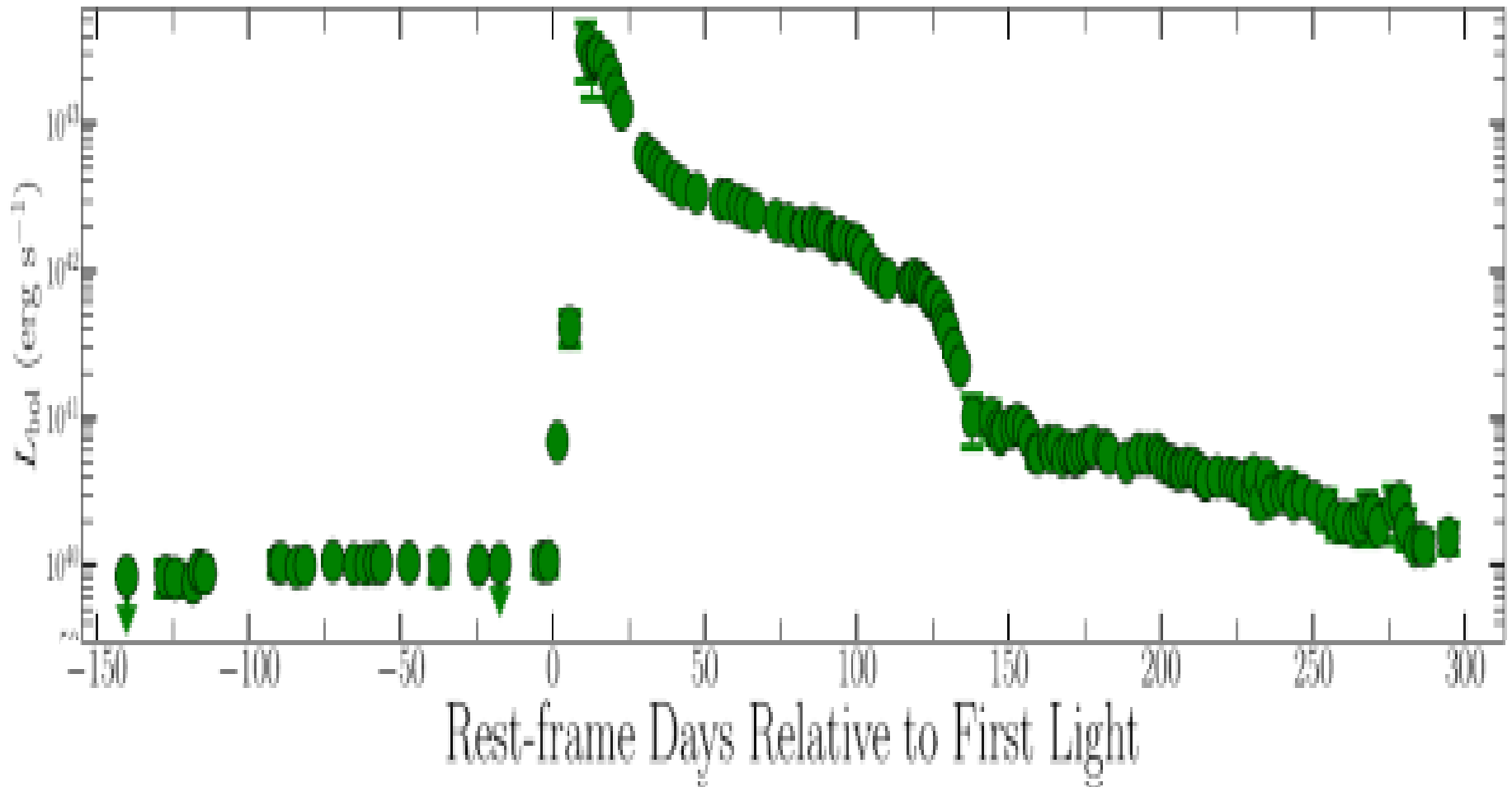
Enhanced mass loss



SN2013fs - Confined CSM



Brightening before the explosion



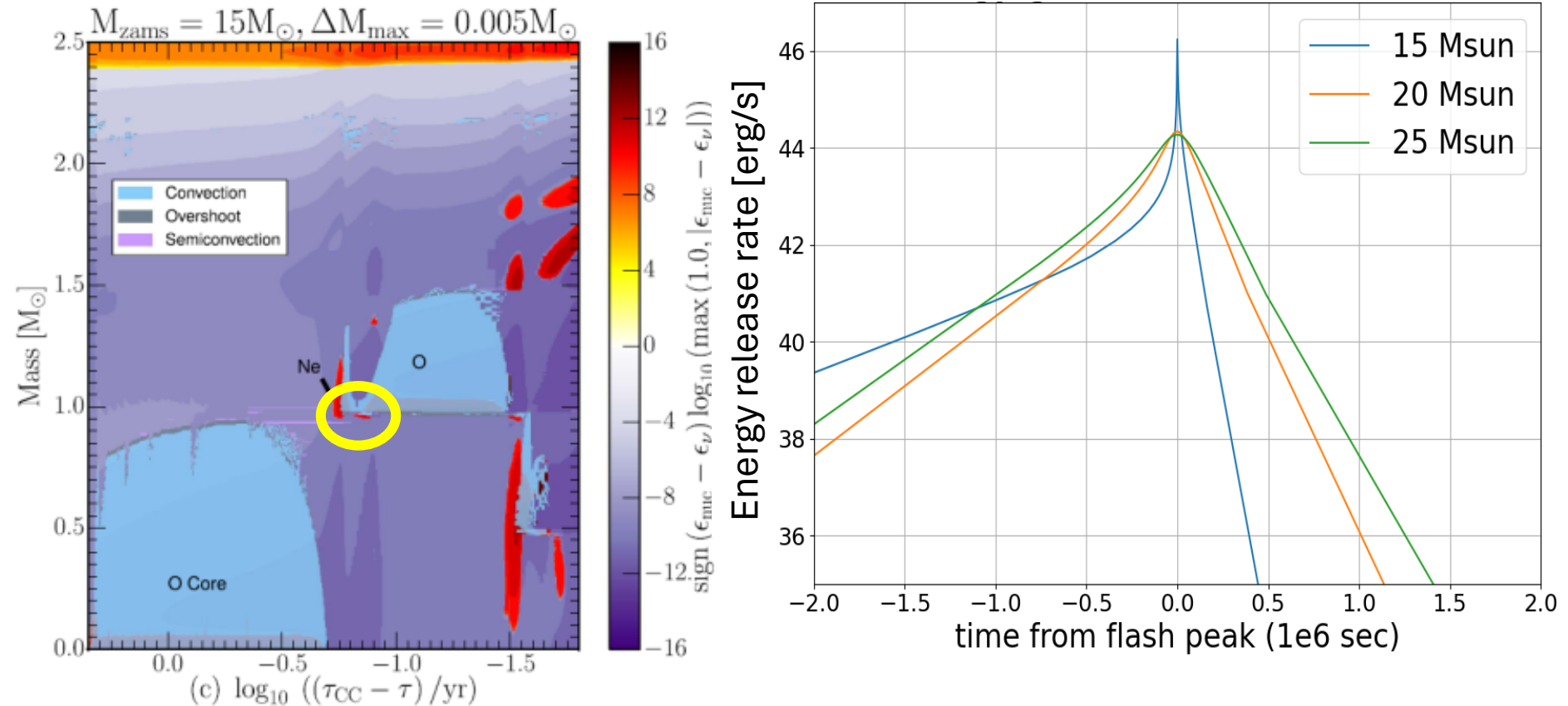
What makes the confined CSM?

Enhanced mass loss just prior to the explosion

Origins?

- Pulsations of red supergiants (Yoon & Cantiello 10)
- Core neutrino emission (Moriya 14)
- Near-surface energy deposition (Quataert + 16)
- Nuclear flash (Woosley & Heger 15)

Oxygen shell flash



Farmer+16

- 5×10^{49} ergs (10% of binding energy of the envelope) is released at the short period.
- The flash takes place when $\epsilon_{\text{O}} > \epsilon_{\nu}$.

Chiba+ in prep.

Summary

- The evolution of stars and the Universe are determined by nuclear reactions.
- Multi-messenger astronomy finds **the synthesis of Lanthanoids in NSMs** and **a black hole merger with masses in a PISN mass gap**.
- Observations of distant galaxies find **high N abundance** and **high F abundance**.
- An enhanced mass loss prior to the collapse is observed. **An O shell flash** might explain the enhanced mass loss.