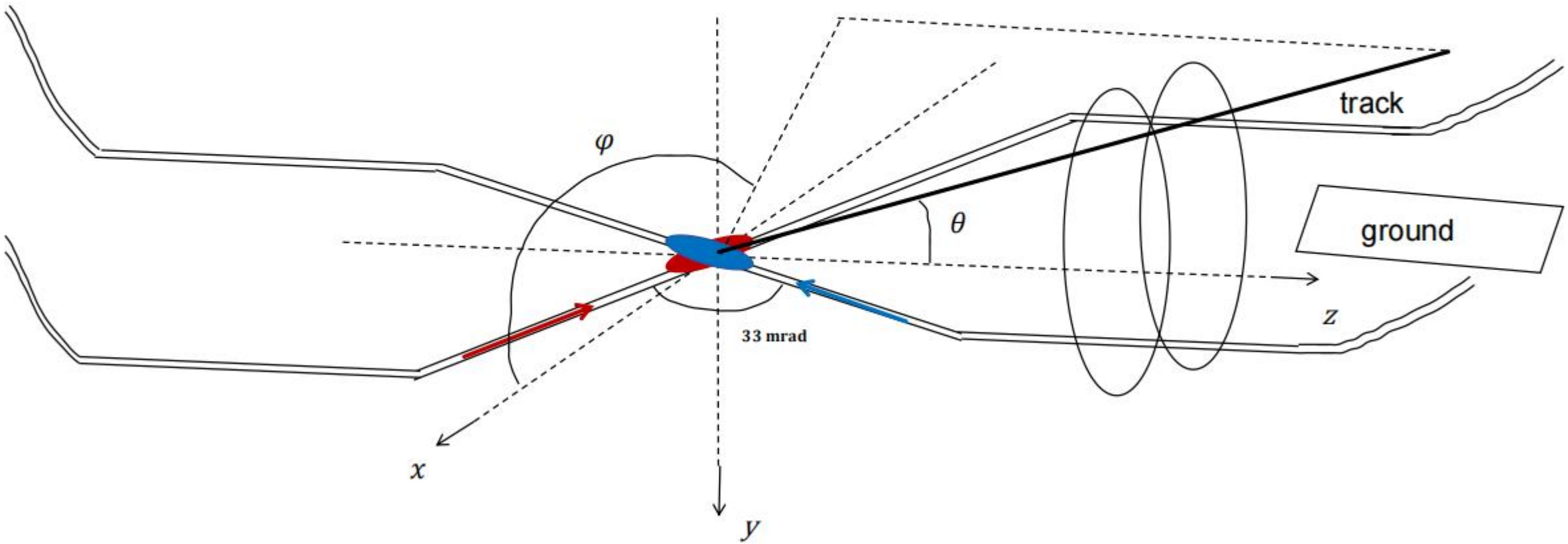
2024.3.12 Toy model of FLM (Ignore *B*)

1. Overview in Lab frame



2. Hits on FLM

(1) CMS frame moves along -x with Lab frame ⇔ Lab frame moves along x with CMS frame

CMS frame *(x, y, z, t)*
Lab frame *(x', y', z', t')*

$$\begin{cases} t' = \gamma\left(t - \frac{v}{c^2}x\right), \\ x' = \gamma(x - vt), \\ y' = y, \\ z' = z, \end{cases}$$

Lab frame: $(E', p'_x, 0, p'_z)$
CMS frame: $(E, 0, 0, p_z)$
Lorentz boost:

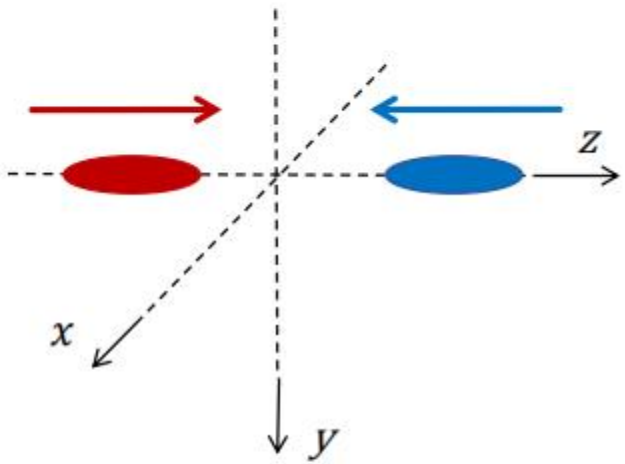
$$\begin{pmatrix} E' \\ p'_x \\ 0 \\ p'_z \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma v & & \\ -\gamma v & v & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} E \\ 0 \\ 0 \\ p_z \end{pmatrix}$$

Using $E^2 - p^2 = E'^2 - p'^2 \approx 0$ and Lorentz boost, we have $\tan\theta = \gamma v$, therefore $v = \tan^2\theta / (1 + \tan^2\theta)$
($\theta = 16.5$ mRad)

```
for(int i = 0; i < N; i++)
    {
        static TRandom3 rndm;
        xi[i] = 0. + rndm.Gaus(0, 6./1000. * gamma);
        yi[i] = 0.;
        zi[i] = 0. + rndm.Gaus(0, 0.36);
        t[i] = (1000. - zi[i])/TMath::Cos(theta);

        xo[i] = xi[i] + TMath::Sin(theta)*TMath::Cos(phi)*t[i];
        yo[i] = yi[i] + TMath::Sin(theta)*TMath::Sin(phi)*t[i];
        zo[i] = 1000.;

        xb[i] = gamma * xo[i] - gamma * v * (zo[i] / (v * TMath::Cos(theta)));
        yb[i] = yo[i];
        zb[i] = zo[i];
        //cout << "xo["<<i<<"]" << xo[i] << endl;
        //cout << "yo["<<i<<"]" << yo[i] << endl;
        //cout << "zo["<<i<<"]" << zo[i] << endl;
        //cout << "t["<<i<<"]" << t[i] << endl;

    }
```

(2) Assuming in CMS frame, IP= $(0, 0, 0)$, $\varphi = 90°$, $\theta = 30$ mRad and FLM at $z = 1$ m

CMS frame (Blue points)

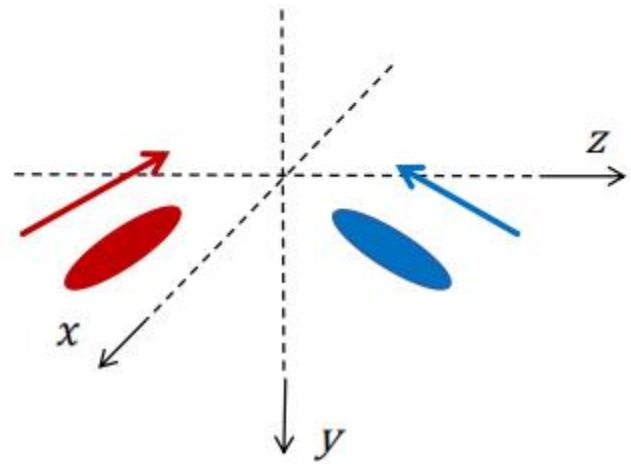


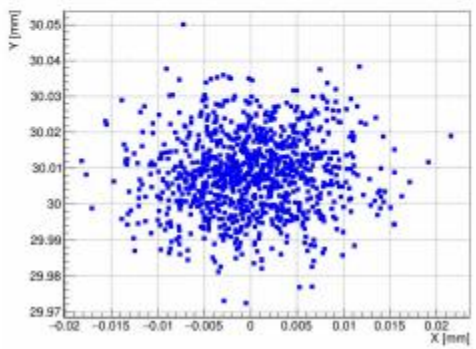
$$\begin{cases} t' = \gamma\left(t - \frac{v}{c^2}x\right), \\ x' = \gamma(x - vt), \\ y' = y, \\ z' = z, \end{cases}$$

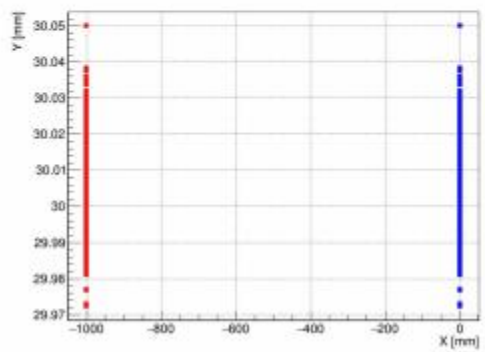
$$\tan 16.5 \text{ mRad} = \gamma v$$

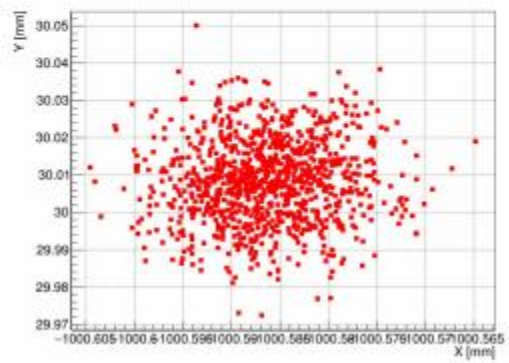
$$t = \frac{z}{v \times \cos\theta}$$

Lab frame (Red points)









3. Summary of the hit positions

Input z deviates 1mm, output y in Lab frame deviates $0.03\ \text{mm}$

```
for(int i = 0; i < N; i++)
    {
        static TRandom3 rndm;
        xi[i] = 0. + rndm.Gaus(0, 6./1000. * gamma);
        yi[i] = 0.;
        zi[i] = 0. + rndm.Gaus(0, 0.36);
        t[i] = (hitposi - zi[i])/TMath::Cos(theta);

        xi2[i] = xi[i];
        yi2[i] = yi[i];
        zi2[i] = 1. + zi[i];
        t2[i] = (hitposi - zi2[i])/TMath::Cos(theta);

        xo[i] = xi[i] + TMath::Sin(theta)*TMath::Cos(phi)*t[i];
        yo[i] = yi[i] + TMath::Sin(theta)*TMath::Sin(phi)*t[i];
        zo[i] = hitposi;

        xo2[i] = xi2[i] + TMath::Sin(theta)*TMath::Cos(phi)*t2[i];
        yo2[i] = yi2[i] + TMath::Sin(theta)*TMath::Sin(phi)*t2[i];
        zo2[i] = hitposi;

        xb[i] = gamma * xo[i] - gamma * v * (zo[i] / (v * TMath::Cos(theta)));
        yb[i] = yo[i];
        zb[i] = zo[i];

        xb2[i] = gamma * xo2[i] - gamma * v * (zo2[i] / (v * TMath::Cos(theta)));
        yb2[i] = yo2[i];
        zb2[i] = zo2[i];

        dy[i] = yb2[i] - yb[i];
        dx[i] = xb2[i] - xb[i];

    }
```

4. Next steps: BHlumi studies of QED scattering and Geant4 studies of multiple scattering and beam pipe $\frac{\mathrm{d}E}{\mathrm{d}x}$