Nuclear Science Computing Center at CCNU

Exploring the Microscopic origins of QCD phase transition

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based on arXiv:2305.10916 & 2312.08860

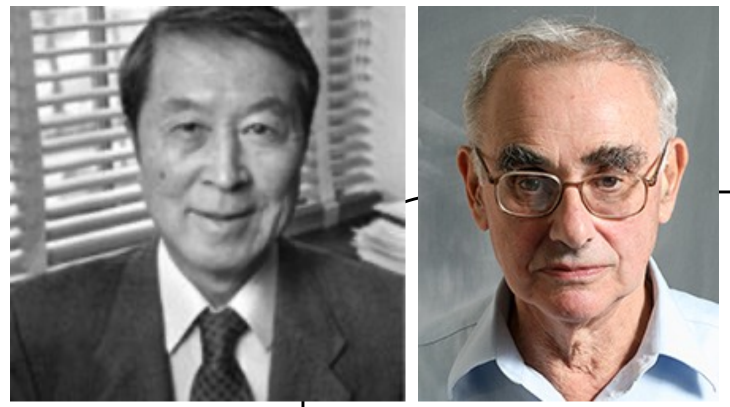
高能理论论坛@ IHEP, CAS
Mar. 25, 2024

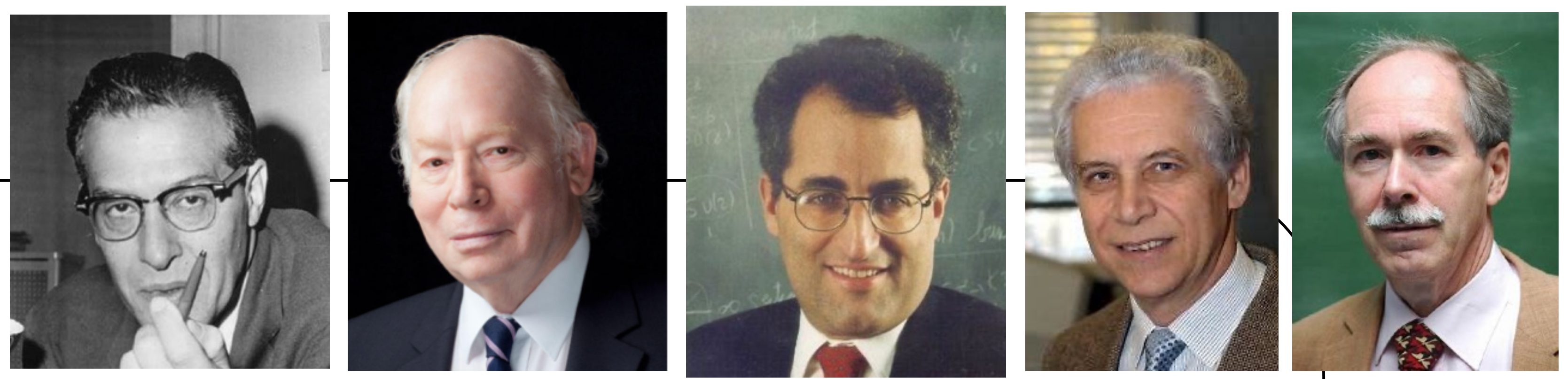
Symmetries of QCD in vacuum

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q}\left[i\gamma^{\mu}(\partial_{\mu} - igA_{\mu}) - m_q\right]q$$

Classical QCD symmetry (m_q=0)

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$$





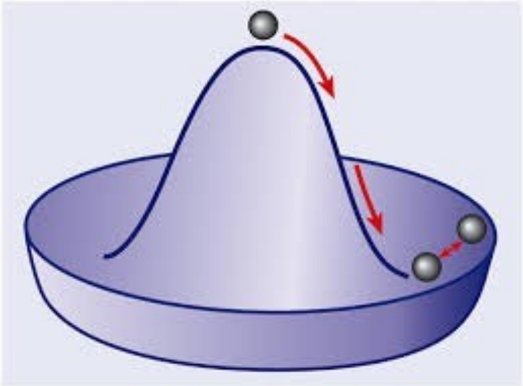
Quantum QCD vacuum (m_q=0)

Chiral condensate:
spontaneous mass generation

Axial anomaly:
quantum violation of $U(1)_A$

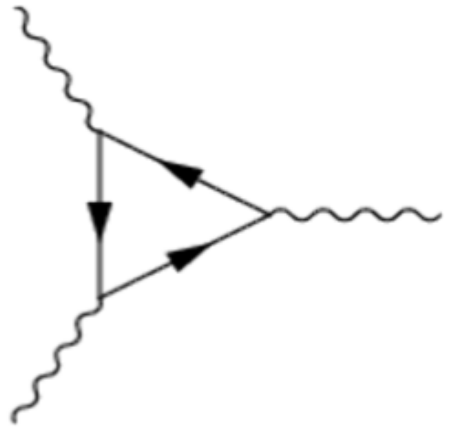
ABJ...

U(1) problem

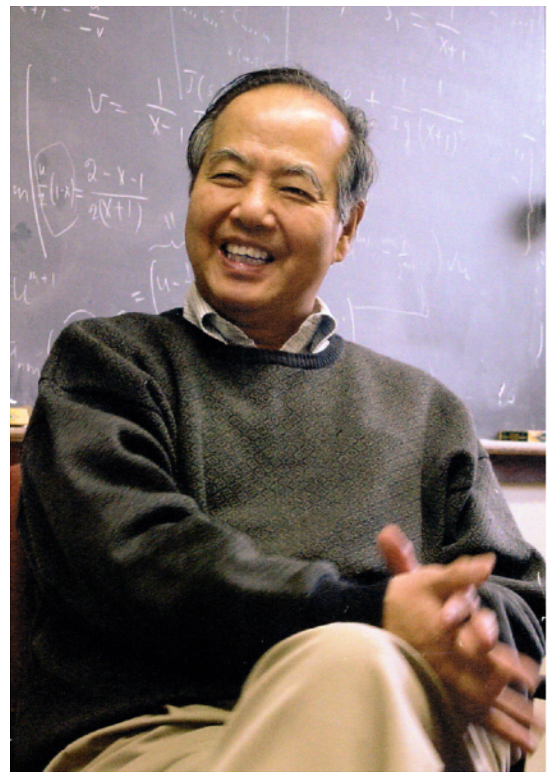


$$\langle \bar{q}_R q_L \rangle \neq 0$$

$$\partial_{\mu} j_5^{\mu} = \frac{g^2 N_f}{16\pi^2} tr(\tilde{F}_{\mu\nu}F^{\mu\nu})$$



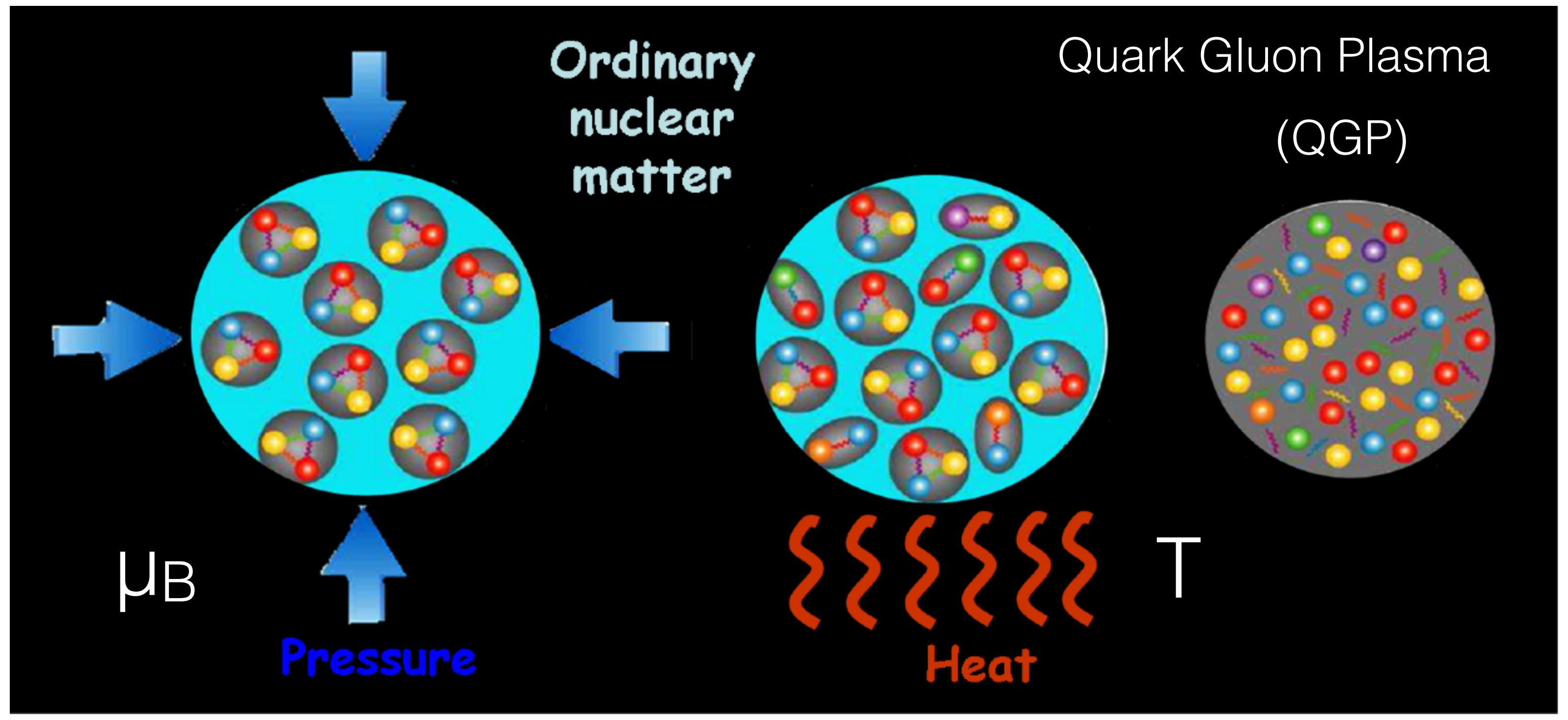
$$SU(N_f)_V \times U(1)_V$$



Missing symmetries & Vacuum excitation

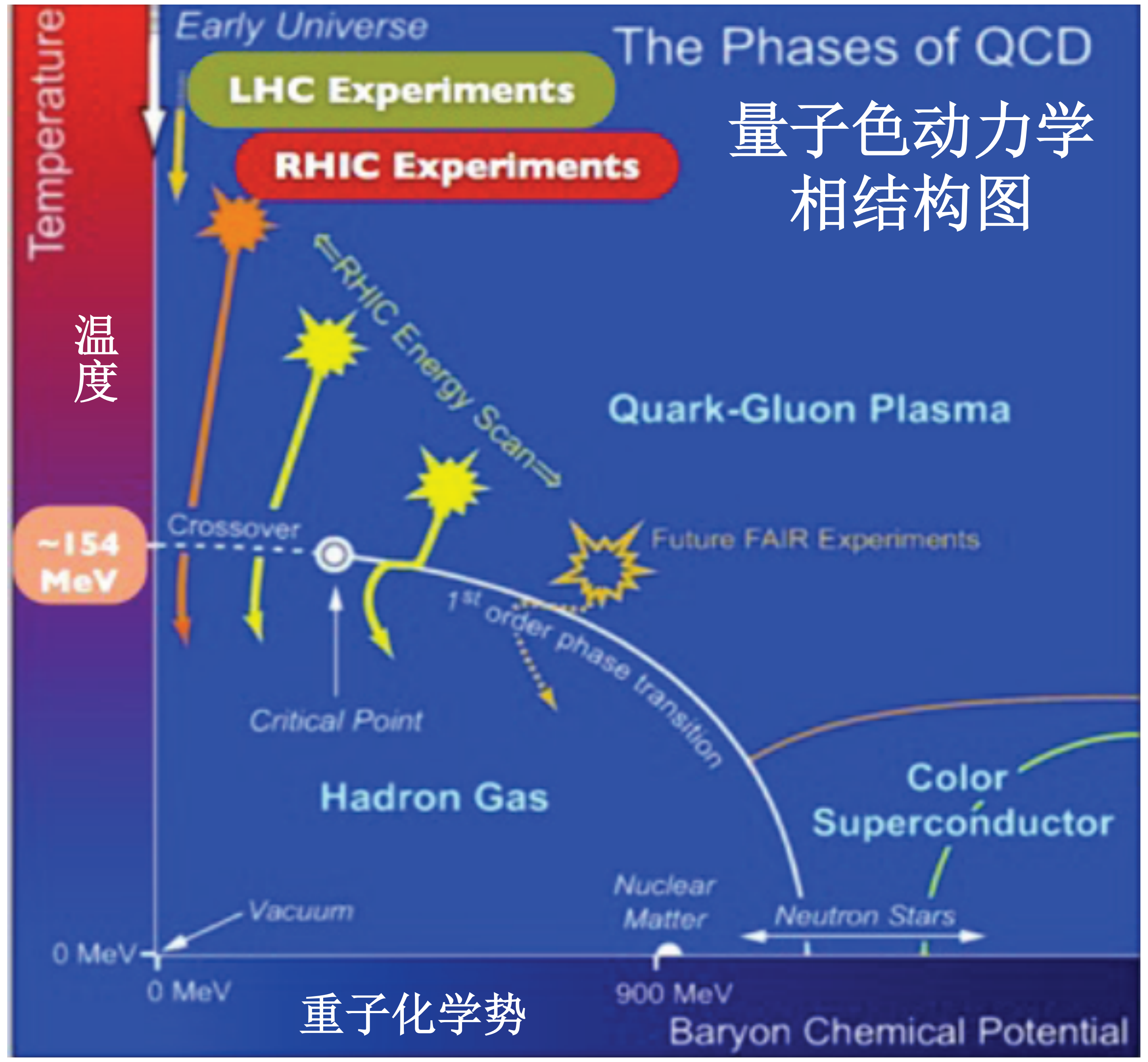
“核子重如牛，对撞生新态。”

Ink painting masterpiece 1986:
"Nuclei as Heavy as Bulls, Through Collision Generate New States of Matter”,
by Li Keran,
reproduced from open source works of
T. D. Lee.



“The whole is more than sum of its parts.”
Aristotle, Metaphysica 10f-1045a

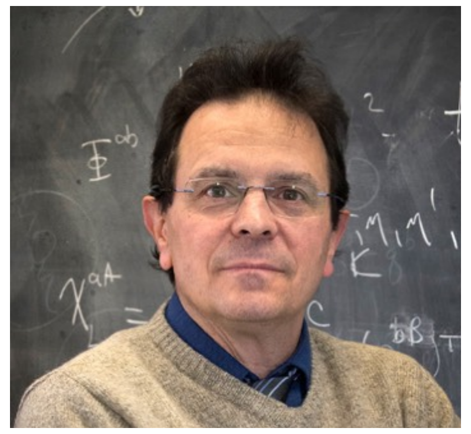
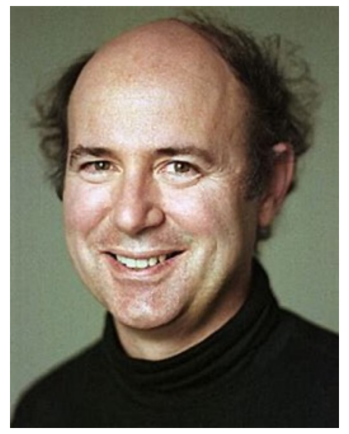
从还原论到整体论



How do symmetries manifest themselves in QCD phase structure?

$$\partial_\mu j_5^\mu = \frac{g^2 N_f}{16\pi^2} tr(\tilde{F}_{\mu\nu} F^{\mu\nu})$$

Landau functional of QCD

Pisarski & Wilczek, PRD 84'

Symmetry: $SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$

Chiral field: $\Phi_{ij} \sim \frac{1}{2}\bar{q}^j(1-\gamma_5)q^i = \bar{q}_R^j q_L^i$

Chiral transformation: $\Phi \to e^{-2i\alpha_A} V_L \, \Phi \, V_R^\dagger$

$$\mathcal{L}_{eff} = \frac{1}{2}\mathrm{tr}\,\partial\Phi^\dagger\partial\Phi + \frac{a}{2}\mathrm{tr}\,\Phi^\dagger\Phi + \frac{b_1}{4!}\left(\mathrm{tr}\,\Phi^\dagger\Phi\right)^2 + \frac{b_2}{4!}\mathrm{tr}\,\left(\Phi^\dagger\Phi\right)^2$$

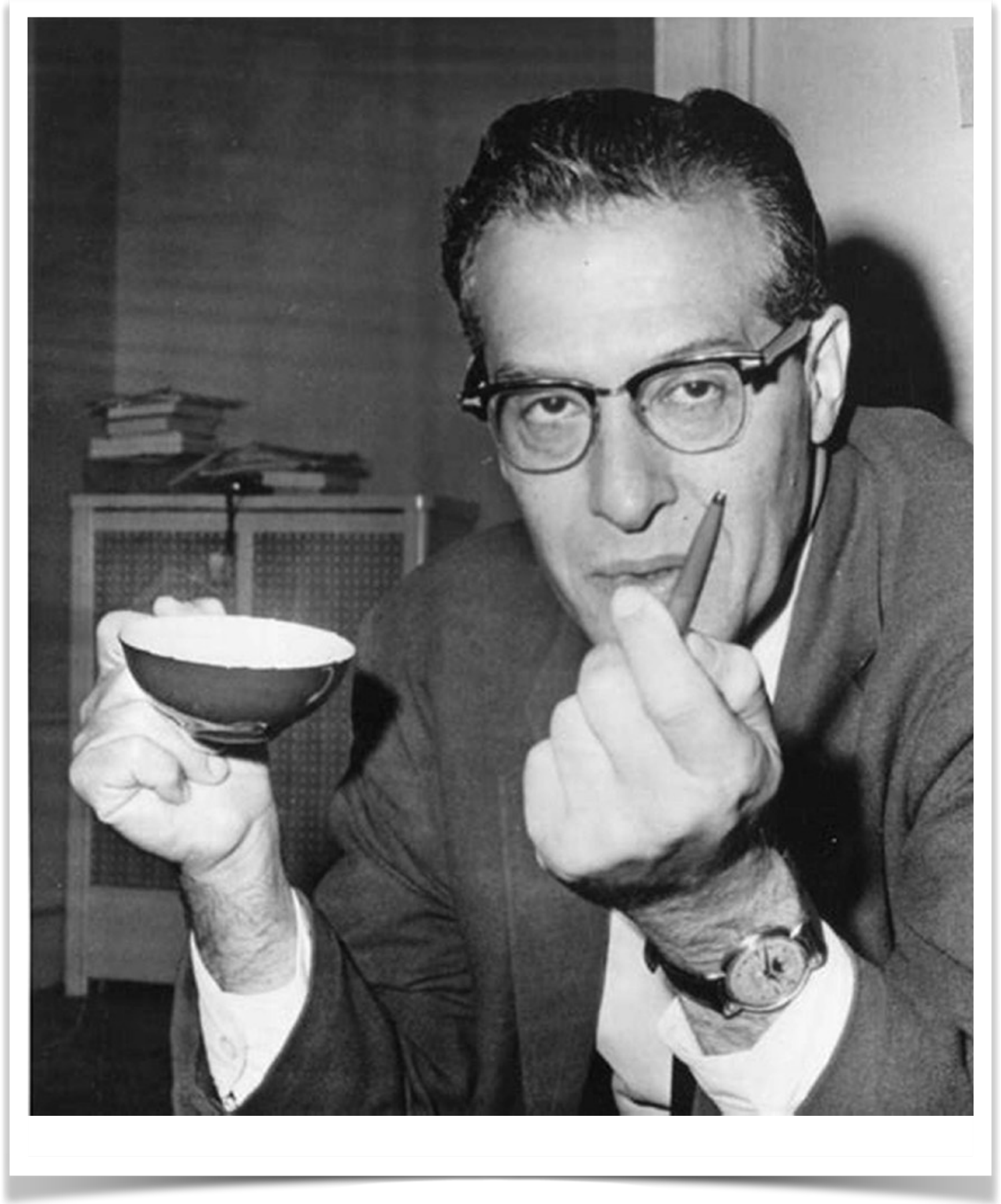
→ $SU(N_f)_L \times SU(N_f)_R \times U(1)_A$

$$- \frac{c}{2}\left(\det\Phi + \det\Phi^\dagger\right)$$

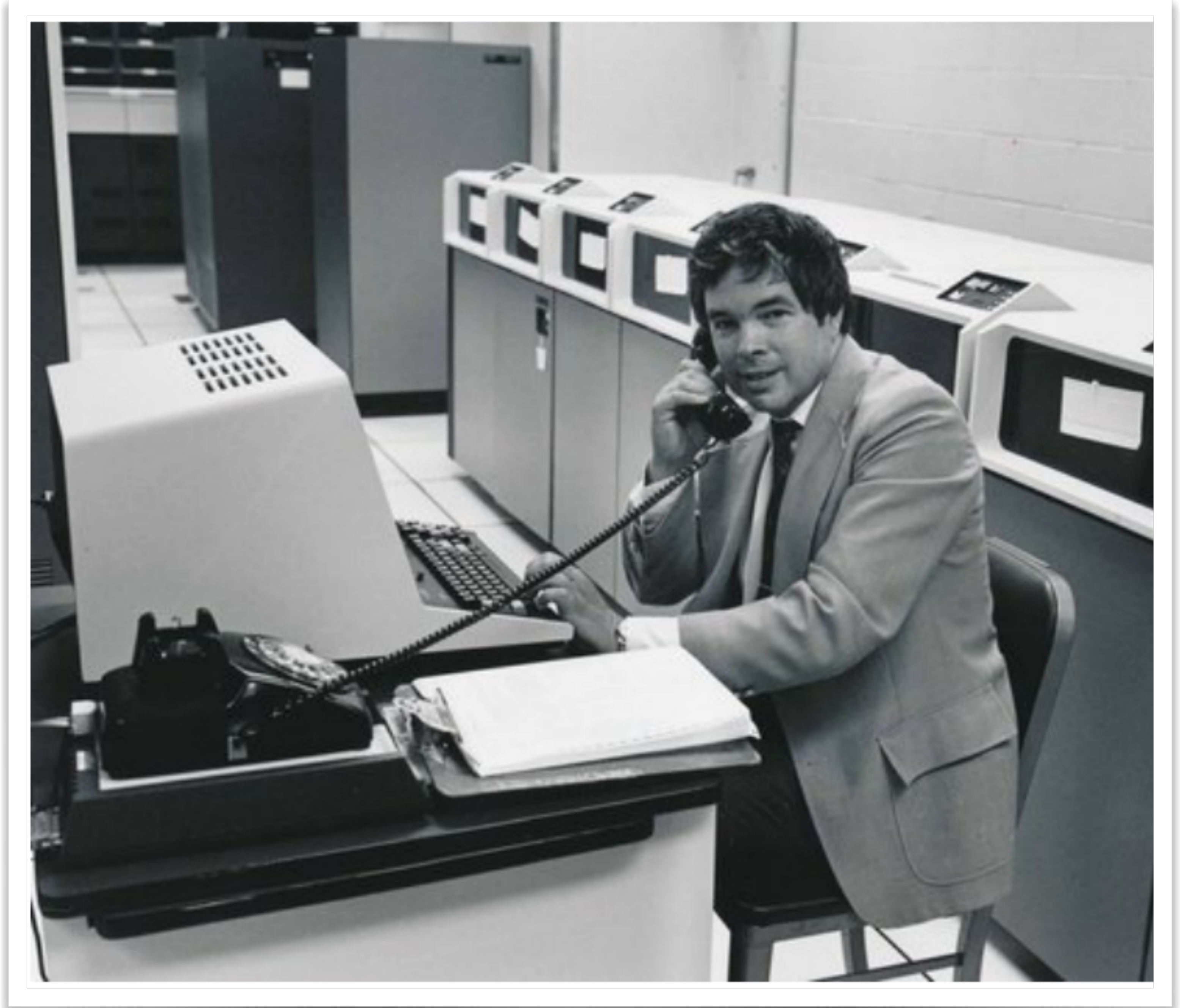
→ $SU(N_f)_L \times SU(N_f)_R$

$$- \frac{d}{2}\mathrm{tr}\,h\left(\Phi + \Phi^\dagger\right).$$

→ Quark mass term

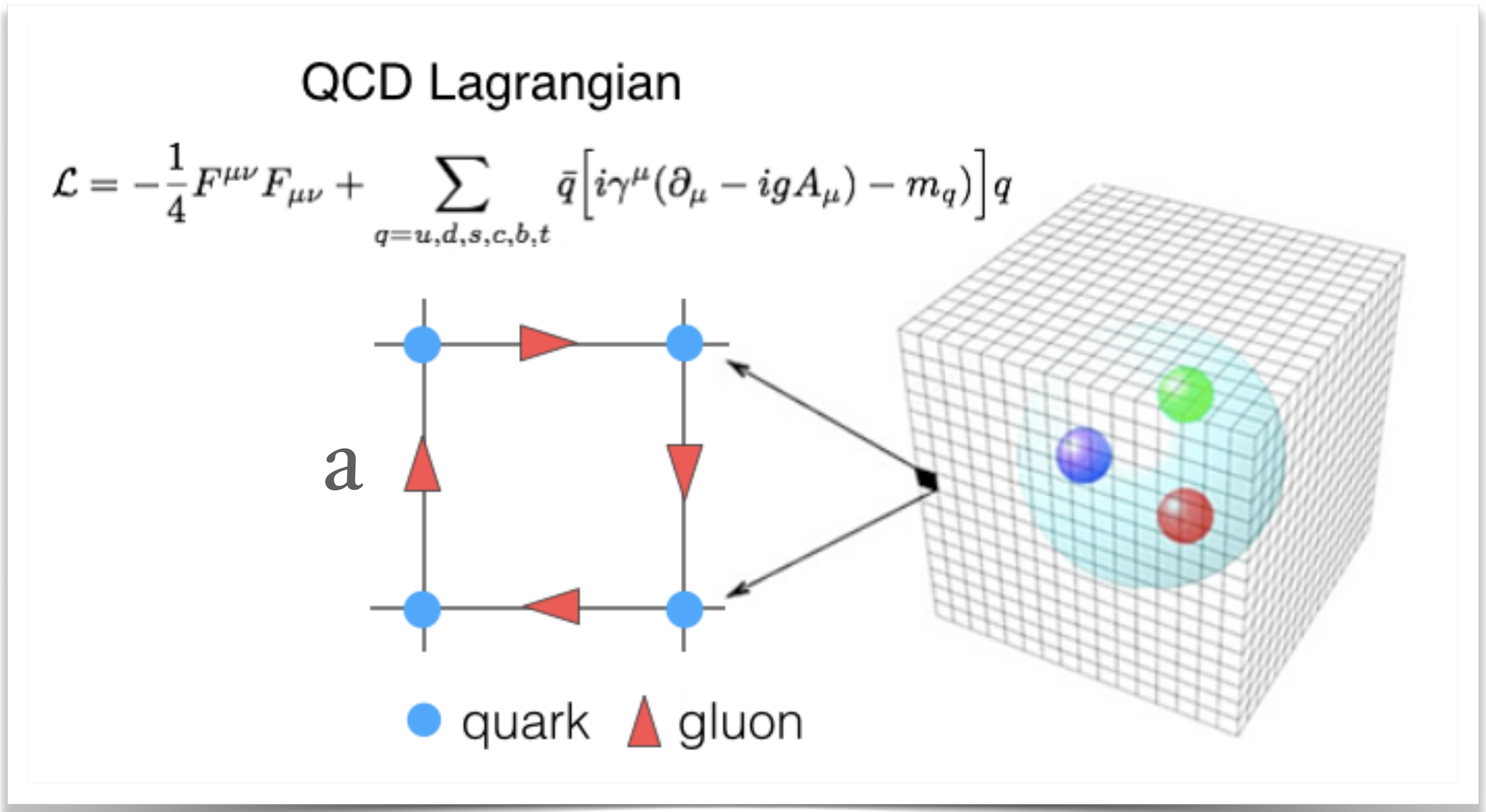


Julian Schwinger:
"physicist who only needs pencil and paper to do physics"
(and coffee)



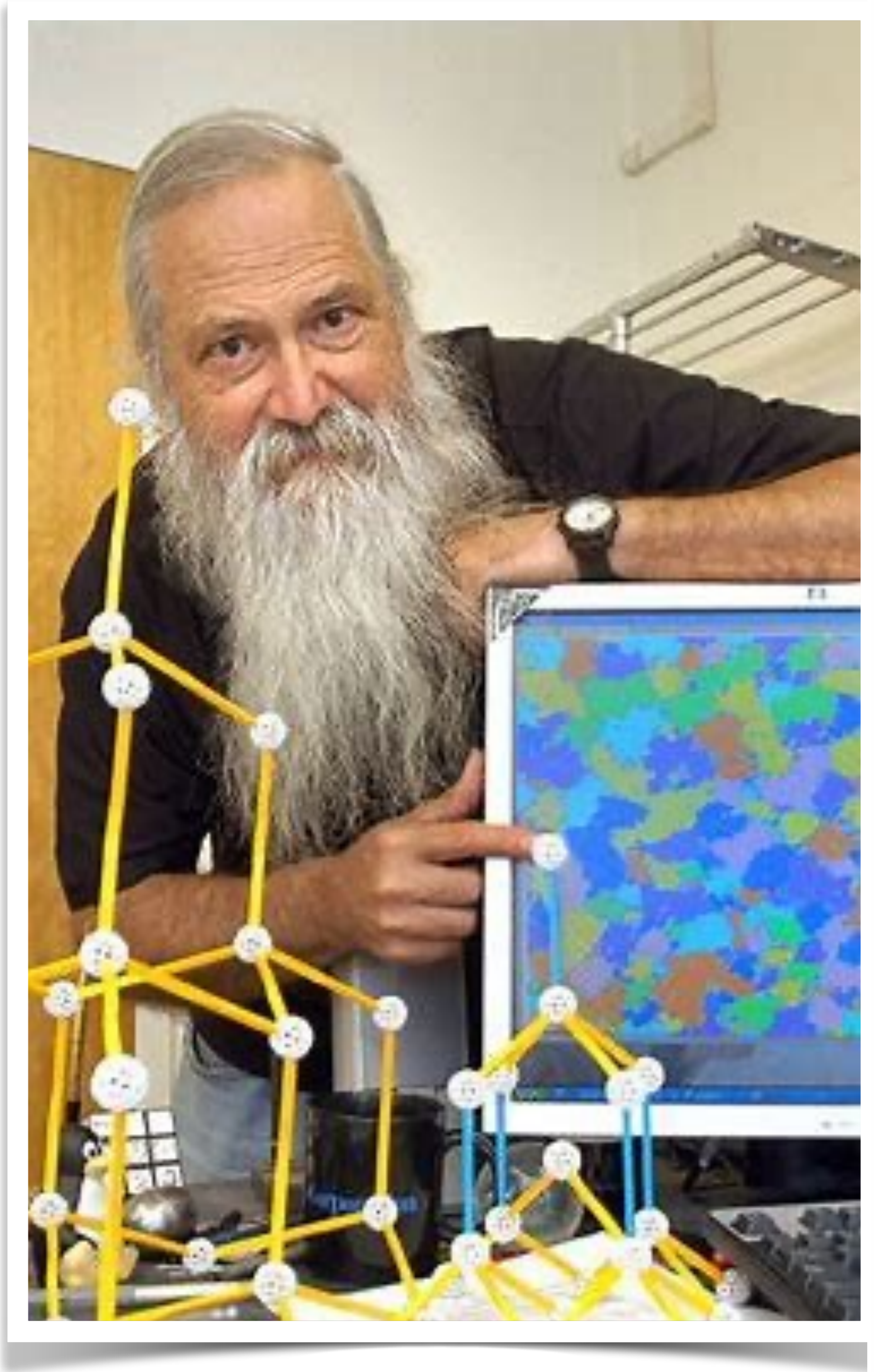
Kenneth G. Wilson

Lattice field theory

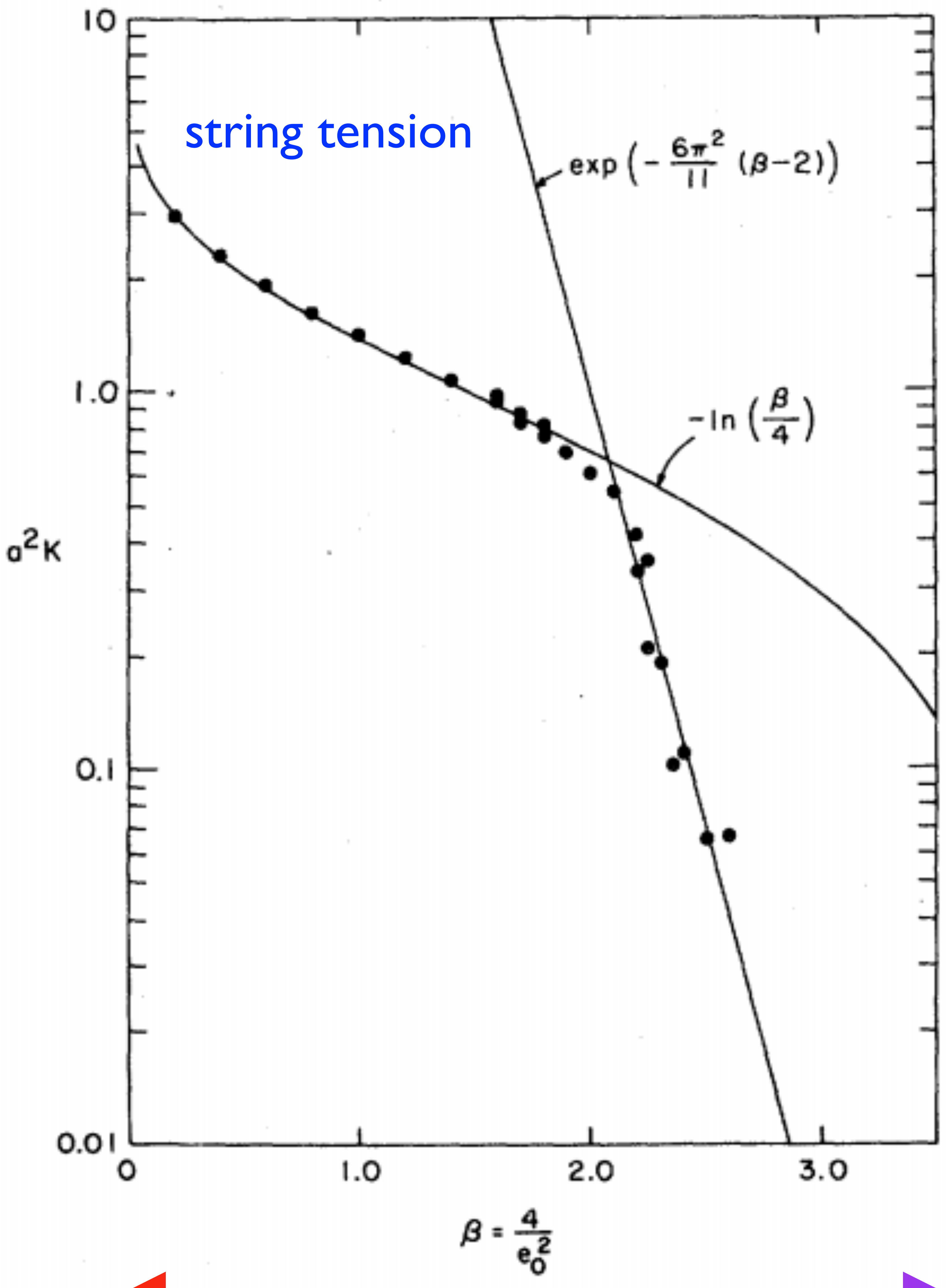


a→0 recovers QCD

Nonzero T: First numerical lattice simulations

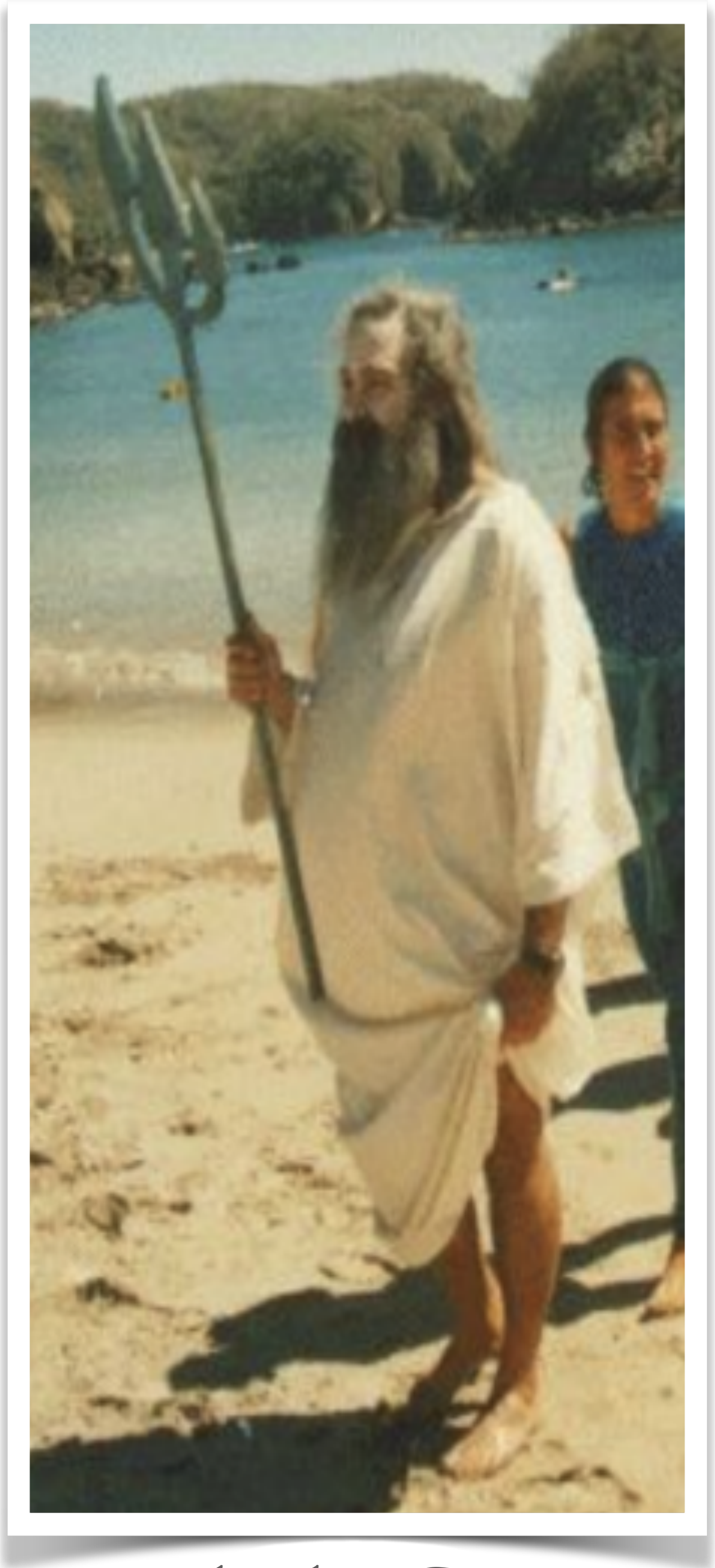


Michale Creutz
@BNL



Low T   High T

https://journals.aps.org/collections/50-years-QCD
PRD 1980, cited by 1181 records

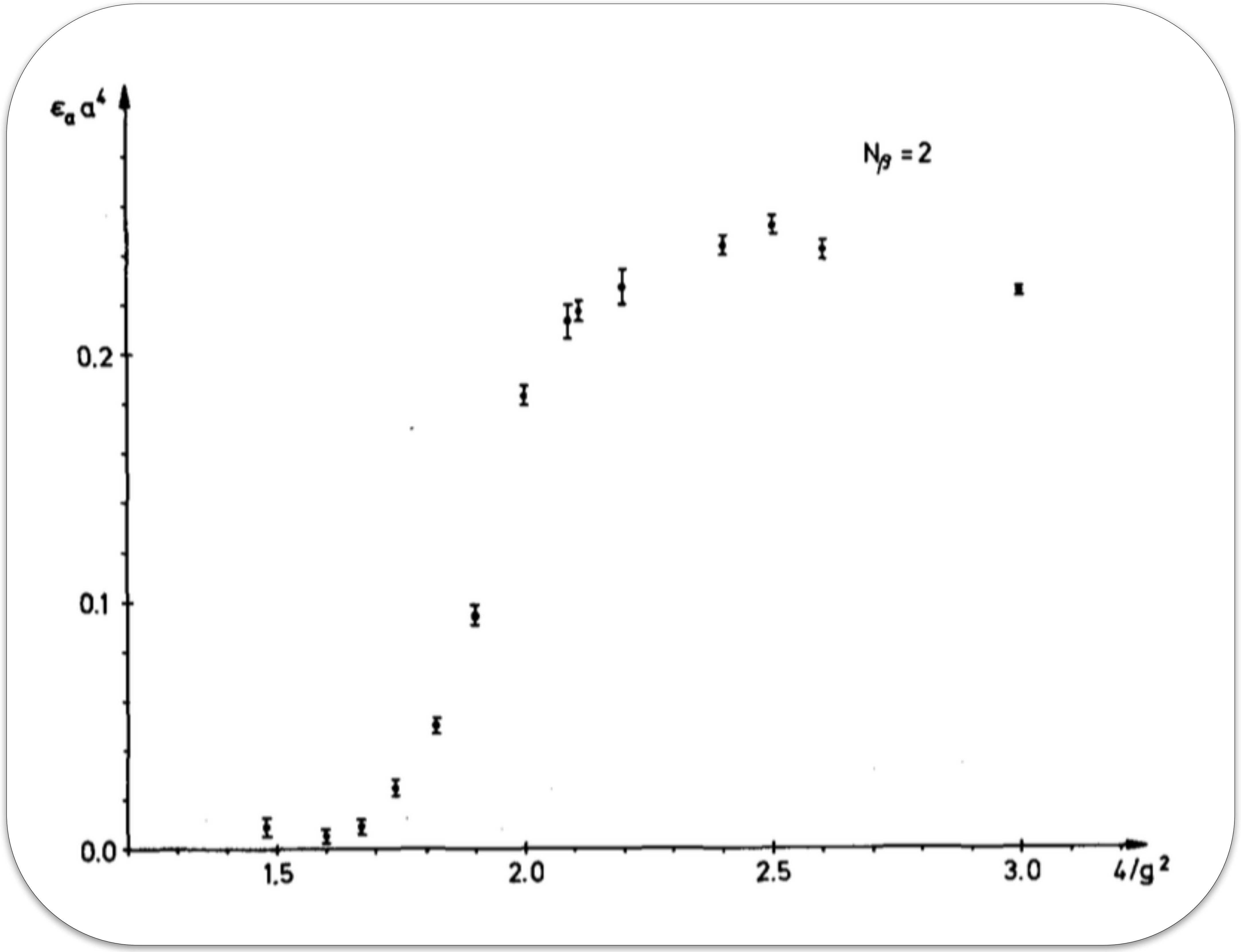


Michale Creutz
on the beach

Spawned golden age in lattice QCD

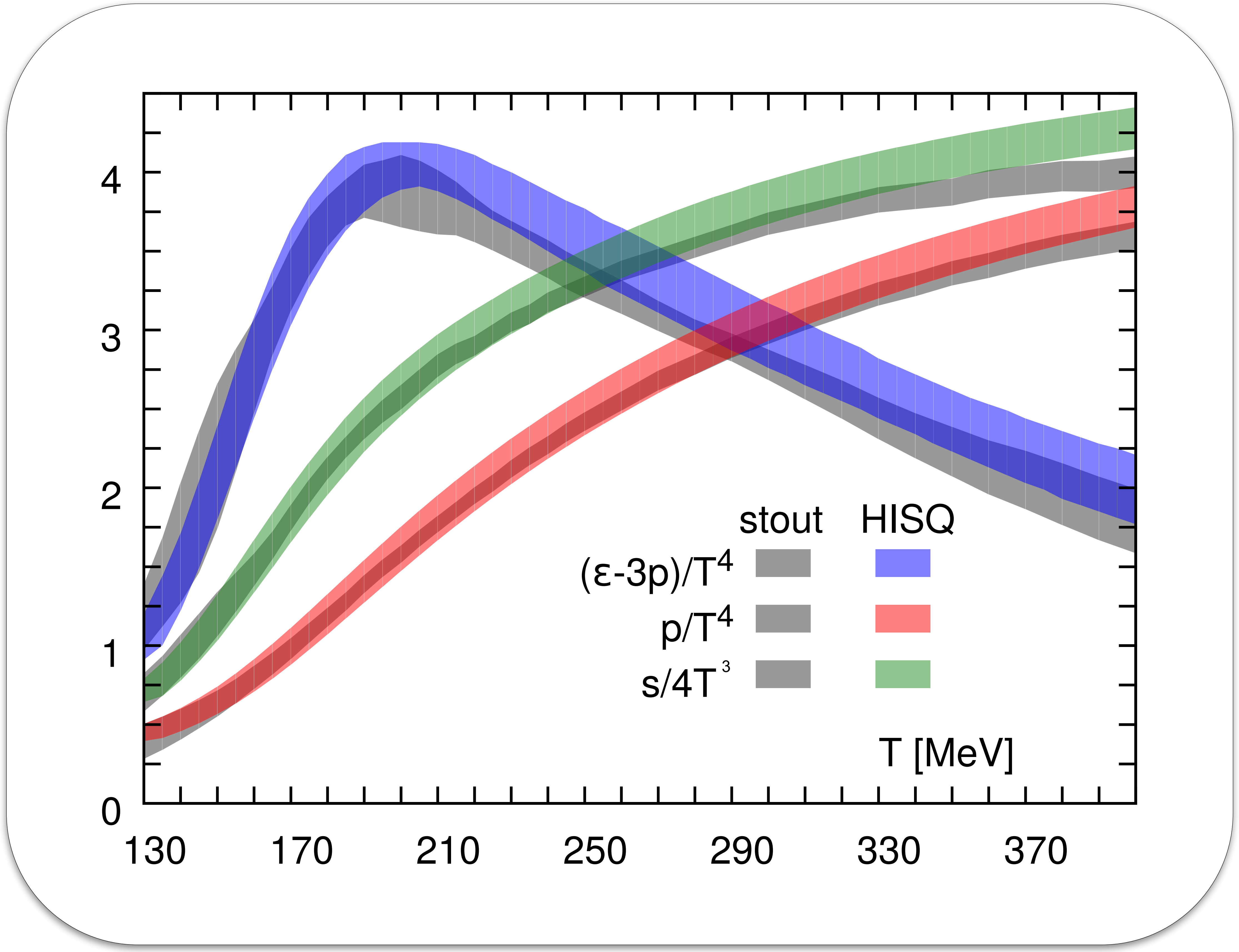
Lattice QCD calculation of EoS at μ_B =0

SU(2) pure gauge; Quenched QCD
at a finite lattice cutoff of Nt=2



J. Engels, F. Karsch, H. Satz, I. Montvay [Bielefeld]
Phys. Lett. B 101 (1981) 89-94

N_f=2+1, physical point
continuum extrapolated



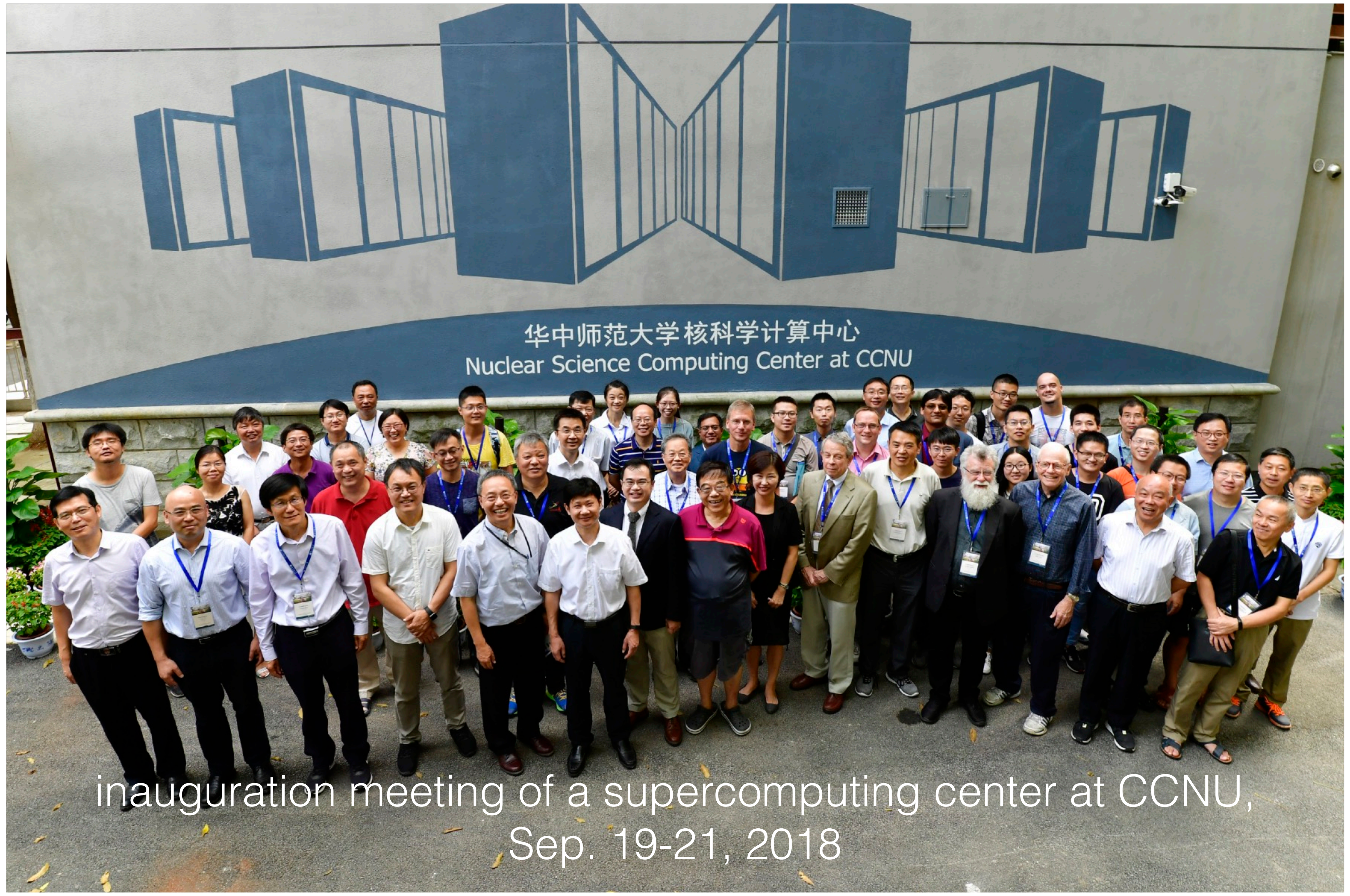
HotQCD, PRD 90 (2014) 094503

华中师范大学核科学计算中心
Nuclear Science Computing Center at CCNU

N: Nuclear **S**: Science **C³**: Color 3 -> QCD

“道生一，一生二，二生三，三生万物。” — 《道德经》老子 600 BC

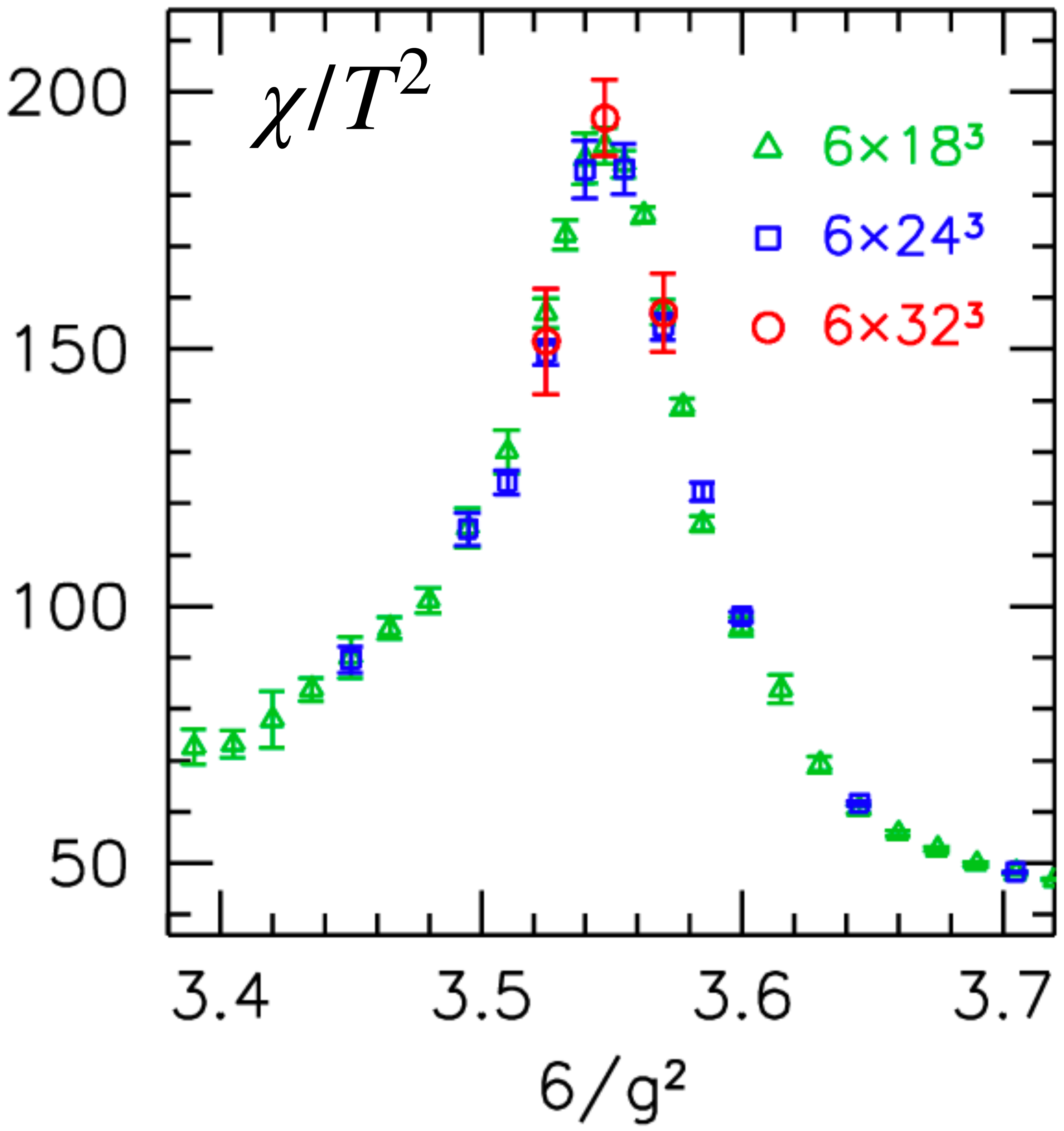
Founded in 2018， 4.7PFlops/s (304 V100 + 216 A800 GPU)， 16 PB



inauguration meeting of a supercomputing center at CCNU,
Sep. 19-21, 2018

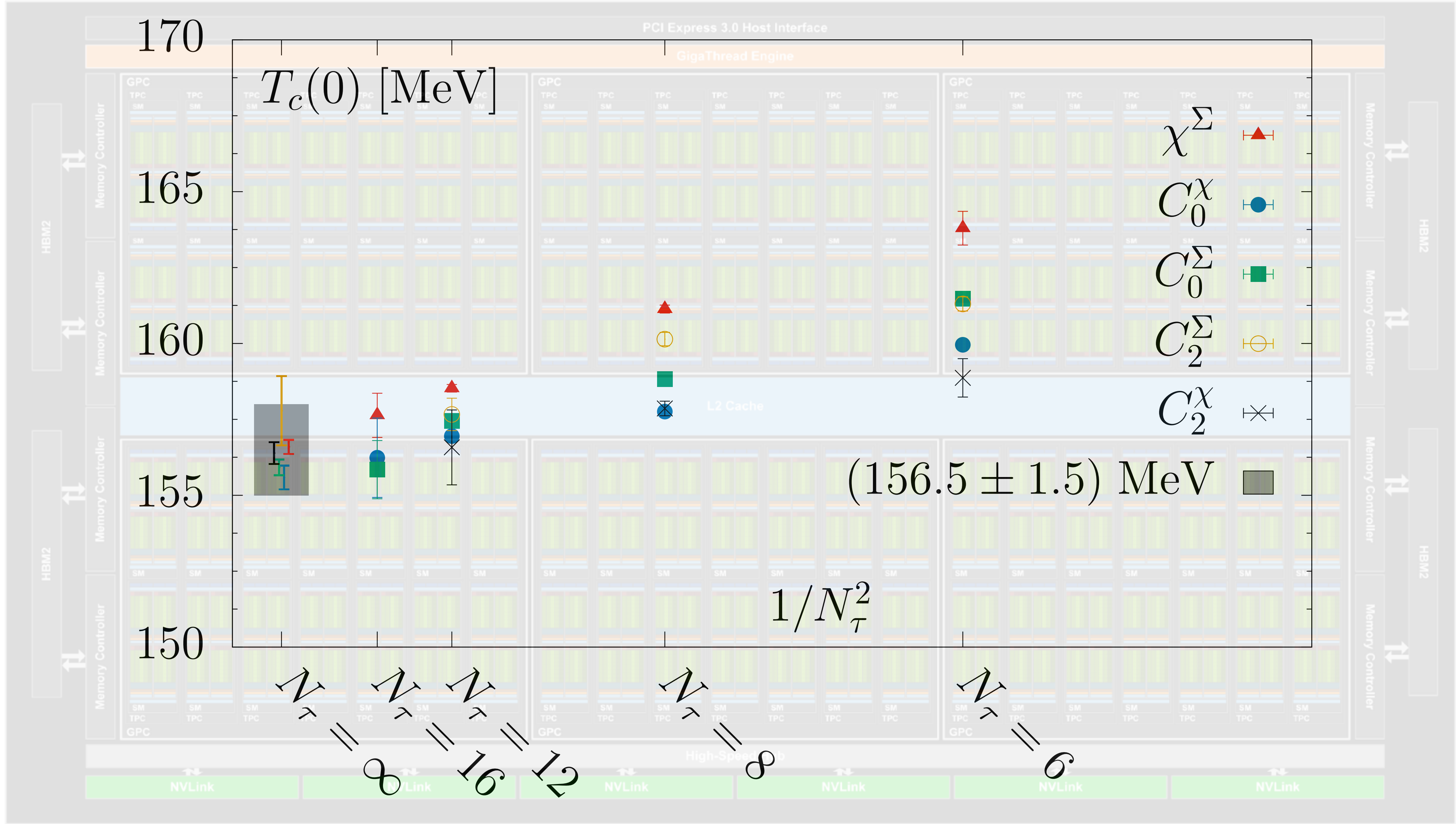
QCD transition at μB=0 in Nature

Not a true phase transition but a rapid crossover



Y. Aoki et al., Nature 443 (2006) 675-678

$T_{pc}(0)$=156.5(1.5)MeV~1.8x10^12 K



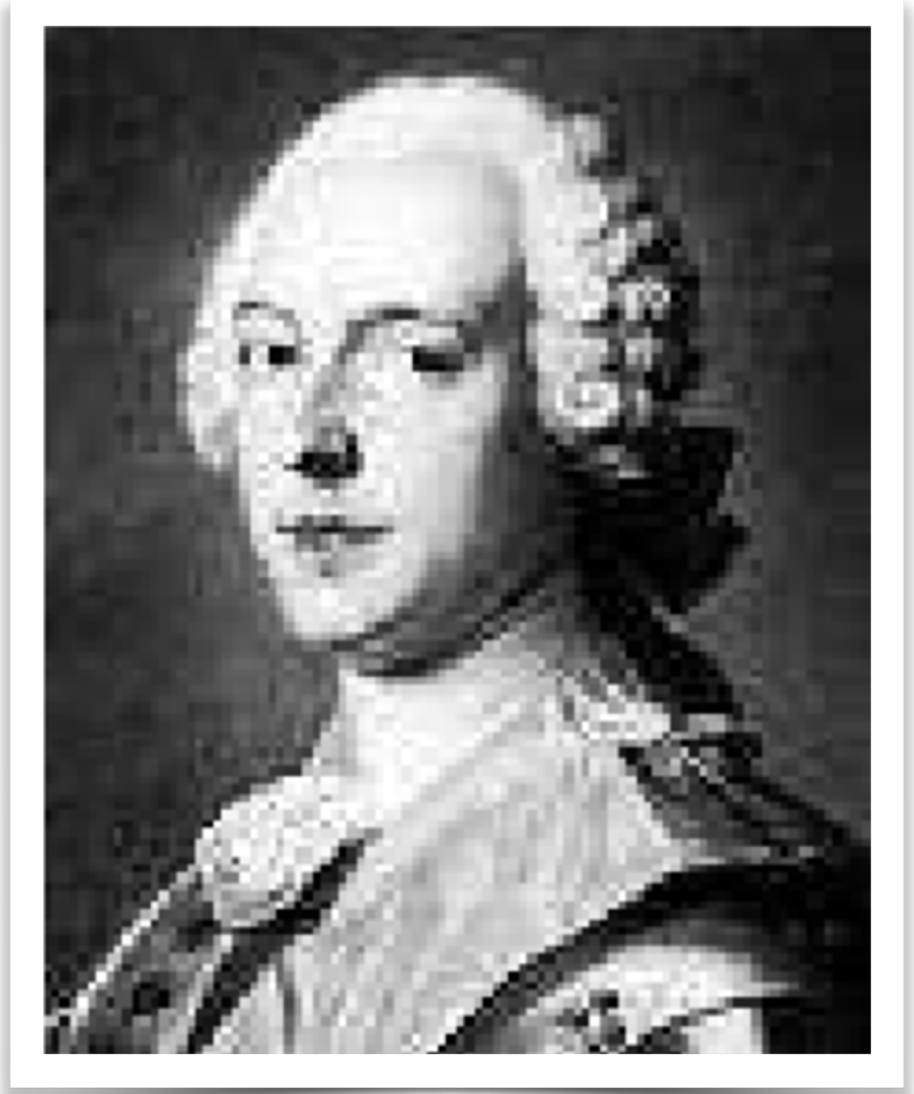
0 ← a

A. Bazavov, HTD, P. Hegde et al. [HotQCD], Phys. Lett. B795 (2019) 15

Made in NSC³

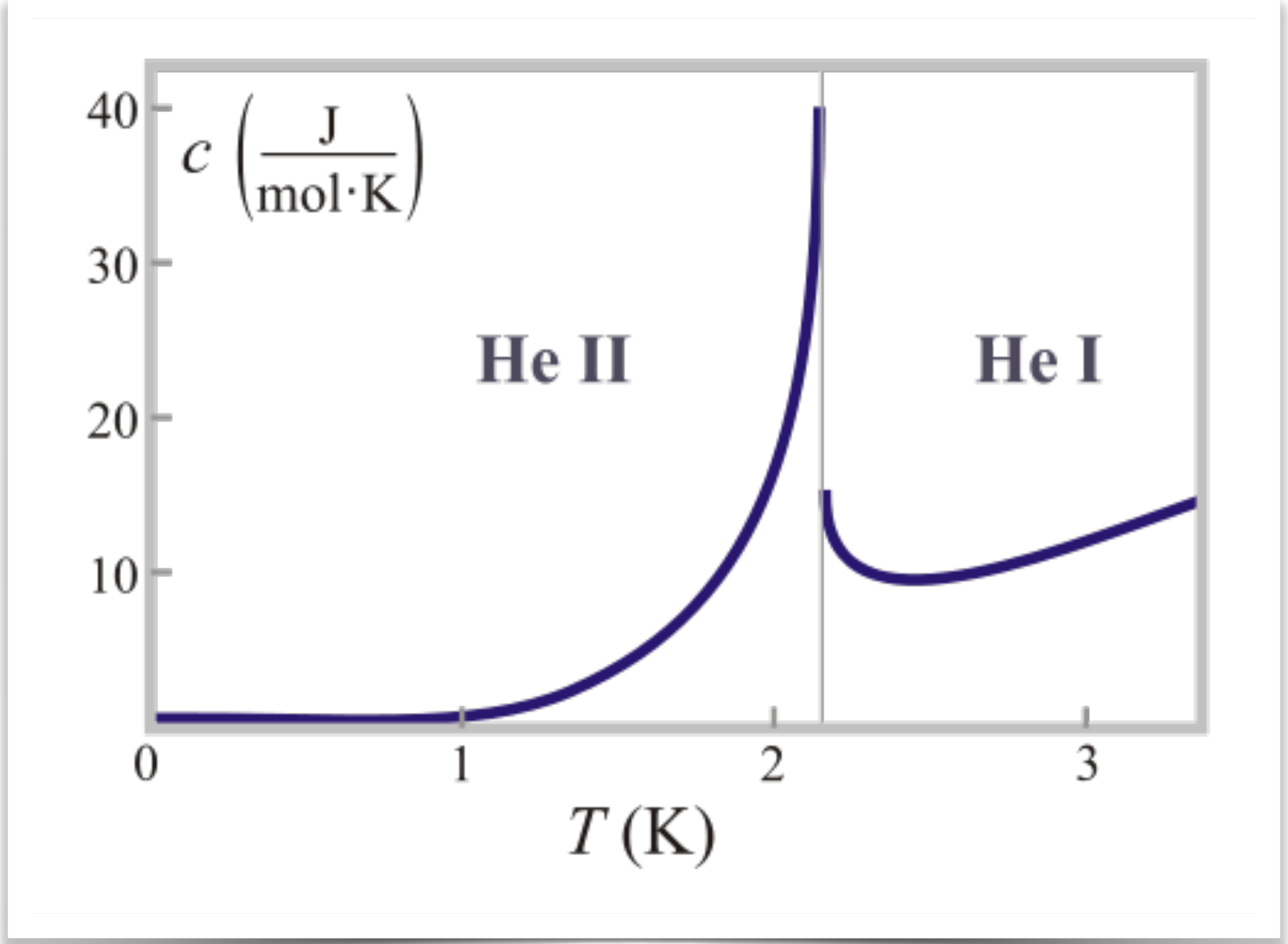
Critical phenomena and universality class

1822: discovered the critical point
of a substance
in his gun barrel experiments

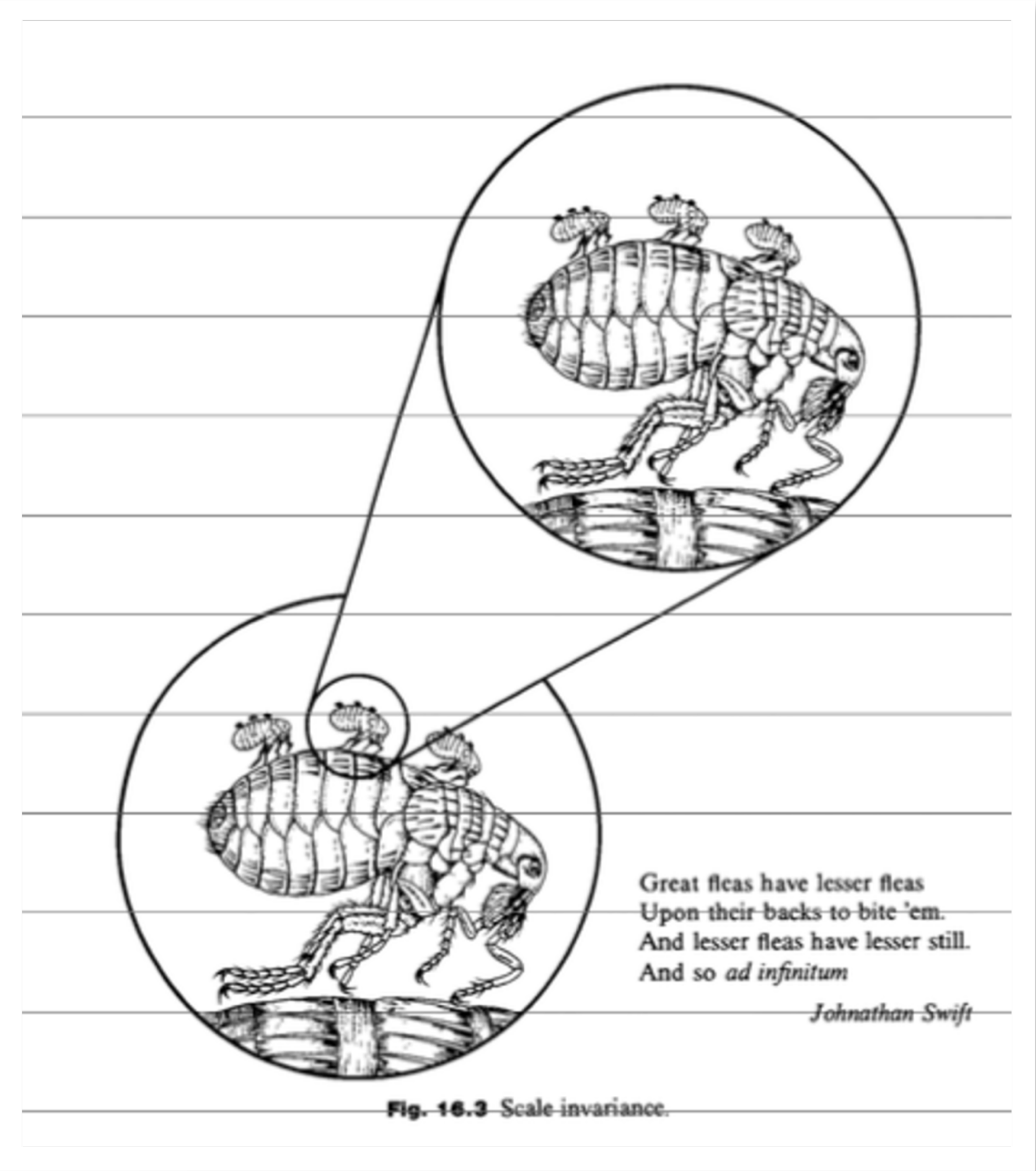


Charles Cagniard de la Tour
1777-1859

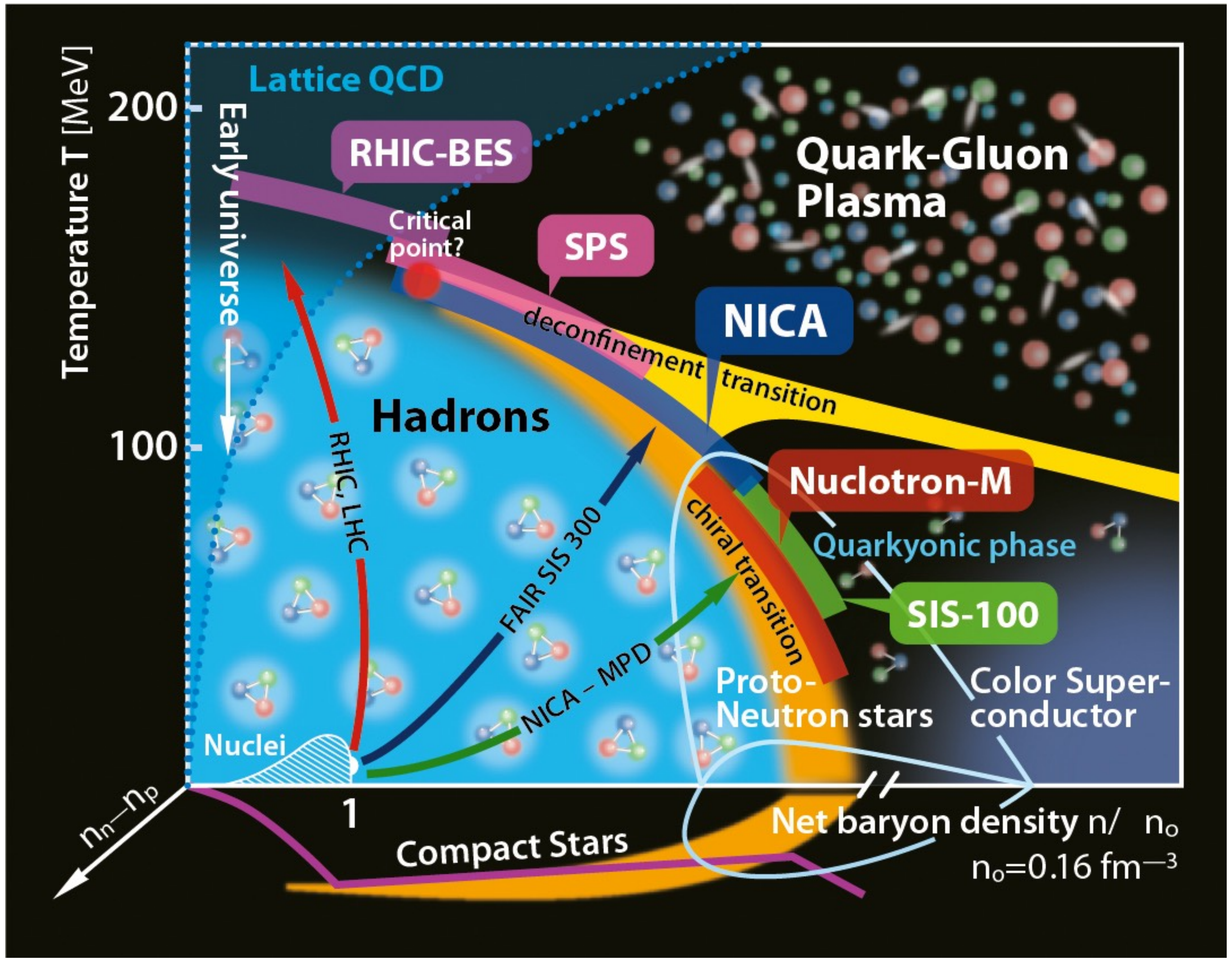
Superfluid transition: λ point
O(2) universality class



https://en.wikipedia.org/wiki/
Lambda_point

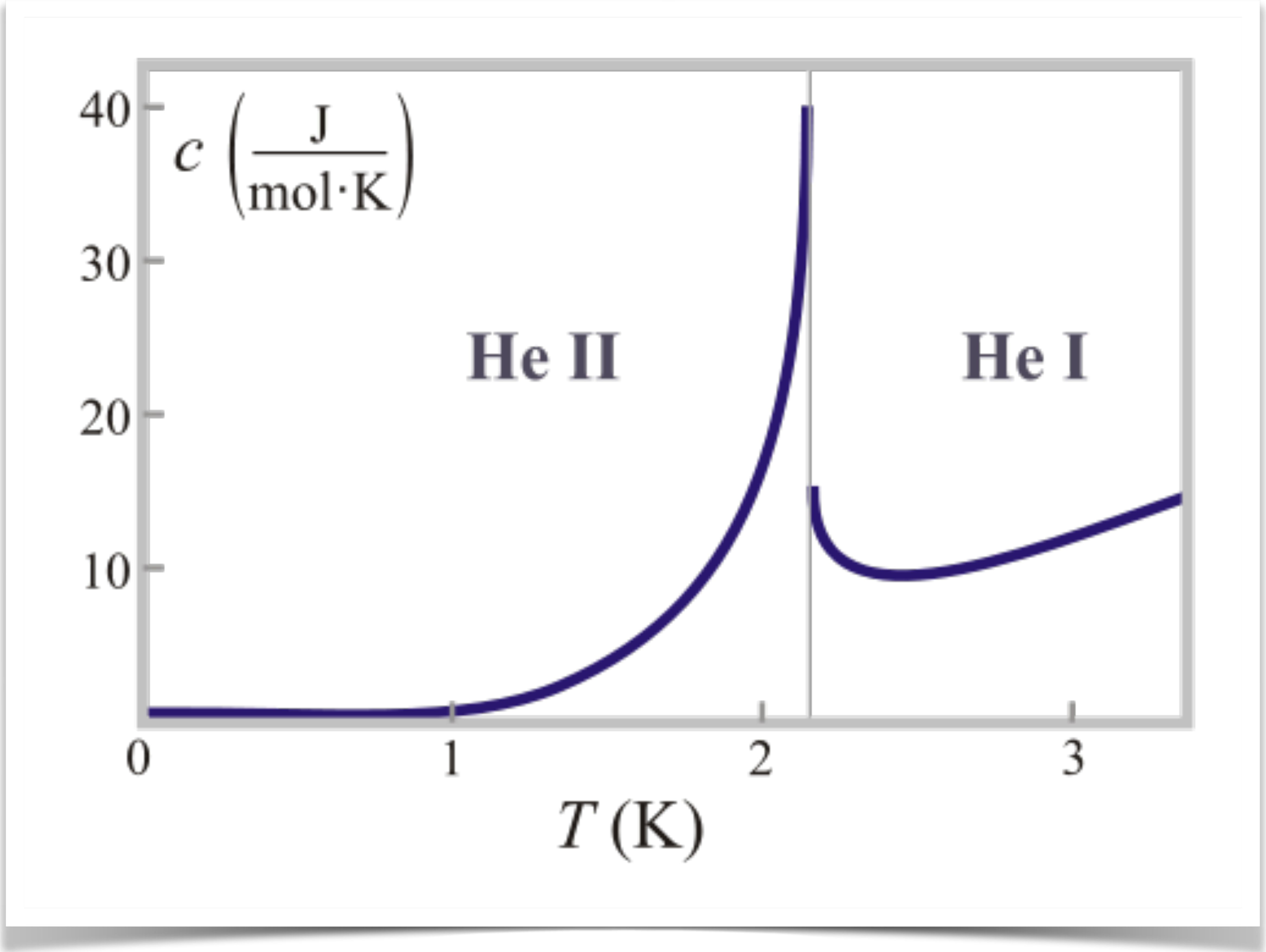


Kerson Huang,
Statistical mechanics



Even if those signatures of criticality are observed an ultimate scientific question will still remain unanswered

How do these universal behaviors at the macroscale arise from the microscopic degrees of freedom, quarks and gluons?



Can we understand the superfluid transition of

Helium4

based on photons and electrons ?

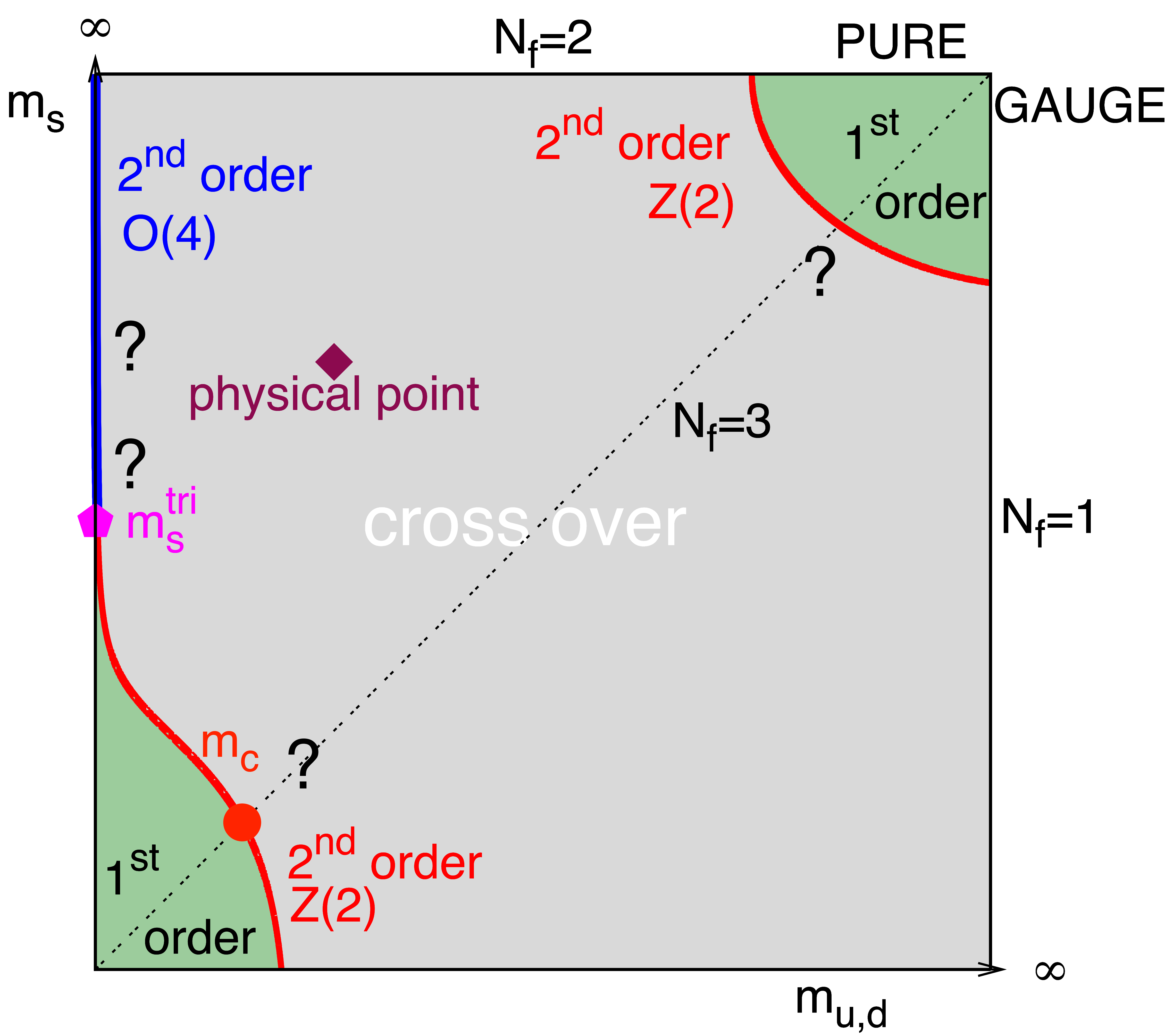
Effective theory: roton, phonon

Can we understand the QCD transition based on quarks and gluons?

Lattice QCD:
first principle, starting from the basic d.o.f. of QCD

QCD criticality at μ_B=0

Columbia plot:
QCD phase diagram in quark mass plane



RG arguments: Pisarski & Wilczek, PRD29 (1984) 338

- m_q=0 or ∞ with N_f=3: a first order phase transition
- Critical lines of 2nd order transition
  N_f=2: O(4) universality class
  N_f=3: Z(2) universality class

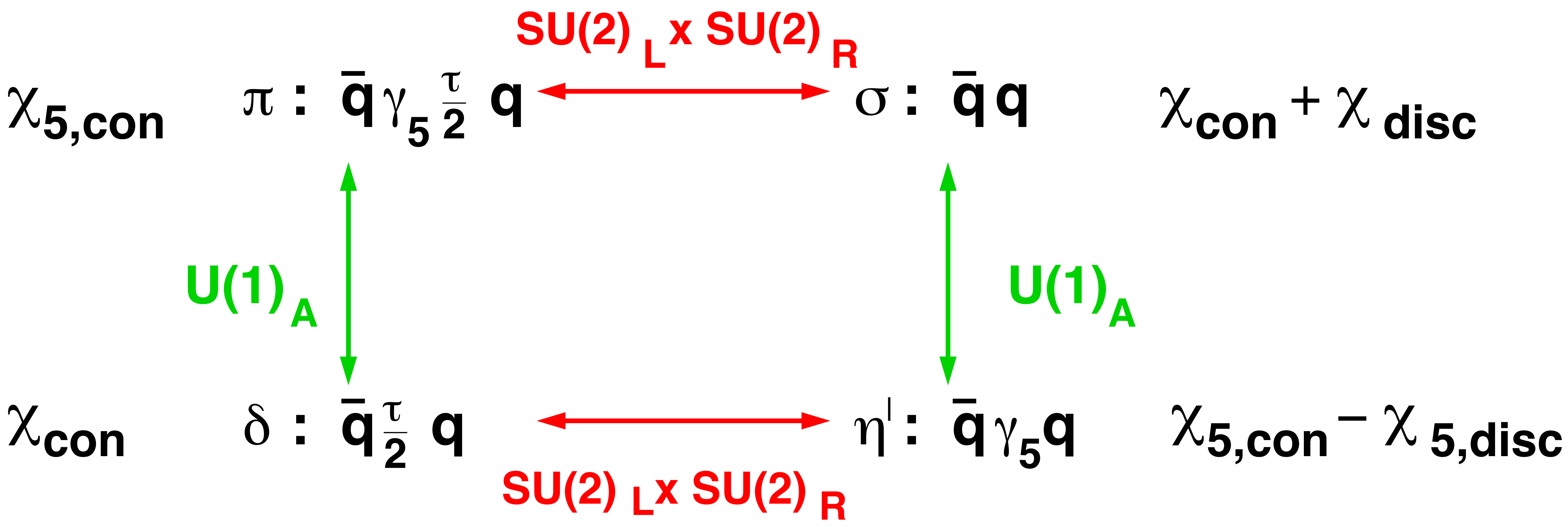
K. Rajagopal & F. Wilczek, NPB 399 (1993) 395
Gavin, Gocksch & Pisarski, PRD 49 (1994) 3079
F. Wilczek IJMPA 7(1992) 3911

- Axial U(1) anomaly in Nf=2 QCD
  If manifested at T_c: 2nd order O(4)
  If not: 1st order or 2nd order $(U(2)_L \otimes U(2)_R/U(2)_V)$

Butti, Pelissetto and Vicari, JHEP 08 (2003) 029
Pelissetto & Vicari, PRD 88 (2013) 105018
Grahl, PRD 90 (2014) 117904

Signatures of symmetry restorations

- Susceptibilities defined as integrated two point correlation functions of the local operators, e.g. $\chi_\pi = \int \mathrm{d}^4x \langle \pi^i(x) \pi^i(0) \rangle$ with $\pi^i(x) = \mathrm{i}\bar{\psi}_l(x)\gamma_5\tau^i\psi_l(x)$



$$\chi_{\rm disc} = \frac{T}{V} \int \mathrm{d}^4x \left\langle \left[\bar{\psi}(x)\psi(x) - \langle \bar{\psi}(x)\psi(x)\rangle\right]^2 \right\rangle$$

Restoration of $SU(2)_L x SU(2)_R$:

$\chi_\pi - \chi_\sigma = 0$

$\chi_\delta - \chi_\eta = 0$

$\Rightarrow \chi_\pi - \chi_\delta = \chi_{disc} = \chi_{5,disc}$

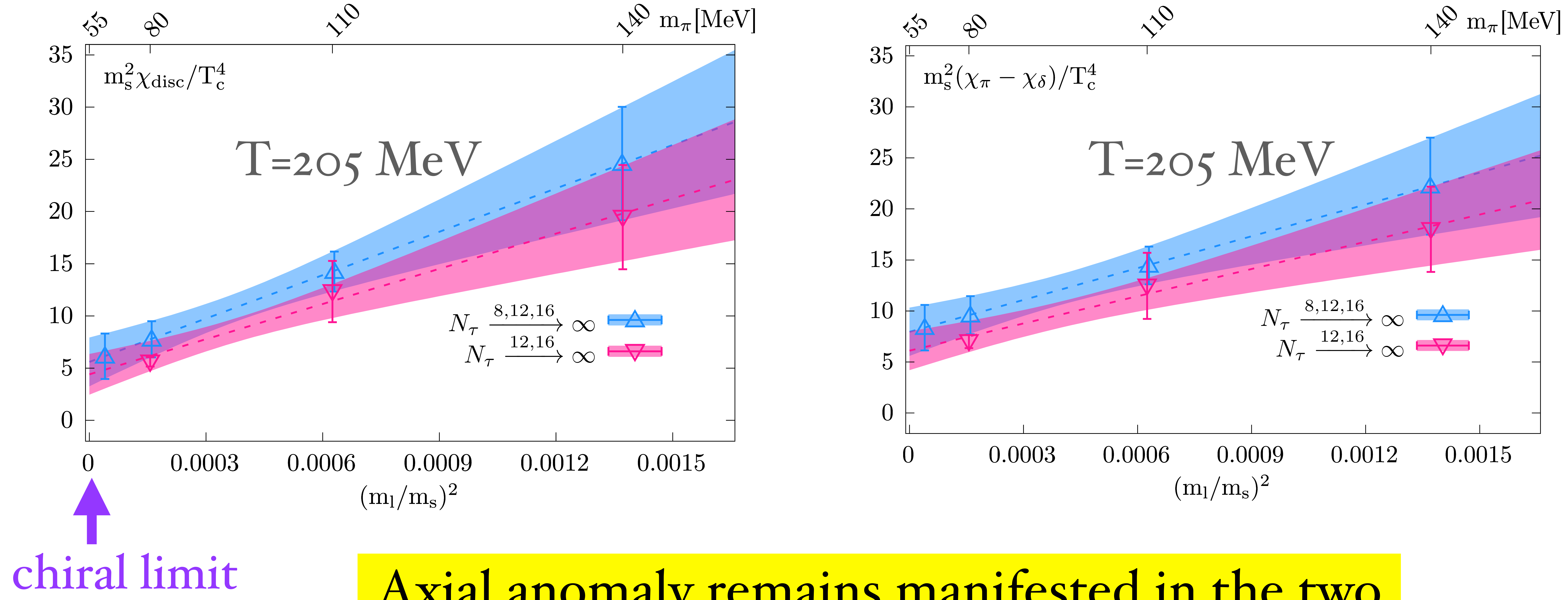
Effective restoration of $U(1)_A$:

$\chi_\pi - \chi_\delta = 0$

$\chi_\sigma - \chi_\eta = 0$

$\Rightarrow \chi_\pi - \chi_\delta = \chi_{disc} = \chi_{5,disc} = 0$

Continuum and chiral extrapolations of two $U_A(1)$ measures



Axial anomaly remains manifested in the two $U(1)_A$ measures at a 2-3 sigma level

Chiral phase transition should be 2nd order belonging to O(4) universality class

HTD, S.-T. Li, A. Tomiya, S. Mukherjee, X.-D. Wang, Y. Zhang, PRL 126 (2021) 082001

Energy levels of quarks: Dirac eigenvalue spectrum ρ

$$\langle\bar{\psi}\psi\rangle = \int_0^\infty \frac{4m_l\,\rho}{\lambda^2+m_l^2}\,\mathrm{d}\lambda\,, \qquad \chi_\pi - \chi_\delta = \int_0^\infty \mathrm{d}\lambda\,\frac{8m_l^2\,\rho}{\left(\lambda^2+m_l^2\right)^2}$$

- Restoration of SU(2)xSU(2) symmetry: $\lim_{m\to 0}\langle\bar{\psi}\psi\rangle = 0$
- Effective restoration of $U(1)_A$ symmetry: $\lim_{m\to 0}(\chi_\pi - \chi_\delta) = 0$
- Possible forms of ρ: $\rho(\lambda,m) = c_0 + c_1\lambda + c_2 m^2 \delta(\lambda) + c_3 m + c_4 m^2 + O(\lambda,m)$

$$\langle\bar{\psi}\psi\rangle = 2c_0\pi - 4c_1 m\ln(m) + 2c_2 m + 2\pi c_3 m + 2\pi c_4 m^2$$

$$\chi_\pi - \chi_\delta = 2c_0\pi/m + 4c_1 + 4c_2 + 2\pi c_3 + 2\pi c_4 m$$

In the chiral limit of $m \to 0$

The c2 term restores SU(2)xSU(2) symmetry but breaks $U(1)_A$ symmetry

Novel method to probe the quark mass dependence of ρ(λ)

$$\frac{\partial \rho(\lambda, m_l)}{\partial m_l} = lim_{\epsilon\to 0}\frac{\rho(\lambda, m_l+\epsilon)-\rho(\lambda, m_l)}{\epsilon} + \mathcal{O}(\epsilon^2)$$

$$\rho_U(\lambda) = \sum_j \delta(\lambda - \lambda_j)$$

$$\rho(\lambda, m_l) = \frac{T}{V Z[\mathcal{U}]}\int \mathcal{D}[\mathcal{U}] e^{-S_G[\mathcal{U}]} \det\left[\not{D}[\mathcal{U}]+m_s\right]\left(\det\left[\not{D}[\mathcal{U}]+m_l\right]\right)^2 \rho_U(\lambda)$$

Partition function: $Z[\mathcal{U}] = \int \mathcal{D}[\mathcal{U}] e^{-S_G[\mathcal{U}]} \det\left[\not{D}[\mathcal{U}]+m_s\right]\left(\det\left[\not{D}[\mathcal{U}]+m_l\right]\right)^2$

$$\det\left[\not{D}[\mathcal{U}]+m_l\right] = \prod_j (+\mathrm{i}\,\lambda_j + m_l)(-\mathrm{i}\,\lambda_j + m_l) = \exp\left(\int_0^\infty \mathrm{d}\lambda\, \rho_U(\lambda) \ln\left[\lambda^2 + m_l^2\right]\right)$$

$$\frac{V}{T}\frac{\partial \rho}{\partial m_l} = \int_0^\infty \mathrm{d}\lambda_2 \frac{4 m_l C_2}{\lambda_2^2 + m_l^2}\,, \quad C_2(\lambda, \lambda_2) = \left\langle \rho_U(\lambda)\rho_U(\lambda_2)\right\rangle - \left\langle \rho_U(\lambda)\right\rangle\left\langle \rho_U(\lambda_2)\right\rangle$$

Computation of $\frac{\partial \rho(\lambda, m_l)}{\partial m_l}$ now: No need of additional LQCD simulations with $m_l + \epsilon$ etc

HTD, S.-T. Li, A. Tomiya, S. Mukherjee, X.-D. Wang, Y. Zhang, PRL126(2021)082001

Relation between ρ derivatives and C_{n+1}

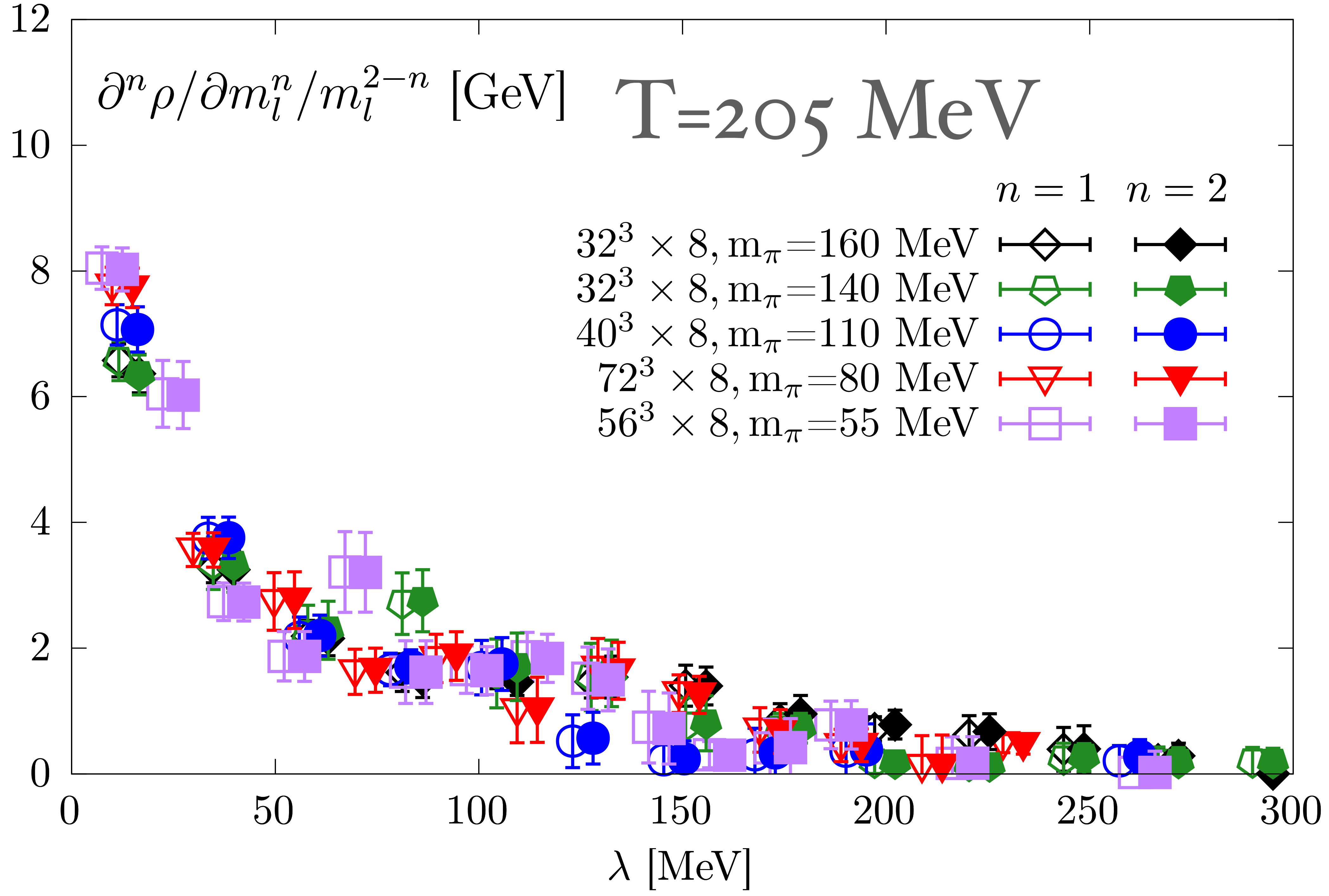
$$\frac{V}{T}\frac{\partial^2\rho}{\partial m_l^2} = \int_0^\infty \mathrm{d}\lambda_2 \, \frac{4(\lambda_2^2 - m_l^2)\, C_2}{(\lambda_2^2 + m_l^2)^2} + \int_0^\infty \mathrm{d}\lambda_2 \, \mathrm{d}\lambda_3 \, \frac{(4m_l)^2\, C_3}{(\lambda_2^2 + m_l^2)(\lambda_3^2 + m_l^2)}$$

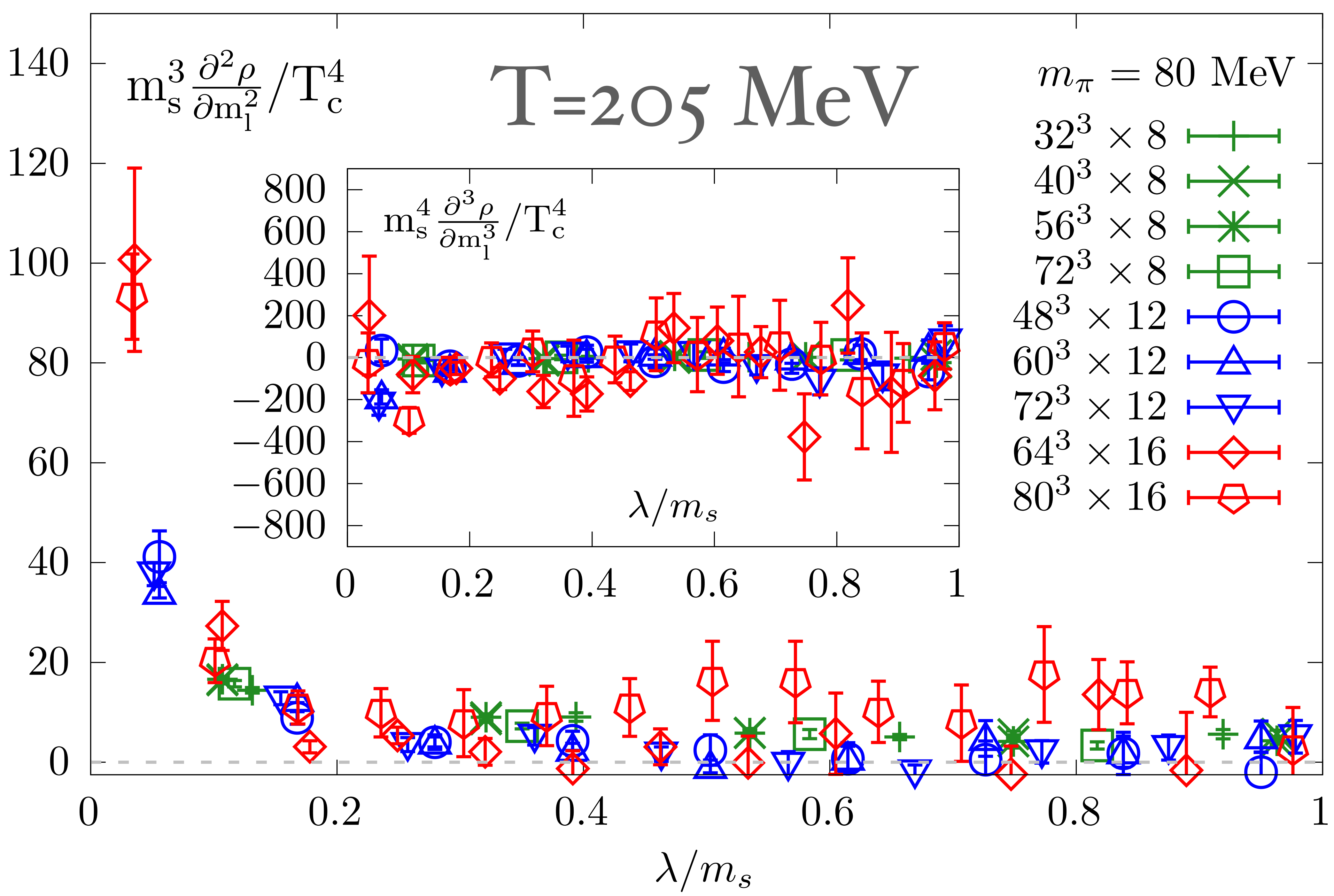
••• •••

••• •••

$$C_n(\lambda_1, \cdots, \lambda_n; m_l) = \left\langle \prod_{i=1}^{n} \left[\rho_U(\lambda_i) - \langle \rho_U(\lambda_i) \rangle\right] \right\rangle$$

Microscopic origin of the axial anomaly from $\partial^n\rho/\partial m^n$ for n=1,2 &3





$$m_l^{-1}\partial\rho/\partial m_l \approx \partial^2\rho/\partial m_l^2$$

$$\partial^3\rho/\partial m_l^3 \approx 0$$

$$\rho(\lambda\to 0, m_l\to 0) \propto m_l^2\delta(\lambda)$$

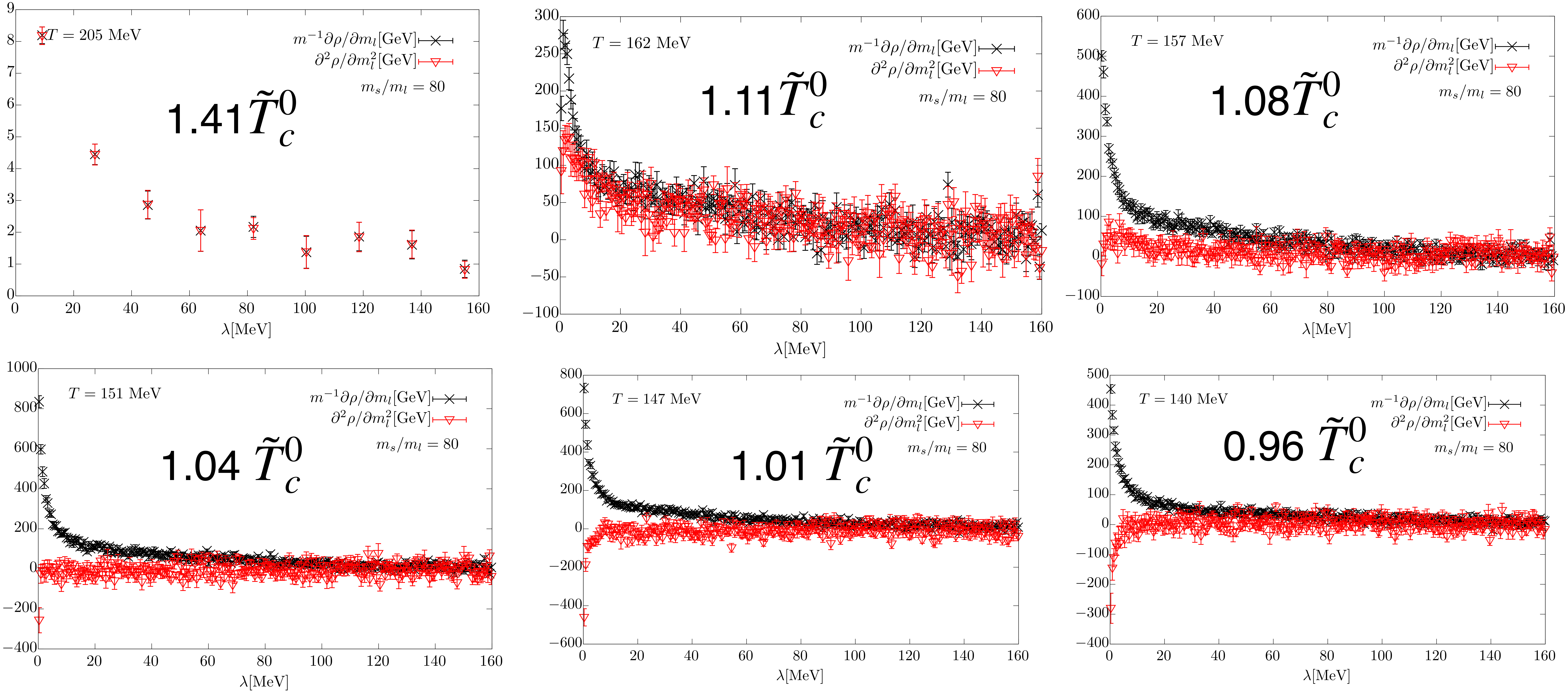
Consistent with dilute instanton gas approximation

HTD, S.-T. Li, A. Tomiya, S. Mukherjee, X.-D. Wang, Y. Zhang, PRL 126 (2021) 082001

Dilute instanton gas approximation does not hold towards T_c

$$\partial^2\rho/\partial m_l \neq m_l^{-1}\partial\rho/\partial m$$

$\tilde{T}_c^0 \equiv T_c^0(N_\tau = 8) \approx 145.6$ MeV



HTD, Wei-Ping Huang, Min Lin et al., PoS LATTICE2021 (2022) 591

Chiral phase transition and universality class

- Behavior of the free energy close to critical lines

$$f(m,T)=f_s(z)+f_{reg}\,, \qquad z=t/h^{1/\beta\delta}$$

$$h=\frac{1}{h_0}\frac{m_l}{m_s} \qquad t=\frac{1}{t_0}\frac{T-T_c}{T_c}$$

- Universality scaling behavior of

Order parameter:

$$M = -\,\partial f_s(t,h)/\partial H = h^{1/\delta} f_1(z)$$

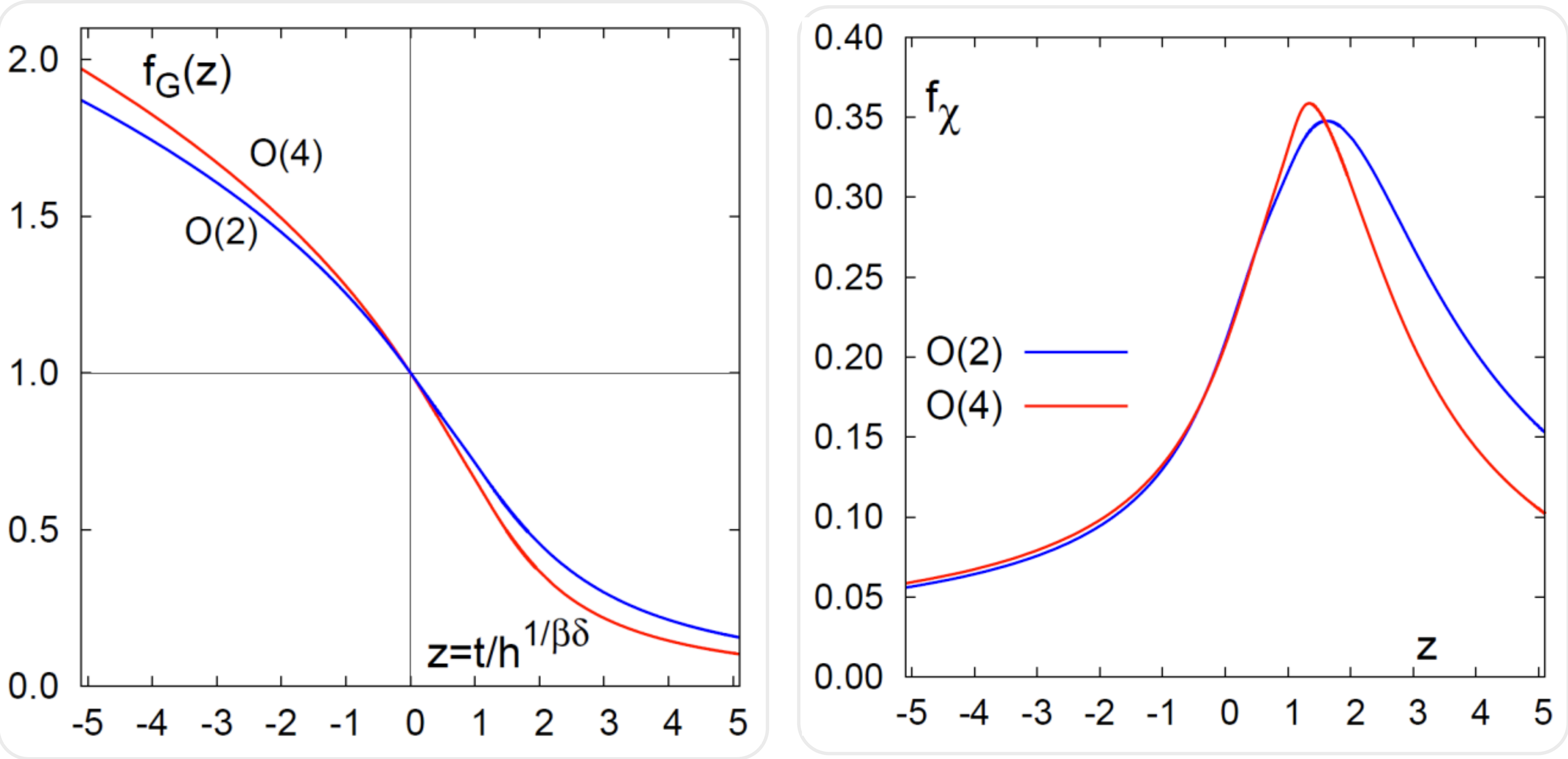
Order parameter susceptibility:

$$\chi_M = -\,\partial M/\partial H = \frac{1}{h_0} h^{1/\delta-1} f_2(z)$$

n-th order susceptibility:

$$\chi_M^n = -\,\partial^n M/\partial H^n = \frac{1}{h_0} h^{1/\delta-n+1} f_n(z)$$

Universal scaling functions



Cumulants of order parameter

- n-th order cumulant of chiral condensate:

$$\mathbb{K}_n(\bar{\psi}\psi) = \frac{T}{V}(-1)^n \frac{\partial^n \mathbb{G}(m_l;\epsilon)}{\partial m_l^n}\bigg|_{\epsilon=m_l}$$

$$\mathbb{G}(m_l;\epsilon) = \ln \left\langle \exp\left\{-m_l \bar{\psi}\psi(\epsilon)\right\}\right\rangle_0$$

$$\bar{\psi}\psi(\epsilon) = 2\ \mathrm{Tr}\ (\not{D}(U)+\epsilon)^{-1} \equiv \frac{4\epsilon}{\lambda^2+\epsilon^2}$$

$$\mathbb{K}_1\left[\bar{\psi}\psi\right] = \frac{T}{V}\langle\bar{\psi}\psi(m_l)\rangle,\quad \mathbb{K}_2\left[\bar{\psi}\psi\right] = \frac{T}{V}\left\langle\left(\bar{\psi}\psi(m_l)\ -\langle\bar{\psi}\psi(m_l)\rangle\right)^2\right\rangle,\cdots$$

- Connection to correlation of Dirac Eigenvalues

$$\rho_U(\lambda) = \sum_j \delta(\lambda-\lambda_j)\ , P_U(\lambda;\epsilon) = \frac{4\epsilon\rho_U(\lambda)}{\lambda^2+\epsilon^2}$$

NEW

$$\mathbb{K}_n(\bar{\psi}\psi) = \int_0^\infty P_n(\lambda)\ \mathrm{d}\ \lambda$$

$$n=1,\ P_1(\lambda) = K_1[P_U(\lambda,m_l)]$$

$$n\geq 2,\ P_n(\lambda) = \int_0^\infty K_1[P_U(\lambda;m_l),P_U(\lambda_2;m_l),\cdots,P_U(\lambda_n;m_l)]\prod_{i=2}^{n}\mathrm{d}\lambda_i$$

A generalized Banks-Casher relation: $\lim_{m\to 0}\mathbb{K}_n(\bar{\psi}\psi) = (2\pi)^n\mathbb{K}_n[\rho_U(0)]$

HTD, W.-P. Huang, S. Mukherjee and P. Petreczky, arXiv:2305.10916, PRL 131 (2023) 161903

Scaling behavior of $\mathbb{K}_n(\bar{\psi}\psi)$

In the proximity of T_c

- n-th order susceptibility of chiral order parameter:

$$\chi_M^n = -\,\partial^n M/\partial H^n = \frac{1}{h_0} m^{1/\delta - n+1} f_n(z) = \mathbb{K}_n(\bar{\psi}\psi) + \text{ other singular parts}$$

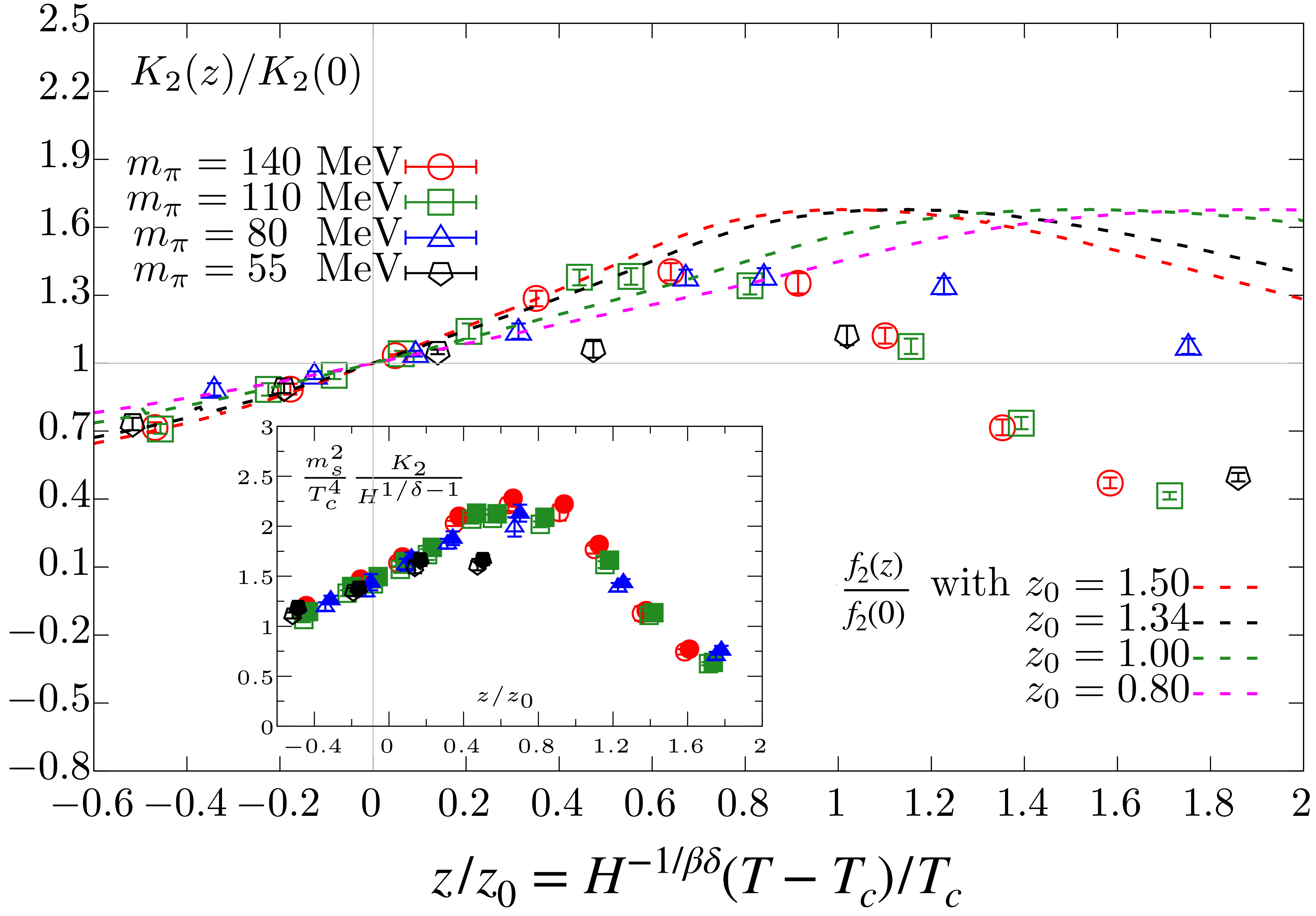
$f_n(z)$: scaling function, $\quad z = z_0(T - T_c)/T_c H^{-1/\beta\delta}$: scaling variable

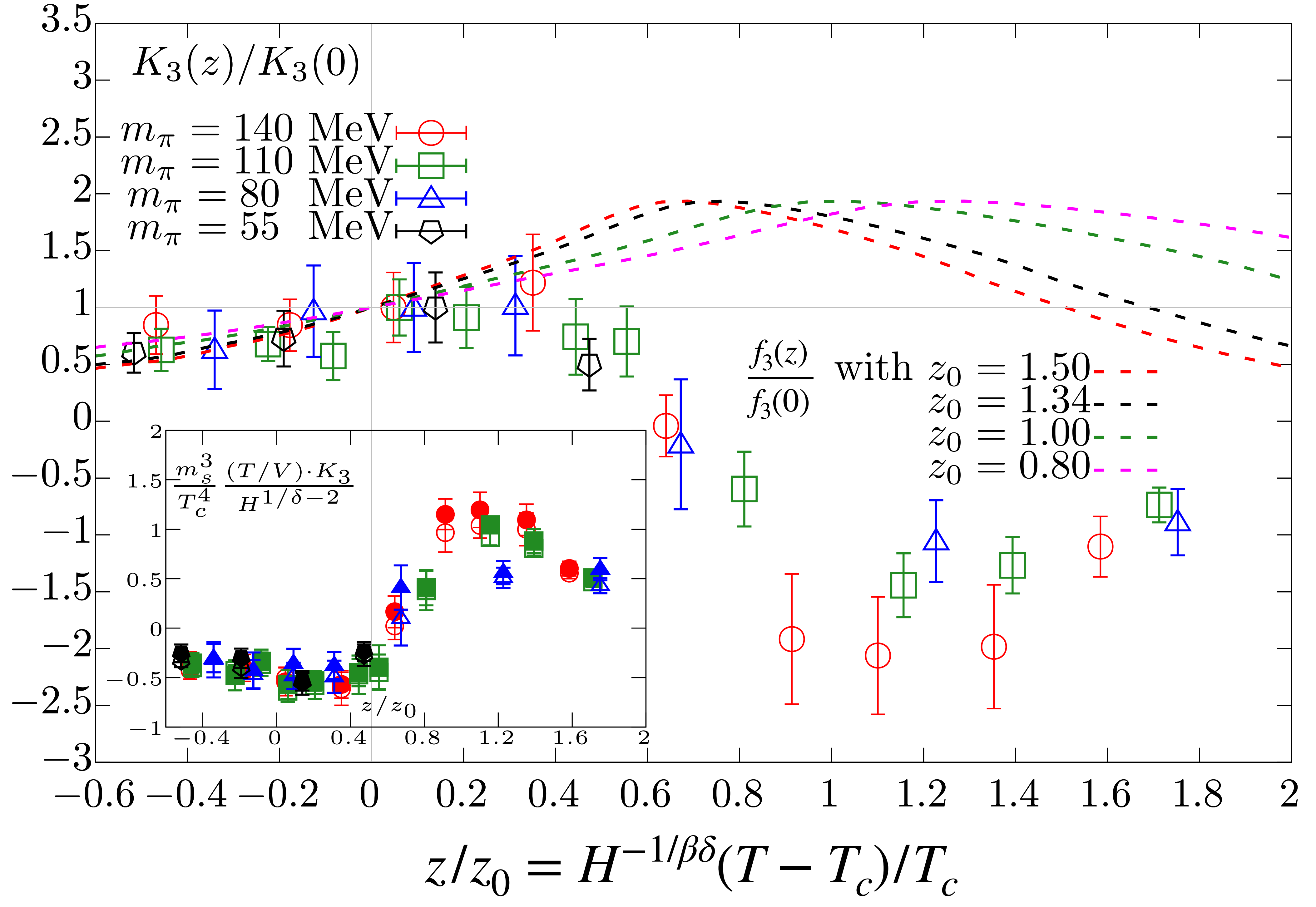
- n-th order cumulant of chiral condensate:

$$\mathbb{K}_n(\bar{\psi}\psi) = \int_0^\infty P_n(\lambda)\,\mathrm{d}\,\lambda \quad \sim m^{1/\delta - n+1} f_n(z)\,?$$

Scaling behavior of $\mathbb{K}_n(\bar{\psi}\psi)$

HISQ, Nt=8, pion mass ranging from 140 to 55 MeV





$\mathbb{K}_2(\bar{\psi}\psi) \sim m_l^{1/\delta-1} f_2(z)$ $\mathbb{K}_3(\bar{\psi}\psi) \sim m_l^{1/\delta-2} f_3(z)$

$$\mathbb{K}_n(\bar{\psi}\psi) = \int_0^\infty P_n(\lambda)\, \mathrm{d}\,\lambda \;\sim m^{1/\delta-n+1} f_n(z)\,!$$

Microscopic encoding of Macroscopic criticality

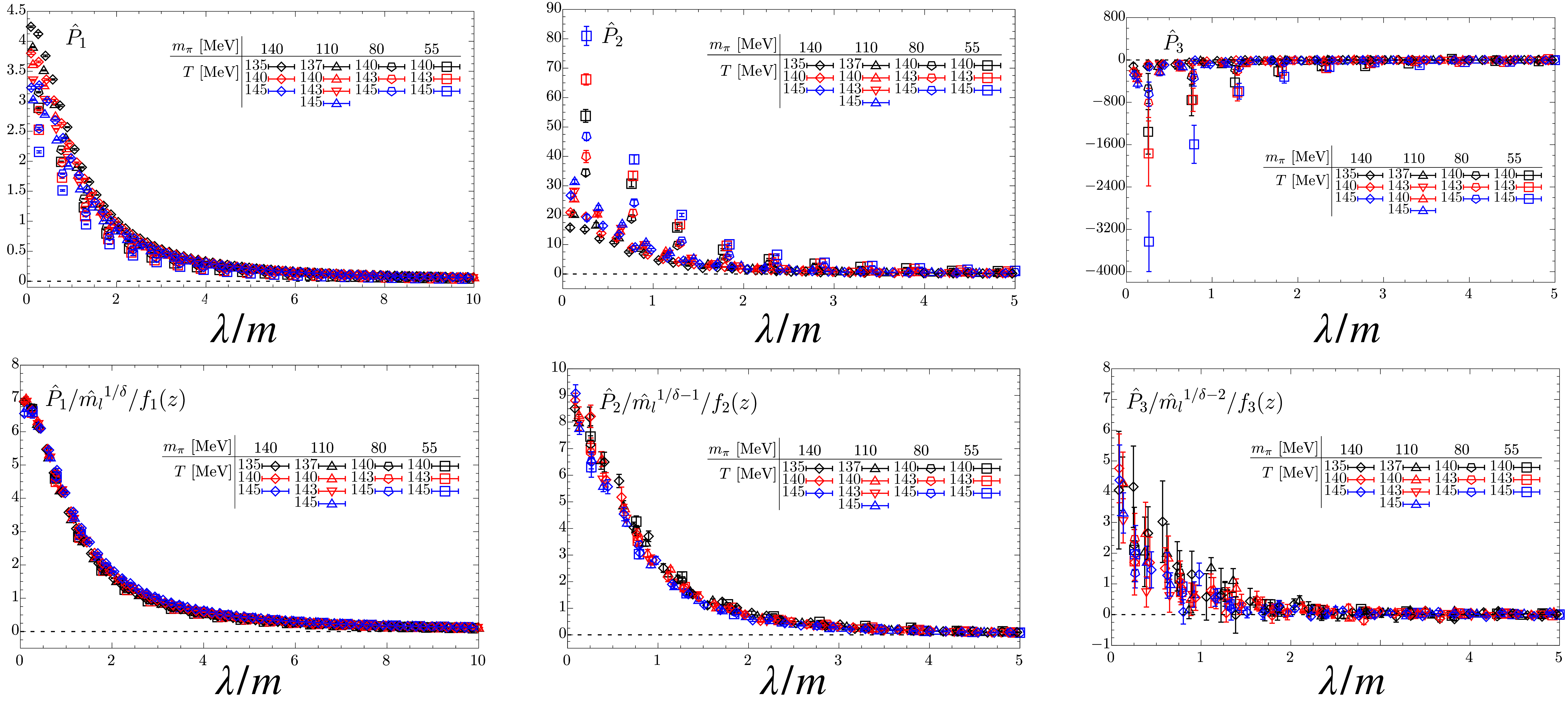
$$\mathbb{K}_n(\bar{\psi}\psi) = \int_0^\infty P_n(\lambda)\,\mathrm{d}\,\lambda \quad \sim m^{1/\delta - n + 1} f_n(z)$$

$$\lim_{m \to 0} \mathbb{K}_n(\bar{\psi}\psi) = (2\pi)^n \mathbb{K}_n[\rho_U(0)]$$

Critical behavior in $\lim_{m \to 0} \mathbb{K}_n(\bar{\psi}\psi)$: must arise from universal behaviors of λ-independent $\mathbb{K}_n[\rho_U(0)]$

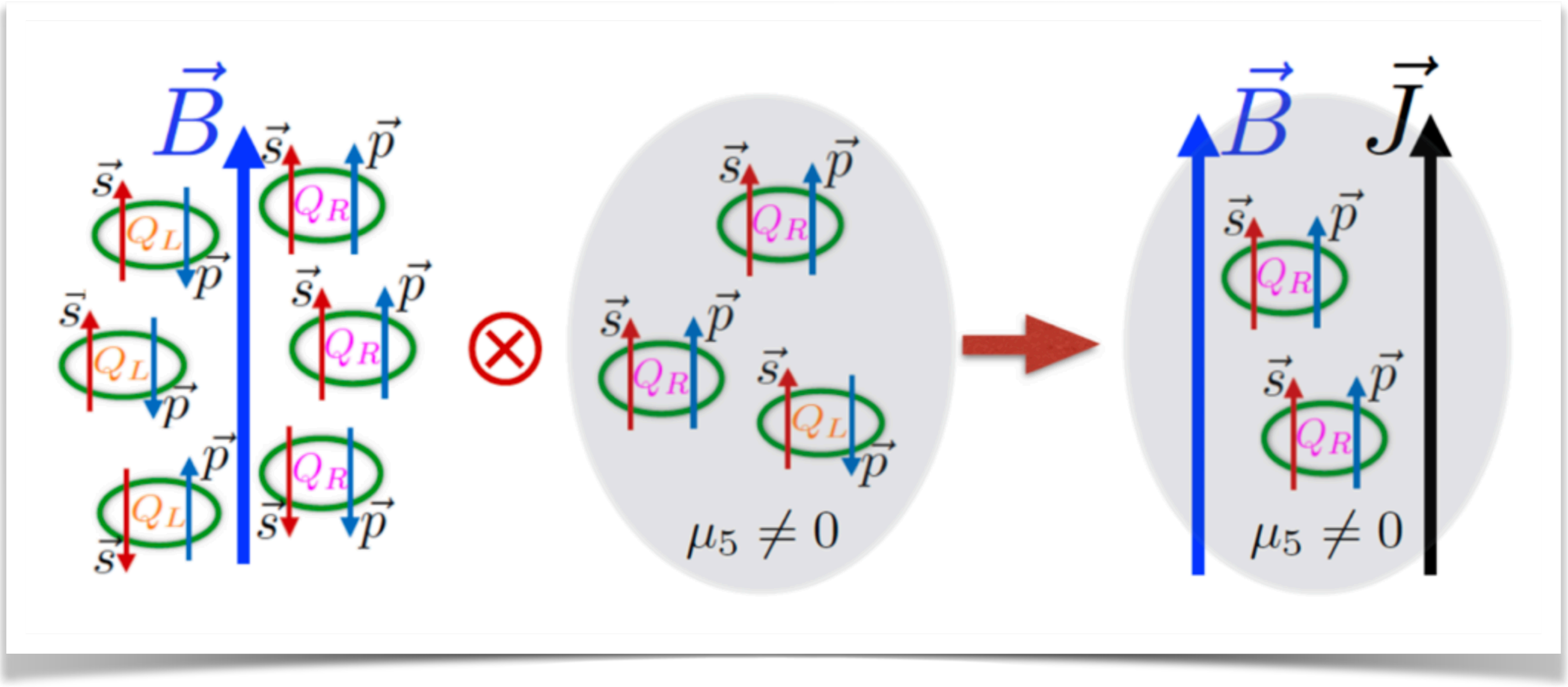
Conjecture: $P_n(\lambda) = m^{1/\delta - n + 1} f_n(z)\ g(\lambda/m)$

Microscopic encoding of Macroscopic criticality



$$P_n(\lambda) = m^{1/\delta - n + 1} f_n(z)\ g(\lambda/m)$$

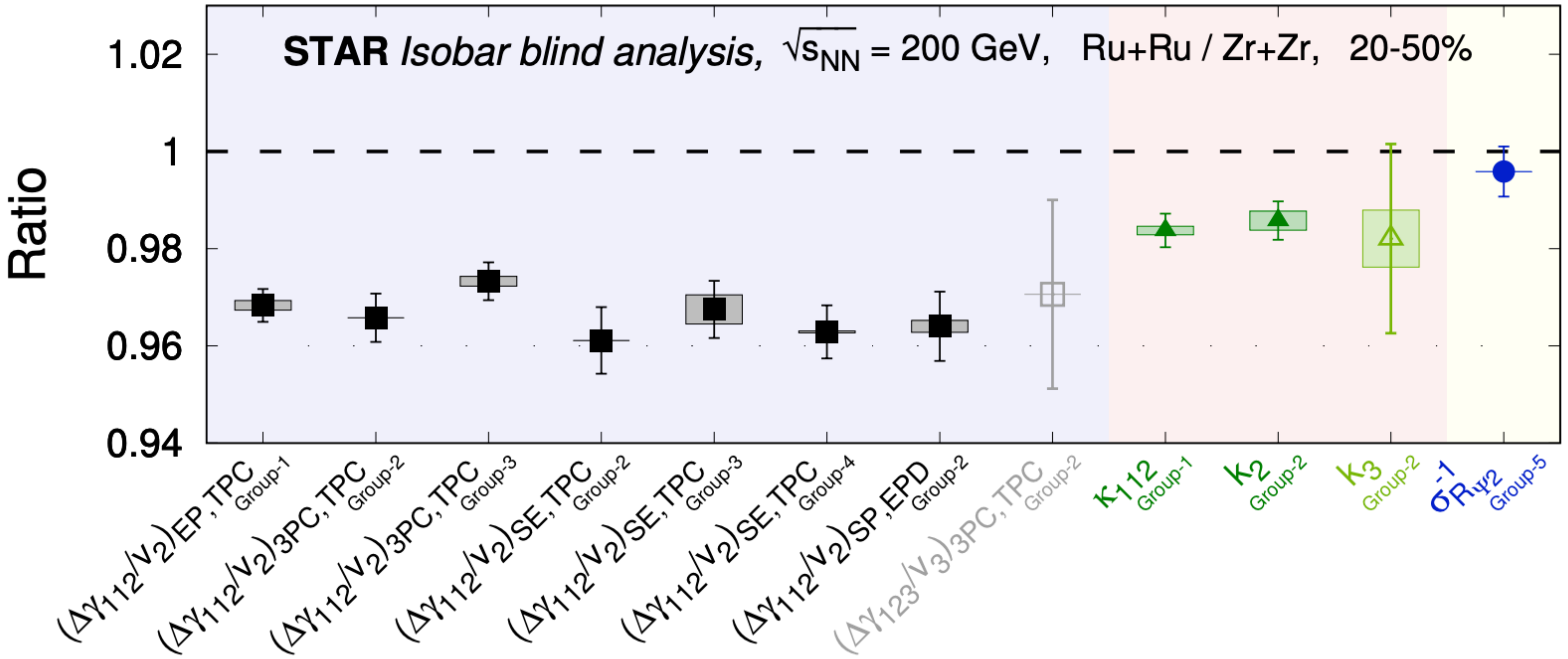
Chiral magnetic effect



See recent reviews e.g.
D.E. Kharzeev and J. Liao, Nature Rev. Phys. 3(2021)55

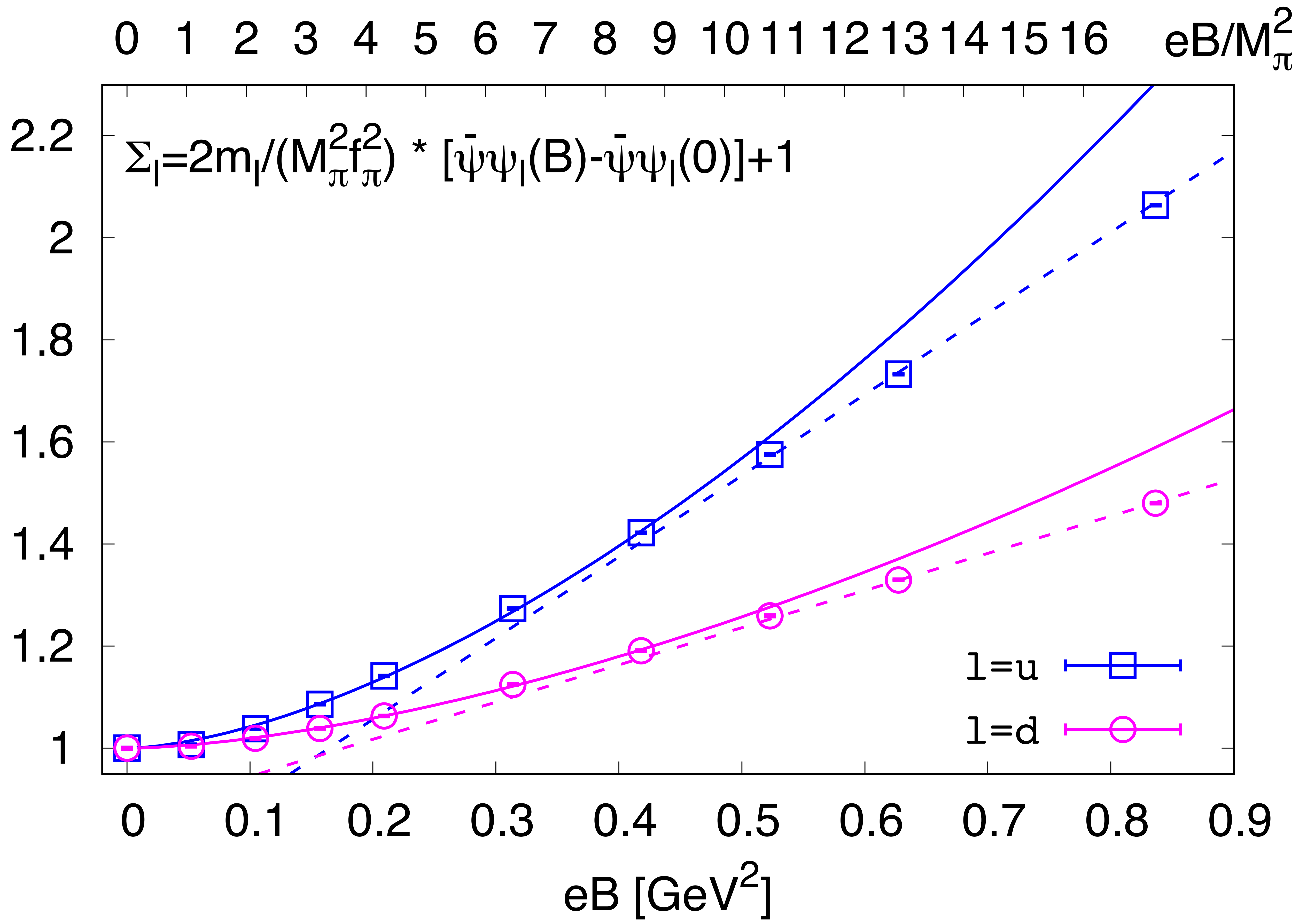
- Axial U(1) anomaly
- Strong magnetic field

Search for the Chiral Magnetic Effect with Isobar Collisions



STAR collaboration, *Phys.Rev.C* 105 (2022) 1, 014901

Isospin symmetry breaking in strong magnetic fields



Not accessible in HIC experiments

HTD, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, PRD 126 (2021) 082001

See also in reviews e.g. M. D'Elia, Lect.NotesPhys.871(2013)181

Fluctuations of net baryon number, electric charge and strangeness

- Taylor expansion of the QCD pressure:

Allton et al., Phys.Rev. D66 (2002) 074507
Gavai & Gupta et al., Phys.Rev. D68 (2003) 034506

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z}(T, V, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

- Taylor expansion coefficients at μ=0 are computable in LQCD

See recent reviews:
LQCD: HTD, F. Karsch, S. Mukherjee, Int. J. Mod. Phys. E 24 (2015) no.10, 1530007
Exp.: X.-F. Luo & N. Xu, Nucl. Sci. Tech. 28 (2017) 112

$$\hat{\chi}_{ijk}^{uds} = \left. \frac{\partial^{i+j+k} p/T^4}{\partial (\mu_u/T)^i \, \partial (\mu_d/T)^j \, \partial (\mu_s/T)^k} \right|_{\mu_{u,d,s}=0}$$

$$\hat{\chi}_{ijk}^{\rm BQS} = \left. \frac{\partial^{i+j+k} p/T^4}{\partial (\mu_{\rm B}/T)^i \, \partial (\mu_{\rm Q}/T)^j \, \partial (\mu_{\rm S}/T)^k} \right|_{\mu_{\rm B,Q,S}=0}$$

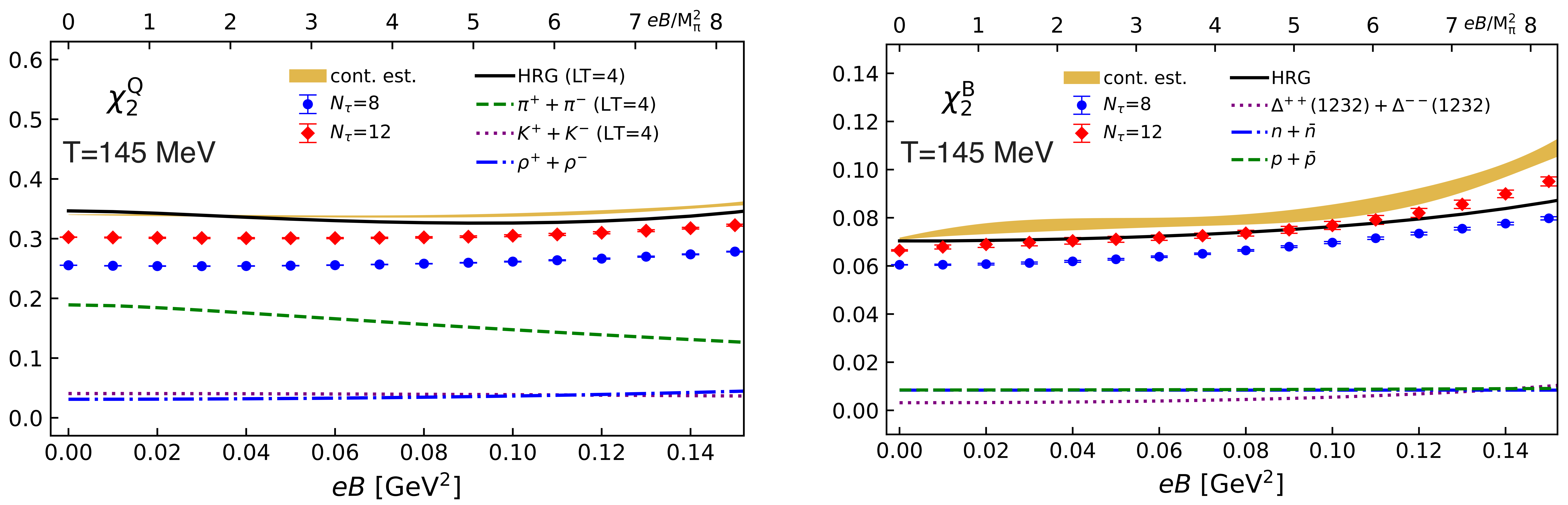
$$\mu_u = \frac{1}{3}\mu_{\rm B} + \frac{2}{3}\mu_{\rm Q} \, , \quad \mu_d = \frac{1}{3}\mu_{\rm B} - \frac{1}{3}\mu_{\rm Q} \, , \quad \mu_s = \frac{1}{3}\mu_{\rm B} - \frac{1}{3}\mu_{\rm Q} - \mu_{\rm S} \, .$$

- At eB=/=0 a lot more need to be explored

HRG: G. Kadam et al., JPG 47 (2020) 125106, Ferreira et al., PRD 98(2018)034003, Fukushima and Hidaka, PRL117 (2016)102301
Bhattacharyya et al., EPL115(2016)62003
PNJL: W.-J. Fu, Phys. Rev. D 88 (2013) 014009

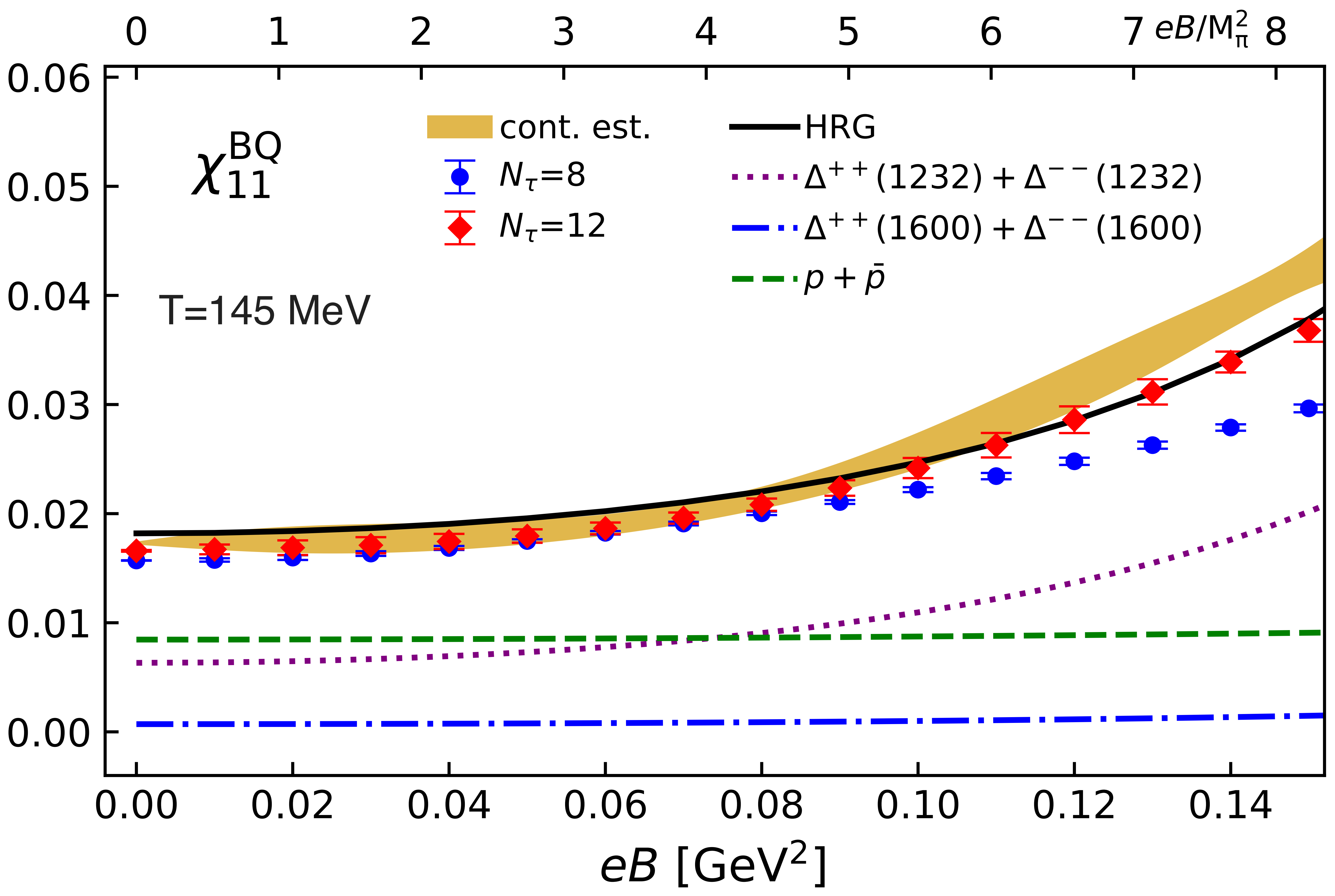
Net baryon number and electric charge fluctuations at T=145 MeV at the physical point

$N_{f=2+1}$ Lattice QCD, $M_\pi(eB=0) = 135$ MeV



HTD, J.-B. Gu, A. Kumar, S.-T. Li, J.-H. Liu, arXiv:2312.08860

Baryon electric charge correlation at T=145 MeV at the physical point



χ_{11}^{BQ}: Magnetometer of QCD

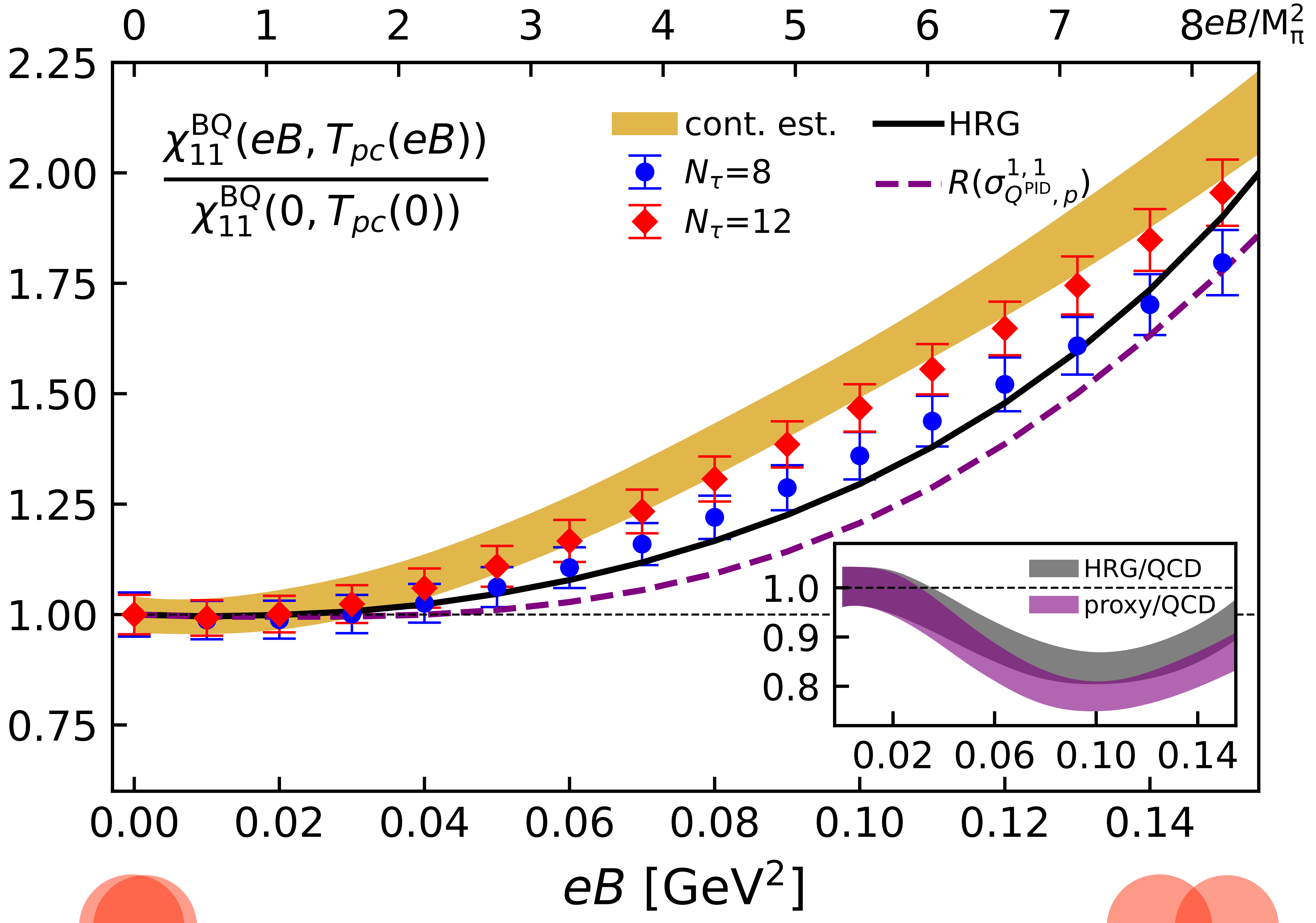
Most of the eB-dependences comes from doubly charged Delta baryons

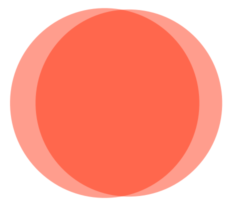
Delta baryons: not-measurable in HIC experiments

$\Delta^{++} \rightarrow p + \pi^{+}$

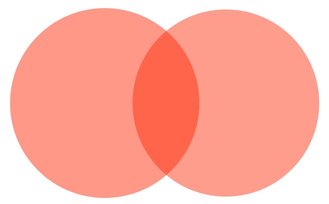
HTD, J.-B. Gu, A. Kumar, S.-T. Li, J.-H. Liu, arXiv:2312.08860

Ratio X(eB)/X(eB=0) for 2nd order diagonal fluctuations along the transition line





Central collision



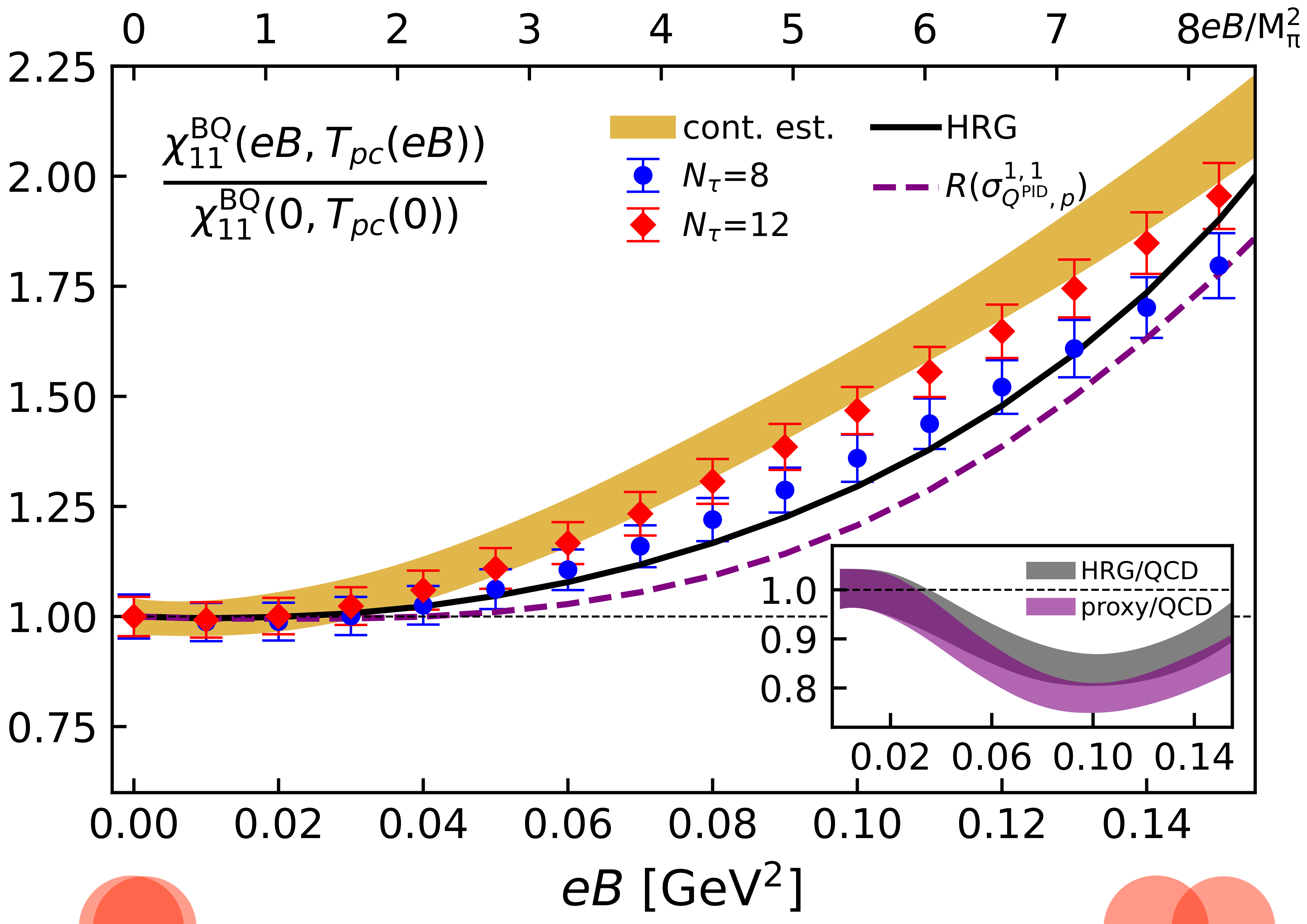
Peripheral collision

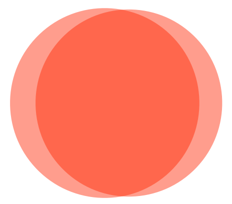
X(eB)/X(eB=0) : Rcp like observable

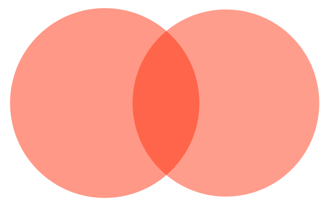
At $eB \lesssim M_\pi^2$: consistent with unity

At $eB \simeq 8M_\pi^2$: ~2 !

Ratio X(eB)/X(eB=0) for 2nd order diagonal fluctuations along the transition line




Central collision


Peripheral collision

Memory carried by the decays of Δ^{++}:

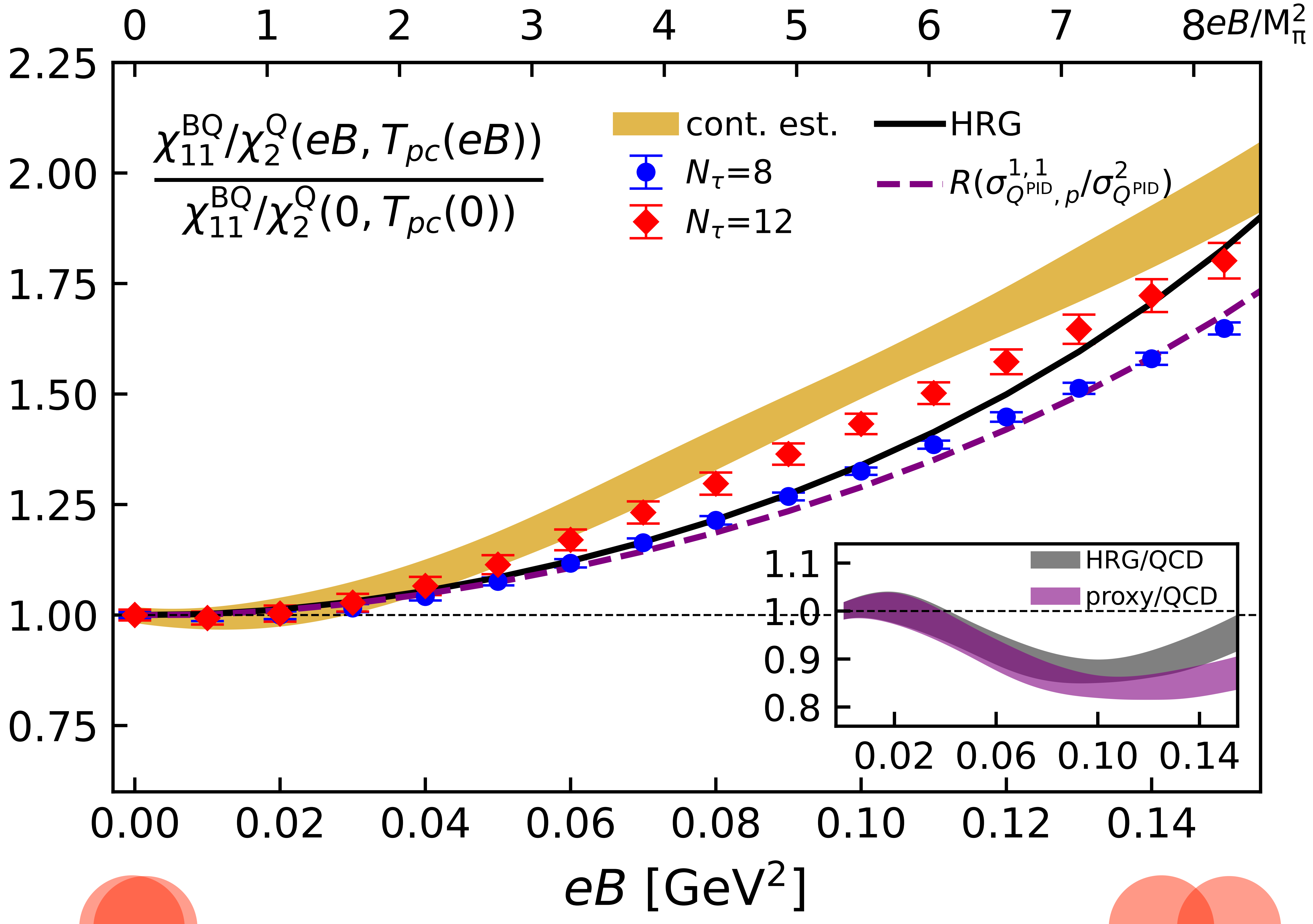
$$\Delta^{++} \rightarrow p + \pi^+$$

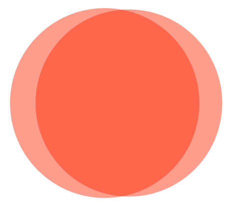
$$\sum_R B_R^l Q_R^m S_R^n I_p^R \rightarrow \sum_{i \in \text{stable}} \sum_R (P_{R\rightarrow i})^p B_i^l Q_i^m S_i^n I_p^R,$$

net-B approximated by Q^{PID}: $\tilde{p}$

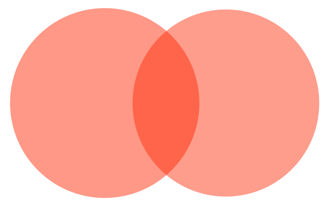
net-Q approximated by Q^{PID}: $\tilde{\pi}^+, \tilde{K}^+, \tilde{p}$

Ratio X(*eB*)/X(*eB*=0) for 2nd order diagonal fluctuations along the transition line





Central collision



Peripheral collision

Memory carried by the decays of Δ^{++}:

$$\Delta^{++} \rightarrow p + \pi^+$$

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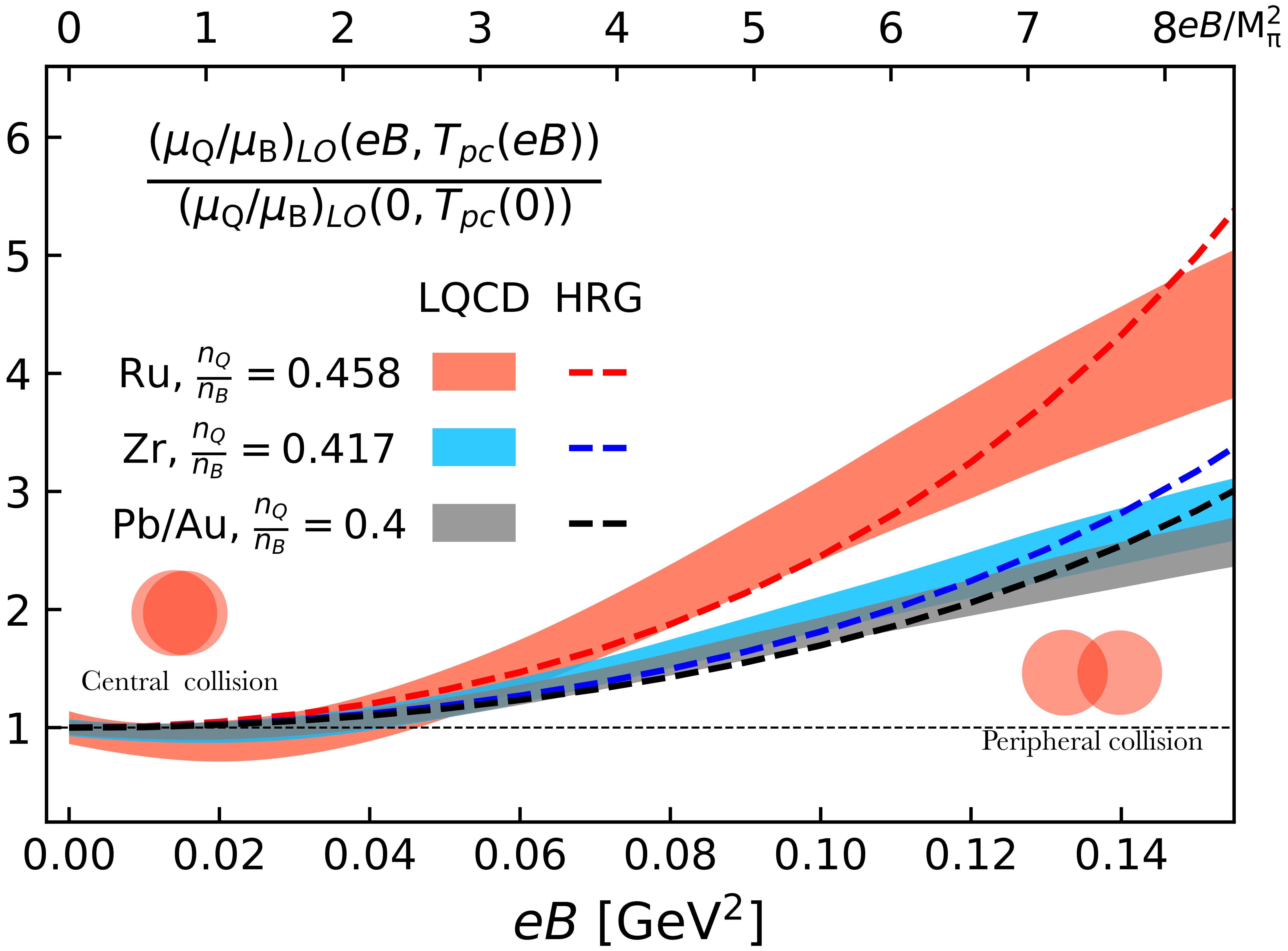
net-B approximated by Q^{PID}: $\tilde{p}$

net-Q approximated by Q^{PID}: $\tilde{\pi}^+, \tilde{K}^+, \tilde{p}$

μ_Q/μ_B in different collision systems



$$\mu_Q/\mu_B = q_1 + q_3\mu_B^2 + \mathcal{O}(\mu_B^4)$$

$$q_1 = \frac{r(\chi_2^B\chi_2^S - \chi_{11}^{BS}\chi_{11}^{BS}) - (\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS})}{(\chi_2^Q\chi_2^S - \chi_{11}^{QS}\chi_{11}^{QS}) - r(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS})}$$

$$r = n_Q/n_B$$

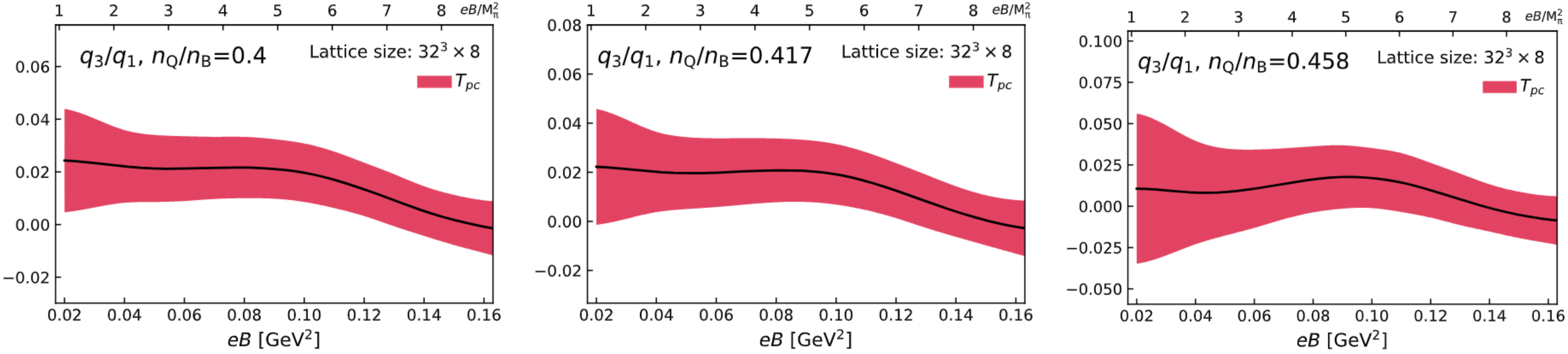
${}^{96}_{44}Ru + {}^{96}_{44}Ru$: r=0.458

${}^{96}_{40}Zr + {}^{96}_{40}Zr$: r=0.417

${}^{208}_{82}Pb + {}^{208}_{82}Pb$: r=0.4

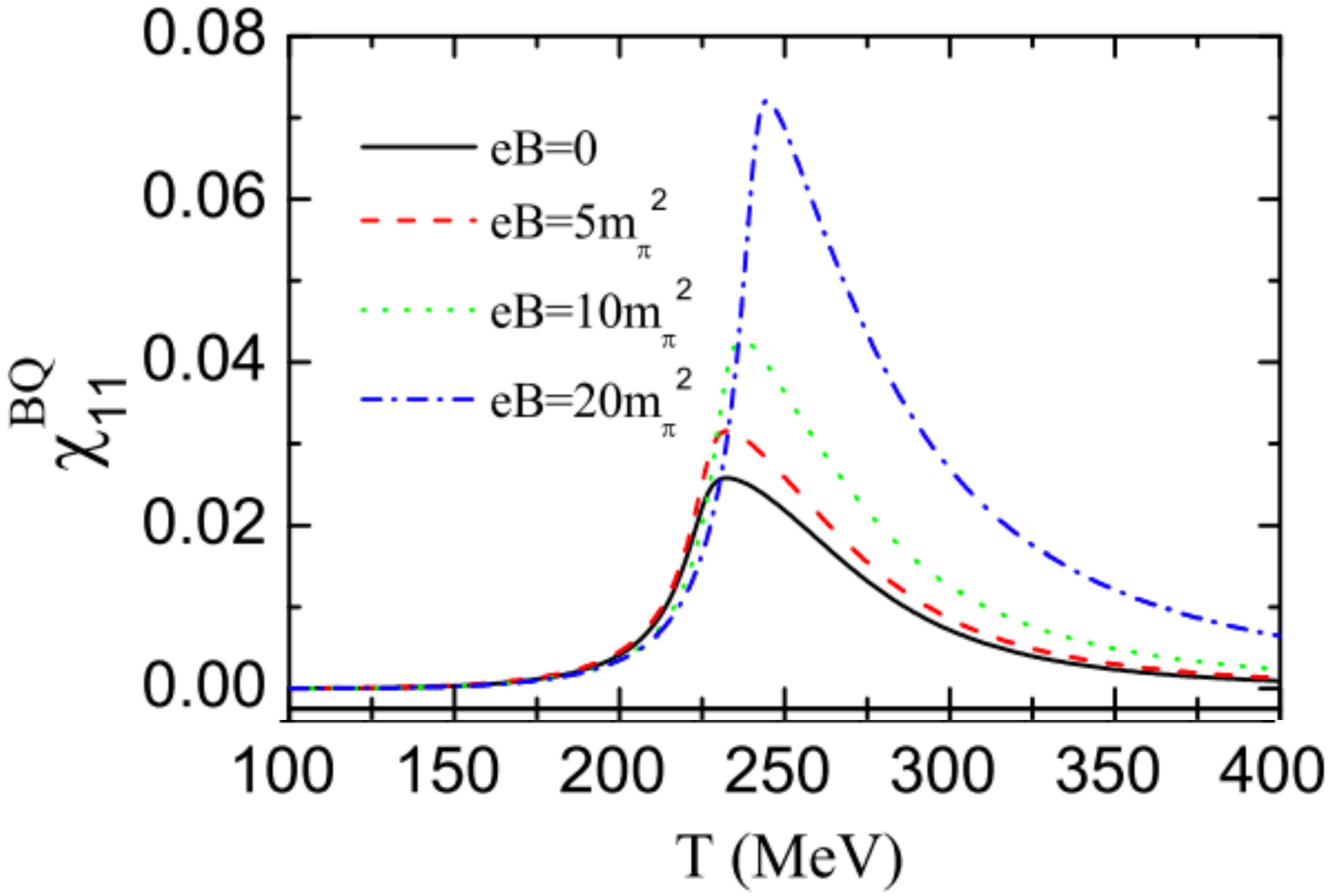
μ_Q/μ_B in different collision systems

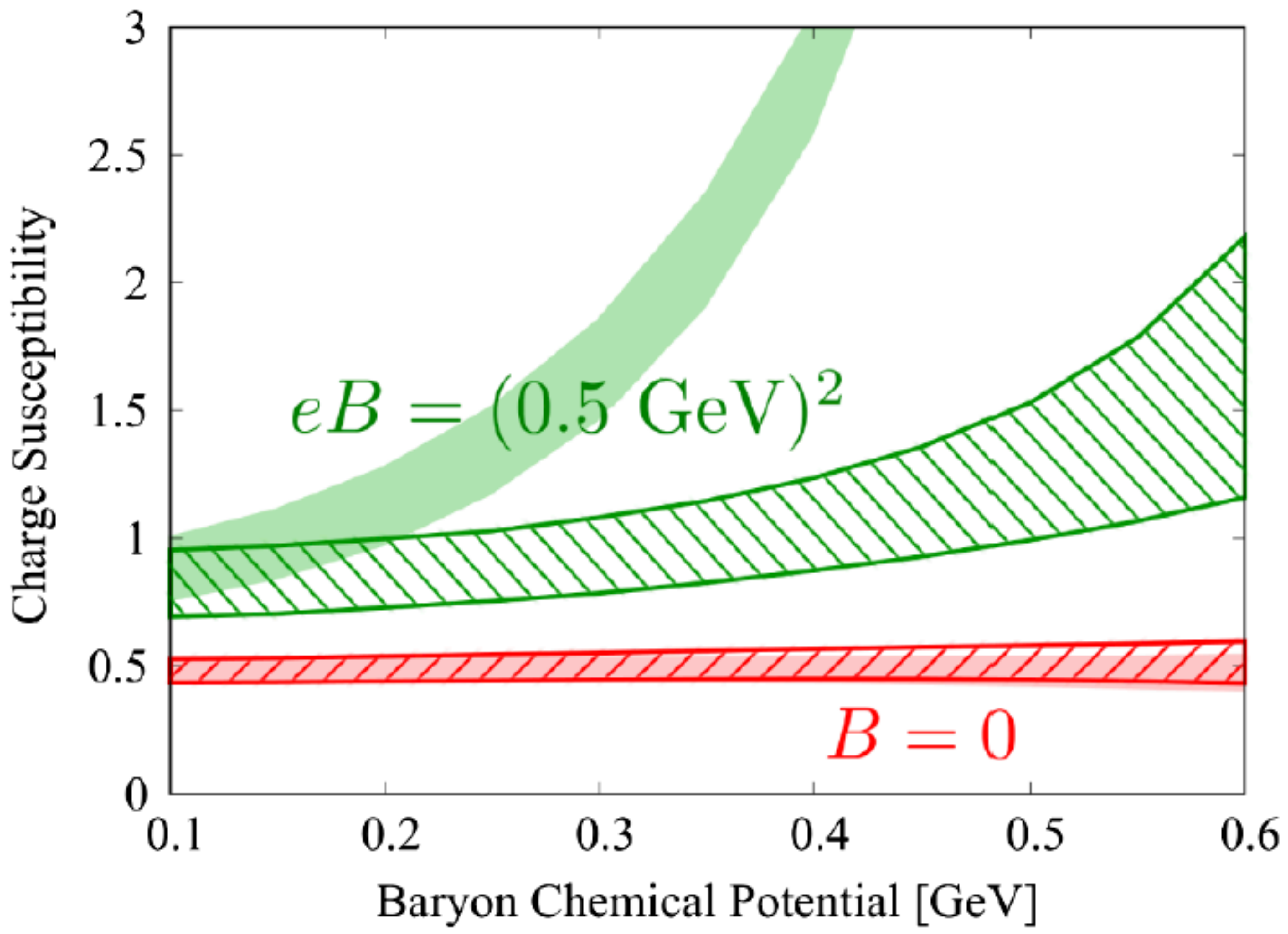
$$\mu_Q/\mu_B = q_1 + q_3\mu_B^2 + \mathcal{O}(\mu_B^4)$$



Negligible next-to-leading order correction

Lattice QCD v.s. effective theory & model studies



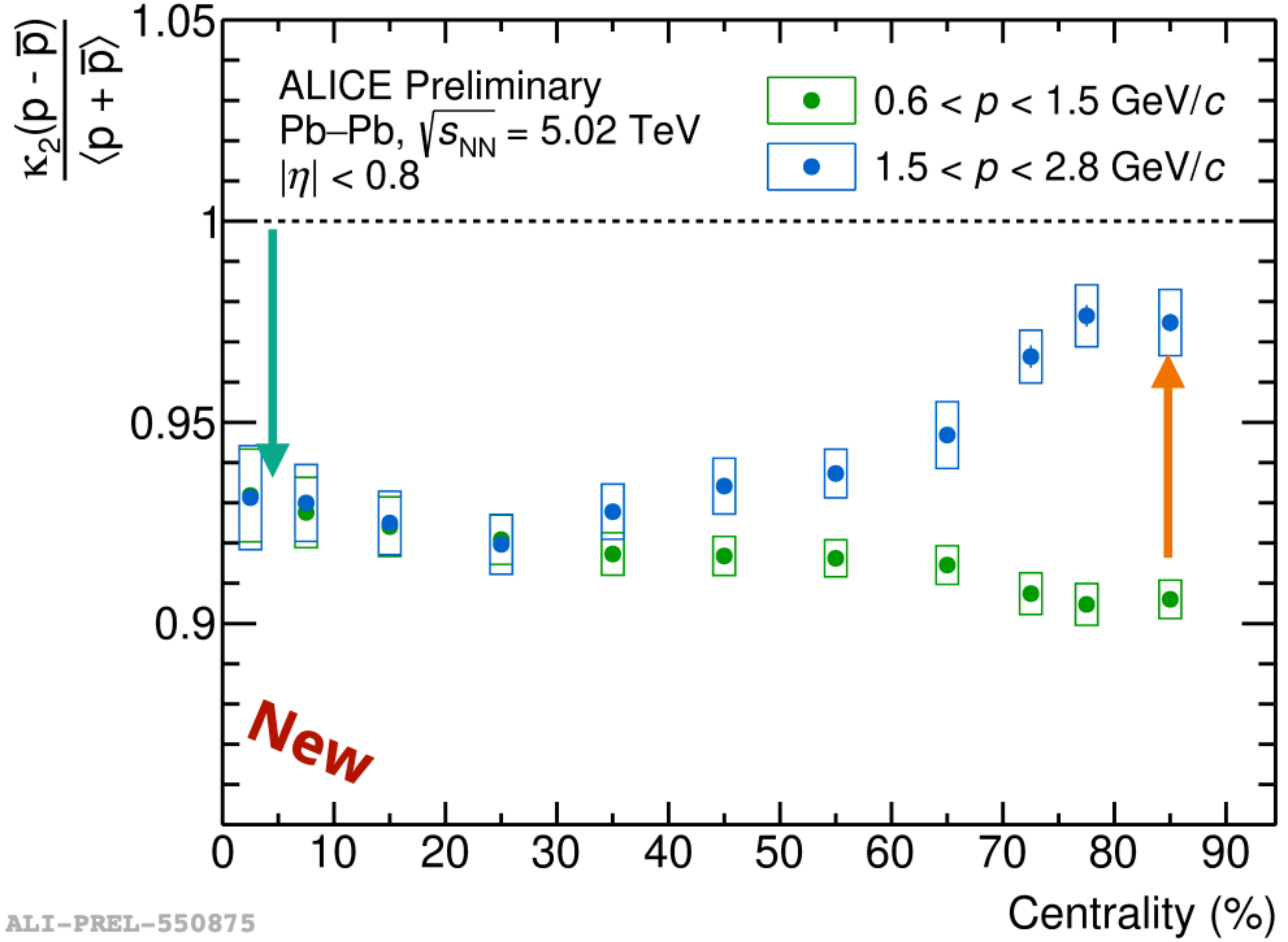


Results obtained from PNJL from
W.-J. Fu, Phys. Rev. D 88 (2013) 014009

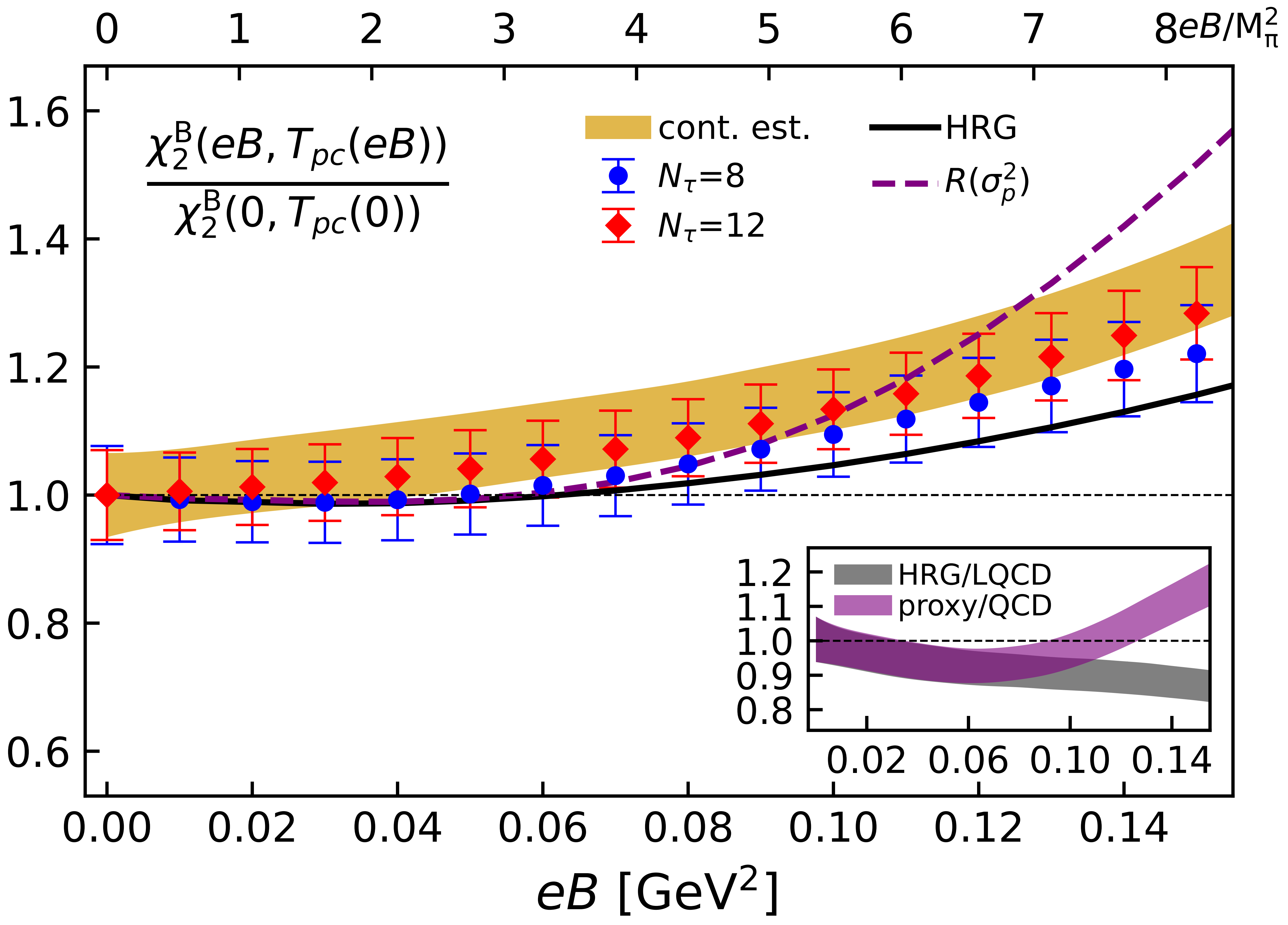
Results obtained from HRG model
K. Fukushima and Y. Hidaka , Phys. Rev. Lett. 117 (2016) 102301

Both above two results are inconsistent with LQCD results!

Lattice QCD v.s. experiments

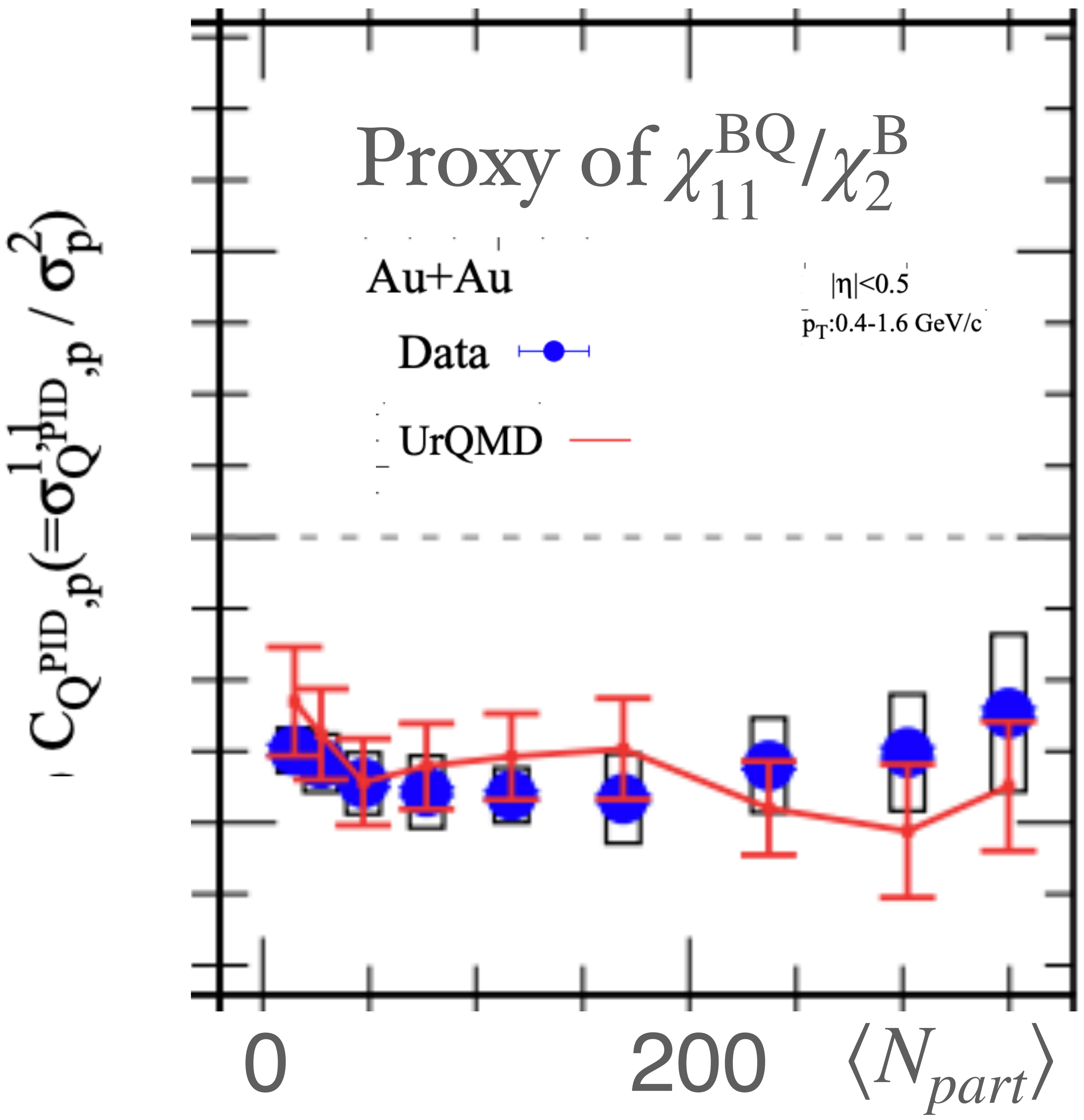


Ilya Fokin, ALICE, Quark Matter 2023

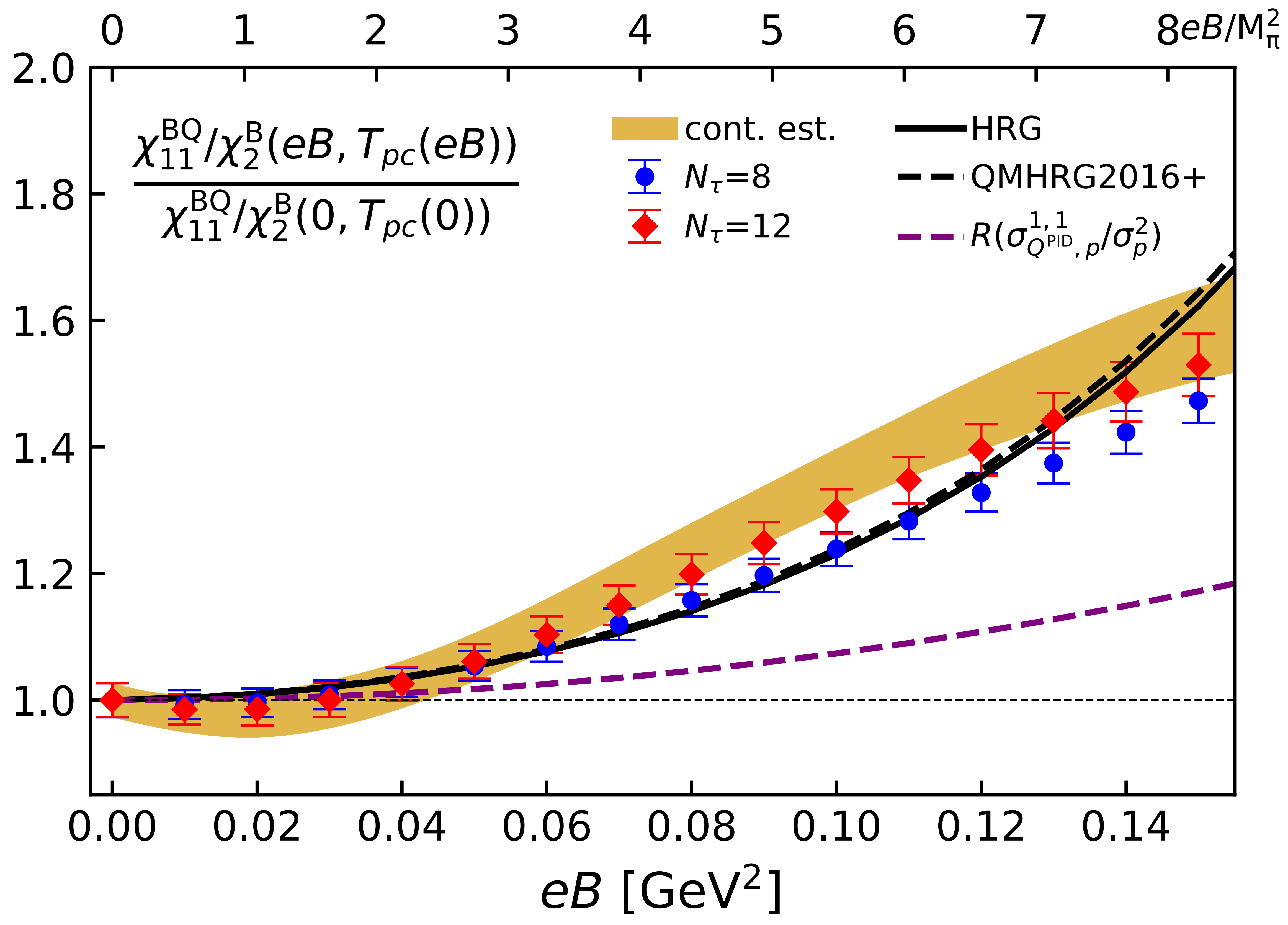


HTD, J.-B. Gu, A. Kumar, S.-T. Li, J.-H. Liu, arXiv:2312.08860

Lattice QCD v.s. experiments



STAR: Phys.Rev. C 105, 029901(E) (2022)



HTD, J.-B. Gu, A. Kumar, S.-T. Li, J.-H. Liu, arXiv:2312.08860

Summary & Conclusion