Cosmological first-order phase transitions without bubbles

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Introduction

In a FOPT, the transition from false vacuum to the true vacuum proceeds via **nucleating bubbles** of the true vacuum, as a result of tunneling process or due to **thermal fluctuations** large enough to jump over the barrier.

Domain wall can arise from spontaneous breaking of a discrete symertry. domain walls will give rise to a rich impact on the dynamics of the cosmological phase transition. For instance, domain walls can catalyze bubble nucleation. However, this realization is still based on the quantum tunneling effect.

In this Letter, the author suggest a competing way to achieve the FOPTs where bubbles nucleation is inefficient or even absolutely prohibited.

Model

As a simple model admitting the domain wall solution and a two-step phase transition, the authors consider the SM extended with a real scalar singlet S.

The effective potential in terms of the two scalar condensates:

$$
\begin{aligned}
V(h,s,T) \; = \; & -\frac{1}{2}\left(\mu_h^2 - c_h T^2\right) h^2 + \frac{1}{4}\lambda_h h^4 \\
& -\frac{1}{2}\left(\mu_s^2 - c_s T^2\right) s^2 + \frac{1}{4}\lambda_s s^4 + \frac{1}{2}\lambda_{hs} h^2 s^2,
\end{aligned} \qquad (1)
$$

Model

at $T = T_{\rm dw} = \sqrt{\mu_s^2/c_s}$,
domain walls are generated, the field configurations interpolating between the $s_\pm$ vacuum are given by

$$s_{\rm dw}(\mathbf{x}, z_0, T) = v_s(T)\tanh[(z - z_0)/L_{\rm dw}(T)],$$
$$h_{\rm dw}(\mathbf{x}, T) = 0,$$
$$L_{\rm dw}(T) = \sqrt{2/(\mu_s^2 - c_s T^2)}.$$

at $T = T_c < T_{\rm dw}$
a new minimum having the same value of V as the $s_\pm$ vacuum appears in the h field direction.

When the temperature drops below Tc,
h vacuum develops into the true vacuum.

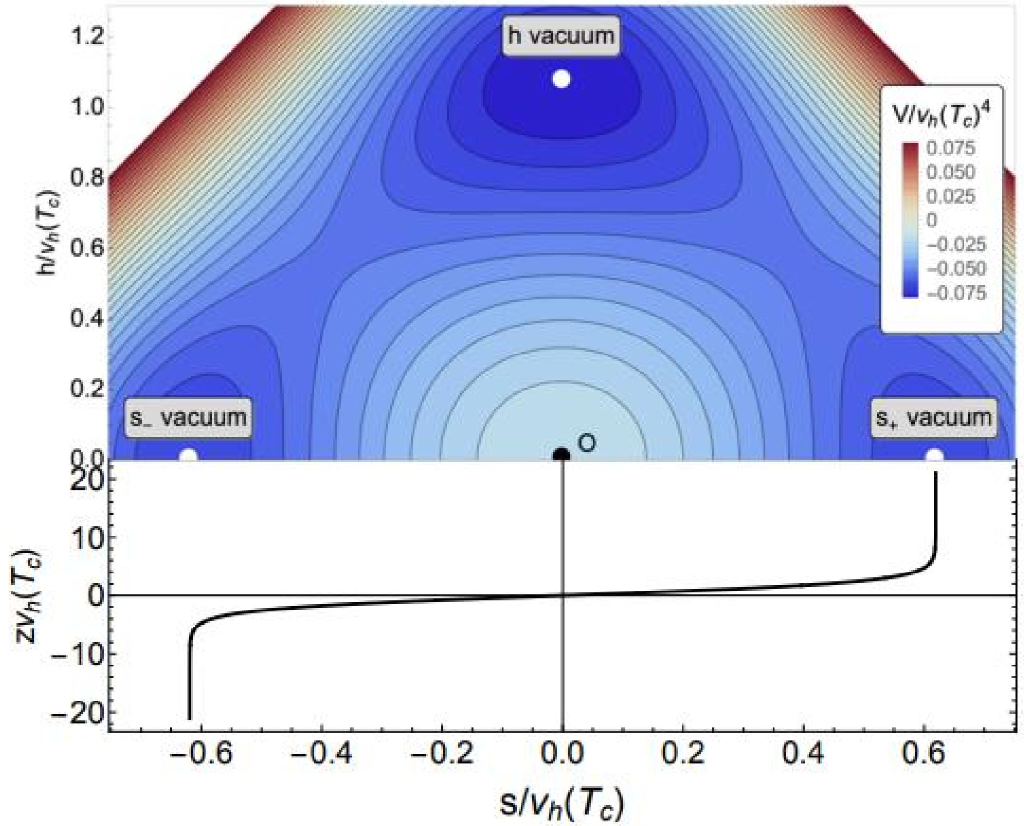

FIG. 1. The potential and the associated static domain wall solution at $T = 75$ GeV $< T_c$ for the BMP-A. There exists a barrier between $s_\pm$ vacuum and h vacuum as a feature of the FOPT.

Numerical simulation of the wall instability

the field configurations governed by the equations of motions of φ = h, s fields,

$$\ddot{\phi} - \nabla^2\phi + \frac{\partial V(h,s)}{\partial \phi} = 0.$$

the authors set the initial profile for the two fields as follows:

$$\begin{aligned} s(\mathbf{x}, t=0) &= s_{\mathrm{dw}}, \quad \dot{s}(\mathbf{x}, t=0) = 0, && (4) \\ h(\mathbf{x}, t=0) &= \delta h(\mathbf{x}, \tau_{\mathrm{ref}}), \quad \dot{h}(\mathbf{x}, t=0) = \delta \dot{h}(\mathbf{x}, \tau_{\mathrm{ref}}), && (5) \end{aligned}$$

where δh is uncorrelated and spatially inhomogeneous perturbations that originate from the vacuum fluctuation on the h field. In the vacuum it obeys the stochastic Gaussian distribution.

on the k-lattice:

$$\mathcal{P}\left(\delta\tilde{h}_{\mathbf{k}}(\tau)\right) = \frac{1}{\sqrt{2\pi\sigma_k^2}} \exp\left(-\frac{\left|\delta\tilde{h}_{\mathbf{k}}(\tau)\right|^2}{2\sigma_k^2}\right)$$

$$\sigma_k^2 = (2\omega_{\mathbf{k}})^{-1} \quad \omega_{\mathbf{k}}^2 = |\mathbf{k}|^2 + V''_{\phi\phi}(h_0, s_0)$$

on the x-lattice

$$\delta h(\mathbf{n}) = \frac{1}{L^3} \sum_{\tilde{\mathbf{n}}} e^{-i\frac{2\pi}{N}\tilde{\mathbf{n}}\cdot\mathbf{n}} \delta\tilde{h}_{\tilde{\mathbf{n}}},$$

Numerical simulation of the wall instability

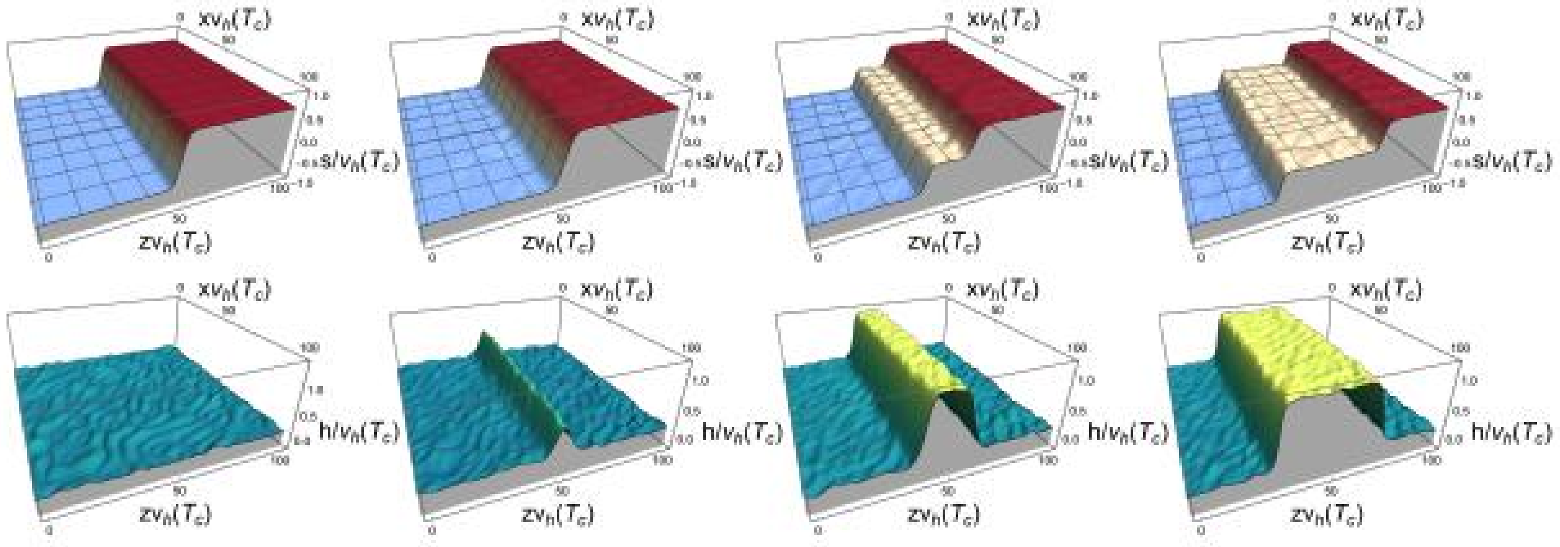

FIG. 2. Dynamical evolution of a planar wall existing in space between the s_+ (red region) and s_- (blue region) domains in the presence of an inhomogeneous fluctuation. The simulation is performed at $T = 75$ GeV $< T_c$. We present the configurations for the s (upper panel) and h (lower panel) fields in the x-z plane at $tv_h(T_c) = 0, 20, 40, 54$ (from left to right).

the domain wall turns into the domain trench, making the entire volume eventually transition to the h vacuum state.

Methods of evaluating the rescue temperature.

to evaluate from which temperature the domain wall becomes the domain trench, consider the homogeneous (spatial-independent) fluctuations $\delta h \equiv \Delta$.

at T = Tc y the h field exhibits a small oscillation around h = 0, s field has no significant change. (upper panel)

at $T \ll T_c$ the h field in the wall region will quickly move to h = vh and meanwhile, the s field will tend to zero.
(lower panel)

By finely adjusting T , we find a critical temperature at which a stable configuration can be achieved in the h field.
We define this critical temperature as the rescue temperature T_{res}.(middle panel)

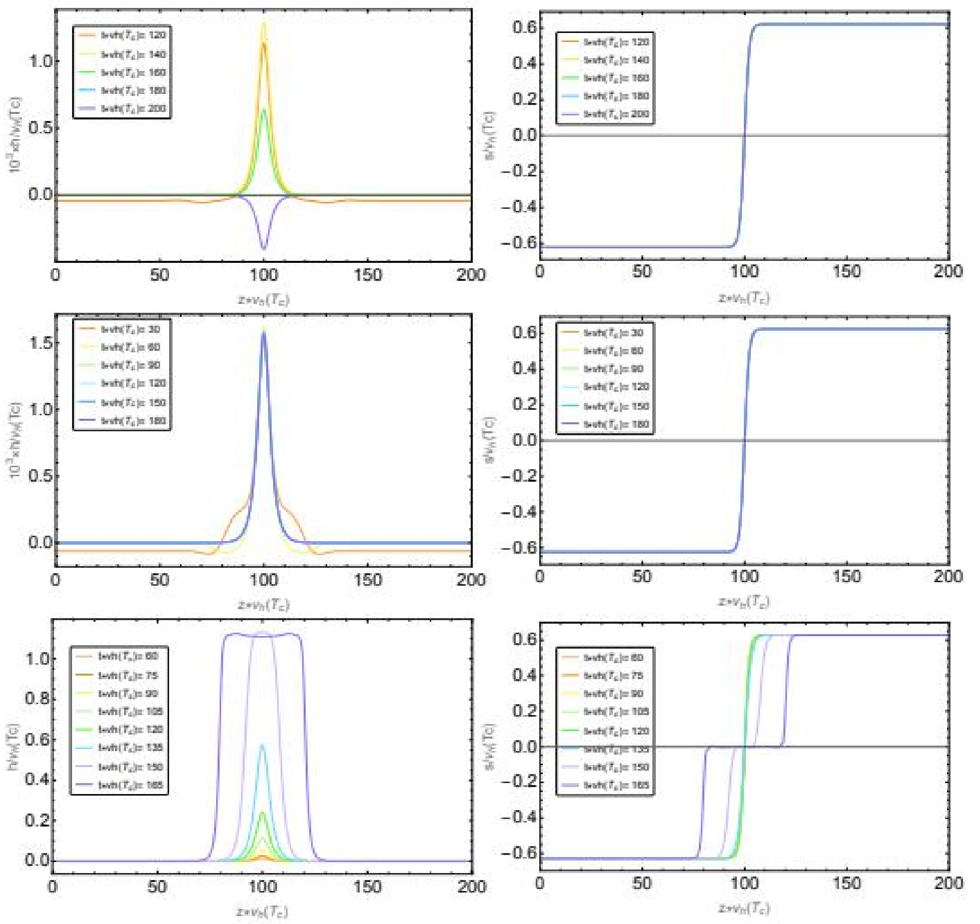

FIG. 3. Time evolution of a pair of planar walls centered at $z_0 = \pm 100/v_h(T_c)$ with a spatially homogeneous fluctuation $\Delta = 5 \times 10^{-3}$ at $T > T_{\rm res}, T \simeq T_{\rm res}, T < T_{\rm res}$ (from top to bottom). The h and s field configurations are shown in the left and right columns, respectively. The negative z axis is omitted due to symmetric configuration.

Methods of evaluating the rescue temperature.

In addition to the lattice simulation, T_{res} can be estimated from the view of energy conservation.

the energy per area deposited into the domain wall, relative to the initial stable wall, is

$$\sigma_V(h_r, T) = \int \Big(V(0, s_{\rm dw}, T) - V(h_r, s_{\rm dw}, T) \Big) dz,$$

the wall tension is characterized by the gradient energy in the unit area associated with the domain wall,

$$\sigma_g(h_r, T) = \int \frac{1}{2} (\partial_z h_r)^2 \, dz.$$

If σ_V exceeds σ_g, this would make possible the destruction of the domain wall. Therefore, we define T_{res} as the highest temperature T that satisfies

$$g(h_r, T) = 0 \qquad g(h_r, T) = \sigma_V(h_r, T) - \sigma_g(h_r, T).$$

at T_{res} the h field configuration appears steady and can be approximately described by a Gaussian wave-packet

$$h_r(z) = A v_h e^{-\frac{(z-z_0)^2}{(\alpha L_{\rm dw})^2}}$$

it is convenient to define a reduce g that is irrelevant to A $\tilde{g}(\alpha, T) = g(h_r, T)/A^2.$

Methods of evaluating the rescue temperature.

BMP	λ_s	λ_{hs}	m_s	$T_{\rm dw}$	T_c	$v_s(T_c)$	$v_h(T_c)$	$L_{\rm dw}(T_c)$
A	1	0.73	168	186	85	113	186	0.01
B	1	0.86	181	203	56	133	222	0.01

seeding a FOPT without bubbles is impossible in the BMP-B since g remains negative.

for BMP-A, Tres corresponds to the temperature at which the function g has exactly one zero root.

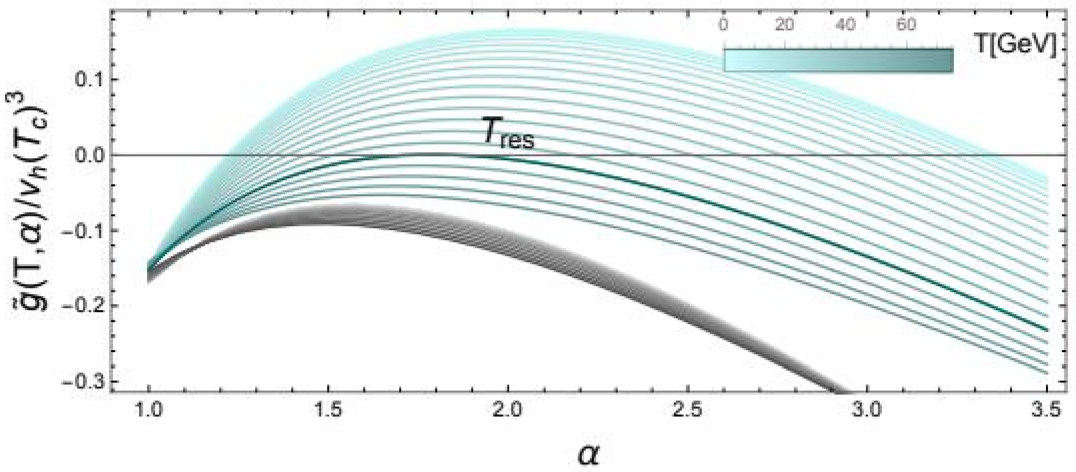

FIG. 5. For the BMP-A (cyan) and BMP-B (grey), we generate a bunch of the $\tilde{g}(\alpha, T)$ curves with the descending value of T from T_c to zero (from bottom to top). BMP-A has a solution for $T_{\rm res}$ indicated in dark line.

Methods of evaluating the rescue temperature.

the results from the inhomogeneous fluctuations(IF) and homogeneous fluctuations (HF) reached satisfactory consistency.

the theoretical calculation (TC) approach predicts a relatively smaller value. the discrepancy may arise from two factors:
we do not model the s-field configuration.
we are unable to accurately determine the size of α.

TABLE II. $T_{\rm res}$ for the BMP-A obtained from various approaches (IF, HF, and TC). The simulation parameters include the size L, the lattice number N^3, the spacing $\Delta x = L/N$ and $\tilde{n}_{\rm cut} = 5$ is taken in the IF. In case that the simulation time is insufficient under the requirement $t_{\rm end}/R = 1$, a small L, corresponding to a small R, may predict a lower $T_{\rm res}$ than the actual value (see HF* results).

Methods	$v_h(T_c)L$	N	$T_{\rm res}$[GeV]	$\sqrt{\langle\delta h^2\rangle}/v_h(T_c)$
IF	200	800	71.4	4×10^{-3}
HF	800	8000	71.8	5×10^{-3}
HF*	200	2000	69.3	5×10^{-3}
TC	-	-	68.6	-

Results & Discussions

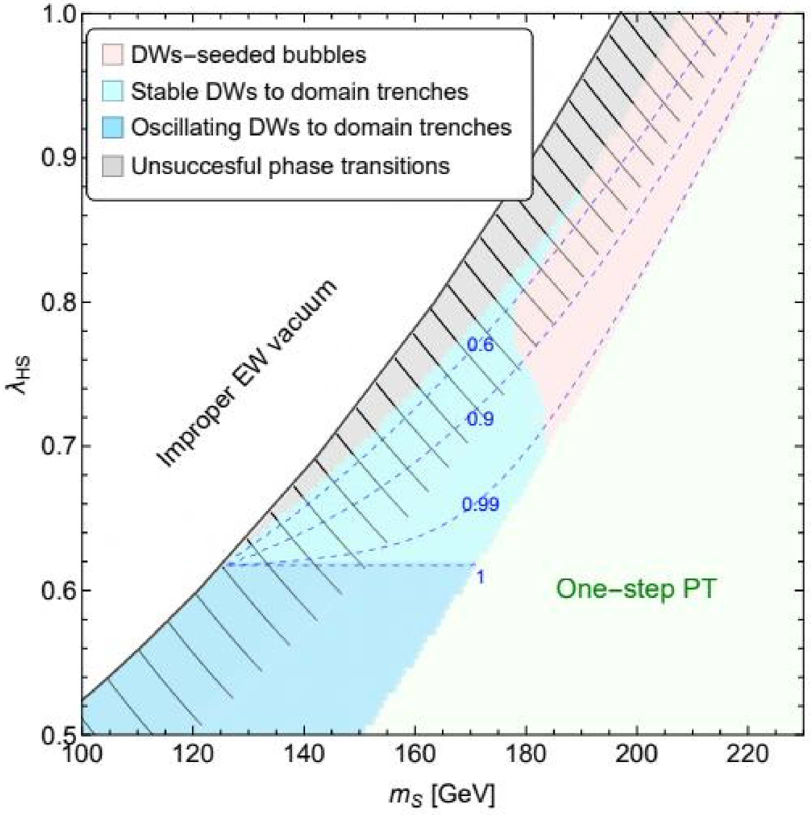

FIG. 6. Two-step phase transitions in the model. Homogeneous nucleation is inefficient in the region marked with backslashes. Cyan and blue regions: transition proceeds with the production of domain trenches from domain walls. Red region: transition proceeds with inhomogeneous bubbles seeded by domain walls. Gray region: eliminated due to unsuccessful phase transition. The blue dashed contours show the value of $T_{\rm res}/T_c$.