Predictive Type-II Neutrino Seesaw Extension of NMSSM from AMSB/GMSB and LFV Constraints

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Why Supersymmetry

SUPERSYMMETRY

Standard particles

SUSY particles

- \triangleright Hierarchy problems.
- \triangleright SUSY GUT.
- ØVacuum stability naturally in SUSY at tree-level.
- \triangleright Possible dark matter candidate.
- \triangleright Many possible baryogenesis mechanism.
- ---Coincidence of DM and Baryon density.
- \triangleright Radiative EW symmetry breaking-driven by RGE.
- **≻ Predictive-the 125 GeV Higgs favored by SUSY.**
- \triangleright Possibly vanishing CC in SUGRA?
- \triangleright Good properties: holomorphic in superpotential...

Next-to-Minimal Supersymmetric Standard model
Singlet extension of MSSM to solve the mu-problem

Ø Singlet extension of MSSM to solve the mu-problem

 \triangleright The most general, renormalizable (R-parity preserving) superpotential

$$
\mathcal{W}_S = \mathcal{W}_{\mathsf{Yukawa}} + \frac{1}{3} \kappa \hat{S}^3 + \mu \hat{H}_u \hat{H}_d + \lambda \hat{S} \hat{H}_u \hat{H}_d + \frac{1}{2} \mu_s \hat{S}^2 + t_s \hat{S} \;,
$$

- \triangleright Discrete Z 3 symmetry can forbide the dimensional terms
- \triangleright Easily accommodate the 125GeV SM-like Higgs by additional treelevel contributions

$$
M_{\text{SM}}^2 \simeq M_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta - \frac{\lambda^2}{\kappa^2} v^2 (\lambda - \kappa \sin 2\beta)^2 + \frac{3m_t^4}{4\pi^2 v^2} \left(\ln \left(\frac{m_T^2}{m_t^2} \right) + \frac{A_t^2}{m_T^2} \left(1 - \frac{A_t^2}{12m_T^2} \right) \right)
$$

 \triangleright Relex the bounds on stop masses

Next-to-Minimal Supersymmetric Standard model
breaking parameters:

Soft SUSY breaking parameters:

$$
-\mathcal{L}_{\text{soft}} = m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 + m_Q^2 |Q^2| + m_U^2 |U_R^2| + m_D^2 |D_R^2| + m_L^2 |L^2| + m_E^2 |E_R^2| + (h_u A_u Q \cdot H_u U_R^c - h_d A_d Q \cdot H_d D_R^c - h_e A_e L \cdot H_d E_R^c + \lambda A_\lambda H_u \cdot H_d S + \frac{1}{3} \kappa A_\kappa S^3 + m_3^2 H_u \cdot H_d + \frac{1}{2} m_S^2 S^2 + \xi_S S + \text{h.c.}) .
$$

Scalar potential: non-trivial to trigger EWSB

$$
V_{\text{Higgs}} = \left[\lambda \left(H_u^+ H_d^- - H_u^0 H_d^0 \right) + \kappa S^2 + \mu' S + \xi_F \right]^2
$$

+
$$
\left(m_{H_u}^2 + |\mu + \lambda S|^2 \right) \left(\left| H_u^0 \right|^2 + \left| H_u^+ \right|^2 \right) + \left(m_{H_d}^2 + |\mu + \lambda S|^2 \right) \left(\left| H_d^0 \right|^2 + \left| H_d^- \right|^2
$$

+
$$
\frac{g_1^2 + g_2^2}{8} \left(\left| H_u^0 \right|^2 + \left| H_u^+ \right|^2 - \left| H_d^0 \right|^2 - \left| H_d^- \right|^2 \right)^2 + \frac{g_2^2}{2} \left| H_u^+ H_u^0 + H_u^0 H_d^{-*} \right|^2
$$

+
$$
m_S^2 |S|^2 + \left(\lambda A_\lambda \left(H_u^+ H_d^- - H_u^0 H_d^0 \right) S + \frac{1}{3} \kappa A_\kappa S^3 + m_3^2 \left(H_u^+ H_d^- - H_u^0 H_d^0 \right) \right)_{\substack{\xi \\ \xi \\ \xi \\ \xi}}^*
$$

+
$$
\frac{1}{2} m_S^2 S^2 + \xi_S S + \text{h.c.}
$$

Collapse of Z3 Domain wall can explain the Nano-HZ gravitational wave signals by NANOGrav.

Fei, et al (2023) PRD

Origin of Soft SUSY Breaking Parameters

Malizable couplings

$$
\mathrm{STr}\mathcal{M}^2=\sum_J(-1)^{2J}(2J+1)\mathcal{M}_J^2=0
$$

Visible Sector

Hidden Sector

 $\langle X_{\text{NL}}\rangle = \theta^2 F_{\text{hid}}$ spontaneous SUSY breaking $\mathcal{L} \supset -\int d^4 \theta \, \frac{\widetilde{m}_i^2}{F^2} \bm{X}_\text{NL}^\dagger \bm{X}_\text{NL} \bm{\Phi}_i^\dagger e^{2 \bm{V}} \bm{\Phi}_i - \bigg(\int d^2 \theta \, \frac{m_\lambda}{2 F_\text{hid}} \bm{X}_\text{NL} \bm{W}^{\alpha a} \bm{W}_\alpha^a \bigg)$ $\label{eq:1} \qquad \qquad + \, \frac{C_i}{F_{\rm hid}} \boldsymbol X_{\rm NL} \boldsymbol{\Phi}_i + \frac{B_{ij}}{2 F_{\rm hid}} \boldsymbol X_{\rm NL} \boldsymbol{\Phi}_i \boldsymbol{\Phi}_j + \frac{A_{ijk}}{6 F_{\rm hid}} \boldsymbol X_{\rm NL} \boldsymbol{\Phi}_i \boldsymbol{\Phi}_j \boldsymbol{\Phi}_k + {\rm h.c.} \bigg).$

soft SUSY-breaking terms in the visible sector

$$
\mathcal{L}_{\text{soft}} = -\widetilde{m}_{i}^{2}|\phi_{i}|^{2} - \left(\frac{m_{\lambda}}{2}\lambda^{a}\lambda^{a} + C_{i}\phi_{i} + \frac{B_{ij}}{2}\phi_{i}\phi_{j} + \frac{A_{ijk}}{6}\phi_{i}\phi_{j}\phi_{k} + \text{h.c.}\right)
$$

Gauge Mediated SUSY Breaking

• In the limit $\alpha_r \to 0$ the theory decouples to two sectors.

Minimal Gauge Mediation: | Hidden

 $\langle X \rangle = X + \theta^2 F,$ $W = \lambda_{ij} X \phi_i \widetilde{\phi}_j$

- Gaugino masses arise at one loop

- Sfermion mass squares arise at two loops (8 graphs).

 $= 1, 2, 3$

- Relations between gaugino masses and sfermion \bullet masses
- Hard for 125 GeV Higgs• Small A-terms
- Hard to generate $\mu \sim B \sim m_{\lambda}$
- $\begin{array}{rcl} \displaystyle m_{\lambda_r} & \sim & \displaystyle \alpha_r \frac{F}{M} \qquad \qquad ^{\bullet} \displaystyle \ \textsf{\textbf{G}} \$

Anomaly Mediated SUSY Breaking News Stroken suger

- \checkmark A special case of gravity mediation---Most ubiqutious
- \checkmark Combination of Kahler mediation and gravitino mediation
- \checkmark Expressions of the soft terms hold at every renormalization scale, exactly, to all orders in perturbation theory. RG-stable
- \checkmark Keep the information of underlying AdS SUSY--m $\{3/2\}$ -inverse radius of AdS space before SUSY breaking.
- \checkmark AMSB is not due to any anomaly of SUSY itself, but due to the need to add local counterterms to preserve SUSY of the 1PI effective action.

 \triangleright Unique input parameter in minimal AMSB: m $\{3/2\}$ --inverse AdS radius & SUSY breaking Ø Too predictive! ----Tachyonic slepton masses. Necessitate additional contributions. \triangleright Flavor conservation

Anomaly Mediated SUSY Breaking

 Simplest derivation in the compensator approach assuming special sequestered form for Kahler potential

RS-formalism

For global SUSY

$$
\mathcal{L}=\int d^4\theta K(\Phi^\dagger,e^V\Phi)+\int d^2\theta \left[W(\Phi)-\frac{i}{16\pi}\tau W^aW_a\right]
$$

can couple to gravity via

$$
S \sim \int d^4x e \left[\int d^4\theta f(\Phi^\dagger, e^V \Phi) \frac{\Sigma^\dagger \Sigma}{M_{Pl}^2} + \int d^2\theta \{ \frac{\Sigma^3}{M_{pl}^3} W(\Phi) - \frac{i}{16\pi} \tau W^a W_a \} \right]
$$

with

$$
f(\Phi^{\dagger}, e^V \Phi) = -3M_{pl}^2 e^{-K/3M_{pl}^2}
$$
, $\tau \equiv \theta_{YM} + i \frac{4\pi}{g^2}$.

After re-scaling of the vierbein with

$$
e^a_\mu \rightarrow e^a_\mu e^{-K/12 M_{Pl}^2} \; ,
$$

we will obtain the resulting Lagrangian with canonical Einstein form. \mathcal{L}

$$
\mathcal{L}_{\text{eff}} = \int d^4 \theta \psi^\dagger e^V \psi \frac{\Sigma^\dagger \Sigma}{M^2} + \int d^2 \theta \frac{\Sigma^3}{M^3} (m_0 \psi^2 + y \psi^3) \n- \frac{i}{16\pi} \int d^2 \theta \tau W^\alpha W_\alpha + h.c. ,
$$
\n
$$
\frac{\Sigma \psi}{M} \to \psi
$$
\n
$$
\mathcal{L}_{\text{eff}} = \int d^4 \theta Z \left(\frac{\mu M}{\Lambda \Sigma}, \frac{\mu M}{\Lambda \Sigma^\dagger} \right) \psi^\dagger e^V \psi
$$
\n
$$
+ \int d^2 \theta y \psi^3 - \frac{i}{16\pi} \int d^2 \theta \tau W^\alpha W_\alpha + h.c.
$$
\n
$$
\tau = i \frac{\tilde{b}}{2\pi} \ln \left(\frac{\mu M}{\Lambda \Sigma} \right)
$$
\n
$$
\sqrt{\langle \Sigma \rangle} = M + \mathcal{F}_{\Sigma} \theta^2
$$
\n
$$
M_{\lambda} = \frac{i}{2\tau} \frac{\partial \tau}{\partial \Sigma} \Big|_{\Sigma = M} \mathcal{F}_{\Sigma} = \frac{bg^2}{16\pi^2} \frac{\mathcal{F}_{\Sigma}}{M}
$$
\n
$$
M_{\widetilde{\psi}}^2 = -\frac{1}{4} \left(\frac{\partial \gamma}{\partial g} \beta_g + \frac{\partial \gamma}{\partial y} \beta_g \right) \frac{|\mathcal{F}_{\Sigma}|^2}{M^2} .
$$

Deflected Anomaly Mediation

- \triangleright Deflect the AMSB Renormalization Group trajectory by thresholds determined by VEV of light fields.
- \triangleright Natural with vector-like thresholds---non-decouple
- \triangleright Possible gauge mediation or Yukawa mediation contributions (with interference terms of mAMSB)
- Slepton masses can change into positive after RGE.
- \triangleright Natural with neutrino mass generation mechanism:
- Messengers: Additional triplets or singlet within complete GUT multiplets
- Interactions: Couplings of lepton to such messengers

Neutrino Seesaw Mechanism

Type II Neutrino Seesaw Mechanism-Non SUSY

Consists of SM Higgs $H \sim (1, 2, 1)$ and an SU(2)_L triplet scalar $\Delta \sim (1, 3, 2)$ The most general renormalizable potential $V = M_{\Delta}^2 Tr \Delta^{\dagger} \Delta - m_H^2 H^{\dagger} H + \frac{\lambda}{4} (H^{\dagger} H)^2 + \lambda_1 (H^{\dagger} H) Tr(\Delta^{\dagger} \Delta)$ The μ-term violates the $+\lambda_2 (Tr \Delta^{\dagger} \Delta)^2 + \lambda_3 Tr (\Delta^{\dagger} \Delta)^2 + \lambda_4 H^{\dagger} \Delta \Delta^{\dagger} H + \mu H^T i \tau_2 \Delta^{\dagger} H$ independent of M_{Δ}

The minimization condition for the scalar potential: $v_{\Delta} \approx \frac{\mu}{M_{\Delta}^2} v^2$,

The neutrino mass: $\mathcal{L} \supseteq -iY_{\nu}L^{T}C\sigma_{2}\Delta L + h.c. \Rightarrow m_{\nu} = Y_{\nu}v_{\Delta} = Y_{\nu}\frac{\mu v^{2}}{M_{\nu}^{2}}$.

Modify the p-parameter: $\rho = \frac{M_W^2}{c^2 M^2} = \frac{v^2 + 2v_\Delta^2}{v^2 + 4v^2} \approx 1.0005$

 v_{Δ} must less than $\mathcal{O}(1)$ GeV . Can be relaxed in Georgi-Machacek model.

For example, see Du &Fei Wang, 2409.20198

lepton number by 2 unit.

SUSY Type II Neutrino Seesaw Mechanism

The superpotential involves two SU(2)_L triplets: $T = \begin{pmatrix} T^0 & -\frac{1}{\sqrt{2}}T^+ \\ -\frac{1}{\sqrt{2}}T^+ & -T^{++} \end{pmatrix}$ $\qquad \bar{T} = \begin{pmatrix} T^{--} & -\frac{1}{\sqrt{2}}T^- \\ -\frac{1}{\sqrt{2}}\bar{T}^- & -\bar{T}^0 \end{pmatrix}$ ----- holomorphism of superpotential $W = \frac{1}{\sqrt{2}} \mathbf{Y}_{T}^{ij} L_{i} T L_{j} + \frac{1}{\sqrt{2}} \lambda_{1} H_{1} T H_{1} + \frac{1}{\sqrt{2}} \lambda_{2} H_{2} \bar{T} H_{2} + M_{T} T \bar{T} + \mu H_{1} H_{2},$ Vanishing of F-terms for triplets gives: $v_{\Delta} = -\frac{\lambda_2 v_2^2}{\sqrt{2}M_{\rm m}}$, $v_{\bar{\Delta}} = -\frac{\lambda_1 v_1^2}{\sqrt{2}M_{\rm m}}$. Subleading contributions of order M_{SUSY} Economical: Tri-scalar couplings involving the triplets determined also by M_T
Soft SUSY breaking parameters: $-\mathcal{L}_{II} \supset m_T^2 |T|^2 + m_T^2 |\bar{T}|^2 + \frac{A_T}{\sqrt{2}} Y_T LTL + \frac{A_{\lambda_1}}{\sqrt{2}} \lambda_1 H_1 TH_1 + \frac{A_{\lambda_2}}{\sqrt{2}} \lambda_2 H_2 \bar{T} H_2$ Soft SUSY breaking parameters: $+ B_T M \overline{T} T + \mathcal{L}_{MSSM}$,

Spoil GUT---needs to be fiited int complete SU(5) representations.

 Adopting universal boundary conditions for sfermions (such as mSUGRA), non-vanishing LFV in the mass matrices of the left-handed sleptons will be triggered through radiative corrections

$$
(\mathbf{m}_{\tilde{L}}^2)_{ij} \propto m_0^2 (\mathbf{Y}_T^{\dagger} \mathbf{Y}_T)_{ij} \log \frac{M_G}{M_T}, \qquad i \neq j, \qquad \text{Br}(l_i \to l_j \gamma) \propto \alpha^3 m_{l_i}^5 \frac{|(\mathbf{m}_{\tilde{L}}^2)_{ij}|^2}{m_{\text{SUSY}}^8} \tan^2 \beta
$$

Type II Neutrino Seesaw Mechanism in NMSSM

MSSM+type II--same difficulties as MSSM after the decouplings of triplets! Go to NMSSM+type II model !

The superpotential in type II+NMSSM

 $W_1 \supseteq W_{\text{NMSSM}} + y_{ij}^L L_j L_i \Delta_T + y_{\Delta}^d \Delta_T H_d H_d$ $+m_T\overline{\Delta}_T\Delta_T+y^u_{\Lambda}\overline{\Delta}_TH_uH_u,$

The scalar potential: The neutrino mass:

$$
W_{\text{NMSSM}} = W_{\text{MSSM}/\mu} + \lambda S H_u H_d + \frac{\kappa}{3} S^3 + \xi_S S + \cdots.
$$

 $V \supset \left| y_{ij}^L L_i L_j + y_\Delta^d H_d H_d + m_T \overline{\Delta}_T \right|^2 + \left| y_\Delta^u H_u H_u + m_T \Delta_T \right|^2$ + $\left|\lambda S H_u + 2 y_{\Delta}^d \Delta_T H_d\right|$ + $\left|\lambda S H_d + 2 y_{\Delta}^u \overline{\Delta}_T H_u\right|$ +

Soft SUSY parametetrs still constribute to neutrino masses!

Large cancellation? Low m_T large μ or A-term ?

 $(m_v)_{ij} = -y_{ij}^L \left[y_{\Delta}^u \frac{v_u^2}{m_T} + y_{\Delta}^d \frac{A_{H_d H_d \Delta_T} v_d^2}{m_T^2} + y_{\Delta}^d \frac{2\mu \tan \beta v_d^2}{m_T^2} \right]$

1. What are the M $_{\rm T}$ scale and SUSY breaking scale ? $\hskip1cm \vert$ Heavy m $_{\rm T}$ as th

Questions:

2. LFV constraints?

Heavy m_T as the messenger scale for deflected AMSB /GMSB

Type-II Seesaw in NMSSM from GMSB/AMSB

Embedding into complete SU(5) representations

$$
\Delta(1,3,1) \subset \mathbf{15}, \quad \overline{\Delta}(1,\overline{3},-1) \subset \overline{\mathbf{15}} \qquad \qquad \overline{\mathbf{15}} = \Delta_{\mathbf{S}}(\mathbf{6},1)_{-2/3} \oplus \Delta_{\mathbf{T}}(\mathbf{1},3)_{1} \oplus \Delta_{(D)}(\mathbf{3},2)_{1/6}.
$$
\n
$$
\text{Relevant superpotential:}
$$
\n
$$
W_{0} \supseteq \frac{y_{15}^{u}_{15}}{2} \overline{\mathbf{15}}_{\Delta} \cdot \mathbf{5}_{H} \cdot \mathbf{5}_{H} + \frac{y_{15}^{d}_{2}}{2} \mathbf{15}_{\Delta} \cdot \overline{\mathbf{5}}_{H} \cdot \overline{\mathbf{5}}
$$

Type-II Seesaw in NMSSM from GMSB

Ordinary GMSB realization of NMSSM----Origin of mu-term & explain Higgs mass!

 $V_{\text{Higgs}}(s) \sim m_S^2 s^2 + \frac{2}{3} \kappa A_\kappa s^3 + \kappa^2 s^4$

Necessary condition for an absolute minimum with $s \neq 0$

$$
A_k^2 \gtrsim 9 \, m_S^2
$$

In general, needs large trilinear couplings A_{κ}

Yukawa deflection contributions: $A_{ab} = -\frac{1}{\sqrt{a^{ij}}\Delta\left(\lambda^*,\lambda_{bij}\right)}\Lambda$ Messenger-matter type interactions! $32\pi^2$ $+\frac{1}{4}d_a^{ij}d_b^{k\ell}\Delta\left(\lambda_{aij}^*\lambda_{cij}\right)\Delta\left(\lambda_{ck\ell}^*\lambda_{bk\ell}\right)-d_a^{ij}C_r^{aij}g_r^2\Delta\left(\lambda_{aij}^*\lambda_{bij}\right)\bigg)\Lambda^2\\ \nonumber \qquad \ \ s\Big(m_S^2+m_S'^2+\mu'^2+2\kappa\xi_F+\kappa A_\kappa s+2\kappa^2s^2+3\kappa s\mu'+\lambda^2(v_u^2+v_d^2)-2\lambda\kappa v_u v_d\Big)\\ \nonumber \qquad \ \ s\Big(m_S^2+m_S'^2+\mu'^2+2\kappa\xi_F+\kappa$ Chacko(2001), Shih,JHEP (2013) $+\xi_s + \xi_F u' - \lambda v_u v_d (A_1 + u') = 0$.

1. Ordinary GMSB predict vanishing A-terms at the messenger scale!

2. S-gauge singlet !

Increase simultaneously A-term and $\mathsf{m}_\mathsf{S}{}^2$, still challenging for GMSB.

Better with a tadpole for S.

Type-II Seesaw in NMSSM from dAMSB

AMSB-naturally large A-terms

Ordinary AMSB realization of NMSSM ---still non-trivial for successful EWSB

Triplets act as messengers

 --messenger-matter contributions in AMSB General discussions: see Fei Wang (2016)

Large A_λ, A_κ needs large λ and κ, $\qquad \qquad \mid$ induce large positive m_S^2 suppressing the singlet VEV

New interactions involving H_u , H_d and triplets will lead to additional contributions to A_{k}

----easily EWSB even without S tadpole $\sqrt{ }$

Numerical Results---GMSB case

Higgs mass: $m_h^2 \simeq M_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta - \frac{\lambda^2}{\mu^2} v^2 (\lambda - \kappa \sin 2\beta)^2$ $+\frac{3m_t^4}{4\pi^2 v^2} \left[\ln \left(\frac{m_{\tilde T}^2}{m_t^2} \right) + \frac{A_t^2}{m_{\tilde T}^2} \left(1 - \frac{A_t^2}{12m_{\tilde T}^2} \right) \right],$

Large A_t -less BG Fine-Tuning.

Lepton Flavor Violation constraints

- \checkmark If sfermion masses being universal at high energy, flavour conservation can be broken in the sfermion masses by radiative effects due to flavor-violating Yukawa couplings
- \checkmark The interactions that generate the neutrino mass also induce LFV in the slepton mass matrices by renormalization effects.
- \checkmark Sfermion masses no longer universal at messenger scale.

SUGRA-type DAMSB/GMSB-type

 \checkmark Non-diagonal sfermion masses for each type at messenger scale!

$$
\Delta(m_{\tilde{L}}^2)_{ij} \propto \left[(y^L)^\dagger y_L \right]_{ij} , \text{ for } i \neq j
$$

- \checkmark LFV interactions decouple with the decouple of messengers
- \checkmark LFV effects from RGE suppressed, generated beyond one-loop

$$
\Delta \left(m_{\tilde{L}}^2 \right)_{ij} \approx -\frac{6}{8\pi^2} \left(3m_{\tilde{L}_L}^2 \right) \left[Y_{ik}^{L\dagger} Y_{kj}^L \right] \log \left(\frac{M_U}{m_T} \right) \sim 0
$$

 \checkmark Relations of LFV processes from $|(\hat{y}^L)^{\dagger} y_L|$

See Joaquim& Rossi,PRL (2006)

Lepton Flavor Violation constraints

$$
(m_{\nu})_{ij} \approx y_{ij}^{L} \frac{y_{\Delta}^{u} v_{u}^{2}}{\sqrt{2}m_{T}}
$$
\n
$$
m_{\nu}^{diag} = U^{T} m_{\nu} U
$$
\n
$$
\left[(y^{L})^{\dagger} y_{L} \right] \approx \left(\frac{\sqrt{2}m_{T}}{y_{\Delta}^{u} v_{u}^{2}} \right)^{2} U (m_{\nu}^{diag})^{2} U^{-1}
$$
\n
$$
= \left(\frac{\sqrt{2}m_{T}}{y_{\Delta}^{u} v_{u}^{2}} \right)^{2} \sum_{k} m_{k}^{2} U_{ik} U_{jk}^{*}.
$$
\nPMNS matrix:

$$
U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}
$$

Note: With messenger scale identified with the heavy triplet scalar scale, leading log contribution being $\Delta(m_{\tilde{L}}^2)_{ij}^{RGE} \propto \frac{1}{(16\pi^2)^2} (m_{\tilde{L}}^2)_{mess} \left[(y^L)^\dagger y_L \right]_{ii} \sim 0 \ , \label{eq:Delta}$

GMSB prediction in our case

$$
m_{\tilde{L}_{L,a}}^{2} = \left(\frac{F_{X}^{2}}{M^{2}}\right) \frac{1}{(16\pi^{2})^{2}} \left[2\left(y_{LL\Delta_{T};a}\right)^{2} \tilde{G}_{LL\Delta_{T};a}^{+} + 3\left(y_{LD\Delta_{3,2};a}\right)^{2} \tilde{G}_{LD\Delta_{3,2};a}^{+} + \left(\frac{3}{2}g_{2}^{4} + \frac{3}{10}g_{1}^{4}\right)7\right]
$$

$$
\Delta(m_{\tilde{L}}^{2})_{ij} \propto \left[(y^{L})^{\dagger}y_{L}\right]_{ij}, \text{ for } i \neq j
$$

whether $(y_{LD\Delta_{3,2}})_{ij} \approx 0$, or $(y_{LD\Delta_{3,2}})_{ij} = (y_{LL\Delta_{T}})_{ij}$.

$$
\frac{Br(l_i \to l_j + \gamma)}{Br(l_i \to l_j + \nu_i \bar{\nu}_j)} \simeq \frac{48\pi^3 \alpha}{G_F^2 m_{SUSY}^4} \left(\frac{g^4 \tan^2 \beta}{(16\pi^2)^2} |\Delta(m_{\tilde{L}}^2)_{ij}|^2\right)
$$

Current best limit from MEG:
\n
$$
Br(\mu^+ \to e^+ \gamma) < 4.2 \times 10^{-13}
$$
, (90% CL)
\n $\implies F_X/m_T > 3 \times 10^{-6} m_T$ For GMSB

Lepton Flavor Violation constraints

$$
\frac{\text{BR}(\tau \to \mu \gamma)}{\text{BR}(\mu \to e \gamma)} \approx \left[\frac{(\mathbf{m}_{\tilde{L}}^2)_{\tau \mu}}{(\mathbf{m}_{\tilde{L}}^2)_{\mu e}} \right]^2 \frac{\text{BR}(\tau \to \mu \nu_{\tau} \bar{\nu}_{\mu})}{\text{BR}(\mu \to e \nu_{\mu} \bar{\nu}_{e})} \approx \begin{cases} 3.48 & \text{(IO)} \\ 3.85 & \text{(NO)} \end{cases} \qquad \frac{\text{BR}(\tau \to \mu \mu \mu)}{\text{BR}(\mu \to eee)} \approx \begin{cases} 1.16 & \text{(IO)} \\ 1.28 & \text{(NO)} \end{cases}
$$

$$
\frac{\text{BR}(\tau \to e\gamma)}{\text{BR}(\mu \to e\gamma)} \approx \left[\frac{(\mathbf{m}_{\tilde{L}}^2)_{\tau e}}{(\mathbf{m}_{\tilde{L}}^2)_{\mu e}} \right]^2 \frac{\text{BR}(\tau \to e\nu_{\tau}\bar{\nu}_e)}{\text{BR}(\mu \to e\nu_{\mu}\bar{\nu}_e)} \approx \begin{cases} 0.15 & (\text{IO}) \\ 0.20 & (\text{NO}) \end{cases} \qquad \frac{\text{BR}(\tau \to eee)}{\text{BR}(\mu \to eee)} \approx \begin{cases} 0.25 & (\text{IO}) \\ 0.33 & (\text{NO}) \end{cases}
$$

 ${\rm BR}(\mu \to e \nu_\mu \bar{\nu}_e) \approx 1$ ${\rm BR}(\tau\to \mu\nu_\tau\bar{\nu}_\mu)\approx 17\%$ $BR(\tau \to e \nu_{\tau} \bar{\nu}_e) \approx 18\%.$

$$
10 \qquad \begin{cases} (Y_T^{\dagger}Y_T)_{12} = \tilde{m}^{-2}(-4.9884258684717194 \times 10^{-14} - 2.707927523936294 \times 10^{-13}i) \\ (Y_T^{\dagger}Y_T)_{13} = \tilde{m}^{-2}(-8.978389780251289 \times 10^{-14} - 2.393322032527885 \times 10^{-13}i) \\ (Y_T^{\dagger}Y_T)_{23} = \tilde{m}^{-2}(-1.2455893961495833 \times 10^{-12} - 4.989638020041978 \times 10^{-15}i) \end{cases}
$$

\n
$$
\begin{cases} (Y_T^{\dagger}Y_T)_{12} = \tilde{m}^{-2}(-2.525515609275667 \times 10^{-13} - 1.442205288833084 \times 10^{-14}i) \\ (Y_T^{\dagger}Y_T)_{12} = \tilde{m}^{-2}(-2.690250638429381 \times 10^{-13} - 1.2757862388391988 \times 10^{-14}i) \end{cases}
$$

NO
$$
(Y_T^{\dagger}Y_T)_{13} = m^{-2}(-2.690250638429381 \times 10^{-2} - 1.2757862388391988 \times 10^{-2}i)
$$

$$
(Y_T^{\dagger}Y_T)_{23} = \tilde{m}^{-2}(1.2038006323809836 \times 10^{-12} + 2.6787676694189162 \times 10^{-16}i)
$$

Similar discussions as for MSSM, Joaquim& Rossi, NPB (2006)

- \triangleright Type II neutrino seesaw extension of NMSSM can origin from dAMSB/GMSB.
- Ø Combination of Type II seesaw with SUSY breaking mechanism in NMSSM can be advantageous.
- \triangleright The identification of messenger scale with triplet scale, can be fairly predictive and non-trivial.
- \triangleright Can set constrains for SUSY breaking parameters by LFV constraints (in addition to predictions on ratios of LFV processes).
- \triangleright Need more LFV works for dAMSB.

Thanks!

Back Up Slides

$$
m_{\tilde{U}_{L,a}^{c}}^{2} = \left(\frac{F_{X}^{2}}{M^{2}}\right) \frac{1}{(16\pi^{2})^{2}} \left[-2y_{t}^{2} \Delta \tilde{G}_{y_{t}} \delta_{a,3} + \left(\frac{8}{3}g_{3}^{4} + \frac{8}{15}g_{1}^{4}\right)7\right],
$$

\n
$$
m_{\tilde{D}_{L,a}^{c}}^{2} = \left(\frac{F_{X}^{2}}{M^{2}}\right) \frac{1}{(16\pi^{2})^{2}} \left[3\left(y_{DD\Delta_{se};a}\right)^{2} \tilde{G}_{DD\Delta_{se};a}^{+} + 2\left(y_{LD\Delta_{3,2};a}\right)^{2} \tilde{G}_{LD\Delta_{3,2};a}^{+} - 2y_{b}^{2} \Delta \tilde{G}_{y_{b}} \delta_{a,3} + \left(\frac{8}{3}g_{3}^{4} + \frac{2}{15}g_{1}^{4}\right)7\right],
$$

\n
$$
m_{\tilde{L}_{L,a}}^{2} = \left(\frac{F_{X}^{2}}{M^{2}}\right) \frac{1}{(16\pi^{2})^{2}} \left[2\left(y_{LL\Delta_{T};a}\right)^{2} \tilde{G}_{LL\Delta_{T};a}^{+} + 3\left(y_{LD\Delta_{3,2};a}\right)^{2} \tilde{G}_{LD\Delta_{3,2};a}^{+} + \left(\frac{3}{2}g_{2}^{4} + \frac{3}{10}g_{1}^{4}\right)7\right],
$$

$$
\tilde{G}_{DD\Delta_{se};a}^{+} = 10 \left(y_{15;\mathbf{a}}^{L} \right)^{2} + \sum_{c} \left(y_{15;\mathbf{c}}^{L} \right)^{2} + (y_{X})^{2}
$$

$$
+ 4y_{b}^{2} \delta_{a,3} - 12g_{3}^{2} - \frac{4}{3} g_{1}^{2},
$$

$$
\tilde{G}_{LD\Delta_{3,2};a}^{+} = 10 \left(y_{15;\mathbf{a}}^{L} \right)^{2} + \sum_{c} \left(y_{15;c}^{L} \right)^{2} + (y_{X})^{2}
$$

$$
+ 2y_{b}^{2} \delta_{a,3} - \frac{16}{3} g_{3}^{2} - 3g_{2}^{2} - \frac{7}{15} g_{1}^{2},
$$

$$
\tilde{G}_{LL\Delta_{T};a}^{+} = 10 \left(y_{15;\mathbf{a}}^{L} \right)^{2} + \left(y_{15}^{d} \right)^{2} + \sum_{c} \left(y_{15;c}^{L} \right)^{2}
$$

$$
+ (y_{X})^{2} - 7g_{2}^{2} - \frac{9}{5} g_{1}^{2},
$$