Rare decays with polarised Λ_b

Michal Kreps

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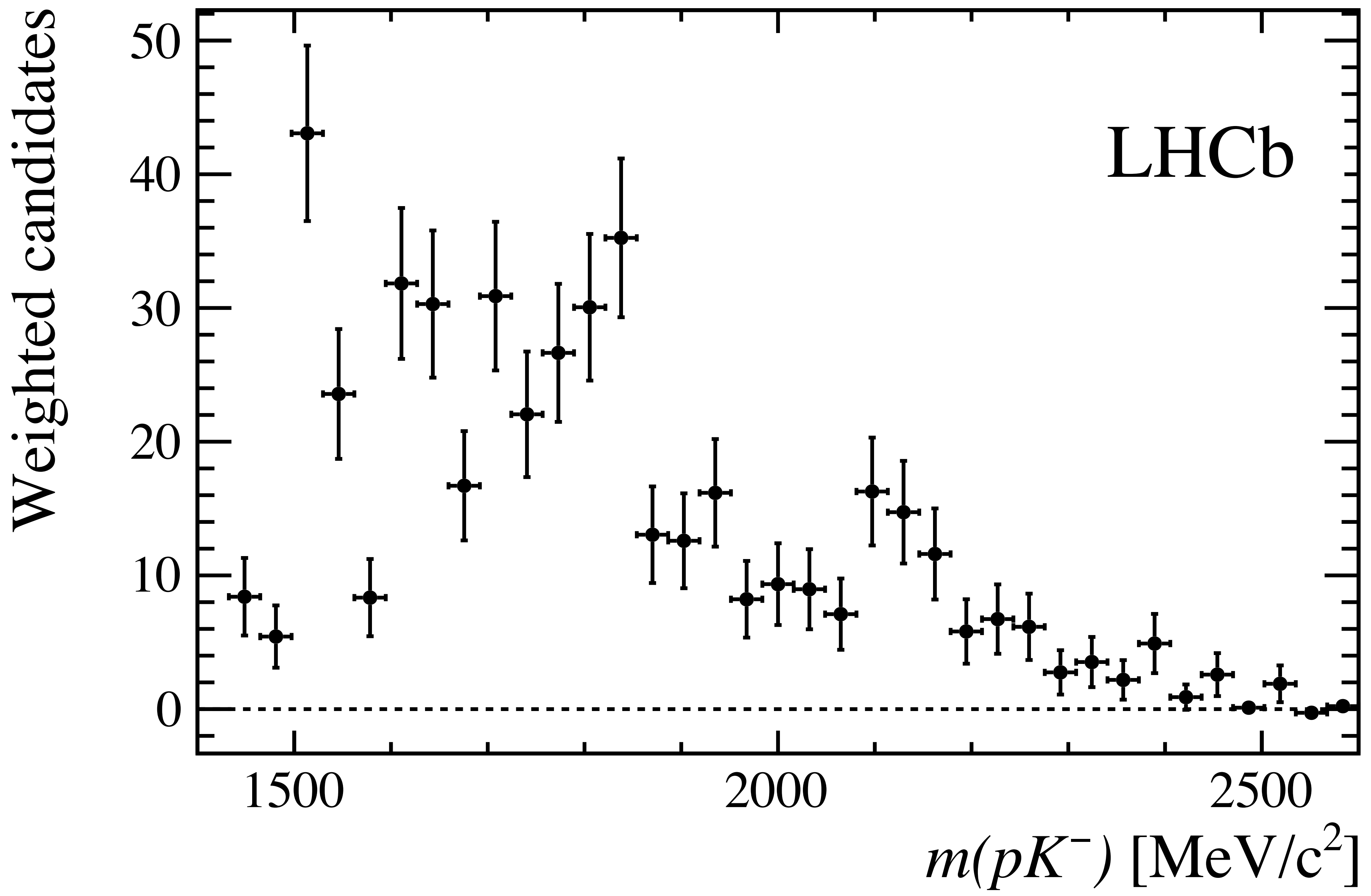
Introduction

- ➡ Decays governed by $b \to sll$ transitions are sensitive probes for new physics
- ➡ Well studied for meson decays
- ➡ Baryon decays provide complementary information
  - ❖ Different spin structure
  - ❖ Differences in hadronic structure
- ➡ Decays $\Lambda_b \to \Lambda\mu\mu$ well studied (JHEP 01 (2015) 155, JHEP 11 (2017) 138 and many others)
- ➡ Decays $\Lambda_b \to \Lambda^*\mu\mu$ with spin 1/2 and 3/2 Λ^* studied previously (1903.10553, JHEP 07 (2020) 002, JHEP 06 (2019) 136, Eur. Phys. J. Plus 136 (2021) 614)

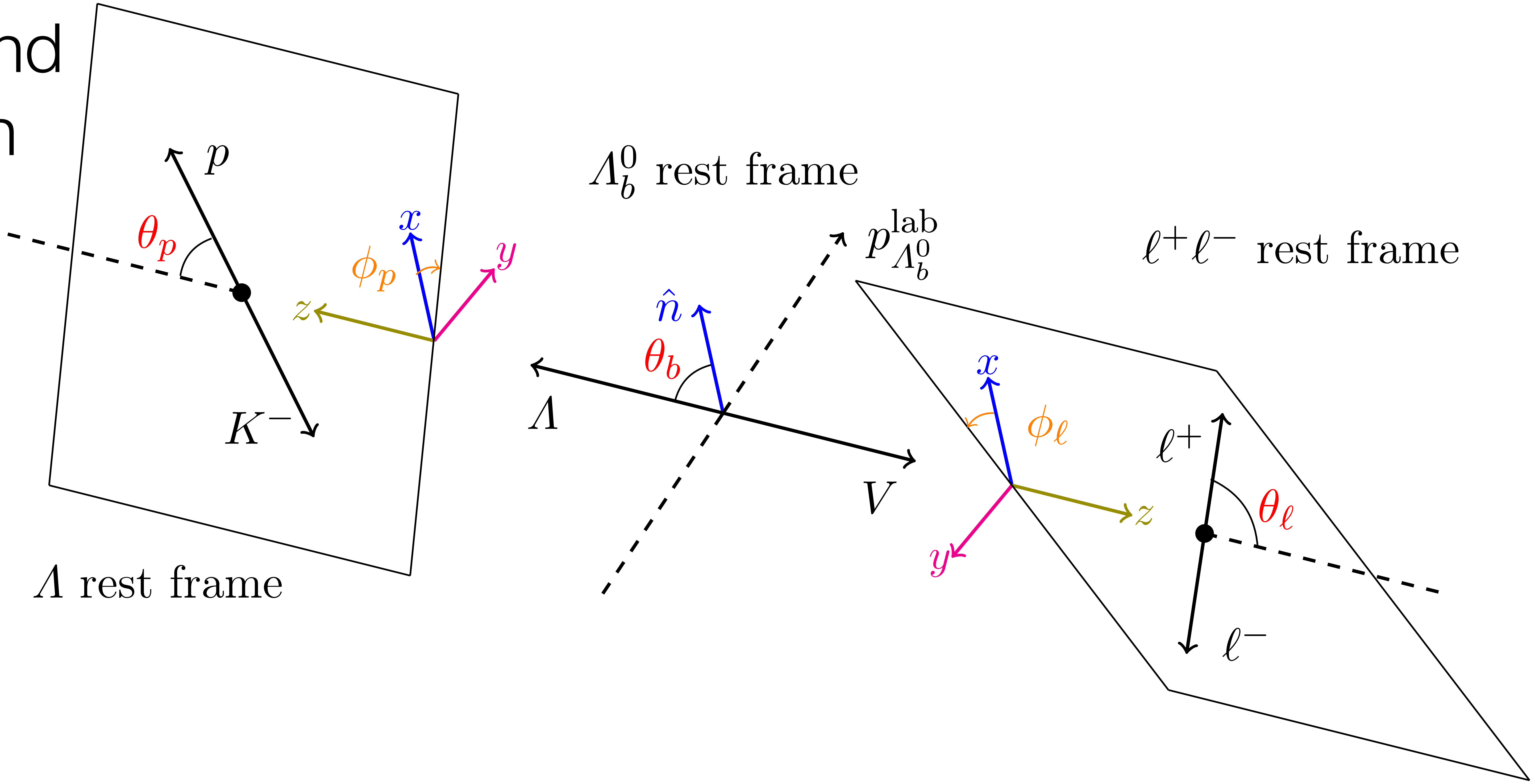
LHCb-PAPER-2019-040

Background subtracted $\Lambda_b \to pK\mu\mu$ in $0.1 < q^2 < 6.0$ GeV^2



Angular distributions

- ➡ With polarised production, 5 angles to describe kinematics
- ➡ Without polarisation, one is sensitive only to $\phi_l + \phi_b$
- ➡ Angle θ should correspond to production polarisation axis
  - ❖ Figure shows case for *pp* collisions with transverse polarisation
  - ❖ For *Z* decays one has to take relevant polarisation axis



Angular distribution

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➡ The full angular distribution with several interfering spin states can be easily written in the helicity formalism

➡ Full decay rate

operators

spin states

$$\frac{d^7\Gamma}{dq^2\, dm_{pK} d\vec{\Omega}} = \frac{1}{m_{\Lambda_b}^2}\frac{N_1^2}{2^6(2\pi)^7}\frac{|\vec{k}||\vec{k}_1||\vec{q}_1|}{\sqrt{q^2}}\sum_{\lambda_b}\mathcal{P}_{\lambda_b}\sum_{\lambda_1,\lambda_2,\lambda_p}\Big|\sum_{\mathcal{O}_i}\sum_{\Lambda}\sqrt{J_\Lambda+\frac{1}{2}}\sum_{\lambda_\Lambda} g_{\lambda_V\lambda_V}$$

Λ_b decay amplitudes $\quad \times \mathcal{H}^{\Lambda,\mathcal{O}_i}_{\lambda_\Lambda,\lambda_V}(q^2, m_{pK})\, d^{1/2}_{\lambda_b,\lambda_\Lambda-\lambda_V}(\theta_b)$

dimuon system amplitudes $\quad \times \tilde{h}^{\mathcal{O}_i,\lambda_V}_{\lambda_1,\lambda_2}(q^2) D^{J_V}_{\lambda_V,\lambda_1-\lambda_2}(\phi_\ell, \theta_\ell, -\phi_\ell)^*$

Λ^* decay amplitudes $\quad \times h^{\Lambda}_{\lambda_\Lambda,\lambda_p}(m_{pK}) D^{J_\Lambda}_{\lambda_\Lambda,\lambda_p}(\phi_p, \theta_p, -\phi_p)^*\Big|^2,$

Wigner d-functions

➡ Several terms will have same angular term, so want to group them

Angular distribution structure

$$\frac{d^7\Gamma}{dq^2\, dm_{pK} d\vec{\Omega}} = \frac{1}{m_{\Lambda_b}^2} \frac{N_1^2}{2^6(2\pi)^7} \frac{|\vec{k}||\vec{k}_1||\vec{q}_1|}{\sqrt{q^2}} \sum_{\lambda_b} \mathcal{P}_{\lambda_b} \sum_{\lambda_1,\lambda_2,\lambda_p} \Big| \sum_{\mathcal{O}_i} \sum_{\Lambda} \sqrt{J_\Lambda + \frac{1}{2}} \sum_{\lambda_\Lambda} g_{\lambda_V \lambda_V} \times \mathcal{H}^{\Lambda,\mathcal{O}_i}_{\lambda_\Lambda,\lambda_V}(q^2, m_{pK}) d^{1/2}_{\lambda_b,\lambda_\Lambda-\lambda_V}(\theta_b) \times \tilde{h}^{\mathcal{O}_i,\lambda_V}_{\lambda_1,\lambda_2}(q^2) D^{J_V}_{\lambda_V,\lambda_1-\lambda_2}(\phi_\ell, \theta_\ell, -\phi_\ell)^* \times h^{\Lambda}_{\lambda_\Lambda,\lambda_p}(m_{pK}) D^{J_\Lambda}_{\lambda_\Lambda,\lambda_p}(\phi_p, \theta_p, -\phi_p)^* \Big|^2 ,$$

Spin-density matrix

$$\frac{1}{2}\begin{pmatrix} 1 + \mathcal{P}_{\Lambda_b^0} \cos\theta_b & \mathcal{P}_{\Lambda_b^0} \sin\theta_b \\ \mathcal{P}_{\Lambda_b^0} \sin\theta_b & 1 - \mathcal{P}_{\Lambda_b^0} \cos\theta_b \end{pmatrix}$$

- ➡ Set of terms without any dependence on polarisation
- ➡ Set of terms proportional to $P_b \cos\theta$ with same amplitude structure as unpolarised terms
- ➡ Set of terms proportional to $P_b \sin\theta$ where amplitude structure is different

Angular basis

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- No unique option how to group terms, pick one based on associated Legendre polynomials ($\Lambda_b \to pK\mu\mu$)
  - Related to angular momentum and makes it easy to keep track of terms
  - Resulting functions are orthogonal (own weights for the method of moments)
  - For $\Lambda_b \to \Lambda\mu\mu$ bases we chose was slightly suboptimal, but relates to Legendre polynomials
- Final basis:

$$f(\vec{\Omega}; l_{\rm lep}, l_{\rm had}, m_{\rm lep}, m_{\rm had}) = 2 n_{l_{\rm lep}}^{m_{\rm lep}} n_{l_{\rm had}}^{m_{\rm had}} P_{l_{\rm lep}}^{|m_{\rm lep}|}(\cos\theta_\ell) P_{l_{\rm had}}^{|m_{\rm had}|}(\cos\theta_p)$$
$$\times \begin{cases} \sin(|m_{\rm lep}|\phi_\ell + |m_{\rm had}|\phi_p) & m_{\rm lep} \leq 0 \text{ and } m_{\rm had} \leq 0 \\ \cos(|m_{\rm lep}|\phi_\ell + |m_{\rm had}|\phi_p) & m_{\rm lep} \geq 0 \text{ and } m_{\rm had} \geq 0 \end{cases}$$

- The angular distribution takes form

$$\frac{32\pi^2}{3} \frac{{\rm d}^7\Gamma}{{\rm d}q^2\,{\rm d}m_{pK}\,{\rm d}\vec{\Omega}} = \sum_{i=1}^{178} K_i(q^2, m_{pK}) f_i(\vec{\Omega})$$

$K_i(q^2,m_{pK})$ are bilinear combinations of products of amplitudes

Anatomy of angular distribution

➡ There are 178 terms when polarisation is allowed to be non-zero

- ❖ 46 of these present also with zero polarisation and have no θ_b dependence (m_{lep}=m_{had})
- ❖ For polarised case, 46 terms have cos θ_b dependence while rest of the angles are same as unpolarised case
- ❖ Remaining terms have sin θ_b dependence with basis functions where m_{lep}≠m_{had}

$\Lambda_b \to pK\mu\mu$

i	parity combination	$J_\Lambda + J'_\Lambda$	single states 1/2	3/2	5/2	Re/Im	V/A	helicity combinations	Eq.
1	same	≥ 1	✓	✓	✓	Re		$J_\Lambda = J'_\Lambda, (\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(62)
2	same	≥ 1	✓	✓	✓	Re	✓	$J_\Lambda = J'_\Lambda, \lambda_V \neq 0, (\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(63)
3	same	≥ 1	✓	✓	✓	Re		$J_\Lambda = J'_\Lambda, (\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(64)
4	opposite	≥ 1				Re		$(\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(66)
5	opposite	≥ 1				Re	✓	$\lambda_V \neq 0, (\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(117)
6	opposite	≥ 1				Re		$(\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(118)
7	same	≥ 2		✓	✓	Re		$(\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(119)
8	same	≥ 2		✓	✓	Re	✓	$\lambda_V \neq 0, (\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(120)
9	same	≥ 2		✓	✓	Re		$(\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(121)
10	opposite	≥ 3				Re		$(\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(122)
11	opposite	≥ 3				Re	✓	$\lambda_V \neq 0, (\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(123)
12	opposite	≥ 3				Re		$(\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(124)
13	same	≥ 4			✓	Re		$(\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(125)
14	same	≥ 4			✓	Re	✓	$\lambda_V \neq 0, (\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(126)
15	same	≥ 4			✓	Re		$(\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(127)
16	opposite	≥ 5				Re		$(\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(128)
17	opposite	≥ 5				Re	✓	$\lambda_V \neq 0, (\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(129)
18	opposite	≥ 5				Re		$(\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(130)
19	opposite	≥ 1				Re			(131)
20	opposite	≥ 1				Re	✓		(132)
21	same	≥ 2		✓	✓	Re			(133)
22	same	≥ 2		✓	✓	Re	✓		(134)
23	opposite	≥ 3				Re		$\lambda_V = 0, \lvert\lambda'_V\rvert = 1$ (all possible $\lambda_\Lambda^{(\prime)}$)	(135)
24	opposite	≥ 3				Re	✓		(136)
25	same	≥ 4			✓	Re			(137)
26	same	≥ 4			✓	Re	✓		(138)
27	opposite	≥ 5				Re			(139)
28	opposite	≥ 5				Re	✓		(140)

Sensitivity to physics

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- For $\Lambda_b \to \Lambda\mu\mu$ we investigated in greater details what can be extracted
- At low-hadronic recoil amplitudes depend on combinations of Wilson coefficients

$$\rho_1^\pm = |C_\mathrm{V} \pm C_\mathrm{V}'|^2 + |C_{10} \pm C_{10}'|^2$$
$$\rho_2 = \mathrm{Re}\left(C_\mathrm{V} C_{10}^* - C_\mathrm{V}' C_{10}'^*\right) - i\mathrm{Im}\left(C_\mathrm{V} C_\mathrm{V}'^* + C_{10} C_{10}'^*\right)$$
$$\rho_3^\pm = 2\mathrm{Re}\left((C_V \pm C_\mathrm{V}')(C_{10} \pm C_{10}')^*\right)$$
$$\rho_4 = |C_\mathrm{V}|^2 - |C_\mathrm{V}'|^2 + |C_{10}|^2 - |C_{10}'|^2 - i\mathrm{Im}\left(C_\mathrm{V} C_{10}^* - C_\mathrm{V}' C_{10}'^*\right)$$

- $\mathrm{Im}(\rho_2)$ only accessible with non-zero polarisation
- One can construct relationships which depend only on short distance physics

$$\frac{K_{16}}{K_{34}} = 2\frac{\mathrm{Re}(\rho_2)}{\mathrm{Im}(\rho_2)}\,, \quad \frac{K_{25}}{K_{22}} = -\frac{\mathrm{Im}(\rho_2)}{\mathrm{Im}(\rho_4)}\,, \quad \frac{K_{23}}{K_{10}} = -\frac{\mathrm{Re}(\rho_4)}{\mathrm{Im}(\rho_4)} P_{\Lambda_b}$$

SM prediction

Obs.	Value	68% interval	Obs.	Value	68% interval
M_1	0.459	[0.453, 0.465]	M_6	0.000	[−0.005, 0.006]
M_2	0.081	[0.071, 0.094]	M_7	−0.025	[−0.034, −0.014]
M_3	−0.005	[−0.014, −0.001]	M_8	−0.003	[−0.016, 0.012]
M_4	−0.280	[−0.290, −0.262]	M_9	0.002	[0.001, 0.002]
M_5	−0.045	[−0.053, −0.037]	M_{10}	0.002	[0.001, 0.002]
M_{11}	−0.366	[−0.383, −0.338]	M_{23}	−0.147	[−0.162, −0.133]
M_{12}	0.071	[0.058, 0.081]	M_{24}	0.132	[0.120, 0.150]
M_{13}	0.001	[−0.010, 0.007]	M_{25}	−0.001	[−0.001, −0.000]
M_{14}	0.243	[0.230, 0.254]	M_{26}	0.004	[0.003, 0.005]
M_{15}	−0.052	[−0.060, −0.045]	M_{27}	0.089	[0.081, 0.099]
M_{16}	0.003	[0.001, 0.009]	M_{28}	−0.089	[−0.100, −0.080]
M_{17}	0.004	[−0.012, 0.018]	M_{29}	0.000	[0.000, 0.000]
M_{18}	0.029	[0.018, 0.037]	M_{30}	0.000	[0.000, 0.000]
M_{19}	−0.001	[−0.002, −0.001]	M_{31}	0.000	[0.000, 0.000]
M_{20}	−0.003	[−0.003, 0.002]	M_{32}	0.075	[0.035, 0.118]
M_{21}	0.002	[0.001, 0.003]	M_{33}	0.007	[0.001, 0.012]
M_{22}	−0.005	[−0.006, −0.003]	M_{34}	0.000	[−0.000, 0.000]

$1 < q^2 < 6$ GeV^2

$P_\Lambda = 1$

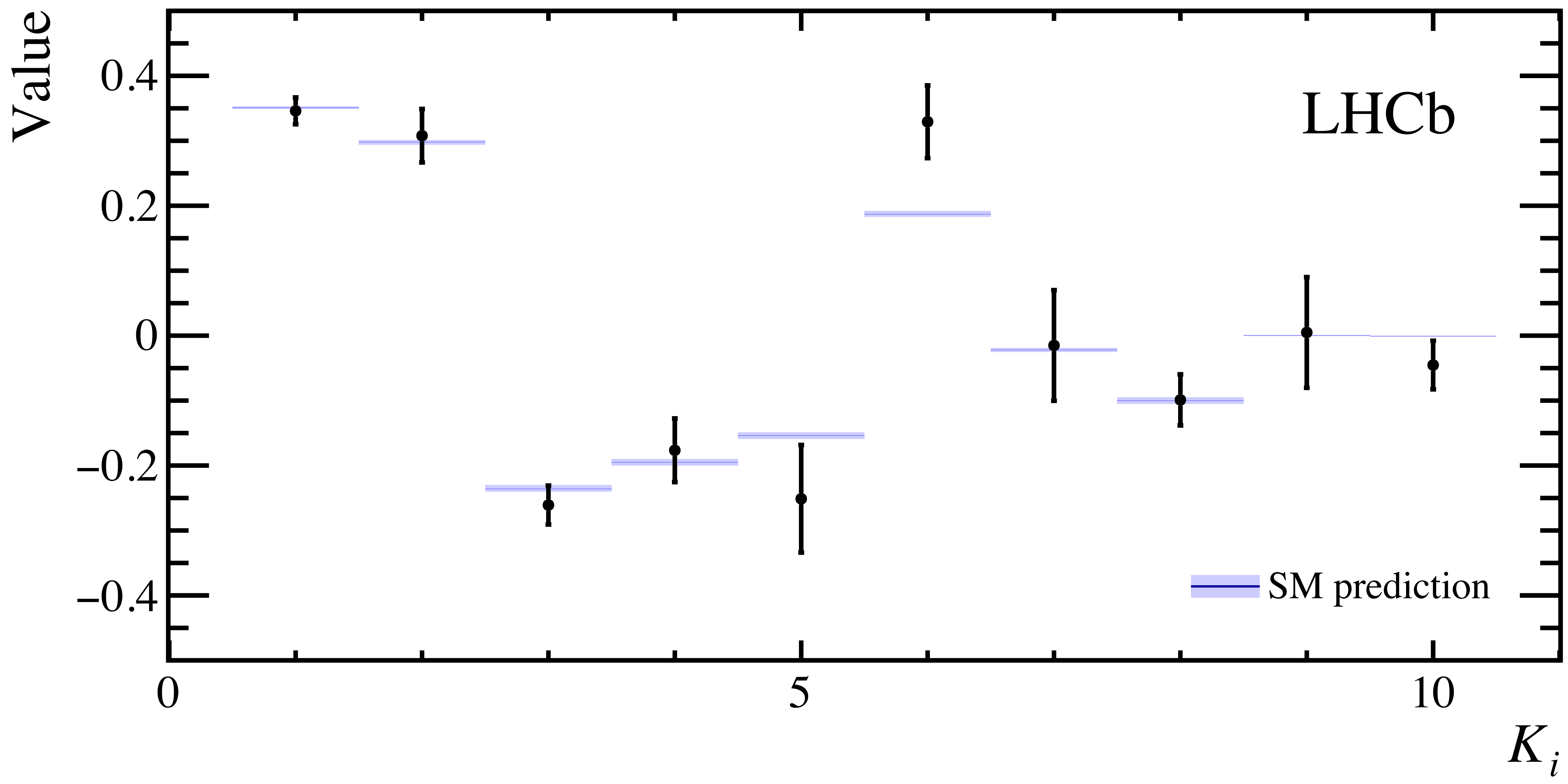
Obs.	Value	68% interval	Obs.	Value	68% interval
M_1	0.351	[0.349, 0.353]	M_6	0.187	[0.183, 0.192]
M_2	0.298	[0.294, 0.301]	M_7	−0.022	[−0.025, −0.019]
M_3	−0.236	[−0.240, −0.230]	M_8	−0.100	[−0.105, −0.095]
M_4	−0.195	[−0.200, −0.190]	M_9	0.000	[0.000, 0.001]
M_5	−0.154	[−0.159, −0.149]	M_{10}	−0.001	[−0.001, −0.000]
M_{11}	−0.064	[−0.069, −0.058]	M_{23}	−0.299	[−0.303, −0.295]
M_{12}	0.240	[0.235, 0.245]	M_{24}	0.337	[0.335, 0.338]
M_{13}	−0.292	[−0.295, −0.288]	M_{25}	−0.001	[−0.001, −0.000]
M_{14}	0.034	[0.031, 0.038]	M_{26}	0.001	[0.000, 0.001]
M_{15}	−0.191	[−0.196, −0.186]	M_{27}	0.221	[0.216, 0.226]
M_{16}	0.151	[0.146, 0.156]	M_{28}	−0.187	[−0.191, −0.183]
M_{17}	0.102	[0.096, 0.107]	M_{29}	0.000	[0.000, 0.000]
M_{18}	0.021	[0.018, 0.024]	M_{30}	−0.001	[−0.001, −0.000]
M_{19}	0.000	[0.000, 0.000]	M_{31}	0.000	[0.000, 0.000]
M_{20}	−0.001	[−0.001, −0.001]	M_{32}	−0.046	[−0.050, −0.043]
M_{21}	0.000	[0.000, 0.001]	M_{33}	−0.053	[−0.056, −0.050]
M_{22}	−0.002	[−0.002, −0.001]	M_{34}	0.000	[0.000, 0.000]

$15 < q^2 < 20$ GeV^2

$P_\Lambda = 1$

For polarisation $P_\Lambda \neq 1$, scale M_{11} — M_{34} by P_Λ

Latest measurement



JHEP09(2018)146

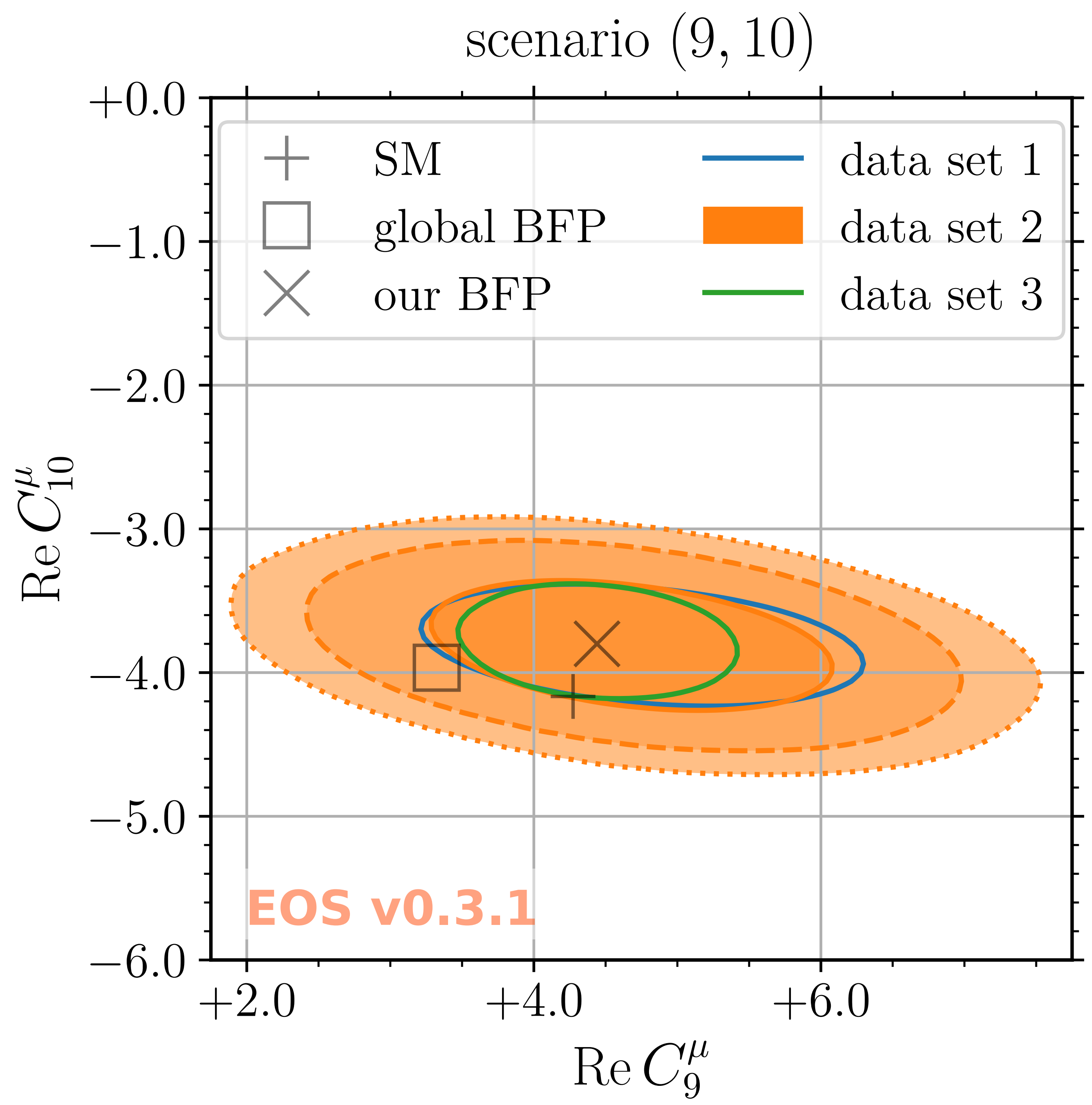
➡ Well compatible with the SM

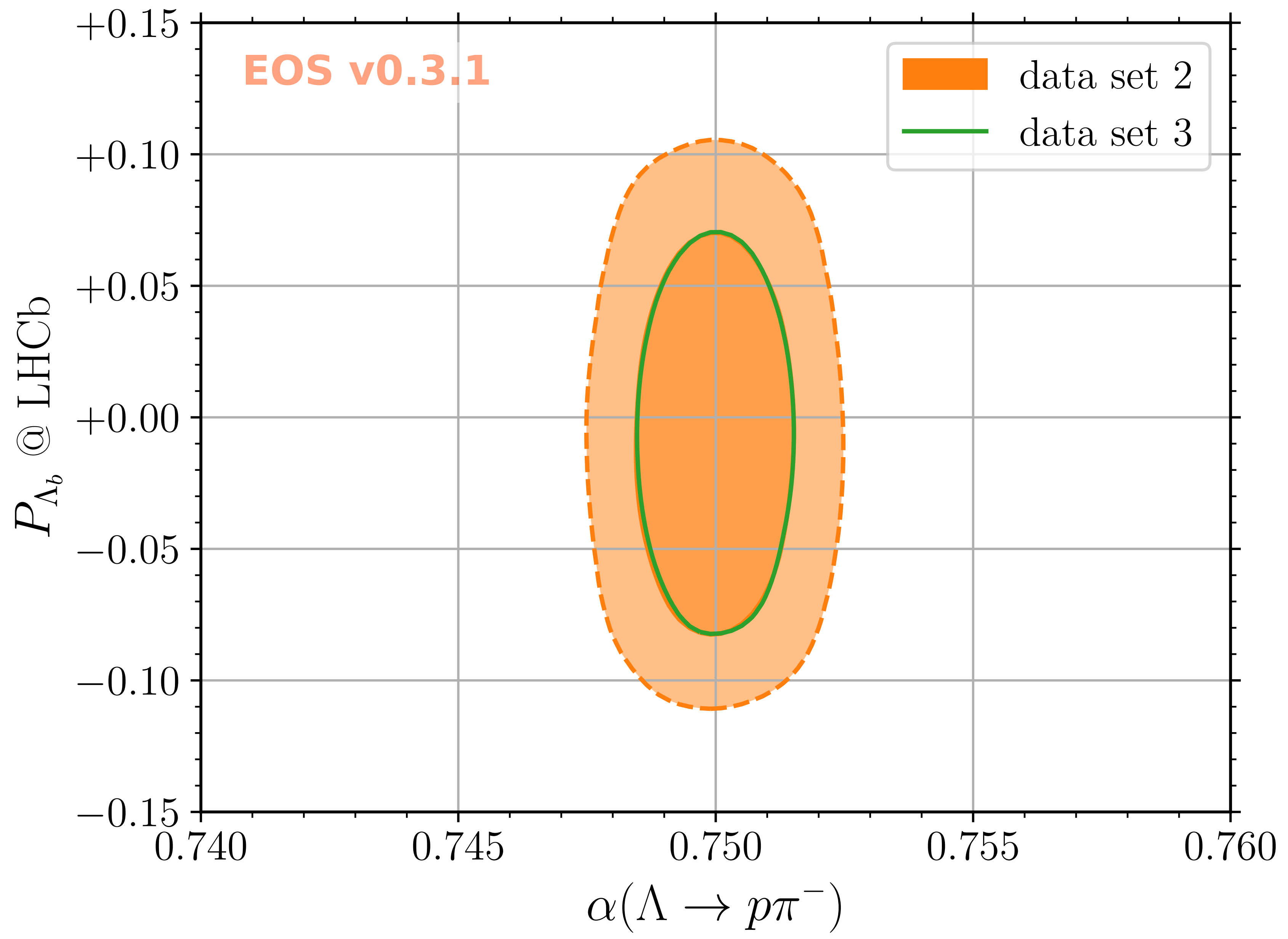
➡ Remaining observables compatible with zero

Global fit

➡ Uses just $\Lambda_b \to \Lambda\mu\mu$ observables and $B_s \to \mu\mu$ branching fraction

➡ Interestingly it constrains production polarisation and Λ decay asymmetry as well as dedicated measurement with $\Lambda_b \to J/\psi\Lambda$

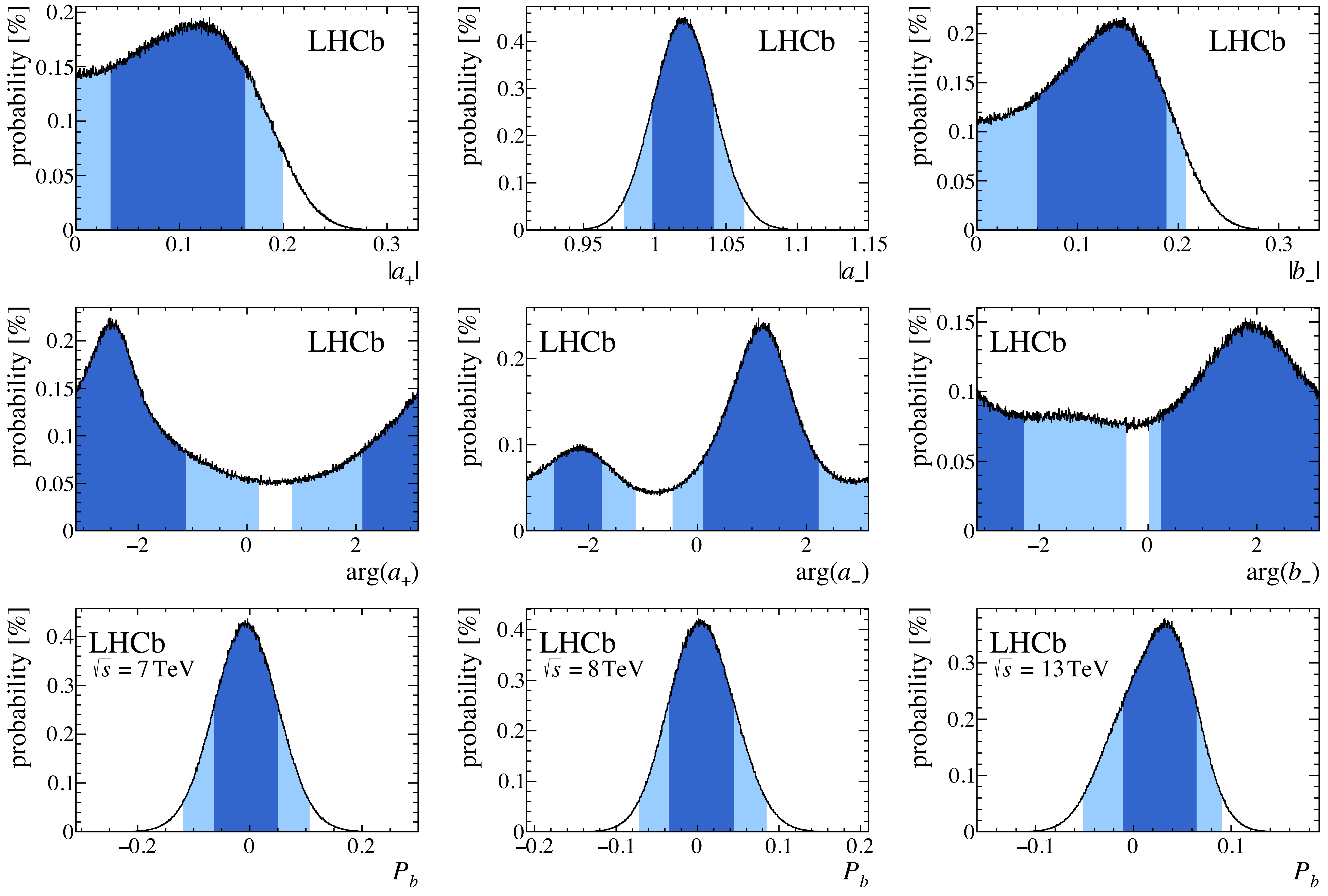




arXiv:1912.05811v1

Production polarisation

- ➡ Measure angular moments in $\Lambda_b \to J/\psi\Lambda$ and then perform Bayesian analysis
- ➡ Uses same dataset as rare decays
- ➡ Polarisation consistent with zero without visible energy dependence



JHEP06(2020)110

Future for $\Lambda_b \to \Lambda\mu\mu$

- When we did work on full distribution, we made crude estimate of precision at LHCb
- $15 < q^2 < 20$ GeV^2
- Pure signal toys without any background
- Just scale yields from published numbers
- Will be able to measure precisely, but many observables give only small effect at LHC

Obs.	Run 1	Run 2	Upgrade	Phase II	Obs.	Run 1	Run 2	Upgrade	Phase II
M_1	0.021	0.011	0.004	0.002	M_{18}	0.071	0.038	0.014	0.006
M_2	0.042	0.023	0.008	0.003	M_{19}	0.156	0.084	0.030	0.012
M_3	0.030	0.016	0.006	0.002	M_{20}	0.071	0.038	0.014	0.006
M_4	0.050	0.026	0.010	0.004	M_{21}	0.090	0.048	0.017	0.007
M_5	0.078	0.042	0.015	0.006	M_{22}	0.041	0.022	0.008	0.003
M_6	0.055	0.030	0.011	0.004	M_{23}	0.089	0.047	0.017	0.007
M_7	0.090	0.048	0.017	0.007	M_{24}	0.036	0.019	0.007	0.003
M_8	0.041	0.022	0.008	0.003	M_{25}	0.156	0.083	0.030	0.012
M_9	0.090	0.048	0.017	0.007	M_{26}	0.071	0.038	0.014	0.006
M_{10}	0.041	0.022	0.008	0.003	M_{27}	0.156	0.083	0.030	0.012
M_{11}	0.051	0.027	0.010	0.004	M_{28}	0.071	0.038	0.014	0.005
M_{12}	0.078	0.041	0.015	0.006	M_{29}	0.097	0.052	0.019	0.008
M_{13}	0.054	0.029	0.010	0.004	M_{30}	0.062	0.033	0.012	0.005
M_{14}	0.088	0.047	0.017	0.007	M_{31}	0.097	0.052	0.019	0.008
M_{15}	0.136	0.073	0.026	0.011	M_{32}	0.062	0.033	0.012	0.005
M_{16}	0.097	0.052	0.019	0.008	M_{33}	0.061	0.033	0.012	0.005
M_{17}	0.156	0.084	0.030	0.012	M_{34}	0.061	0.033	0.012	0.005

- LHCb Phase II corresponds to about 50k reconstructed events

$\Lambda_b \to pK\mu\mu$ details

- 1D distribution in θ_l has usual form, K_2 generates lepton A_{FB}
  - Usual contributions, just adds Λ^* helicity 3/2 in addition to 1/2
- 1D distribution in θ_p gets larger number of terms
  - Includes odd terms in cos θ_p which vanish for single resonance
  - With interference, A_{FB} generated also on hadron side with K_4, K_{10} and K_{16} contributing

$$\frac{d^3\Gamma}{dq^2\, dm_{pK}\, d\cos\theta_\ell} = \frac{\sqrt{3}}{2}K_1 + \boxed{\frac{3}{2}K_2\cos\theta_\ell} + \frac{\sqrt{15}}{4}K_3(3\cos^2\theta_\ell - 1)$$

$$\begin{aligned}\frac{d^3\Gamma}{dq^2\, dm_{pK}\, d\cos\theta_p} &= \frac{\sqrt{3}}{2}K_1 - \frac{\sqrt{15}}{4}K_7 + 9\frac{\sqrt{3}}{16}K_{13} \\ &+ \boxed{\left(\frac{3}{2}K_4 - 3\frac{\sqrt{21}}{4}K_{10} + 15\frac{\sqrt{33}}{16}K_{16}\right)\cos\theta_p} \\ &+ \left(3\frac{\sqrt{15}}{4}K_7 - 45\frac{\sqrt{3}}{8}K_{13}\right)\cos^2\theta_p \\ &+ \boxed{\left(5\frac{\sqrt{21}}{4}K_{10} - 35\frac{\sqrt{33}}{8}K_{16}\right)\cos^3\theta_p} \\ &+ \frac{105\sqrt{3}}{16}K_{13}\cos^4\theta_p + \boxed{\frac{63\sqrt{33}}{16}K_{16}\cos^5\theta_p}\,.\end{aligned}$$

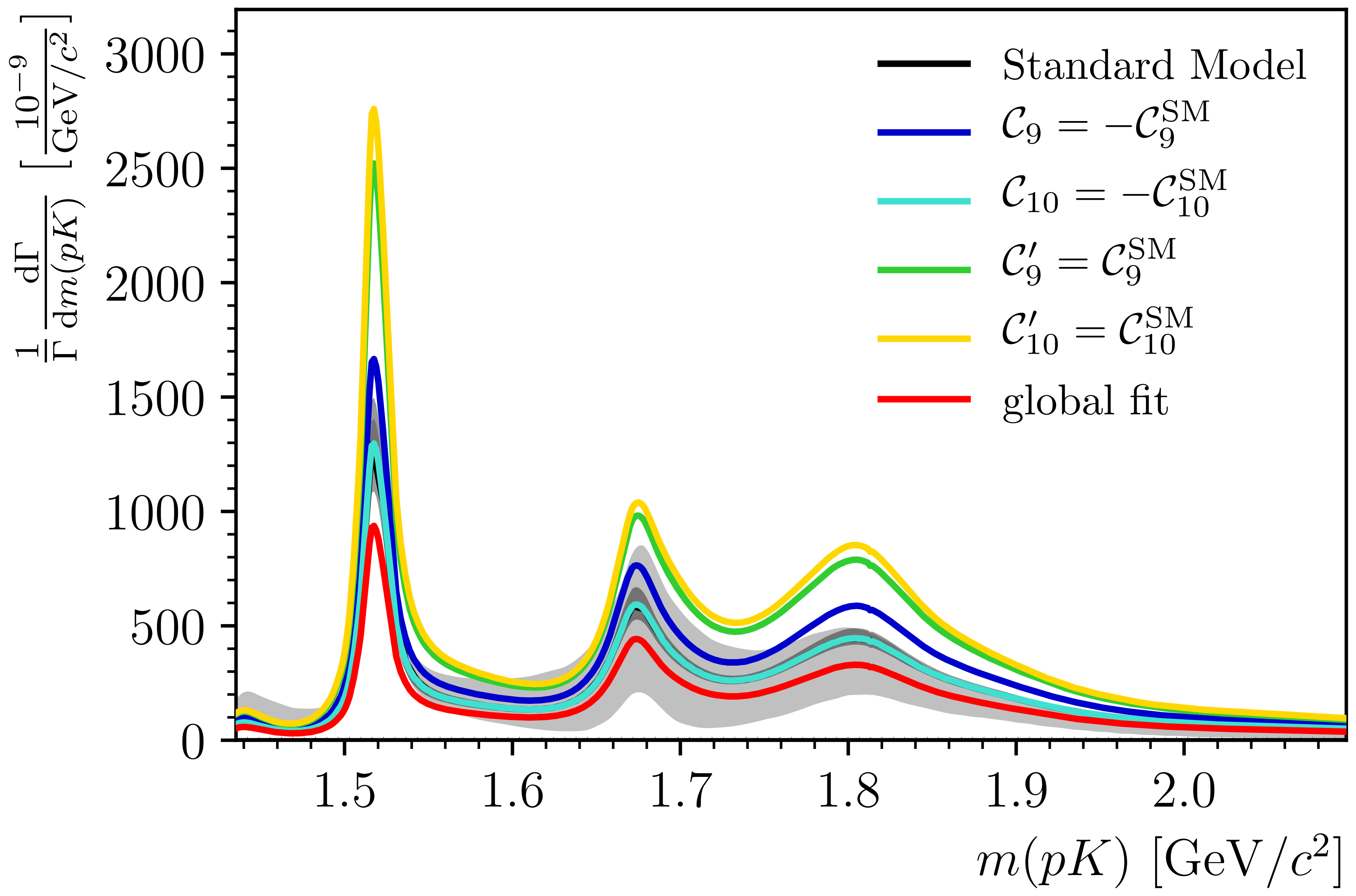
Numerical studies

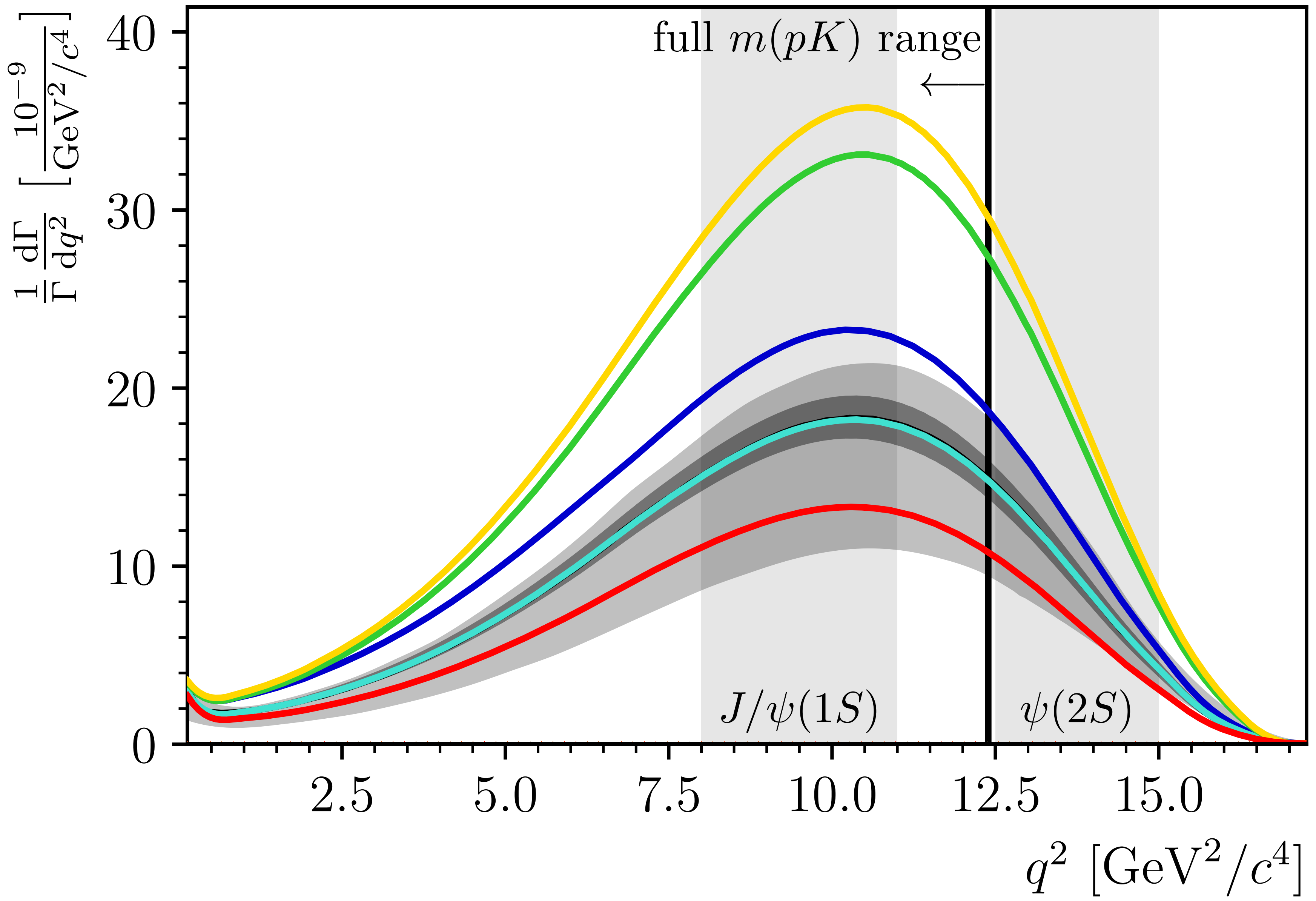
- ➡ Use SM Wilson coefficients used in JHEP 05 (2013) 137
- ➡ Use all well established states for which prediction for form-factors exists
  - ❖ Form-factors based on quark-model from Int. J. Mod. Phys. A 30 (2015) 1550172
- ➡ Most of the resonances modelled by relativistic Breit-Wigner
- ➡ $\Lambda(1405)$ uses Flattè model
- ➡ Investigated scenarios:
  - ➡ Flip C_9/C_{10} or add right C_9'/C_{10}'
  - ➡ Global fit in Eur. Phys. J. C 82 (2022) 326

resonance	m_Λ [GeV/c^2]	Γ_Λ [GeV/c^2]	$2J_\Lambda$	P_Λ	$\mathcal{B}(\Lambda \to N\overline{K})$
$\Lambda(1405)$	1.405	0.051	1	−	0.50
$\Lambda(1520)$	1.519	0.016	3	−	0.45
$\Lambda(1600)$	1.600	0.200	1	+	0.15 – 0.30
$\Lambda(1670)$	1.674	0.030	1	−	0.20 – 0.30
$\Lambda(1690)$	1.690	0.070	3	−	0.20 – 0.30
$\Lambda(1800)$	1.800	0.200	1	−	0.25 – 0.40
$\Lambda(1810)$	1.790	0.110	1	+	0.05 – 0.35
$\Lambda(1820)$	1.820	0.080	5	+	0.55 – 0.65
$\Lambda(1890)$	1.890	0.120	3	+	0.24 – 0.36
$\Lambda(2110)$	2.090	0.250	5	+	0.05 – 0.25

Ensemble of resonances

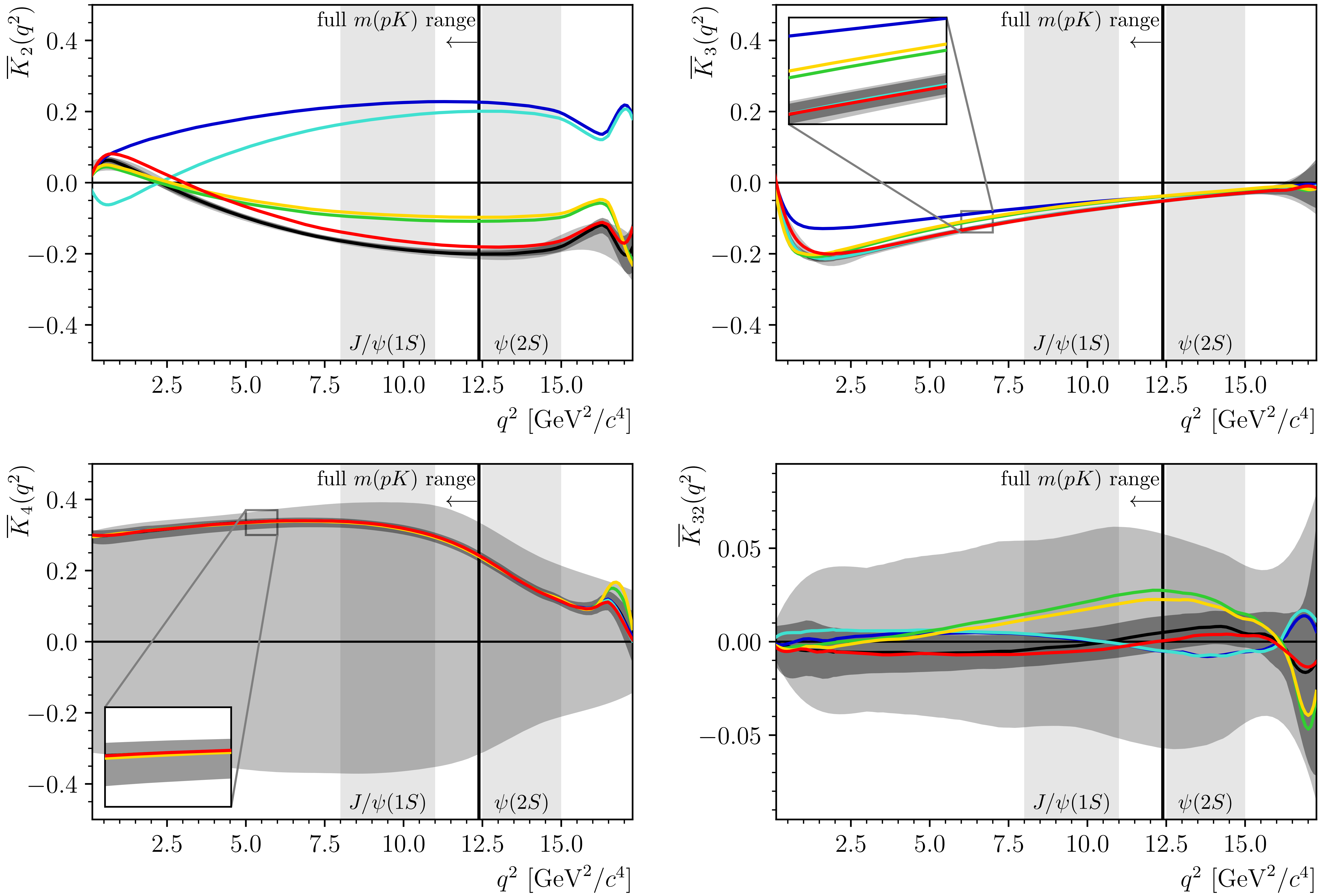
- ➡ Investigate sensitivity of observables with ensemble of different Λ resonances
- ➡ Strong phases of all Λ resonances set to 0 (π/2 at the pole)
- ➡ Additional uncertainty from strong phases by varying them between -π and π





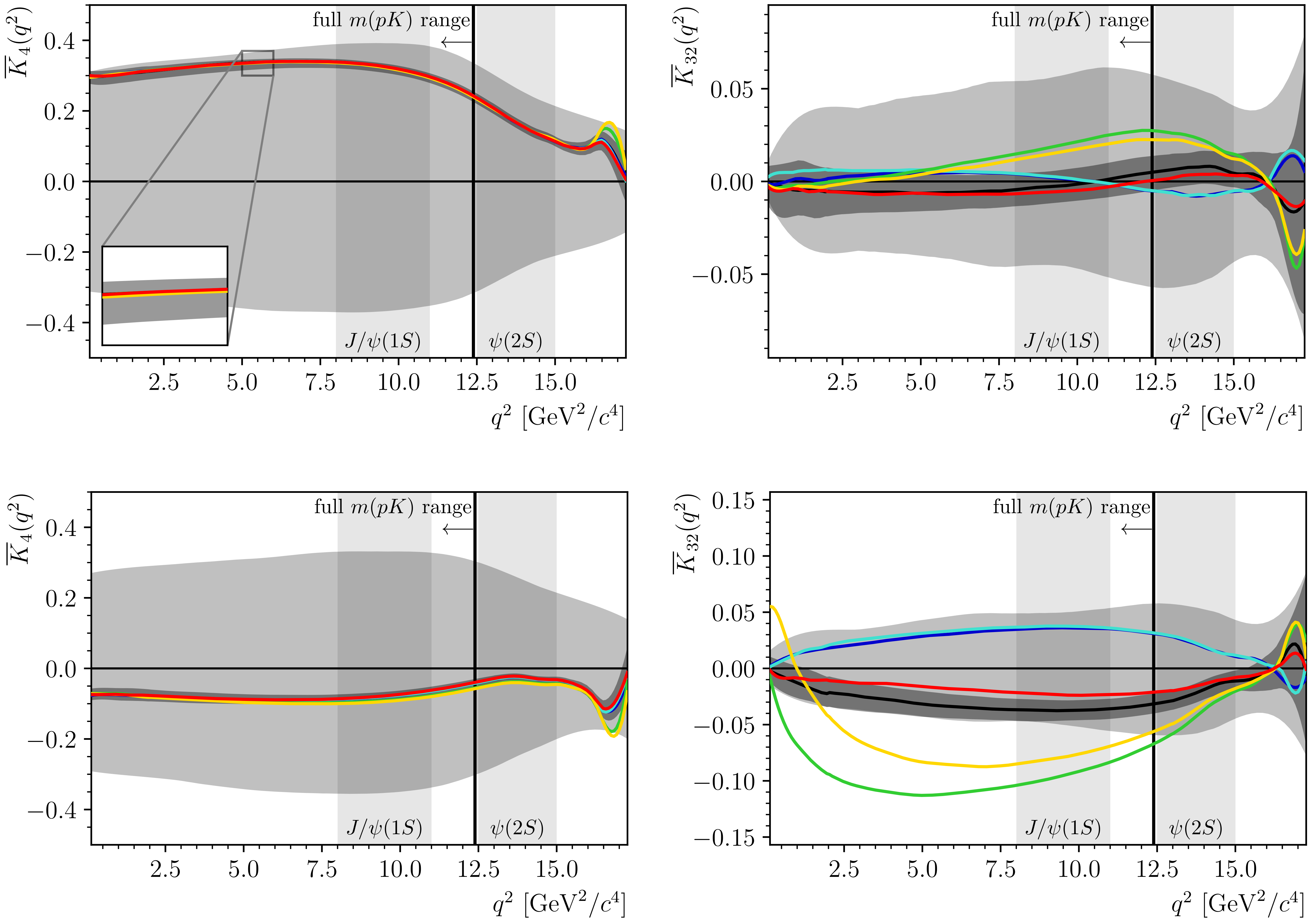
Ensemble of resonances

- ➡ Some cases give good sensitivity to new physics without effects from strong phases
- ➡ Some observables like K_4 has little sensitivity to new physics, but large effect from strong phases
- ➡ Several observables like K_{32} sensitive to new physics but require knowledge of strong phases



Ensemble of resonances

- ➡ Particular example of effect of strong phases
- ➡ Set strong phase of spin-3/2 resonances to π while keeping rest to 0
- ➡ Very large effects on K_4 and K_{32}
  - ❖ K_{32} shows significantly different behaviour
- ➡ We have all ingredients but as polarisation at LHCb is small, we never looked into details



$\Lambda_b \to pK\mu\mu$ measurement

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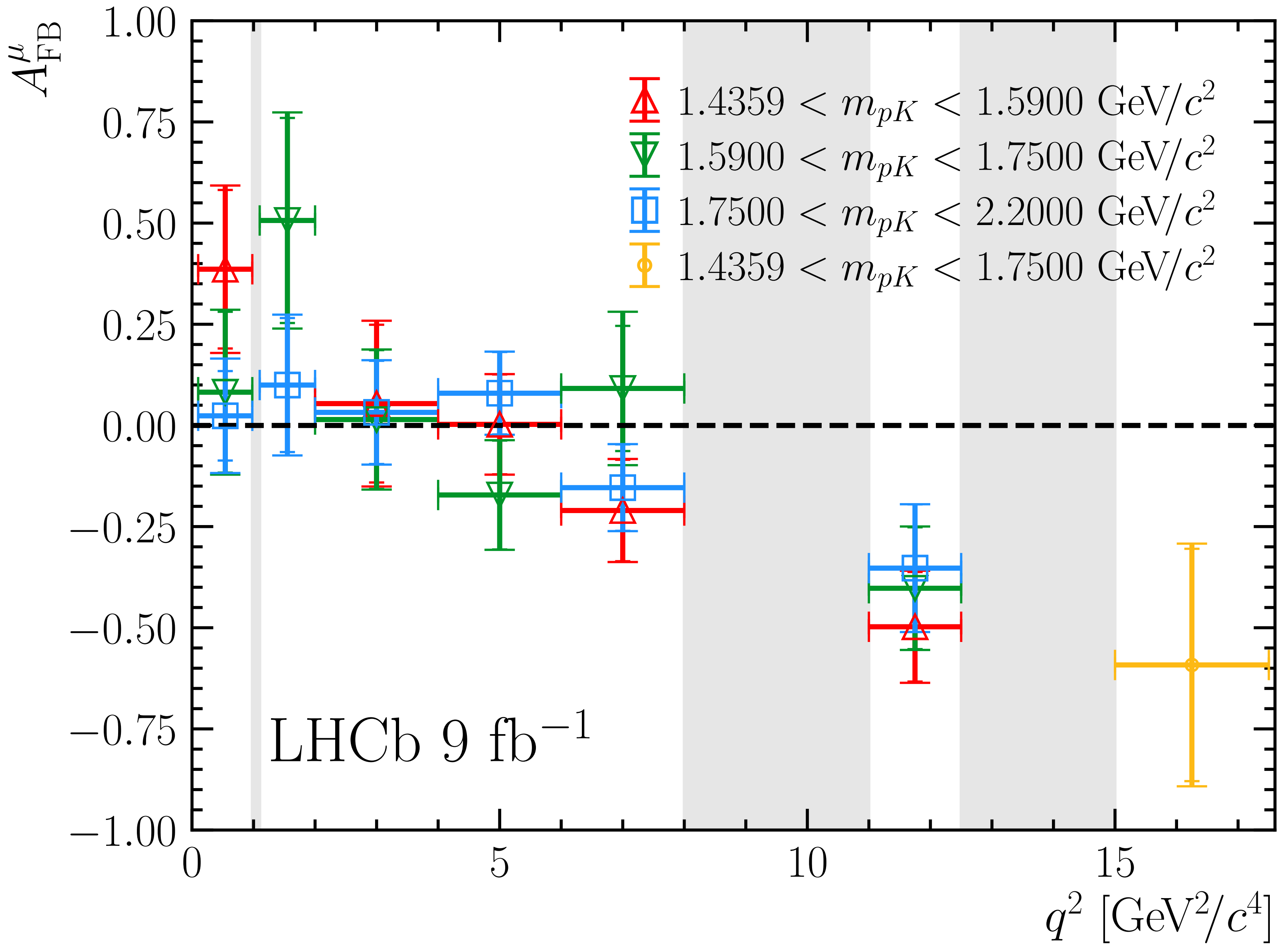
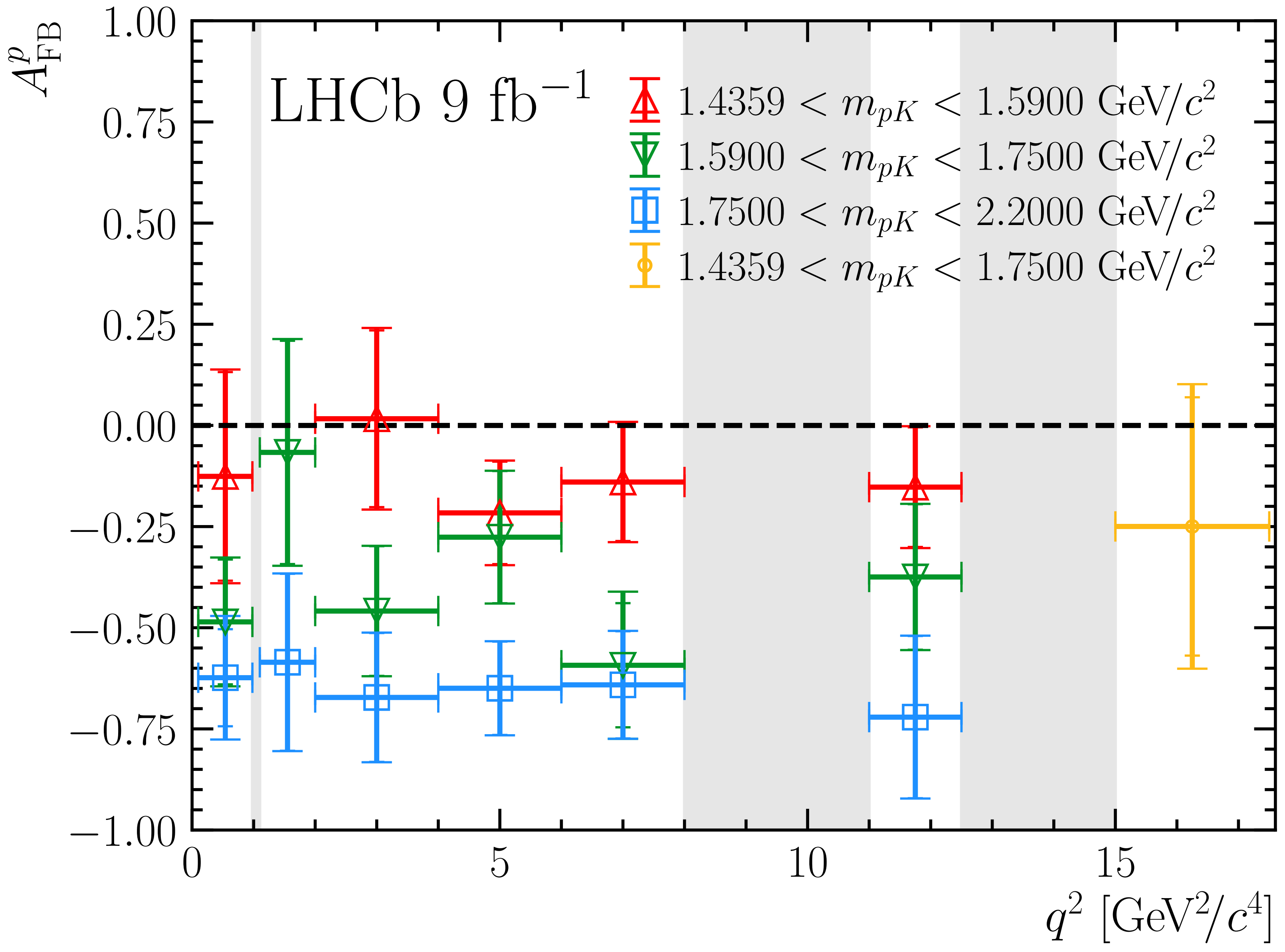
- ➡ Unpolarised observables measured at LHCb with Runs 1 and 2 data
- ➡ Interpretation is not trivial without detailed understanding of hadronic contributions
- ➡ But interference of various resonances introduces more observables





arXiv:2409.1262

Summary

- There is interesting physics to be extracted from rare Λ_b decays
- With 10^{12} Z bosons we expect about 15k decays for BF 10^{-6}
- Size of the sample will likely be smaller than ultimate LHCb sample
- But with polarisation possibly being about 0.5 (10 times of that at LHCb), there is possibility to complement LHCb measurements
  - Larger uncertainty, but also on 10 times larger effect
  - Assumes that the polarisation axis does not align to make relevant terms zero
- There might be other interesting options with higher BF decays, but generally there are not many studies done
  - People interested will likely need to do work to understand whether polarisation brings benefits

Summary

- Tom Blake and myself would be interested to look into question what can be gained by 10^{12} Z decays, but currently do not have enough bandwidth to do study on our own
  - Anja Beck who did lot on $\Lambda_b \to pK\mu\mu$ is still in physics and she might do some work on this, but again, not as a main work
- One should work out how well one can do measurement at Z pole and also look what impact such measurement would have
- If somebody is interested, get in touch we can discuss some collaboration to look into these questions

Backup

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Why $\Lambda_b \to \Lambda\mu\mu$

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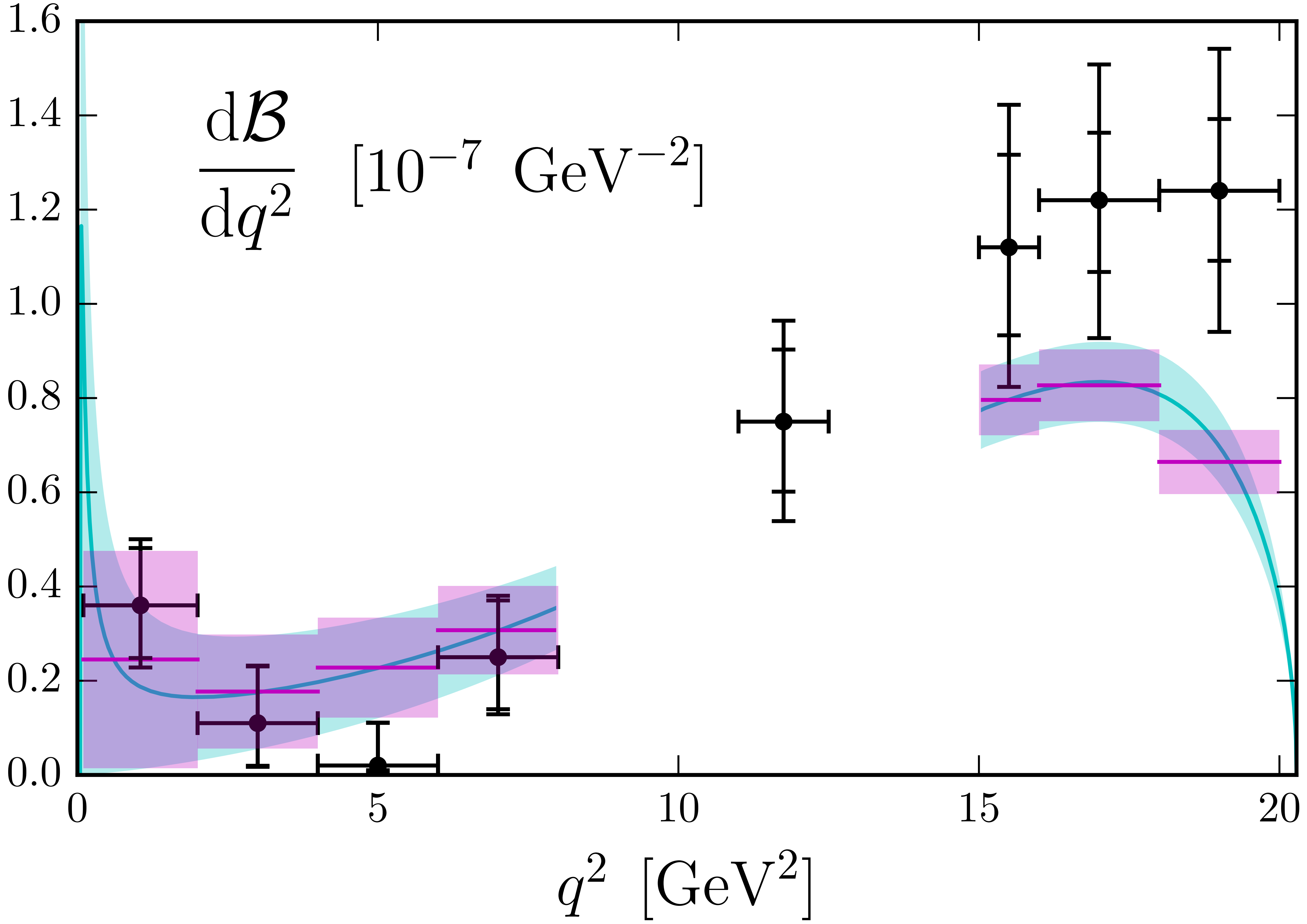
- ➡ Provides rich angular structure thanks to non-zero spin of initial state
- ➡ Λ baryon is very long lived and can be easily treated as stable particle in calculations
- ➡ Both experimentally and theoretically very clean from any interference and backgrounds
- ➡ If produced polarised, it offers access to information not available with mesons
- ➡ Con: Long Λ lifetime decreases detection efficiency, so statistics is usually smaller than similar meson decays

Differential branching fraction

- Measured at LHCb with Run 1 data
- Theory prediction is currently more precise than experiment
- Experimentally measured relative to $\Lambda_b \to J/\psi\Lambda$ for which we do not have good BF
- No significant signal below J/ψ yet

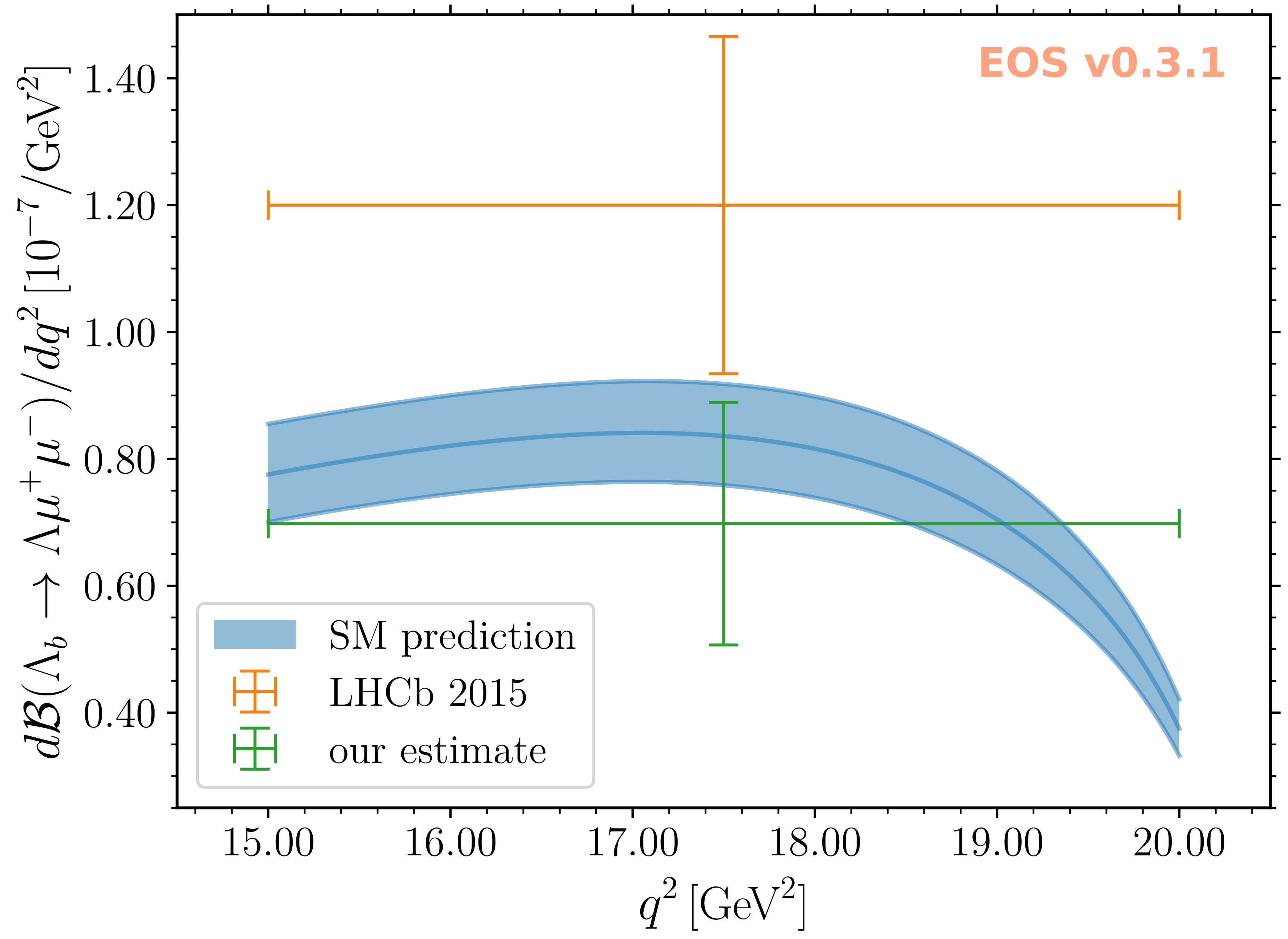
arXiv:1602.01399v2



Experimental normalisation

- Measurements for $\Lambda_b \to J/\psi\Lambda$ come from Tevatron which measured $\dfrac{f_\Lambda}{f_d}\dfrac{B(\Lambda_b \to J/\psi\Lambda)}{B(B^0 \to J/\psi K_S)}$
- Best number comes from D0
- One needs also fragmentation fraction, in past one would average LEP and Tevatron
- But there is pT dependence, which means that averaging LEP and Tevatron is not good
- Needs measurement of both ingredients from same experiment ⇒ ongoing at LHCb

arXiv:1912.05811v1



Angular distributions

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- Up to some constants, angular distribution in unpolarised case is

$$
\begin{aligned}
K(q^2, \cos\theta_\ell, \cos\theta_\Lambda, \phi) = & \left(K_{1ss}\sin^2\theta_\ell + K_{1cc}\cos^2\theta_\ell + K_{1c}\cos\theta_\ell\right) \\
& + \left(K_{2ss}\sin^2\theta_\ell + K_{2cc}\cos^2\theta_\ell + K_{2c}\cos\theta_\ell\right)\cos\theta_\Lambda \\
& + \left(K_{3sc}\sin\theta_\ell\cos\theta_\ell + K_{3s}\sin\theta_\ell\right)\sin\theta_\Lambda\sin\phi \\
& + \left(K_{4sc}\sin\theta_\ell\cos\theta_\ell + K_{4s}\sin\theta_\ell\right)\sin\theta_\Lambda\cos\phi\,.
\end{aligned}
$$

- Specific features :
  - We can still define fraction of longitudinally polarised dilepton system
  - There is non-zero hadron side forward-backward asymmetry thanks to weak decay of Λ with significant differences between two amplitudes α_Λ=…
  - One can also construct combined forward-backward asymmetry

Angular distributions

➡ One can take ratios of observables to construct quantities which in first order are sensitive only to:

- Form factors

$$\frac{2\,K_{2ss}}{K_{2cc}} = 1 + \frac{m_{\Lambda_b}^2 - m_{\Lambda}^2}{q^2} \frac{f_0^V f_0^A}{f_\perp^V f_\perp^A}\,,$$

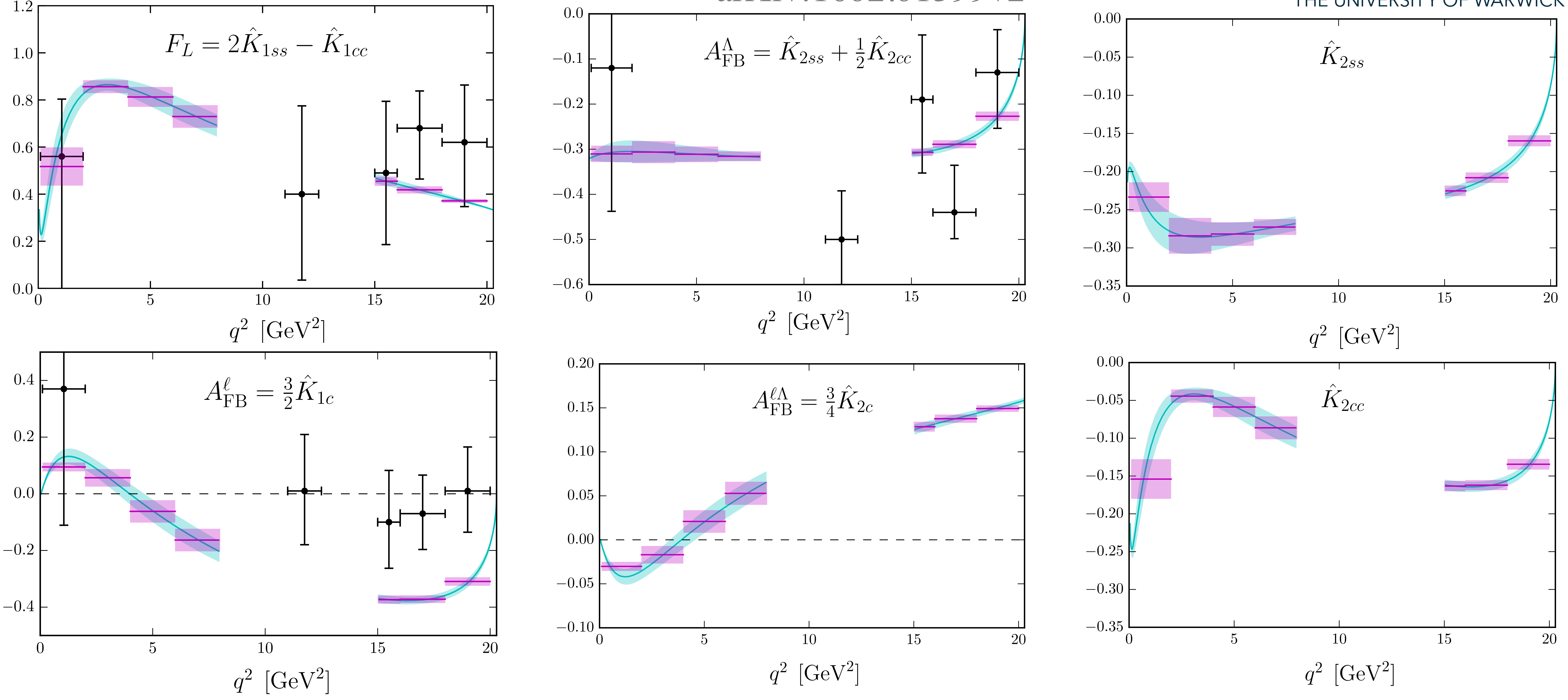
$$\frac{2\,K_{4sc}}{K_{2cc}} = \frac{m_{\Lambda_b} + m_{\Lambda}}{\sqrt{q^2}} \frac{f_0^V}{f_\perp^V} - \frac{m_{\Lambda_b} - m_{\Lambda}}{\sqrt{q^2}} \frac{f_0^A}{f_\perp^A}\,.$$

- Short-scale physics

$$X_1 \equiv \frac{K_{1c}}{K_{2cc}} = -\frac{\mathrm{Re}\,\{\rho_2\}}{\alpha\,\mathrm{Re}\,\{\rho_4\}}\,,$$

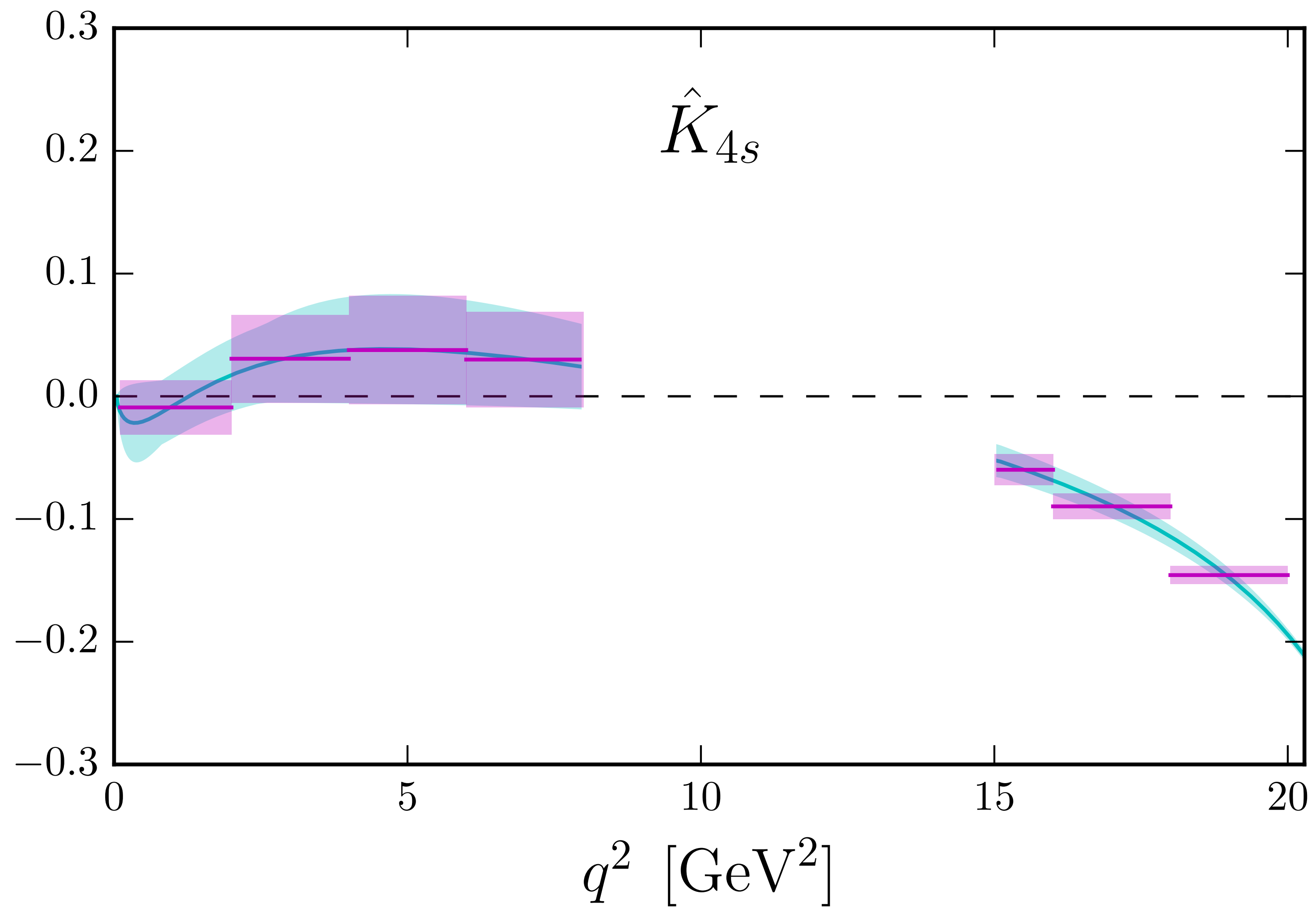
Predictions

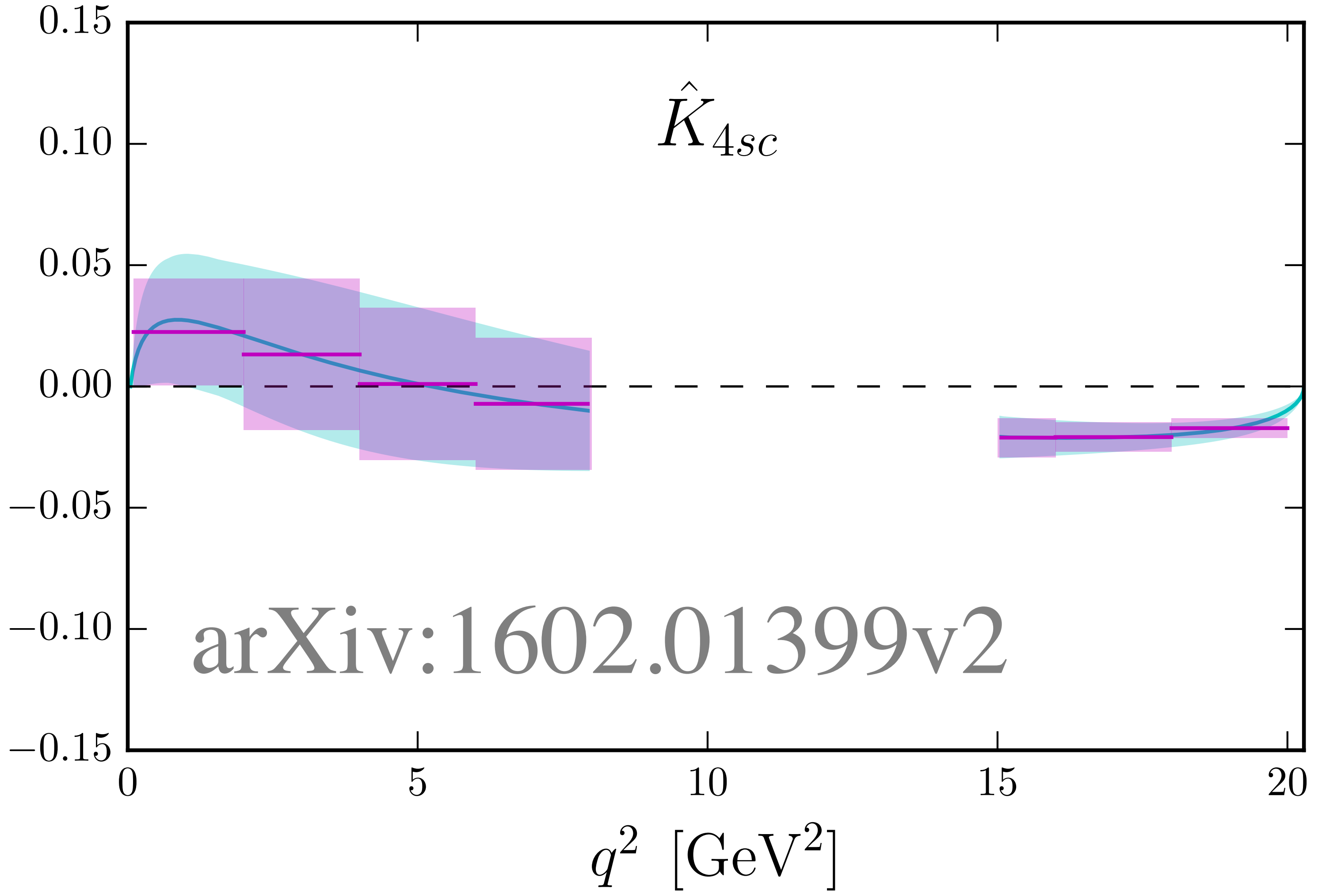
arXiv:1602.01399v2



Predictions

- ➡ Predictions are generally reasonably precise
- ➡ Measurements on these plots come from very early analysis when we were figuring out what we should be actually doing
- ➡ With Tom Blake we extended work to polarised case, which adds another 24 observables
  - ❖ 10 have same structure as unpolarised case, just being multiplied by production polarisation
  - ❖ 14 are proportional to production polarisation and give access to more information





Latest measurement

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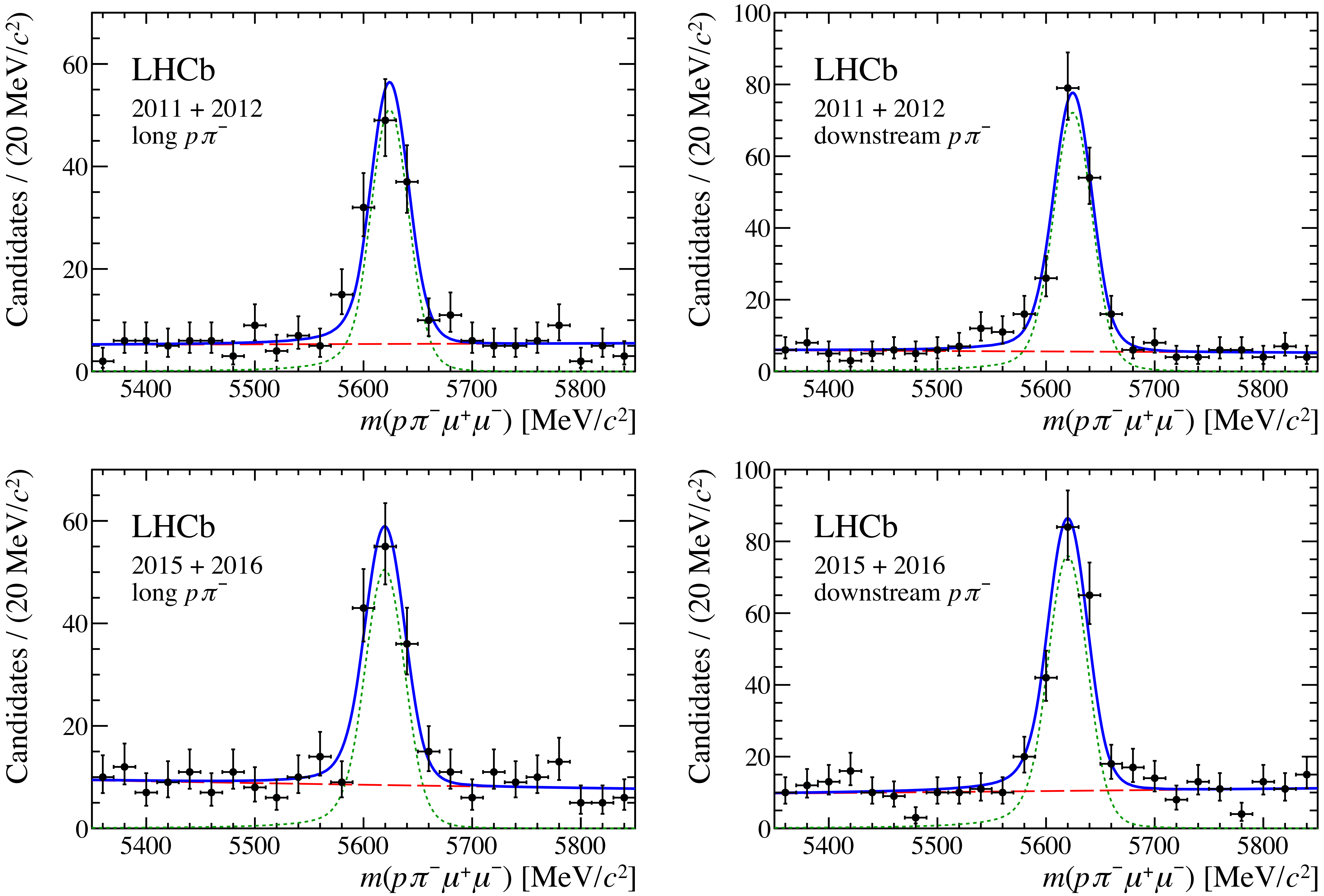
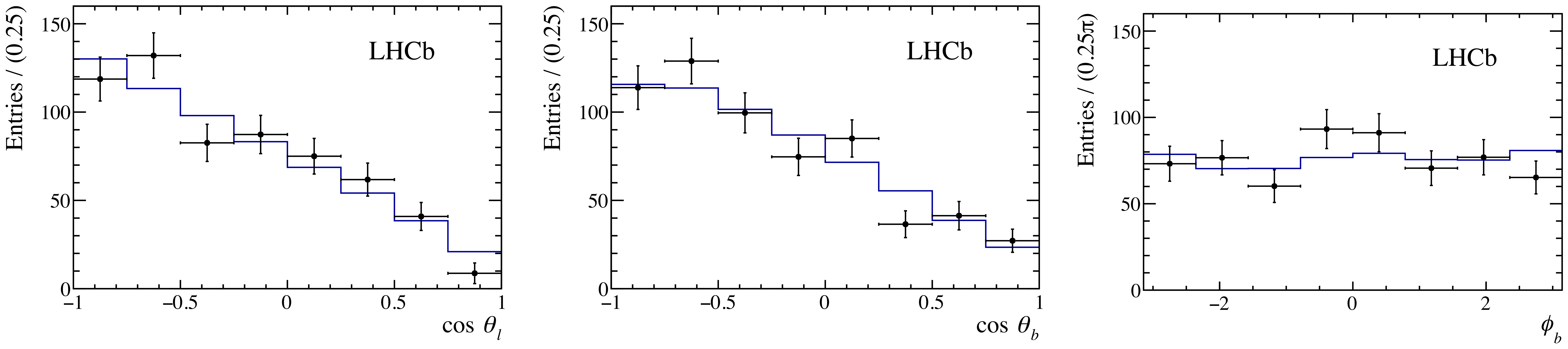
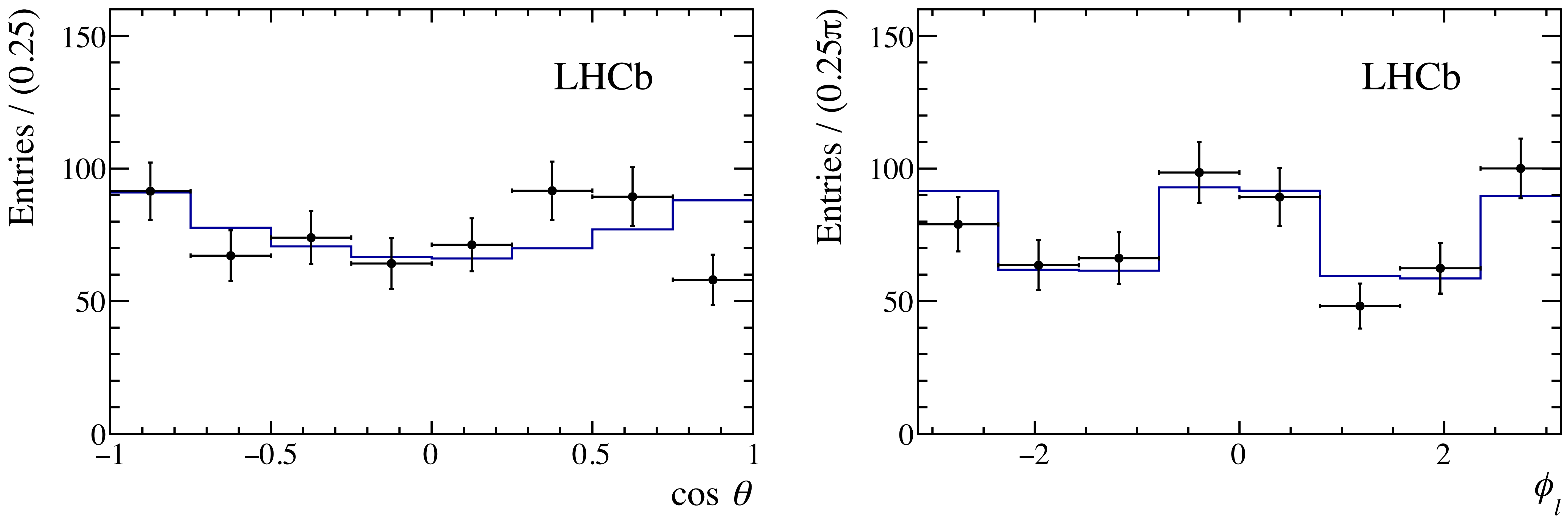
- ➡ Uses Run 1 and part of Run 2 data from LHCb
- ➡ Measured only $15 < q^2 < 20$ GeV^2 bin as this is the only one having significant yield
- ➡ About 610 signal decays
- ➡ Used method of moments
  - ❖ Luckily, otherwise would run to troubles with value of α_Λ



Latest measurement





Source	Uncertainty $[10^{-3}]$	
	Range among K_i	Mean
Simulated sample size	3–22	9
Efficiency parameterisation	1–13	4
Data-simulation differences	2–16	6
Angular resolution	1–11	4
Beam crossing angle	1–8	4
Signal mass model	1–4	2

How to get polarised sample

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- If there is enough interest in observables accessible only with polarisation, we can try to play some tricks
  - We measured polarisation only integrated over large η-p_T region, but it does not have to be constant
  - One can look for Λ_b coming from decays which itself could introduce polarisation
    - Obvious choice for LHCb would be Σ_b^* but my intuition is that it will not help
    - Top quark decays might be interesting, W in such case is polarised and so would be b-quark, this would be more suitable for ATLAS and CMS
- Each idea would need dedicated study whether it would work
- Each idea would mean lower statistics, on the other hand, one does not need to do all observables

What to expect

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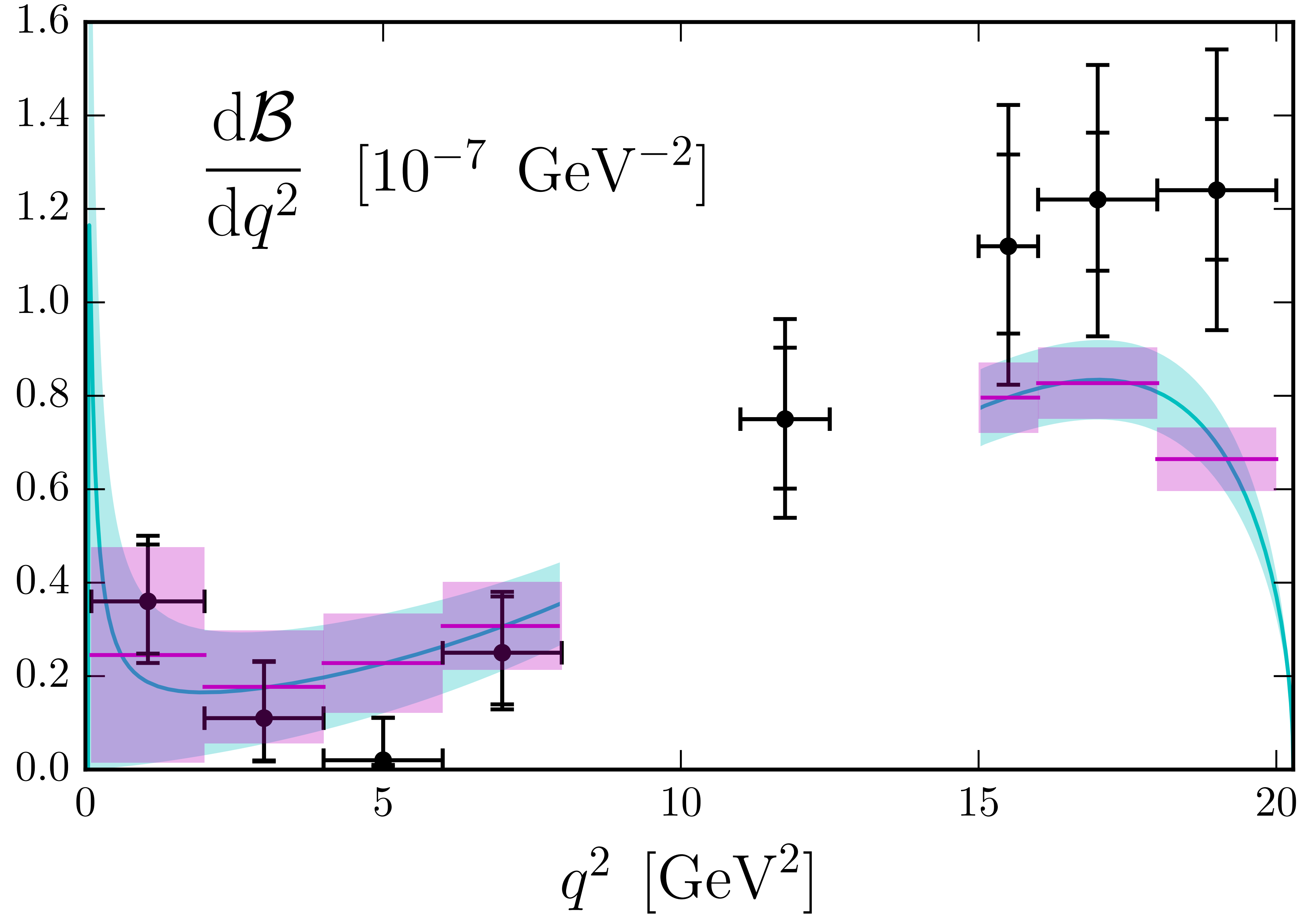
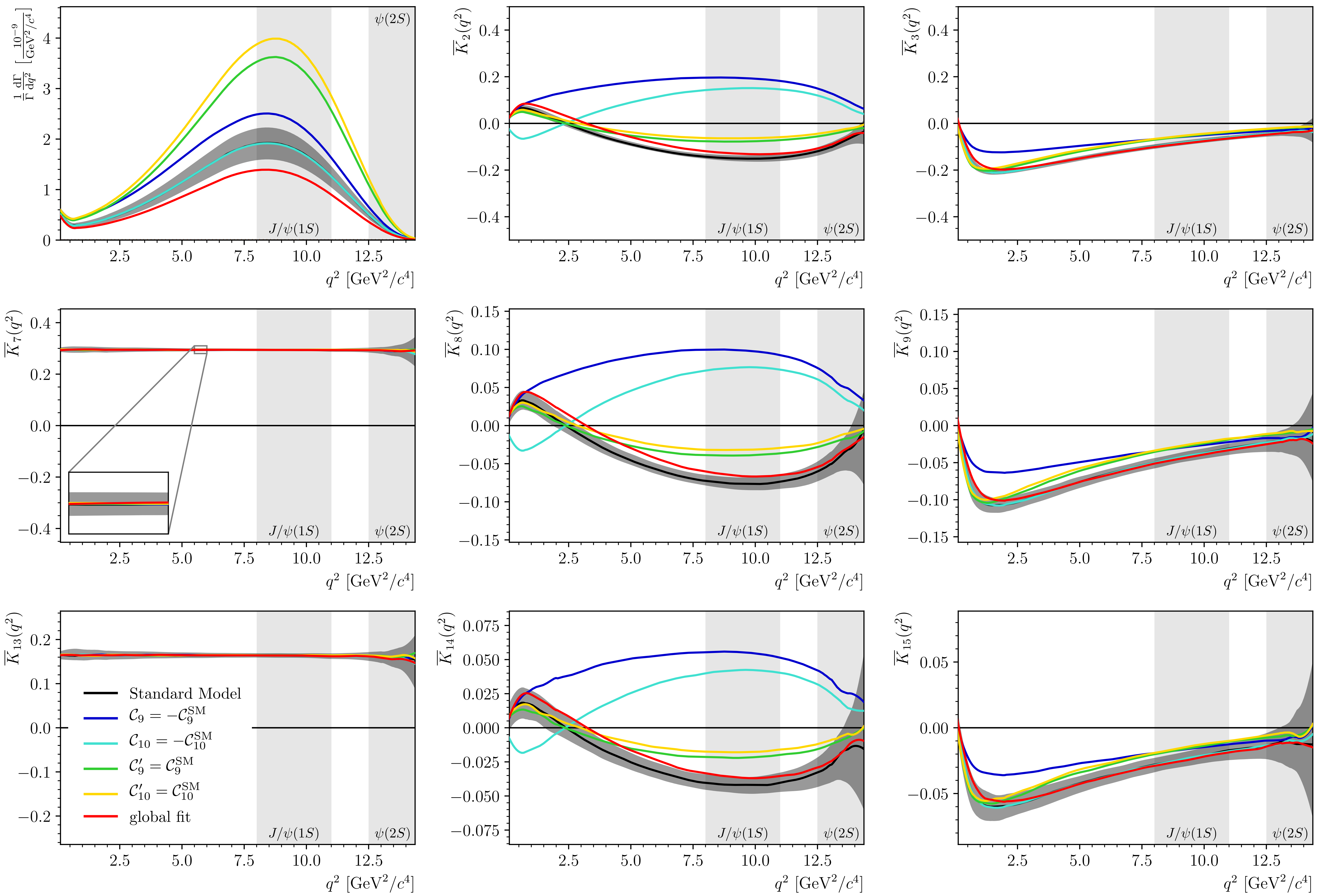
- LHCb is working on update of $\Lambda_b \to \Lambda\mu\mu$ branching fraction with Run 1+2 data
- Good chance to see signal in more q^2 bins, we have about 4 times more data in Run 2
- Not yet clear what we can do with angular observables below J/ψ
- Want to look back to polarisation measurement to see whether there is at least some indication of non-zero polarisation somewhere



Isolated spin 5/2 resonance

- ➡ Only isolated $\Lambda(1820)$
- ➡ Grey band shows uncertainty from:
  - ❖ Form-factor
  - ❖ Widths etc.
  - ❖ Non-factorisable corrections
- ➡ Often need rather large change in Wilson coefficients for effects larger than uncertainties



Helicity amplitudes

$$\mathcal{H}^{\Lambda,7^{(\prime)}}_{\lambda_\Lambda,\lambda_V}(q^2,m_{pK}) = -\frac{2m_b}{q^2}\frac{\mathcal{C}^{\rm eff}_{7^{(\prime)}}}{2}\, e^{i\delta_\Lambda}\left(H^{\Lambda,T}_{\lambda_\Lambda,\lambda_V} \mp H^{\Lambda,T5}_{\lambda_\Lambda,\lambda_V}\right)$$

$$\mathcal{H}^{\Lambda,9^{(\prime)}}_{\lambda_\Lambda,\lambda_V}(q^2,m_{pK}) = \frac{\mathcal{C}^{\rm eff}_{9^{(\prime)}}}{2}\, e^{i\delta_\Lambda}\left(H^{\Lambda,V}_{\lambda_\Lambda,\lambda_V} \mp H^{\Lambda,A}_{\lambda_\Lambda,\lambda_V}\right)$$

$$\mathcal{H}^{\Lambda,10^{(\prime)}}_{\lambda_\Lambda,\lambda_V}(q^2,m_{pK}) = \frac{\mathcal{C}_{10^{(\prime)}}}{2} e^{i\delta_\Lambda}\left(H^{\Lambda,V}_{\lambda_\Lambda,\lambda_V} \mp H^{\Lambda,A}_{\lambda_\Lambda,\lambda_V}\right)$$

$$H^{\Lambda,\Gamma^\mu}_{\lambda_\Lambda,\lambda_V} = \varepsilon^*_\mu(\lambda_V)\langle\Lambda|\bar{s}\Gamma^\mu b|\Lambda_b^0\rangle$$

$$\langle\Lambda|\bar{s}\Gamma^\mu b|\Lambda_b^0\rangle = \bar{u}(k,\lambda_\Lambda)\left[X_{\Gamma 1}(q^2)\gamma^\mu + X_{\Gamma 2}(q^2)v_p^\mu + X_{\Gamma 3}(q^2)v_k^\mu\right]u(p,\lambda_b)$$

Spin 1/2

$$\langle\Lambda|\bar{s}\Gamma^\mu b|\Lambda_b^0\rangle = \bar{u}_\alpha(k,\lambda_\Lambda)\left[v_p^\alpha\left(X_{\Gamma 1}(q^2)\gamma^\mu + X_{\Gamma 2}(q^2)v_p^\mu + X_{\Gamma 3}(q^2)v_k^\mu\right) + X_{\Gamma 4}(q^2)g^{\alpha\mu}\right]u(p,\lambda_b)$$

Spin 3/2

$$\langle\Lambda|\bar{s}\Gamma^\mu b|\Lambda_b^0\rangle = \bar{u}_{\alpha\beta}(k,\lambda_\Lambda)v_p^\alpha\Big[v_p^\beta\left(X_{\Gamma 1}(q^2)\gamma^\mu + X_{\Gamma 2}(q^2)v_p^\mu + X_{\Gamma 3}(q^2)v_k^\mu\right) + X_{\Gamma 4}(q^2)g^{\beta\mu}\Big]u(p,\lambda_b)\,.$$

Spin 5/2

Amplitude combinations

i	parity combination	$J_\Lambda + J'_\Lambda$	single states 1/2	3/2	5/2	Re/Im	V/A	helicity combinations	Eq.
1	same	≥ 1	✓	✓	✓	Re		$J_\Lambda = J'_\Lambda, (\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(62)
2	same	≥ 1	✓	✓	✓	Re	✓	$J_\Lambda = J'_\Lambda, \lambda_V \neq 0, (\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(63)
3	same	≥ 1	✓	✓	✓	Re		$J_\Lambda = J'_\Lambda, (\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(64)
4	opposite	≥ 1				Re		$(\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(66)
5	opposite	≥ 1				Re	✓	$\lambda_V \neq 0, (\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(117)
6	opposite	≥ 1				Re		$(\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(118)
7	same	≥ 2		✓	✓	Re		$(\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(119)
8	same	≥ 2		✓	✓	Re	✓	$\lambda_V \neq 0, (\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(120)
9	same	≥ 2		✓	✓	Re		$(\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(121)
10	opposite	≥ 3				Re		$(\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(122)
11	opposite	≥ 3				Re	✓	$\lambda_V \neq 0, (\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(123)
12	opposite	≥ 3				Re		$(\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(124)
13	same	≥ 4			✓	Re		$(\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(125)
14	same	≥ 4			✓	Re	✓	$\lambda_V \neq 0, (\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(126)
15	same	≥ 4			✓	Re		$(\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(127)
16	opposite	≥ 5				Re		$(\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(128)
17	opposite	≥ 5				Re	✓	$\lambda_V \neq 0, (\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(129)
18	opposite	≥ 5				Re		$(\lambda_\Lambda, \lambda_V) = (\lambda_\Lambda, \lambda_V)'$	(130)

Amplitude combinations

19	opposite	≥ 1			Re			(131)
20	opposite	≥ 1			Re	✓		(132)
21	same	≥ 2	✓	✓	Re			(133)
22	same	≥ 2	✓	✓	Re	✓		(134)
23	opposite	≥ 3			Re			(135)
24	opposite	≥ 3			Re	✓	$\lambda_V = 0, \lvert\lambda'_V\rvert = 1$ (all possible $\lambda_\Lambda^{(\prime)}$)	(136)
25	same	≥ 4		✓	Re			(137)
26	same	≥ 4		✓	Re	✓		(138)
27	opposite	≥ 5			Re			(139)
28	opposite	≥ 5			Re	✓		(140)

Amplitude combinations

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29	opposite	≥ 1			Im			(141)
30	opposite	≥ 1			Im	✓		(142)
31	same	≥ 2	✓	✓	Im			(143)
32	same	≥ 2	✓	✓	Im	✓		(67)
33	opposite	≥ 3			Im			(144)
34	opposite	≥ 3			Im	✓	$\lambda_V = 0, \lvert\lambda'_V\rvert = 1$ (all possible $\lambda_\Lambda^{(\prime)}$)	(145)
35	same	≥ 4		✓	Im			(146)
36	same	≥ 4		✓	Im	✓		(147)
37	opposite	≥ 5			Im			(148)
38	opposite	≥ 5			Im	✓		(149)
39	same	≥ 2	✓	✓	Re			(150)
40	opposite	≥ 3			Re			(151)
41	same	≥ 4		✓	Re			(152)
42	opposite	≥ 5			Re			(153)
43	same	≥ 2	✓	✓	Im		$\lvert\lambda_V^{(\prime)}\rvert = 1,\ \lambda_\Lambda = \pm 1/2, \lambda'_\Lambda = \mp 3/2$	(154)
44	opposite	≥ 3			Im			(155)
45	same	≥ 4		✓	Im			(156)
46	opposite	≥ 5			Im			(157)

Explicit expressions for observables

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$$\mathcal{A}^{Q,V}_{\lambda_\Lambda,\lambda_V} = N \sum_{\Lambda} \sum_{i=7^{(\prime)},9^{(\prime)}} \mathcal{H}^{\Lambda,\mathcal{O}_i}_{\lambda_\Lambda,\lambda_V} h^{\Lambda}_{\lambda_\Lambda,1/2} \, ,$$

$$\mathcal{A}^{Q,A}_{\lambda_\Lambda,\lambda_V} = N \sum_{\Lambda} \sum_{i=10^{(\prime)}} \mathcal{H}^{\Lambda,\mathcal{O}_i}_{\lambda_\Lambda,\lambda_V} h^{\Lambda}_{\lambda_\Lambda,1/2} \, ,$$

$$K_1 = \frac{1}{\sqrt{3}} \sum_{Q} \sum_{\lambda_\Lambda,\lambda_V} \left(\left| \mathcal{A}^{Q,V}_{\lambda_\Lambda,\lambda_V} \right|^2 + V \longleftrightarrow A \right)$$

$$K_2 = -\sum_{Q} \sum_{\lambda=\pm 1} \lambda \cdot \mathrm{Re} \left[\mathcal{A}^{Q,A*}_{\frac{3}{2}\lambda,\lambda} \mathcal{A}^{Q,V}_{\frac{3}{2}\lambda,\lambda} + \mathcal{A}^{Q,A*}_{\frac{1}{2}\lambda,\lambda} \mathcal{A}^{Q,V}_{\frac{1}{2}\lambda,\lambda} \right]$$

$$K_3 = \frac{1}{2\sqrt{15}} \sum_{Q} \sum_{\lambda=\pm 1} \left(\left| \mathcal{A}^{Q,V}_{\frac{3}{2}\lambda,\lambda} \right|^2 + \left| \mathcal{A}^{Q,V}_{\frac{1}{2}\lambda,\lambda} \right|^2 - 2 \left| \mathcal{A}^{Q,V}_{\frac{1}{2}\lambda,0} \right|^2 \right) + V \longleftrightarrow A$$

$$\begin{aligned} K_4 = \frac{1}{105} \sum_{\lambda=\pm 1} \mathrm{Re}\Big[+ \lambda \Big(& +35 \mathcal{A}^{\frac{1}{2}^+,V*}_{\frac{1}{2}\lambda,0} \mathcal{A}^{\frac{1}{2}^-,V}_{\frac{1}{2}\lambda,0} + 35 \mathcal{A}^{\frac{1}{2}^+,V*}_{\frac{1}{2}\lambda,\lambda} \mathcal{A}^{\frac{1}{2}^-,V}_{\frac{1}{2}\lambda,\lambda} \\ & +21 \mathcal{A}^{\frac{3}{2}^+,V*}_{\frac{3}{2}\lambda,\lambda} \mathcal{A}^{\frac{3}{2}^-,V}_{\frac{3}{2}\lambda,\lambda} + 7 \mathcal{A}^{\frac{3}{2}^+,V*}_{\frac{1}{2}\lambda,0} \mathcal{A}^{\frac{3}{2}^-,V}_{\frac{1}{2}\lambda,0} + 7 \mathcal{A}^{\frac{3}{2}^+,V*}_{\frac{1}{2}\lambda,\lambda} \mathcal{A}^{\frac{3}{2}^-,V}_{\frac{1}{2}\lambda,\lambda} \\ & +3 \mathcal{A}^{\frac{5}{2}^+,V*}_{\frac{1}{2}\lambda,0} \mathcal{A}^{\frac{5}{2}^-,V}_{\frac{1}{2}\lambda,0} + 3 \mathcal{A}^{\frac{5}{2}^+,V*}_{\frac{1}{2}\lambda,\lambda} \mathcal{A}^{\frac{5}{2}^-,V}_{\frac{1}{2}\lambda,\lambda} + 9 \mathcal{A}^{\frac{5}{2}^+,V*}_{\frac{3}{2}\lambda,\lambda} \mathcal{A}^{\frac{5}{2}^-,V}_{\frac{3}{2}\lambda,\lambda} \Big) \\ & + 84 \mathcal{A}^{\frac{3}{2}^+,V*}_{\frac{3}{2}\lambda,\lambda} \mathcal{A}^{\frac{5}{2}^-,V}_{\frac{3}{2}\lambda,\lambda} + 70\sqrt{2} \mathcal{A}^{\frac{1}{2}^+,V*}_{\frac{1}{2}\lambda,0} \mathcal{A}^{\frac{3}{2}^-,V}_{\frac{1}{2}\lambda,0} + 70\sqrt{2} \mathcal{A}^{\frac{1}{2}^+,V*}_{\frac{1}{2}\lambda,\lambda} \mathcal{A}^{\frac{3}{2}^-,V}_{\frac{1}{2}\lambda,\lambda} \\ & + 42\sqrt{6} \mathcal{A}^{\frac{3}{2}^+,V*}_{\frac{1}{2}\lambda,0} \mathcal{A}^{\frac{5}{2}^-,V}_{\frac{1}{2}\lambda,0} + 42\sqrt{6} \mathcal{A}^{\frac{3}{2}^+,V*}_{\frac{1}{2}\lambda,\lambda} \mathcal{A}^{\frac{5}{2}^-,V}_{\frac{1}{2}\lambda,\lambda} \Big] \\ & + (V \longleftrightarrow A) + (P_\Lambda \longrightarrow -P_\Lambda) \, , \end{aligned}$$

$$\begin{aligned} K_{32} = -\frac{1}{7\sqrt{10}} \sum_{\lambda=\pm 1} \mathrm{Im}\Big[& + 4\sqrt{3} \mathcal{A}^{\frac{5}{2}^+,V*}_{\frac{1}{2}\lambda,0} \mathcal{A}^{\frac{5}{2}^+,A}_{\frac{3}{2}\lambda,\lambda} + 7\sqrt{2} \mathcal{A}^{\frac{3}{2}^+,V*}_{\frac{1}{2}\lambda,0} \mathcal{A}^{\frac{3}{2}^+,A}_{\frac{3}{2}\lambda,\lambda} \\ & - \lambda \left(3\sqrt{3} \mathcal{A}^{\frac{5}{2}^+,V*}_{\frac{1}{2}\lambda,0} \mathcal{A}^{\frac{3}{2}^+,A}_{\frac{3}{2}\lambda,\lambda} + \sqrt{2} \mathcal{A}^{\frac{3}{2}^+,V*}_{\frac{1}{2}\lambda,0} \mathcal{A}^{\frac{5}{2}^+,A}_{\frac{3}{2}\lambda,\lambda} \right) \\ & + 5\lambda \left(\mathcal{A}^{\frac{3}{2}^+,V*}_{\frac{1}{2}\lambda,0} \mathcal{A}^{\frac{5}{2}^+,A}_{-\frac{1}{2}\lambda,-\lambda} + \left(\frac{3}{2} \longleftrightarrow \frac{5}{2} \right) \right) \\ & + 7\sqrt{2} \left(\mathcal{A}^{\frac{1}{2}^+,V*}_{\frac{1}{2}\lambda,0} \mathcal{A}^{\frac{5}{2}^+,A}_{-\frac{1}{2}\lambda,-\lambda} + \sqrt{2} \mathcal{A}^{\frac{1}{2}^+,V*}_{\frac{1}{2}\lambda,0} \mathcal{A}^{\frac{5}{2}^+,A}_{\frac{3}{2}\lambda,\lambda} - \left(\frac{5}{2} \longleftrightarrow \frac{1}{2} \right) \right) \\ & + 7\lambda \left(\sqrt{3} \mathcal{A}^{\frac{1}{2}^+,V*}_{\frac{1}{2}\lambda,0} \mathcal{A}^{\frac{3}{2}^+,A}_{-\frac{1}{2}\lambda,-\lambda} + \mathcal{A}^{\frac{1}{2}^+,V*}_{\frac{1}{2}\lambda,0} \mathcal{A}^{\frac{3}{2}^+,A}_{\frac{3}{2}\lambda,\lambda} + \left(\frac{3}{2} \longleftrightarrow \frac{1}{2} \right) \right) \Big] \\ & + (V \longleftrightarrow A) + (P_\Lambda \longrightarrow -P_\Lambda) \; . \end{aligned}$$

Wilson coefficients

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- ➡ SM Wilson coefficients used in JHEP 05 (2013) 137
- ➡ Global fit from Eur. Phys. J. C 82 (2022) 326
  - ❖ Consistent with existing measurements in $b \to sll$

	Standard Model	global fit
$\mathcal{C}_1$	-0.2632	
$\mathcal{C}_2$	1.0111	
$\mathcal{C}_3$	-0.0055	
$\mathcal{C}_4$	-0.0806	
$\mathcal{C}_5$	0.0004	
$\mathcal{C}_6$	0.0009	
$\mathcal{C}_7$	-0.3120	-0.3120
$\mathcal{C}_9$	4.0749	2.9949
$\mathcal{C}_{10}$	-4.3085	-4.1585
$\mathcal{C}_{7'}$	0.0000	0.0000
$\mathcal{C}_{9'}$	0.0000	0.1600
$\mathcal{C}_{10'}$	0.0000	-0.1800