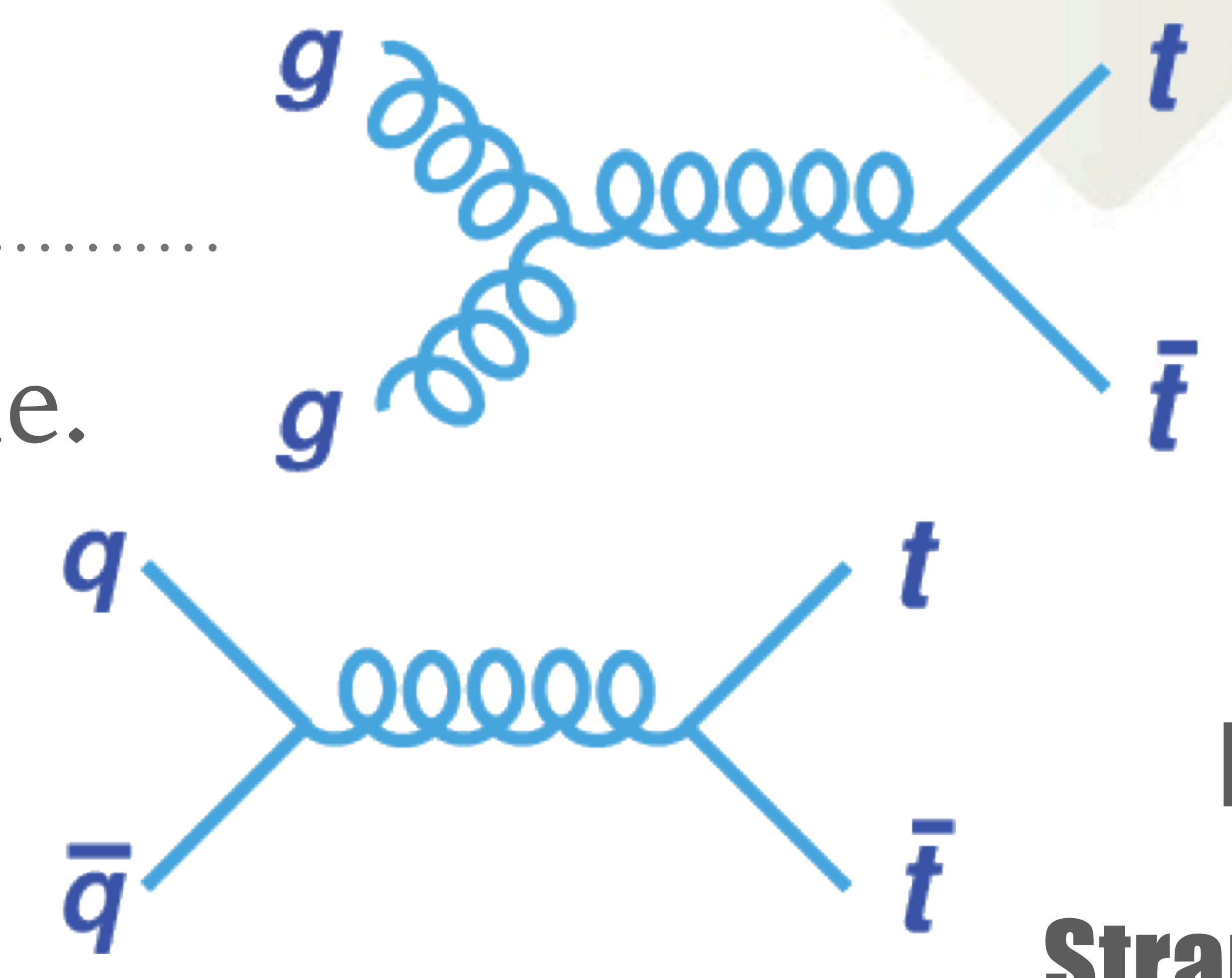
The 2024 international workshop on the high energy Circular Electron Positron Collider
Oct 22-27, 2024

TOP QUARK SPIN CORRELATION & ENTANGLEMENT @CMS

Kai-Feng Chen (NTU)

On Behalf of the CMS Collaboration

TOP QUARK PRODUCTIONS AT THE LHC

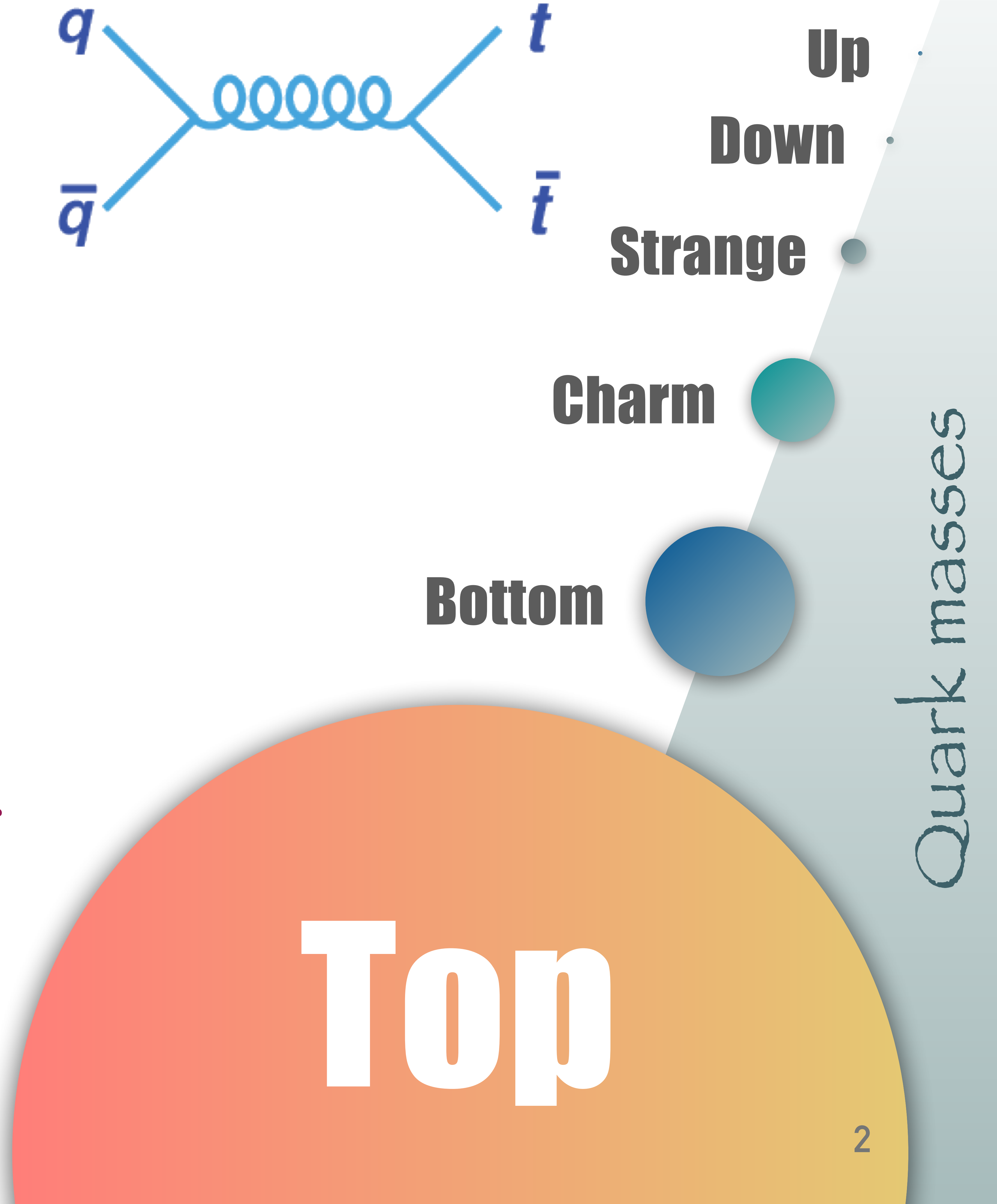


- Top quark is the **most massive** known fundamental particle.
  - $\sim$36 times heavier than the bottom quark. Any reason?
- Top quark is **extremely short lived**:

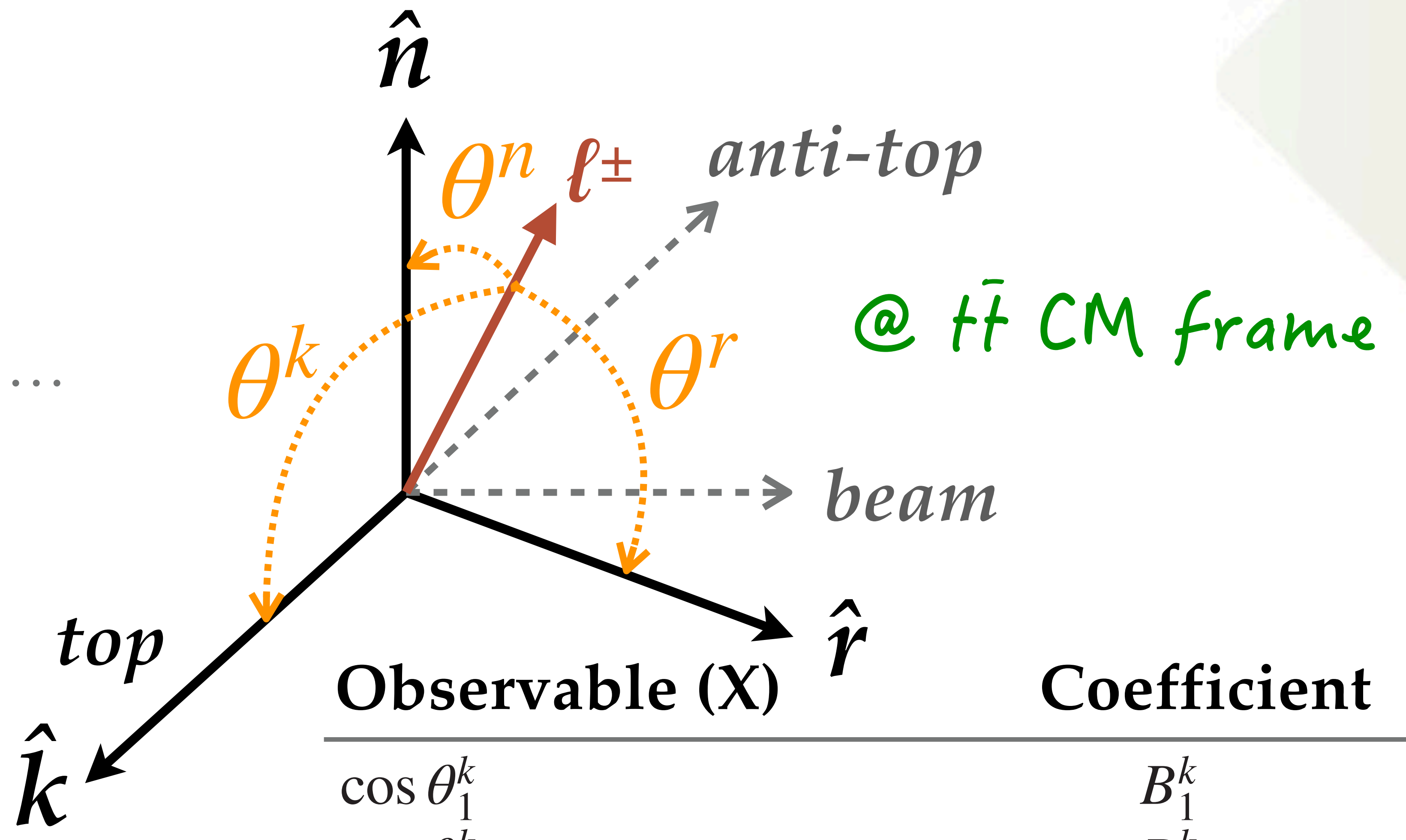
$$\underbrace{\frac{1}{m_t}}_{\substack{\text{production}\\ 10^{-27}s}} < \underbrace{\frac{1}{\Gamma_t}}_{\substack{\text{decay}\\ 10^{-25}s}} < \underbrace{\frac{1}{\Lambda_{\rm QCD}}}_{\substack{\text{hadronization}\\ 10^{-24}s}} < \underbrace{\frac{m_t}{\Lambda_{\rm QCD}}}_{\substack{\text{spin-flip}\\ 10^{-21}s}}$$

  - Decays before its hadronization & spin decorrelation

    $\Rightarrow$ Allow to probe bare quark properties!
  - The decay preserves the spin information in the **angular distribution of the decay products**.
- **Top pair productions at LHC are ideal for studying polarization and spin correlation!**



TOP POLARIZATION & SPIN CORRELATION



- Measuring $t\bar{t}$ polarization and spin correlation in the **helicity basis**: $\{\hat{k}, \hat{r}, \hat{n}\}$
- Angular distribution for the decay daughters:

  e.g. dilepton $t\bar{t}$ events

Polarization

Spin correlation

$$\frac{1}{\sigma}\frac{d^4\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2}(1 + \mathbf{B_1}\cdot\hat{\ell}_1 + \mathbf{B_2}\cdot\hat{\ell}_2 - \hat{\ell}_1\cdot\mathbf{C}\cdot\hat{\ell}_2)$$

$$\mathbf{B_{1,2}} = \begin{pmatrix} x \\ x \\ x \end{pmatrix} \qquad \mathbf{C} = \begin{pmatrix} x & x & x \\ x & x & x \\ x & x & x \end{pmatrix}$$

- Spin information is fully described by the polarization vectors and correlation matrix elements.
- These coefficients can be probed by 1D angular distributions individually:

$$\frac{1}{\sigma}\frac{d\sigma}{dX} = \frac{1}{2}(1 + [\text{Coefficient}]\cdot X)f(X)$$

Observable (X)	Coefficient	Coeff. func
$\cos\theta_1^k$	B_1^k	b_k^+
$\cos\theta_2^k$	B_2^k	b_k^-
$\cos\theta_1^r$	B_1^r	b_r^+
$\cos\theta_2^r$	B_2^r	b_r^-
$\cos\theta_1^n$	B_1^n	b_n^+
$\cos\theta_2^n$	B_2^n	b_n^-
$\cos\theta_1^{k*}$	B_1^{k*}	b_k^+
$\cos\theta_2^{k*}$	B_2^{k*}	b_k^-
$\cos\theta_1^{r*}$	B_1^{r*}	b_r^+
$\cos\theta_2^{r*}$	B_2^{r*}	b_r^-
$\cos\theta_1^k\cos\theta_2^k$	C_{kk}	c_{kk}
$\cos\theta_1^r\cos\theta_2^r$	C_{rr}	c_{rr}
$\cos\theta_1^n\cos\theta_2^n$	C_{nn}	c_{nn}
$\cos\theta_1^r\cos\theta_2^k + \cos\theta_1^k\cos\theta_2^r$	$C_{rk} + C_{kr}$	c_{rk}
$\cos\theta_1^r\cos\theta_2^k - \cos\theta_1^k\cos\theta_2^r$	$C_{rk} - C_{kr}$	c_n
$\cos\theta_1^n\cos\theta_2^r + \cos\theta_1^r\cos\theta_2^n$	$C_{nr} + C_{rn}$	c_{nr}
$\cos\theta_1^n\cos\theta_2^r - \cos\theta_1^r\cos\theta_2^n$	$C_{nr} - C_{rn}$	c_k
$\cos\theta_1^n\cos\theta_2^k + \cos\theta_1^k\cos\theta_2^n$	$C_{nk} + C_{kn}$	c_{kn}
$\cos\theta_1^n\cos\theta_2^k - \cos\theta_1^k\cos\theta_2^n$	$C_{nk} - C_{kn}$	$-c_r$
$\cos\varphi$	D	$-(c_{kk} + c_{rr} + c_{nn})/3$
$\cos\varphi_{\text{lab}}$	$A_{\cos\varphi}^{\text{lab}}$	$\cdots$
$\lvert\Delta\phi_{\ell\ell}\rvert$	$A_{\lvert\Delta\phi_{\ell\ell}\rvert}$	$\cdots$

MEASURING TOP QUARK SPINS

Ref. CMS PRD 100 (2019) 072002

- In the SM $t\bar{t}$ production is **~unpolarized**:

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta^i_{1,2}} = \frac{1}{2}(1 + \mathbf{B}^{\mathbf{i}}_{\mathbf{1,2}} \cdot \cos\theta^i_{1,2})$$

~Null polarization vectors; flat angular distributions.

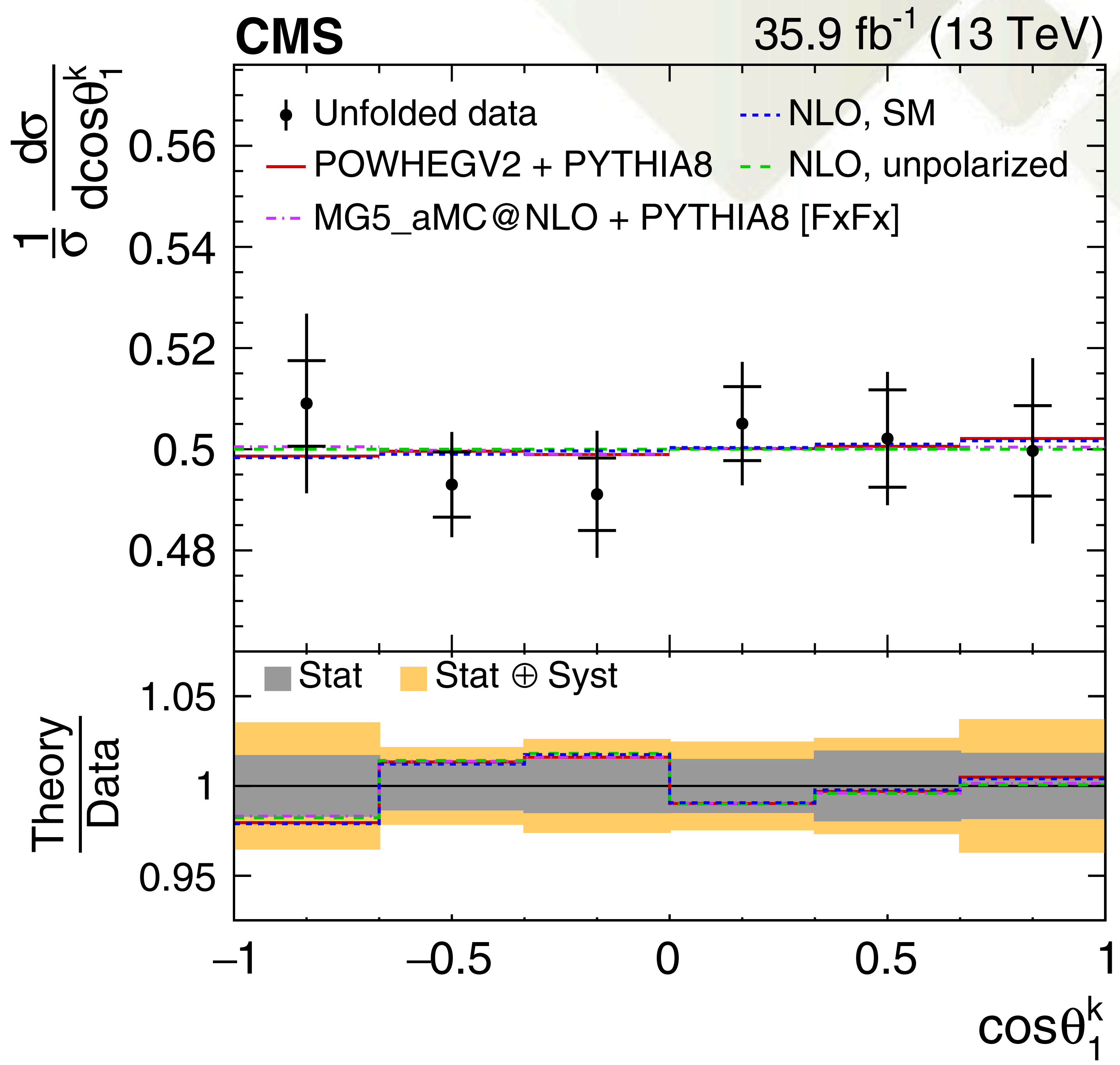
- However top/anti-top spins are strongly correlated:
  - Rich spin correlation structure!
  - Spin correlation coefficient ***D*** is related to the ***C*** matrix diagonal terms.
  - Sensitivity to the alignment of top/anti-top quark spins $\Rightarrow$ access to **entanglement measurement**!

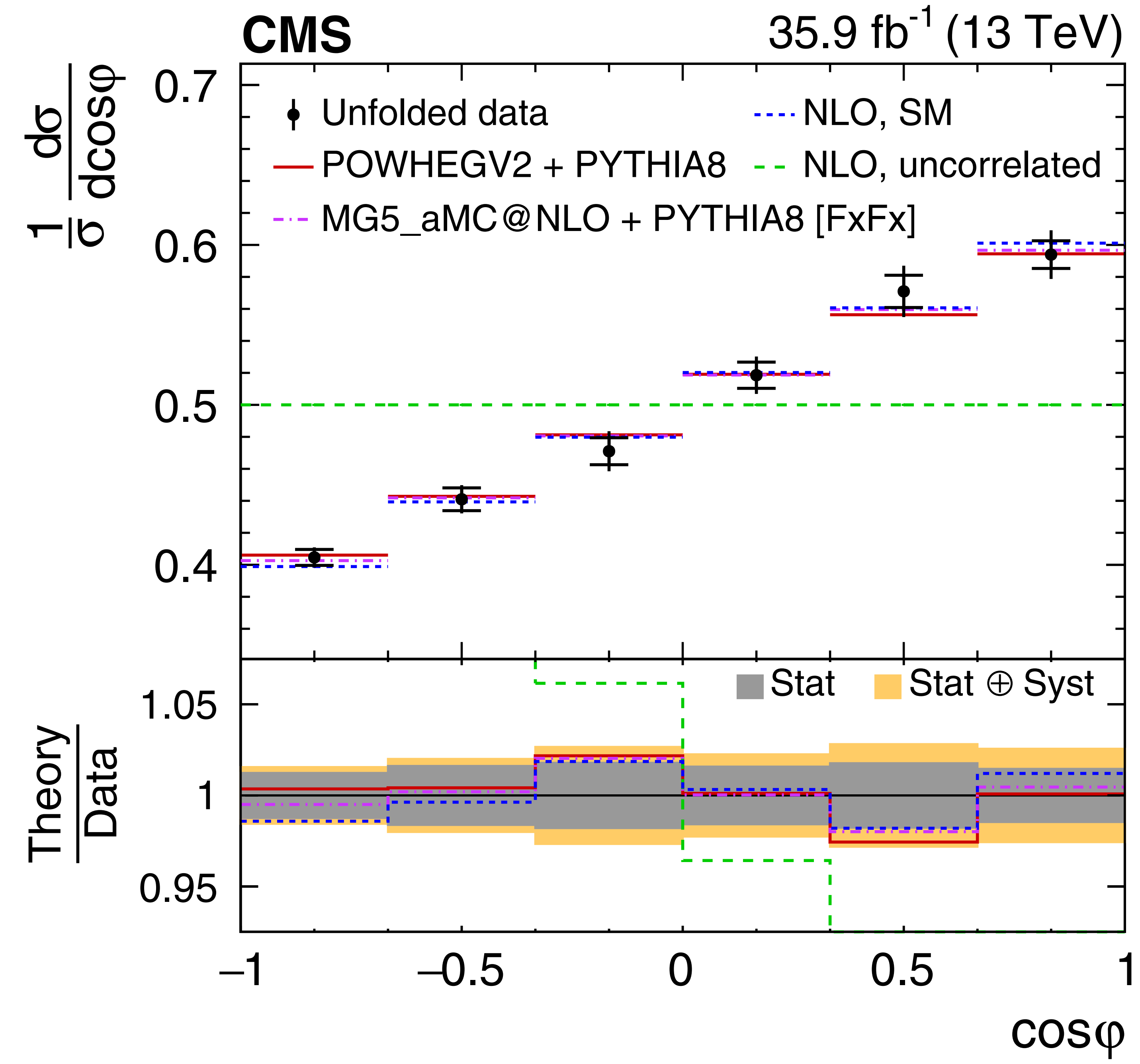
$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\phi} = \frac{1}{2}(1 - \mathbf{D} \cdot \cos\phi)$$

Non-zero spin correlation variables; most precise observable!

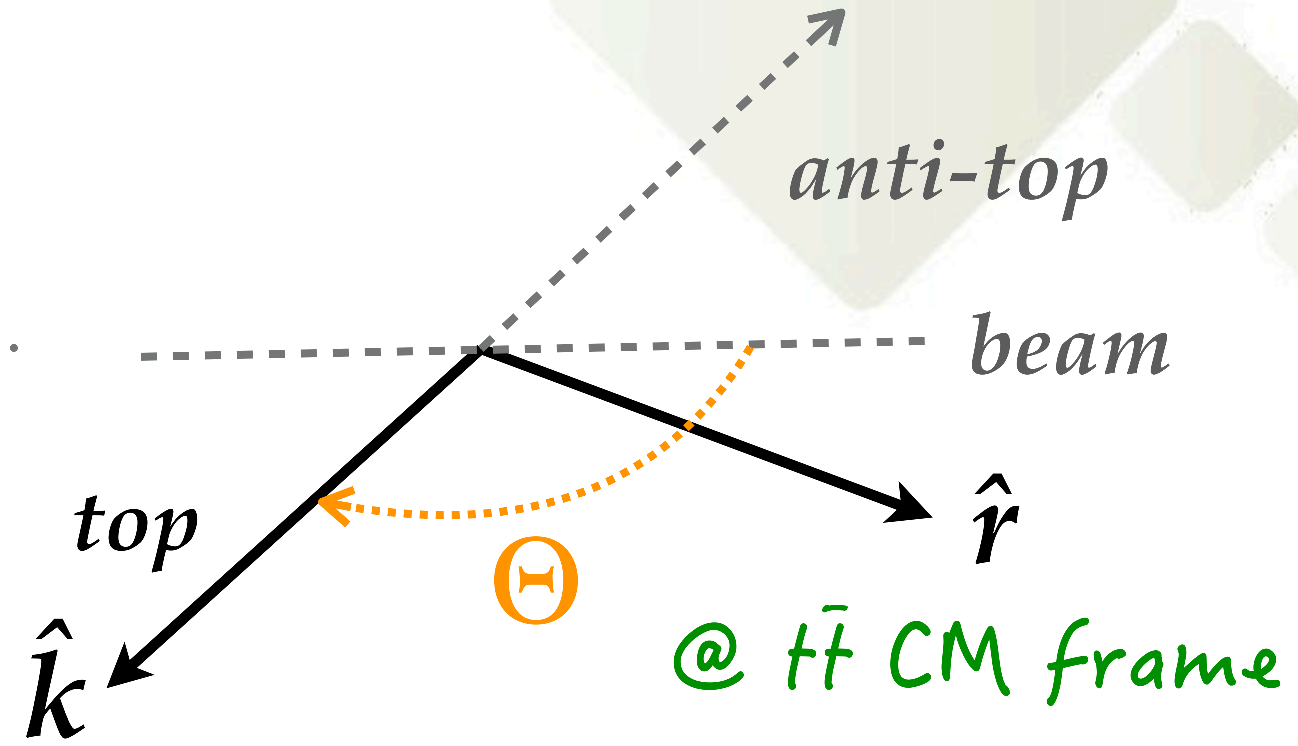
$$\cos\phi = \hat{\ell}_1 \cdot \hat{\ell}_2 \qquad \mathbf{D} = -\frac{\mathrm{Tr}[C]}{3} = -\left(\frac{C_{kk} + C_{rr} + C_{nn}}{3}\right)$$

@ $t\bar{t}$ CM frame





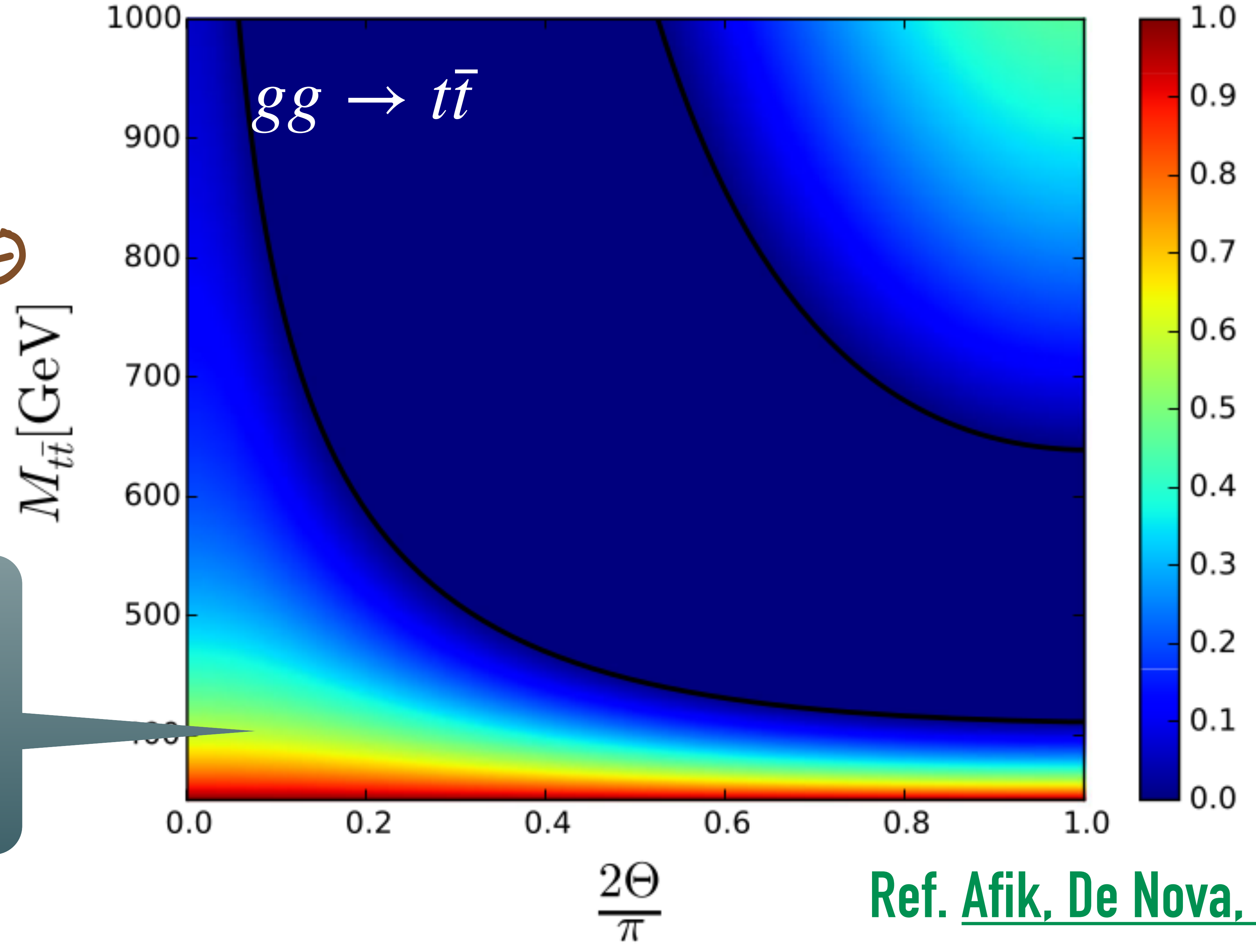
TOP QUARKS ARE ENTANGLED OR NOT?

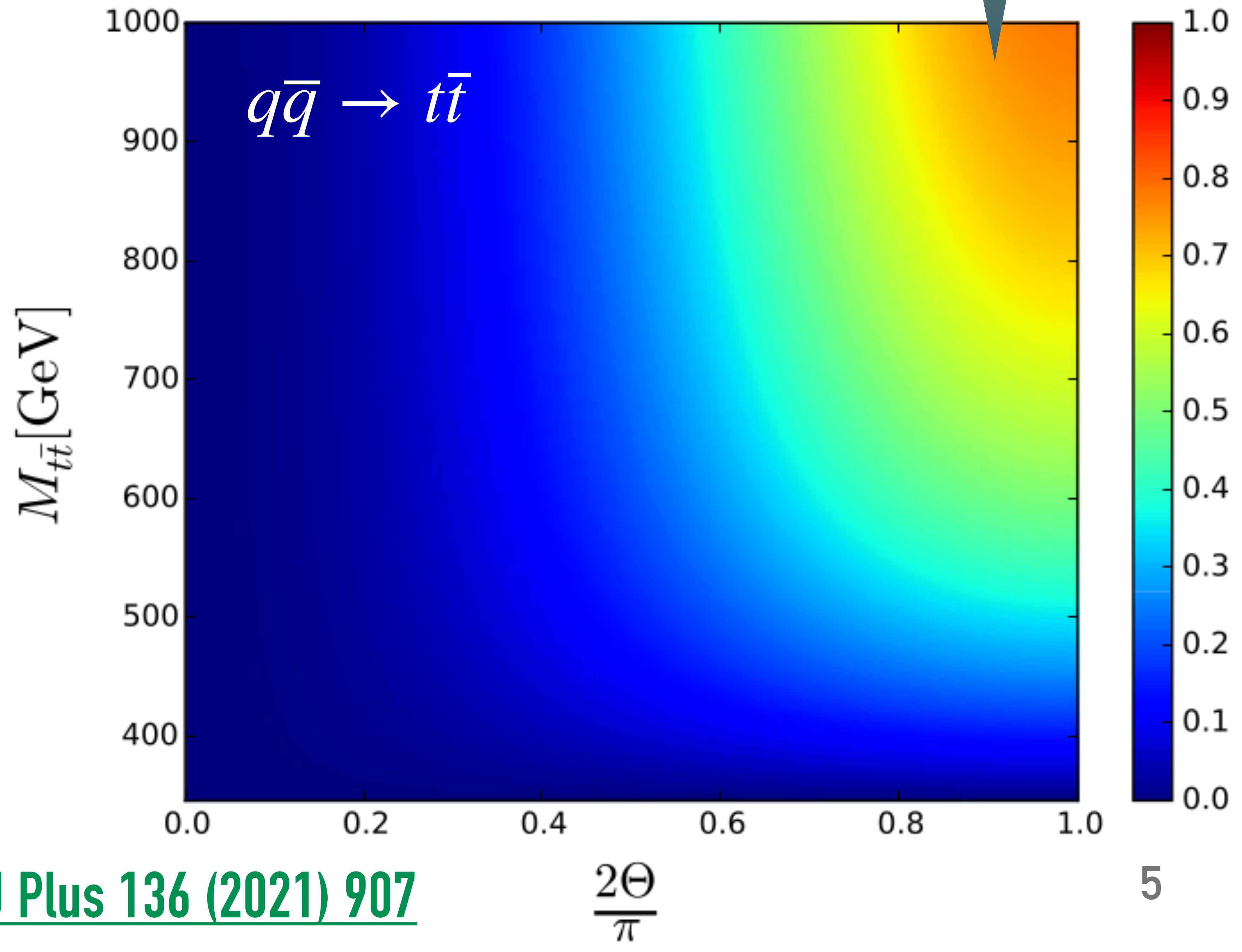


- **Entanglement = inseparability of quantum states**
- Can be studied with spin correlations observables.
- The SM predicts entangled of top/anti-top:
  - **near the production threshold** in g-g fusion production;
  - **boosted region** for central production of $t\bar{t}$.

The strength depends on production modes, $M(t\bar{t})$, and top scattering angle Θ

Relatively higher velocity top quarks → space-like separated events

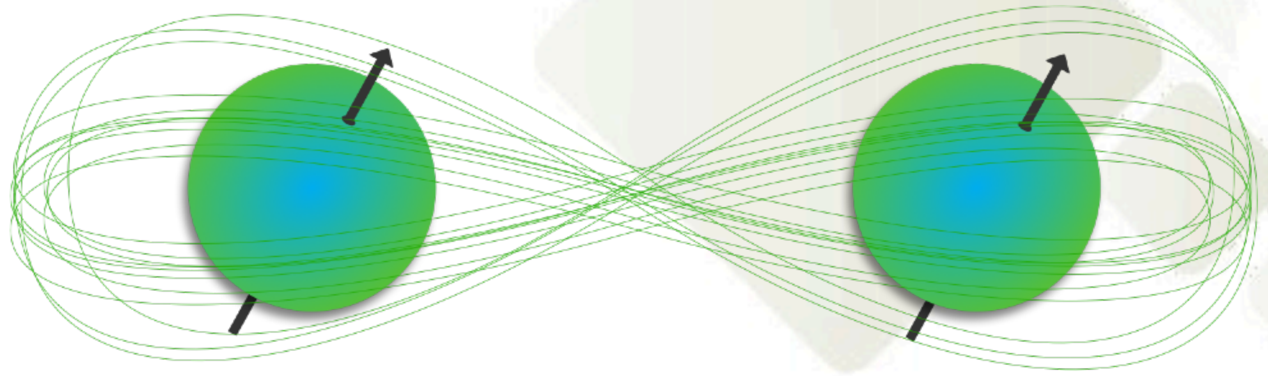




Relatively lower velocity top quarks → time-like separated events

Ref. Afik, De Nova, EPJ Plus 136 (2021) 907

DILEPTON ANALYSIS FOR LOW M($t\bar{t}$)



- The **Peres-Horodecki criterion**:

Ref. Peres PRL 77 (1996) 1413
Horodecki PLA 232 (1997) 5

  - For $t\bar{t}$ system, states are separable if eigenvalues of the spin density matrix are all positive;
  - Top quarks are entangled in a certain phase space if at least one negative eigenvalue!
- A sufficient condition to observe entanglement in top quarks is:

$$\Delta E = C_{nn} + |C_{rr} + C_{kk}| > 1$$

Ref. Afik, De Nova, EPJ Plus 136 (2021) 907

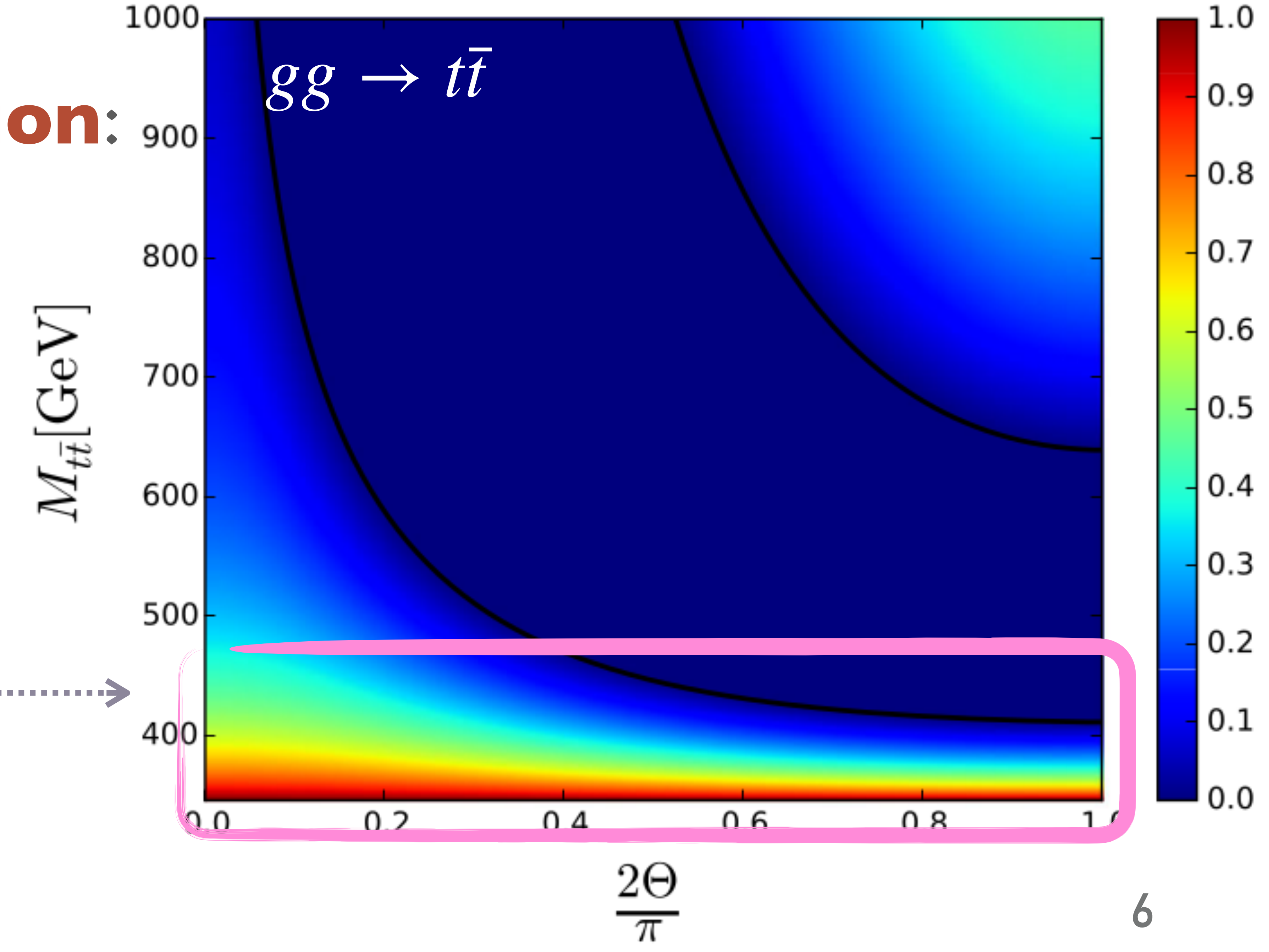
- Consider a **dilepton $t\bar{t}$ analysis focused in low m($t\bar{t}$) region**:
  - Top spin information 100% goes to charged leptons!
  - Spin-singlet state ⇒ positive C_{rr} and C_{kk}:

$$\Delta E = C_{nn} + C_{rr} + C_{kk} = \mathrm{Tr}[C] = -3\mathbf{D} > 1$$

  - The sufficient condition is then translated as:

$$\mathbf{D} = -\frac{\mathrm{Tr}[\mathbf{C}]}{3} < -\frac{1}{3}$$

Measure the proxy D to verify top entanglement!



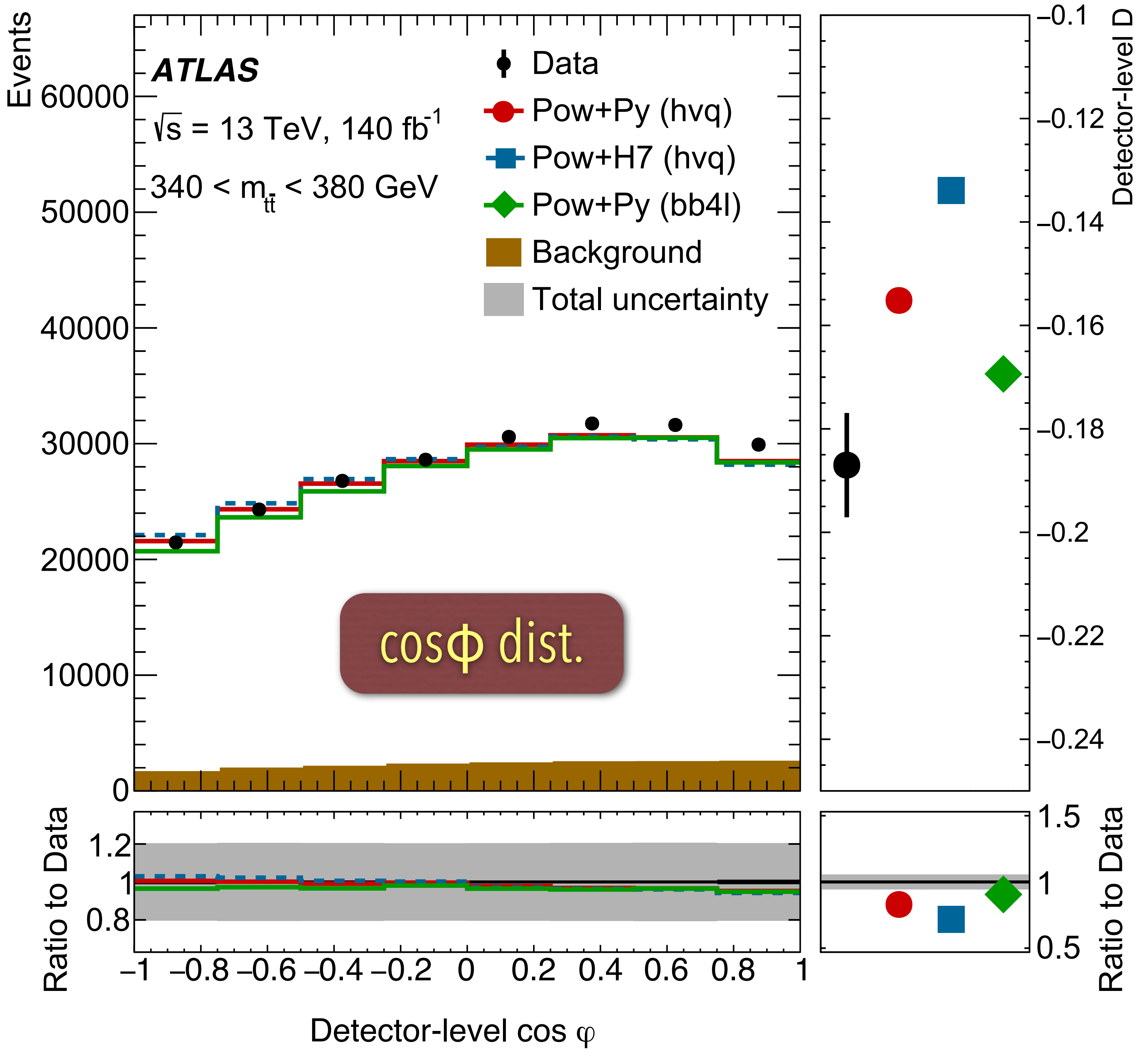
ATLAS OBSERVATION OF TOP ENTANGLEMENT AT LOW M($t\bar{t}$)

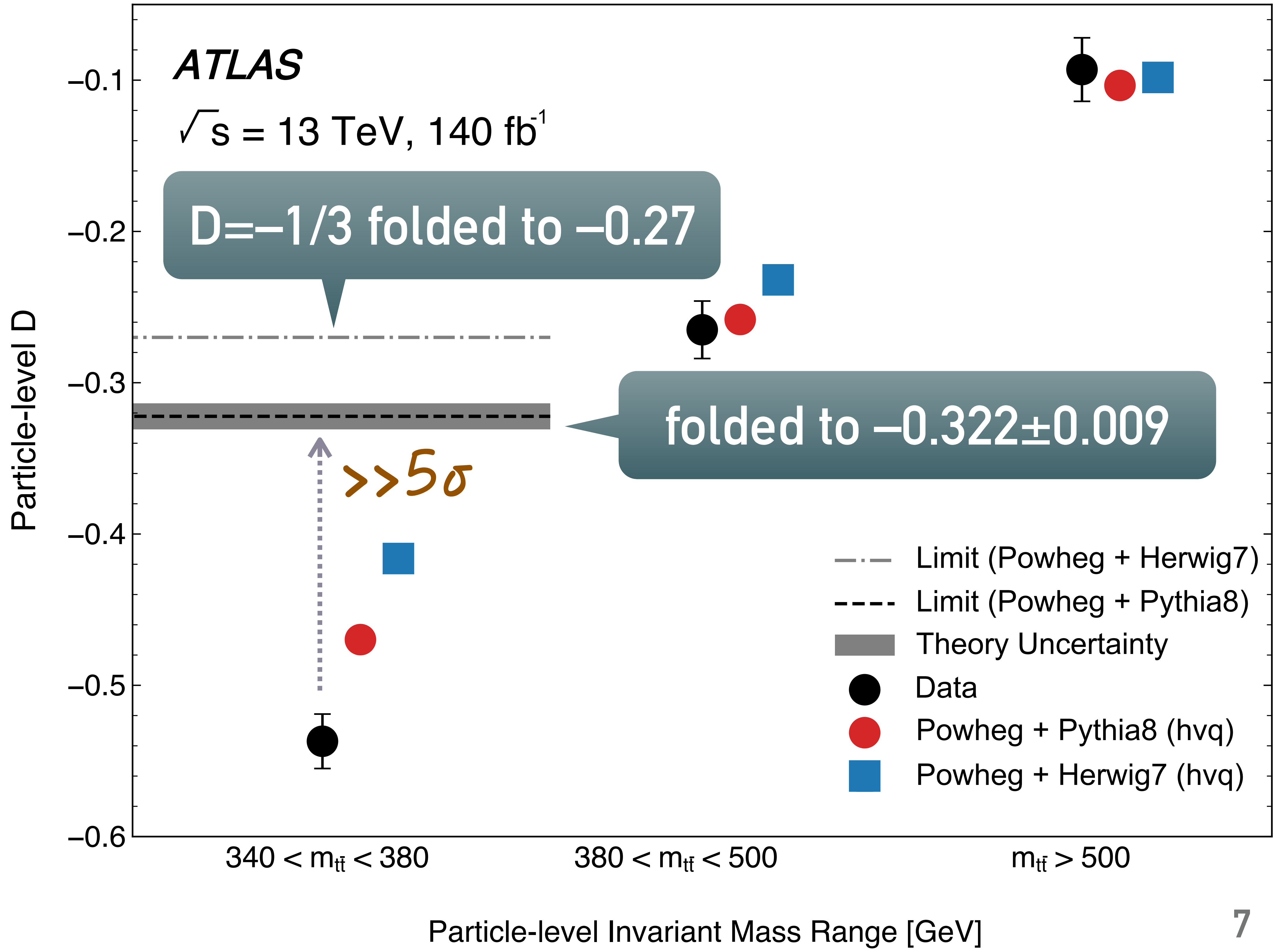
- ATLAS performed a analysis using very clean eμ+jets events with 140 fb^{-1} data.
- A measurement of proxy D at particle level in the region of $340 < m(t\bar{t}) < 380$ GeV clearly away from the threshold.
- **Observation of entanglement in $t\bar{t}$!**

$$D^{\text{particle}}_{\text{obs}} = -0.547 \pm 0.002(\text{stat.}) \pm 0.021(\text{syst.})$$

Ref. [ATLAS Nature 633 (2024) 542]

Read more about this!
[https://atlas.cern/Updates/Briefing/Top-Entanglement]



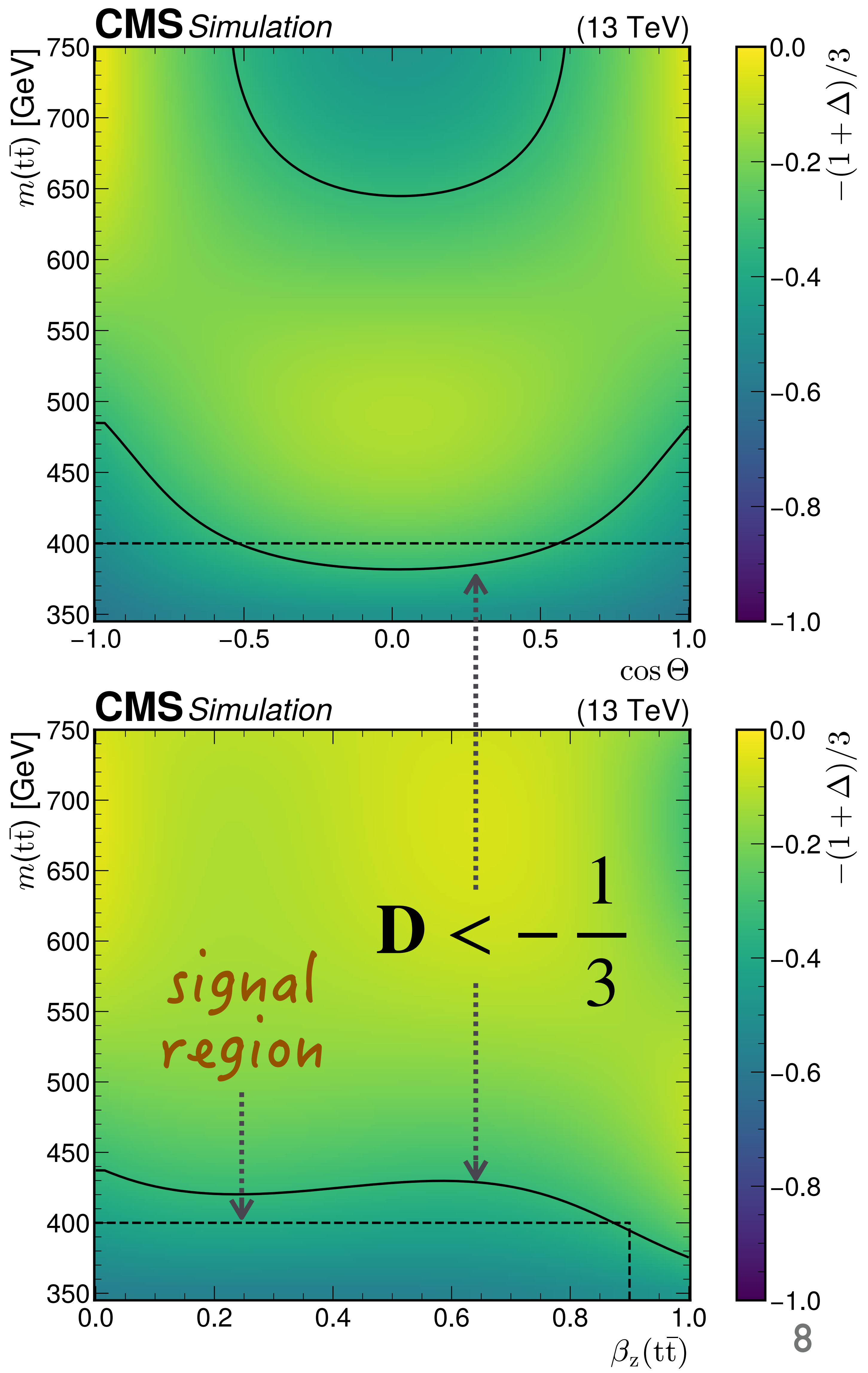


How about CMS analysis?

CMS ANALYSIS STRATEGY: DILEPTON

- Based on 36.3 fb^{-1} data collected in 2016.
- **Strength of entanglement depends on the phase-space**:
  - Scan over **m(tt̄)** vs **cosΘ** vs **βz(tt̄)** to determine most sensitive region + minimizing total uncertainty.
  - Focus on low-mass region: **345 < m(tt̄) < 400 GeV**.
  - Increase gg/qq̄ fraction with cut on the velocity along the beam: $\beta_z(t\bar{t}) = \left| \frac{p_z^t - p_z^{\bar{t}}}{E^t - E^{\bar{t}}} \right| < 0.9$ Ref. Aguilar-Saavedra, Casas, EPJC 82 (2022) 666
- The tt̄ four-momentum are calculated using a kinematic reconstruction algorithm. Ref. CMS JHEP 02 (2019) 149
- The distribution of **helicity angle** $\cos\phi = \hat{\ell}_1 \cdot \hat{\ell}_2$ is measured and perform a profiled maximum likelihood fit to extract the *D* value of the events in the signal region.

Entanglement vs phase-space

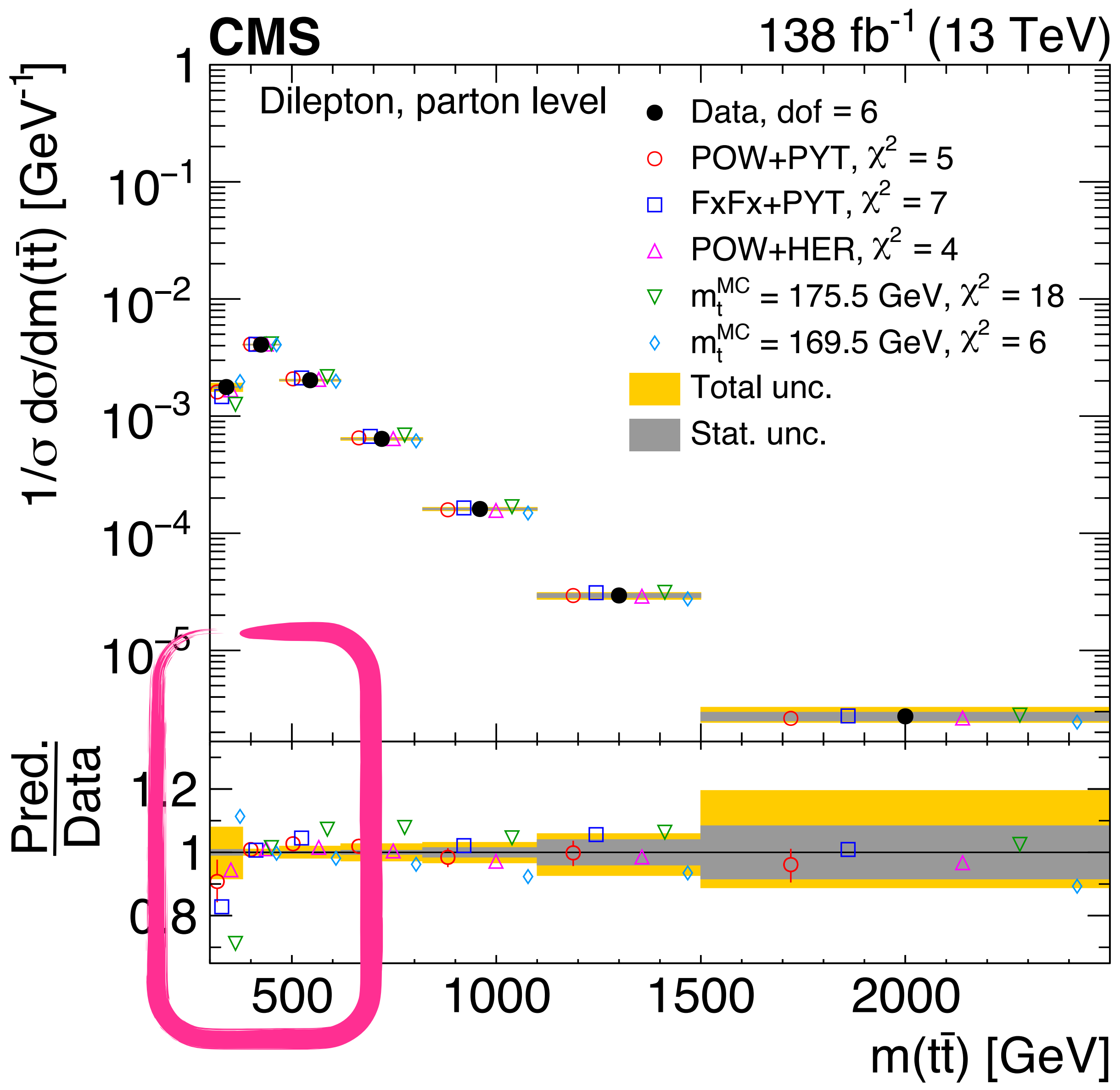


NEAR THRESHOLD $t\bar{t}$ MODELING

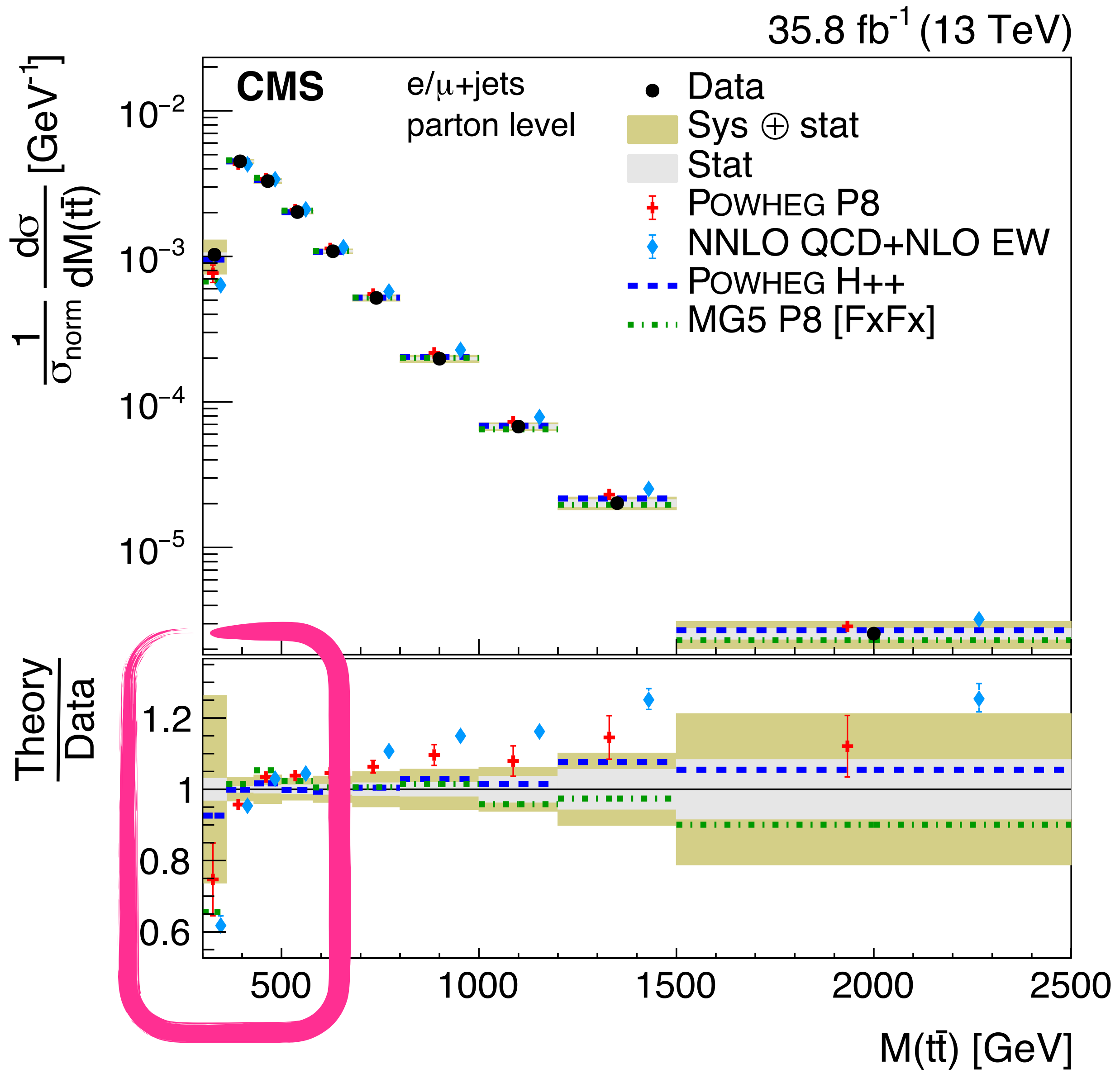
- ➤ The $t\bar{t}$ simulated samples are described by **PowhegBox+Pythia8(NLO)**:
  - ○ EWK corrections at NLO with HATHOR included. Ref. HATHOR Comput. Phys. Commun. 182 (2011) 1034
  - ○ Reweighed to NNLO QCD calculations. Ref. Mazzitelli at el PRL 127 (2021) 062001
- ➤ However **mis-modeling of the events near the threshold** has been observed in several CMS/ATLAS differential cross section analyses already:

O(10%) discrepancy, consistent between dilepton and lepton+jets analyses from both CMS and ATLAS

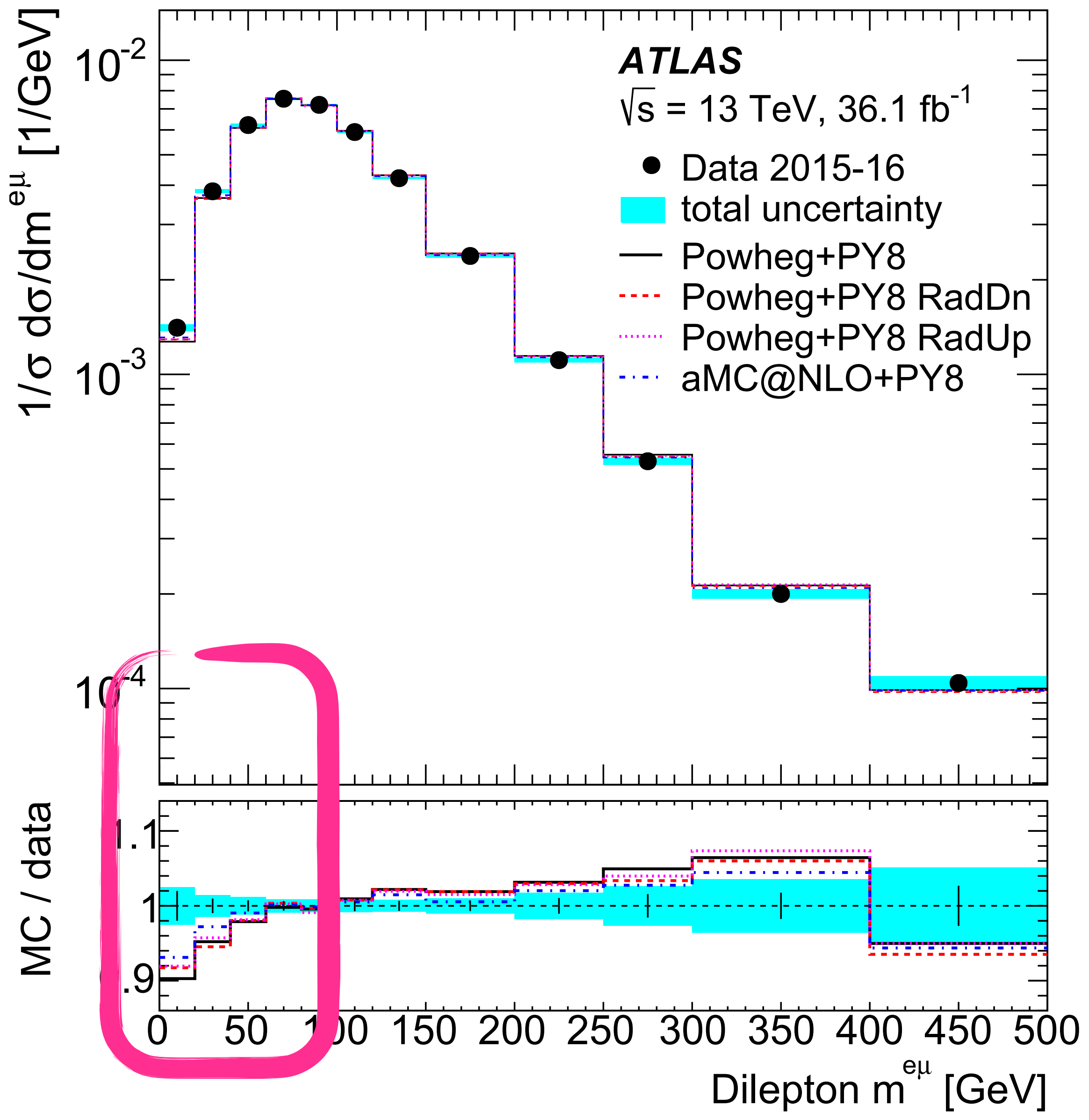
Ref. CMS arXiv:2402.08486



Ref. CMS PRD 97 (2018) 112003



Ref. ATLAS EPJC 80 (2020) 528

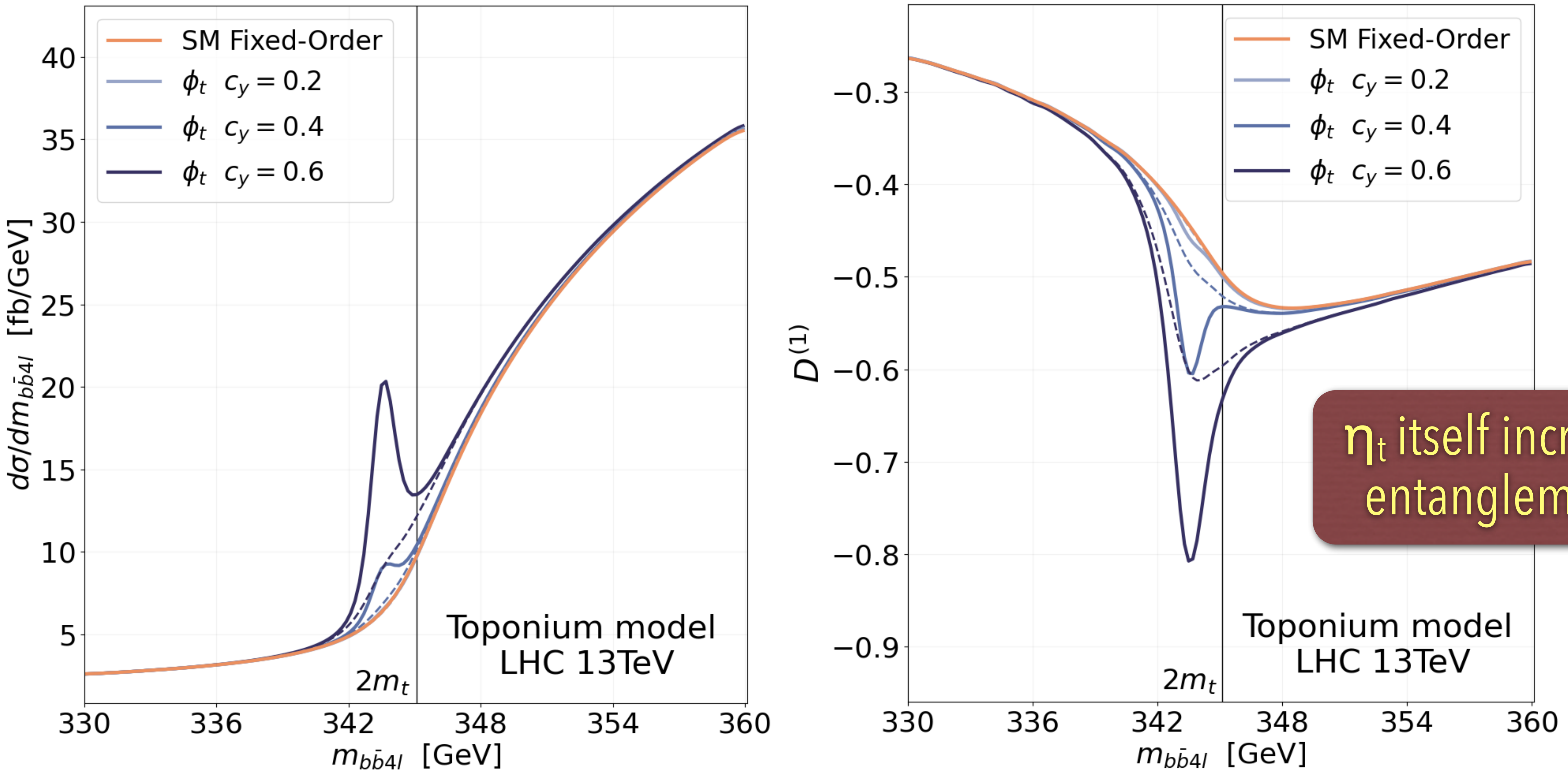


NEAR THRESHOLD $t\bar{t}$ MODELING (CONT.)

Not considered in ATLAS analysis, although the conclusion should not be affected.

- Consider a potential **"toponium"** contribution:
  - NRQCD predicts a quasi-bound state at 343 GeV and a width of 7 GeV. Ref. Ju et al JHEP 06 (2020) 158
  - **Modifies both the invariant mass and spin correlations.** Ref. Maltoni et al JHEP 03 (2024) 099
- Toponium model generated with MG5 aMC@NLO(LO)+Pythia8: Ref. Fuks et al, PRD 104 (2021) 034023
  - Consider a pseudoscalar colour singlet spin-0 state η_t.
  - Improves the modeling near the threshold!

Perform the full entanglement measurement w/ and w/o toponium model included.



η_t itself increase entanglement

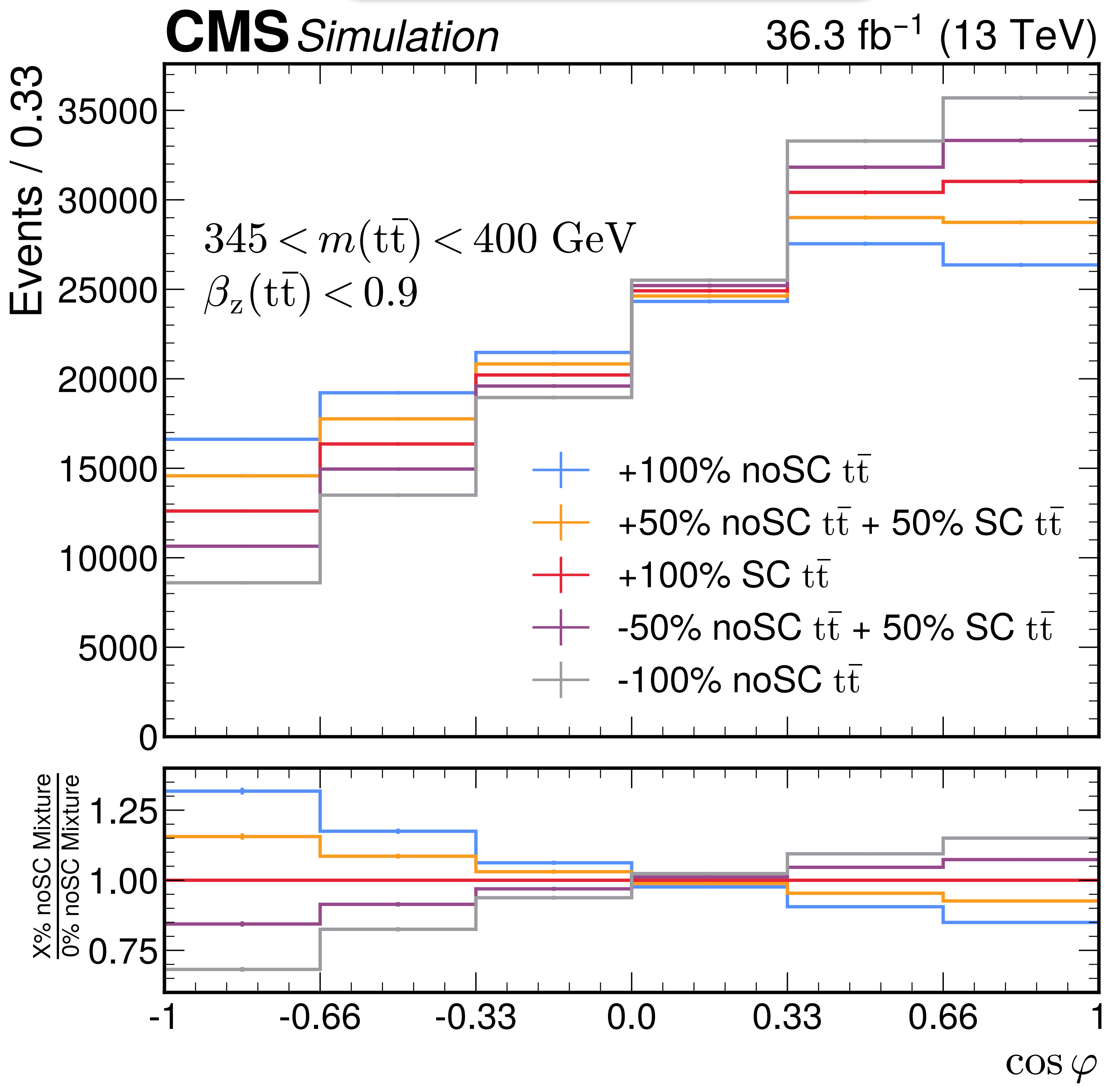
See also the CMS $H/A \to t\bar{t}$ analysis! (HIG-22-013)

- Strong excess above the background if no η_t considered.
- $\sigma(\eta_t) = 7.14 \pm 0.77$ pb
- See L.Jeppe's talk @ TOP2024 for details.

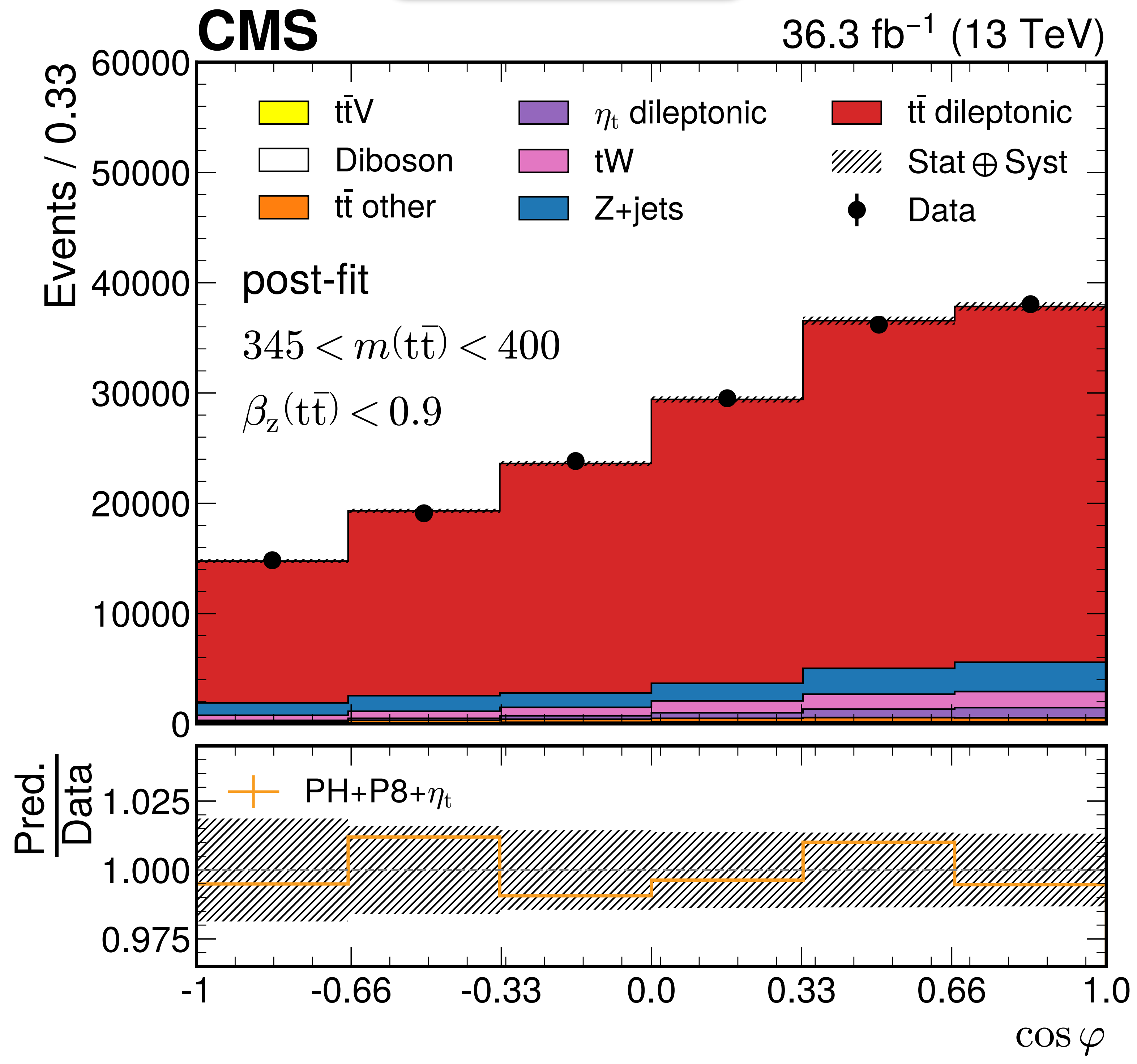
EXTRACTION OF ENTANGLEMENT

- **Binned profiled likelihood fit** to the distribution with ~48k signal candidates:
  - Signal templates with different proxy $\boldsymbol{D}$ value are derived from **mixed samples generated w/ and w/o spin correlations** $\Rightarrow$ any possible value of $\boldsymbol{D}$ between ± 1 is accessible.
  - All systematic effects included as nuisance parameters.

Signal templates vs D



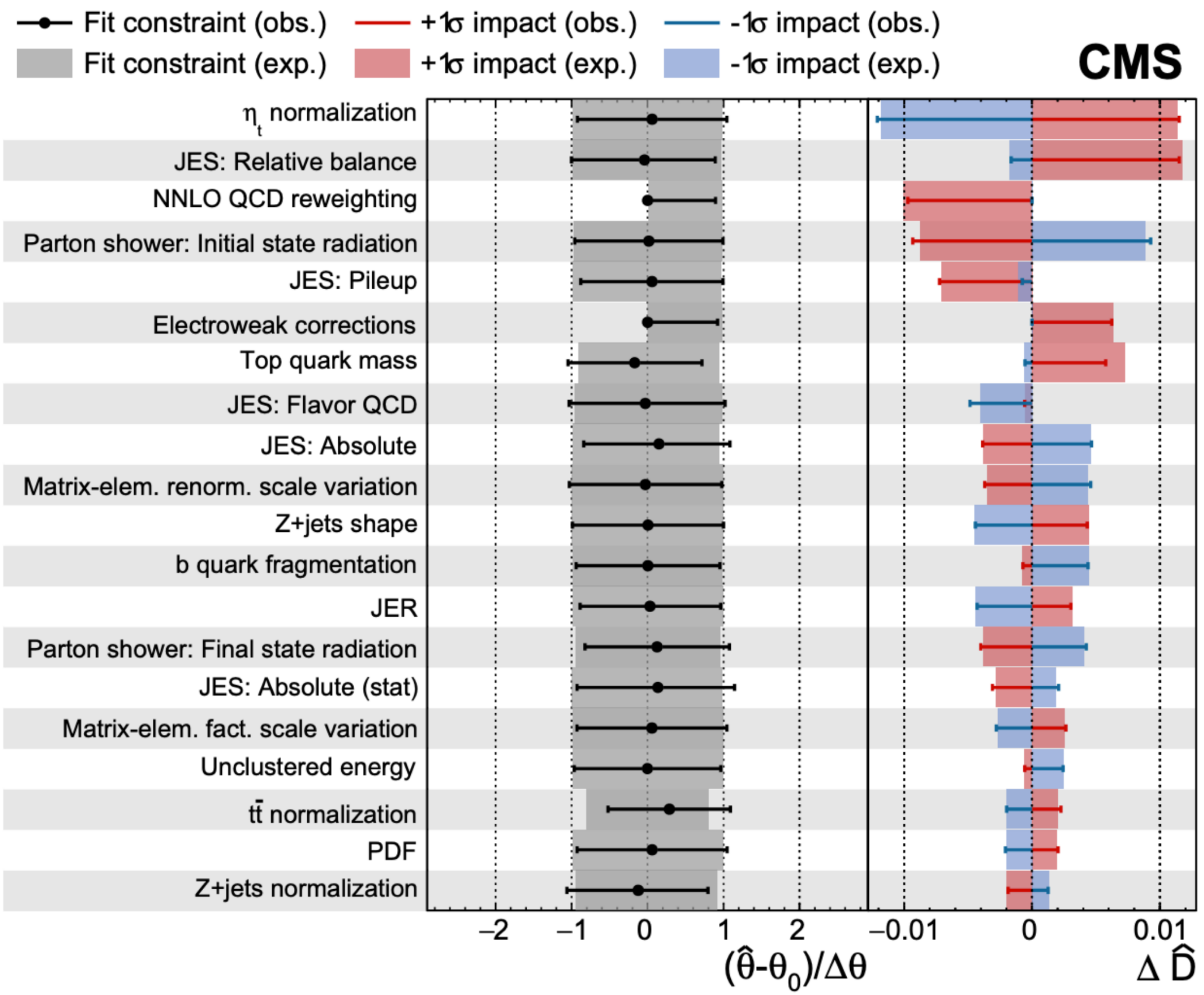
Post-fit cosΦ



Excellent description of the events: good agreement with the SM predictions.

SYSTEMATIC UNCERTAINTIES

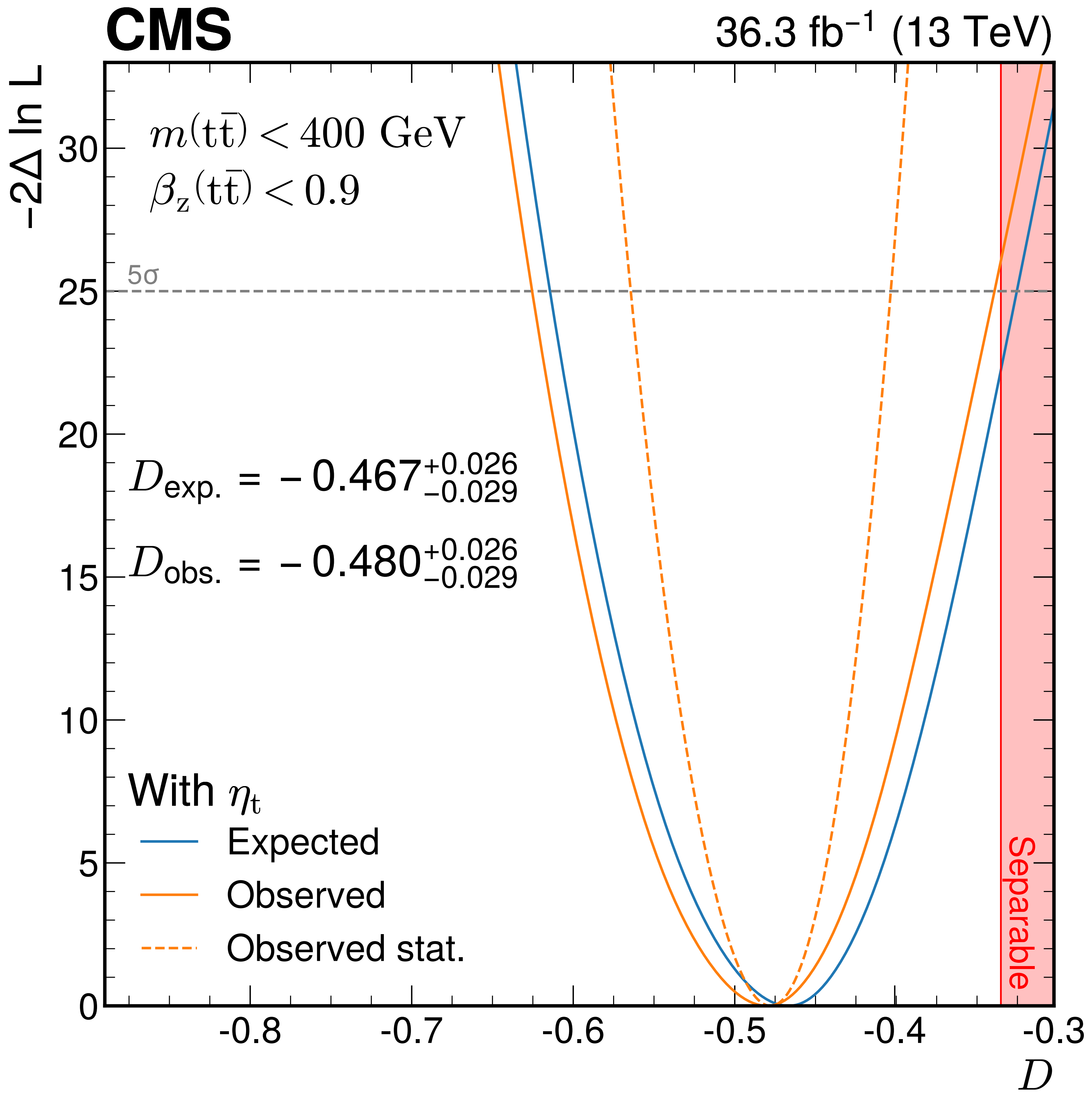
- As the analysis heavily rely on the knowledge for the near-threshold $t\bar{t}$ events, the uncertainties associated with signal **$t\bar{t}$ modeling** are unavoidable.
- Uncertainties related to **Toponium**:
  - Toponium cross section varied by ±50% due to missing the octet contributions.
  - Binding energy uncertainty varied by ±0.5 GeV.
- Other leading uncertainties:
  - Jet energy scale
  - NNLO QCD reweighing
  - Parton Shower

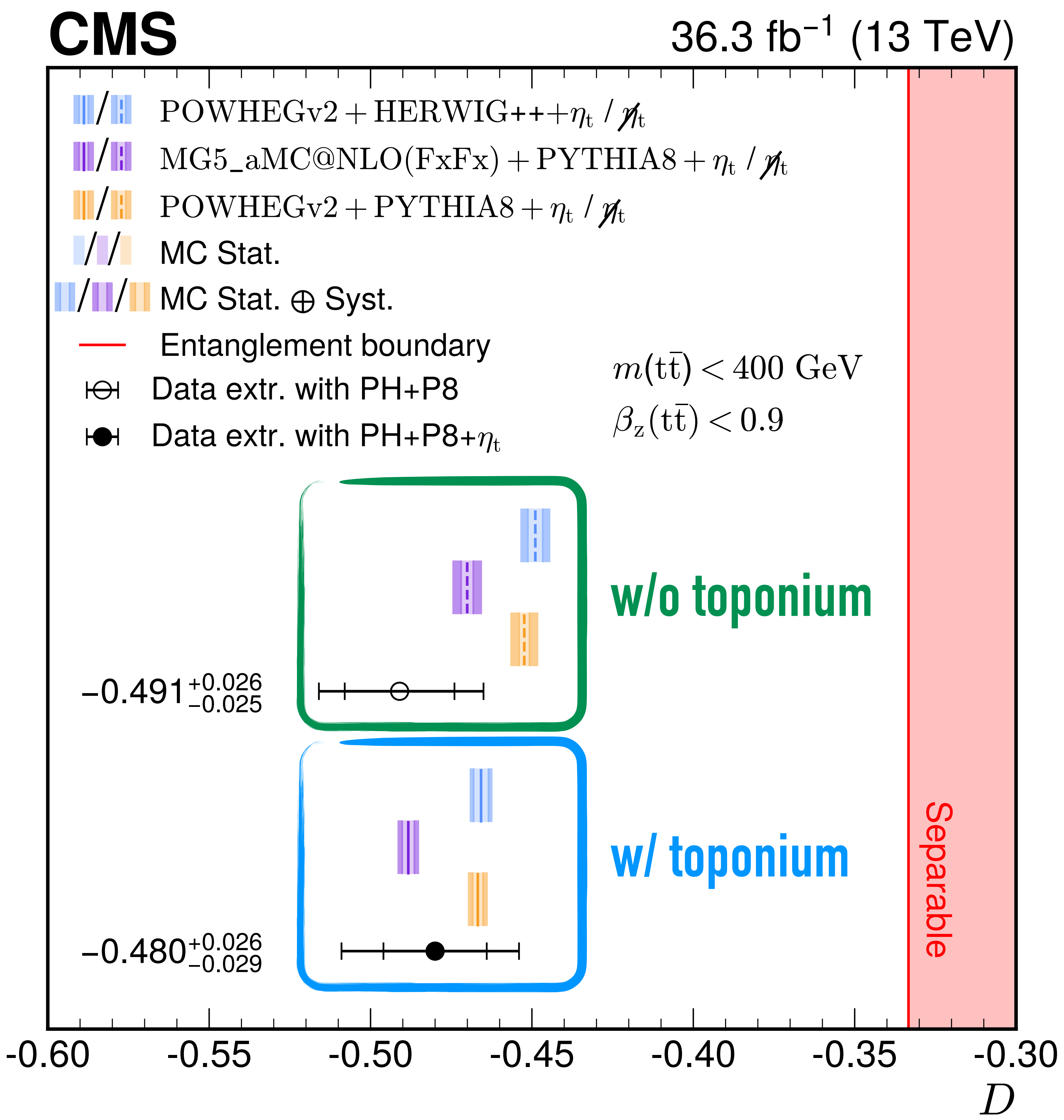


OBSERVATION OF $t\bar{t}$ ENTANGLEMENT AT THRESHOLD

Ref. CMS arXiv:2406.03976, Accepted by Rep. on Progress in Physics

- Scan of the likelihood value versus ***D*** at parton level, with all detector effects accounted.
- If toponium contribution is not included:
  - Pushes to a more negative ***D*** value / stronger entanglement.
  - **~1.5σ tension with respect to the expectation.**





Observation of top quarks being entangled at $t\bar{t}$ threshold!

$$D_{obs} = -0.480^{+0.016}_{-0.017}(\text{stat.})^{+0.020}_{-0.023}(\text{syst.})$$

$$D_{exp} = -0.467^{+0.016}_{-0.017}(\text{stat.})^{+0.021}_{-0.024}(\text{syst.})$$

Significance: 5.1σ Obs (4.7σ Expt.)

LEPTON+JETS ANALYSIS

$$\Omega = (\sin\theta_p \cos\phi_p, \sin\theta_p \sin\phi_p, \cos\theta_p)$$

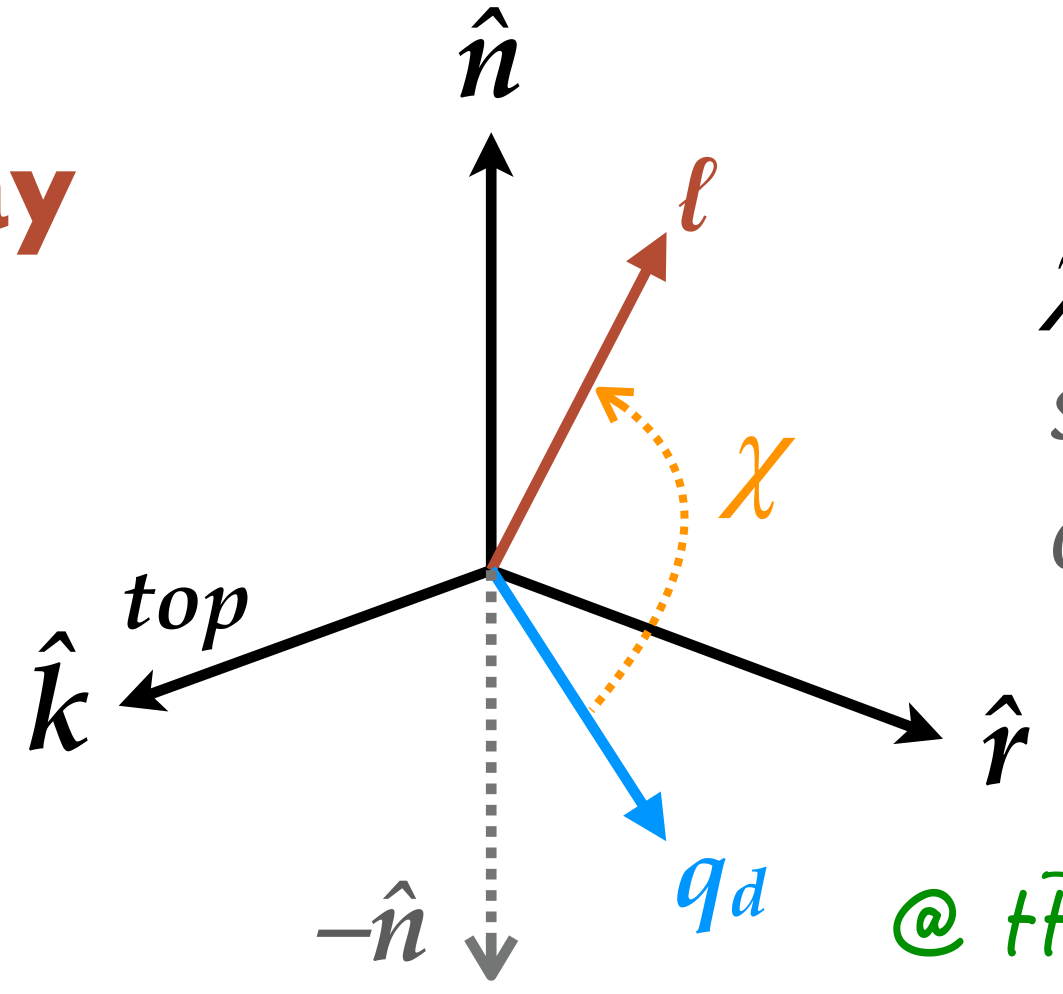
- Consider the differential cross section for decay products $p(\bar{p})$ of $\mathrm{t}(\bar{\mathrm{t}})$ along the direction $\Omega(\bar{\Omega})$:

Polarization — Spin correlation

$$\frac{1}{\sigma}\frac{d^4\sigma}{d\Omega d\bar{\Omega}} \propto (1 + \kappa \mathbf{P}\cdot\Omega + \bar{\kappa}\bar{\mathbf{P}}\cdot\bar{\Omega} - \kappa\bar{\kappa}\Omega\cdot\mathbf{C}\cdot\bar{\Omega})$$

$$\mathbf{P}(\bar{\mathbf{P}}) = \begin{pmatrix} x \\ x \\ x \end{pmatrix} \qquad \mathbf{C} = \begin{pmatrix} x & x & x \\ x & x & x \\ x & x & x \end{pmatrix}$$

- The charged lepton and down-type quark decaying from W bosons are used, which has a large spin analyzing power κ (~unity at LO).
- Entangled or not via the **Peres-Horodecki criterion**: $\Delta E = C_{nn} + |C_{rr} + C_{kk}| > 1$
- Two analysis approaches:
  - Use the **full angular information of decay products** to measure spin matrix and then derive *ΔE*.
  - Measure **proxies D and D̃** via simpler 1D χ and $\tilde{\chi}$ angular distributions.



$\tilde{\chi} = \chi$ *with an inverted sign of n-component of one decay product*

@ $t\bar{t}$ CM frame

LEPTON+JETS ANALYSIS (CONT.)

- Study the angular distribution of χ for at **low m($t\bar{t}$) region** for gg fusion events:

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\chi} \propto 1 + \mathbf{D}\,\kappa\bar{\kappa}\cdot\cos\chi \;\Rightarrow\; \Delta E = -3\mathbf{D} > 1 \;\Rightarrow\; \boxed{\mathbf{D} < -\frac{1}{3}}$$

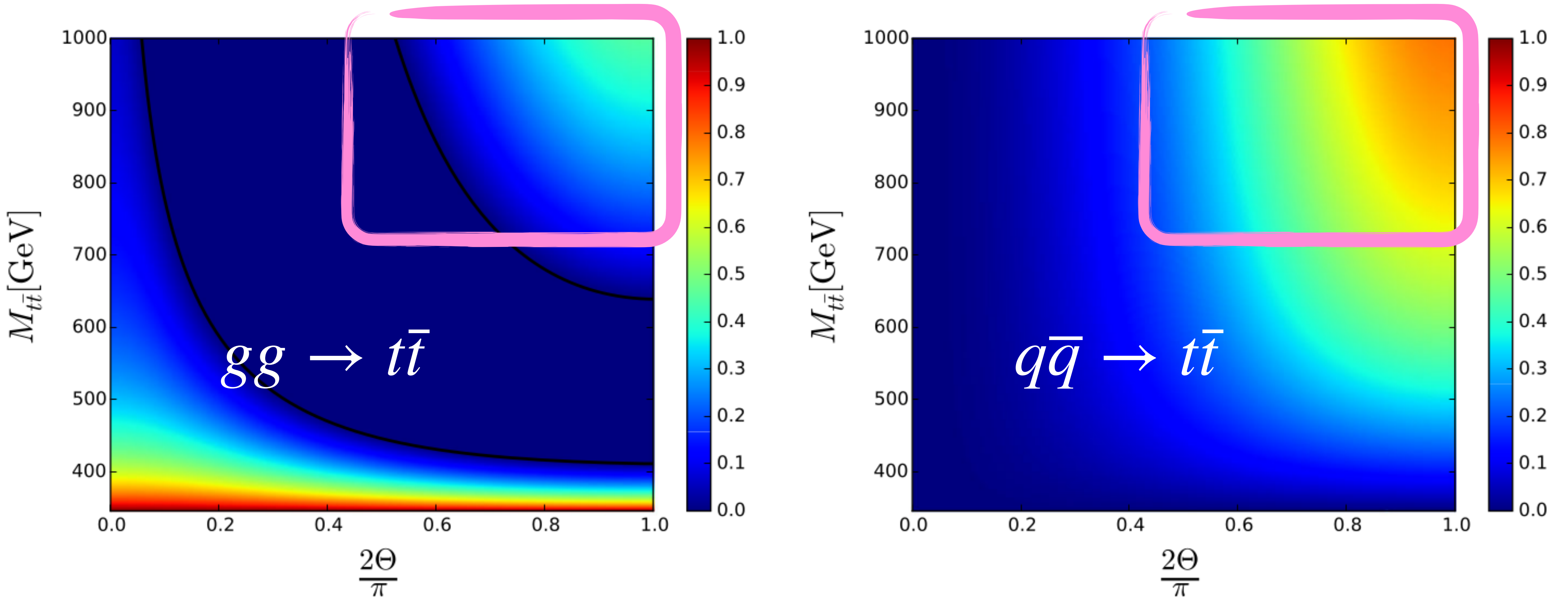
Basically the same as the dilepton analysis

- The entanglement in a spin-triplet state at **high m($t\bar{t}$) and central region**, via both $q\bar{q}$ & gg productions, can be probed with the $\tilde{\chi}$ distribution:

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\tilde{\chi}} \propto 1 + \tilde{\mathbf{D}}\,\kappa\bar{\kappa}\cdot\cos\tilde{\chi}$$

- With negative C_{rr} and C_{kk}, the sufficient condition is then translated as:

$$\Delta E = C_{nn} - C_{rr} - C_{kk} = 3\tilde{\mathbf{D}} > 1 \quad or \quad \boxed{\tilde{\mathbf{D}} > \frac{1}{3}}$$

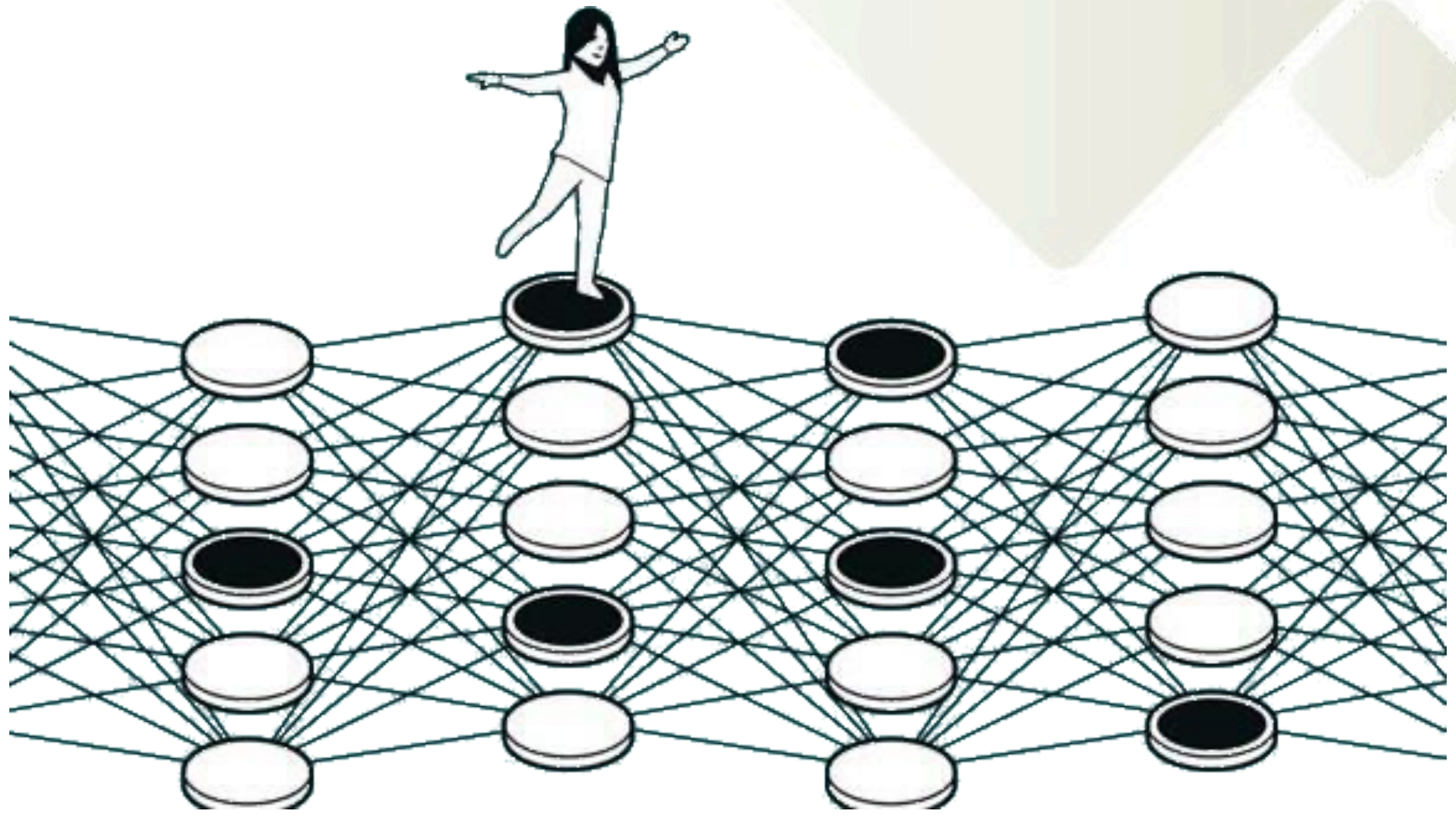


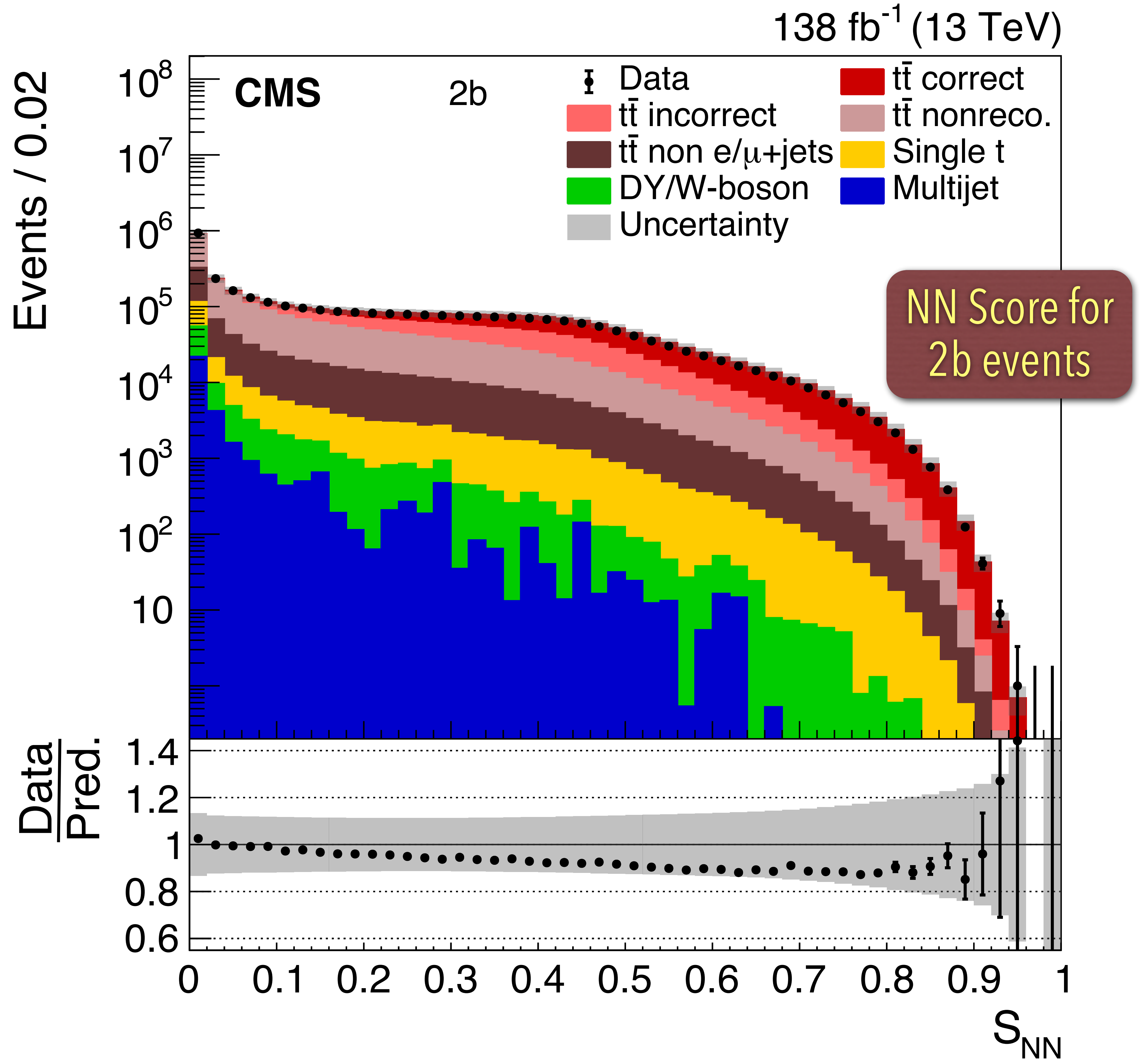
Measure the proxies D & $\tilde{\text{D}}$ to prob entanglement!

Ref. Baumgart, Tweedie, JHEP 03 (2013) 117
Aguilar-Saavedra, Casas, EPJC 82 (2022) 666

CMS ANALYSIS STRATEGY & EVENT RECONSTRUCTION

- Based on full Run-2 138 fb^{-1} data at 13 TeV.
- Target: **determine the full correlation matrix & polarization vectors + measure D & $\tilde{D}$**.
  - Differential measurements in bins of $\mathbf{m(t\bar{t})}$, $\mathbf{p_T(t)}$, and/or the top scattering angle $|\mathbf{cos\Theta}|$
  - Inclusive measurement by combining differential bins.
- Nominal $t\bar{t}$ sample is prepared with the same prescription: PowhegBox+Pythia8 + EWK corrections/NNLO weights.
- Reconstruction of the $t\bar{t}$ system is performed using an **artificial NN**:
  - For **identifying detector-level physics objects and in particular up/down jet assignments**.
  - Inputs include lepton kinematics, missing energy, jet kinematics, b-tagging scores (up to 8 jets).





EVENT CATEGORIZATION

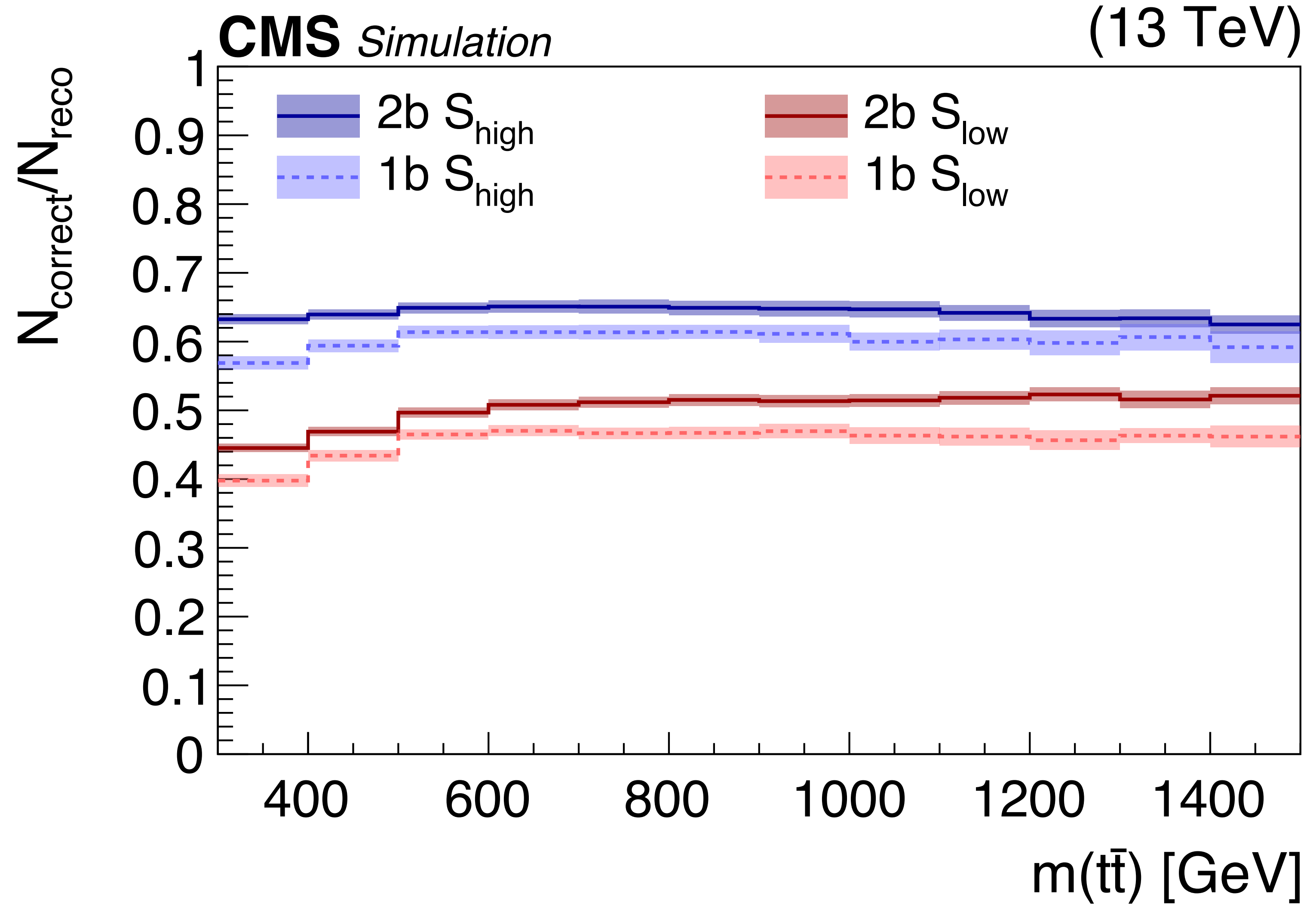
- ANN is trained with *"correct permutation"* vs. *"wrong permutations"* ⇒ the permutation with the highest score is used in the final analysis.
- Events divided into categories based on **lepton flavor (e/μ)**, **number of b-tagged jets**, and the **S_{NN} score**.
  - Drop the events with low S_{NN} score (<0.1) to avoid the events with low fraction of correctly assigned permutation & larger background contribution.

4 event classes

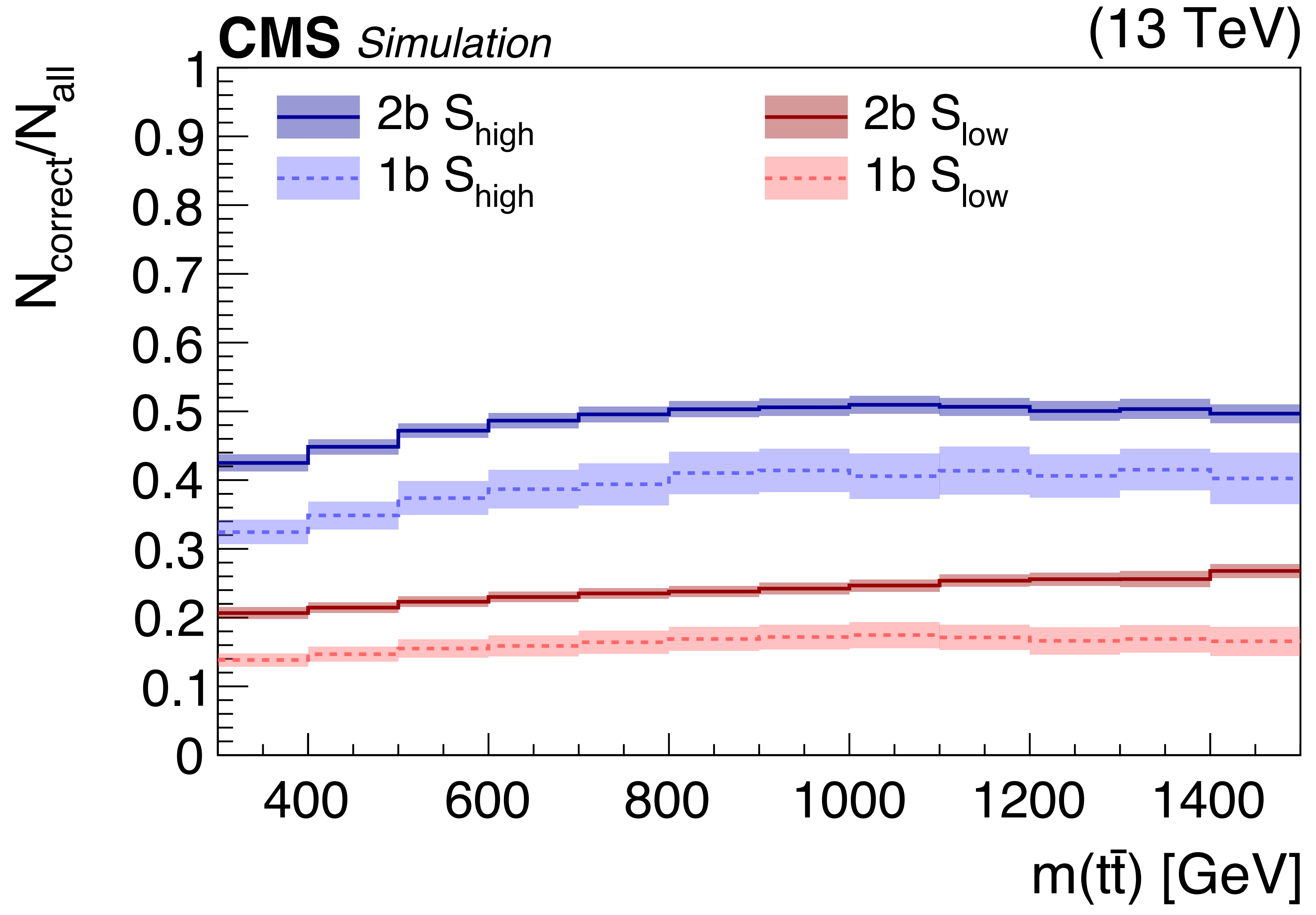
# of b-tags	S_{low}	S_{high}
2b	$0.1 < S_{NN} < 0.36$	$0.36 < S_{NN}$
1b	$0.1 < S_{NN} < 0.30$	$0.30 < S_{NN}$

- **Reaches ~50% correct jet assignment rate (*including the down-type quark assignment too!*)**.

Reconstruction efficiency vs. m(tt̄)



Fraction of correct recon. vs. m(tt̄)



EXTRACTION OF POLARIZATION & SPIN CORRELATION

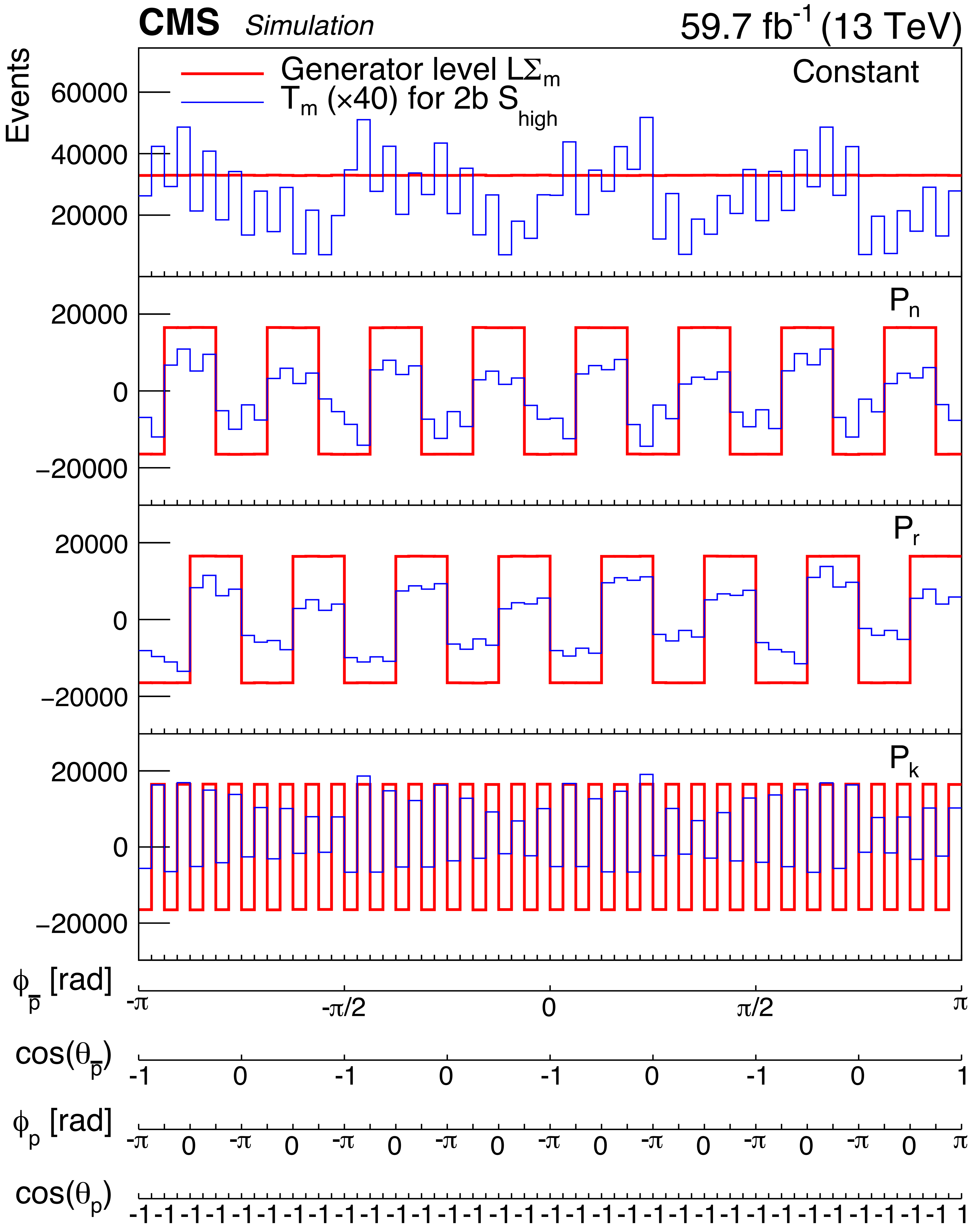
$L\Sigma_m$: gen-level template
T_m: detector-level template
vs (θ_p, ϕ_p)

- Differential cross section Σ_{tot} as a **linear combination of template functions Σ_m** × **P & C coefficients Q_m**:

$$\Sigma_{tot} = \Sigma_0 + \sum_{m=1}^{15} Q_m \Sigma_m$$

$$\Sigma_m \propto \sigma_{norm} \{ \sin\theta_p \cos\phi_p, \sin\theta_p \sin\phi_p, \ldots, \cos\theta_p \cos\theta_{\bar{p}} \}$$

- Coefficients Q_m are extracted by fitting Σ_{tot} in bins of $m(t\bar{t})$ vs $|\cos\Theta|$ or $p_T(t)$ vs $|\cos\Theta|$.
- Reweighting technique (Σ_m/Σ_{tot}) to evaluate the **detector-level template T_m** for each Q_m.
- The **gen-level template Σ_m** should be independent from top kinematics, but not the case of T_m.
- The binning should be narrow enough to **minimize the bias due to top kinematics dependence** of T_m within a bin.

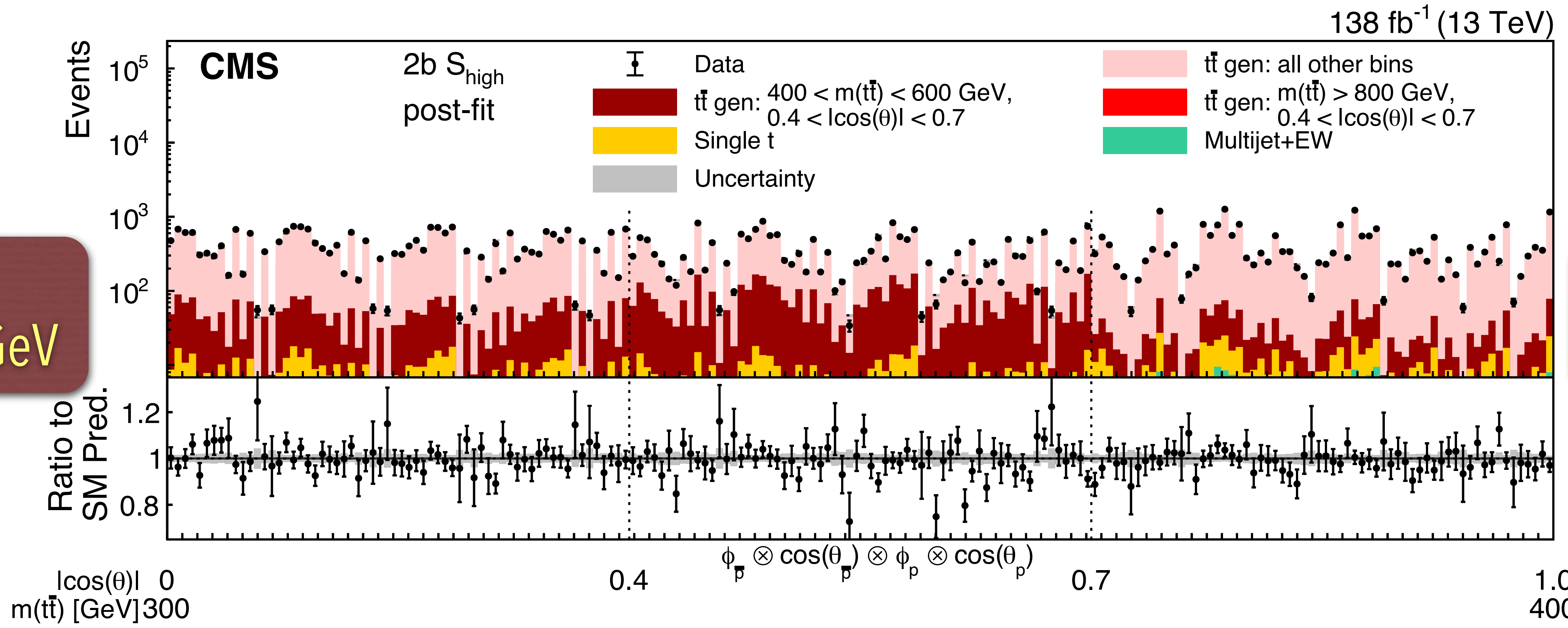


400 < m(tt̄) < 600 GeV, |cosΘ|<0.4

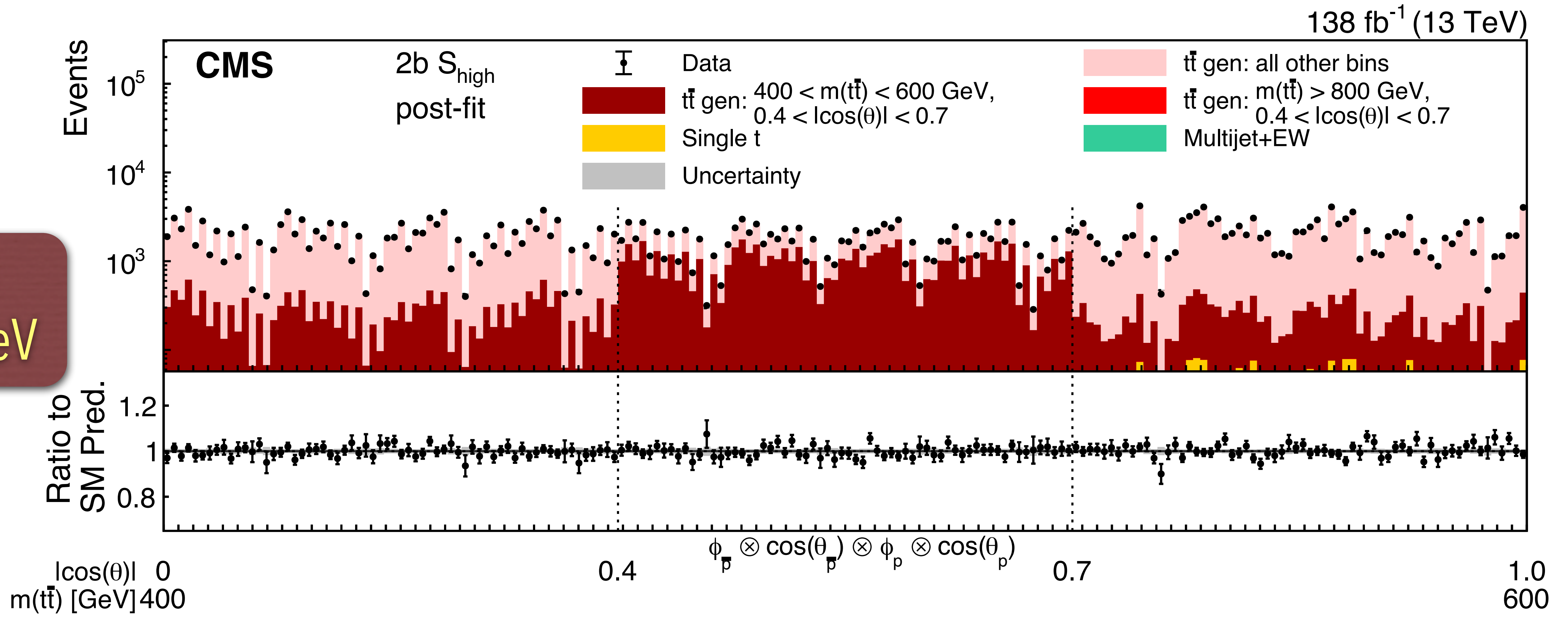
FULL POLARIZATION & SPIN MATRIX FITS

- **A maximum likelihood fit** combining the information of the 4 event classes × 4 data-taking periods = 16 categories in total.
- Post-fit distributions for full matrix analysis in $\mathbf{m(t\bar{t})}$ **vs** $\mathbf{|cos\Theta|}$ bins:

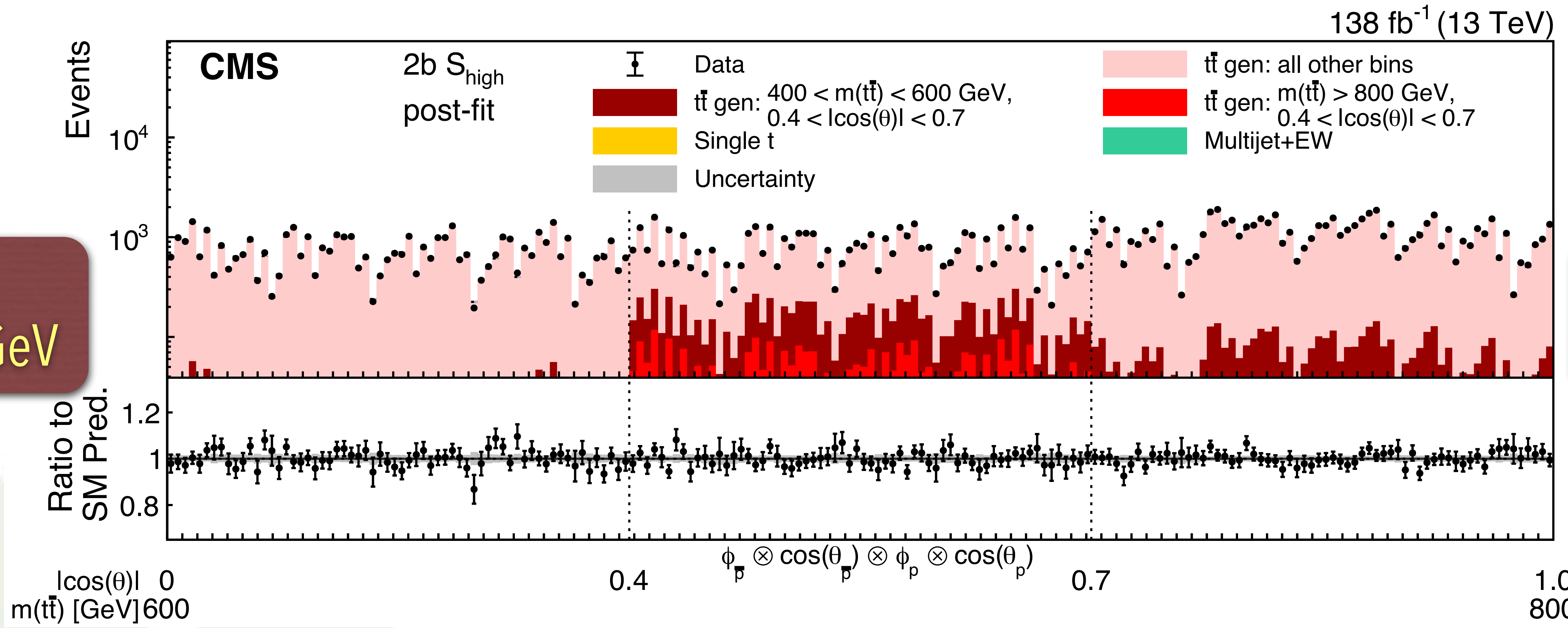
$m(t\bar{t})$: 300-400 GeV



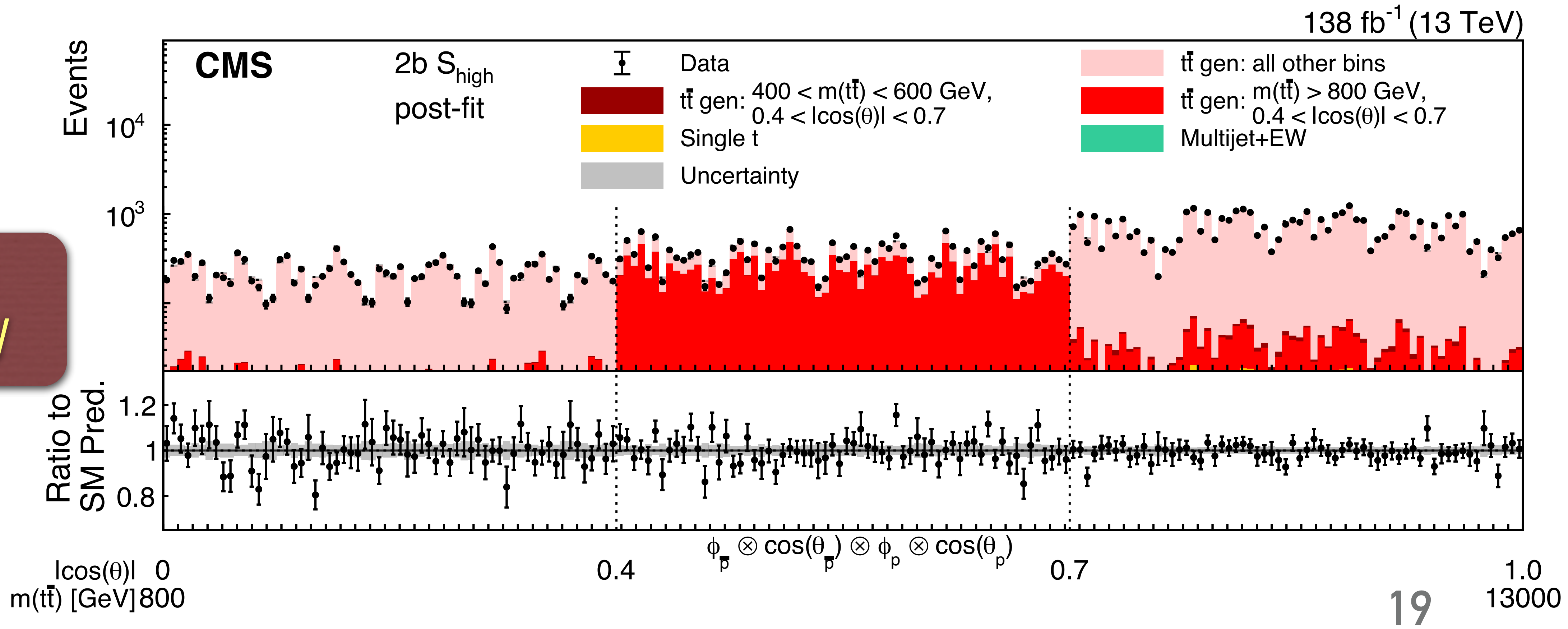
$m(t\bar{t})$: 400-600 GeV



$m(t\bar{t})$: 600-800 GeV



$m(t\bar{t})$: >800 GeV



RESULTS: FULL POLARIZATION & SPIN MATRIX

- **Inclusive** & Differential measurements in bins of $m(t\bar{t})$ vs $|\cos\Theta|$ and $p_T(t)$ vs $|\cos\Theta|$.
  - Evaluation of full correlation matrix and polarization vectors.
  - Good agreement with SM prediction!

Ref. CMS arXiv:2409.11067
Accepted by PRD

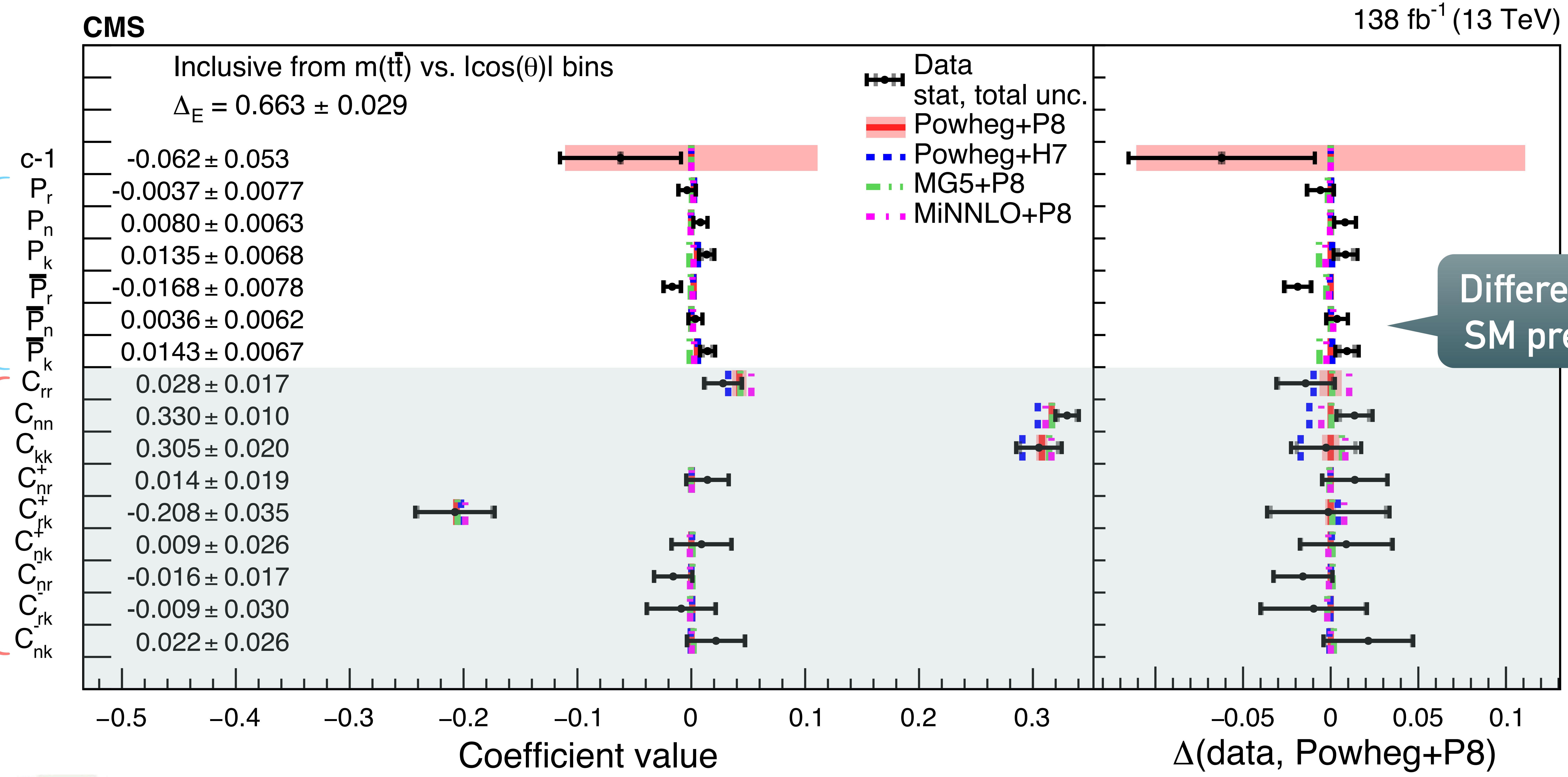
Inclusive measurement by combining $m(t\bar{t})$ vs $|\cos\Theta|$ bins

$$\mathbf{P}(\overline{\mathbf{P}}) = \begin{pmatrix} x \\ x \\ x \end{pmatrix}$$

Polarization

$$\mathbf{C} = \begin{pmatrix} x & x & x \\ x & x & x \\ x & x & x \end{pmatrix}$$

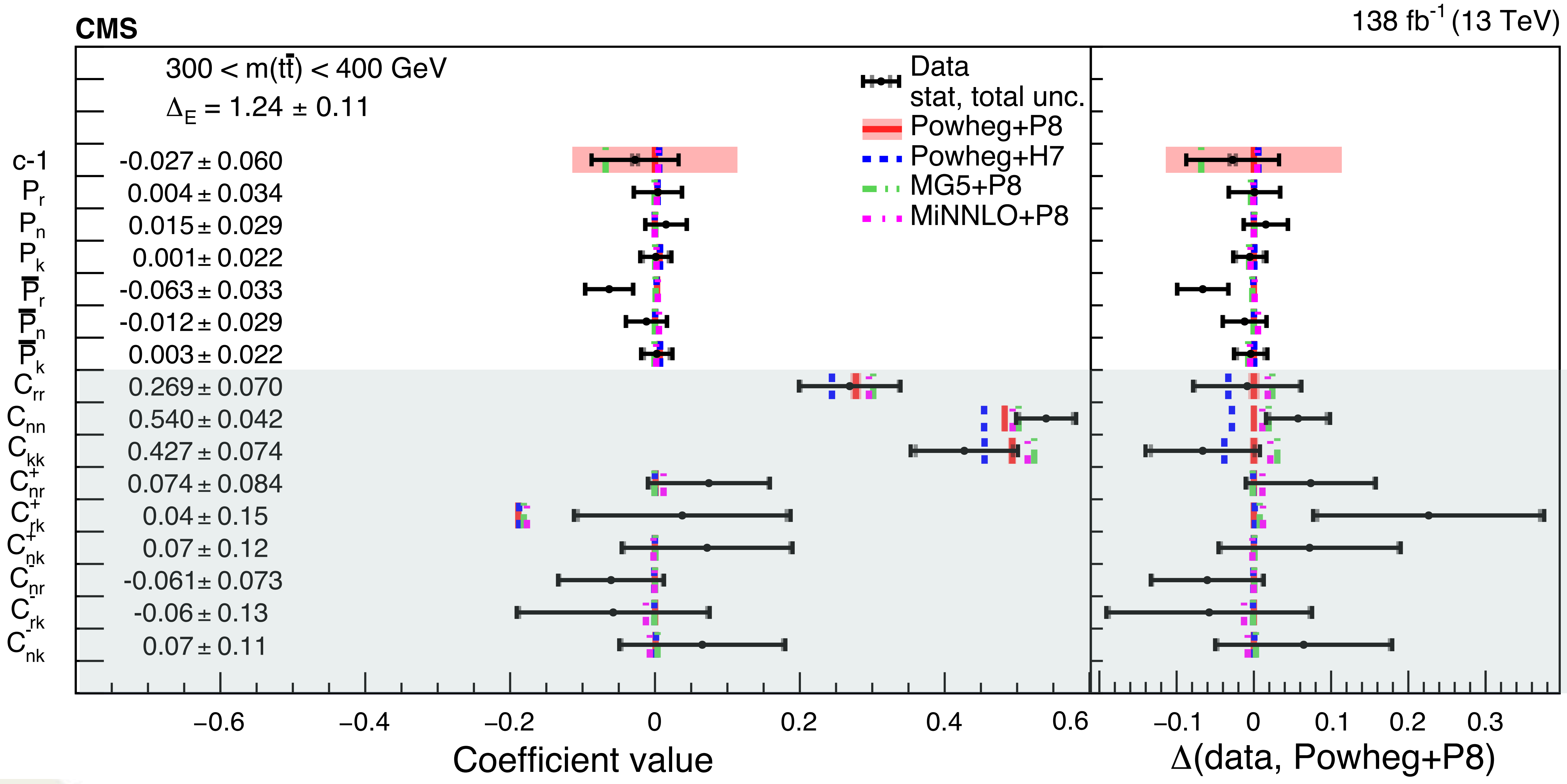
Spin correlation



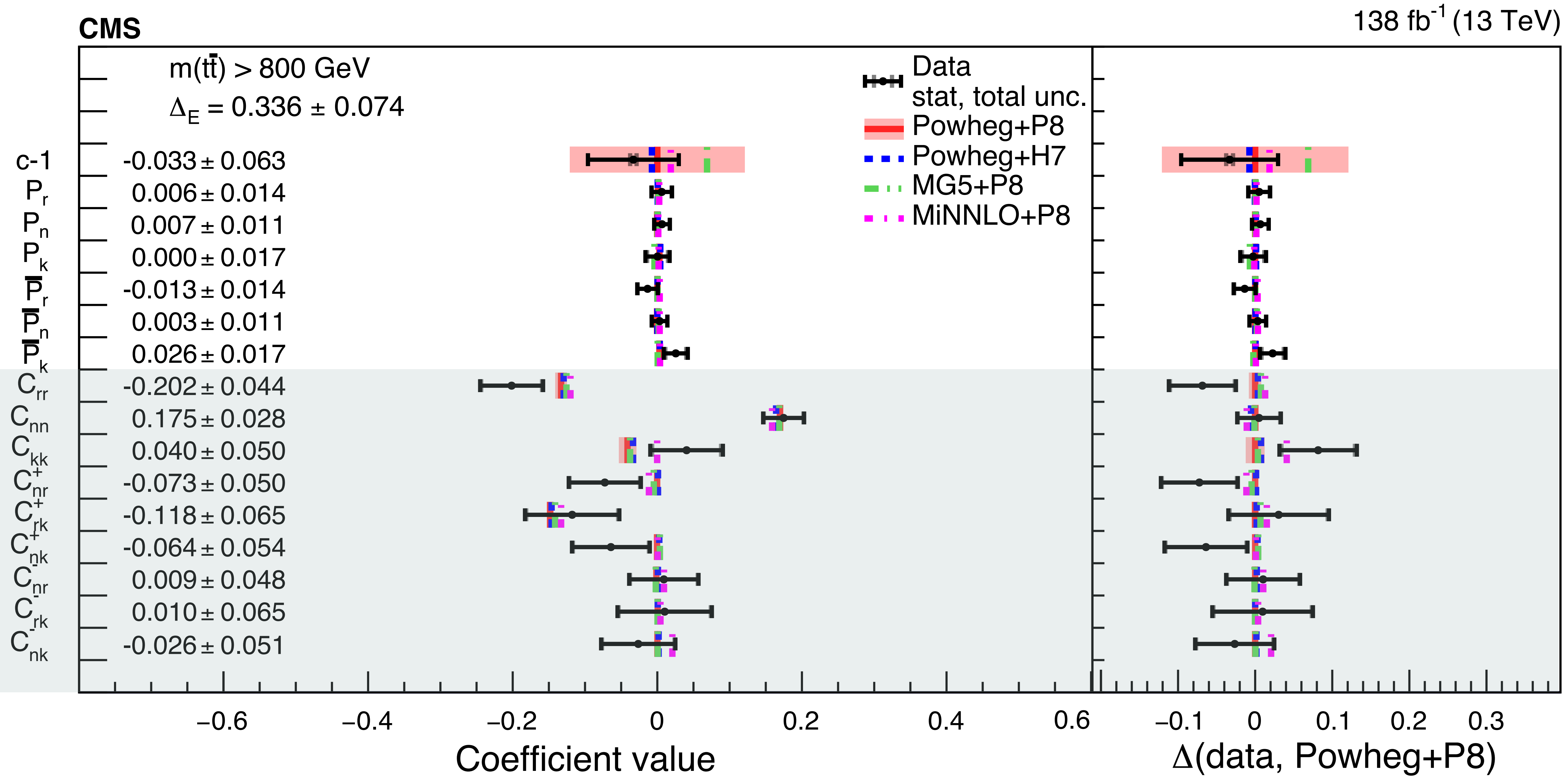
Difference w.r.t. SM predictions

RESULTS: FULL POLARIZATION & SPIN MATRIX (CONT.)

- Inclusive & **Differential measurements** in bins of $\mathbf{m(t\bar{t})}$ **vs** $\mathbf{|cos\Theta|}$ and $\mathbf{p_T(t)}$ **vs** $\mathbf{|cos\Theta|}$.
  - Good agreement with SM prediction still!
  - **First time in high $\mathbf{m(t\bar{t})}$ region!**



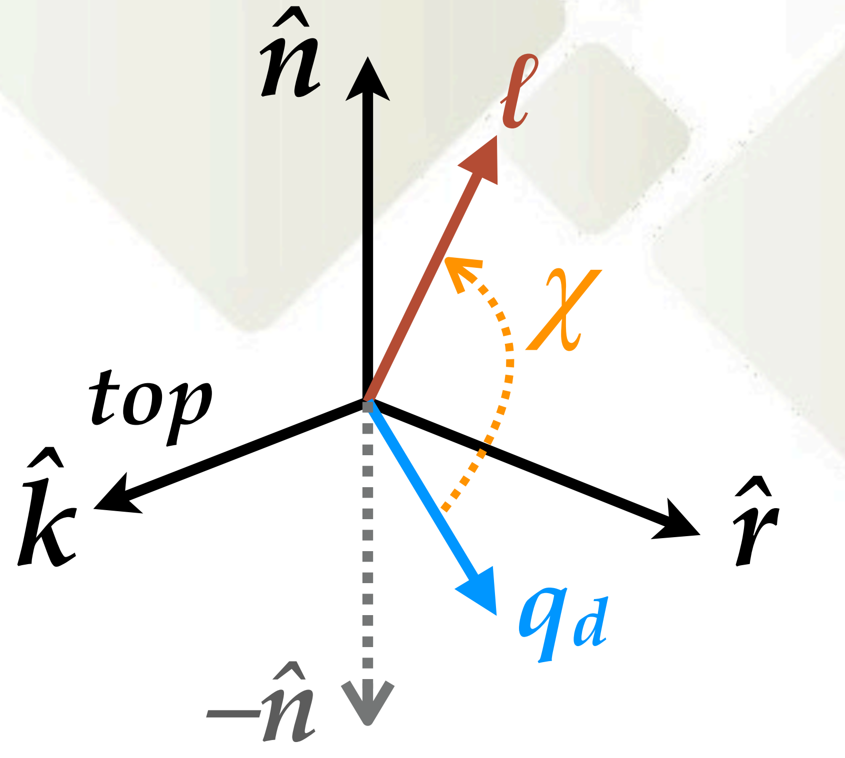
Differential measurement in m(tt̄)<400 GeV



Differential measurement in m(tt̄)>800 GeV

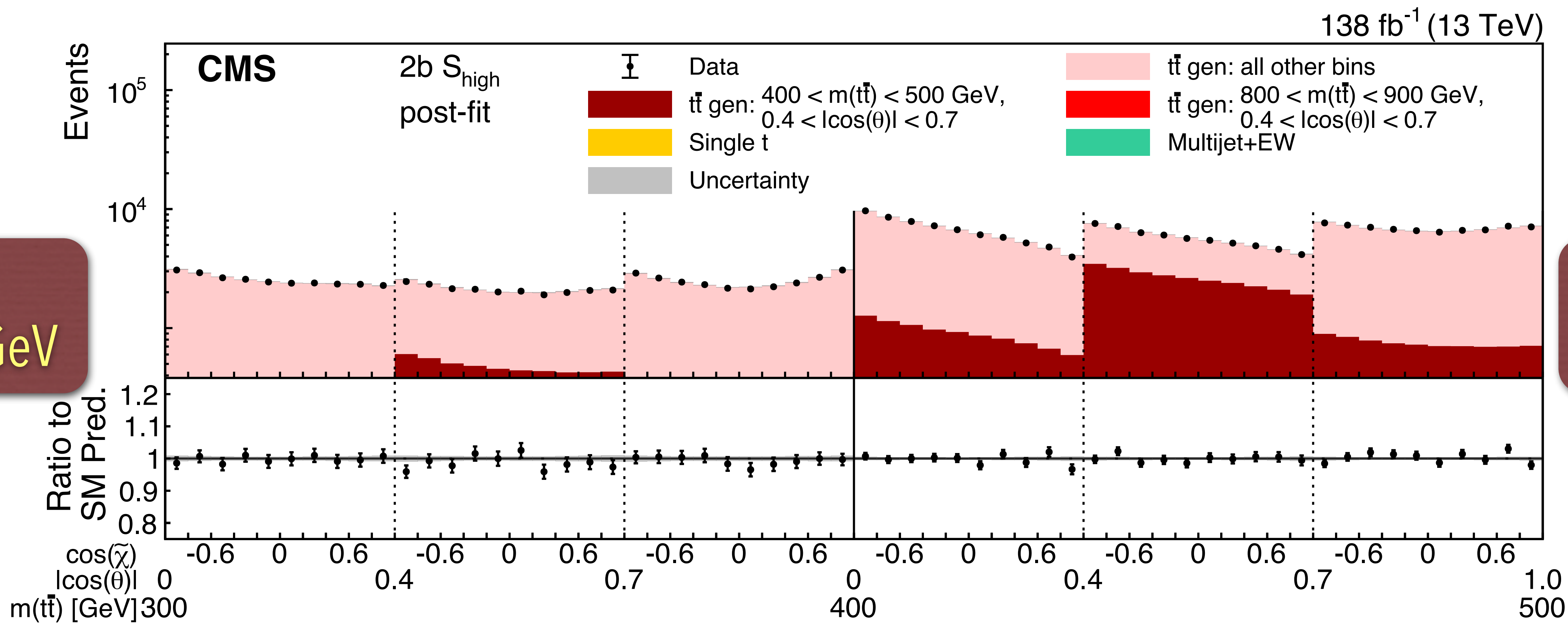
Spin correlations do depend on the kinematics, ie. m(tt̄) / pT(t) / |cosΘ|

ENTANGLEMENT FITS

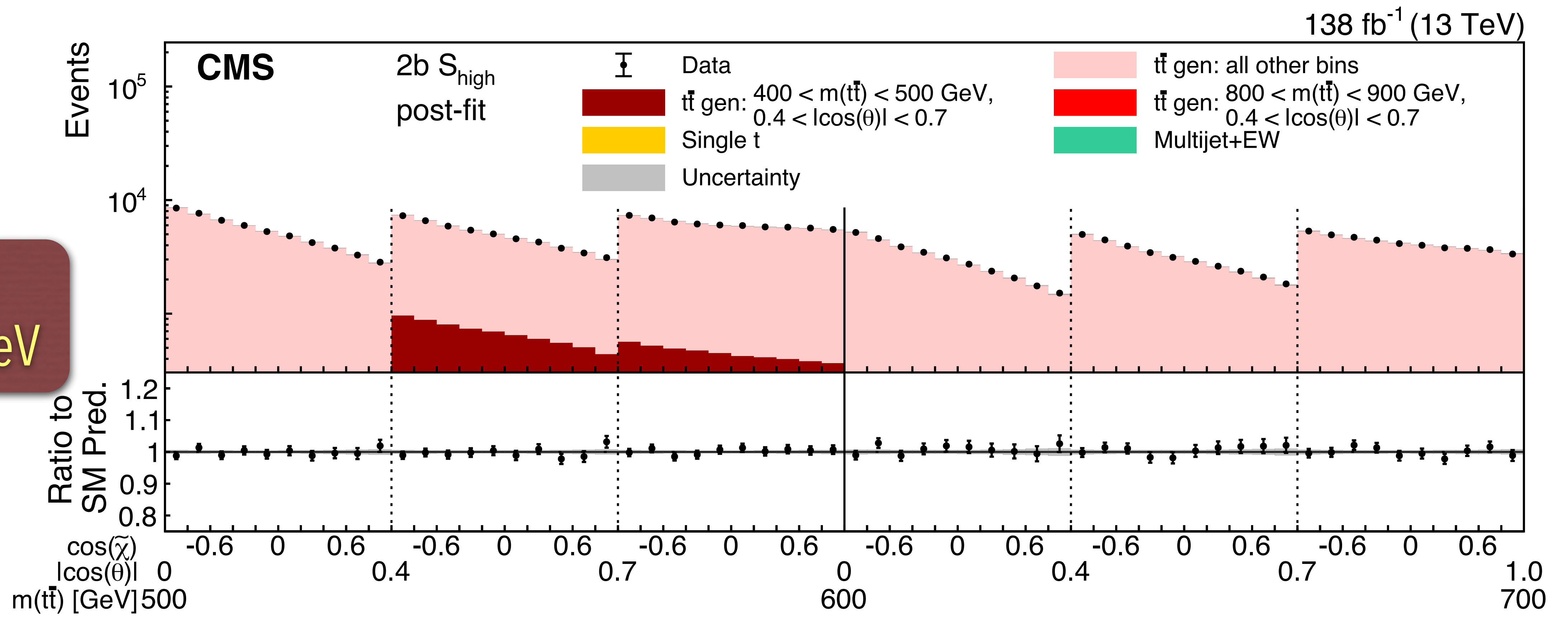


- Likelihood fits to **$\cos\chi$ & $\cos\tilde{\chi}$ distributions** with the detector-level templates to determine the value of D & $\tilde{\mathrm{D}}$ proxies for the entanglement analysis.
- Post-fit distributions in $\mathbf{m(t\bar{t})}$ **vs** $|\mathbf{cos\Theta}|$ bins:

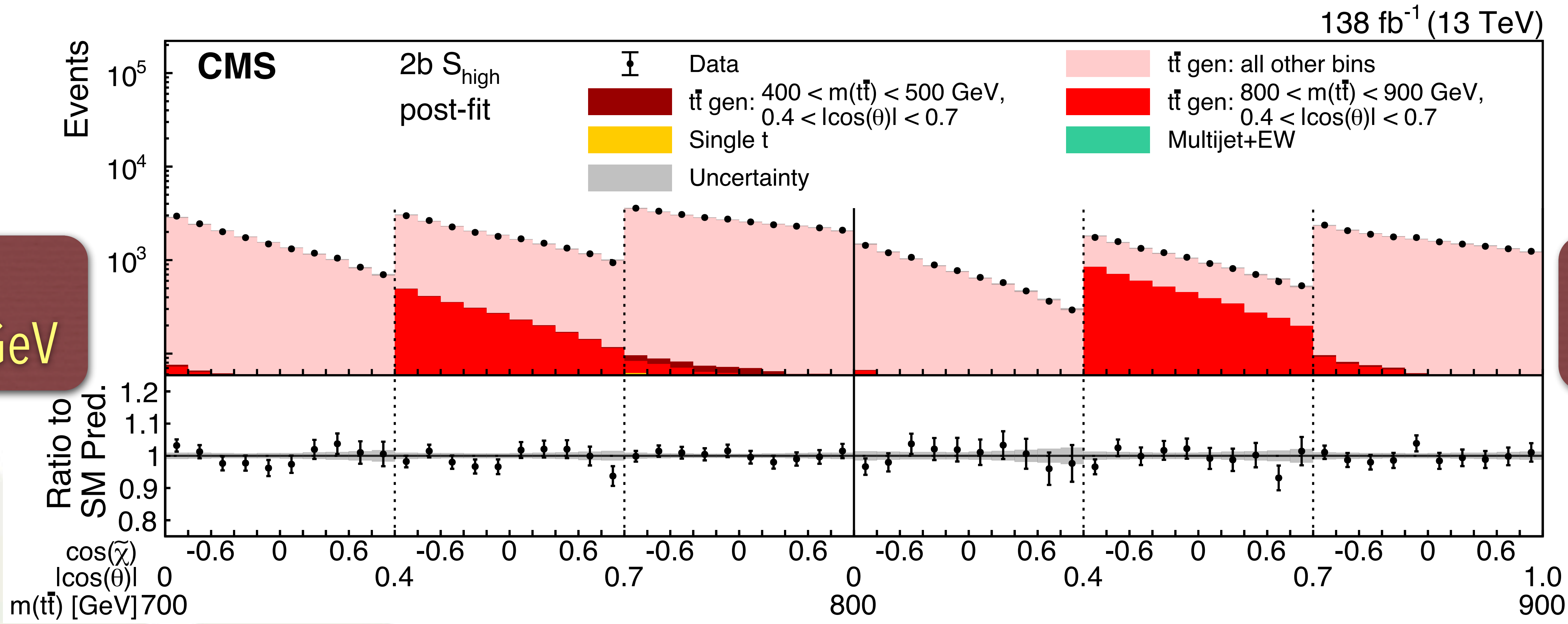
$\cos(\tilde{\chi})$: 300-400 GeV



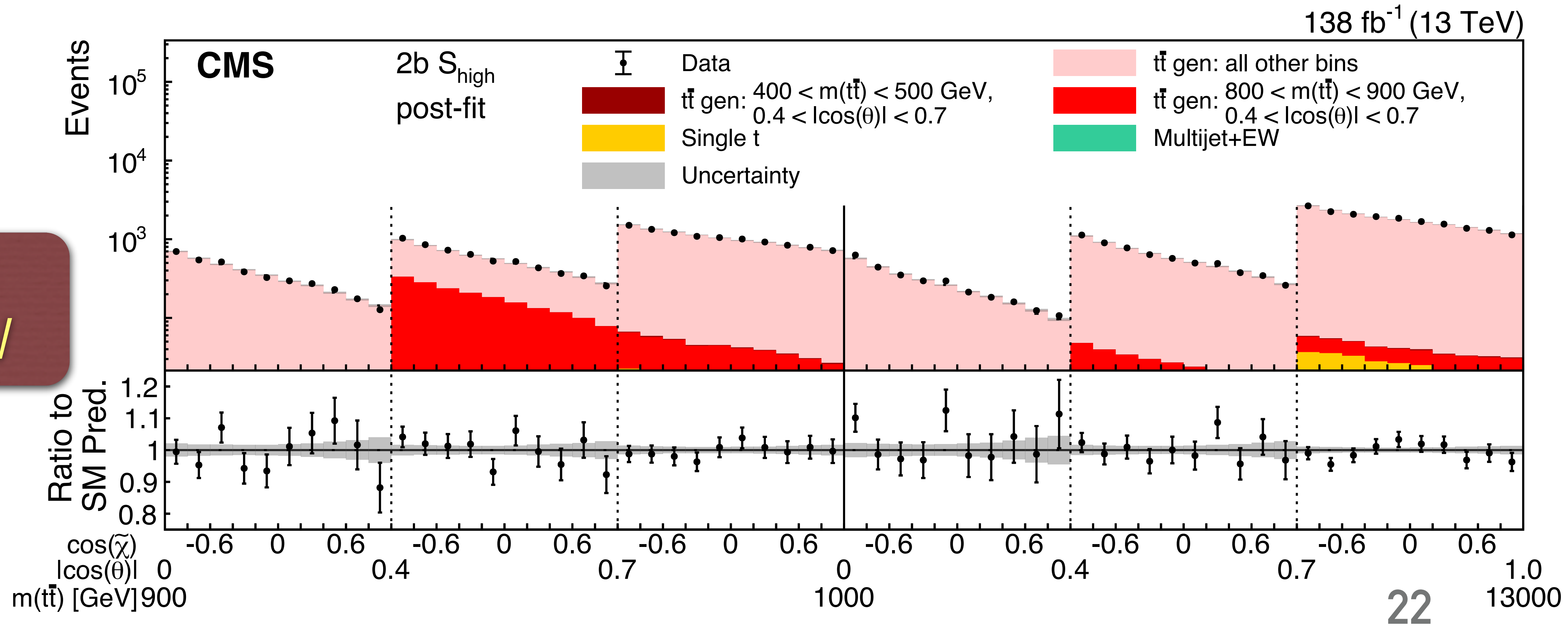
$\cos(\tilde{\chi})$: 400-600 GeV



$\cos(\tilde{\chi})$: 600-800 GeV

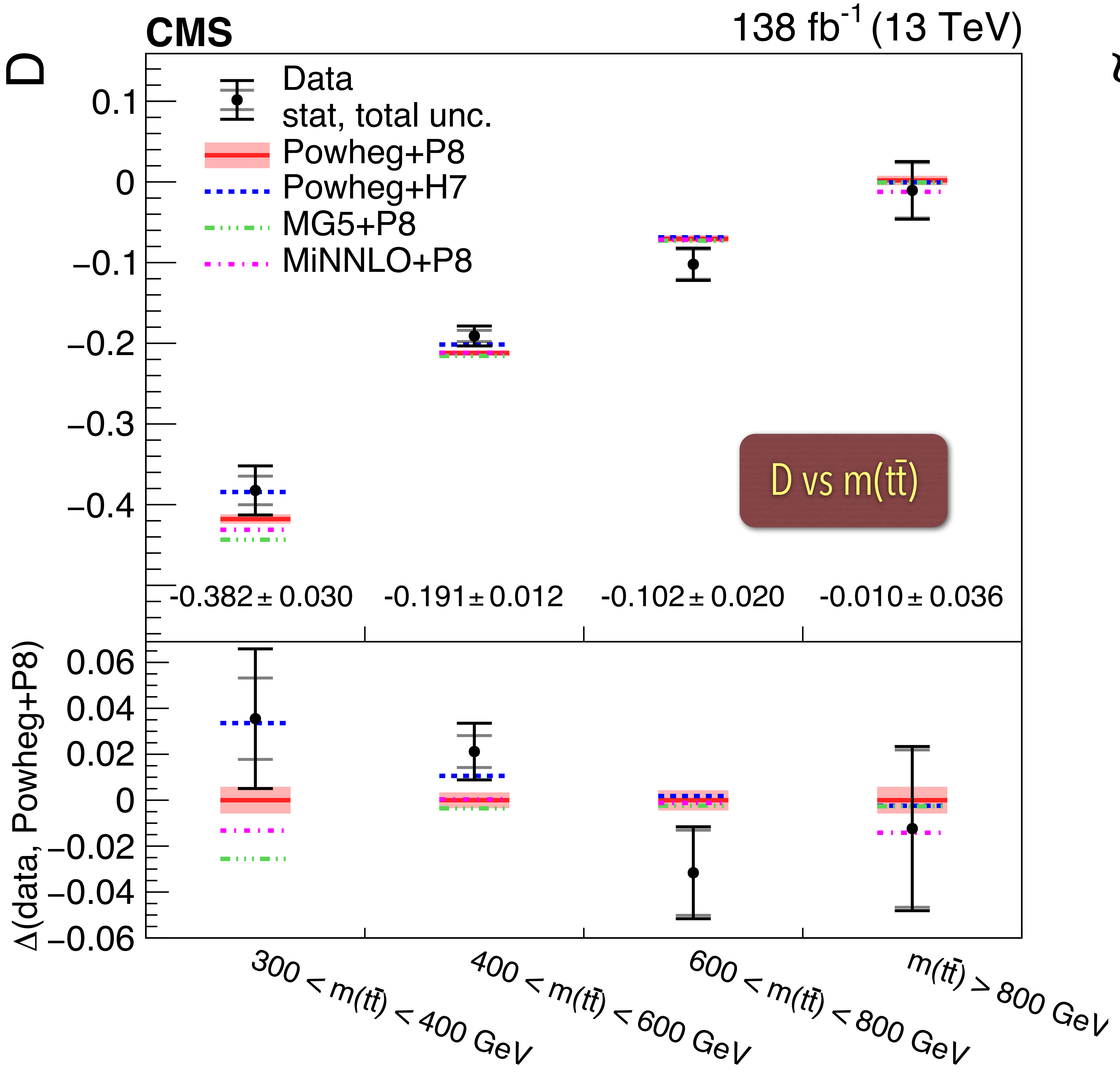


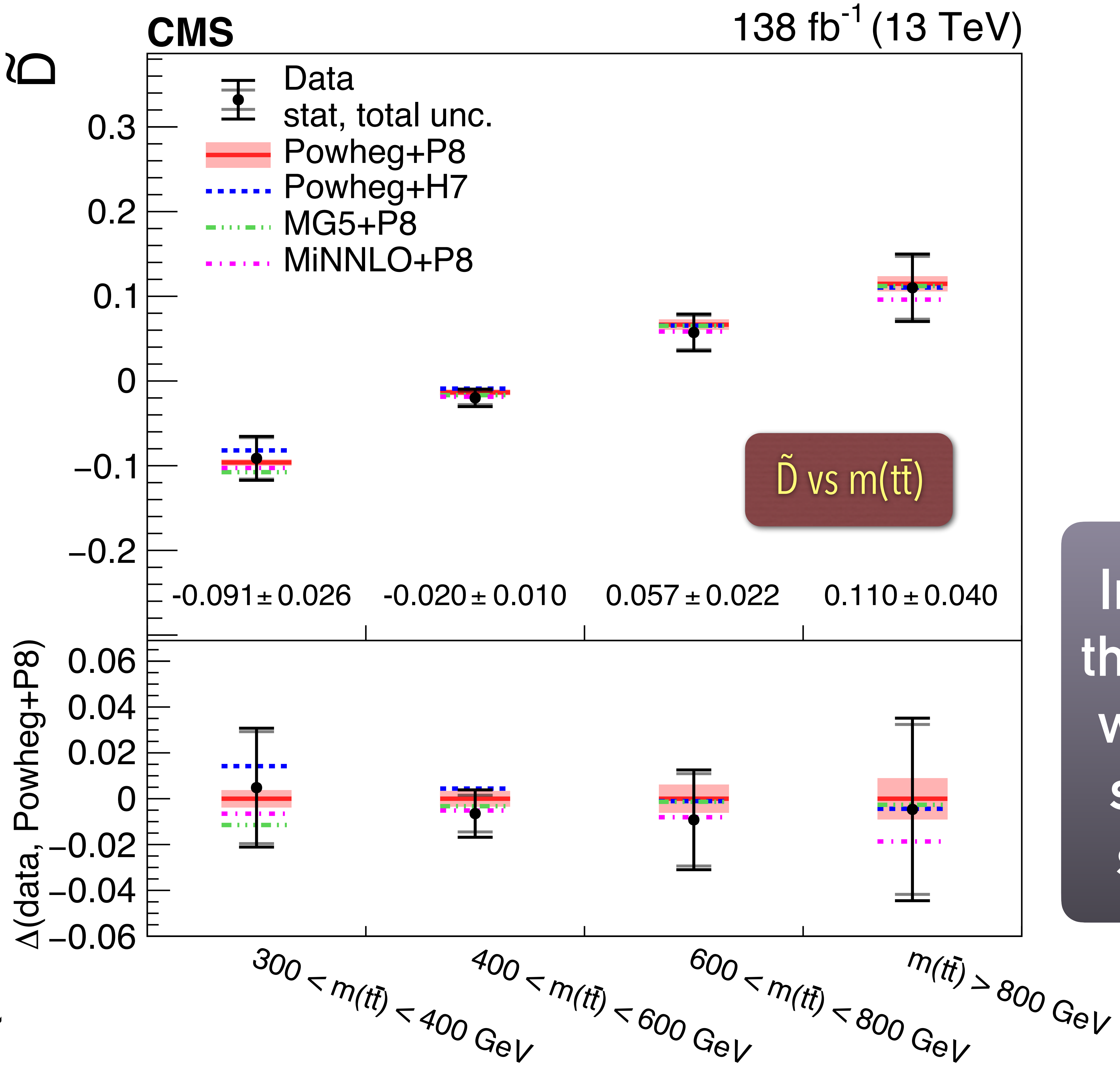
$\cos(\tilde{\chi})$: >800 GeV



RESULTS: ENTANGLEMENT FITS

- Inclusive & **differential measurements** of D & $\tilde{D}$ proxies in bins of $\mathbf{m(t\bar{t})}$ **vs** $\mathbf{|\cos\Theta|}$ and $\mathbf{p_T(t)}$ **vs** $\mathbf{|\cos\Theta|}$.
  - Good agreement with SM prediction as well!

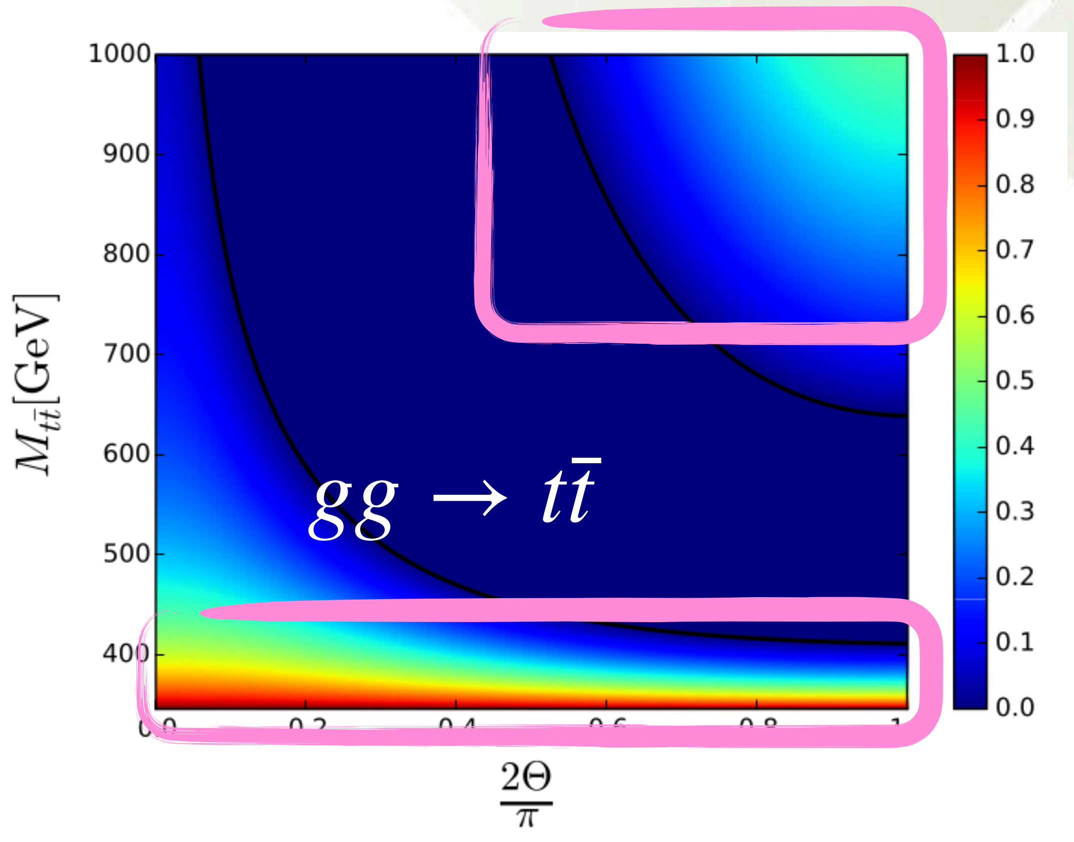




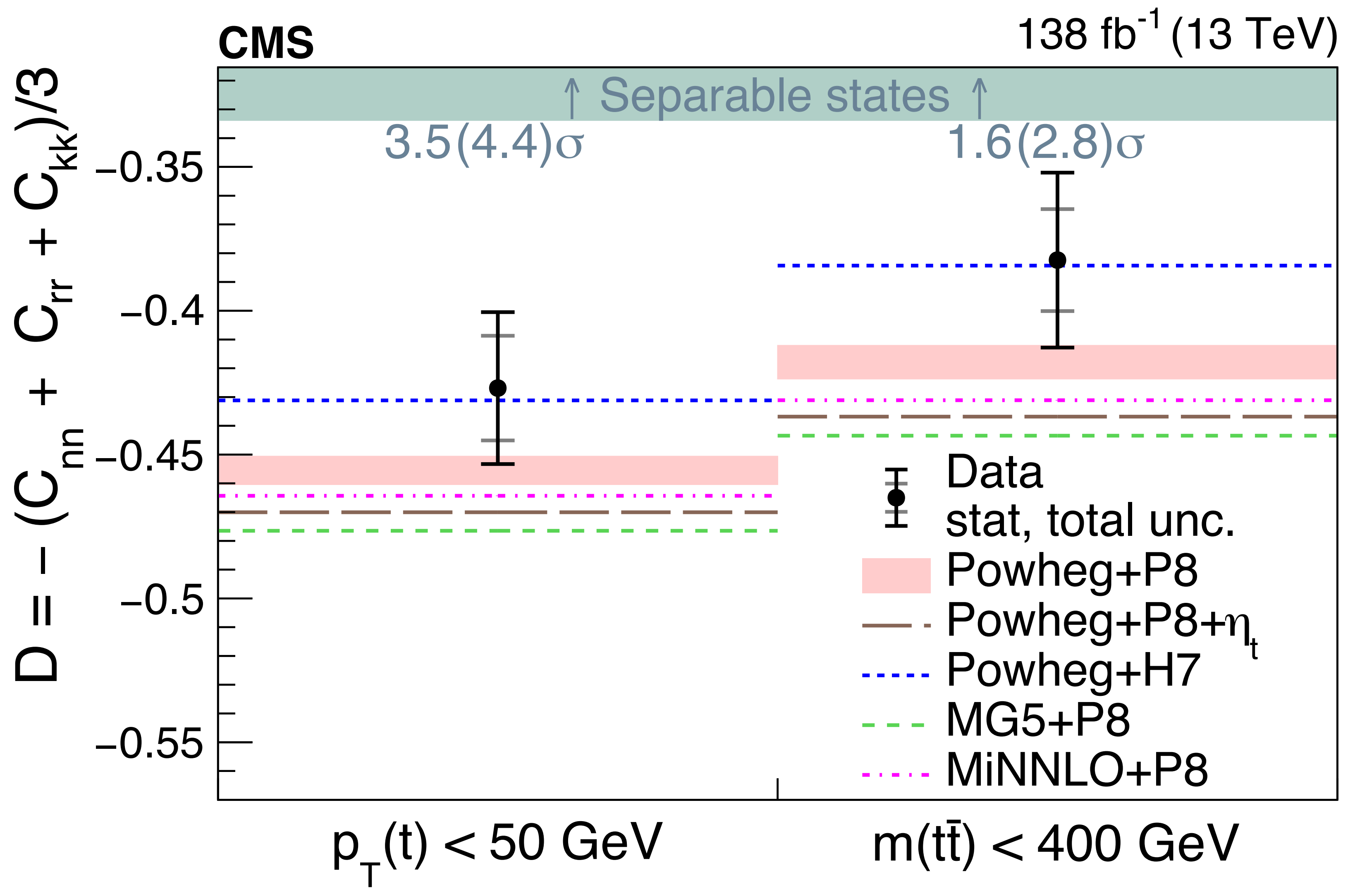
In order to “see” the entanglement we have to go to specific phase-space regions!

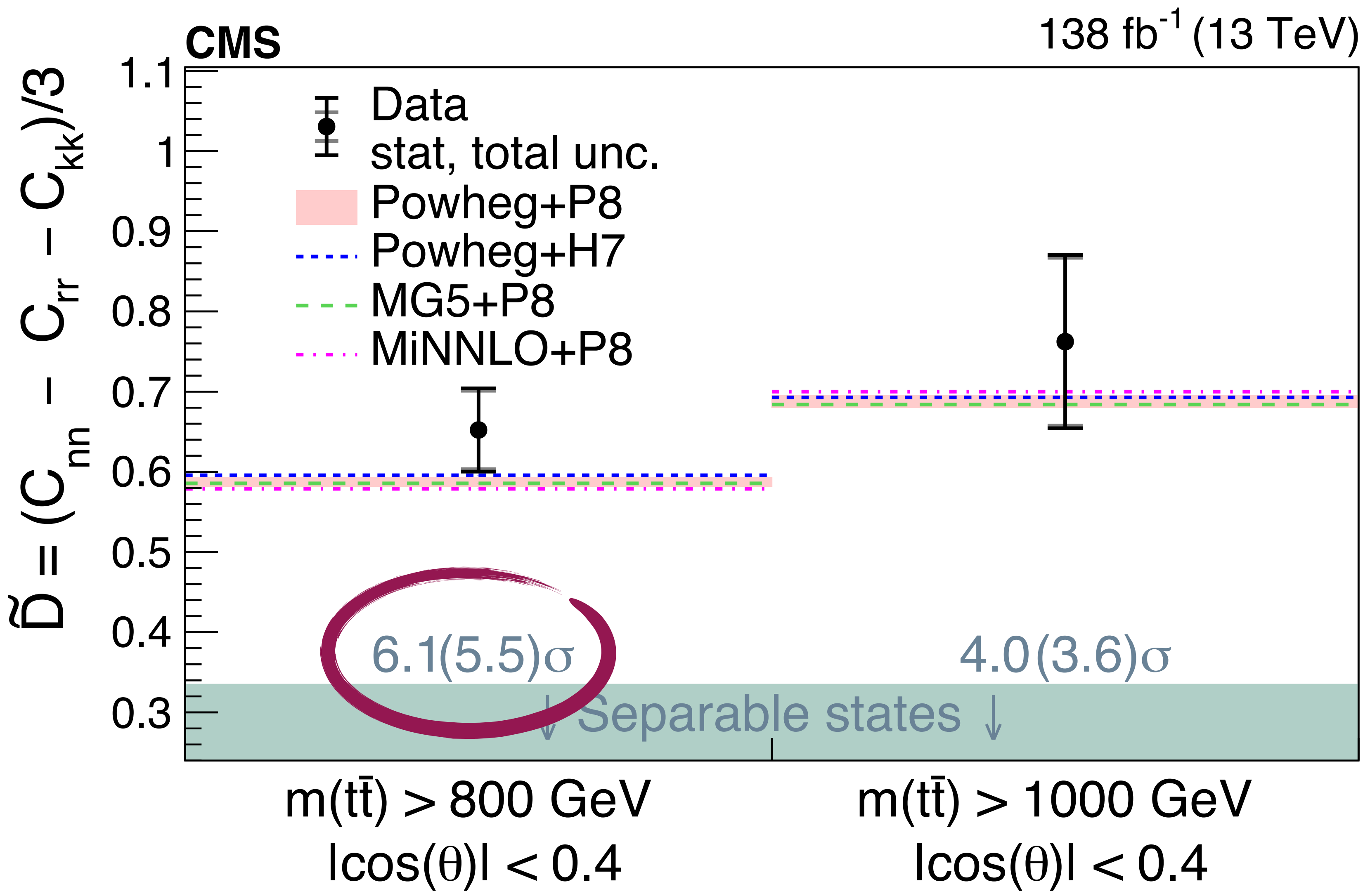
ENTANGLED OR NOT?

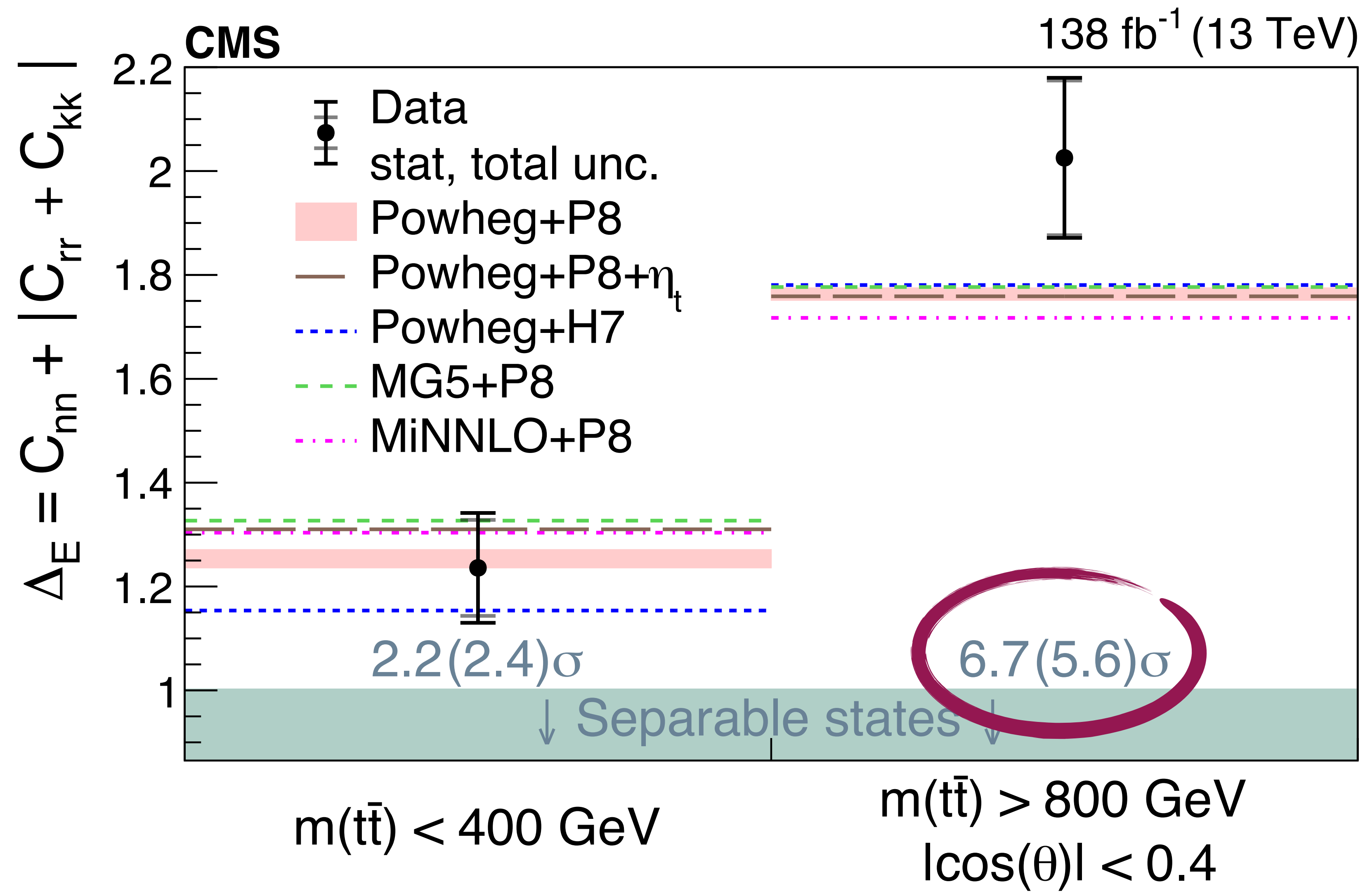
- Test of entanglement with **D & $\tilde{D}$ proxies** or with **full correlation matrix** in specific phase-space regions.
  - Observation of **$t\bar{t}$ entanglement at high $m(t\bar{t})$** for the first time!
  - Modest sensitivity for low $m(t\bar{t})$.



Best sensitivity here!







Low $p_T(t)$ or low $m(t\bar{t})$ test with $\mathbf{D} < -\frac{1}{3}$

High $m(t\bar{t})$ & central test with $\tilde{\mathbf{D}} > \frac{1}{3}$

Test of entanglement $\Delta E > 1$ with full matrix result

SPACE-LIKE EVENTS: ENTANGLED OR NOT?

- Fraction of **space-like** separation events increases with $m(t\bar{t})$:
  - **For $m(t\bar{t})$ > 800GeV the fraction of space-like separated decays is ~90%.**
- A more stringent criterion for entanglement that cannot be explained exchange of information at $v \leq c$ alone (“*critical entanglement*”) is defined:

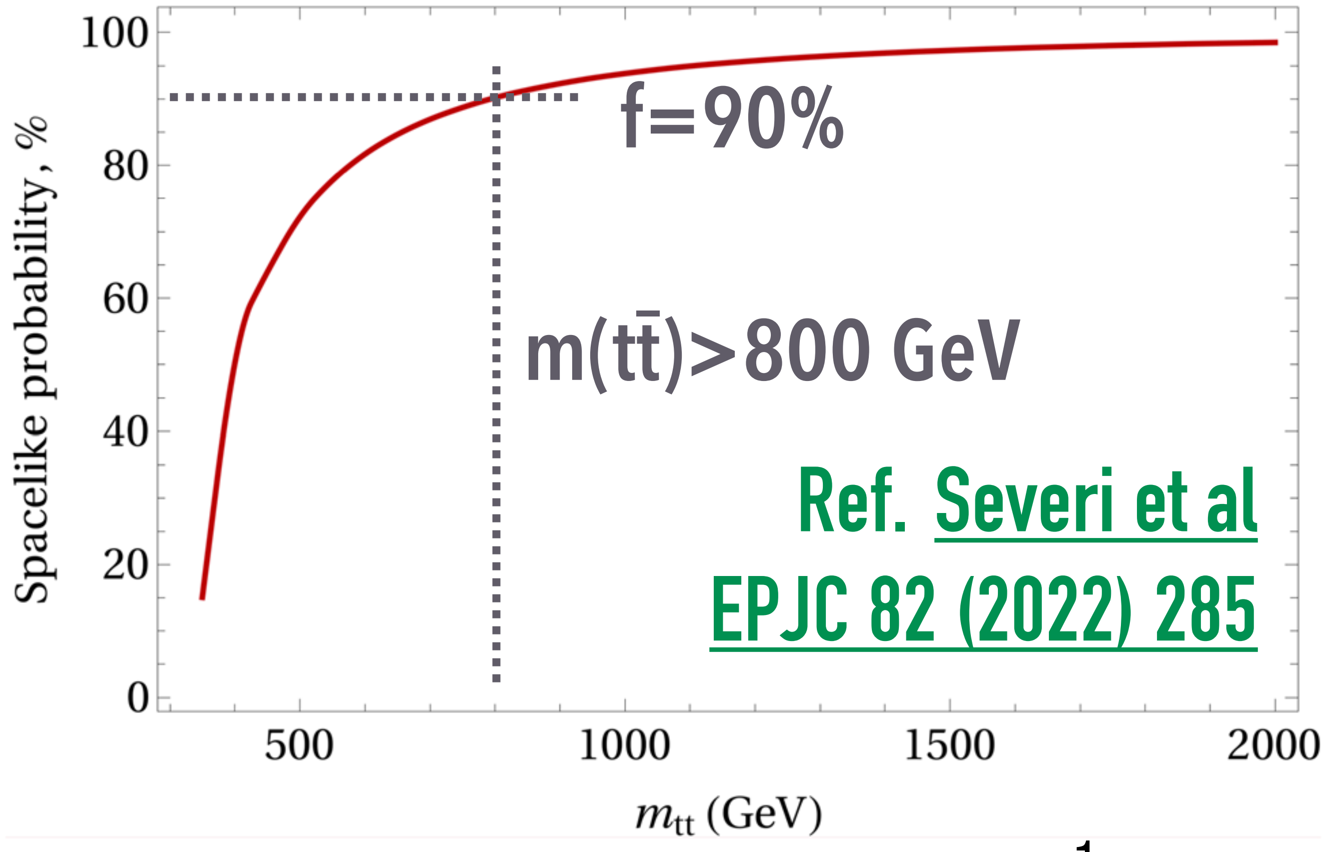
$$\Delta E_{\mathrm{crit}} = f \Delta E_{\mathrm{sep}} + (1-f)\Delta E_{\max} \approx 1.2$$

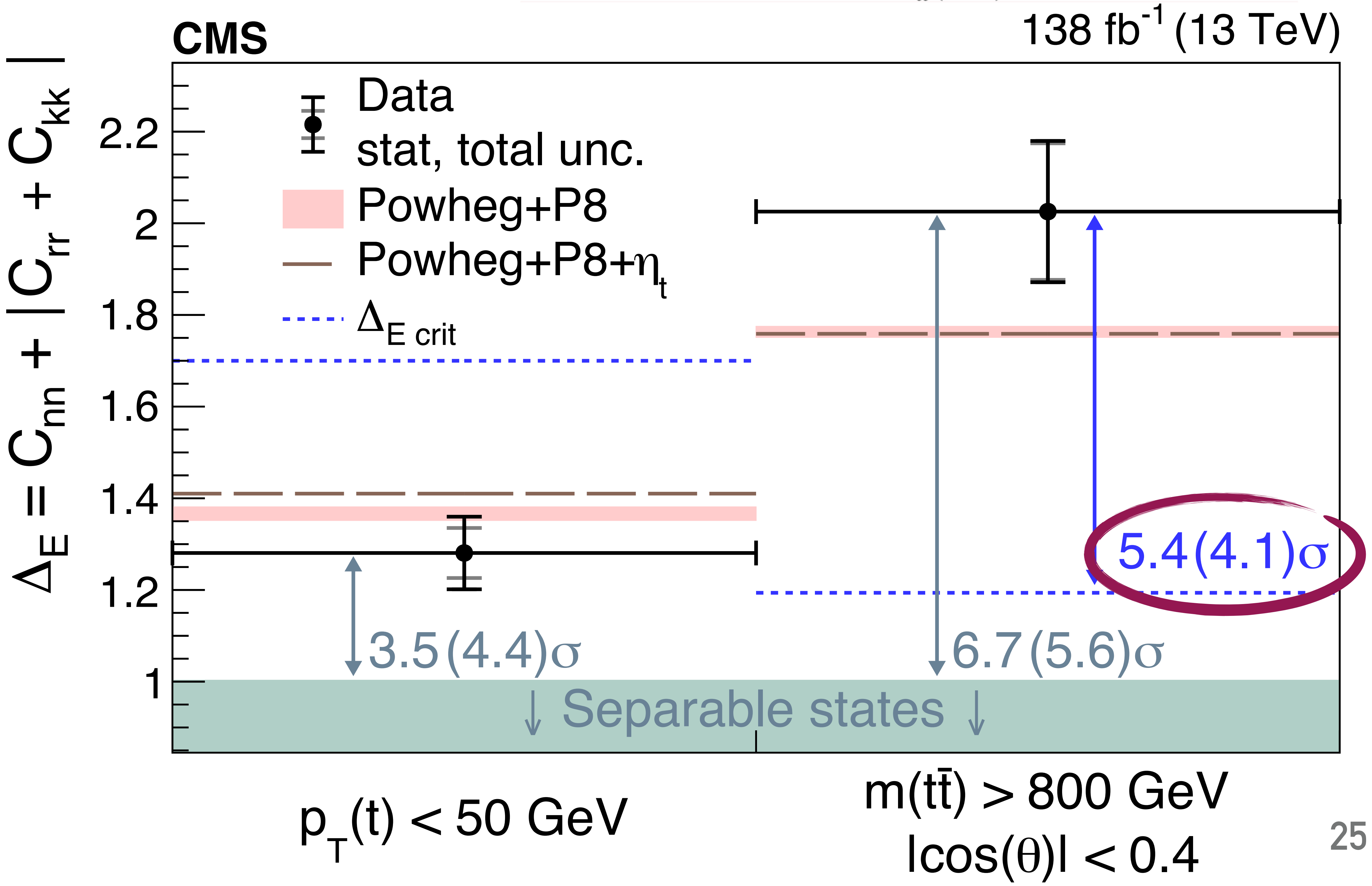
Regular criterion for from space-like events: $\Delta E_{sep} = 1$

Maximized contribution from time-like events: $\Delta E_{max} = 3$

Criterion for entangled space-like events $\Delta E > \Delta E_{\mathrm{crit}}$

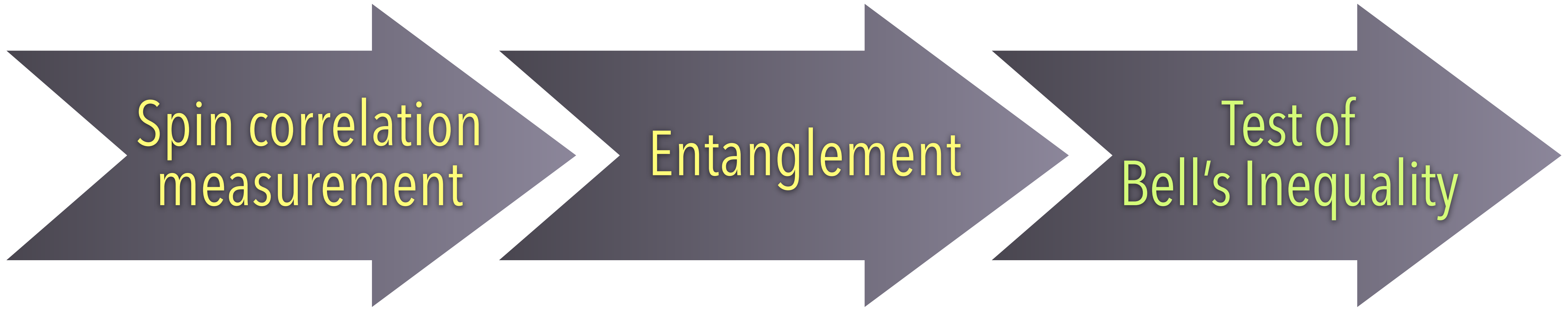
space-like fraction vs $m(t\bar{t})$



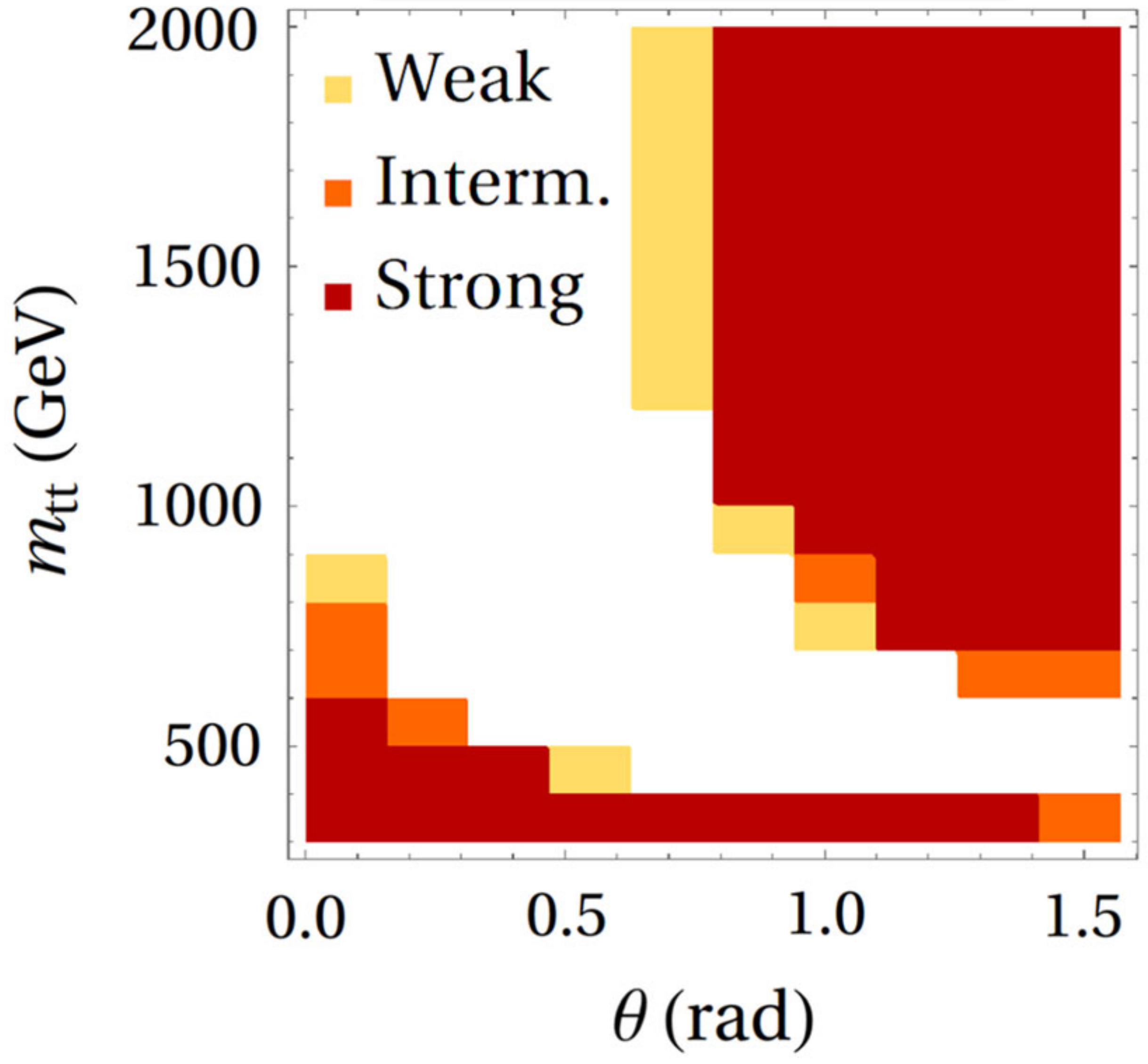


WHAT'S THE NEXT?

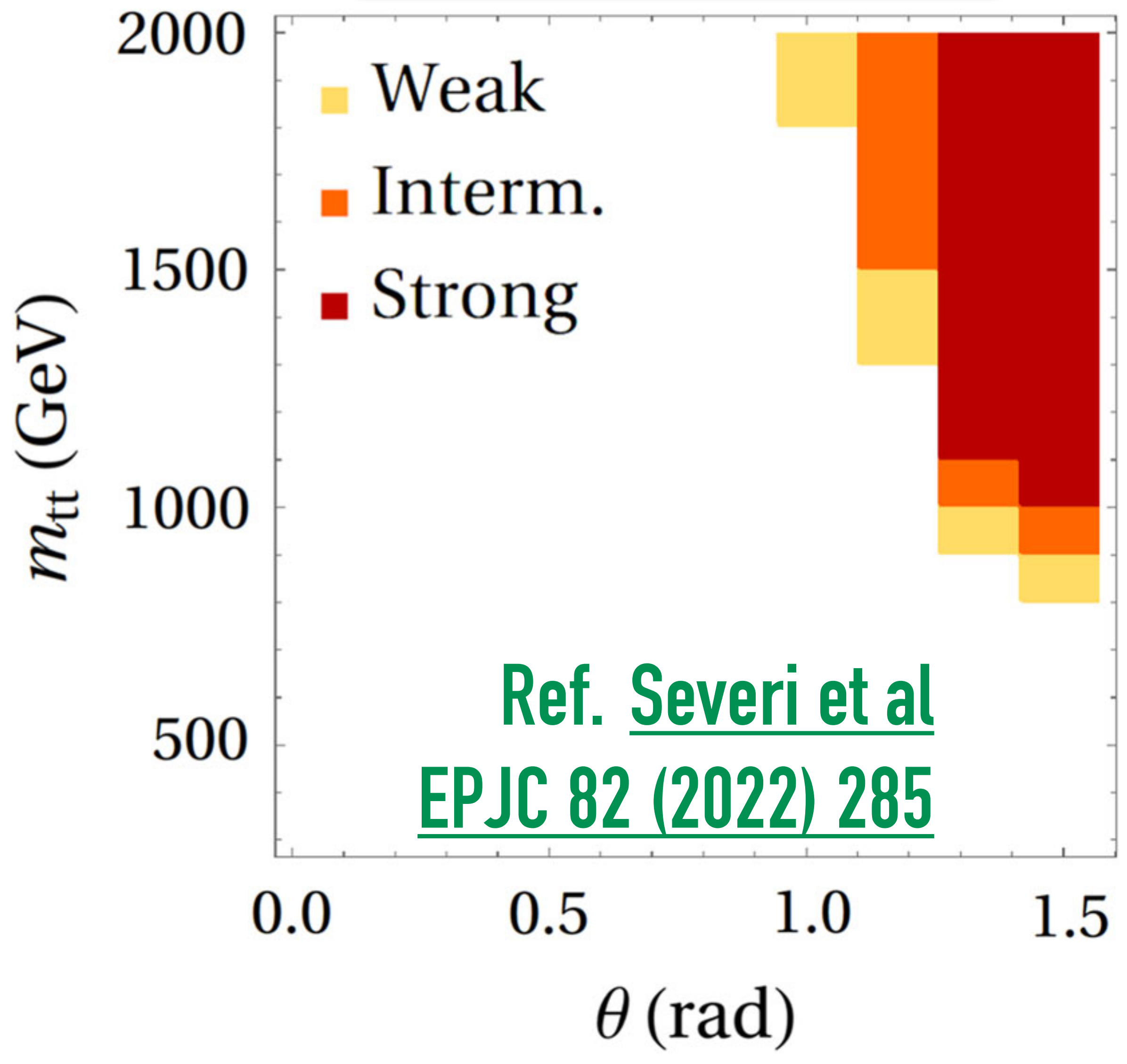
- Observation of $t\bar{t}$ entanglement is just the beginning of the journey!
- Analysis of **high m($t\bar{t}$) region** has the potential to find the violation of Bell's Inequality!
  - Phrased in terms of Clauser, Horne, Shimony, and Holt (CHSH) inequality. Ref. Clauser et al PRL 23(15) (1969) 880
  - For $t\bar{t}$ the criterion can be expressed as: $\sqrt{2}\,|-C_{rr}+C_{nn}| \leq 2$
  - To be examined in the near future with a much larger data set!



Level of Entanglement



Test of Bell's Inequality



Ref. Severi et al EPJC 82 (2022) 285

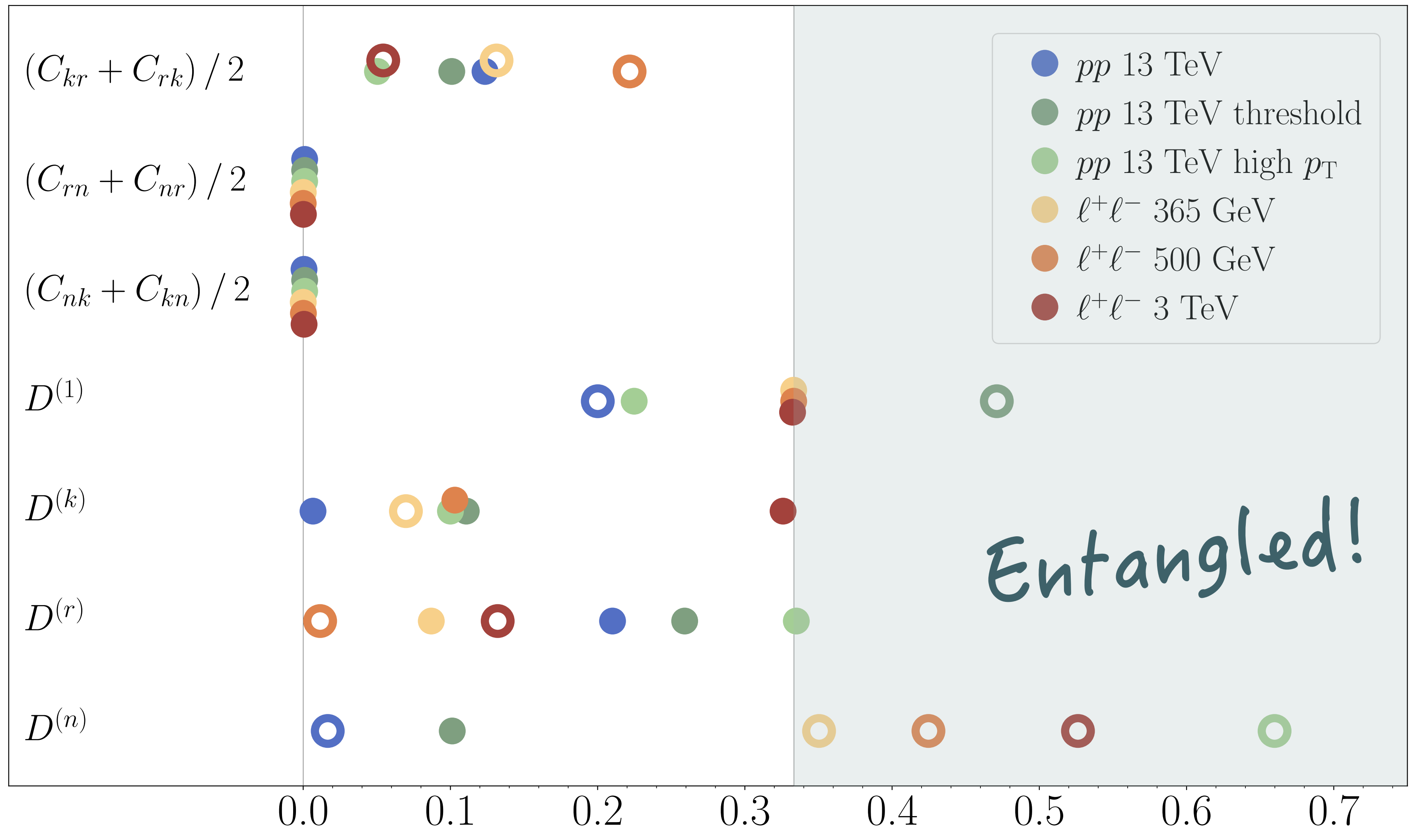
Better to go to even higher mass region!

SUMMARY & OUTLOOK

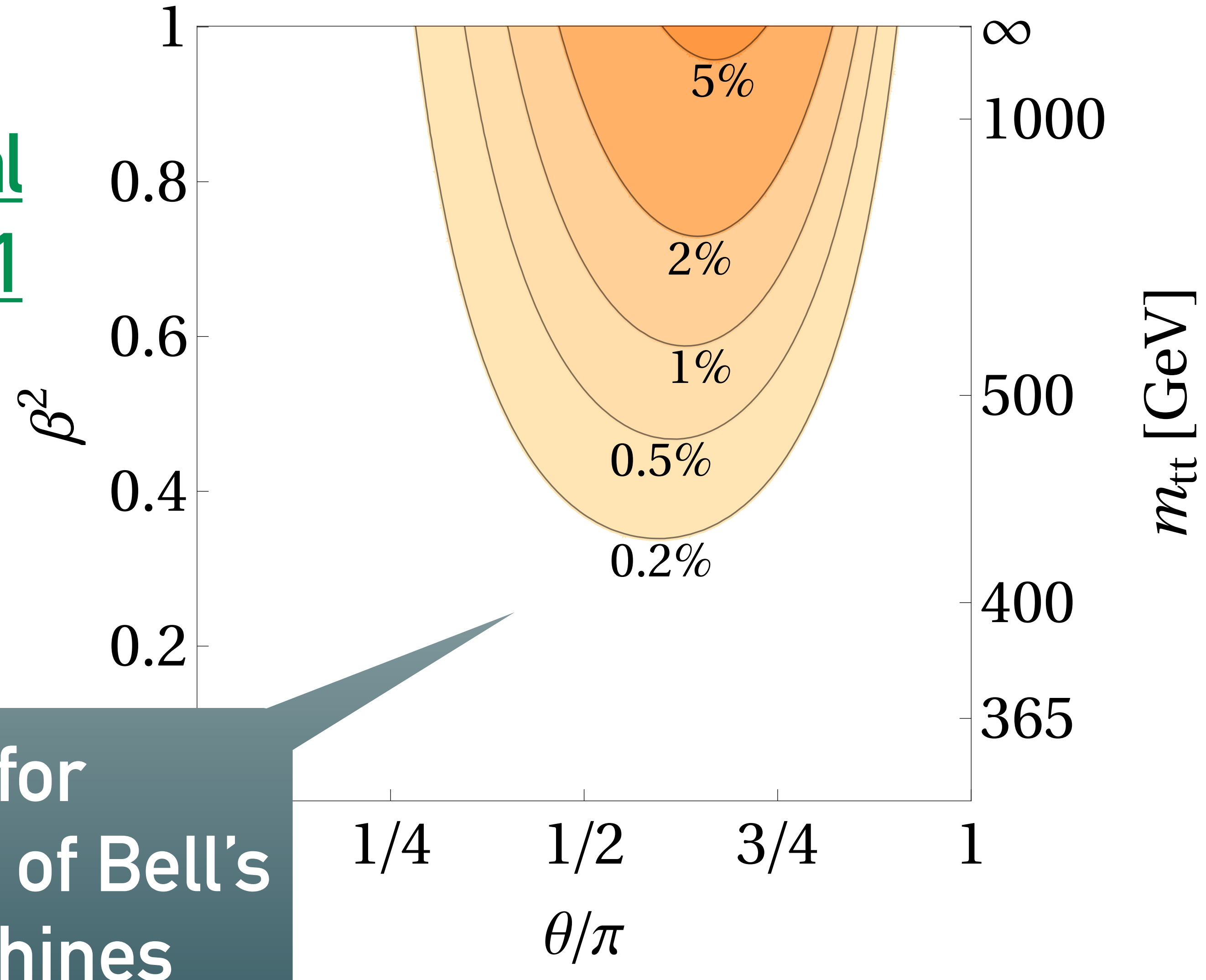
Also see the related talks yesterday!

- **Top quark entanglement has been observed:**
  - A potential "toponium" bound state at the threshold improves the data modeling.
  - Using the lepton+jets events at high m(tt̄) confirms the space-like separated events are entangled.
- **At future e⁺e⁻ machines:**
  - A much cleaner environment for better understanding of top quark properties & modeling (*including the threshold effects*)!
  - Similar formulation can be introduced for spin correlation studies; level of entanglement depends on m(tt̄) as well –– **higher $\sqrt{s}$ is preferred!**

Prediction of entanglement markers



Ref. Maltoni et al JHEP 09 (2024) 001



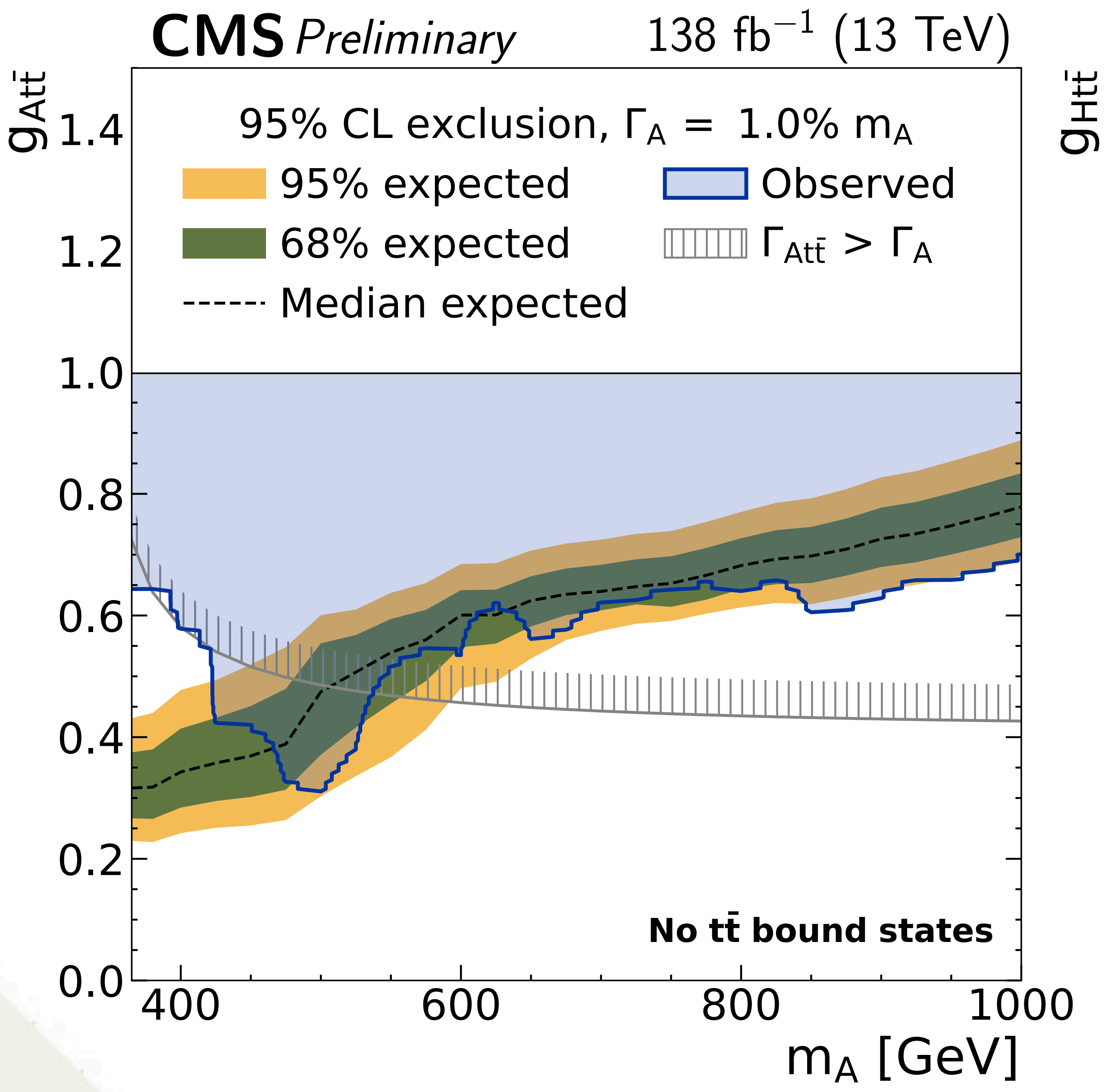
Precision required for establishing the violation of Bell's Inequality at e⁺e⁻ machines


BACKUP

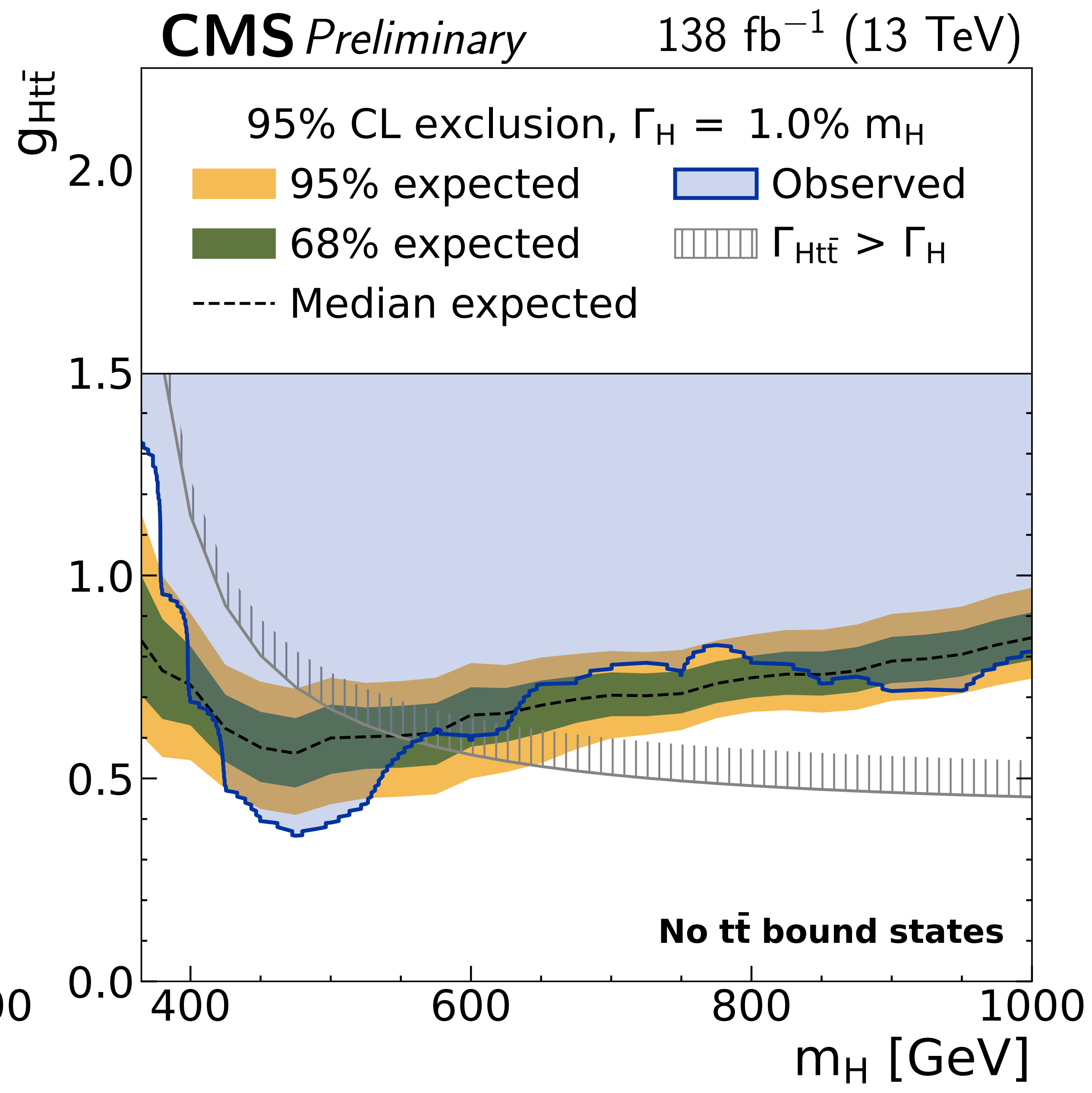
CMS SEARCH FOR H/A → $t\bar{t}$

- Heavy A/H to $t\bar{t}$ dilepton and lepton+jets, 138 fb^{-1} at CMS.
- **Significant excess at $t\bar{t}$ threshold**, favors the pseudoscalar signal hypothesis.
- Perform fits to the invariant masse and helicity angle distributions.

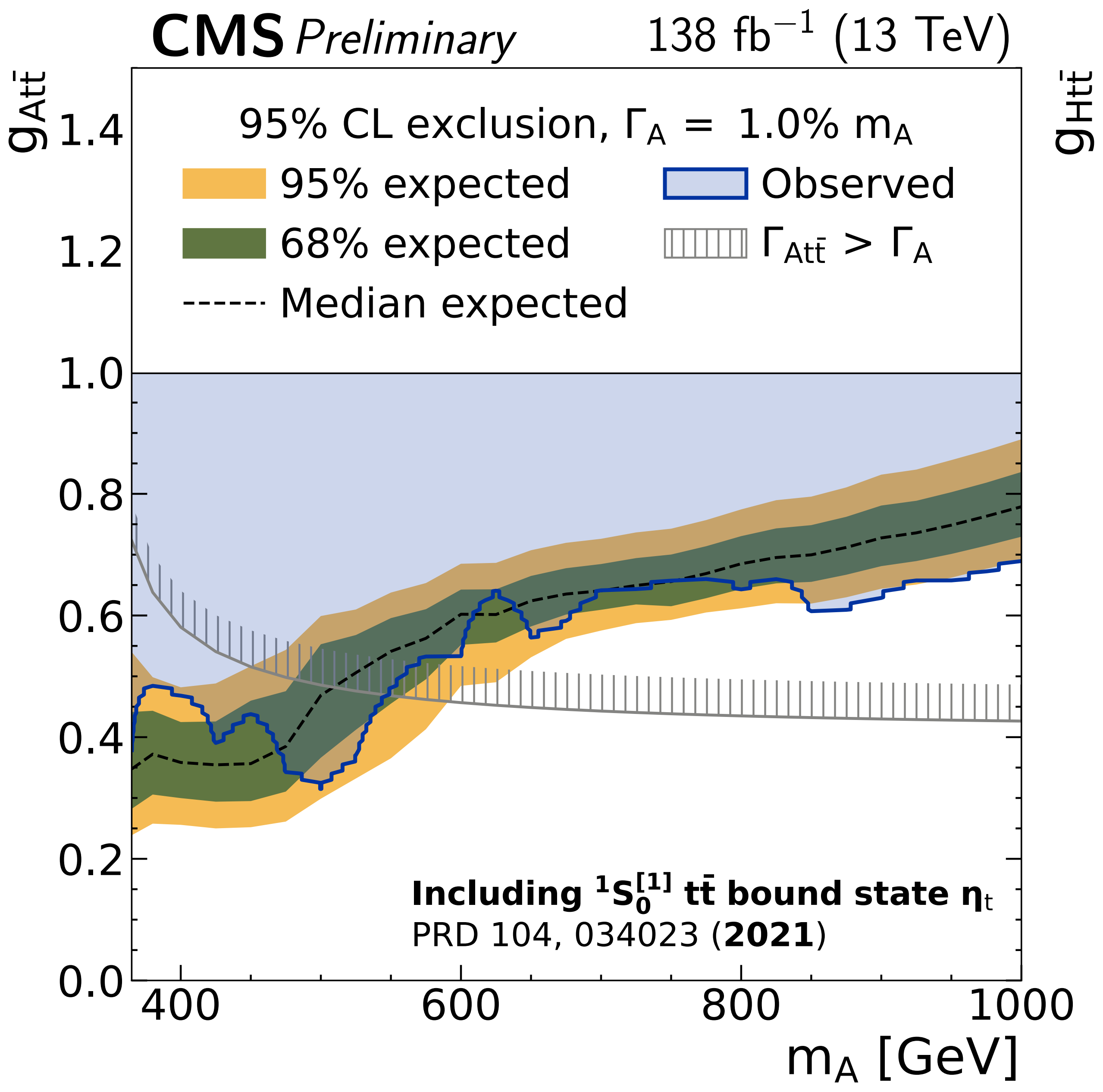
Ref. CMS-PAS-HIG-22-013



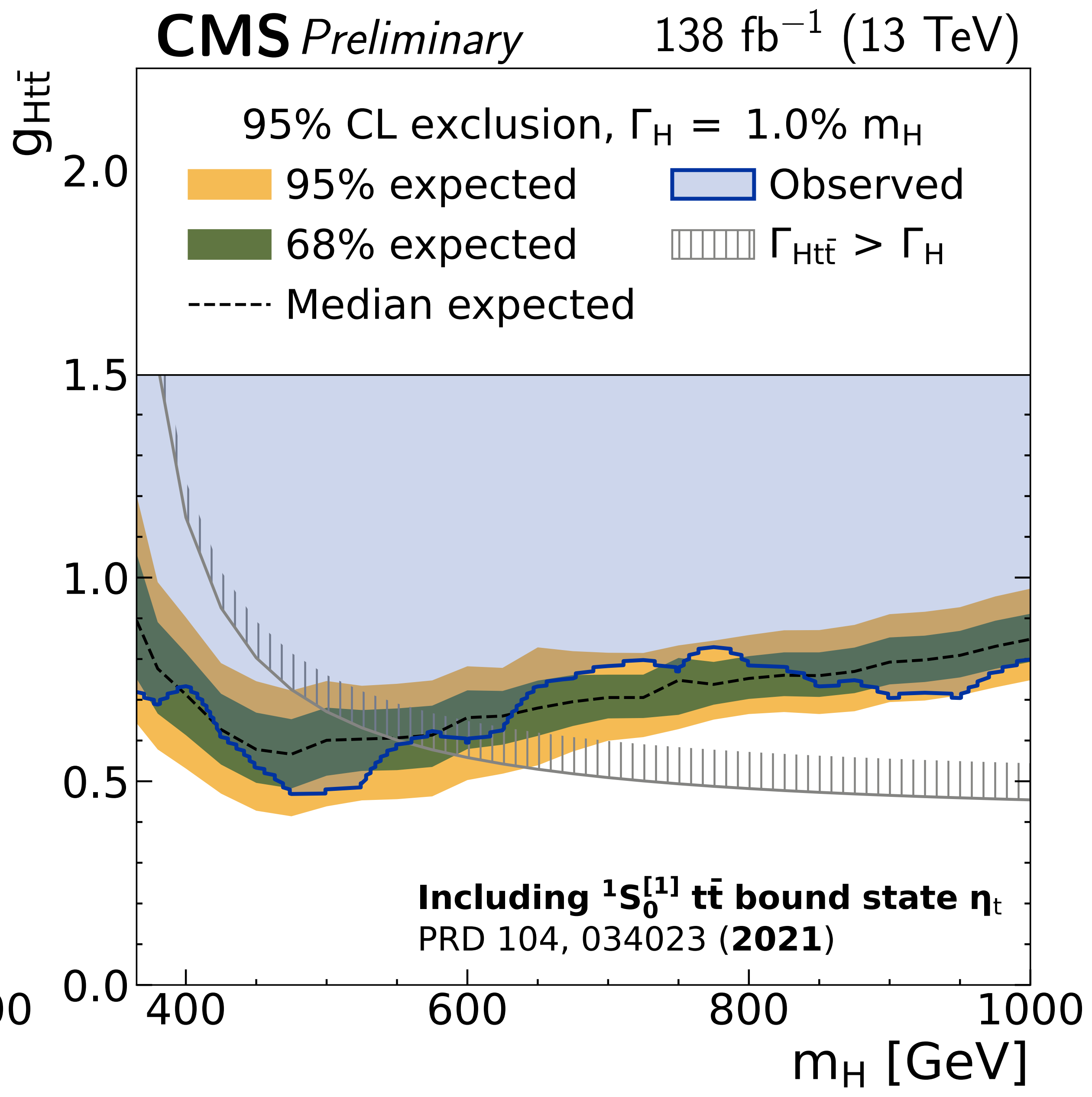
A, 1% width, w/o η(t)



H, 1% width, w/o η(t)



A, 1% width, w/ η(t)



H, 1% width, w/ η(t)

If interpret the excess as a simplified η_t resonance:
$\sigma(\eta_t) = 7.14 \pm 0.77$ pb

The excess is removed such an η_t resonance is included as background.