Testing Resonance Schemes @ Lepton Colliders

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Tsung-Dao Lee Institute

Outline

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1) Breit-Wigner vs Energy-Dependent Schemes

Z Lineshape Scan

Forward-Backward Asymmetries

2) Breit-Wigner vs Theoretical Schemes

Z Lineshape Scan

WW Threshold Scan + Fermi Constant

3) Summary

Breit-Wigner Resonance

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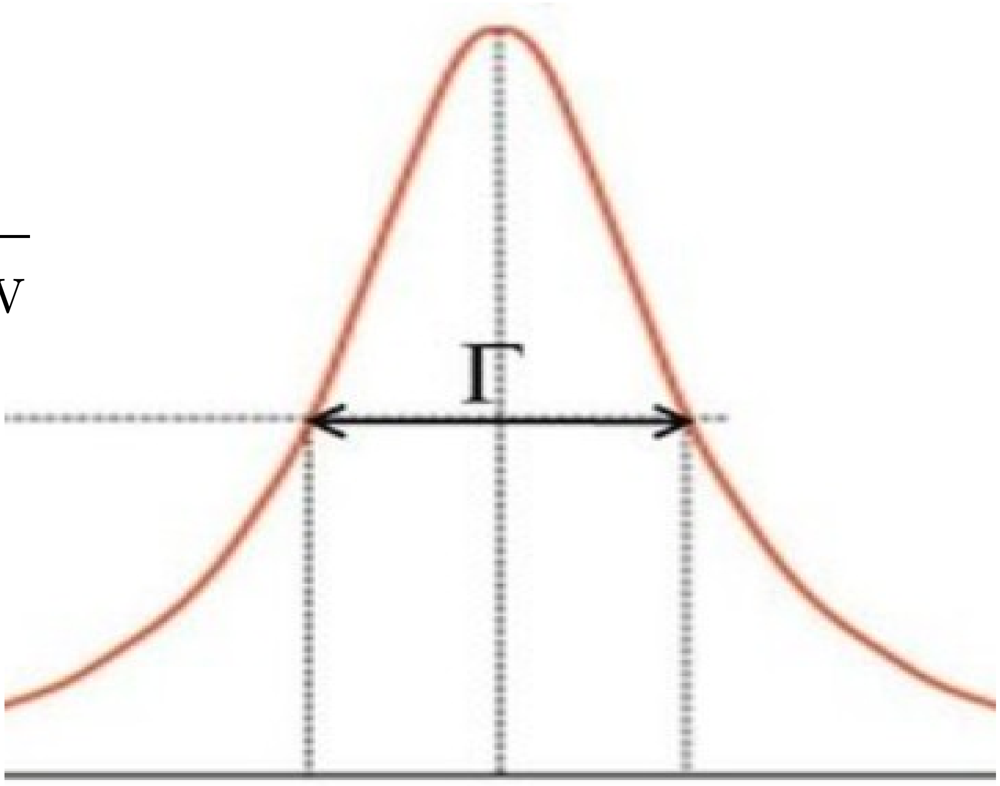
$$\mathcal{D}_{\rm BW} \sim \frac{1}{p^2 - m^2_{\rm BW} + i m_{\rm BW} \Gamma_{\rm BW}}$$



$$|\mathcal{D}_{\rm BW}|^2 \sim \frac{1}{(p^2 - m^2_{\rm BW})^2 + m^2_{\rm BW} \Gamma^2_{\rm BW}}$$

Resonance peak is at exactly the mass m_{BW}

Energy-Dependent Scheme

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Decay width is defined as

$$\Gamma \equiv \frac{1}{2m} \int |\mathcal{M}|^2 d\Omega \quad \text{where} \quad \int |\mathcal{M}|^2 d\Omega \propto p^2$$

$$m_{\text{parent}} \gg m_{\text{daughters}}$$

Decay width scales with virtuality p^2 !

$$\Gamma(p^2) \equiv \frac{p^2}{m_{\text{ED}}^2} \Gamma_{\text{ED}} \quad \Rightarrow \quad \mathcal{D}_{\text{ED}} \sim \frac{1}{p^2 - m_{\text{ED}}^2 + i \frac{p^2 \Gamma_{\text{ED}}}{m_{\text{ED}}}}$$

Berends, Burgers, Hollik & van Neerven [PLB88]
Bardin, Leike, Riemann & Sachwitz [PLB88]
Bardin, Bilenky, Mitselmakher, Riemann & Sachwitz [ZPC89]

Breit-Wigner vs Energy-Dependent

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$$|\mathcal{D}_{\rm ED}|^2 \sim \frac{\mathcal{Z}}{(p^2 - \mathcal{Z} m^2_{\rm ED})^2 + \mathcal{Z}^2 m^2_{\rm ED} \Gamma^2_{\rm ED}}$$

$$m_{\rm BW} = \sqrt{\mathcal{Z}} m_{\rm ED}$$

$$\Gamma_{\rm BW} = \sqrt{\mathcal{Z}} \Gamma_{\rm ED}$$

$$g_{\rm BW} = \mathcal{Z}^{1/4} g_{\rm ED}$$

$$\mathcal{Z} \equiv \frac{1}{1 + \frac{\Gamma^2_{\rm ED}}{m^2_{\rm ED}}}$$

$$1 - \sqrt{\mathcal{Z}} \sim 3.7 \times 10^{-4}$$

For both Z & W

$$|\mathcal{D}_{\rm BW}| \sim \frac{1}{(p^2 - m^2_{\rm BW})^2 + m^2_{\rm BW} \Gamma^2_{\rm BW}}$$

Equivalent! **?**

How to distinguish them?

Z-γ Interference Term

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$$|\mathcal{M}_Z|^2 \sim |\mathcal{D}_{\rm ED}|^2 \sim \frac{\mathcal{Z}}{(p^2 - \mathcal{Z} m_{\rm ED}^2)^2 + \mathcal{Z}^2 m_{\rm ED}^2 \Gamma_{\rm ED}^2}$$

Equivalence happens for the Z mediated contribution!

$$(\mathcal{M}_Z \mathcal{M}_\gamma^*)_{\rm BW} \propto \frac{g_{Z,{\rm BW}}^2 e^2 (s - m_{\rm BW}^2)}{[(s - m_{\rm BW}^2)^2 + m_{\rm BW}^2 \Gamma_{\rm BW}^2]\, s}$$

$$(\mathcal{M}_Z \mathcal{M}_\gamma^*)_{\rm ED} \propto \frac{\mathcal{Z} g_{Z,{\rm ED}}^2 e^2 (s - m_{\rm ED}^2)}{[(s - \mathcal{Z} m_{\rm ED}^2)^2 + \mathcal{Z}^2 m_{\rm ED}^2 \Gamma_{\rm ED}^2]\, s}$$

No simultaneous equivalence for the interference term!

Z-γ Interference Term

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$$\sigma_{Z\gamma} = \frac{\alpha Q_e Q_f}{6} \frac{(g_{eL} + g_{eR})(g_{fL} + g_{fR})(s - m_Z^2)}{(s - m_Z^2)^2 + \Gamma_Z^2 m_Z^2}$$

$$\left.\begin{aligned} g_{\ell L} &= -\frac{1}{2} + s_w^2 \\ g_{\ell R} &= s_w^2 \end{aligned}\right\} \Rightarrow \quad g_{\ell L} + g_{\ell R} = -\frac{1}{2} + 2 s_w^2 \sim \mathcal{O}(1\%)$$

The interference term is unfortunately suppressed!

Forward-Backward Asymmetry

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$$\sigma_{Z\gamma} = \frac{\alpha Q_e Q_f}{6} \frac{(g_{eL} + g_{eR})(g_{fL} + g_{fR})(s - m_Z^2)}{(s - m_Z^2)^2 + \Gamma_Z^2 m_Z^2}$$

$$\sigma_F - \sigma_B \propto \frac{3s}{64\pi}(g_{eL}^2 - g_{eR}^2)(g_{fL}^2 - g_{fR}^2)$$

$$+\frac{3Q_e Q_f \alpha(s - M_Z^2)}{8} (g_{eL} - g_{eR}) (g_{fL} - g_{fR})$$

The interference term is enhanced!

Z Lineshape Scan

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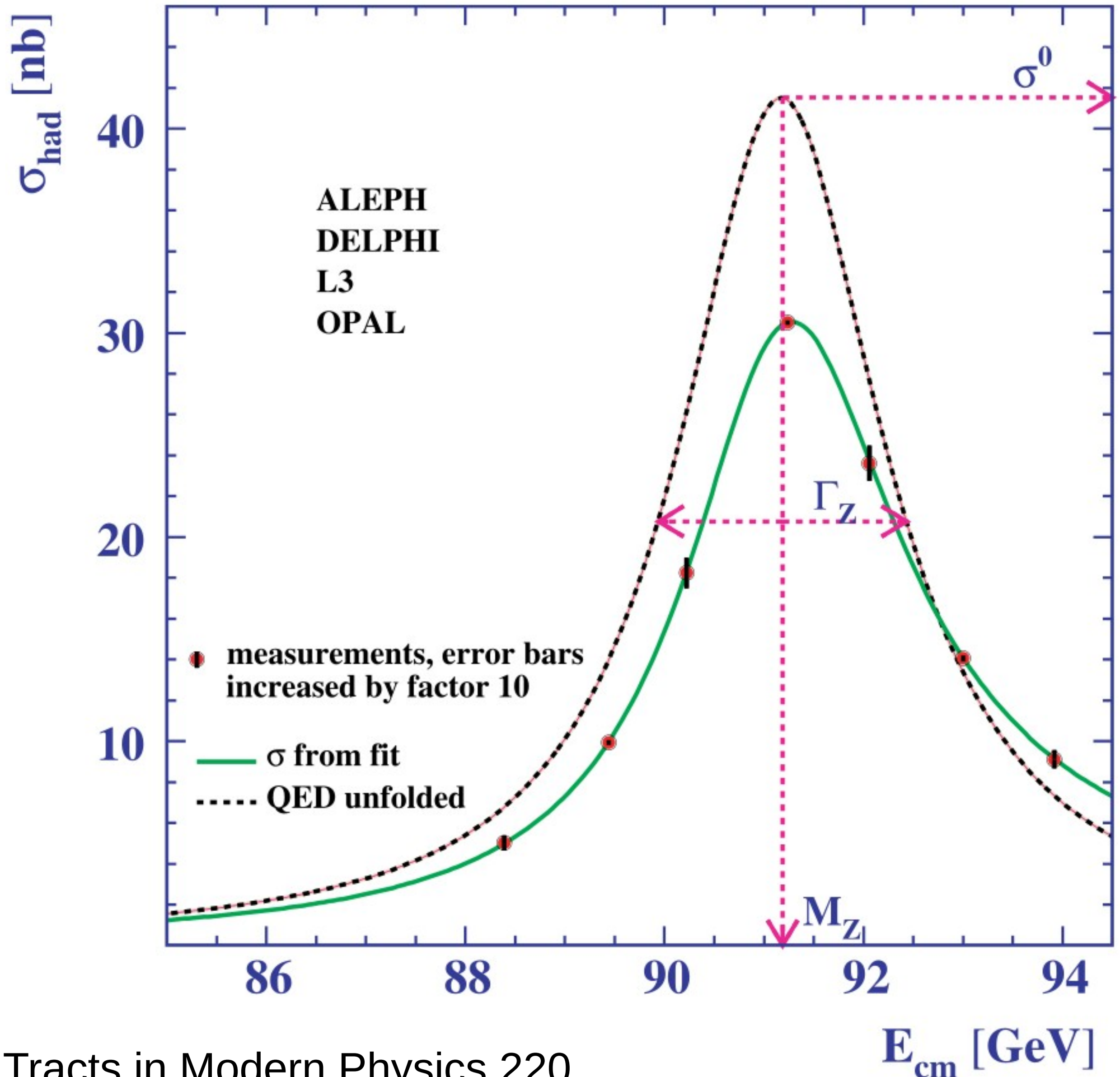

Roth, Springer Tracts in Modern Physics 220

Z Lineshape Scan + A_{FB}

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$$\sigma_{\rm had} \equiv \sum_q \sigma^q_F + \sigma^q_B \qquad A^f_{FB} \equiv \frac{\sigma^f_F - \sigma^f_B}{\sigma^f_F + \sigma^f_B}$$

Required precision: $\Delta m_Z \lesssim 34\,\mathrm{MeV}$ $\quad \Delta\Gamma_Z \lesssim 0.9\,\mathrm{MeV}$

$\Delta\chi^2_{\rm min}$	ALEPH	DELPHI	L3	OPAL	Combined
$\sigma_{\rm had}$	0.106	0.0083	0.0127	0.0529	0.183
$\sigma_{\rm had} + A^\mu_{FB}$	8.34	10.1	5.70	14.1	37.9
$\sigma_{\rm had} + A^{b,c}_{FB}$	1.19	0.878	0.010*	0.734	2.97
$\sigma_{\rm had} + A^{\mu,b,c}_{FB}$	8.42	9.67	5.58*	13.3	37.0

CEPC: **490** ($\sigma_{\rm had} + A^\mu$), **5.1×10^5** ($\sigma_{\rm had} + A^\mu + A^q$)

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Quantum Description

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A fundamental particle is described by wave function

Schodinger Equation

$$\psi \longrightarrow e^{-iEt}\psi$$

This form is Intrinsically for stable particles!

How about unstable particles?

$$E \longrightarrow E - \frac{i}{2}\Gamma$$

Then the wave function decays with time

$$\psi(t) \propto e^{-\frac{1}{2}\Gamma t}$$

Willenbrock & Valencia [PLB91]
Willenbrock [2203.11056]

Physical Pole for Unstable Particle

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It is much more convenient to define mass & decay width in the rest frame.

$$m - \frac{i}{2}\Gamma$$

A physical pole can be generally parametrized as

$$\frac{1}{p^2 - \mu^2} \xrightarrow{\mu \equiv m - \frac{i}{2}\Gamma} \frac{1}{p^2 - m^2 + \frac{1}{4}\Gamma^2 + im\Gamma}$$

Resonance peak is at $m^2 - \frac{1}{4}\Gamma^2$

Willenbrock & Valencia [PLB91]
Willenbrock [2203.11056]

Breit-Wigner vs Theoretical

$$\mathcal{D}_{\rm BW} \sim \frac{1}{p^2 - m_{\rm BW}^2 + i m_{\rm BW} \Gamma_{\rm BW}}$$

$$\frac{m_{\rm th}}{m_{\rm BW}} = \frac{\Gamma_{\rm BW}}{\Gamma_{\rm th}} \simeq 1 + \frac{\Gamma_Z^2}{8 m_Z^2} \equiv R_Z$$

Equivalent!

$$\mathcal{D}_{\rm th} \sim \frac{1}{p^2 - m_{\rm th}^2 + \frac{1}{4}\Gamma_{\rm th}^2 + i m_{\rm th} \Gamma_{\rm th}}$$

Reciprocal Scaling Behaviors

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$$\frac{1}{p^2 - m_{\rm BW}^2 + i m_{\rm BW}\Gamma_{\rm BW}} \quad \text{vs} \quad \frac{1}{p^2 - m_{\rm th}^2 + \frac{1}{4}\Gamma_{\rm th}^2 + i m_{\rm th}\Gamma_{\rm th}}$$

Being extracted from data, the imaginary parts should be the same!

$$\Gamma \propto \frac{1}{m}$$

Theoretical predictions should be $\Gamma_Z \propto m_Z$

$$\Gamma_Z = \frac{m_Z}{24\pi}\beta_f \left[g_L^2 + g_R^2 - (g_L^2 - 6 g_L g_R + g_R^2)\frac{m_f^2}{m_Z^2} \right]$$

$$\beta_f \equiv \sqrt{1 - 4m_f^2/m_Z^2}$$

Constrained Fit

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$$\chi^2_Z \equiv \left(\frac{m_{\rm th}/R_Z - m_Z}{\Delta m_Z} \right)^2 + \left(\frac{\Gamma_{\rm th} R_Z - \Gamma_Z}{\Delta \Gamma_Z} \right)^2$$

Required precision: $\Delta m_Z \lesssim 8.4\,\mathrm{MeV} \quad \Delta \Gamma_Z \lesssim 0.23\,\mathrm{MeV}$

	Δm_Z (MeV)	$\Delta\Gamma_Z$ (MeV)
LEP	2.1	2.3
CEPC	0.1	0.025
FCC-ee	0.1	0.025
ILC	0.2	0.12

$\chi^2_{Z,\min} = 326$ (CEPC & FCC-ee) $\quad 14.3$ (ILC)

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WW Threshold Scan

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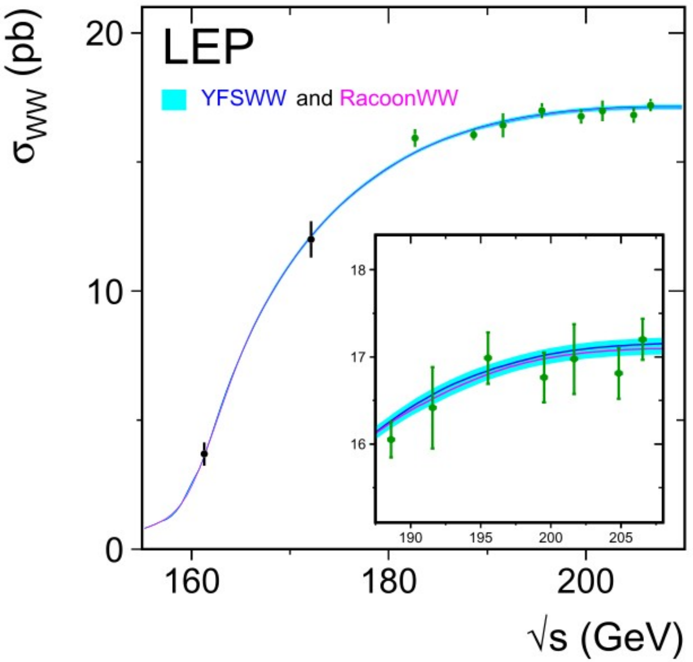
$$\chi^2_W \equiv \left(\frac{m_{W,\text{th}}/R_W - m_W}{\Delta m_W} \right)^2 + \left(\frac{\Gamma_{W,\text{th}} R_W - \Gamma_W}{\Delta \Gamma_W} \right)^2$$

$$\Gamma_W = \frac{m_W}{24\pi} g^2 \beta_{12} \left[1 - \frac{m_1^2 + m_2^2}{2 m_W^2} - \frac{(m_1^2 - m_2^2)^2}{2 m_W^4} \right]$$

Unfortunately, the mass & decay with precisions are not enough!

	Δm_W (MeV)	$\Delta \Gamma_W$ (MeV)
CEPC	0.5	2.0
FCC-ee	0.4	1.2
ILC	2.4	2.0



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Fermi Constant G_F

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$$G_{F,\mathrm{BW}} \approx \frac{g^2}{4\sqrt{2} m^2_{W,\mathrm{BW}}} \left(1 - \frac{\overline{\Gamma}^2_W}{2 m^2_W} \right)$$

Momentum transfer in muon decay $p^2 < m^2_\mu$

$$\overline{\Gamma}_W = \frac{p^2}{m^2_W} \frac{2}{5} \Gamma_W(m^2_W) \sim 10^{-7} \Gamma_W(m^2_W)$$

Fermi constant is free of decay width @ tree level

$$G_{F,\mathrm{BW}} \approx \frac{g^2}{4\sqrt{2} m^2_{W,\mathrm{BW}}} \approx \frac{g^2}{4\sqrt{2} m^2_{W,\mathrm{th}}}$$

Fermi constant fixes m_W to the same value!

WW Threshold + Fermi Constant

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$$\chi^2_W \equiv \left(\frac{m_{W,\mathrm{th}}/R_W - m_W}{\Delta m_W}\right)^2 + \left(\frac{\Gamma_{W,\mathrm{th}} R_W - \Gamma_W}{\Delta \Gamma_W}\right)^2 + \left(\frac{G_{F,\mathrm{th}} - G_F}{\Delta G_F}\right)^2$$

	Δm_W (MeV)	$\Delta \Gamma_W$ (MeV)
CEPC	0.5	2.0
FCC-ee	0.4	1.2
ILC	2.4	2.0

	$\Delta\chi^2_{min}$
CEPC	169
FCC-ee	263
ILC	7.3

$$G_F = 1.166\,378\,8\,(6) \times 10^{-5}\,\mathrm{GeV}^{-2}$$

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Summary

Breit-Wigner

$$\frac{1}{p^2 - m_{\rm BW}^2 + i m_{\rm BW} \Gamma_{\rm BW}}$$

m, Γ, g changes

Z lineshape + A_{FB}

m & Γ changes

Z: reciprocal scaling

W: + Fermi constant G_F

Energy-Depedent

$$\frac{1}{p^2 - m_{\rm ED}^2 + i \frac{p^2 \Gamma_{\rm ED}}{m_{\rm ED}}}$$

Theoretical

$$\frac{1}{p^2 - m_{\rm th}^2 + \frac{1}{4}\Gamma_{\rm th}^2 + i m_{\rm th} \Gamma_{\rm th}}$$

1. Lepton collider can test one important aspect of quantum & field theories.

2. Resonance scheme can affect precision measurements.

《 Physics Letters A 》

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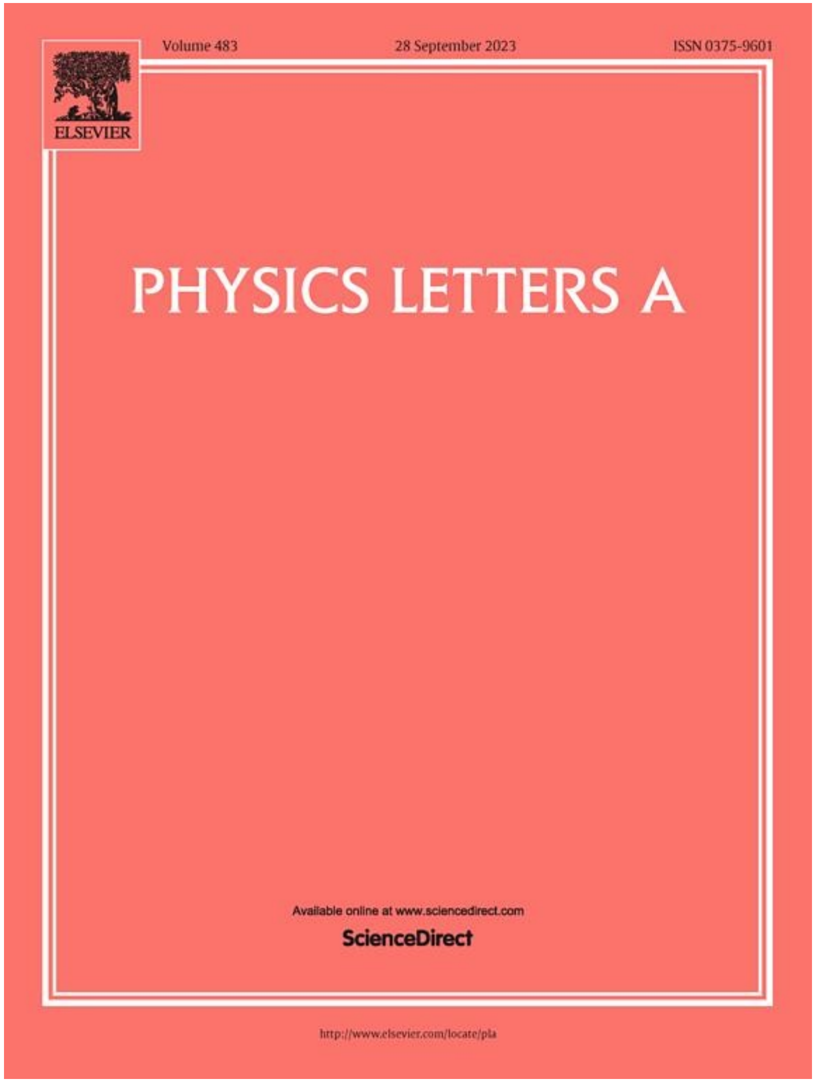

Aims & Scope

- Nonlinear science,
- Statistical physics,
- Mathematical and computational physics,
- AMO and physics of complex systems,
- Plasma and fluid physics,
- Optical physics,
- General and cross-disciplinary physics,
- Biological physics and nanoscience,
- Astrophysics, Particle physics and Cosmology.

PLA now also accepts submissions on Particle, Astrophysics, and Cosmology! You are welcome to submit.