Testing Bell Inequalities and Probing Quantum Entanglement at CEPC

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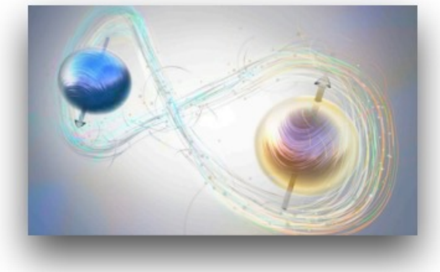
Peking University

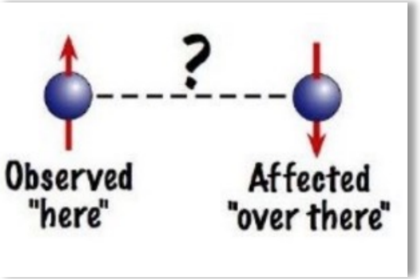
Entanglement in QM

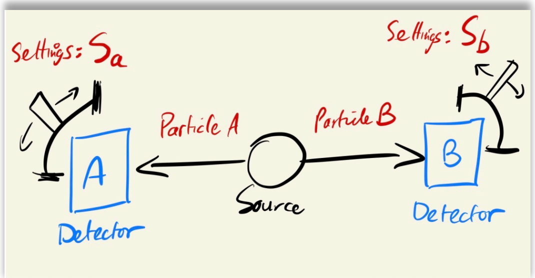
- Qubit = two-level quantum system $|0\rangle, |1\rangle$: most simple quantum system
- Two qubits: the most simple example of quantum correlations.
- A quantum state of two subsystems A and B is separable when its density matrix:

$$\rho = \sum_i p_i \rho_A^i \otimes \rho_B^i$$

- Non-separability of a quantum state = entanglement.
  - entangled states cannot be described by independent superpositions.
  - measuring particle spin in an entangled system immediately reveals the spin state of the second particle even when casually separated.





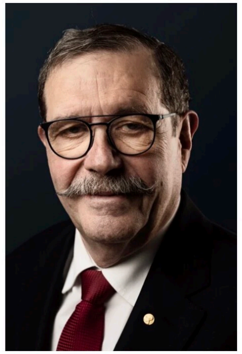


Entanglement in HEP:

Several experimental tests carried out since 1972

- mostly with electrons and photons at low energy
- Interest in repeating these tests with massive systems at high energy.

The Nobel Prize in Physics 2022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Alain Aspect
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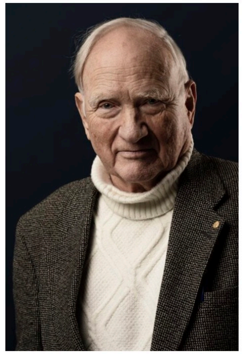
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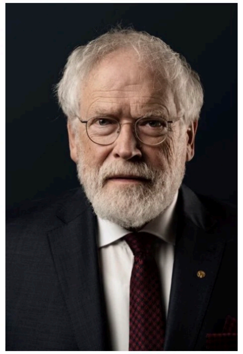
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Anton Zeilinger
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The Nobel Prize in Physics 2022 was awarded jointly to Alain Aspect, John F. Clauser and Anton Zeilinger "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"

Entanglement at the LHC

- LHC can provide a unique TeV environment to study entanglement and violation of Bell's inequalities:
  - simplest qubits at LHC: $t\tilde{t}$.
- First observation of entanglement in $t\tilde{t}$ by ATLAS at 2023. [https://doi.org/10.1038/s41586-024-07824-z]
- The first observation at CMS a few months ago in the dilepton events. [https://doi.org/10.48550/arXiv.2406.03976].
- Recently first observation in lepton+jets events by CMS - first time with casually separated top quarks at high $m_{t\tilde{t}}$. [CMS-PAS-TOP-23-007]

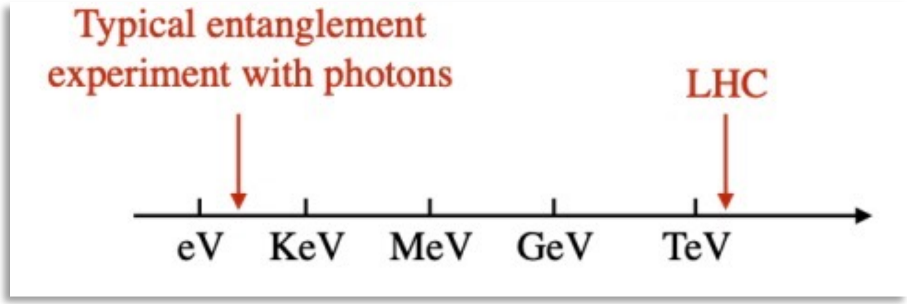
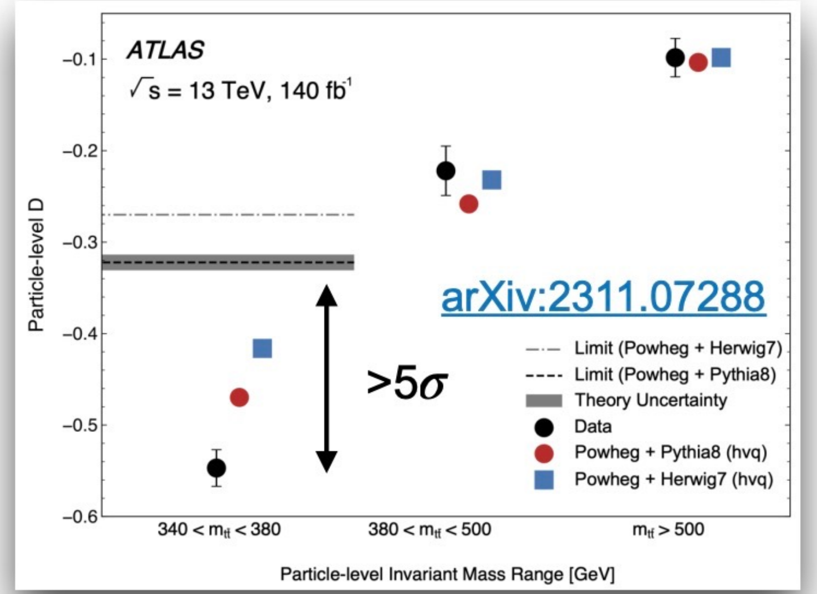



Image from ATLAS physics briefing



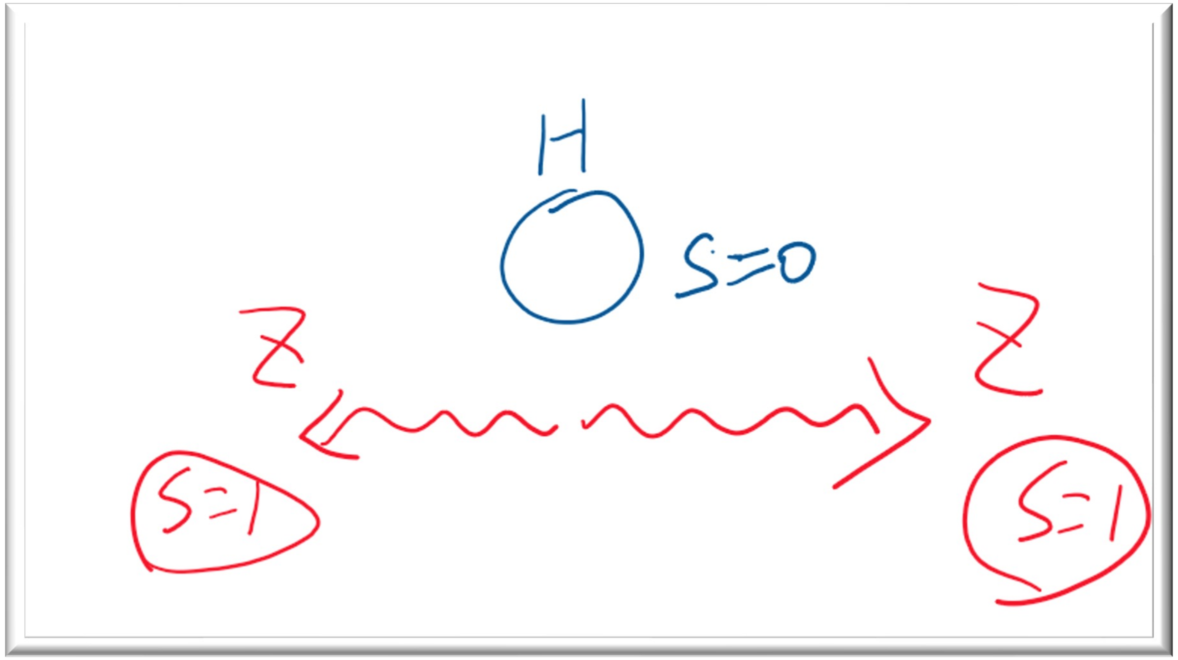
Theory and Methods------Density Matrix

- The polarization density matrix(PDM) can be reconstructed from the angular distributions of the decay products:

$$\rho = |\Psi_{ZZ}\rangle\langle\Psi_{ZZ}| = |\Phi\rangle\langle\Phi|$$

$$|\Phi\rangle = \sum c_{ij}|ij\rangle \rightarrow \sum \mathcal{M}(\lambda_1, \lambda_2)|\lambda_1, \lambda_2\rangle$$

Ψ_Z has three polarization states: +1, 0, -1



- Parametrization from using the irreducible tensor operators:

$$\rho = \frac{1}{9}\left[\mathbb{1}_3 \otimes \mathbb{1}_3 + A^1_{LM} T^L_M \otimes \mathbb{1}_3 + A^2_{LM} \mathbb{1}_3 \otimes T^L_M + C_{L_1 M_1 L_2 M_2} T^{L_1}_{M_1} \otimes T^{L_2}_{M_2}\right]$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_+ \, d\Omega_-} = \left(\frac{3}{4\pi}\right)^2 \mathrm{Tr}\left[\rho_{V_1 V_2}\left(\Gamma_1 \otimes \Gamma_2\right)\right]$$

Production **Decay**

All coefficients → Quantum Tomography

- No direct spin measurements: inferred by angular distributions.
- Both the state before decay & the final state decay products inherit the SAME quantum information.

Theory and Methods------Density Matrix

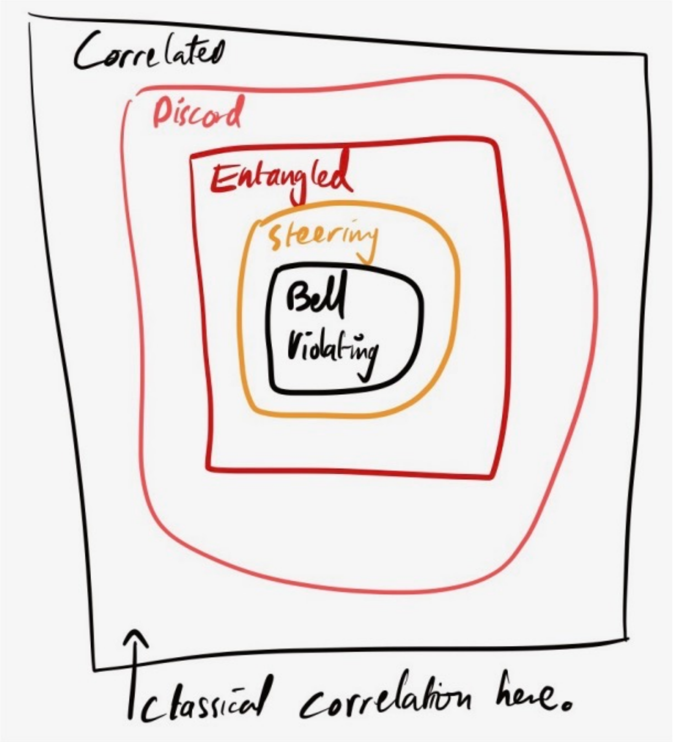
➢ The decaying density matrix:

$$\Gamma(\theta,\phi)=\frac{1}{4}\begin{pmatrix} 1+\cos^2\theta-2\eta_\ell\cos\theta & \frac{1}{\sqrt{2}}(\sin 2\theta-2\eta_\ell\sin\theta)e^{i\phi} & (1-\cos^2\theta)e^{i2\phi} \\ \frac{1}{\sqrt{2}}(\sin 2\theta-2\eta_\ell\sin\theta)e^{-i\phi} & 2\sin^2\theta & -\frac{1}{\sqrt{2}}(\sin 2\theta+2\eta_\ell\sin\theta)e^{i\phi} \\ (1-\cos^2\theta)e^{-i2\phi} & -\frac{1}{\sqrt{2}}(\sin 2\theta+2\eta_\ell\sin\theta)e^{-i\phi} & 1+\cos^2\theta-2\eta_\ell\cos\theta \end{pmatrix}$$

simplified

➢ The density matrix:

$$\rho=\begin{bmatrix} 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&\frac{1}{6}(\sqrt{2}A^1_{2,0}+2)&0&\frac{1}{3}C_{2,1,2,-1}&0&\frac{1}{3}C_{2,2,2,-2}&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&\frac{1}{3}C_{2,1,2,-1}&0&\frac{1}{3}(1-\sqrt{2}A^1_{2,0})&0&\frac{1}{3}C_{2,1,2,-1}&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&\frac{1}{3}C_{2,2,2,-2}&0&\frac{1}{3}C_{2,1,2,-1}&0&\frac{1}{6}(\sqrt{2}A^1_{2,0}+2)&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0 \end{bmatrix}$$



➢ The coefficients can be expressed using the differential cross section making use of the orthogonal property of the spherical harmonics:

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y_L^M(\Omega_j)d\Omega_j=\frac{B_L}{4\pi}A^j_{LM}, \qquad j=1,2;$$

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y_{L_1}^{M_1}(\Omega_1)Y_{L_2}^{M_2}(\Omega_1)d\Omega_1 d\Omega_2=\frac{B_{L_1}B_{L_2}}{4\pi}C_{L_1M_1L_2M_2}.$$

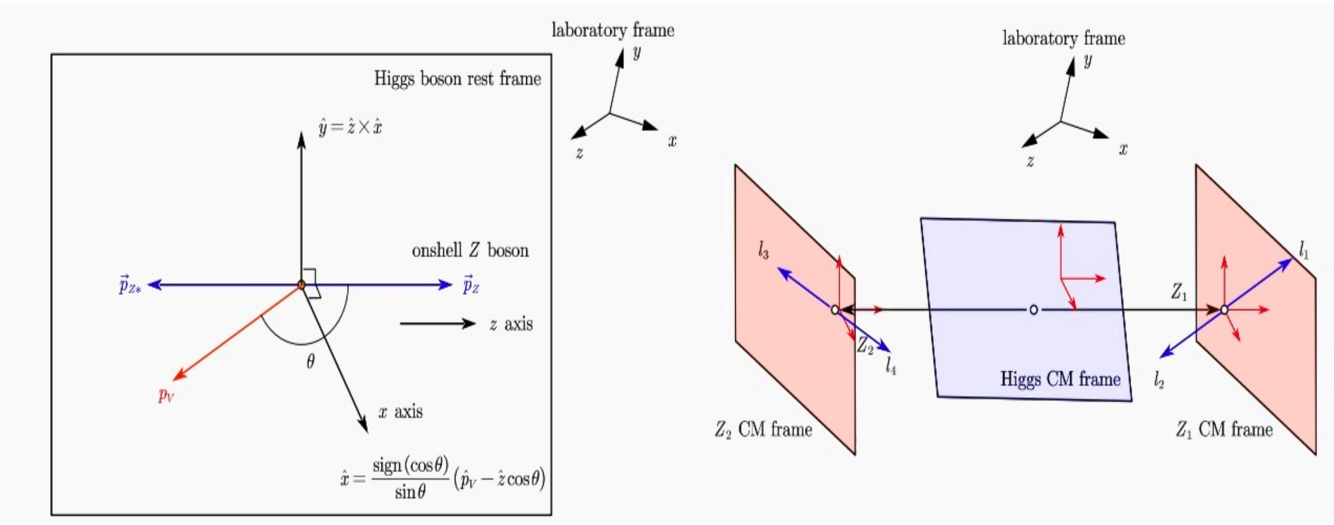
In two spin-1 massive bosons' system:

- The z-axis is the direction of the on-shell Z boson's 3-momentum.
- The $\hat{x}$ axis is in the production plane: $\hat{x} = \frac{sign(cos\theta)(\hat{p}_p - cos\theta\hat{z})}{sin\theta}, \hat{p}_p = (0,0,1)$
- The $\hat{y} = \hat{z}\times\hat{x}$
- J_Z is the polarization operator.
- The eigenstates of J_Z is the basis of the spin space.

Two Lorentz Transformation:

- Higgs rest frame → determine Z axis
- Z boson rest frame(boost along Z vector) → lepton's polar angles

Obtain :(θ_1, φ_1) in Z_1 rest frame, (θ_2, φ_2) in Z_2 rest frame. The coefficients can A^I_{LM} *and* $C_{L_1M_1L_2M_2}$ can be calculated



$$\rho = \frac{1}{9}\left[\mathbb{1}_3 \otimes \mathbb{1}_3 + A^1_{LM} T^L_M \otimes \mathbb{1}_3 + A^2_{LM} \mathbb{1}_3 \otimes T^L_M + C_{L_1M_1L_2M_2} T^{L_1}_{M_1} \otimes T^{L_2}_{M_2}\right]$$

Theory and Methods------Bell Inequalities

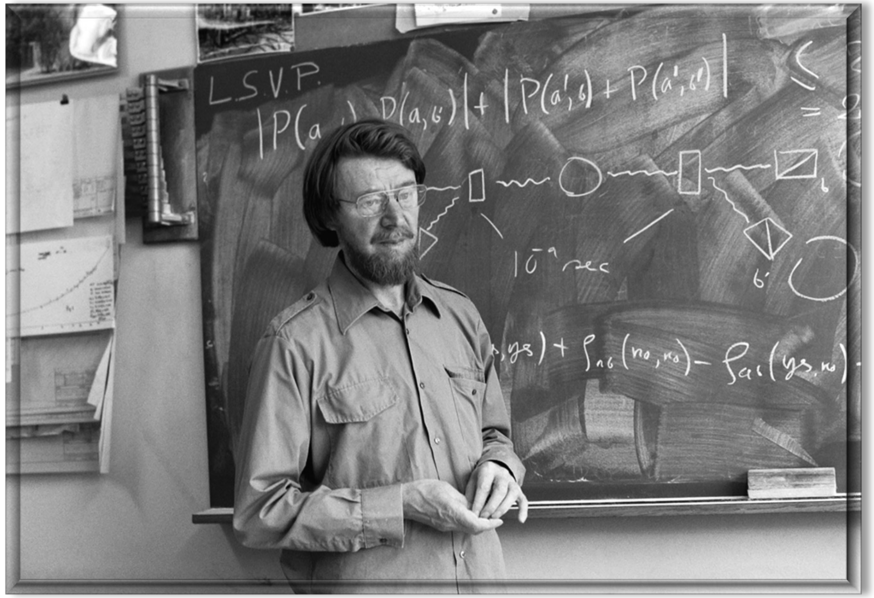
- The most original form of Bell inequalities (Clauser-Horne-Shimony-Holt Inequality):

$$P(A_1 B_1 | AB, \lambda) = P(A_1 | A, \lambda) P(B_1 | B, \lambda)$$

Classical local hidden variable theory:

$$\boldsymbol{I_3 = \langle O_{Bell} \rangle = Tr\{\rho O_{Bell}\} \leq 2}$$

ρ : Polarization density matrix (PDM)



- More general form (Collins-Gisin-Linden-Massar-Popescu Inequality):

$$\mathcal{I}_d = \sum_{k=0}^{[d/2]-1} \left(1 - \frac{2k}{d-1}\right)\{+[P(A_1 = B_1 + k) + P(B_1 = A_2 + k + 1) + P(A_2 = B_2 + k) + P(B_2 = A_1 + k)] - [P(A_1 = B_1 - k - 1) + P(B_1 = A_2 - k) + P(A_2 = B_2 - k - 1) + P(B_2 = A_1 - k - 1)]\}$$

3-dimensional form:

$$\mathcal{I}_3 = P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) - [P(A_1 = B_1 - 1) + P(B_1 = A_2) + P(A_2 = B_2 - 1) + P(B_2 = A_1 - 1)].$$

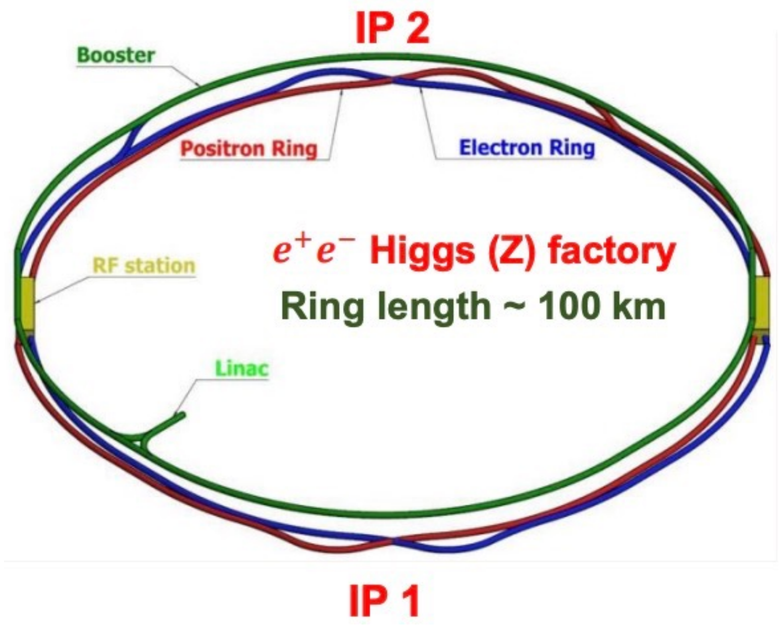
- The expectation value of the Bell operator can be written as:

$$\mathcal{B} = \left[\frac{2}{3\sqrt{3}}\left(T_1^1 \otimes T_1^1 - T_0^1 \otimes T_0^1 + T_1^1 \otimes T_{-1}^1\right) + \frac{1}{12}\left(T_2^2 \otimes T_2^2 + T_2^2 \otimes T_{-2}^2\right) + \frac{1}{2\sqrt{6}}\left(T_2^2 \otimes T_0^2 + T_0^2 \otimes T_2^2\right) - \frac{1}{3}\left(T_1^2 \otimes T_1^2 + T_1^2 \otimes T_{-1}^2\right) + \frac{1}{4} T_0^2 \otimes T_0^2\right] + \text{h.c.}.$$

- Bell inequality expectation value can be calculated:

$$\mathcal{I}_3 = \frac{1}{36}\left(18 + 16\sqrt{3} - \sqrt{2}\left(9 - 8\sqrt{3}\right) A_{2,0}^1 - 8\left(3 + 2\sqrt{3}\right) C_{2,1,2,-1} + 6 C_{2,2,2,-2}\right)$$

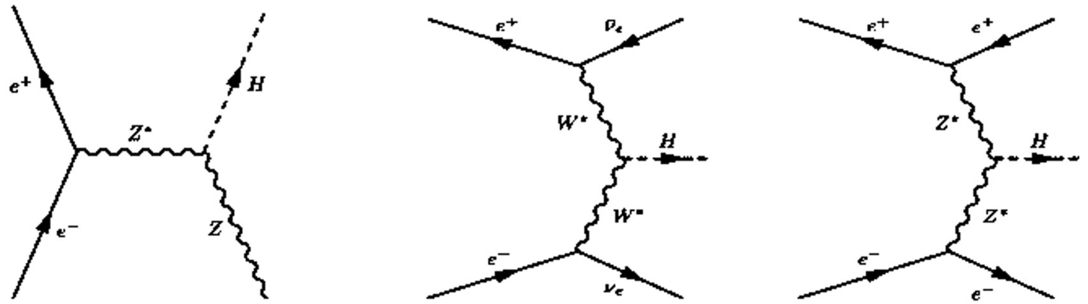
Circular Electron–Positron Collider(CEPC)

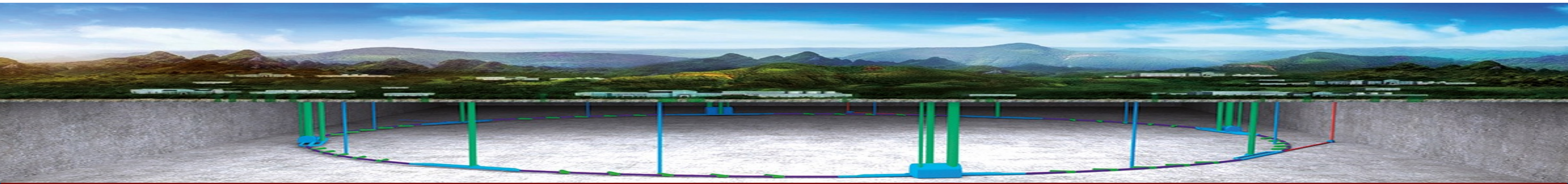


- Lepton collider has a much cleaner backgrounds and simpler final states than the Hadron collider.
- Such a Higgs factory can also be a factory for top, Z, and W.
- CEPC can be upgraded to a ~100 TeV pp collider in the future (SppC).

Three processes to generate Higgs boson at CEPC:

Higgsstrahlung, WW fusion, ZZ fusion

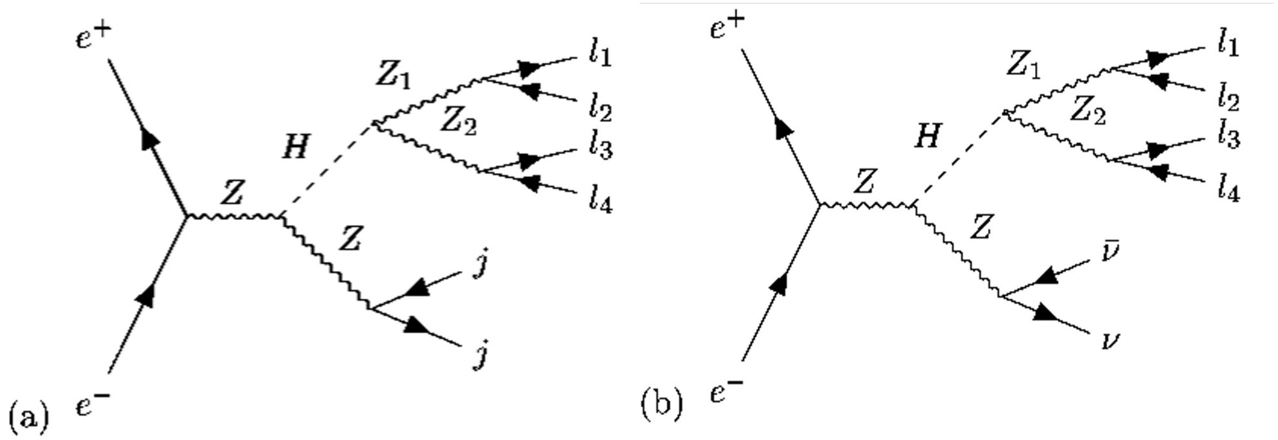


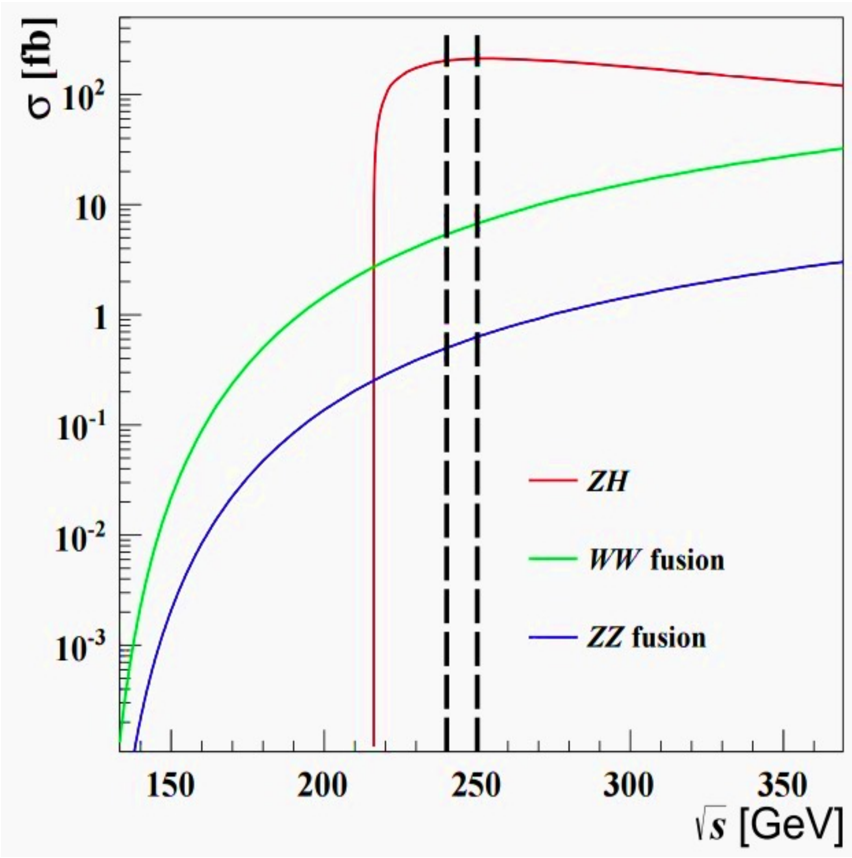


In our analysis:

The signal process:

$$e^+e^- \rightarrow ZH, H \rightarrow ZZ^* (Z^*: \text{off} - \text{shell Z boson})$$



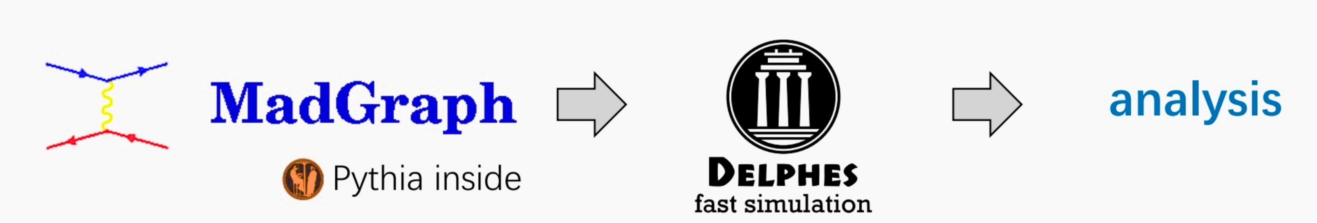


Backgrounds for this process:

- $e^+e^- \rightarrow ZZ$
- $e^+e^- \rightarrow ZZZ$
- $e^+e^- \rightarrow \ell^+\ell^- H$

- This process is the main dominant to generate Higgs at $\sqrt{s} = 250\text{GeV}$.
- Consider two channels depending on the Z boson decay.
- Both semi-leptonic and pure-leptonic channels are not complicated to analyze.
- Through the four leptons (θ, φ) to calculate I_3 and coefficients

The results of the simulation



- Both signal and backgrounds are simulated with MadGraph5_aMC@NLO
- Showered and hadronized by Pythia8
- Use DELPHES version 3.0 to simulate the detector effects with the default card for CEPC.

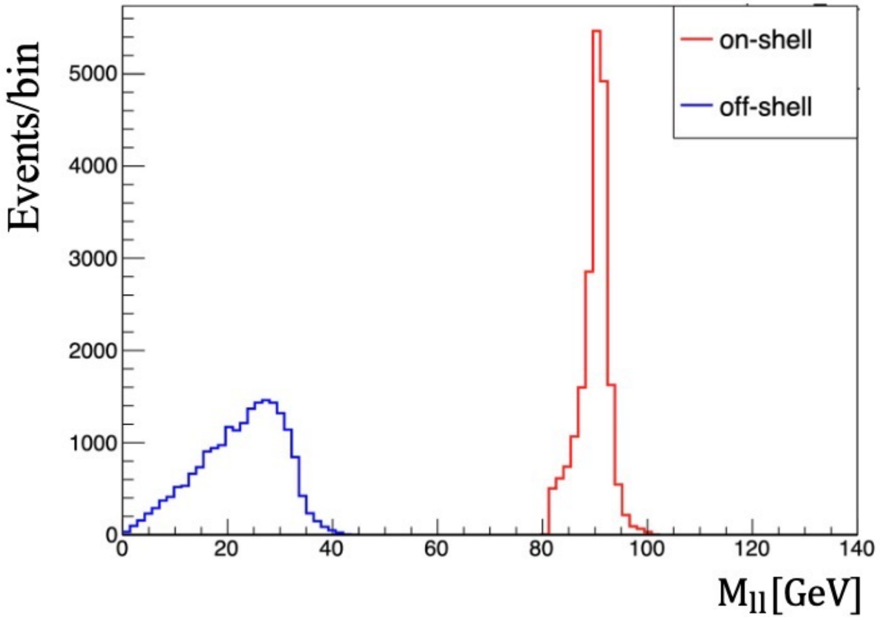
Two final states:

The pure-leptonic state channel:

$$e^+e^- \to ZH, (H \to Z\,\ell^+\ell^-, Z \to \ell^+\ell^-), Z \to \nu_\ell \tilde{\nu}_\ell$$

The semi-leptonic state channel:

$$e^+e^- \to ZH, (H \to Z\,\ell^+\ell^-, Z \to \ell^+\ell^-), Z \to jj$$



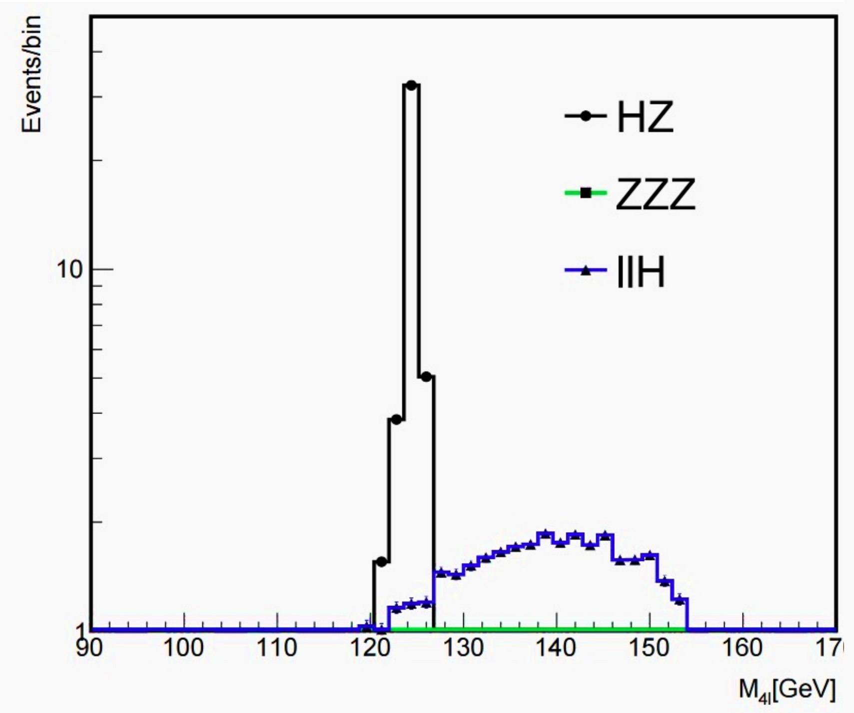
The distributions of variables:

The invariant mass of four leptons in the final states:

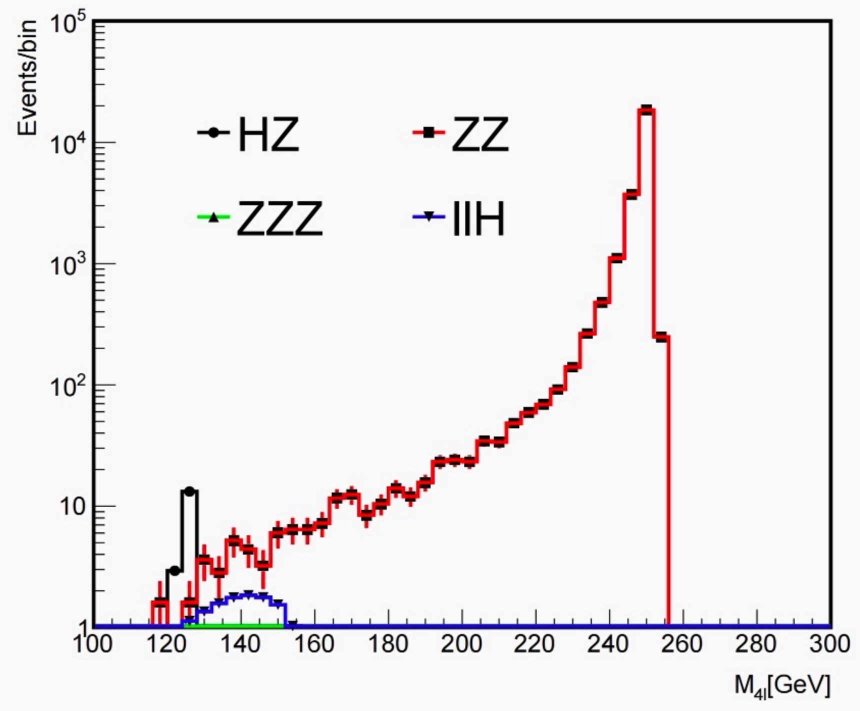
The signal can be separated from backgrounds easily, so we can only use the signal to calculate I_3 and coefficients: C_{212-1}, C_{222-2}.

- If $I_3 \geq 2$: the existence of the violation of Bell inequality.
- If $C_{212-1}, C_{222-2} \neq 0$: the existence of quantum entanglement.

Semi-leptonic channel



Pure-leptonic channel



Selections table

N_leptons	**==4**
N_jets(for semi-leptonic)	**==2**
$p_{T,\ell}$	**> 15GeV**
$p_{T,j}$	**> 10GeV**
$\lvert\eta_\ell\rvert$	**< 2.5**
$\lvert\eta_j\rvert$	**< 5.0**

Final results

- Consider the Luminosity: $\mathcal{L} = 50ab^{-1}$.
- set a series of pseudo-experiments according to the expected number of events.
- Set four different lower mass limits: $M_{Z*} \in [0, 10, 20, 30]GeV$.

The semi-leptonic channel

M_z^* [GeV]	$\mathcal{I}_3$	C_{212-1}	C_{222-2}
0	$2.823 \pm 0.640(1.29\sigma)$	$-1.080 \pm 0.420(2.57\sigma)$	$0.637 \pm 0.559(1.14\sigma)$
10	$2.913 \pm 0.692(1.32\sigma)$	$-1.126 \pm 0.451(2.50\sigma)$	$0.677 \pm 0.598(1.13\sigma)$
20	$3.092 \pm 0.800(1.37\sigma)$	$-1.225 \pm 0.514(2.38\sigma)$	$0.761 \pm 0.734(1.04\sigma)$
30	$3.048 \pm 1.816(0.58\sigma)$	$-1.160 \pm 1.192(0.97\sigma)$	$0.875 \pm 1.338(0.65\sigma)$

The pure-leptonic channel

M_z^*[GeV]	$\mathcal{I}_3$	C_{212-1}	C_{222-2}
0	$2.713 \pm 1.167(0.61\sigma)$	$-1.008 \pm 0.745(1.35\sigma)$	$0.608 \pm 0.931(0.65\sigma)$
10	$2.780 \pm 1.328(0.59\sigma)$	$-1.044 \pm 0.849(1.23\sigma)$	$0.644 \pm 1.038(0.62\sigma)$
20	$2.936 \pm 1.455(0.64\sigma)$	$-1.119 \pm 0.940(1.19\sigma)$	$0.754 \pm 1.083(0.70\sigma)$
30	$3.016 \pm 2.465(0.41\sigma)$	$-1.129 \pm 1.616(0.70\sigma)$	$0.905 \pm 1.617(0.56\sigma)$

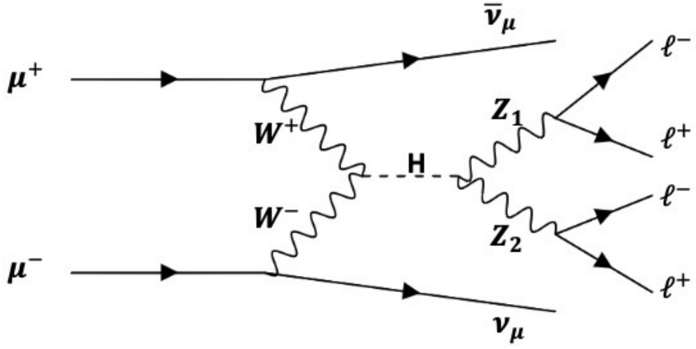
Numerical Simulation------Muon Collider

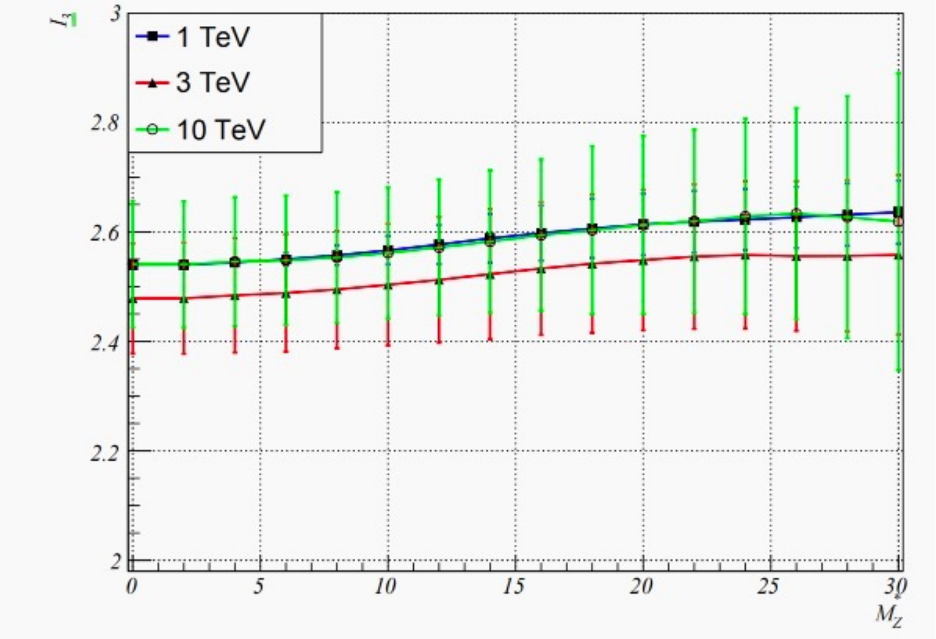
Based on arXiv:2408.05429 (accepted by JHEP)

- **muon–muon collisions are cleaner than proton-proton collisions and thus can lead to higher effective c.m. energy.**
- **muon collider could be much smaller and cheaper than a functionally equivalent proton collider.**
- **massive muons emit much less synchrotron radiation than electrons.**

VBS process for QE:

$$\mu^+\mu^- \to \upsilon_\mu \tilde{\upsilon}_\mu H, H \to ZZ$$





Final results

- Consider the Luminosity: $\mathcal{L} = 30\text{ab}^{-1}$.
- Set a series of pseudo-experiments according to the expected number of events.
- Set three collision energy experiments: $\sqrt{s} \in [1, 3, 10]$TeV, four different lower mass limits: $M_{Z*} \in [0, 10, 20, 30]$GeV.
  - The quantum entanglement can be probed with a significance of around 4σ.
  - The violation of the Bell inequality can be tested up to 2σ level.

$\sqrt{s} = 1$ TeV

M_Z^* (GeV)	I_3	$C_{2,1,2,-1}$	$C_{2,2,2,-2}$
0.000	2.563 ± 0.325	-0.928 ± 0.216	0.527 ± 0.164
10.000	2.596 ± 0.335	-0.943 ± 0.220	0.553 ± 0.179
20.000	2.654 ± 0.373	-0.977 ± 0.248	0.574 ± 0.192
30.000	2.663 ± 0.508	-0.979 ± 0.334	0.589 ± 0.248

$\sqrt{s} = 3$ TeV

M_Z^* (GeV)	I_3	$C_{2,1,2,-1}$	$C_{2,2,2,-2}$
0.000	2.467 ± 0.217	-0.871 ± 0.121	0.493 ± 0.377
10.000	2.499 ± 0.225	-0.891 ± 0.135	0.502 ± 0.390
20.000	2.538 ± 0.254	-0.908 ± 0.163	0.536 ± 0.365
30.000	2.543 ± 0.342	-0.890 ± 0.216	0.606 ± 0.423

$\sqrt{s} = 10$ TeV

M_Z^* (GeV)	I_3	$C_{2,1,2,-1}$	$C_{2,2,2,-2}$
0.000	2.539 ± 0.312	-0.930 ± 0.196	0.466 ± 0.232
10.000	2.569 ± 0.295	-0.946 ± 0.194	0.482 ± 0.217
20.000	2.616 ± 0.321	-0.969 ± 0.218	0.514 ± 0.219
30.000	2.644 ± 0.517	-0.943 ± 0.334	0.527 ± 0.280

Summary and Outlook

- **We have finished a complete simulation analysis about Testing Bell Inequalities and Probing Quantum Entanglement through H → ZZ at CEPC.**
  - ✓ Consider two final states and corresponding backgrounds.
  - ✓ Obtain variable distributions to determine using signal to calculate I_3 and coefficients: C_{212-1}, C_{222-2}.
  - ✓ The quantum entanglement can be measured with a significance up to 2σ in the semi-leptonic signal channel and 1σ in the pure-leptonic signal channel.
  - ✓ The significance of the Bell inequality violation can be probed up to 1σ in semi-leptonic channel.

- **In the future, more work about testing Bell inequalities and probing quantum entanglement at CEPC:**

$\boldsymbol{H \rightarrow W^{+}W^{-}}$**,……**

Thanks for your attention!

Back-up

The detectors can't measure the spin:

No direct spin measurement: inferred by angular distributions

- while momenta (exp observables) is -> possible to construct LHVT (Local Hidden Variables Theory) to mimic all observables
- ECAL is not able to measure photon spin; a new dedicated ECAL in the future may.
- Testing BI to test QM's consistently
- It is not to test QM fundamentals, however, instead, one can do **Quantum Tomography @ collider**

The significance is not high:

- It's a new topic in collider physics; we will do more research in CEPC. QE is a new idea which provides a new variables to collider physics.
- Calculate Quantum behaviors, get Quantum information...
- BSM may affect production or decay。
- Higher energy of CEPC,,,,,,in SppC