EW corrections at future colliders

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SEZIONE DI BOLOGNA

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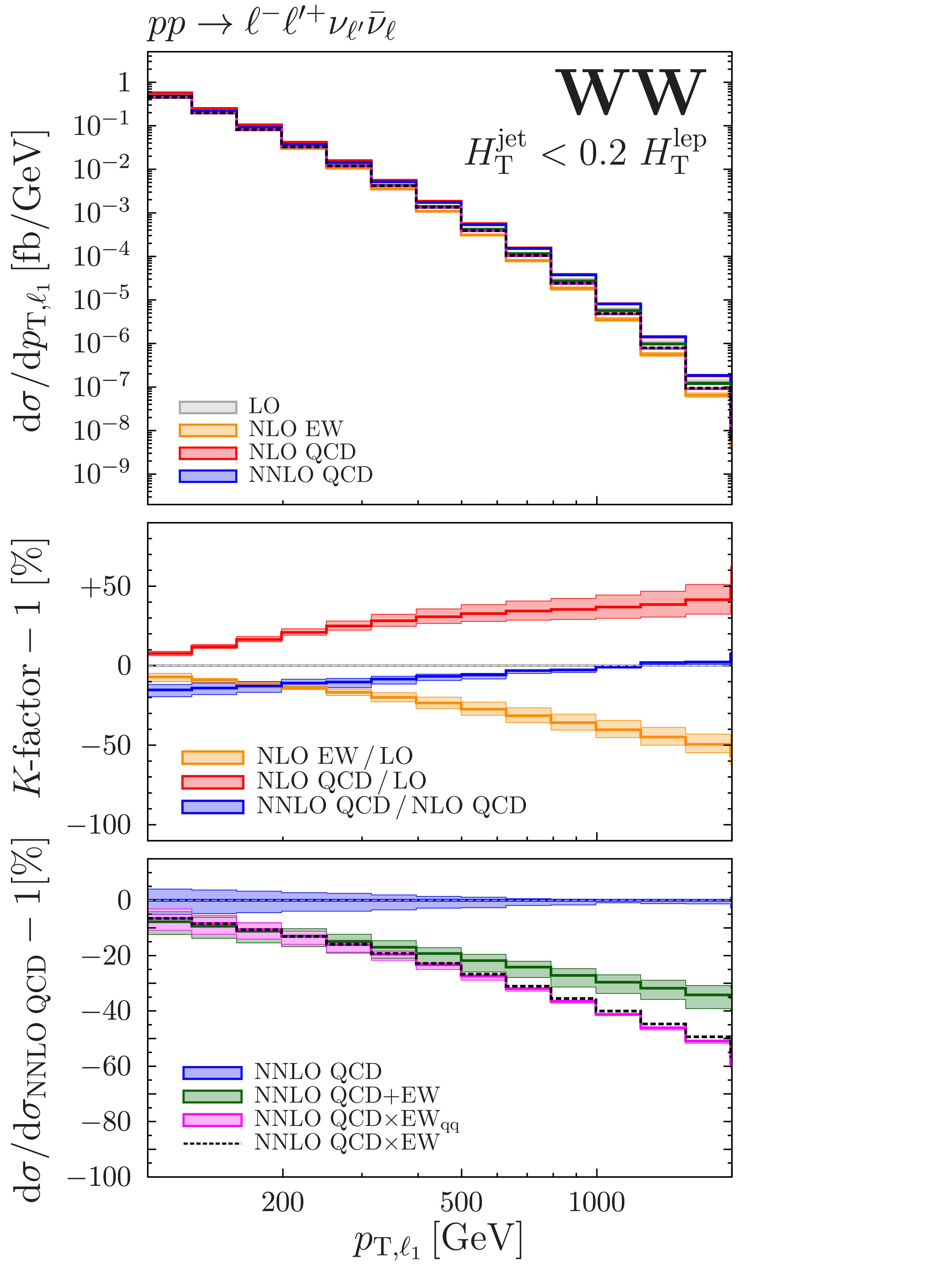
EW corrections are already relevant at the LHC: an example



Grazzini, Kallweit, Lindert, Pozzorini, Wiesemann '19

$$
\begin{aligned}
\Delta\mathcal{L}_{\rm BSM} = {} & \delta g^Z_{uL}\left[Z^\mu \bar{u}_L\gamma_\mu u_L + \frac{\cos\theta_W}{\sqrt{2}}(W^{+\mu}\bar{u}_L\gamma_\mu d_L + \text{h.c.}) + \ldots\right] + \delta g^Z_{uR}\left[Z^\mu \bar{u}_R\gamma_\mu u_R\right] \\
& + \delta g^Z_{dL}\left[Z^\mu \bar{d}_L\gamma_\mu d_L - \frac{\cos\theta_W}{\sqrt{2}}(W^{+\mu}\bar{u}_L\gamma_\mu d_L + \text{h.c.}) + \ldots\right] + \delta g^Z_{dR}\left[Z^\mu \bar{d}_R\gamma_\mu d_R\right] \\
& + ig\cos\theta_W\delta g^Z_1\left[Z^\mu(W^{+\nu}W^-_{\mu\nu} - \text{h.c.}) + Z^{\mu\nu}W^+_\mu W^-_\nu + \ldots\right] \\
& + ie\delta\kappa_\gamma[(A_{\mu\nu} - \tan\theta_W Z_{\mu\nu})W^{+\mu}W^{-\nu} + \ldots],
\end{aligned}
$$

Banerjee, Reichelt, Spannowsky '24

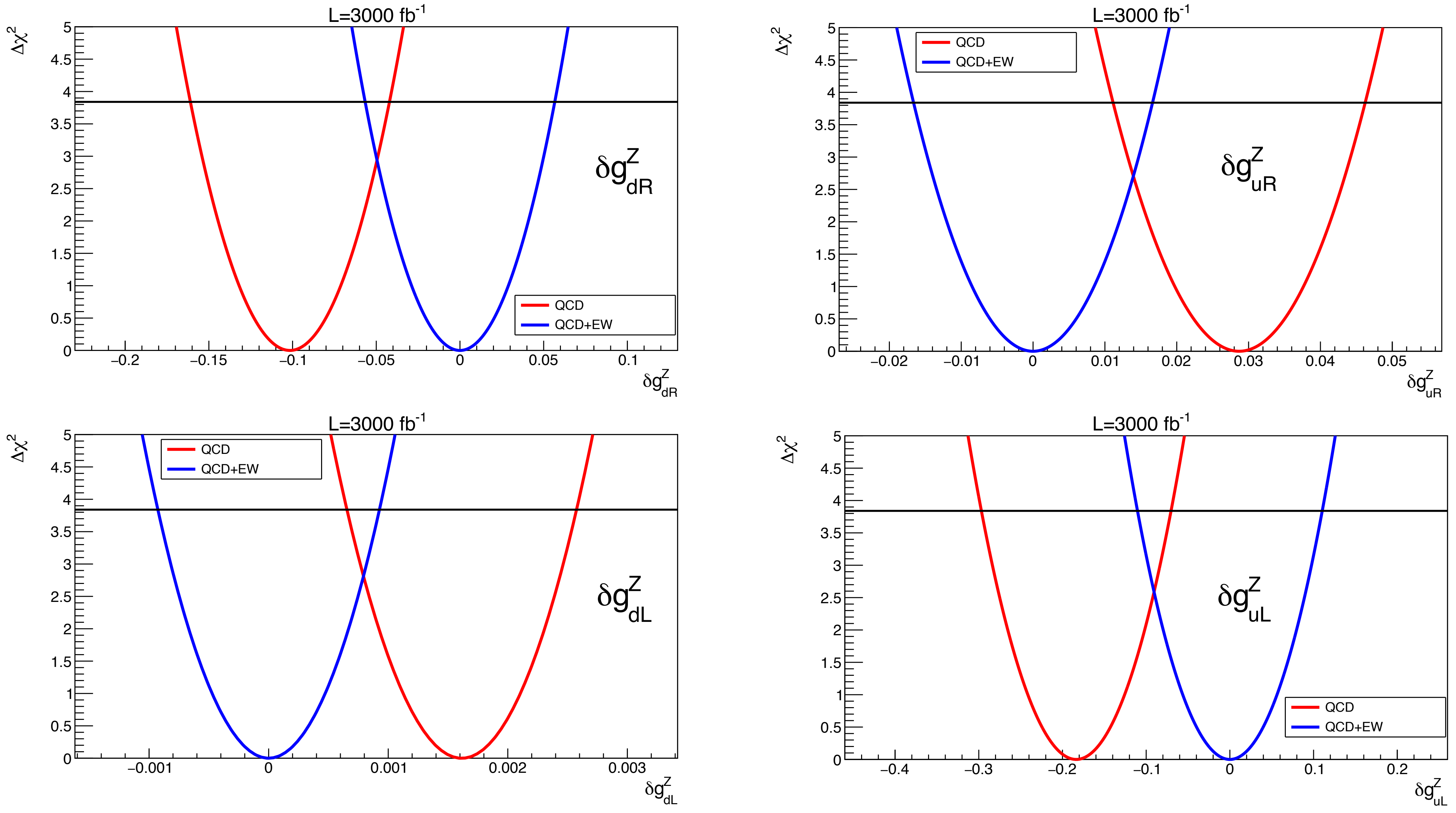


Figure 3: Comparison of the one-dimensional bounds at 95% C.L. between theory assumptions of SM@(NLO QCD) + SMEFT@LO and SM@(NLO QCD + approximate NLO EW) + SMEFT@LO. The expectation assumption is the same, SM@(NLO QCD + approximate NLO EW) for both theory assumptions. The other parameters have not been marginalised over and we consider $\Delta\chi^2 = 3.84$ for a one-parameter χ^2 fit to present our allowed regions.

Future Colliders vs. EW corrections

(1) Higher Precision

EW corrections become even more relevant. NLO EW is not sufficient and higher orders are necessary.

(2) Higher Energy

EW corrections become larger (Sudakov).

EW corrections become relevant both for signal/bkg or BSM/SM, not only for precision studies.

For Precision: see **(1)**.

Future Colliders vs. EW corrections

(1) Higher Precision

EW corrections become even more relevant. NLO EW is not sufficient and higher orders are necessary.

EW is the new **QCD**

EW corrections cannot be neglected at futures collider.

(2) Higher Energy

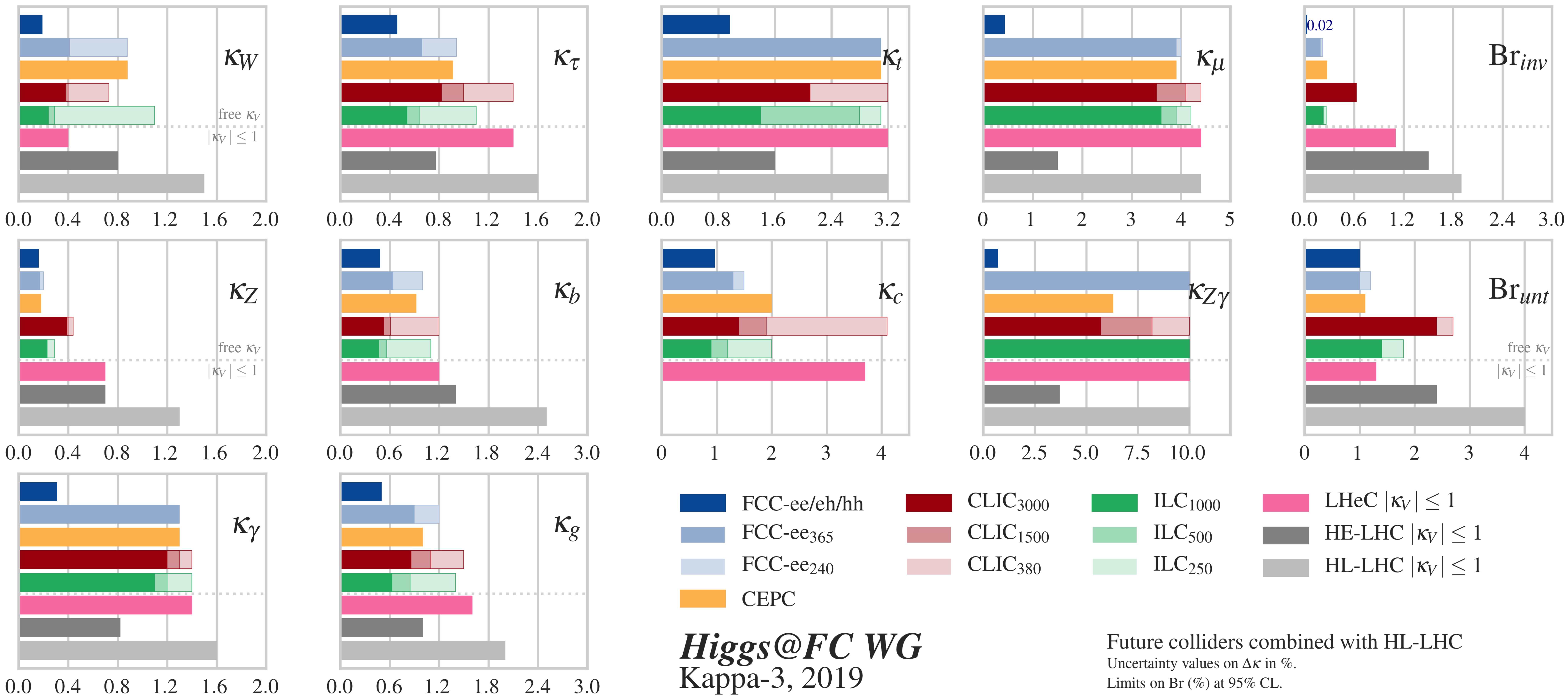
EW corrections become larger (Sudakov).

EW corrections become relevant both for signal/bkg or BSM/SM, not only for precision studies.

For Precision: see **(1)**.

Higher Precision

Precision: the Higgs case

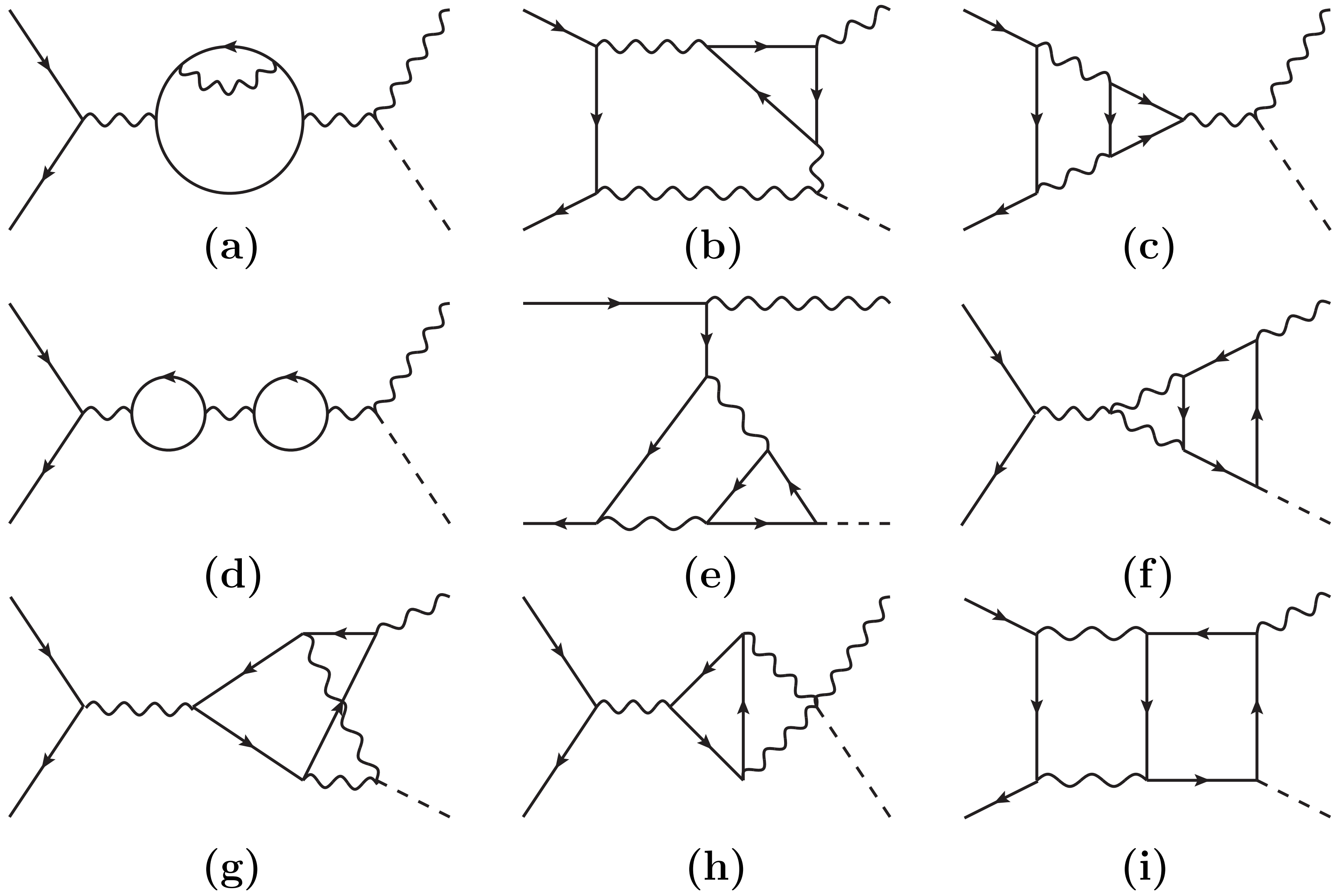


Sub-percent precision is expected and so NLO EW is not enough.

$e^+e^- \to ZH$: NNLO EW $\mathcal{O}(\alpha^2)$

Contributions from fermion loops at NNLO

Freitas, Song '22
Freitas, Song, Xie '24



		$\alpha(0)$ scheme		G_μ scheme
σ^{LO} [fb]		222.96		239.18
σ^{NLO} [fb]	+3.1%	229.89	-2.9%	232.08
σ^{NNLO} [fb]	+0.7%	231.55	+0.3%	232.74
$\mathcal{O}(\alpha^2_{N_f=2})$		1.88		0.73
$\mathcal{O}(\alpha^2_{N_f=1})$		-0.23		-0.07

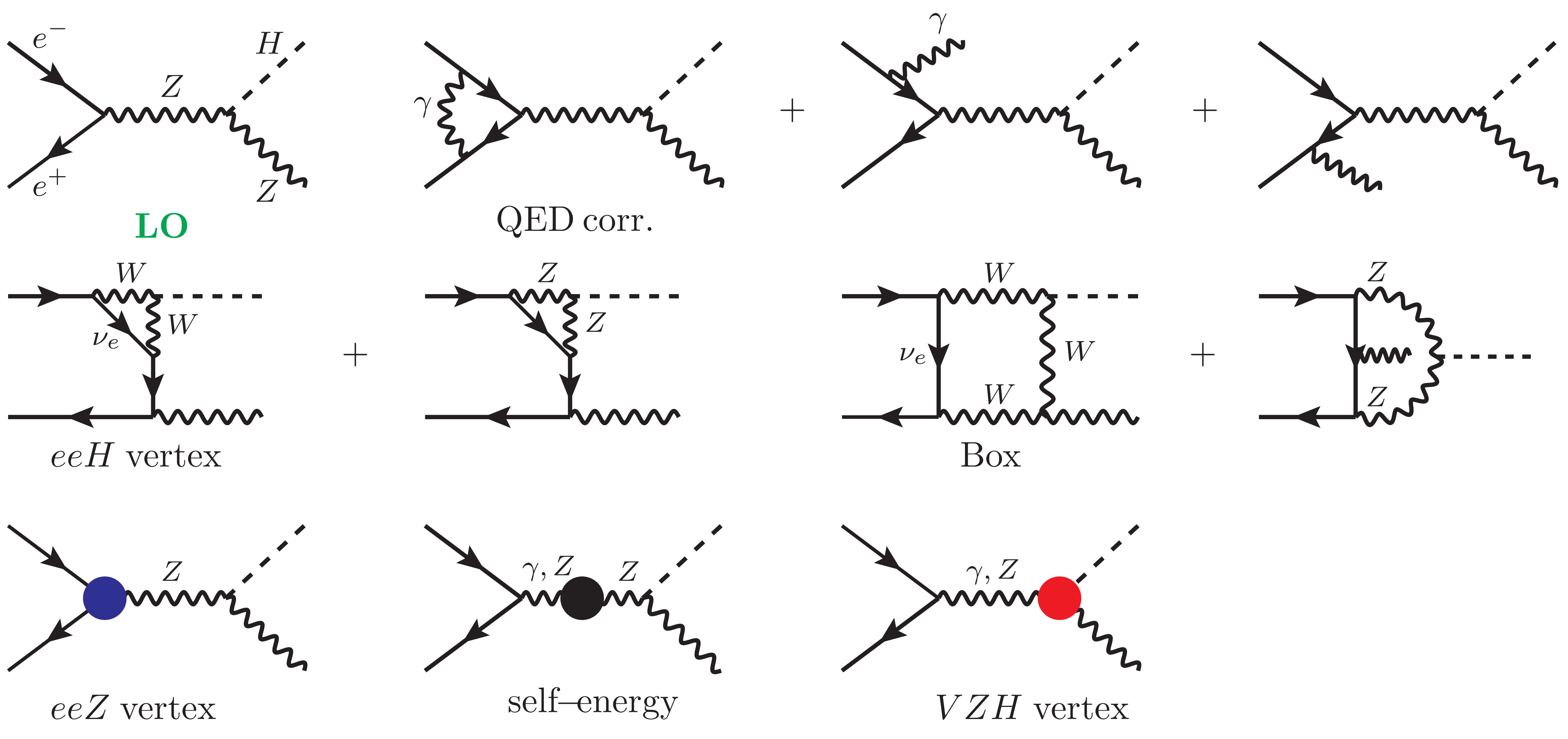
$\sqrt{S} = 240$ GeV

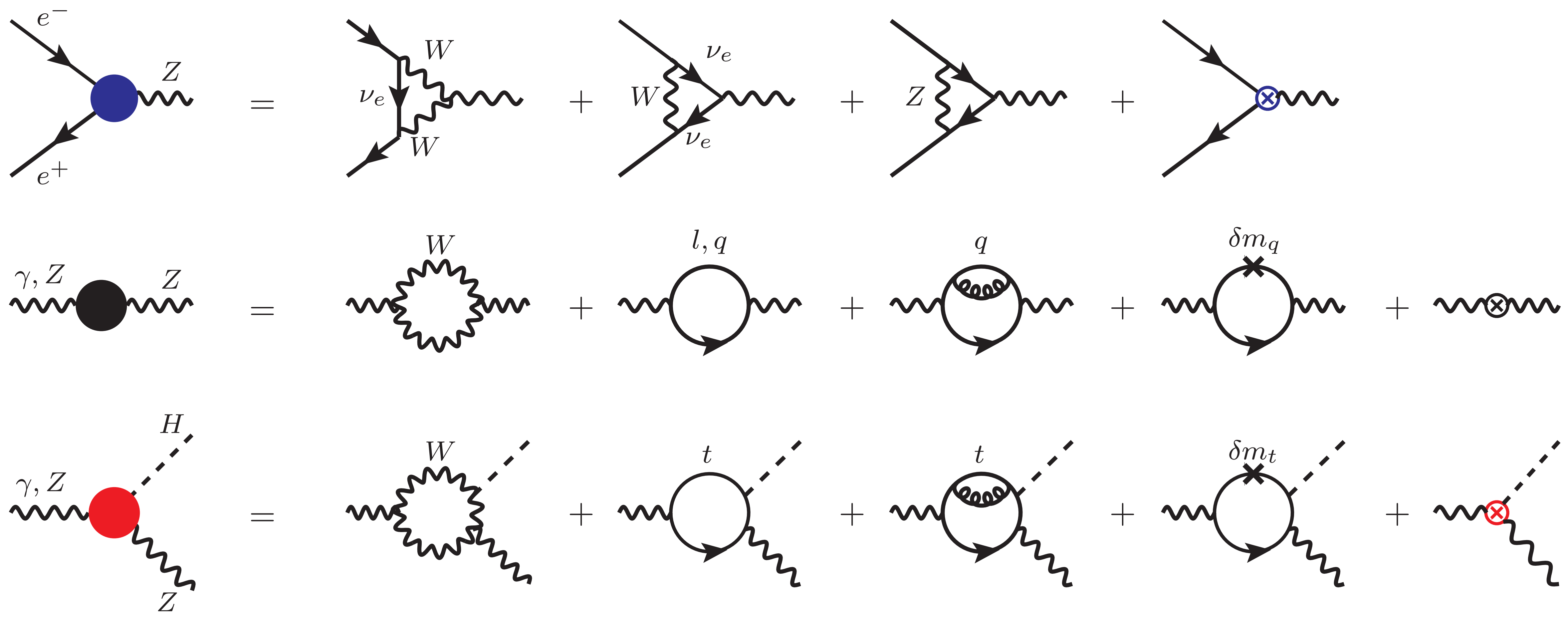
Small corrections, but necessary for matching the experimental precision.

The choice of the renormalisation scheme is relevant.

This is only one of the effects: e.g. PDFs and ISR are not taken into account.

$e^+e^- \to ZH$: mixed NNLO QCD-EW $\mathcal{O}(\alpha_s\alpha)$





Gong, Li, Xu, Yang, Zhao '17

Sun, Feng, Jia, Sang '24

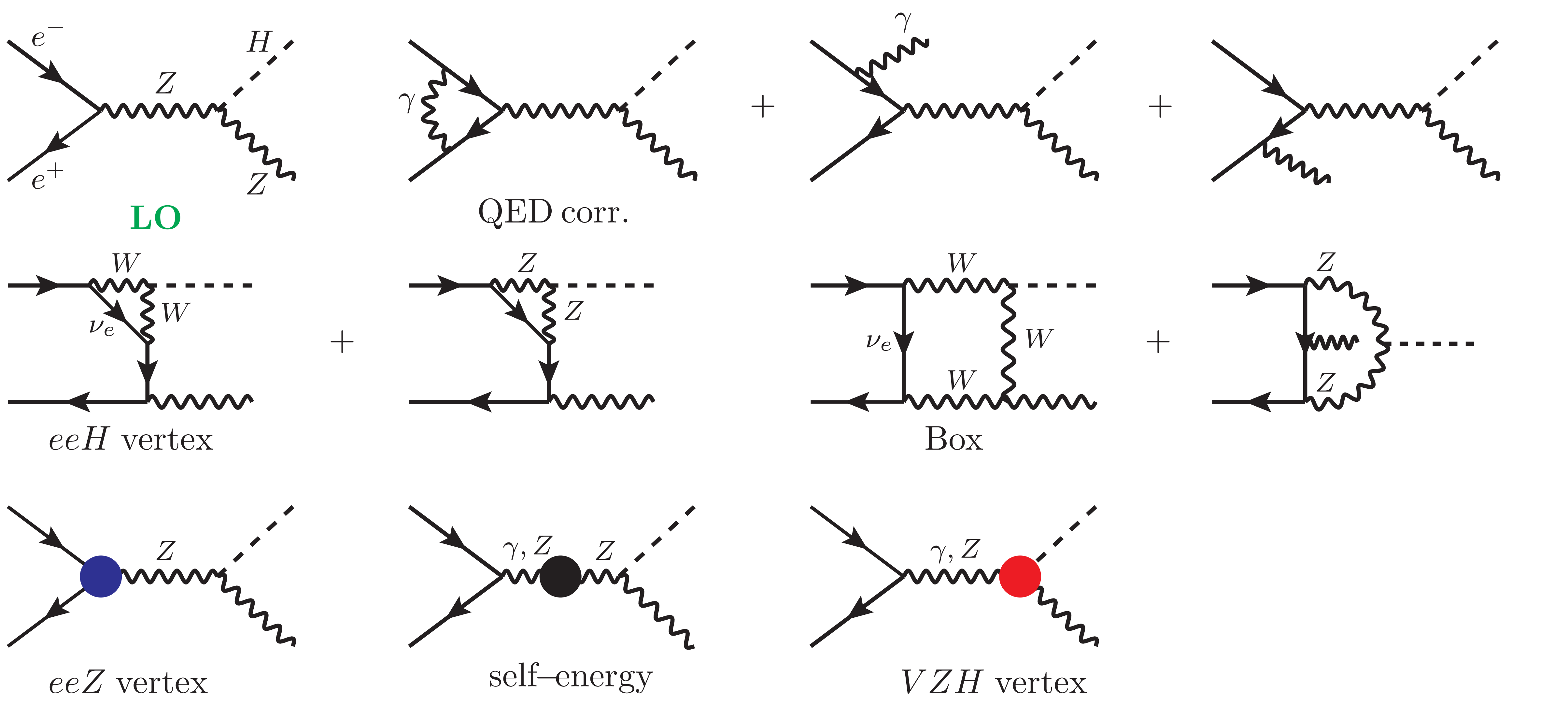
TABLE I. Total cross sections at various collider energies in the $\overline{\rm MS}$ scheme.

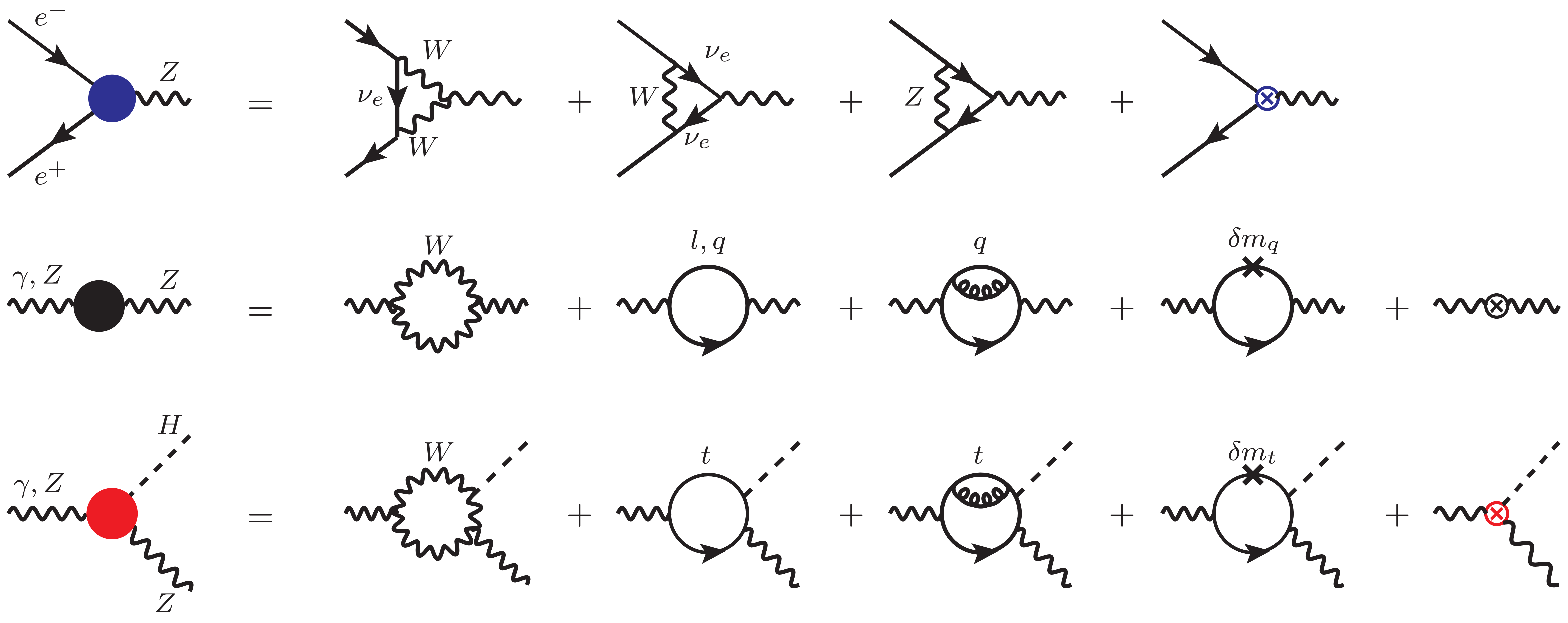
	$\sqrt{s}$ (GeV)	$\sigma_{\rm LO}$ (fb)	$\sigma_{\rm NLO}$ (fb)	$\sigma_{\rm NNLO}$ (fb)	$\sigma^{\rm exp}_{\rm NNLO}$ (fb)
+1.1%	240	256.3(9)	228.0(1)	230.9(4)	230.9(4)
+1.1%	250	256.3(9)	227.3(1)	230.2(4)	230.2(4)
+1.1%	300	193.4(7)	170.2(1)	172.4(3)	172.4(3)
+1.3%	350	138.2(5)	122.1(1)	123.9(2)	123.6(2)
+0.6%	500	61.38(22)	53.86(2)	54.24(7)	54.64(10)

TABLE II. Total cross sections at various collider energies in the $\alpha(m_Z)$ scheme.

	$\sqrt{s}$ (GeV)	$\sigma_{\rm LO}$ (fb)	$\sigma_{\rm NLO}$ (fb)	$\sigma_{\rm NNLO}$ (fb)	$\sigma^{\rm exp}_{\rm NNLO}$ (fb)
+1.1%	240	252.0	228.6	231.5	231.5
+1.1%	250	252.0	227.9	230.8	230.8
+1.1%	300	190.0	170.7	172.9	172.9
+1.3%	350	135.6	122.5	124.2	124.0
+0.6%	500	60.12	54.03	54.42	54.81

$e^+e^- \to ZH$: mixed NNLO QCD-EW $\mathcal{O}(\alpha_s\alpha)$





Gong, Li, Xu, Yang, Zhao '17
Sun, Feng, Jia, Sang '24

TABLE III. Convergence of the $1/m_t^2$ expansion for the mixed QCD-EW corrections in the $\overline{\text{MS}}$ scheme with $\mu = \sqrt{s}/2$.

$\sqrt{s}$ (GeV)	$\mathcal{O}(m_t^2)$	$\mathcal{O}(m_t^0)$	$\mathcal{O}(m_t^{-2})$	$\mathcal{O}(m_t^{-4})$
240	81.8%	16.2%	1.4%	0.4%
250	81.7%	16.1%	1.5%	0.5%
300	80.0%	15.2%	2.1%	1.1%
350	69.7%	12.6%	2.7%	2.1%
500	137%	18.6%	17.3%	31.1%

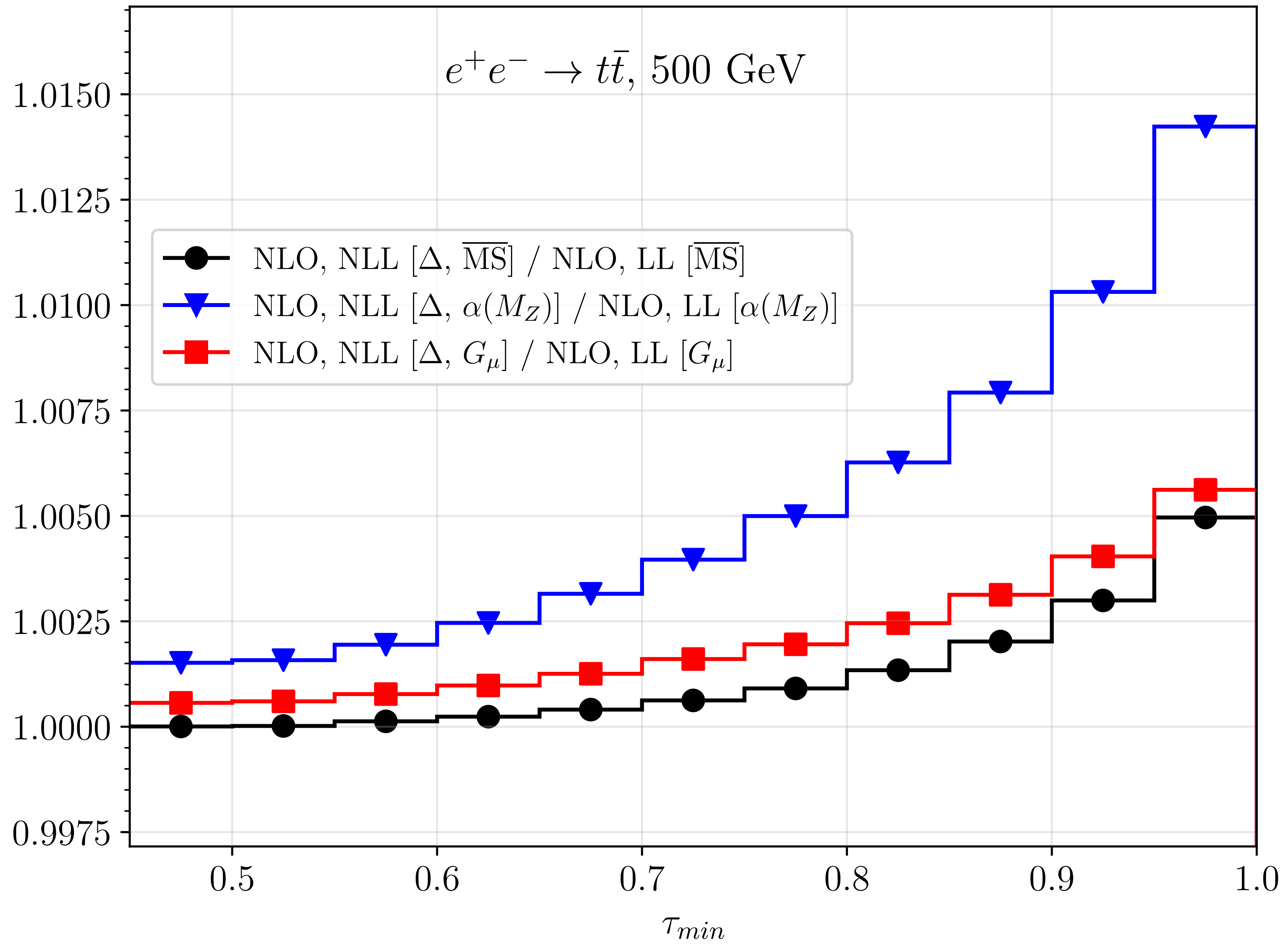
Expansion in powers of m_t works well only for energies up to $2m_t$

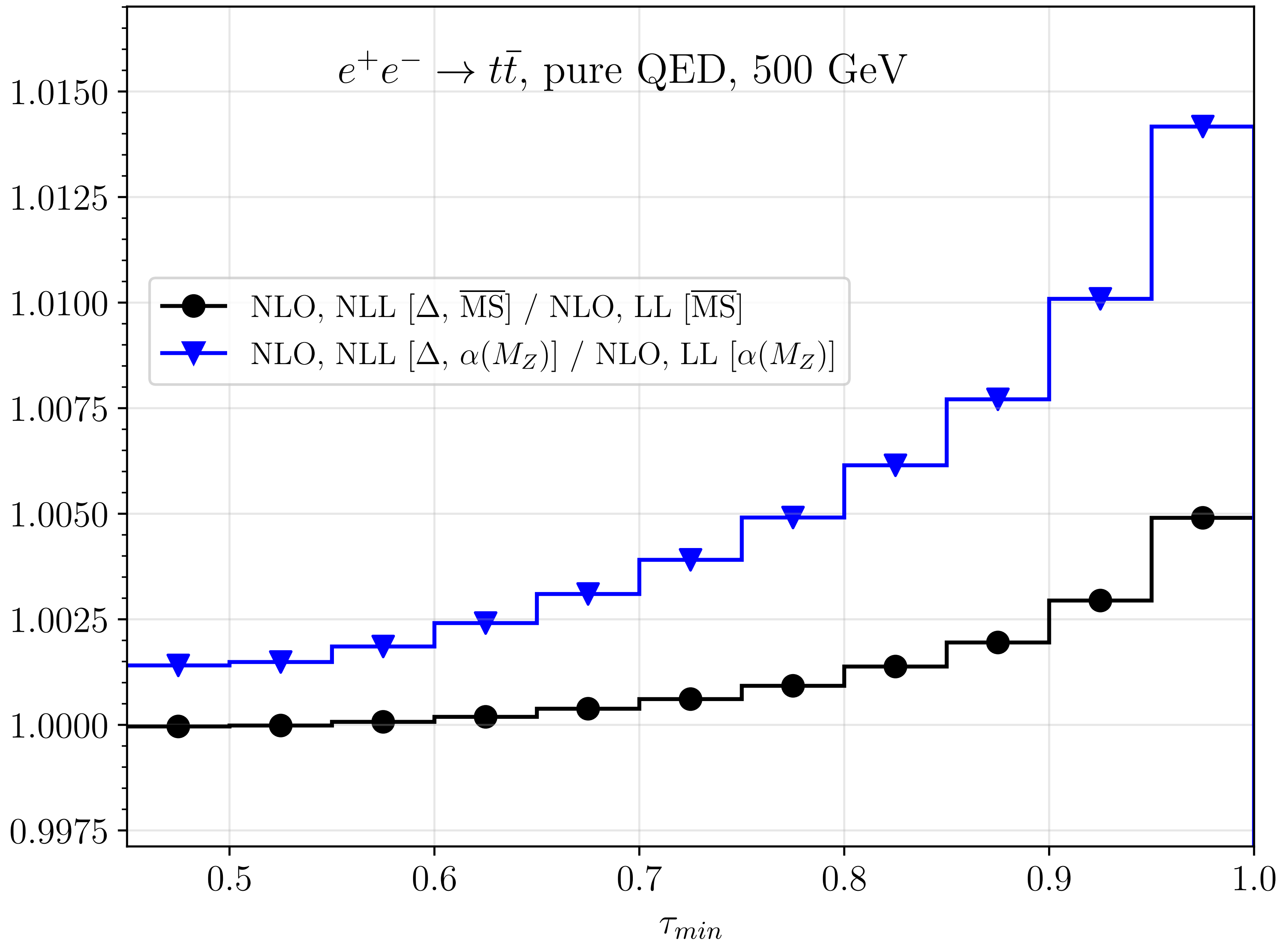
Small but necessary for matching the experimental precision.

PDFs at NLL accuracy in QED

Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao '22
Bertone, Cacciari, Frixione, Stagnitto '19
Frixione '19

$$\sigma(\tau_{min}) = \int d\sigma\, \Theta\left(\tau_{min} \leq \frac{M^2_{p\bar{p}}}{s}\right), \qquad p = q\,, t\,, W^+$$





ISR radiation resummed within the PDFs of e in the e.

NLO EW with NLL and LL PDFs can have ~ 1% differences: **small but necessary for matching the 1% precision.**
Renormalisation scheme relevant too, factorisation scheme less relevant for x-sections.

PDFs at NLL accuracy in QED

Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao '22
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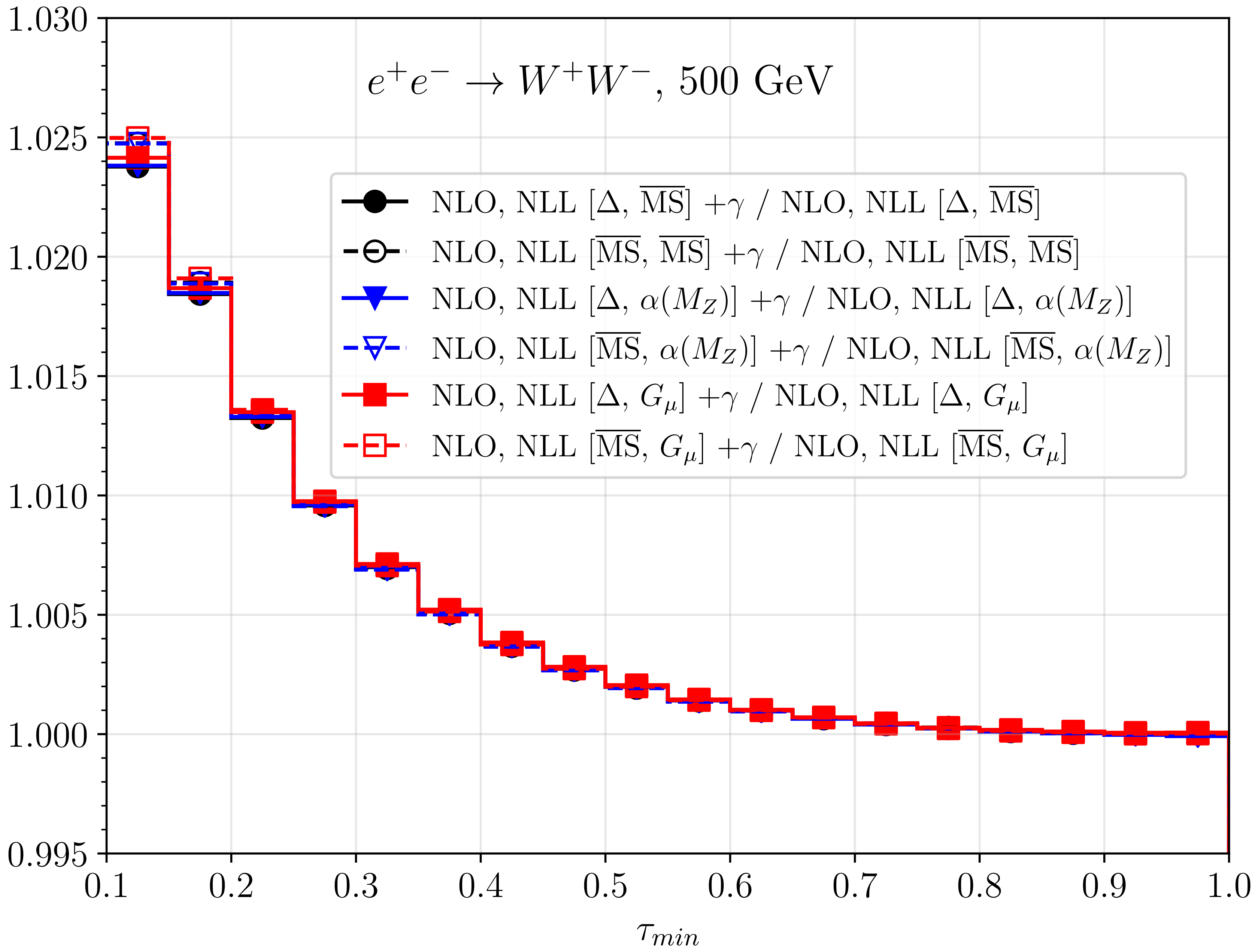
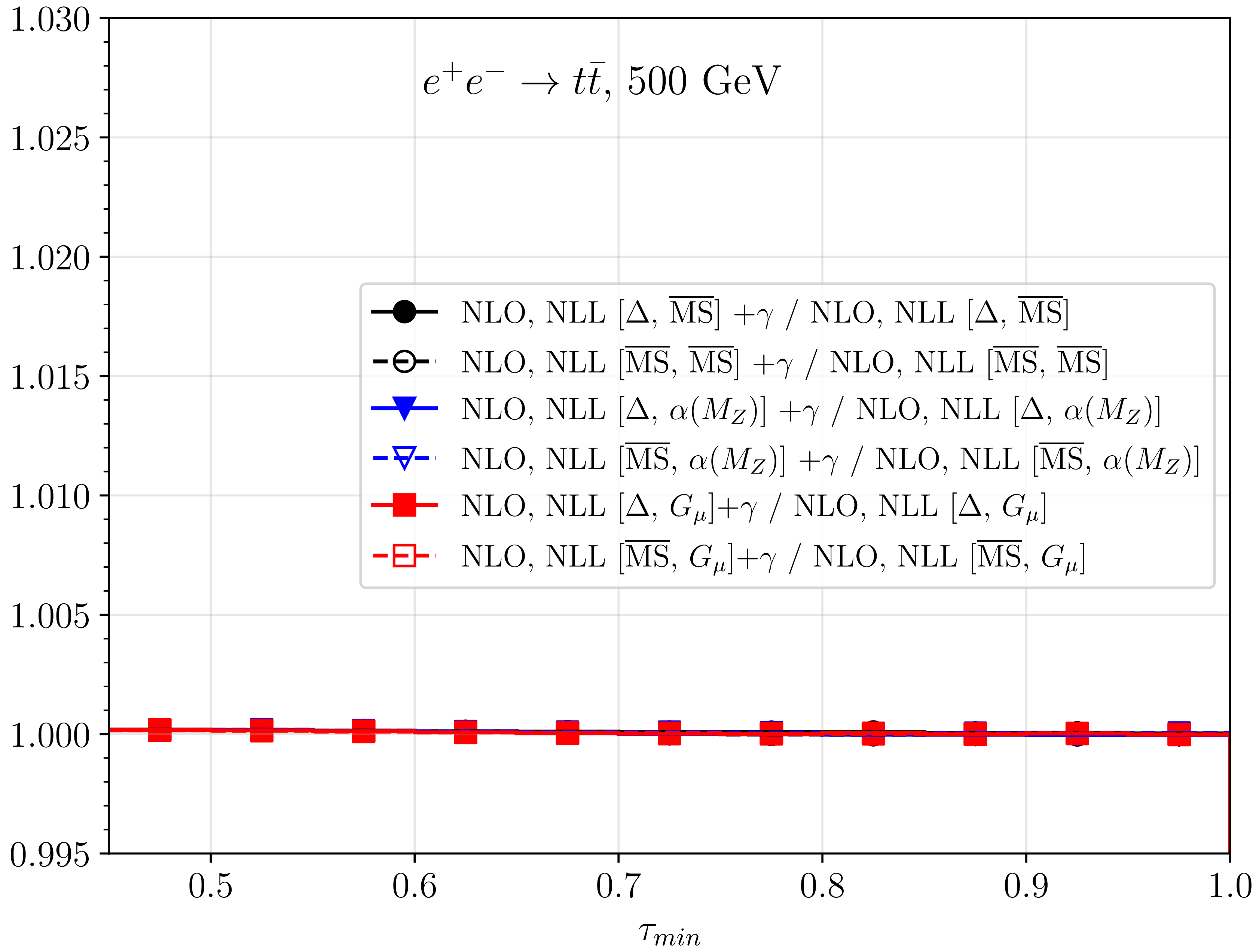




ISR radiation resummed within the PDFs of e in the e. **But γ in the e too!**

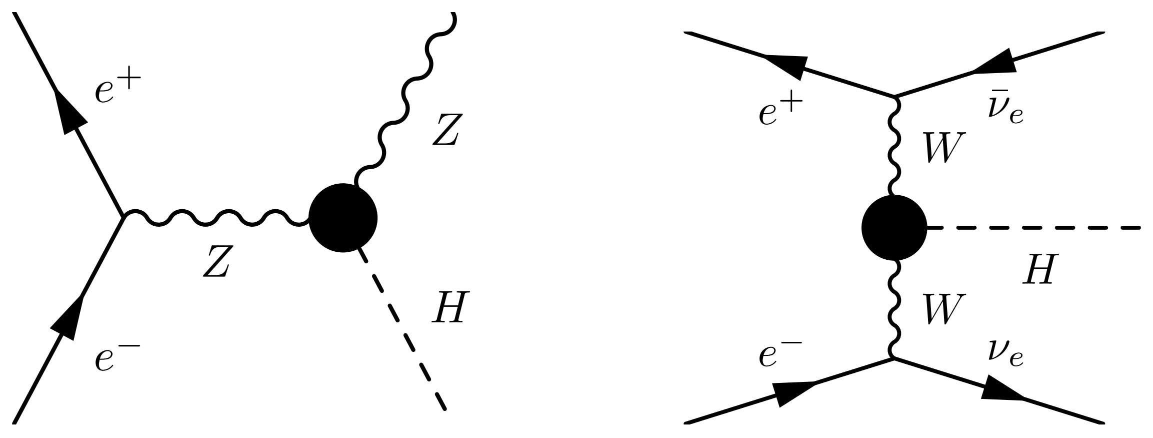
Regardless of the scheme used, photon initiated processes can be relevant: **small but necessary for matching the 1% precision.**

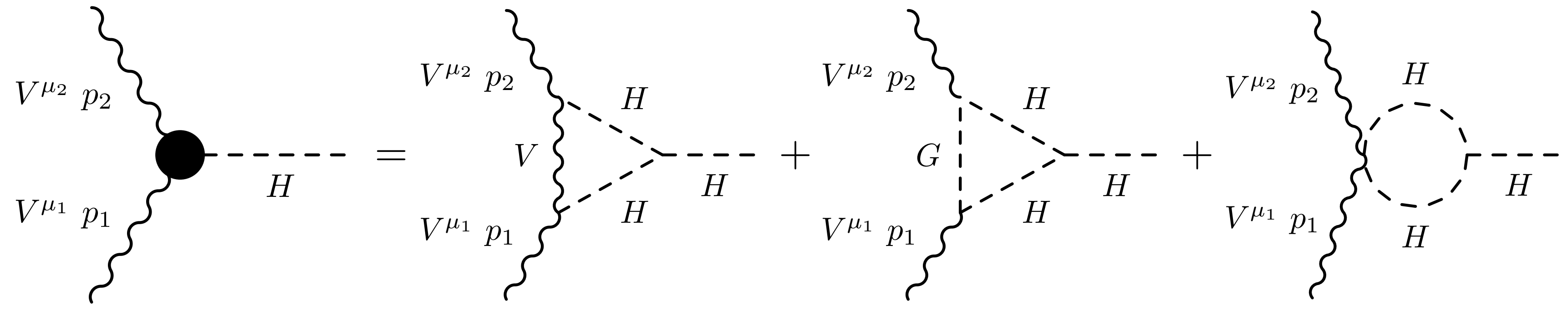
NLO EW as a probe of new physics: H self couplings

Process	λ_3	λ_4
ZH, $\nu_e\bar{\nu}_e H$ (WBF)	one-loop	~~two-loop~~
ZHH, $\nu_e\bar{\nu}_e HH$ (WBF)	tree	one-loop
$ZHHH$, $\nu_e\bar{\nu}_e HHH$ (WBF)	tree	tree

$$V(\Phi) = V^{\rm SM}(\Phi) + V^{\rm NP}(\Phi) \qquad V^{\rm NP}(\Phi) \equiv \sum_{n=3}^{\infty} \frac{c_{2n}}{\Lambda^{2n-4}} \left(\Phi^\dagger \Phi - \frac{1}{2}v^2\right)^n$$

HHH from single-H at one loop

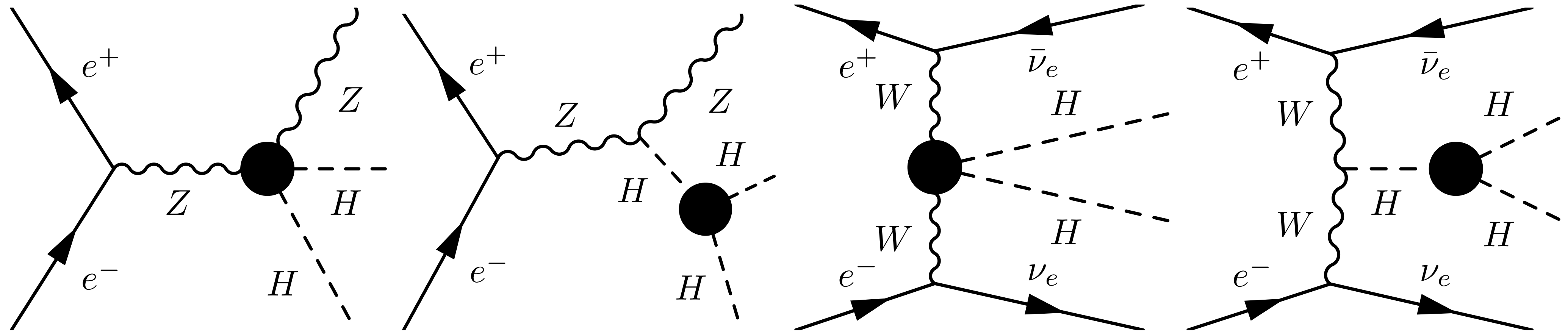


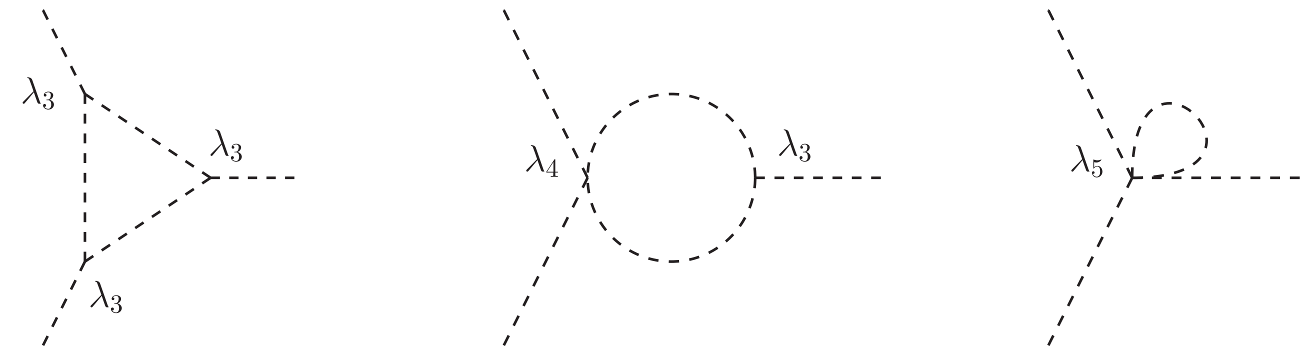


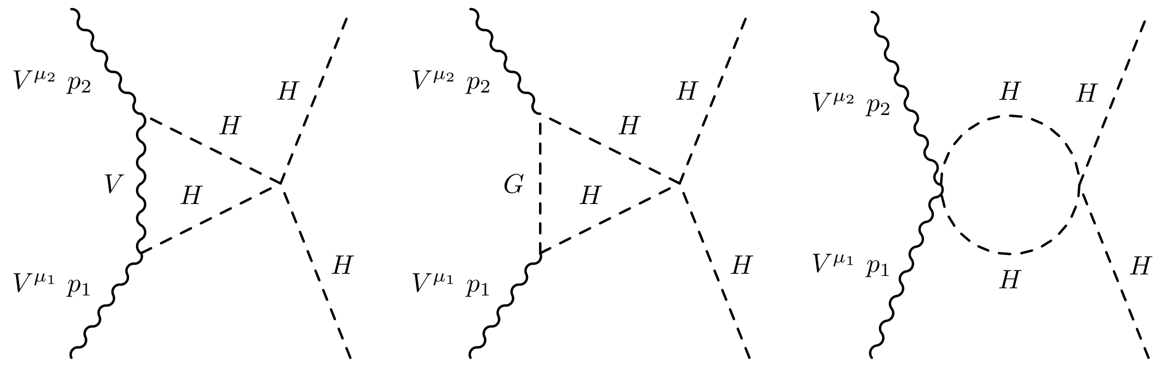
$$\kappa_3 \equiv \frac{\lambda_3}{\lambda_3^{\rm SM}} = 1 + \frac{c_6 v^2}{\lambda \Lambda^2} \equiv 1 + \bar{c}_6,$$

Maltoni, DP, Zhao '18

HHHH from double-H at one loop







$$\kappa_4 \equiv \frac{\lambda_4}{\lambda_4^{\rm SM}} = 1 + \frac{6 c_6 v^2}{\lambda \Lambda^2} + \frac{4 c_8 v^4}{\lambda \Lambda^4} \equiv 1 + 6\bar{c}_6 + \bar{c}_8$$

HHH from single Higgs

	$\sqrt{\hat{s}}$ [GeV]	process	ϵ [%]	C_1 [%]	$\bar{c}_6(\pm 1\sigma)$	$\bar{c}_6(\pm 2\sigma)$
CEPC	250	ZH	0.51	1.6	$(-0.38, 0.42) \cup$ ~~$(8.0, 8.8)$~~	$(-0.73, 0.88) \cup$ ~~$(7.5, 9.1)$~~
FCC-ee	240	ZH	0.4	1.8	$(-0.26, 0.28) \cup$ ~~$(9.4, 9.9)$~~	$(-0.51.0.57) \cup$ ~~$(9.1, 10.2)$~~
	240	WBF H	2.2	0.66	$(-2.81, 5.1)$	$(-4.3, 6.6)$
	350	WBF H	0.6	0.65	$(-1.15, 3.4)$	$(-1.89, 4.1)$
ILC	250	ZH	0.71	1.6	$(-0.52, 0.59) \cup$ ~~$(7.8, 8.9)$~~	$(-0.98, 1.3) \cup$ ~~$(7.1, 9.4)$~~
	500	WBF H	0.23	0.63	$(-0.56, 2.7)$	$(-0.97, 3.1)$
	1000	WBF H	0.33	0.61	$(-0.78, 2.7)$	$(-1.3, 3.3)$
CLIC	350	ZH	1.65	0.59	$(-2.48, 4.3)$	$(-3.80, 5.6)$
	1400	WBF H	0.4	0.61	$(-0.91, 2.9)$	$(-1.50, 3.5)$
	3000	WBF H	0.3	0.59	$(-0.75, 2.6)$	$(-1.26, 3.1)$

$$\sigma_{\rm NLO}^{\rm pheno}(H) = \sigma_{\rm LO} + \sigma_1 \bar{c}_6 + \sigma_2 \bar{c}_6^2 \,.$$

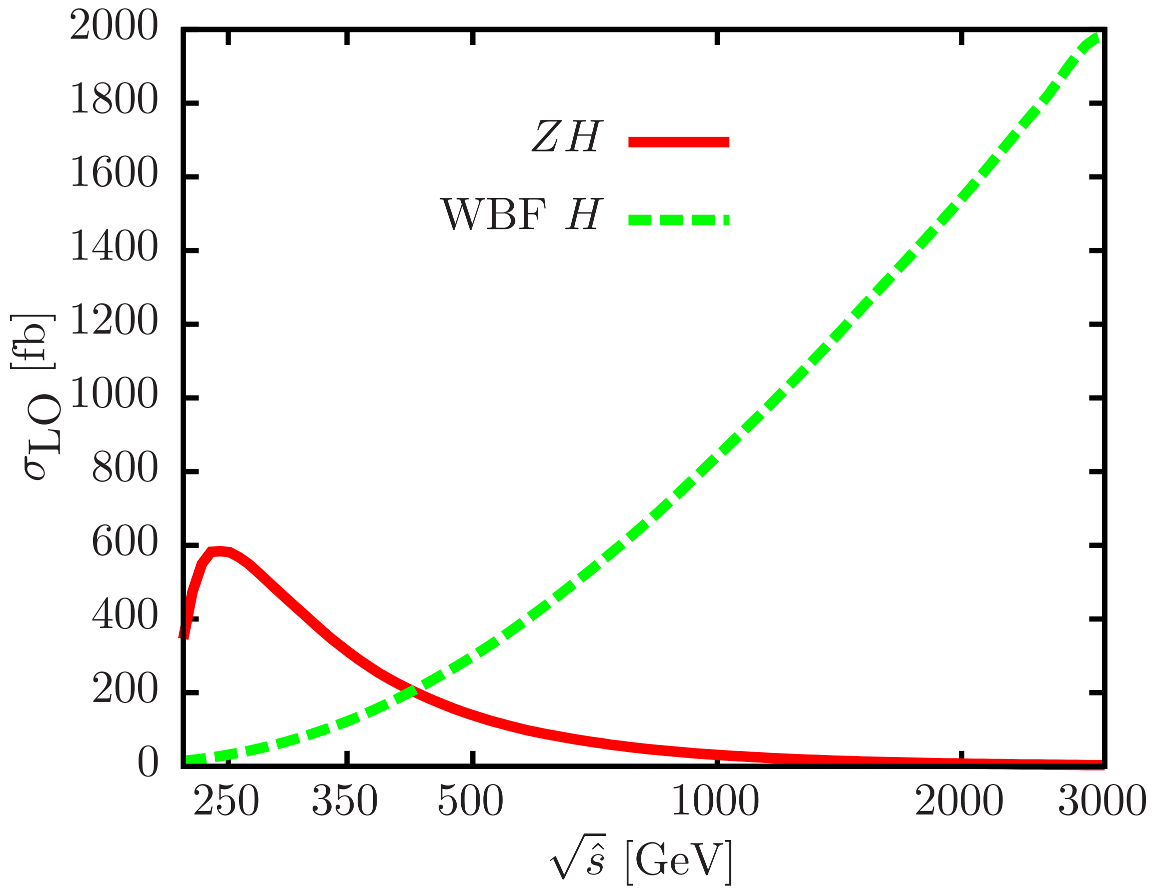
$$\delta\sigma(H) \equiv \frac{\sigma_{\rm NLO}^{\rm pheno} - \sigma_{\rm LO}}{\sigma_{\rm LO}} = \frac{\sigma_1 \bar{c}_6 + \sigma_2 \bar{c}_6^2}{\sigma_{\rm LO}} = (\kappa_3 - 1) C_1 + (\kappa_3^2 - 1) C_2 \,,$$

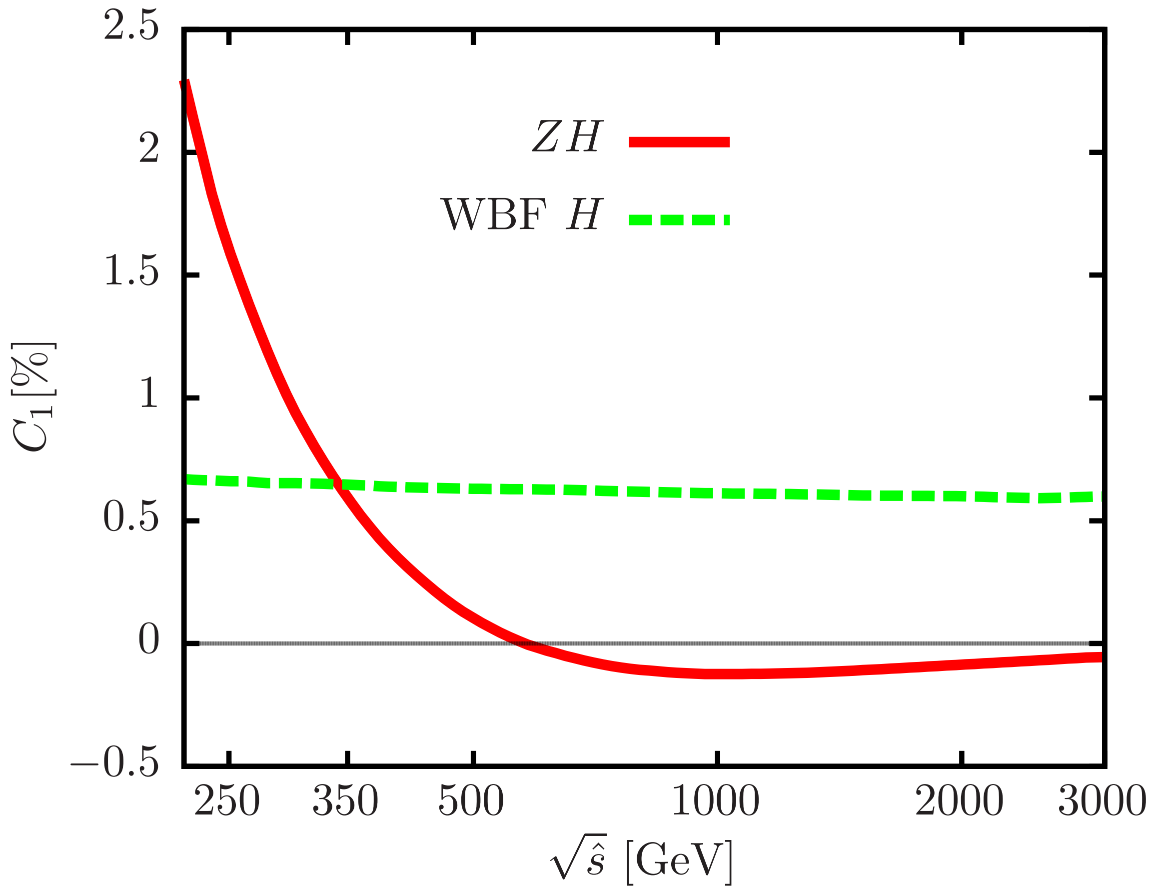
$$C_2 = \delta Z_H^{{\rm SM},\lambda} \,,$$

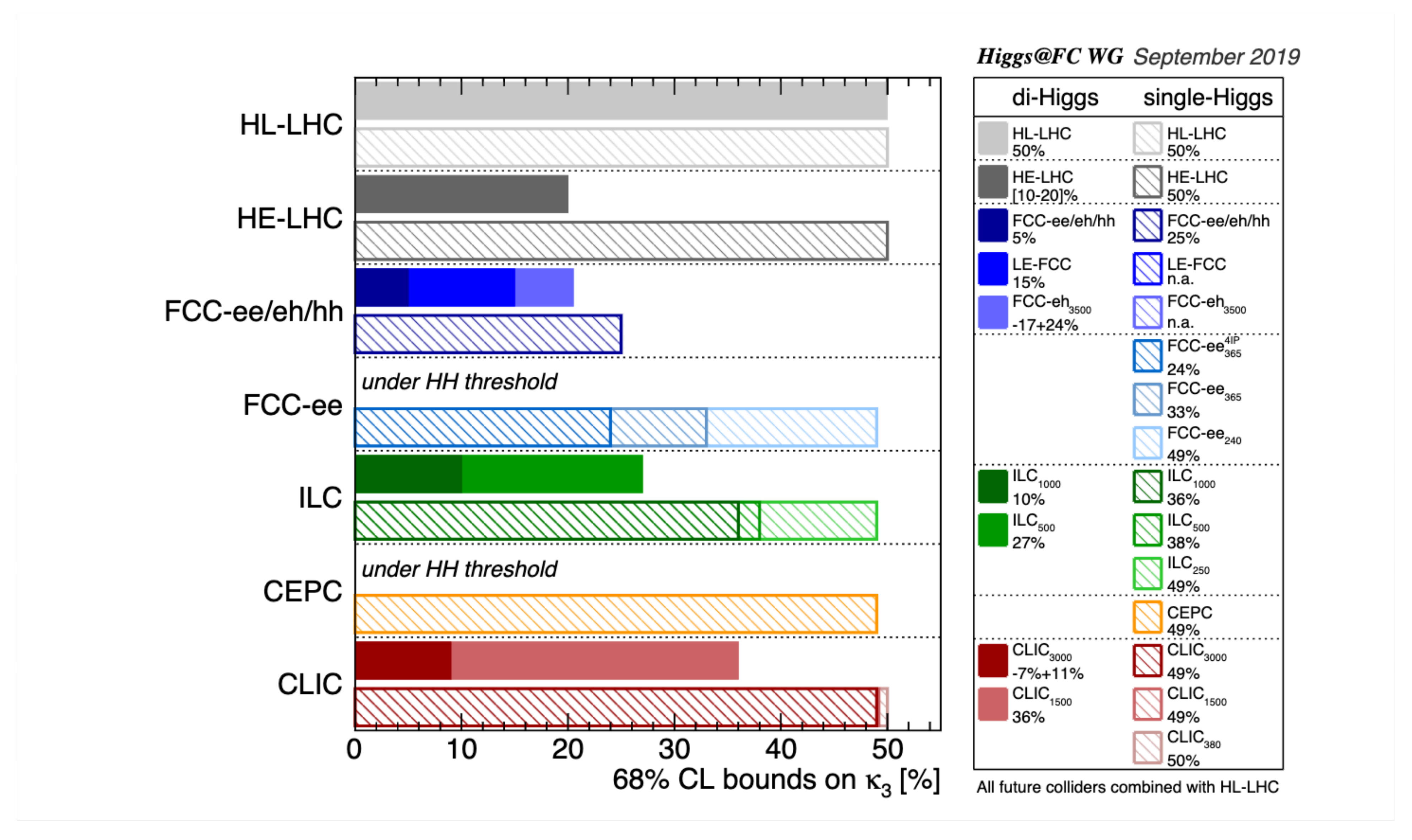
Maltoni, DP, Zhao '18

see also McCullough '13

Lower energy, ZH larger and C_1 too.







NLO EW and SMEFT in ZH

Asteriadis, Dawson, Giardino, Szafron '24

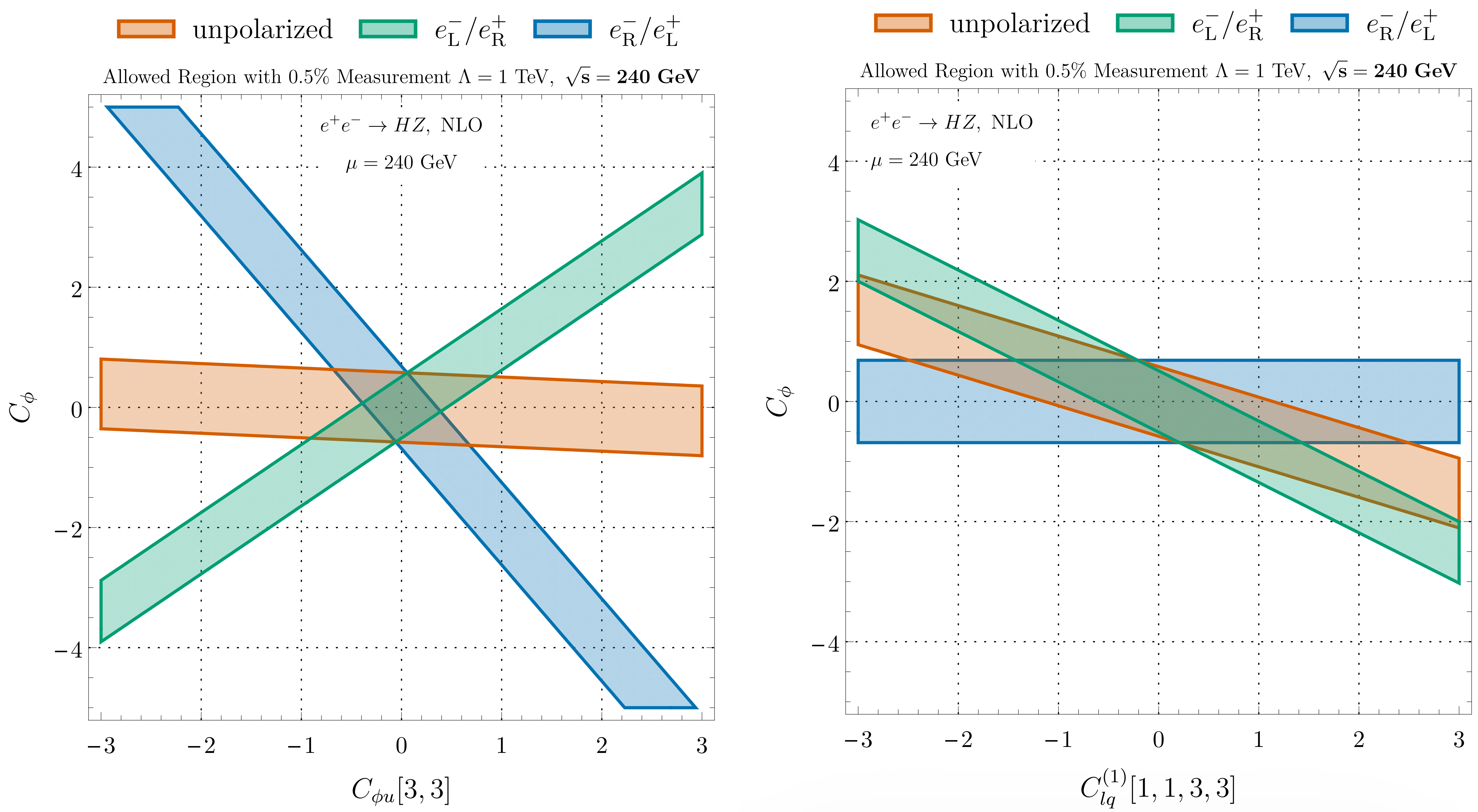
Other operators enter via EW loop corrections. An example here:

$(\varphi^\dagger\varphi)^3$

but also

$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{u}_p\gamma^\mu u_r)$

$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$



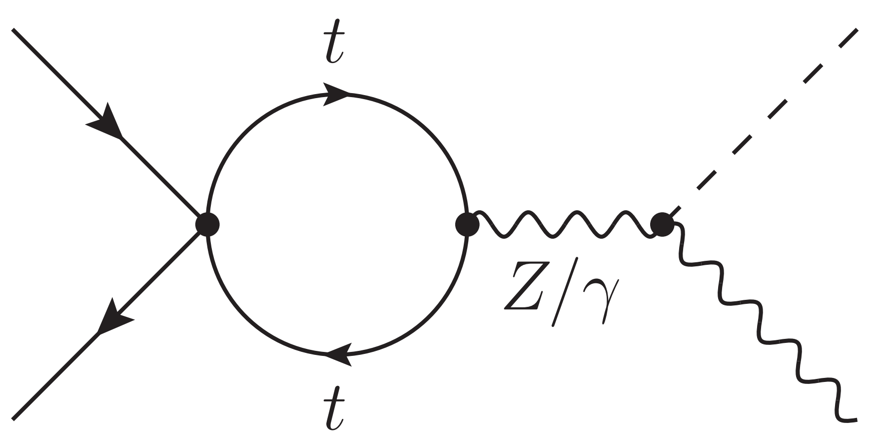


Figure 9. Contributions from modifications of the Higgs tri-linear coupling C_ϕ on the cross-section for $e^+e^- \to ZH$ correlated with those from $C_{\phi u}[3,3]$, which modifies the $Zt\bar{t}$ vertex, and from $C^{(1)}_{lq}[1,1,3,3]$ vertex which modifies the $e^+e^-t\bar{t}$ interaction. The sensitivity to a 0.5% measurement at $\sqrt{s} = 240$ GeV is shown. Note that there is no sensitivity to C_ϕ, $C_{\phi u}[3,3]$ or $C^{(1)}_{lq}[1,1,3,3]$ at tree level.

Higher Energy

(muon collider / 100 TeV pp)

NLO EW corrections at high energies

NLO EW corrections for energies of the order of few TeVs are as large as (or even more than) NLO QCD corrections at the LHC. Origin: **EW Sudakov logarithms.**

EW corrections should be considered not only for precision physics, since they give $\mathcal{O}(10-100\%)$ effects. This includes also BSM scenarios.

$\mu^+\mu^- \to X, \sqrt{s} = 3$ TeV	$\sigma^{\text{incl}}_{\text{LO}}$ [fb]	$\sigma^{\text{incl}}_{\text{NLO}}$ [fb]	δ_{EW} [%]
W^+W^-Z	$3.330(2) \cdot 10^1$	$2.568(8) \cdot 10^1$	$-22.9(2)$
W^+W^-H	$1.1253(5) \cdot 10^0$	$0.895(2) \cdot 10^0$	$-20.5(2)$
ZZZ	$3.598(2) \cdot 10^{-1}$	$2.68(1) \cdot 10^{-1}$	$-25.5(3)$
HZZ	$8.199(4) \cdot 10^{-2}$	$6.60(3) \cdot 10^{-2}$	$-19.6(3)$
HHZ	$3.277(1) \cdot 10^{-2}$	$2.451(5) \cdot 10^{-2}$	$-25.2(1)$
HHH	$2.9699(6) \cdot 10^{-8}$	$0.86(7) \cdot 10^{-8}$ *	
$W^+W^-W^+W^-$	$1.484(1) \cdot 10^0$	$0.993(6) \cdot 10^0$	$-33.1(4)$
W^+W^-ZZ	$1.209(1) \cdot 10^0$	$0.699(7) \cdot 10^0$	$-42.2(6)$
W^+W^-HZ	$8.754(8) \cdot 10^{-2}$	$6.05(4) \cdot 10^{-2}$	$-30.9(5)$
W^+W^-HH	$1.058(1) \cdot 10^{-2}$	$0.655(5) \cdot 10^{-2}$	$-38.1(4)$
$ZZZZ$	$3.114(2) \cdot 10^{-3}$	$1.799(7) \cdot 10^{-3}$	$-42.2(2)$
$HZZZ$	$2.693(2) \cdot 10^{-3}$	$1.766(6) \cdot 10^{-3}$	$-34.4(2)$
$HHZZ$	$9.828(7) \cdot 10^{-4}$	$6.24(2) \cdot 10^{-4}$	$-36.5(2)$
$HHHZ$	$1.568(1) \cdot 10^{-4}$	$1.165(4) \cdot 10^{-4}$	$-25.7(2)$

3 TeV Muon Collider

WHIZARD

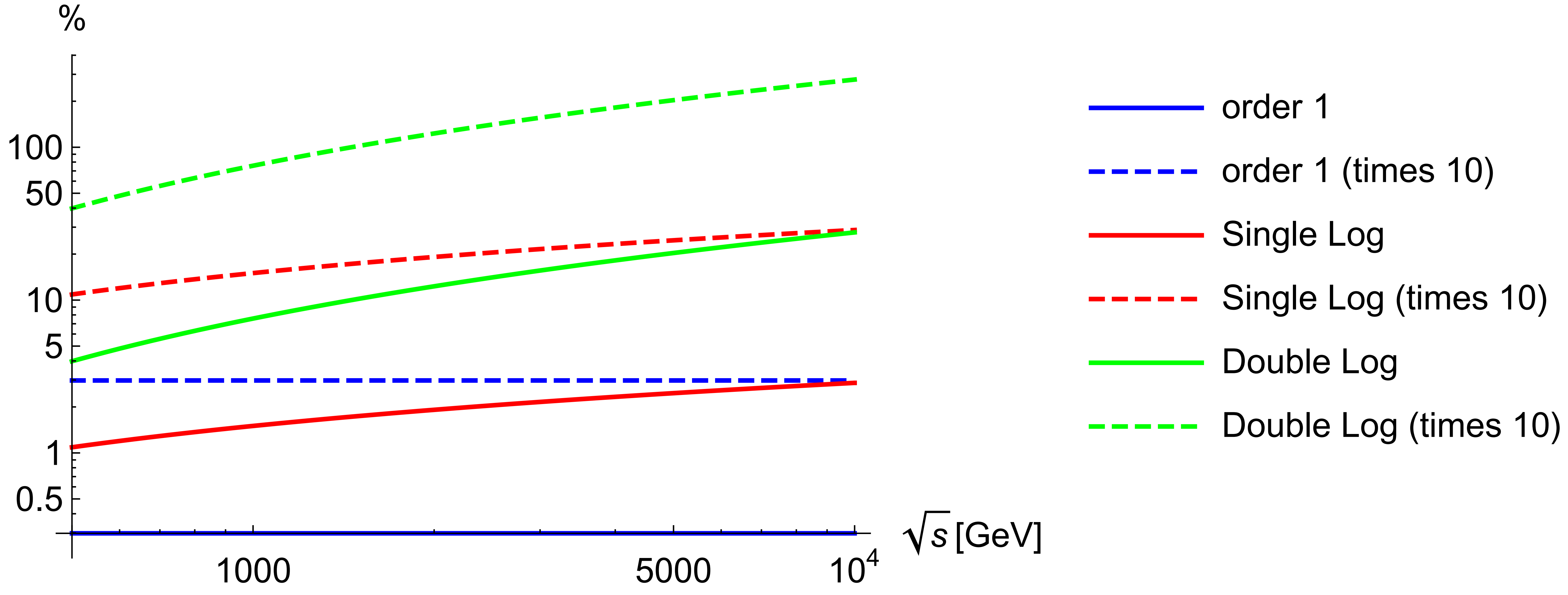
Bredt, Kilian, Reuter, Steinemeier '22

How large are expected to be the EW Sudakov at 1 loop?

$$\mathcal{O}(1) \to \frac{\alpha}{4\pi s_w^2} \sim 0.3\,\%\,,\quad \textbf{Single Log} \to \frac{\alpha}{4\pi s_w^2}\log(s/m_W^2),$$

$$\textbf{Double Log} \to \frac{\alpha}{4\pi s_w^2}\log^2(s/m_W^2)$$

Relative corrections in NLO EW



Taking into account only DL, and not SL, is not safe for partonic energies up to 10 TeV.

Just a representative example of a process

The estimate done via the variation of a factor of 10 is actually conservative.

$$\delta^{\mathrm{RR,ew}}_{\mathrm{e^+e^-}\to\mu^+\mu^-} = -2.58\,L(s) - 5.15\left(\log\frac{t}{u}\right) l(s) + 0.29\,l_{\mathrm{Z}} + 7.73\,l_{\mathrm{C}} + 8.80\,l_{\mathrm{PR}},$$

$$\delta^{\mathrm{RL,ew}}_{\mathrm{e^+e^-}\to\mu^+\mu^-} = -4.96\,L(s) - 2.58\left(\log\frac{t}{u}\right) l(s) + 0.37\,l_{\mathrm{Z}} + 14.9\,l_{\mathrm{C}} + 8.80\,l_{\mathrm{PR}},$$

Denner Pozzorini '01

Future Colliders: are EW Sudakov logarithms a good and robust approximation for EW corrections at high energies?

Currently: exact NLO EW automated for SM but not for BSM.

Since EW corrections are expected to be relevant also for BSM, can we safely use the high-energy Sudakov approximation?

MadGraph5_aMC@NLO: EW corrections for FC

NLO EW hadron colliders: *Frederix, Frixione, Hirshi, DP, Shao, Zaro '18*

NLO EW e^+e^- colliders: *Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao '22*

One-loop EW Sudakov alone: *DP, Zaro '21*

one-loop EW virtual corrections $\mathcal{O}(\alpha)$

=

α **[Sudakov Logs $\mathcal{O}(-\log^k(s/m_W^2), k = 1,2)$ +**

constant term $\mathcal{O}(1)$ +

mass-suppressed terms $\mathcal{O}(m_W^2/s)$]

Having separately exact NLO EW and EW Sudakov logarithms is possible to study the goodness of the high-energy approximation(s). **SM as a test case!**

Master formula (Denner&Pozzorini)

Denner Pozzorini '01

Born amplitude: $\mathcal{M}_0^{i_1 \dots i_n}(p_1, \dots, p_n)$

One-loop EW Sudakov corrections: $\delta \mathcal{M}^{i_1 \dots i_n}(p_1, \dots, p_n) = \mathcal{M}_0^{i'_1 \dots i'_n}(p_1, \dots, p_n)\delta_{i'_1 i_1 \dots i'_n i_n}$

other tree-level amplitudes — the logs

$$\delta = \delta^{\mathrm{LSC}} + \delta^{\mathrm{SSC}} + \delta^{\mathrm{C}} + \delta^{\mathrm{PR}}$$

eikonal approximation of soft EW boson exchange

- **Leading Soft-Collinear** (δ^{LSC}): It depends only on s and it is the only term involving double logarithms.
- **Subleading Soft-Collinear** (δ^{SSC}): The only one involving ratios of s with other invariants and also angular dependences.
- **Collinear** (δ^{C})
- **Parameter renormalis.** (δ^{PR}): In an on-shell scheme, the dependence on the UV regularisation scale cancels. No μ_r dependence is left.

The logs inside the δ^i have always the form:

$$L(|r_{kl}|, M^2) \equiv \frac{\alpha}{4\pi} \log^2 \frac{|r_{kl}|}{M^2}$$

$$l(|r_{kl}|, M^2) \equiv \frac{\alpha}{4\pi} \log \frac{|r_{kl}|}{M^2}$$

$$M = M_W, M_Z, m_f, \lambda, \dots$$
$$r_{kl} \equiv (p_k + p_l)^2$$

ASSUMPTIONS:

$$r_{kl} \equiv (p_k + p_l)^2 \simeq 2 p_k p_l \gg M_W^2 \simeq M_H^2, m_t^2, M_W^2, M_Z^2$$

the high-energy limit

$$r_{kl}/r_{k'l'} \simeq 1$$

All invariants $\simeq s$. Reasonable, but $r_{kl} = s$ is impossible.

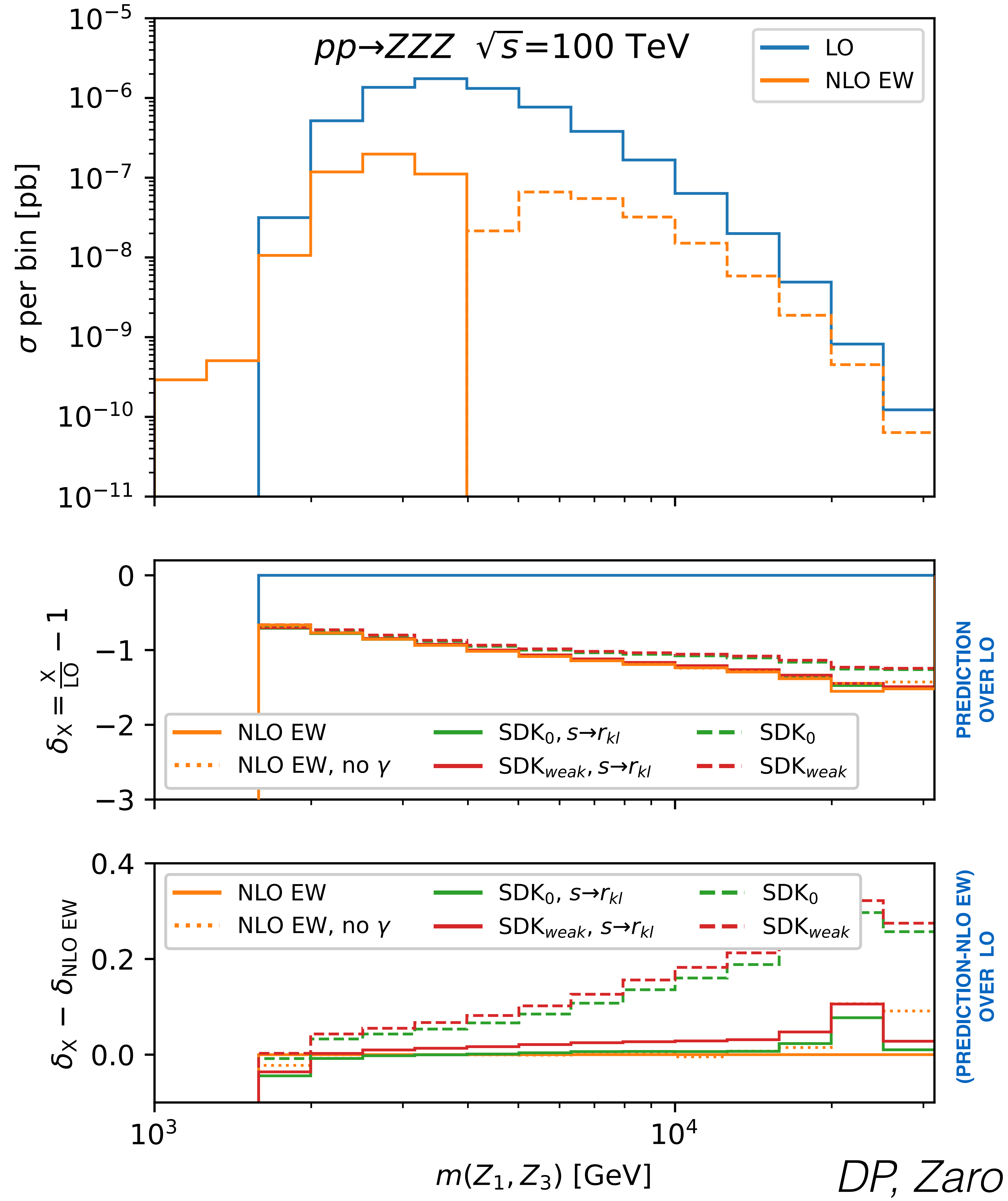
Our revisitation:

DP, Zaro '21

Logs among invariants: Logs like $\log(t/s)$ taken into account.

$\mathrm{SDK_{Weak}}$ scheme: A purely Weak (no QED) scheme for improving approximation of IR-finite physical observables. Different to the more common $\mathrm{SDK_0}$ scheme that has been used in the literature.

ZZZ production at 100 TeV **FCC-hh/SppC**



DP, Zaro '21

$$p_T(Z_i) > 1\ \text{TeV}, \quad |\eta(Z_i)| < 2.5, \quad m(Z_i, Z_j) > 1\ \text{TeV}, \quad \Delta R(Z_i, Z_j) > 0.5.$$

Orange: NLO EW, (**dotted**: NLO EW no γ PDF)
Green = SDK_0, **Red** = $\mathrm{SDK}_{\mathrm{weak}}$
Dashed: standard approach for amplitudes.
Solid: our formulation (more angular information)

Reference Prediction:
Red-solid line

$\mathrm{SDK}_{\mathrm{weak}}$ and SDK_0 not so relevant for neutral final state).

Only the solid lines, having more angular information, correctly capture NLO EW.

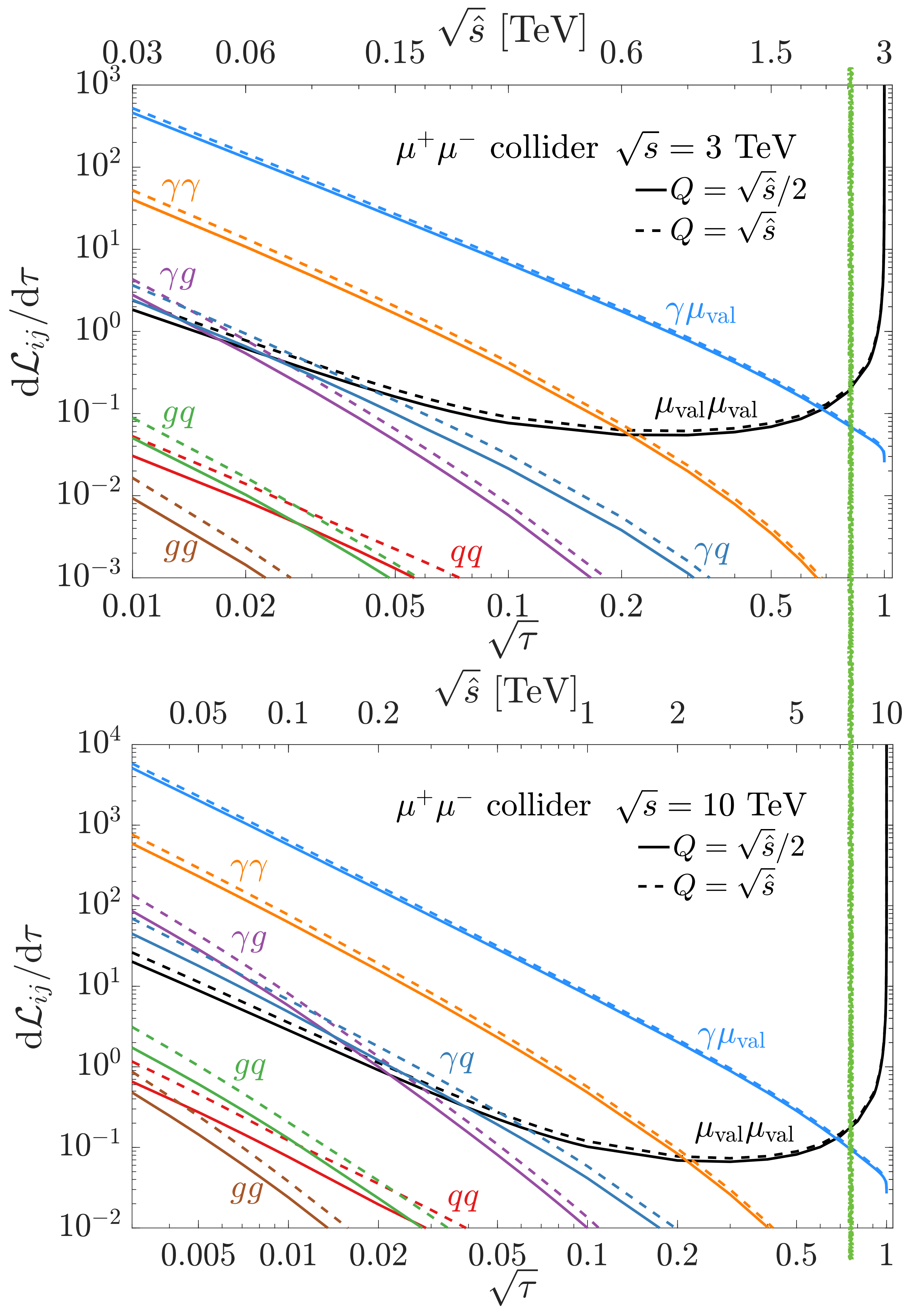
One cannot forget terms as $\log^2[m^2(Z_2, Z_3)/s]$

Larger invariant -> larger correction

The muon collider case

Ma, DP, Zaro '24

$\mu^+\mu^- \longrightarrow F$, where F is a generic final state involving W, Z, t, H.
Thus we select direct production, with no VBF contributions.



Han, Ma, Xie '20, '21

We require $m(F) > 0.8\sqrt{S}$, so that neither VBF nor PDFs other than μ are relevant.

We apply further experimentally motivated cuts for each X, Y particle in F:

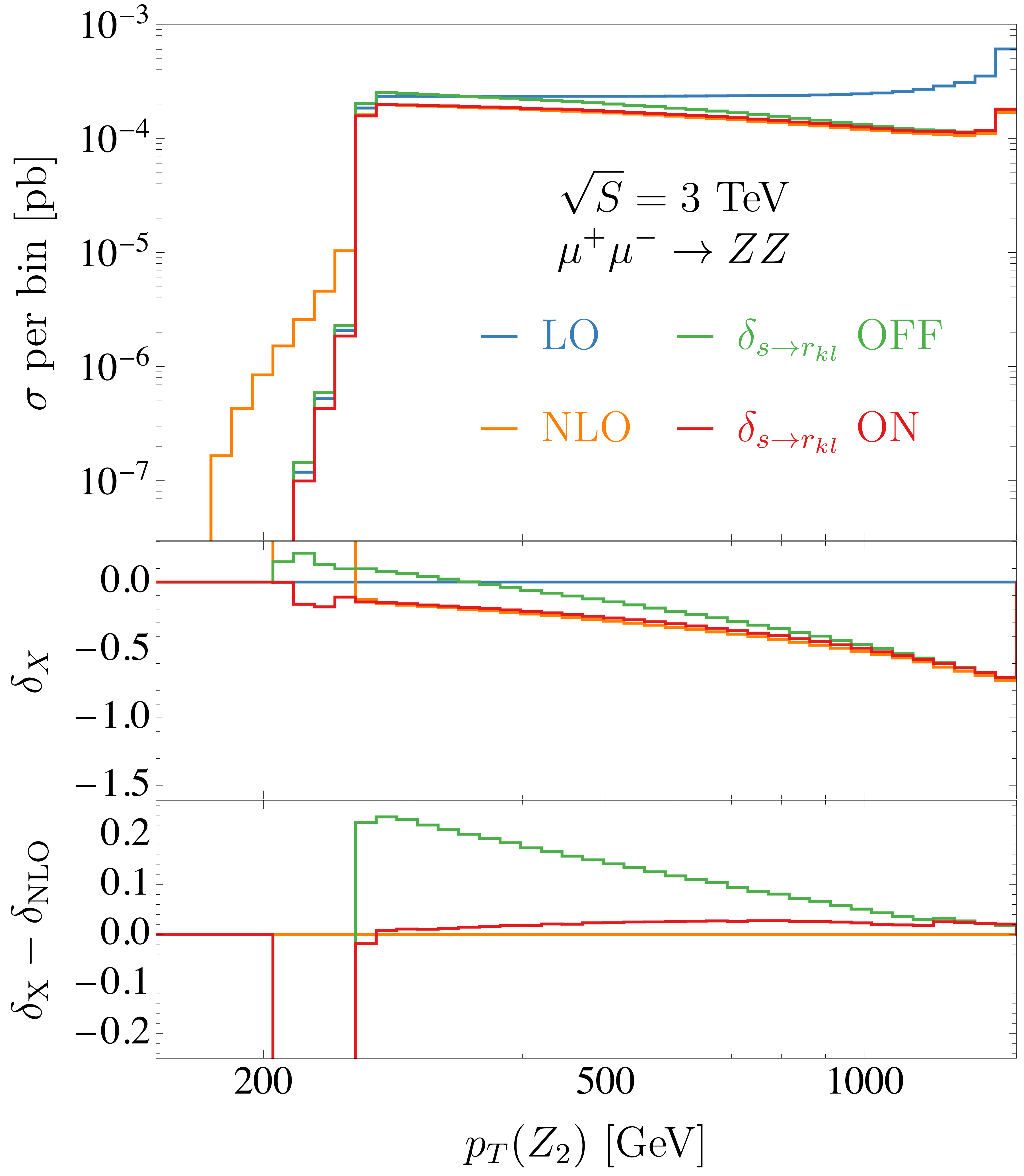
$$p_T(X) > 100 \text{ GeV},\ |\eta(X)| < 2.44,\ \Delta R(X, Y) > 0.4$$

And we recombine photons with charged (also massive) particles.

The μ PDF in the μ is peaked at Bjorken-x=1, therefore: Collider $S \simeq$ partonic s

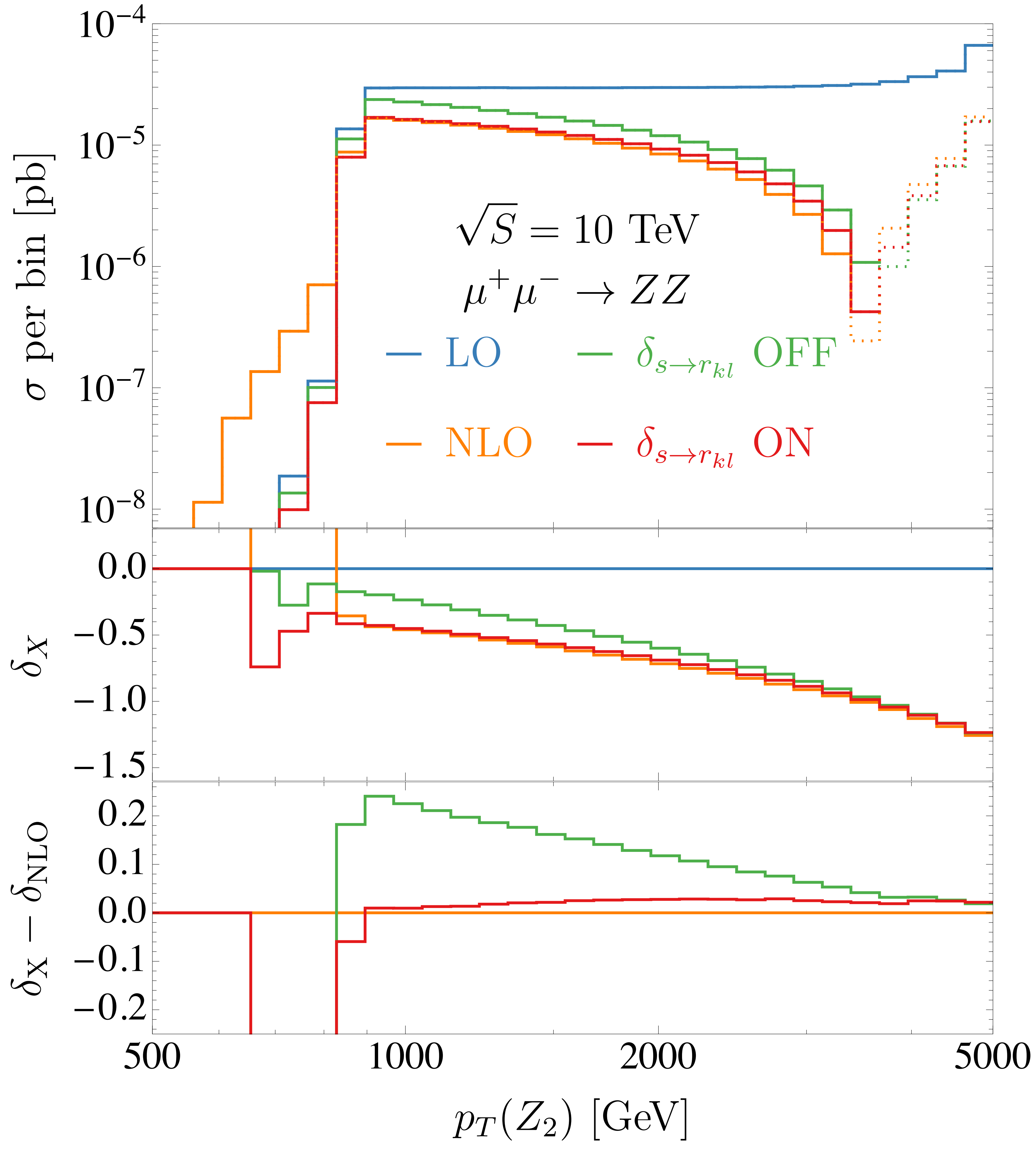
ZZ

Ma, DP, Zaro '24



Sudakov logs **capture NLO EW corrections** up to the % level, but only if all the logs of the form $\log(t/s)$ are taken into account.

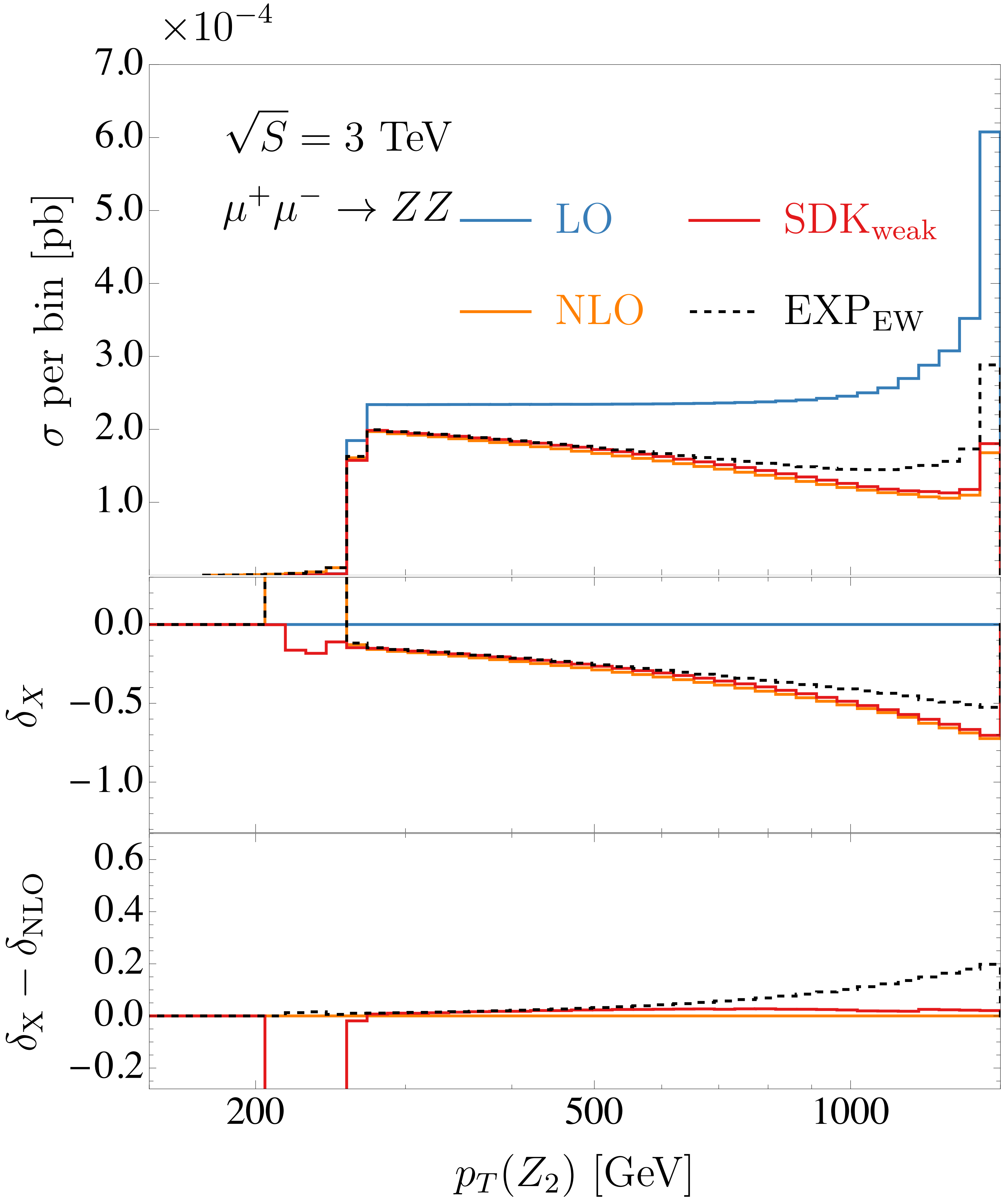
Green: logs of the form $\log^2(t/s)$ or $\log(t/s)$ ignored.



ZZ

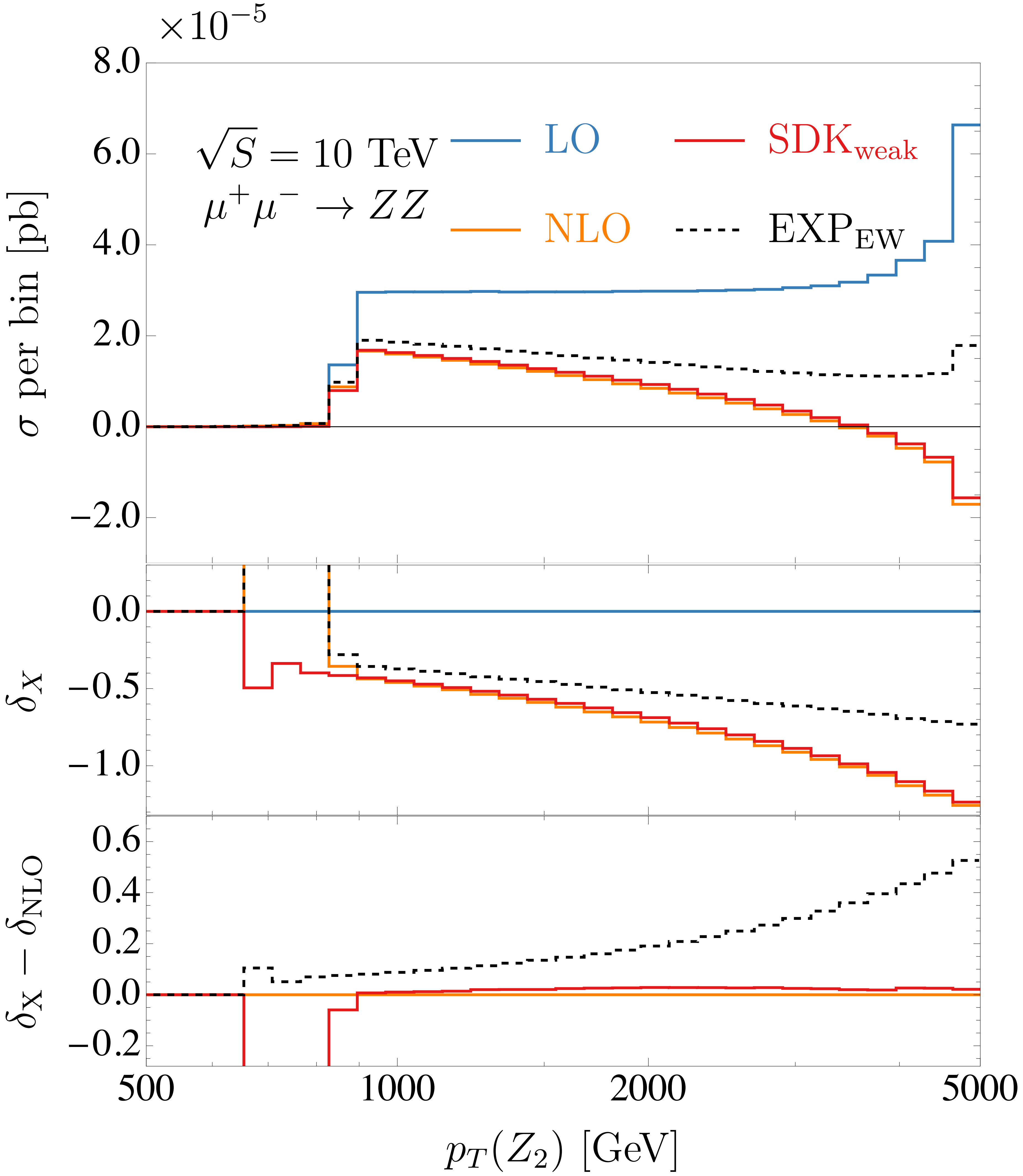
Ma, DP, Zaro '24

$$\sigma_{\mathrm{EXP_{EW}}} \equiv \left(\sigma_{\mathrm{LO}}\, e^{\delta_{\mathrm{SDK_{weak}}}}\right) + \left(\sigma_{\mathrm{NLO_{EW}}} - \sigma_{\mathrm{SDK_{weak}}}\right) = \sigma_{\mathrm{NLO_{EW}}} + \mathcal{O}(\alpha^2) \times \sigma_{\mathrm{LO}}.$$



Exponentiation as an approximation of proper resummation.

At 10 TeV resummation is unavoidable for sensible predictions, and it is necessary for precision at 3 TeV.



What about extra radiation of Z (and H)?

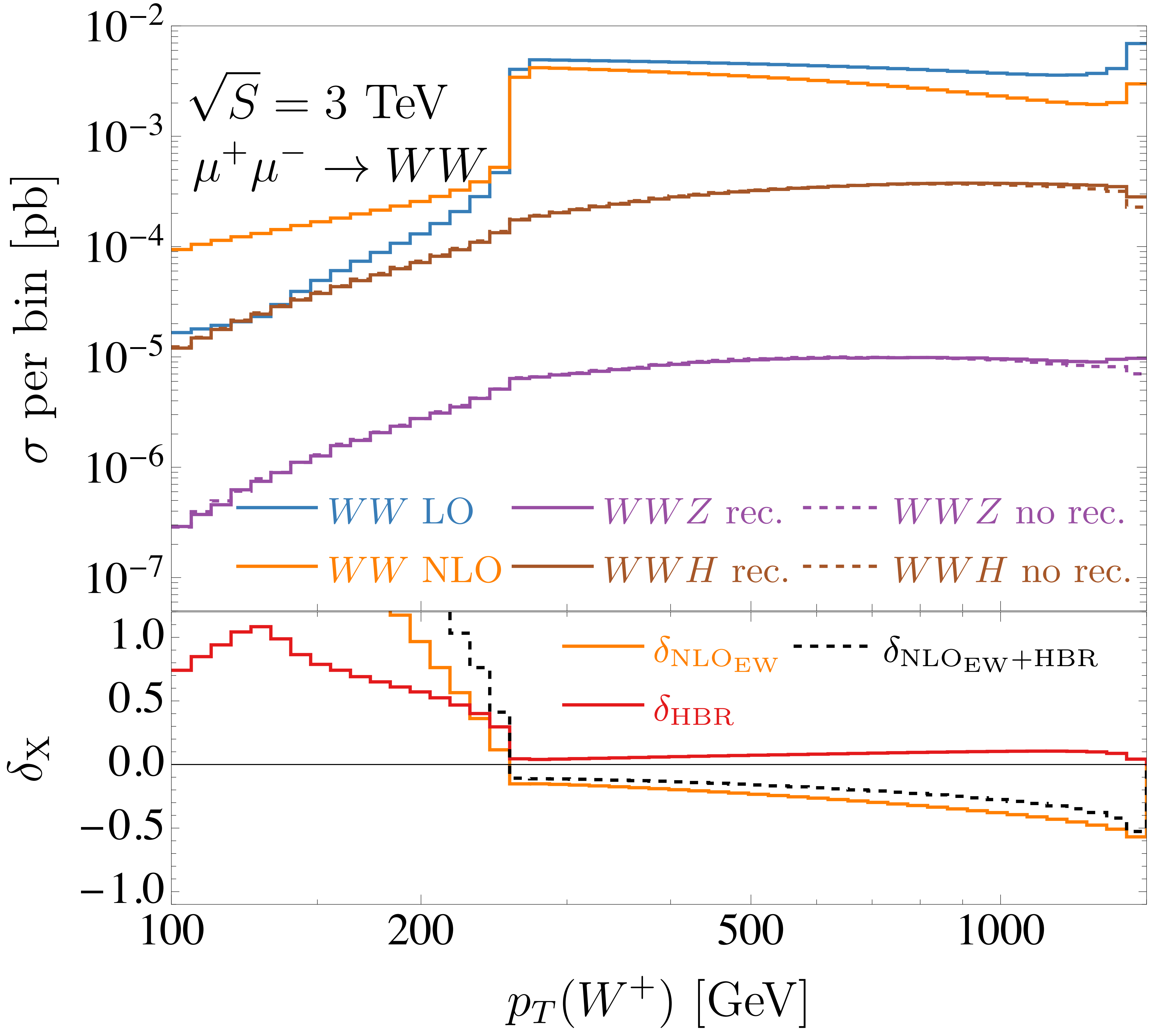
We know that unlike QCD in virtual+real there is not the exact cancellation of logarithms.

But a cancellation is still present, how much large?

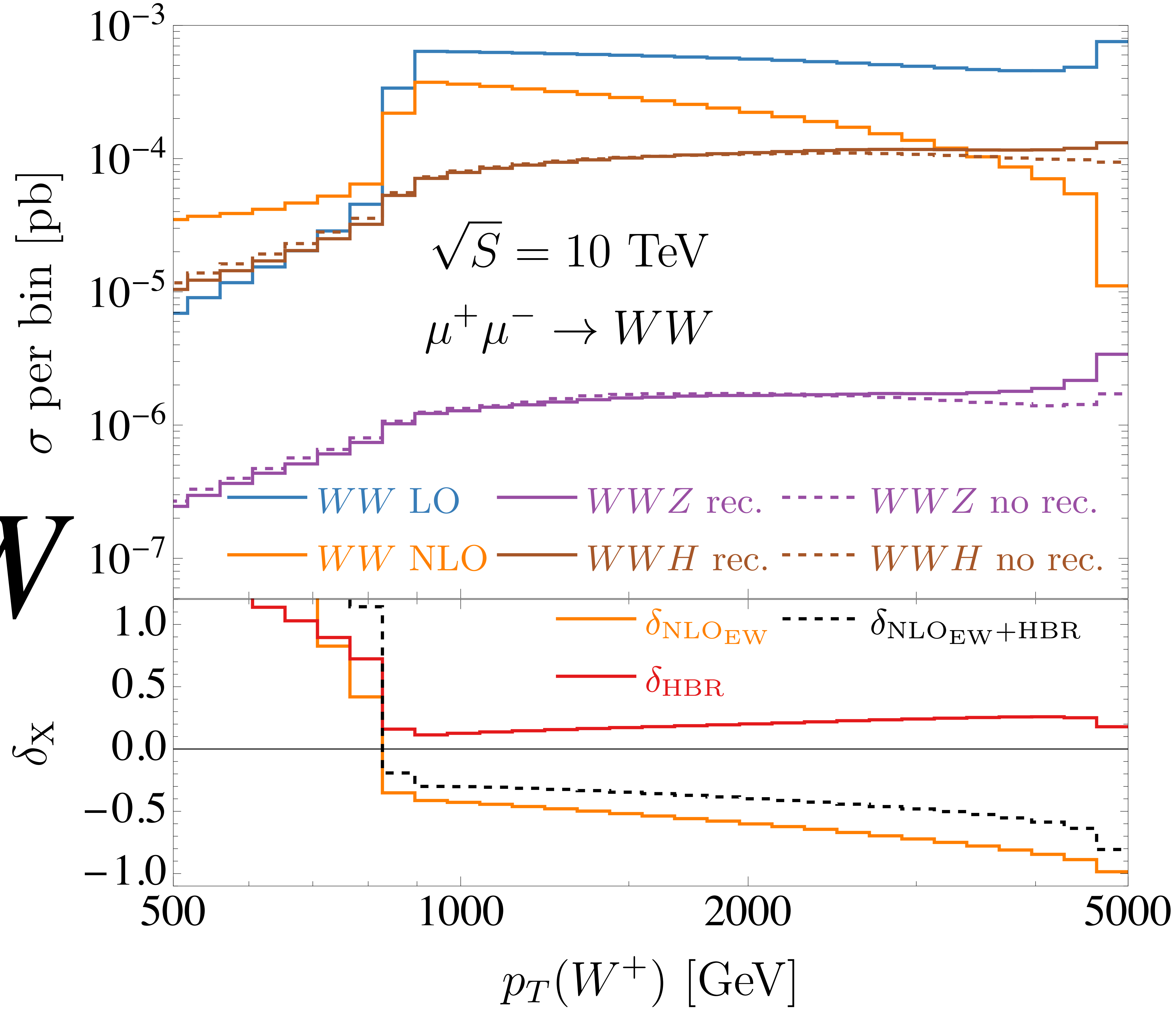
Is it really Heavy-Boson-Radiation (HBR) leading to $\mathcal{O}(1)$ corrections?

EW is the new **QCD**,
but it is not exactly as the QCD!

Very small effects from Z and H radiation, especially in the bulk: $p_T(W) \simeq \sqrt{S}/2$



WW



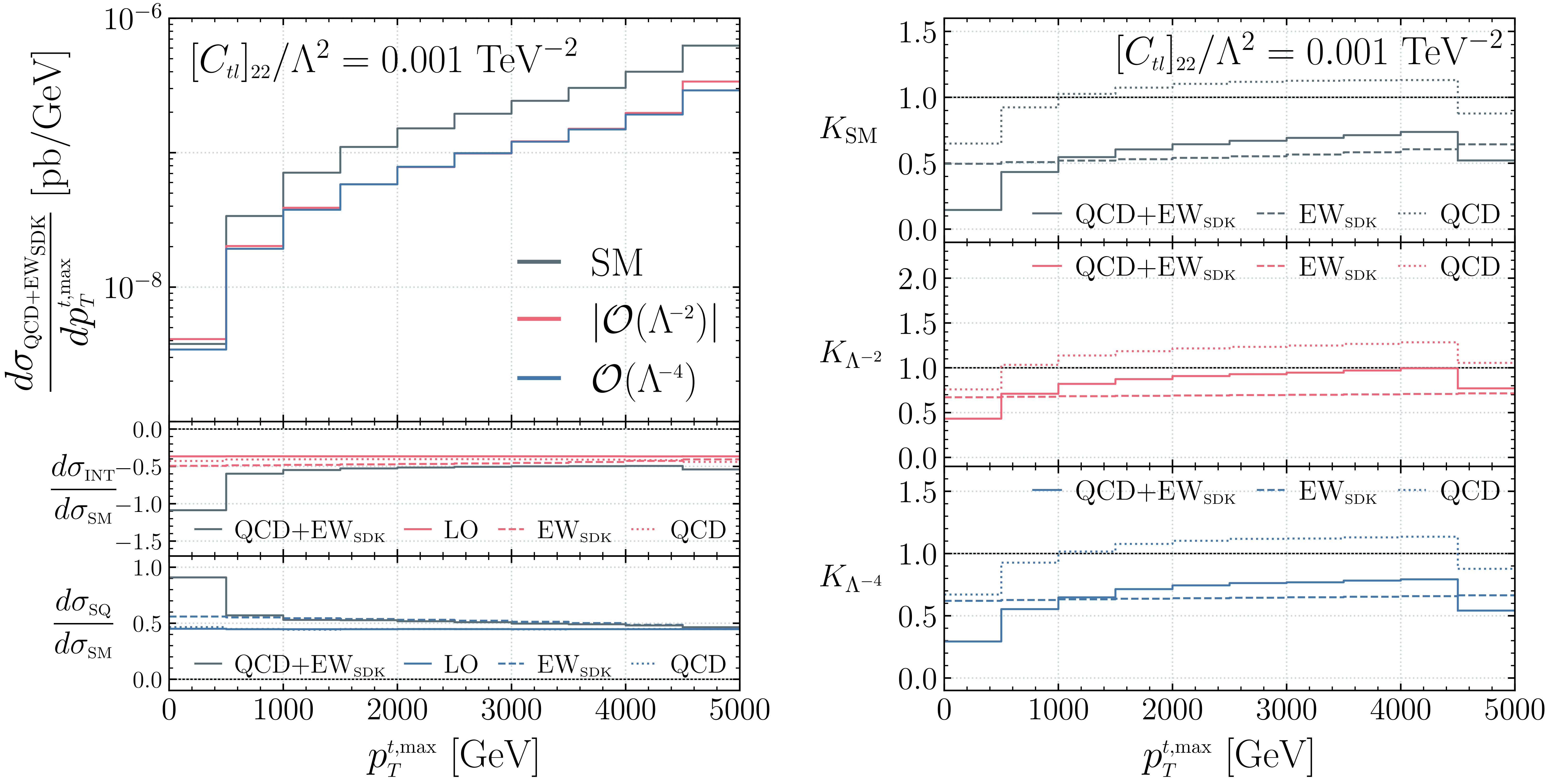
It is a general pattern: radiation of heavy bosons is much less important than loops!

Ma, DP, Zaro '24

EW Sudakov and SMEFT: $t\bar{t}$

Only Four-Fermion operators are considered in the study.

$$\mathcal{O}_{tl} = (\bar{t}\gamma^{\mu}t)(\bar{l}_i\gamma_{\mu}l_i)$$



10 Tev μ-coll

Both QCD and EW corrections are different for SM, SM-SMEFT interference, and SMEFT^2 contributions of dim-6.

QCD and EW cancel each other: both are important.

K-factors can be different in SM and BSM!

El Faham, Mimasu, DP, Severi, Vryonidou, Zaro: in preparation

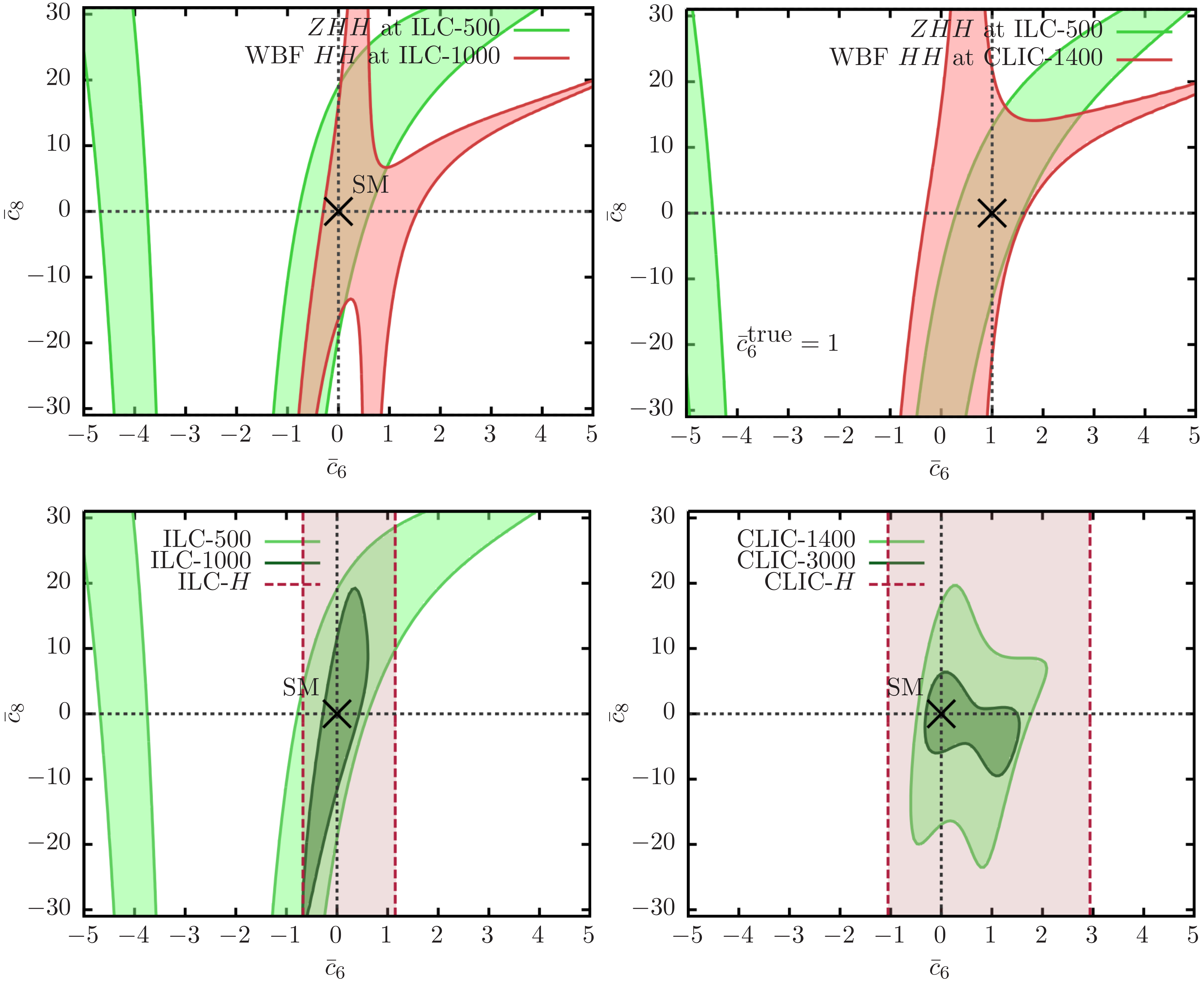
CONCLUSION

- EW corrections are mandatory for phenomenology at future colliders, especially for high energies. **Not only for the SM also for BSM!**
- For precision: both **NNLO ($\alpha_s\alpha$ and α^2) corrections and NNL accuracy for PDFs** are mandatory.
- EW corrections open up **sensitivity to new (BSM) interactions**: HHH(H) in single(double) H.
- **Sudakov logs** are the dominant contribution of EW corrections at high energy (**muon Collider**) and they are a **good approximation** of them, but only **IF**: single logs present, logs among invariants present and other features not discussed here in the plot.
- **Heavy-Boson Radiation** has an impact, but not always so large and typically **smaller than** the **virtual** contributions.
- **Resummation may be mandatory for sensible results** in many configurations and in general for precision.

EXTRA SLIDES

Results

Maltoni, DP, Zhao '18



NLO EW: some open questions/issues

Resummation?

When is it necessary to resum EW (Sudakov) corrections?

BSM?

What features of NLO EW corrections are universal and can be extended to the BSM case?

Heavy Boson Radiation (HBR)?

What should one do with Z,W radiation? Experimental set-up may impact the calculation result.

PDFs or VBF with matrix elements?

If PDFs involve weak effects, weak counter terms in NLO EW corrections should be included. Resum logs or keep power corrections? Both?

What are EW Sudakov logarithms?

QCD: virtual and real terms are separately IR divergent ($1/\epsilon$ poles). In physical cross sections the contributions are combined and poles cancel.

QED: same story, but I can also regularise IR divergencies via a photon-mass λ. So $1/\epsilon$ poles $\rightarrow \log(Q^2/\lambda^2)$, where Q is a generic scale.

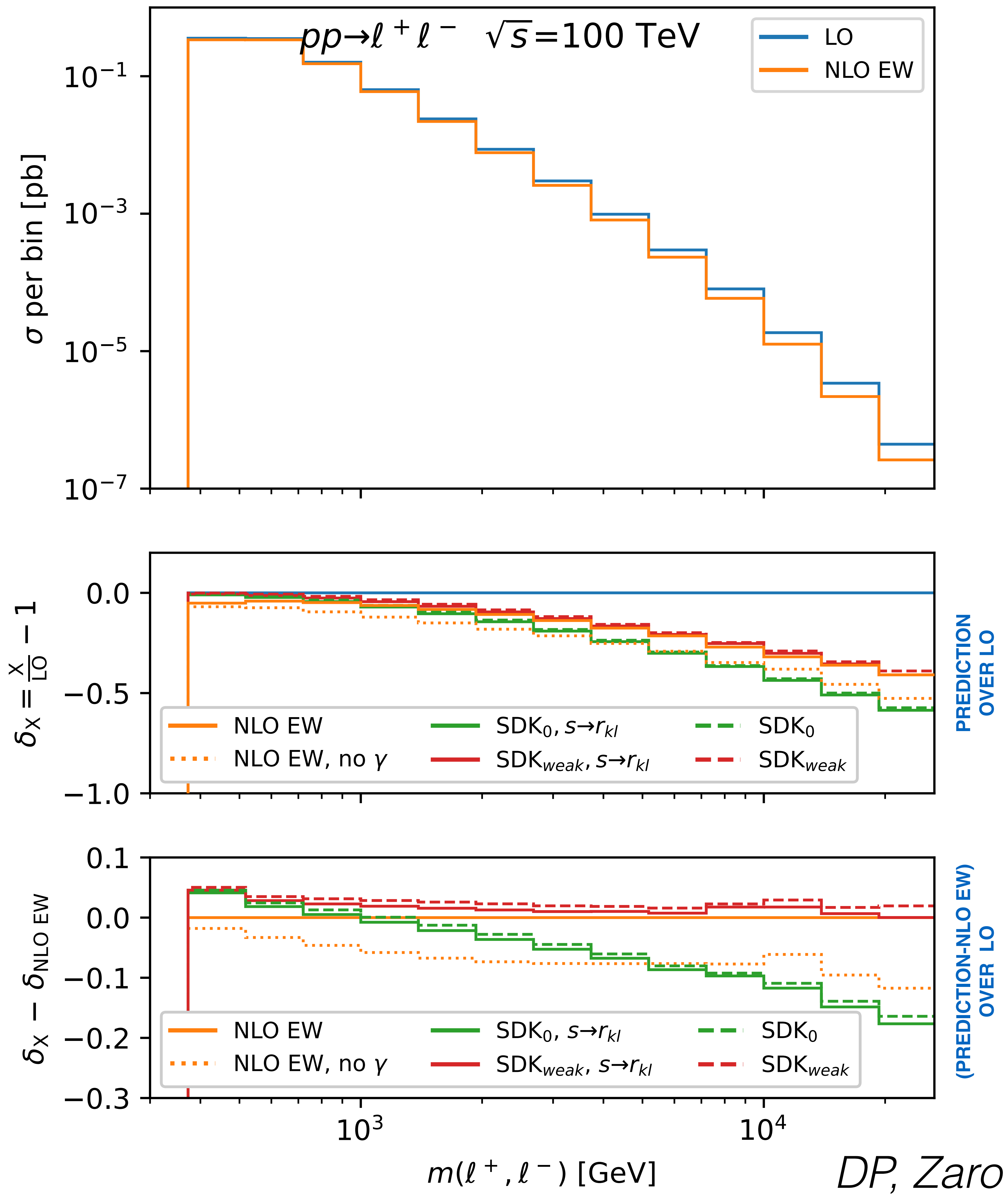
EW: with weak interactions $\lambda \rightarrow m_W, m_Z$ and W and Z radiation are typically not taken into account, which is anyway IR-safe.

Therefore, at high energies EW loops induce corrections of order

$$-\alpha^k \log^n(s/m_W^2)$$

where k is the number of loops and $n \leq 2k$. These logs are physical. Even including the real radiation of W and Z, there is not the full cancellation of this kind of logarithms.

e^+e^- production at 100 TeV **FCC-hh**



DP, Zaro '21

$p_T(\ell^{\pm}) > 200\ \text{GeV}\,, \qquad |\eta(\ell^{\pm})| < 2.5\,, \qquad m(\ell^+,\ell^-) > 400\ \text{GeV}\,, \qquad \Delta R(\ell^+,\ell^-) > 0.5\,.$

Orange: NLO EW, (**dotted**: NLO EW no γ PDF)
Green = SDK_0, **Red** = $\mathrm{SDK}_{\mathrm{weak}}$
Dashed: standard approach for amplitudes.
Solid: our formulation (more angular information)

Reference Prediction:
Red-solid line

Only the $\mathrm{SDK}_{\mathrm{weak}}$ approach correctly captures the NLO EW prediction.

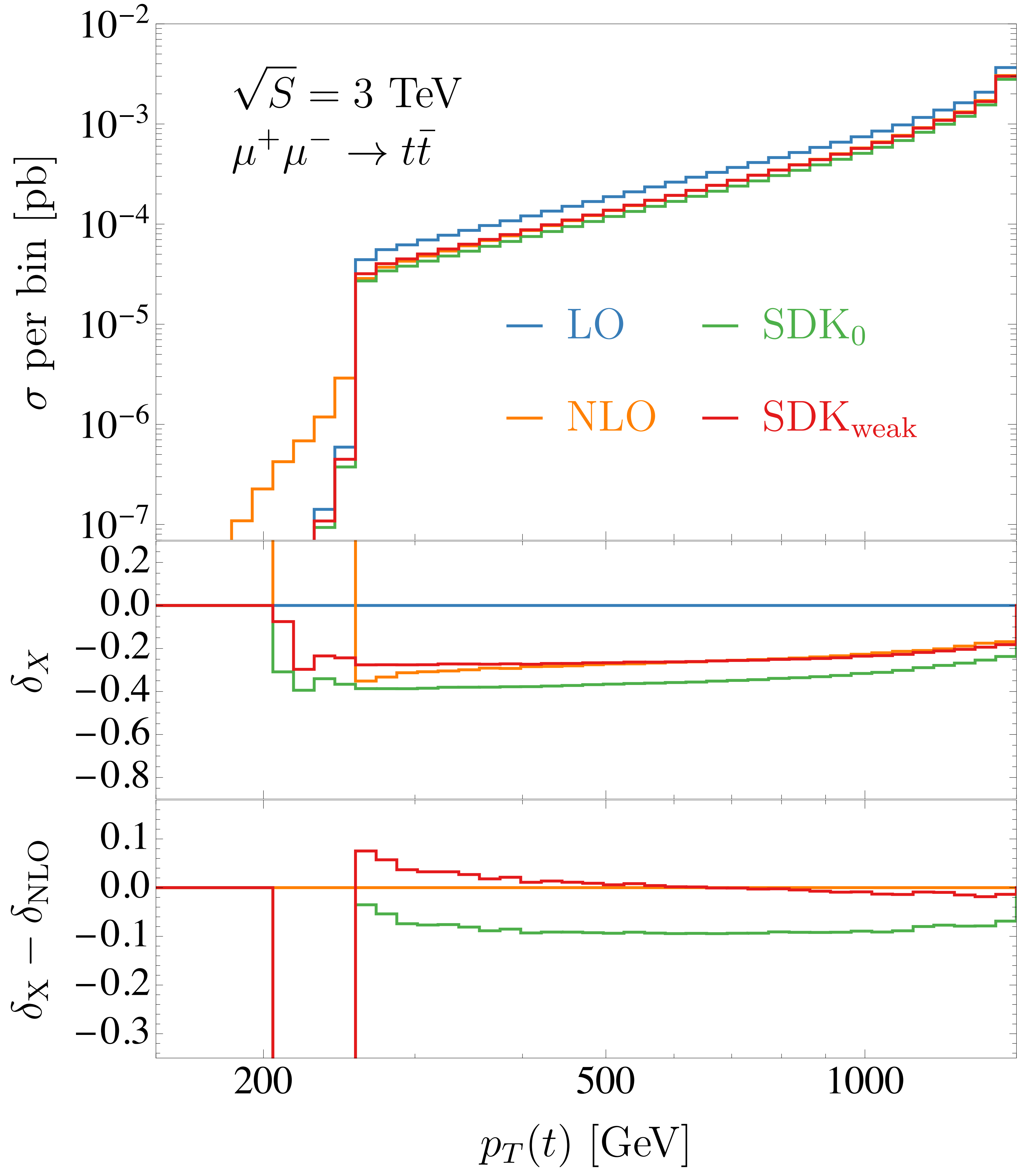
Solid and dashed very similar.

Photon PDF cannot be ignored.

Larger invariant -> larger correction

$t\bar{t}$

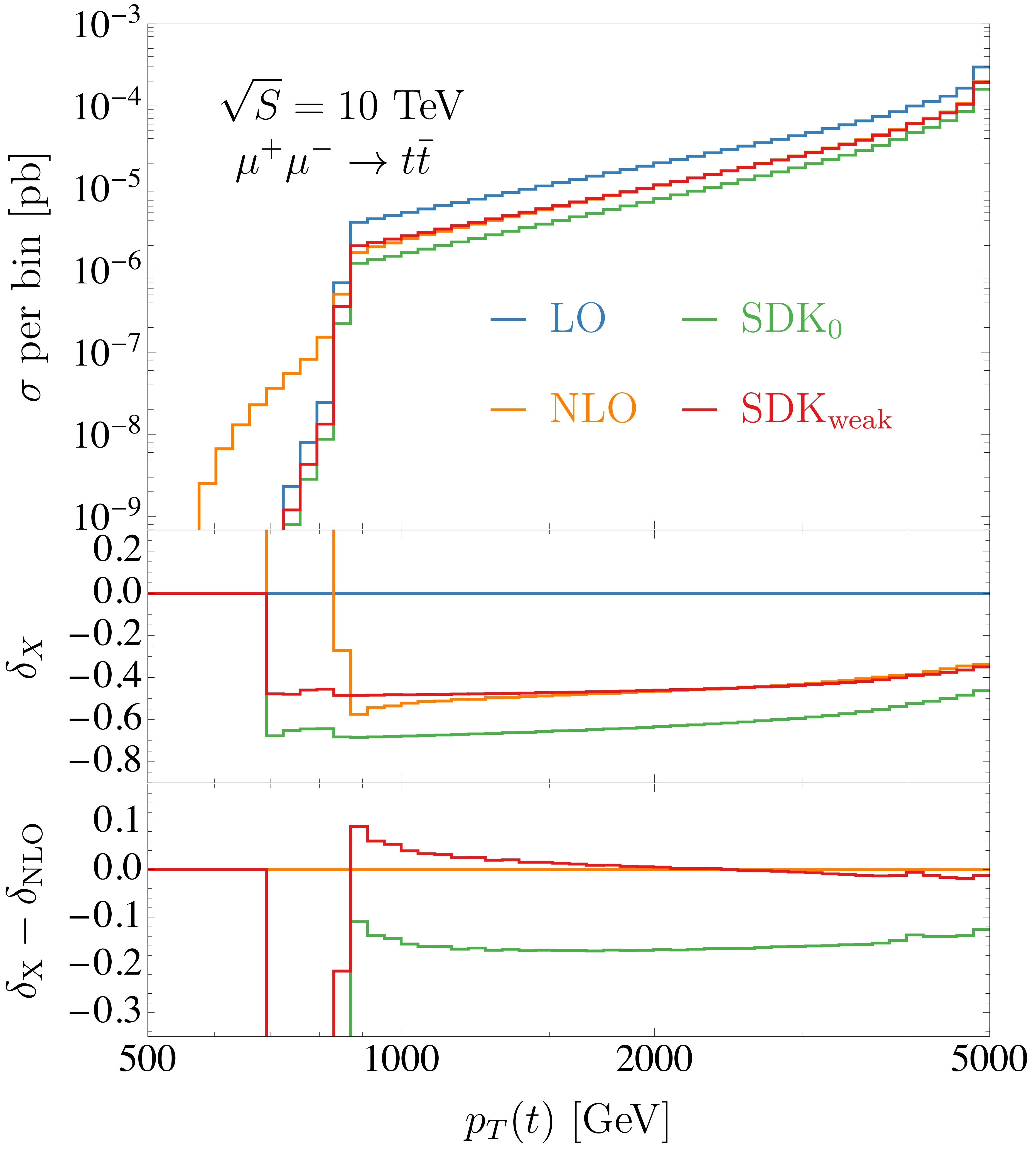
Ma, DP, Zaro **TODAY**



For smaller p_T , larger corrections.

Sudakov (in the SDK$_{\rm weak}$ scheme) **capture NLO EW corrections** up to the % level.

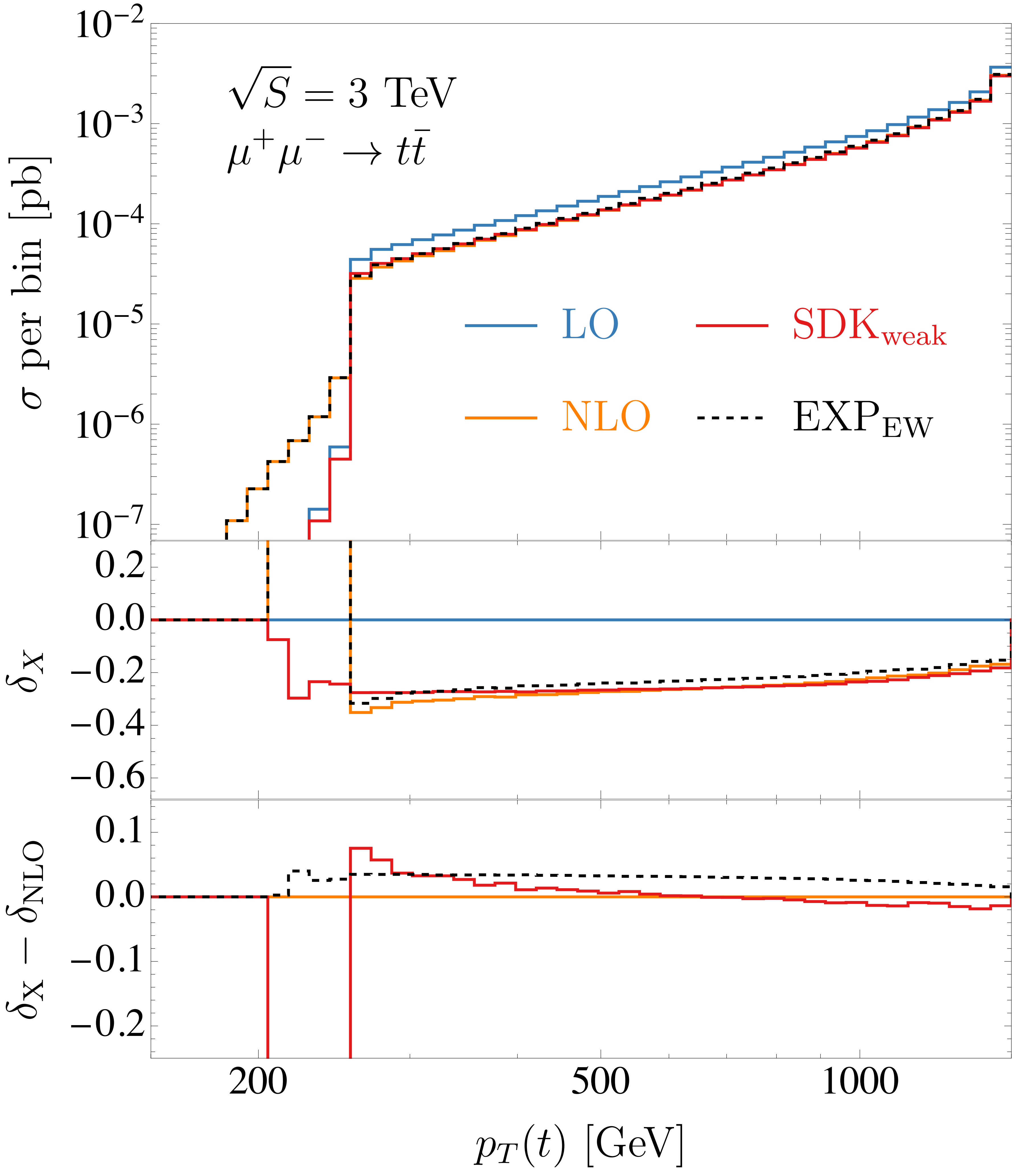
If double logs are written in the form $\log^2(s/m_W^2)$, the shapes observed here are all arising from **single logs**.



$t\bar{t}$

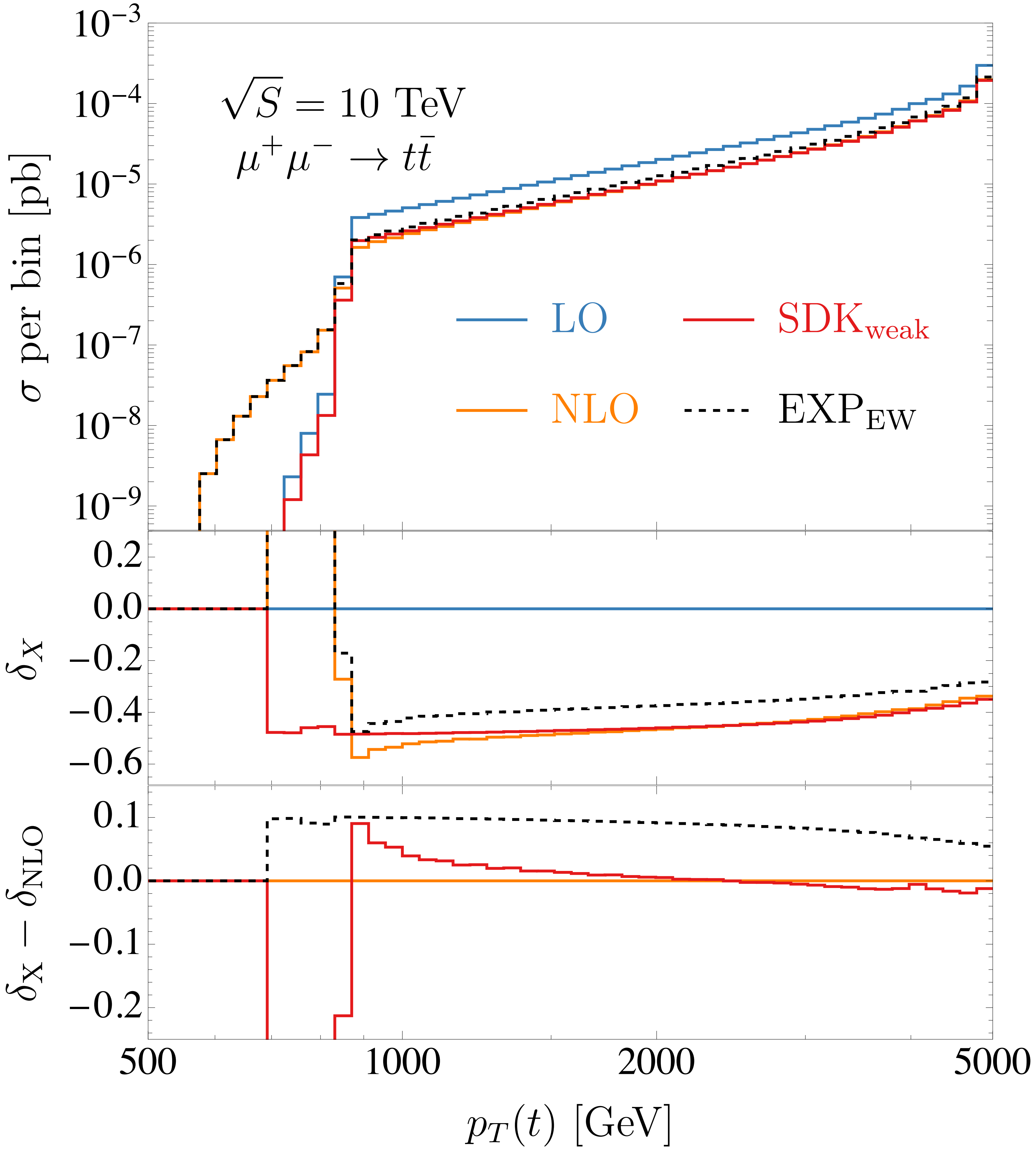
Ma, DP, Zaro **TODAY**

$$\sigma_{\mathrm{EXP_{EW}}} \equiv \left(\sigma_{\mathrm{LO}}\, e^{\delta_{\mathrm{SDK_{weak}}}}\right) + \left(\sigma_{\mathrm{NLO_{EW}}} - \sigma_{\mathrm{SDK_{weak}}}\right) = \sigma_{\mathrm{NLO_{EW}}} + \mathcal{O}(\alpha^2) \times \sigma_{\mathrm{LO}}.$$



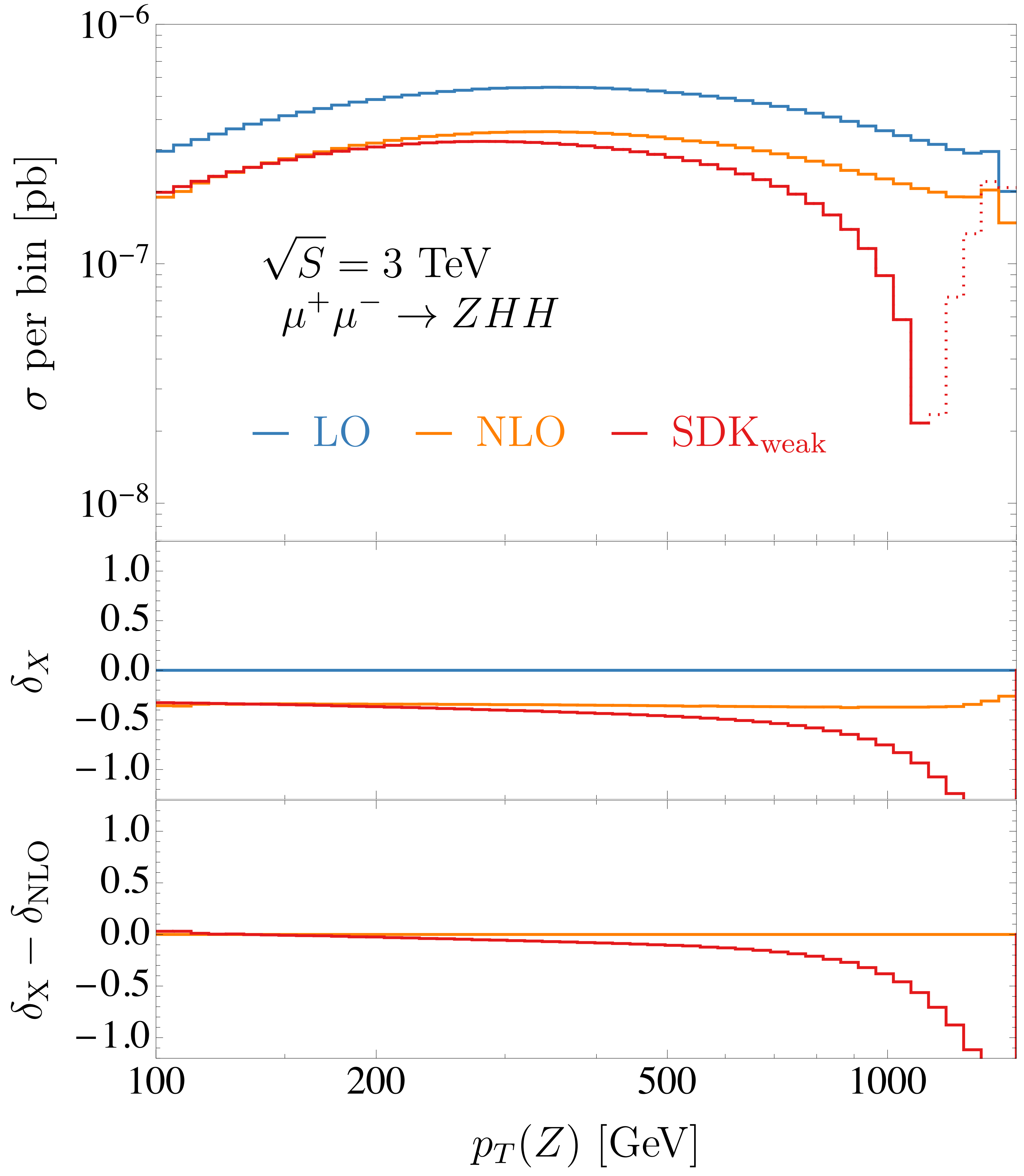
Exponentiation as an approximation of proper resummation.

Unlike ZZ, for $t\bar{t}$ **also at 10 TeV resummation is** necessary only for precision.



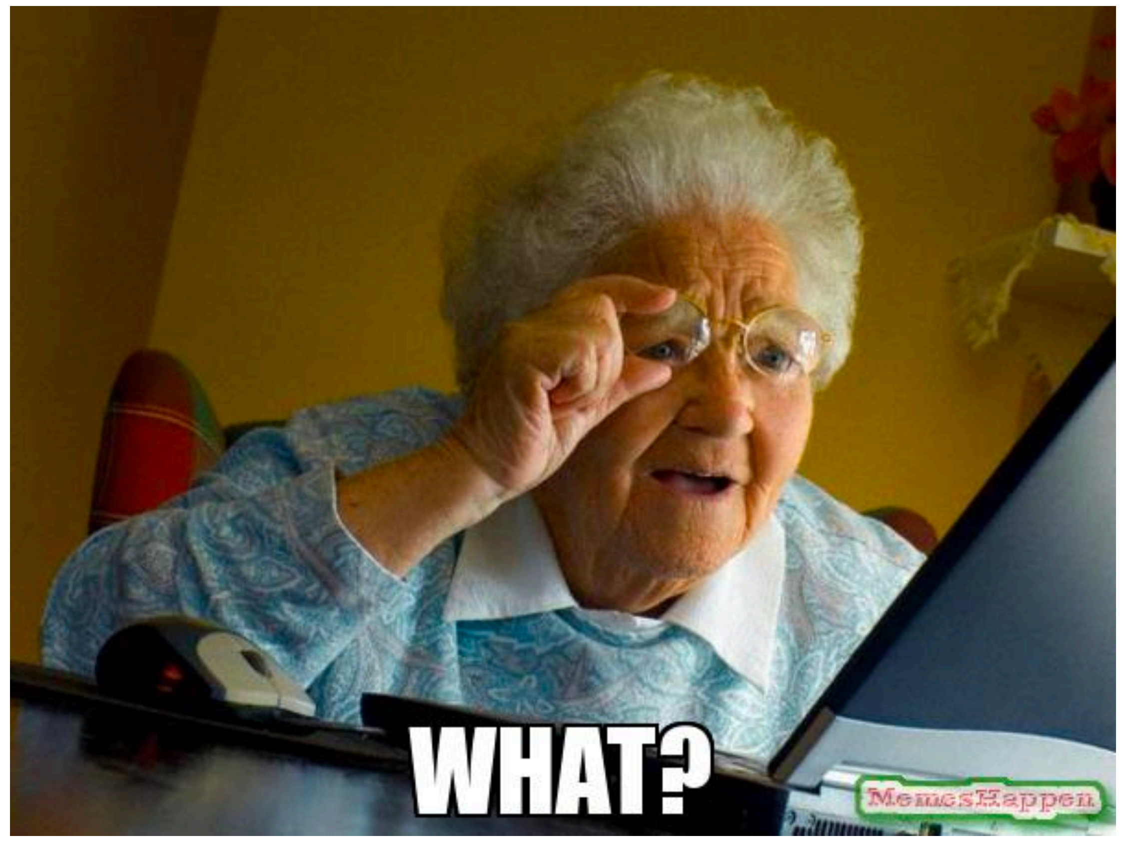
Sudakov may completely fail: ZHH

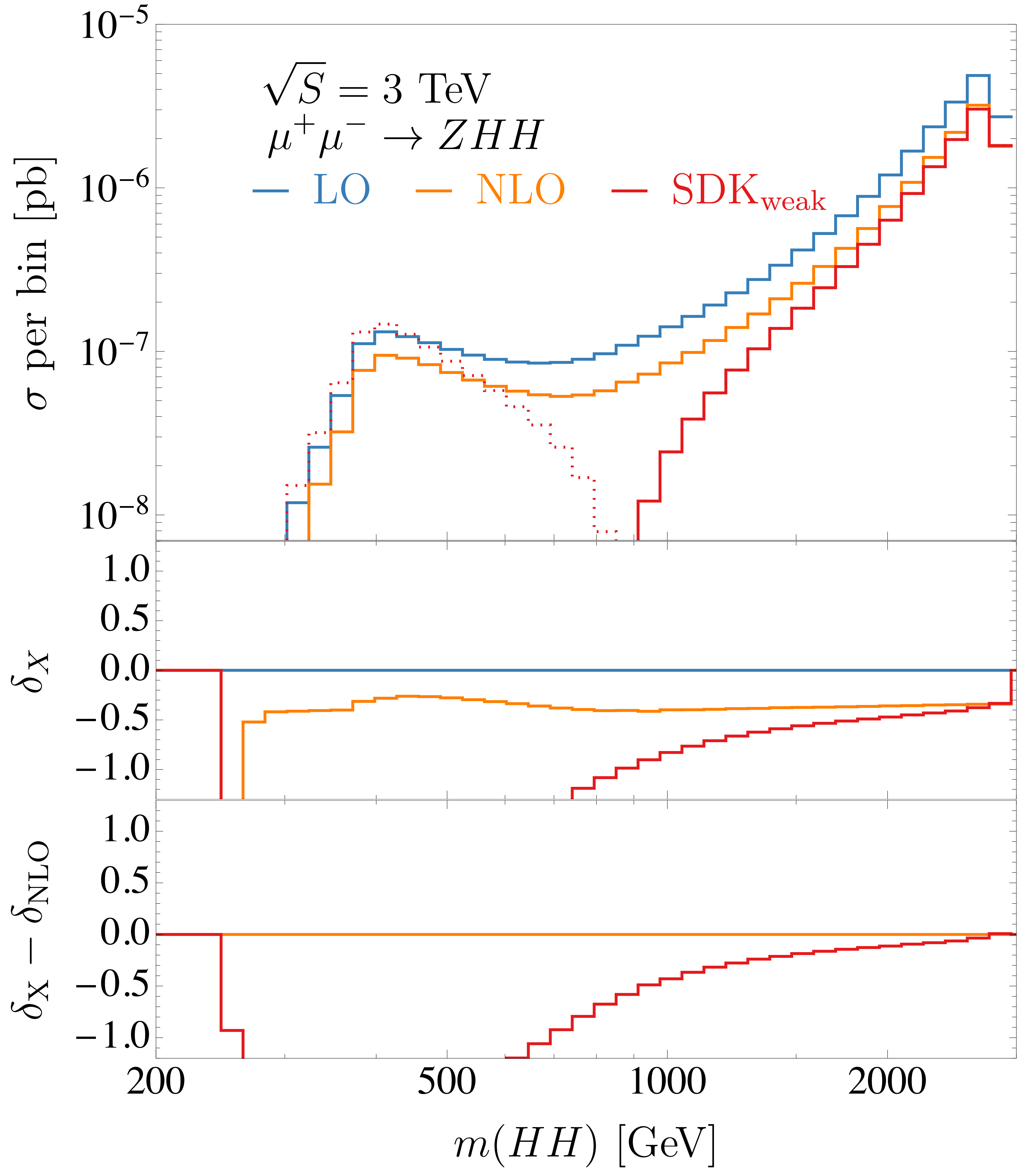
Ma, DP, Zaro **TODAY**



NLO EW corrections are flat.

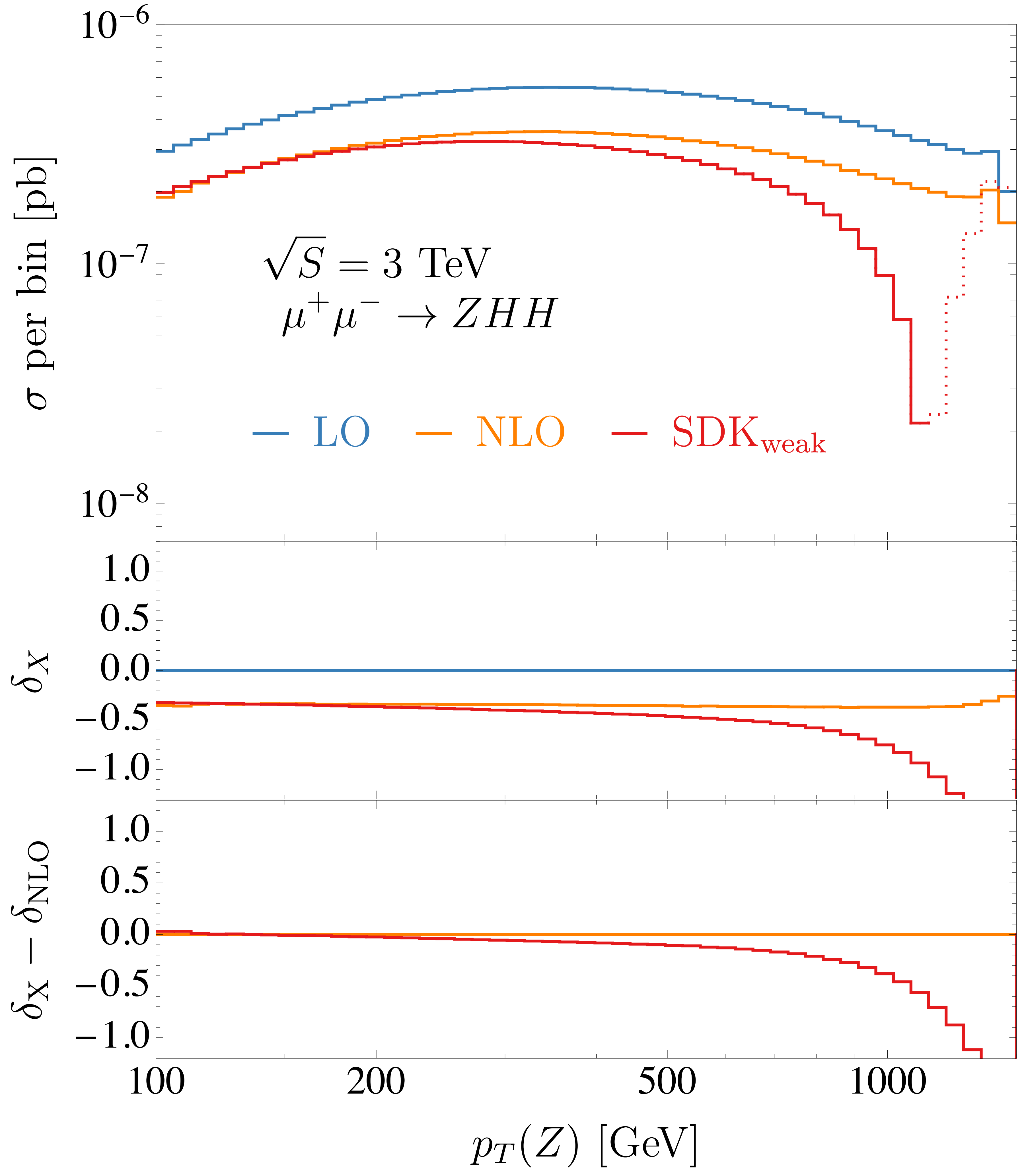
Sudakov logarithms work **very well at low pt and very bad at high pt.**





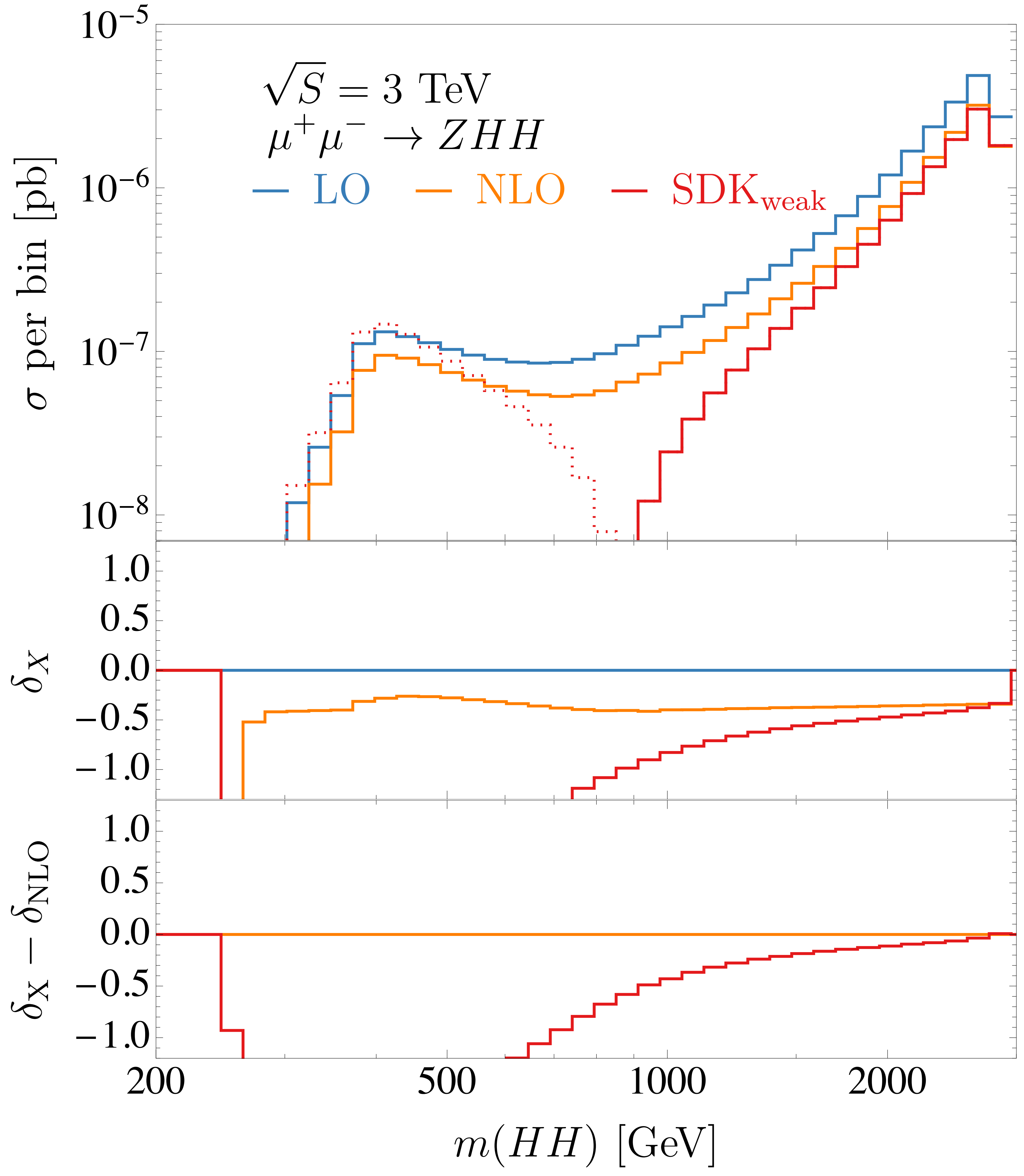
Sudakov may completely fail: ZHH

Ma, DP, Zaro **TODAY**

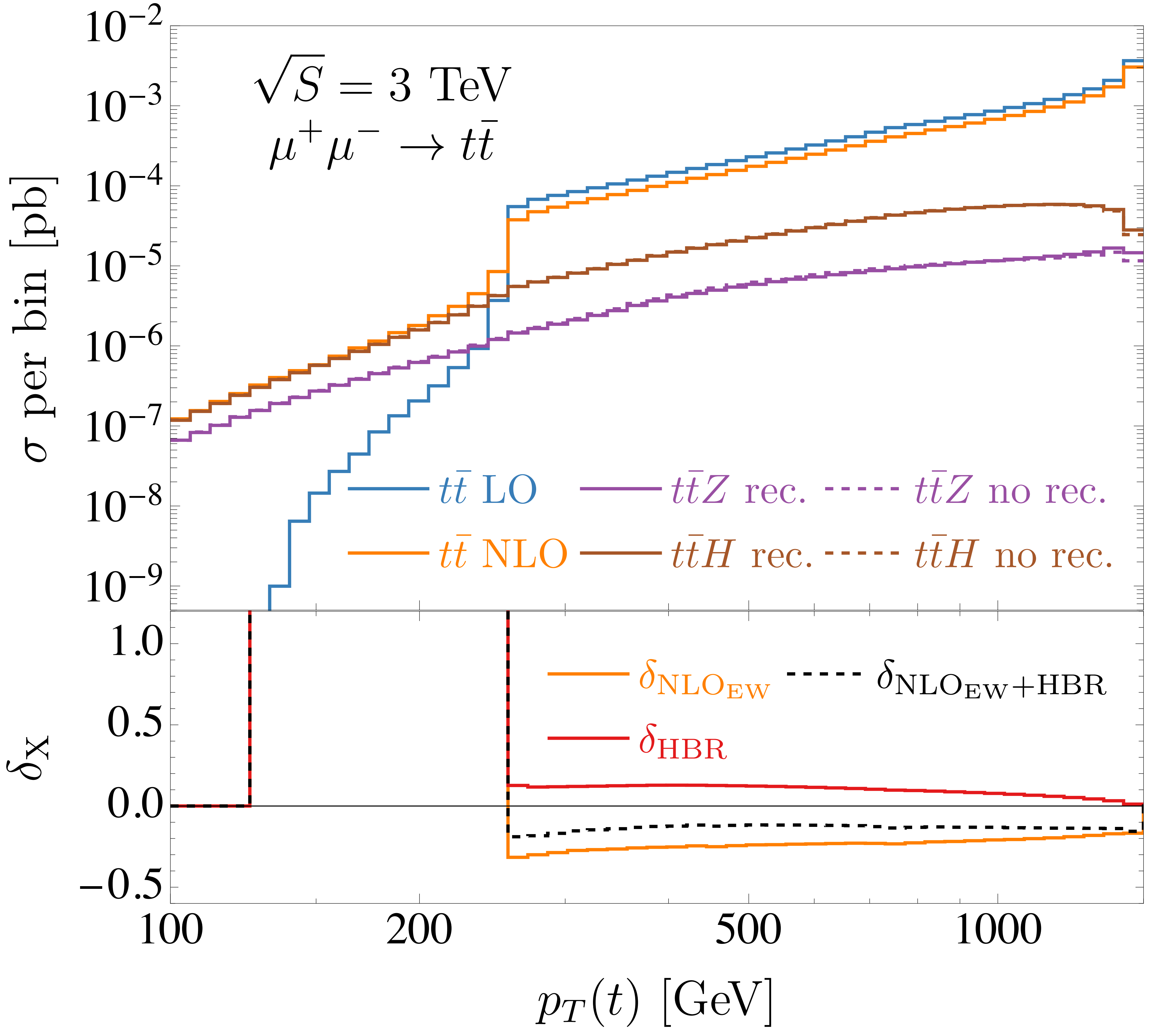


For High pt of the Z boson, the two Higgs can have very small ΔR and so small $m(HH)$, recoiling against the Z.

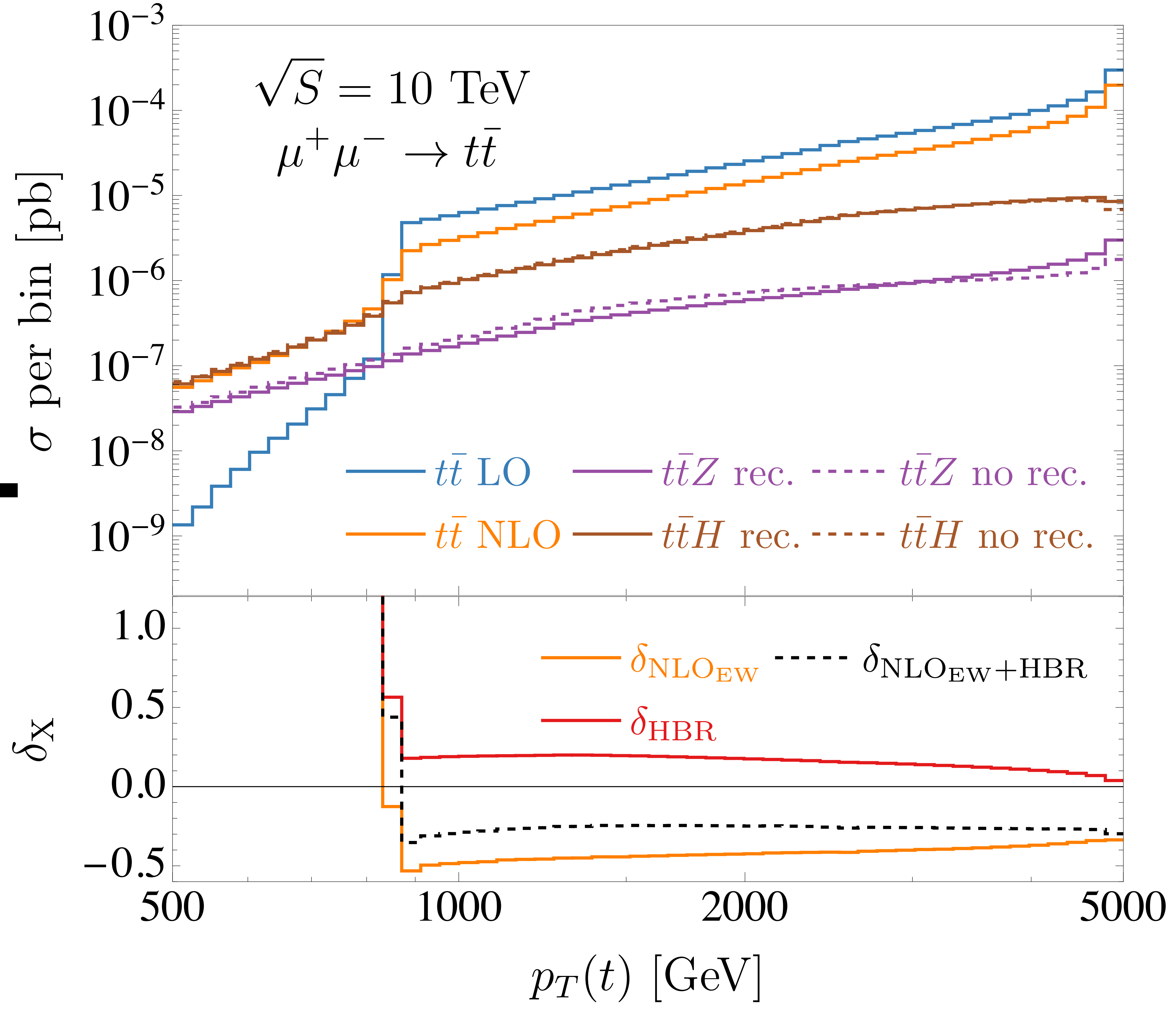
In that configuration, formally **mass suppressed** terms $\sim \dfrac{v}{m(H_1H_2)}$ can become numerically sizeable, and the **DP algorithm fails**.



Very small effects from Z and H radiation, especially in the bulk: $p_T(t) \simeq \sqrt{S}/2$



$t\bar{t}$



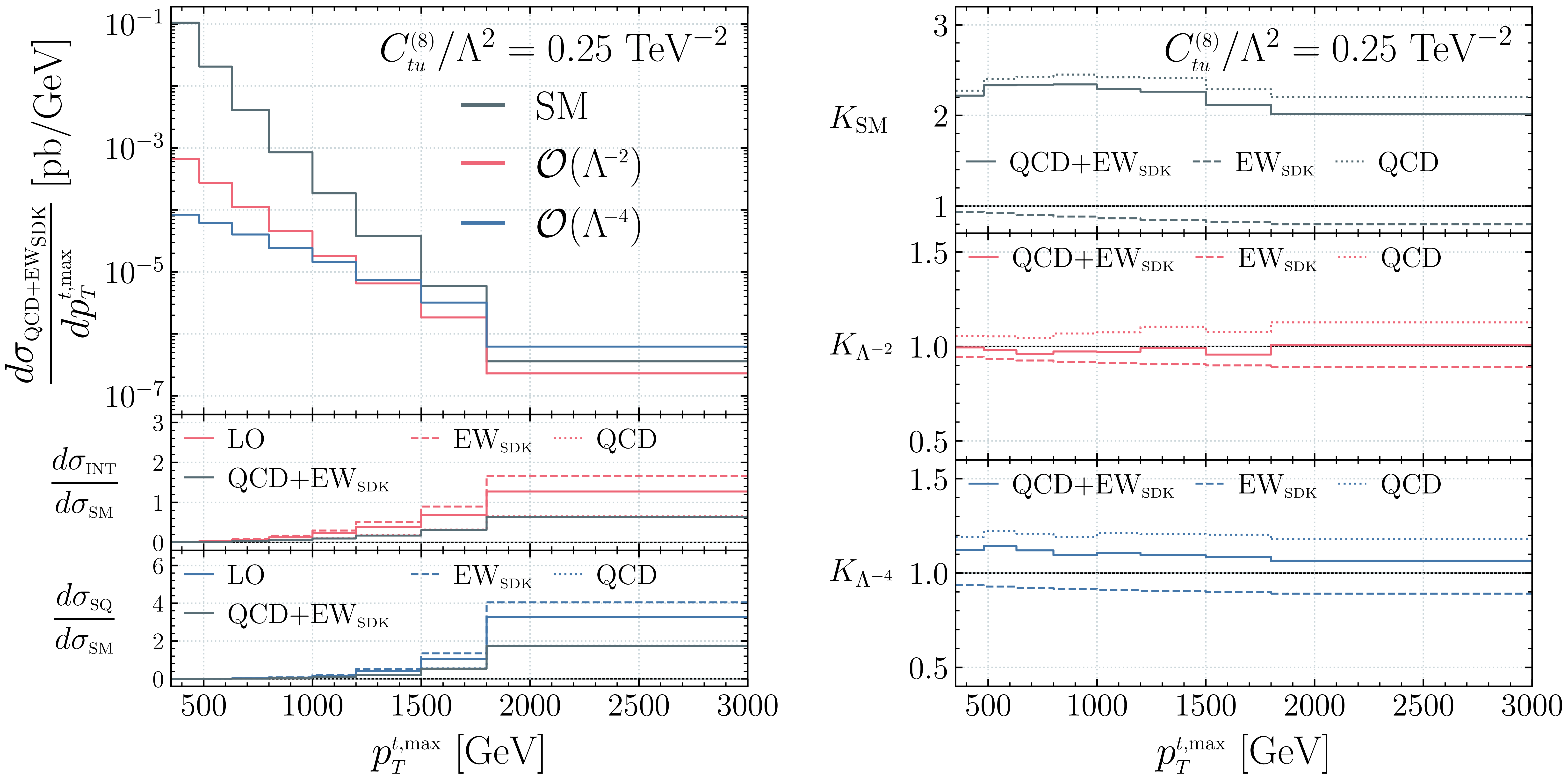
Notice that in order to allow more phase space we required just $m(F) > 0.5\sqrt{S}$.
Still HBR << NLO EW in absolute value.

Ma, DP, Zaro '24

EW Sudakov and SMEFT: $t\bar{t}$

Only Four-Fermion operators are considered in the study.

$$\mathcal{O}_{tu}^{8} = (\bar{t}\gamma^{\mu}T^{A}t)(\bar{u}_i\gamma_{\mu}T^{A}u_i)$$



LHC

Both QCD and EW corrections are different for SM, SM-SMEFT interference, and SMEFT^2 contributions of dim-6.

QCD and EW cancel each other: both are important.

K-factors can be different in SM and BSM!

El Faham, Mimasu, DP, Severi, Vryonidou, Zaro: in preparation

Our revisitation and automation: Amplitude level

We have **revisited** and **automated** in aMG5 the **Denner&Pozzorini algorithm** for the evaluation of one-loop EW Sudakov corrections to amplitudes (*Denner, Pozzorini '01*). In particular we have introduced the **following novelties**.

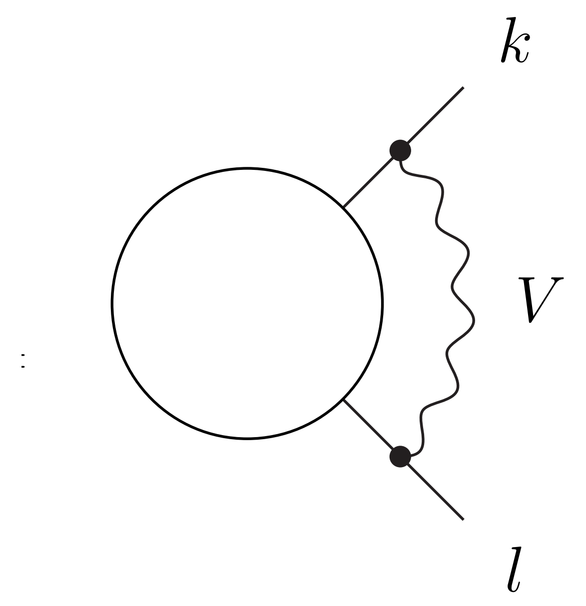
- **IR QED** divergencies are dealt with **via D**imensional **R**egularisation, with strictly massless photons and light fermions.

- **Additional logarithms** that involve ratios between invariants, and therefore **angular** dependences, are taken into account.

- We correctly take into account an **imaginary term** that was **previously omitted** in the literature. Relevant for $2 \to n$ processes with $n > 2$

- Moving to the level of interferences of tree and one-loop amplitudes, we take into account NLO EW contributions originating from **QCD loops on top of subleading LO terms**.

Derivation of LSC and SSC



Denner&Pozzorini

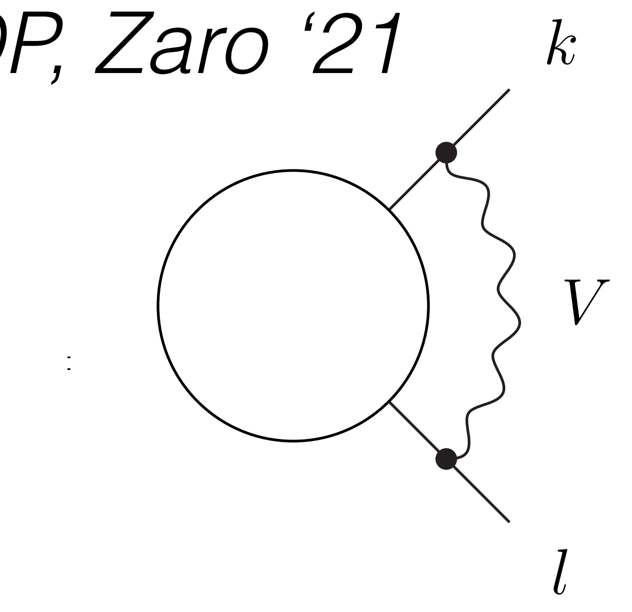
$$L(|r_{kl}|, M^2) = L(s, M^2) + 2l(s, M^2)\log\frac{|r_{kl}|}{s} + L(|r_{kl}|, s)$$

$$= \underbrace{L(s) + 2l(s)\log\frac{M_W^2}{M^2}}_{\text{LSC}} + \underbrace{2l(s)\log\frac{|r_{kl}|}{s}}_{\text{SSC}} + \cdots$$

$$L(s) \equiv L(s, M_W^2) \quad \text{and} \quad l(s) \equiv l(s, M_W^2)$$

The relation $r_{kl} = r_{k'l'} = s$ is used in all logs, unless they multiply $l(s)$.

DP, Zaro '21



Our approach: ~~$r_{kl} = r_{k'l'} = s$~~ **in the expressions**

$$C_0(p_k, p_l, M, M_k, M_l) \;\Rightarrow\; L(|r_{kl}|, M^2) - 2i\pi\Theta(r_{kl})l(|r_{kl}|, M^2)$$

Previously omitted imaginary term

$$L(|r_{kl}|, M^2) - 2i\pi\Theta(r_{kl})l(|r_{kl}|, M^2) =$$

$$= L(s, M^2) + 2l(s, M^2)\left(\log\frac{|r_{kl}|}{s} - i\pi\Theta(r_{kl})\right) + L(|r_{kl}|, s) - 2i\pi\Theta(r_{kl})l(|r_{kl}|, s) =$$

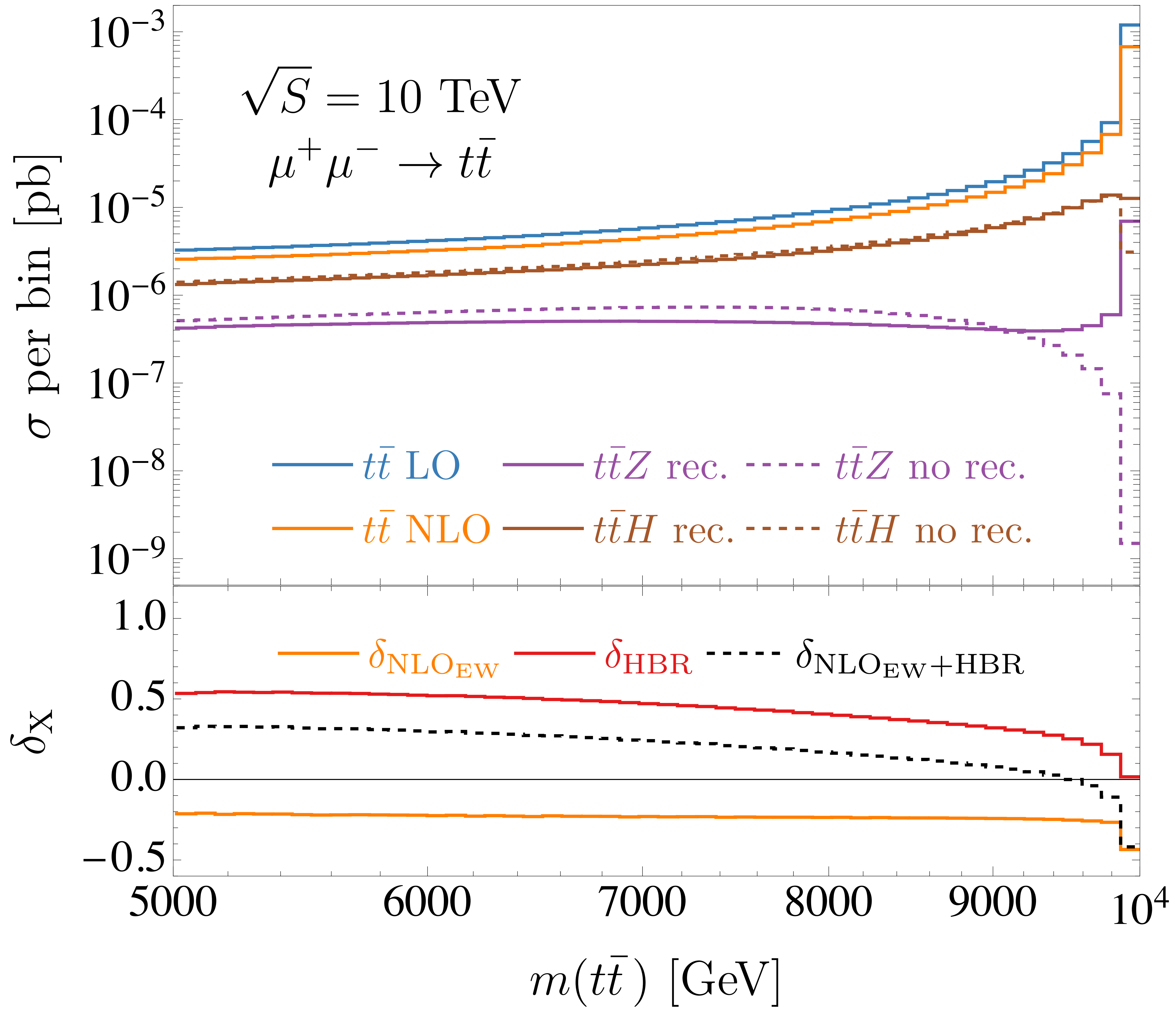
$$= \underbrace{L(s) + 2l(s)\log\frac{M_W^2}{M^2}}_{\text{LSC}} + \underbrace{2l(s)\left(\log\frac{|r_{kl}|}{s} - i\pi\Theta(r_{kl})\right)}_{\text{SSC}} +$$

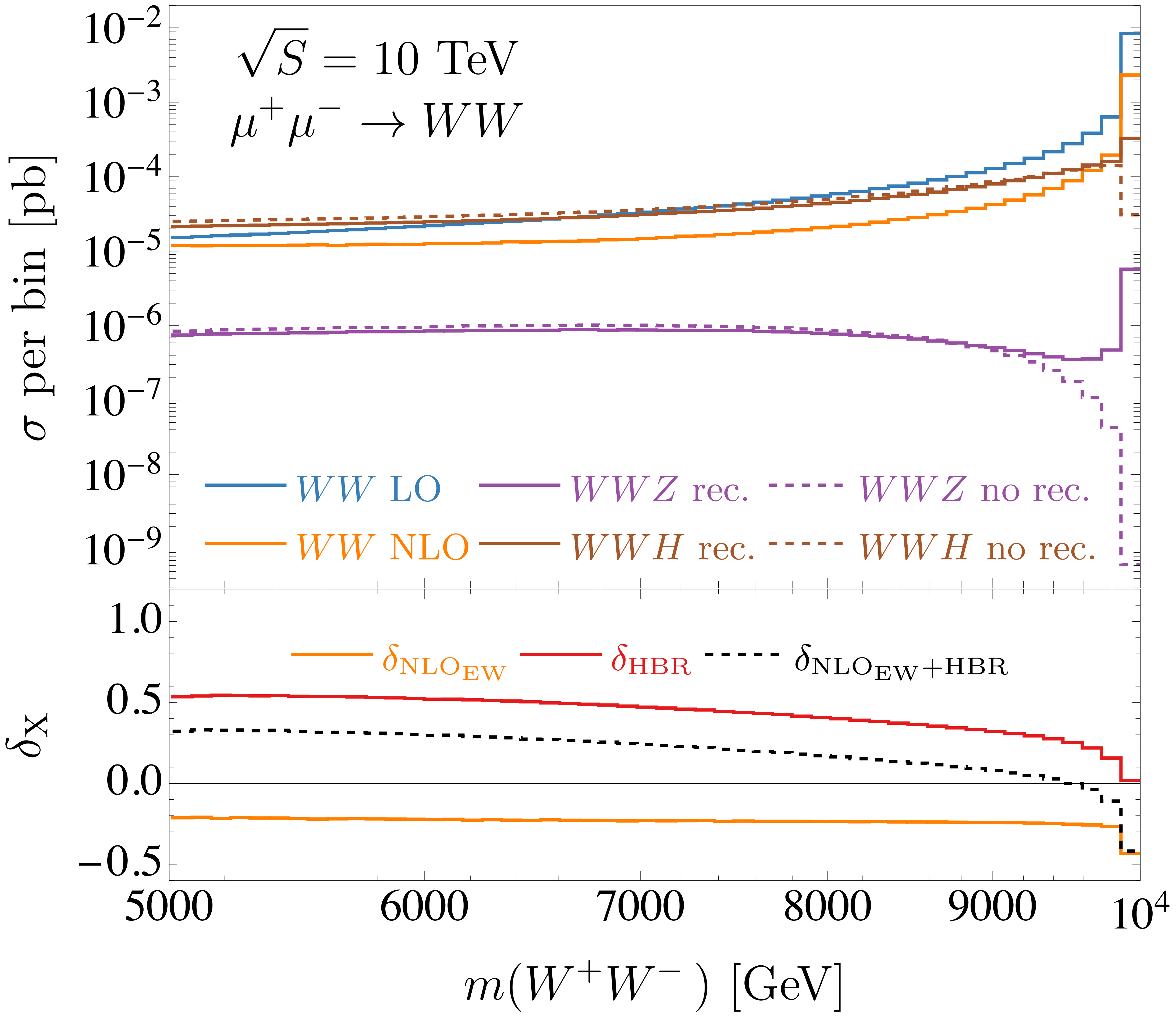
$$\underbrace{2l(M_W^2, M^2)\log\frac{|r_{kl}|}{s} + L(|r_{kl}|, s) - 2i\pi\Theta(r_{kl})l(|r_{kl}|, s)}_{\text{SSC}^{s\to r_{kl}}} + \cdots$$

The conceptual derivation relies on the assumption $s = r_{kl}$, but is not actually used in the expressions. Therefore, further angular dependencies are taken into account.

New angular dependences via ratios among invariants

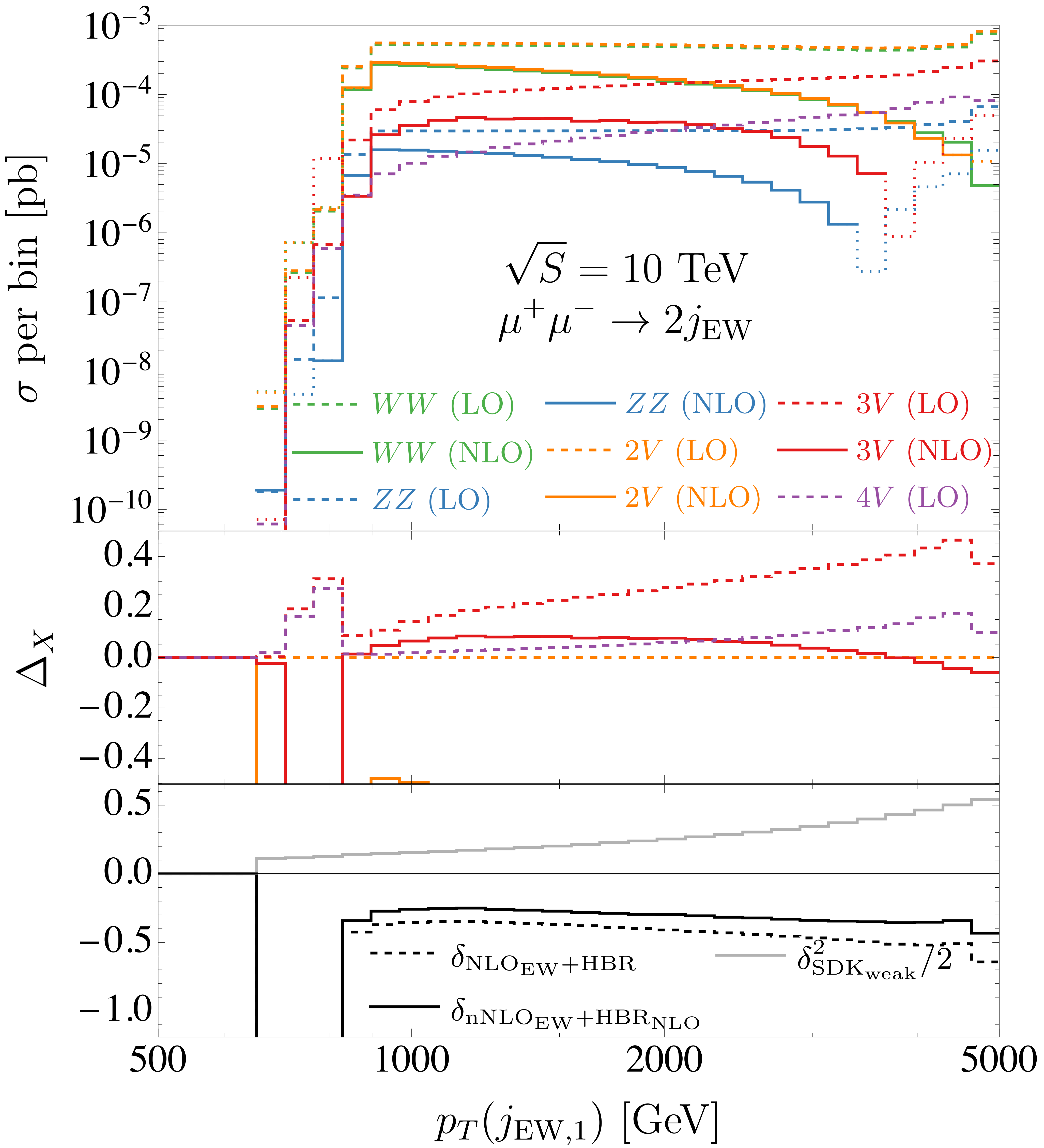
more on the very small effects from Z and H radiation





Ma, DP, Zaro '24

EW jets



$$\sigma_X(2j_{\rm EW}) \equiv \sigma_X(2V) \qquad \text{for } X = \text{LO, NLO EW, SDK}_{\rm weak}$$

$$\sigma_{\rm HBR}(2j_{\rm EW}) \equiv \sigma_{\rm LO}(3V)\,,$$

$$\sigma_{\rm NLO_{EW}+HBR}(2j_{\rm EW}) \equiv \sigma_{\rm NLO_{EW}}(2V) + \sigma_{\rm LO}(3V)$$

$$\sigma_{\rm nNLO_{EW}+HBR_{NLO}}(2j_{\rm EW}) \equiv \sigma_{\rm LO}(2V)\left(1 + \delta_{\rm NLO_{EW}} + \frac{\delta^2_{\rm SDK_{weak}}}{2}\right) + \sigma_{\rm NLO_{EW}}(3V) + \sigma_{\rm LO}(4V)\,.$$

$$\Delta_X(2V) \equiv \frac{\sigma_X(2V) - \sigma_{\rm LO}(2V)}{\sigma_{\rm LO}(2V)}$$

$$\Delta_X(3V) \equiv \frac{\sigma_X(3V)}{\sigma_{\rm LO}(2V)}$$

$$\Delta_X(4V) \equiv \frac{\sigma_X(4V)}{\sigma_{\rm LO}(2V)}$$

It is a general pattern: radiation of heavy bosons is much less important than loops!

Ma, DP, Zaro '24

Cross-sections: our approach.

FOR WHAT EW SUDAKOV ARE USEFUL?
For providing a very **good approximation of NLO EW** in the **high-energy** limit.

HOW SHOULD ONE PERFORM THE CALCULATION IN THE HIGH-ENERGY LIMIT?
Photons have to be **always clustered with massless charged particle for IR**-safety reasons. But from an experimental point of view, **at high energy also clustering tops and W bosons with photons** is very reasonable, either if you imagine to tag heavy object directly or via their massless decay products.

The **QED Logs,** involving s and λ^2 (or Q^2), **cancel against their real-emission counterparts and PDF counterterms**. The only one surviving are those from tops in vacuum polarisation for external (not tagged) photons, both in the initial and final state:

$$\mathrm{SDK}_{\mathrm{weak}}$$

Almost all the contributions of QED are removed
(e.g. $C_{\mathrm{EW}}(k) \to C_{\mathrm{EW}}(k) - Q_k^2$, $Q_k^2 = 0$),
but NOT in the parameter renormalisation δ^{PR}.

DP, Zaro '21

Implementation

Born amplitude: $\mathcal{M}_0^{i_1 \ldots i_n}(p_1, \ldots, p_n)$

One-loop EW Sudakov corrections: $\delta\mathcal{M}^{i_1 \ldots i_n}(p_1, \ldots, p_n) = \mathcal{M}_0^{i'_1 \ldots i'_n}(p_1, \ldots, p_n)\delta_{i'_1 i_1 \ldots i'_n i_n}$

other tree-level amplitudes — the logs

Born process: $\varphi_{i_1}(p_1) \ldots \varphi_{i_n}(p_n) \to 0$

$$Z \longrightarrow \chi\,, \quad W^{\pm} \longrightarrow \phi^{\pm}\,,$$

GBE theorem for longitudinal W and Z bosons.

$$\varphi_{i_1}(p_1) \ldots \varphi_{i'_k} \ldots \varphi_{i_n}(p_n) \to 0\,,$$

$$Z \longleftrightarrow A\,, \quad H \longleftrightarrow \chi\,.$$

Relevant for LSC and C contributions.

$$\varphi_{i_1}(p_1) \ldots \varphi_{i'_k} \ldots \varphi_{i'_l} \ldots \varphi_{i_n}(p_n) \to 0$$

$$f_\sigma \longleftrightarrow f_{-\sigma}\,, \quad H \longleftrightarrow \phi^{\pm}\,, \quad \chi \longleftrightarrow \phi^{\pm}\,, \quad A \longleftrightarrow W^{\pm}\,, \quad Z \longleftrightarrow W^{\pm}\,.$$

Relevant for SSC charged contributions.

Amplitudes with one or 2 different external particles w.r.t. the Born have to be generated.

Organisation of the logs in the algorithm

Two examples: LSC and C for fermions

$$\delta^{\mathrm{LSC}}_{i'_k i_k}(k) = -\frac{1}{2}\left[C^{\mathrm{ew}}_{i'_k i_k}(k) L(s) - 2(I^Z(k))^2_{i'_k i_k} \log\frac{M_Z^2}{M_W^2} l(s) + \delta_{i'_k i_k} Q_k^2 L^{\mathrm{em}}(s,\lambda^2,m_k^2)\right]$$

Casimir for the entire $SU(2)_L \times U(1)_B$

Charge for $U(1)_{QED}$

$$\delta^{\mathrm{C}}_{f_\sigma f_{\sigma'}}(f^\kappa) = \delta_{\sigma\sigma'}\left\{\left[\frac{3}{2}C^{\mathrm{ew}}_{f^\kappa} - \frac{1}{8s_{\mathrm{w}}^2}\left((1+\delta_{\kappa\mathrm{R}})\frac{m^2_{f_\sigma}}{M_{\mathrm{W}}^2} + \delta_{\kappa\mathrm{L}}\frac{m^2_{f_{-\sigma}}}{M_{\mathrm{W}}^2}\right)\right] l(s) + Q^2_{f_\sigma} l^{\mathrm{em}}(m^2_{f_\sigma})\right\},$$

$$L(s) \equiv L(s, M_W^2) \quad \text{and} \quad l(s) \equiv l(s, M_W^2)$$

$$l^{\mathrm{em}}(m_f^2) := \frac{1}{2} l(M_{\mathrm{W}}^2, m_f^2) + l(M_{\mathrm{W}}^2, \lambda^2) \qquad L^{\mathrm{em}}(s,\lambda^2,m_k^2) := 2l(s)\log\left(\frac{M_{\mathrm{W}}^2}{\lambda^2}\right) + L(M_{\mathrm{W}}^2,\lambda^2) - L(m_k^2,\lambda^2)$$

The full EW is present between s and M_W^2, while only QED is present between M_W^2 and λ^2.

So the QED contribution is split between the intervals $(s, M_W^2) + (M_W^2, \lambda^2)$. But the division at M_W^2 is simply determined by convenience, in parallel with the weak case. In this case M_W^2 is just a technical parameter and not a physical quantity.

Cross-sections: standard approach in the literature

SDK_0

Two examples: LSC and C for fermions

$$\delta^{\mathrm{LSC}}_{i'_k i_k}(k) = -\frac{1}{2}\left[C^{\mathrm{ew}}_{i'_k i_k}(k) L(s) - 2(I^Z(k))^2_{i'_k i_k}\log\frac{M_Z^2}{M_W^2}\, l(s) + \delta_{i'_k i_k} Q_k^2 \ldots \lambda^2, m_k^2)\right]$$

Casimir for the entire
$SU(2)_L \times U(1)_B$

$$\delta^{\mathrm{C}}_{f_\sigma f_{\sigma'}}(f^\kappa) = \delta_{\sigma\sigma'}\left\{\left[\frac{3}{2}C^{\mathrm{ew}}_{f^\kappa} - \frac{1}{8s_{\mathrm{w}}^2}\left((1+\delta_{\kappa\mathrm{R}})\frac{m^2_{f_\sigma}}{M_{\mathrm{W}}^2} + \delta_{\kappa\mathrm{L}}\frac{m^2_{f_{-\sigma}}}{M_{\mathrm{W}}^2}\right)\right] l(s) + Q^2_{f_\sigma} \ldots {}^2_{f_\sigma})\right\}$$

$L(s) \equiv L(s, M_W^2)$ and $l(s) \equiv l(s, M_W^2)$

The logarithms between M_W^2 and the infrared scale are simply removed. Equivalently in the case of DR, logarithms involving M_W^2 and the IR regulator Q^2.

Easy, but not very well motivated.

We will denote in the following this approach as SDK_0.