Energy Energy correlators in DIS

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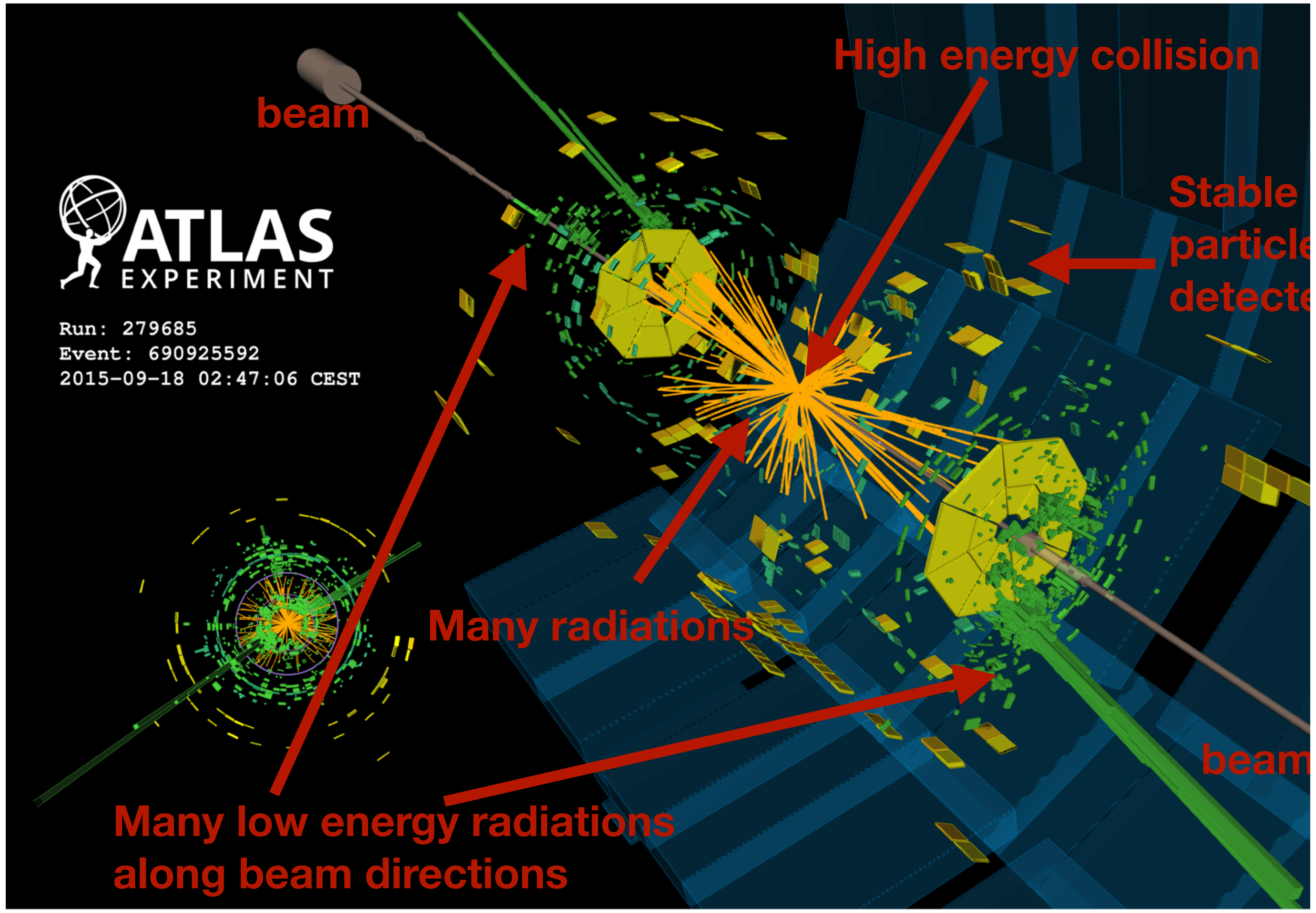
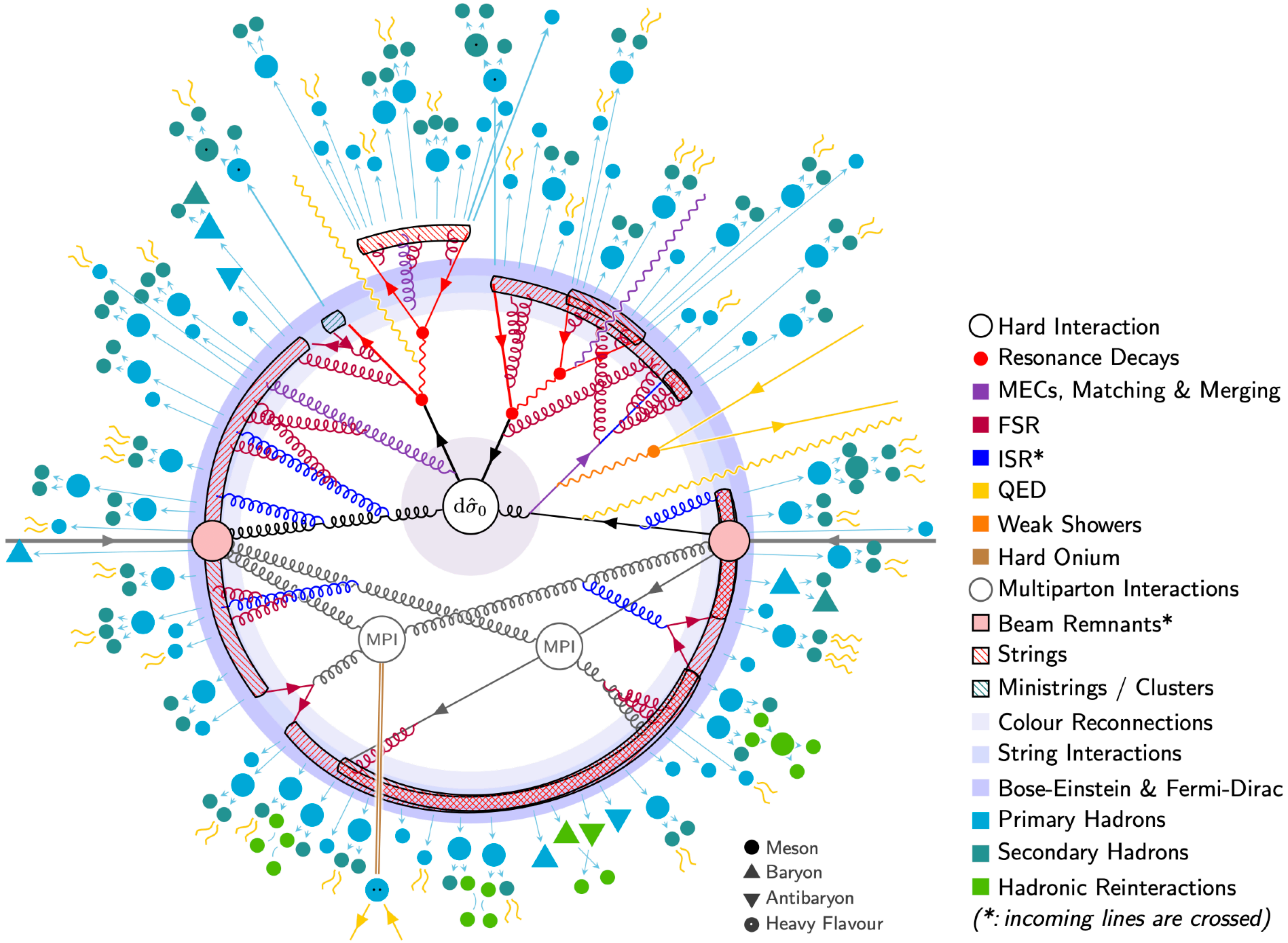
Fang, Gao, HTL, Shao, arXiv:2409.09248
Cao, HTL, Mi, PRD, 2024
Gao, HTL, Moult, Zhu, JHEP, 2024
HTL, Vitev, Zhu, JHEP, 2020
HTL, Makris, Vitev, PRD, 2020
Gao, HTL, Moult, Zhu, PRL, 2019

Rich Phenomenology due to QCD



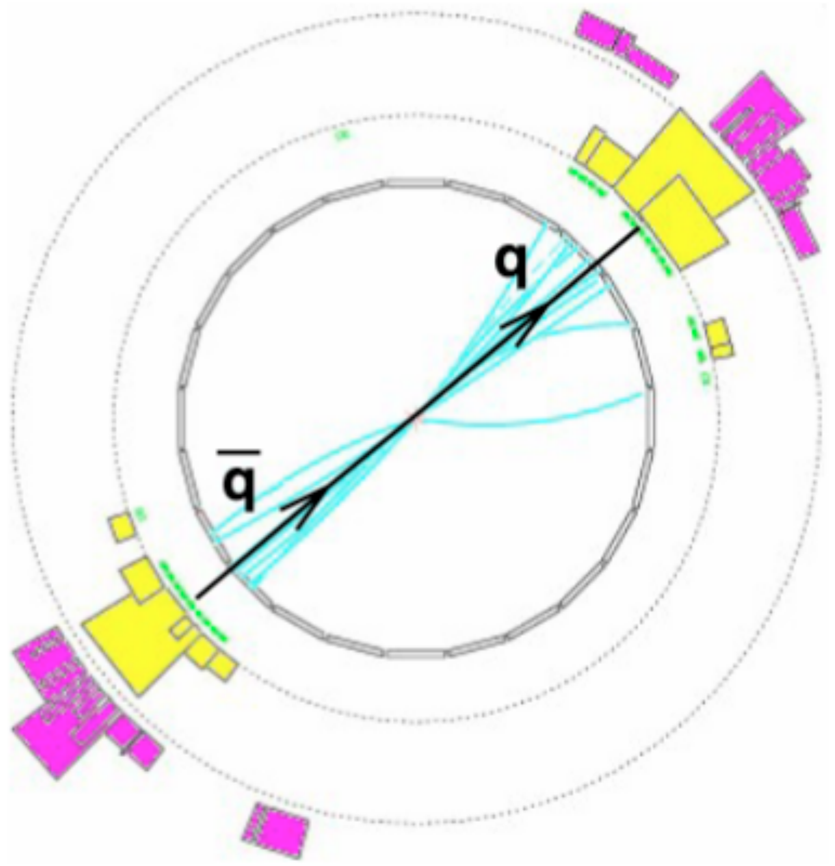


From PYTHIA 8.3

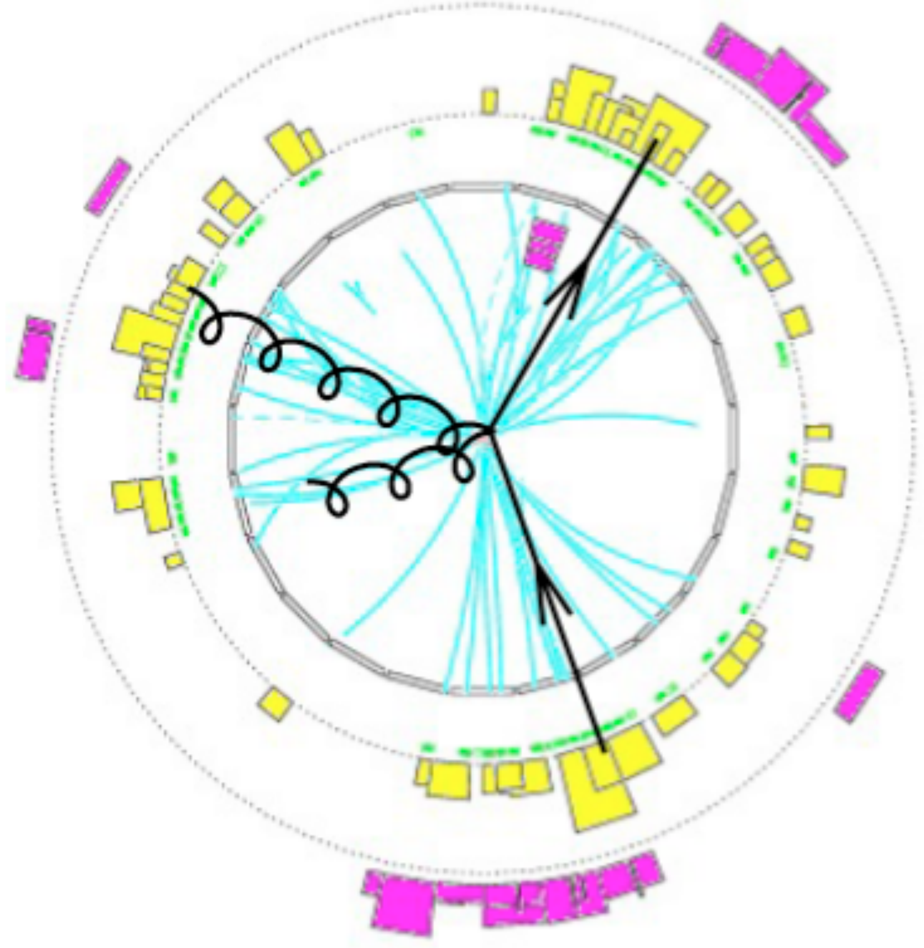
Event Shape

Event Shape: Most basic class of final-state observables

Thrust: $T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$



T=1



Broader distributions

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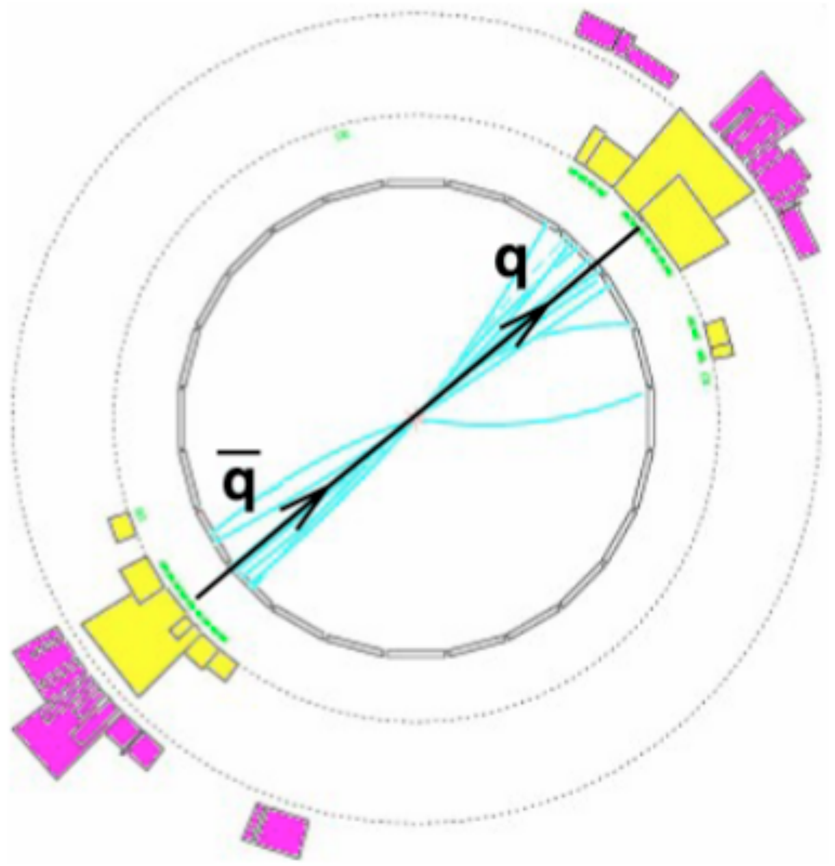

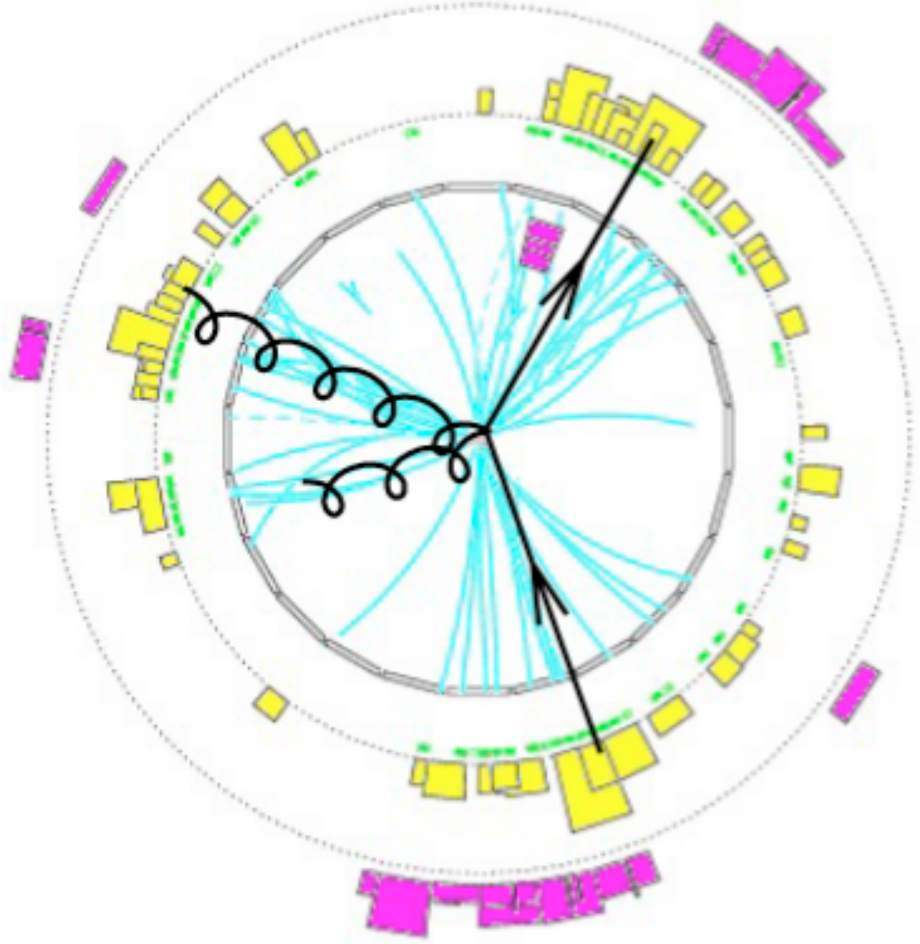

T=1

- **parametrizing geometrical properties of the energy and momentum flow**
- **sensitive to the flow of radiations in a scattering event**
- **distribution of deviation from leading-order event**
- **extensively investigated at e^+e^- collider and in DIS**

Event shape observables have long provided useful insights into the underlying dynamics of quantum chromodynamics

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Applications of Event shape: serve as a QCD laboratory, a tool for QCD study

Usually defined as normalized distribution which will reduce the sensitivity to calibration and luminosity

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- most importantly: a crucial role in measuring the strong coupling
- testing and tuning the parton showers
- tuning non-perturbative components of Monte Carlo event generators
- developing and testing insight into perturbative QCD
- searching new physics, such as deriving constraints on potential new colored particle

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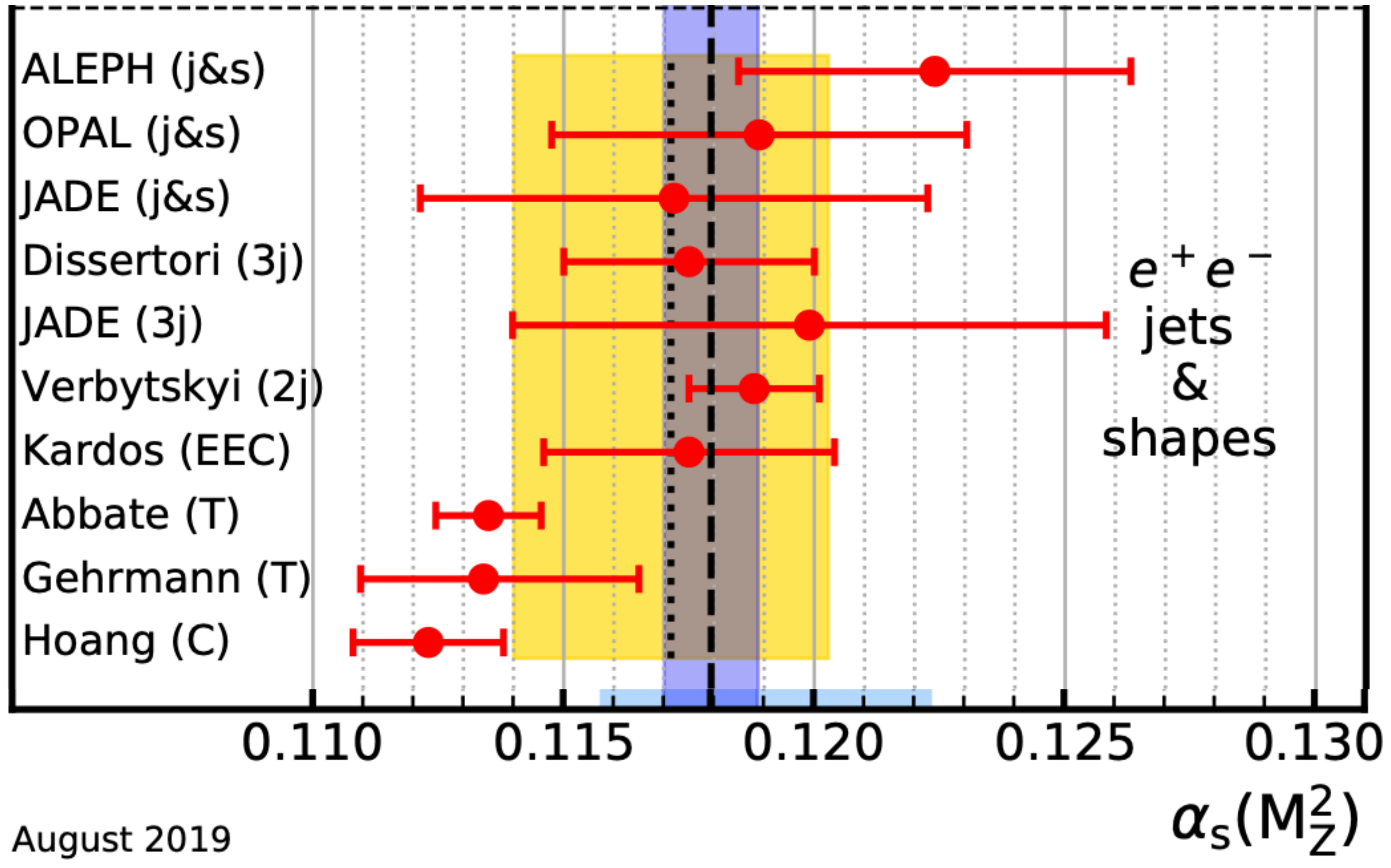
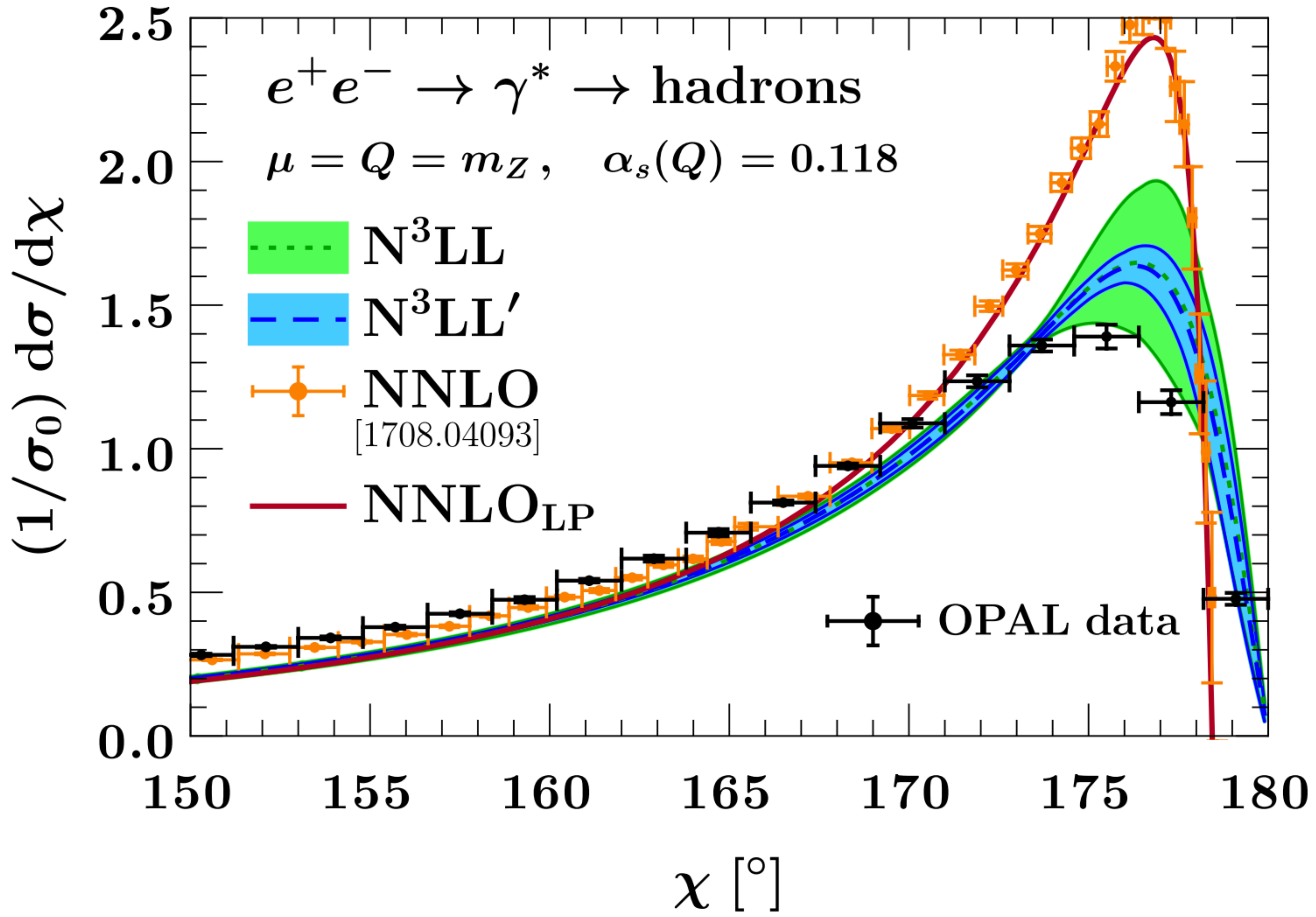
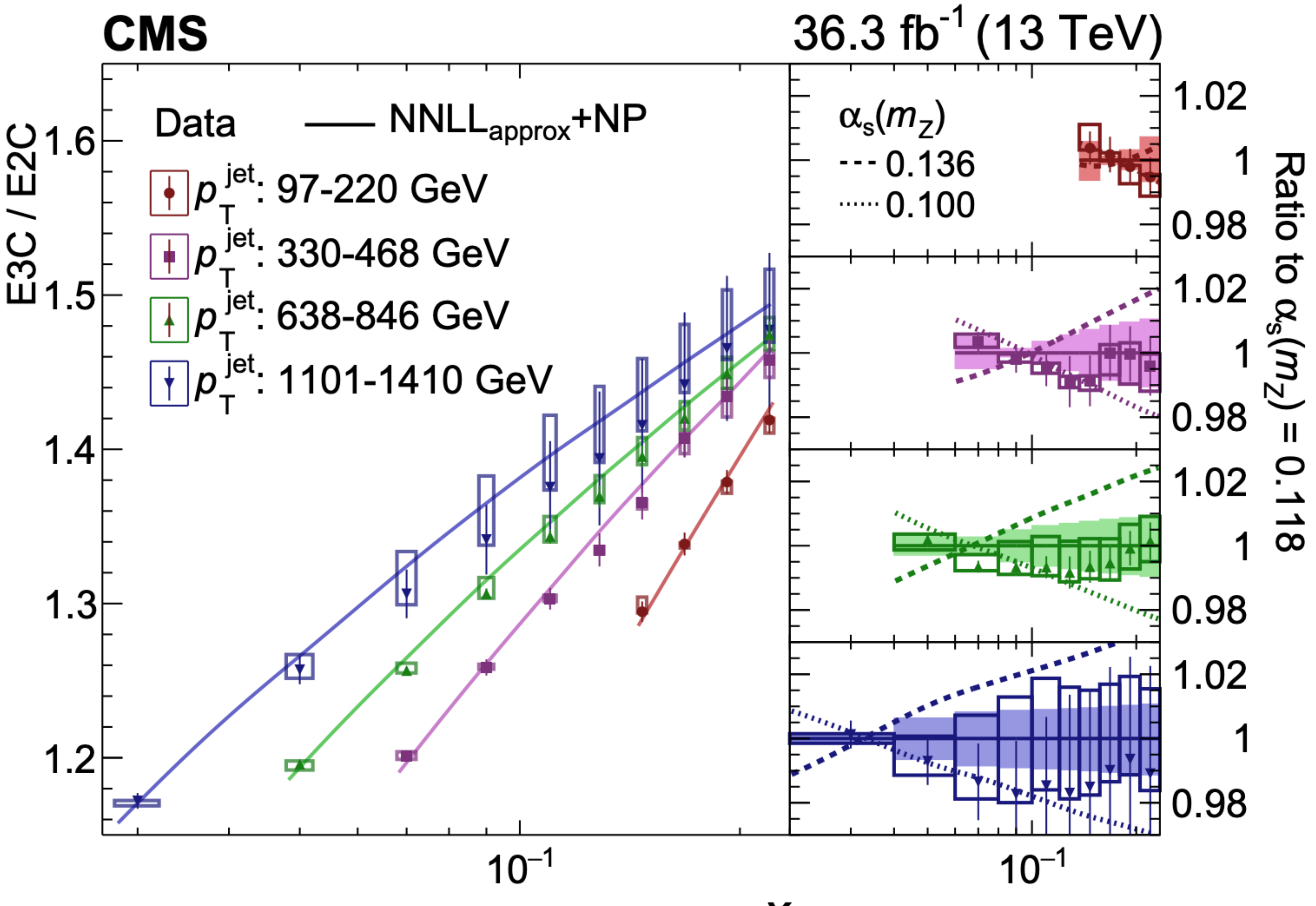


Event Shape

EEC/TEEC is a class of event shape variables



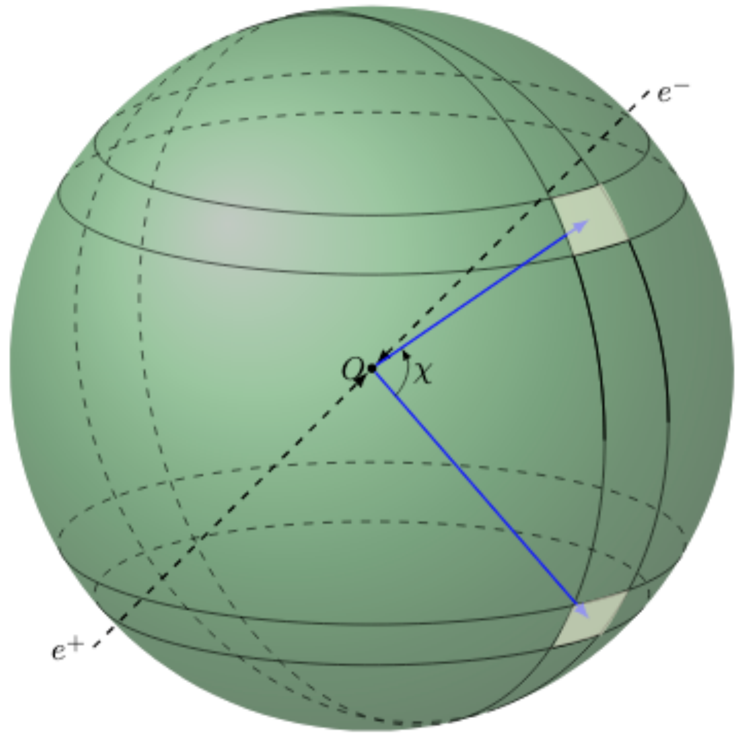


CMS, 2402.13864

see Zhen Lin's talk

EEC and TEEC

e^+e^- Collisions

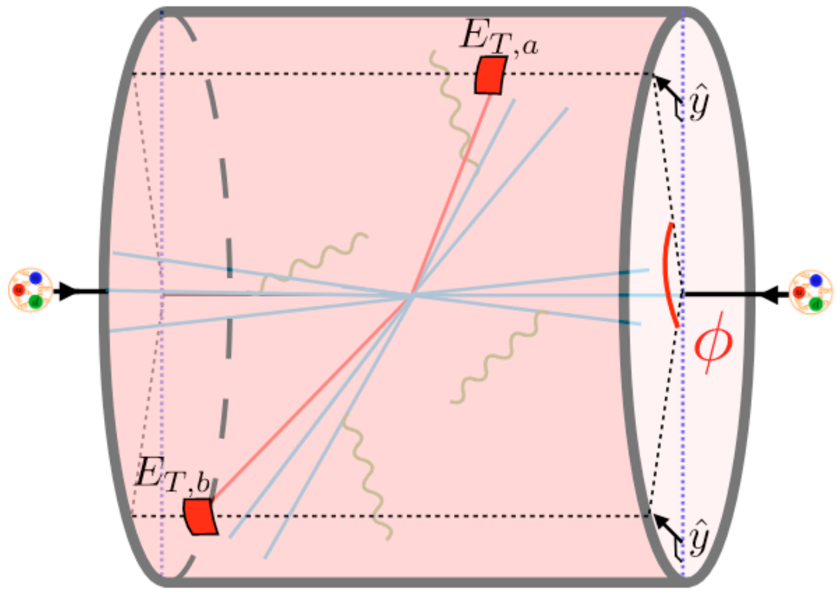


$$\mathrm{EEC} = \sum_{a,b} \int d\sigma_{V\to a+b+X} \frac{2E_a E_b}{Q^2 \sigma_{\mathrm{tot}}} \delta(\cos(\theta_{ab}) - \cos(\chi))$$

- sum over all the jets for each event
- sum over all the particles for each event

Basham et al 1978
Moult, Zhu, 2018

Hadronic initial state



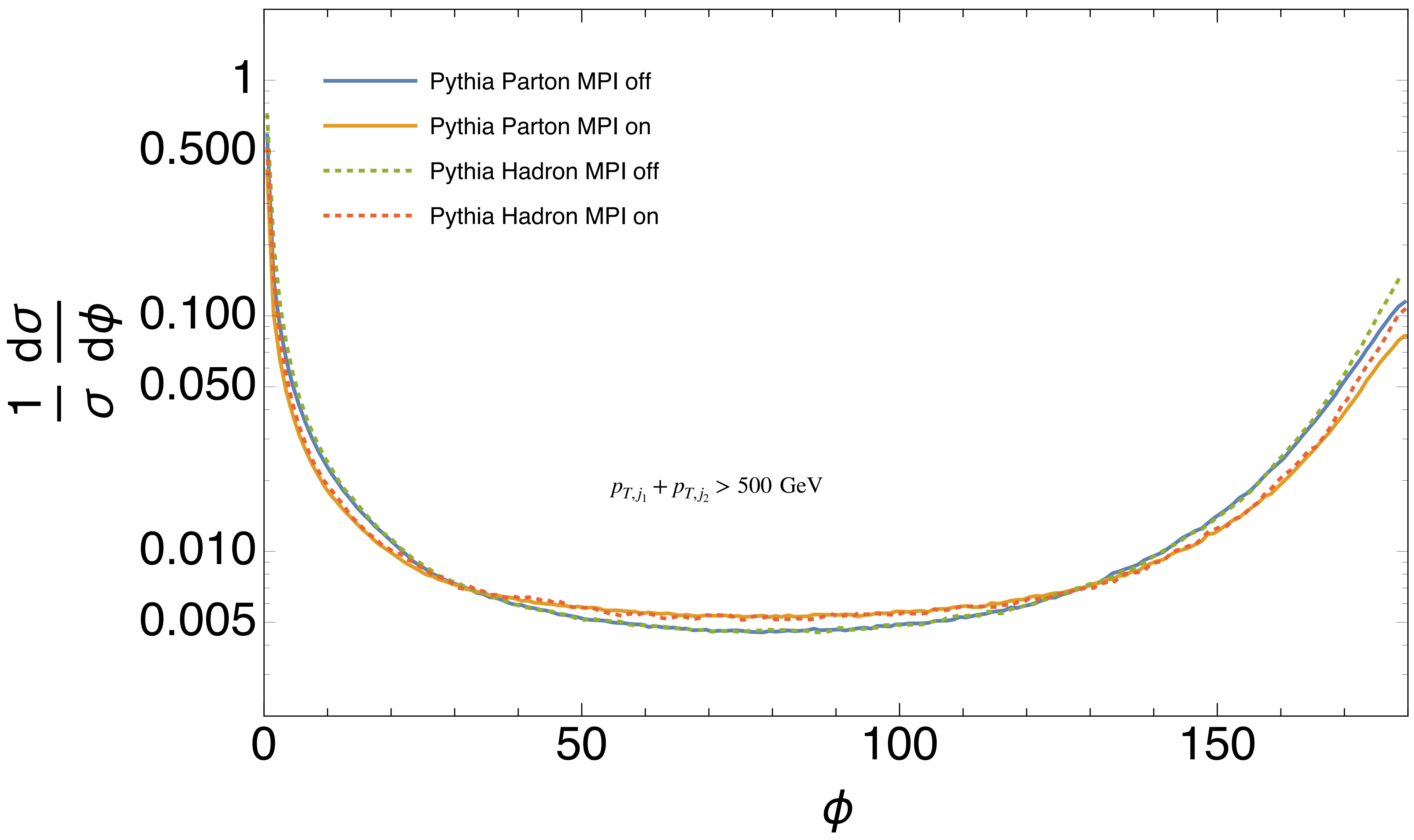
observable

$$\mathrm{TEEC} = \sum_{a,b} \int d\sigma_{pp\to a+b+X} \frac{2E_{T,a} E_{T,b}}{|\sum_i E_{T,i}|^2} \delta(\cos\phi_{ab} - \cos\phi)$$

- weighted cross section
- the soft radiation does not contribute directly to the observable at leading power
- soft gluon contributes only via recoil

Ali et al 1984
Gao, HTL, Moult, Zhu, 2019, 2023

TEEC at LHC

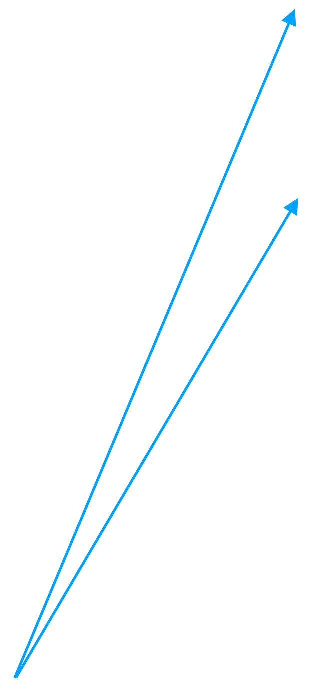


Pythia Parton MPI off
Pythia Parton MPI on
Pythia Hadron MPI off
Pythia Hadron MPI on
$p_{T,j_1} + p_{T,j_2} > 500$ GeV
$\frac{1}{\sigma}\frac{d\sigma}{d\phi}$
1
0.500
0.100
0.050
0.010
0.005
0
50
100
150
ϕ

TEEC at LHC

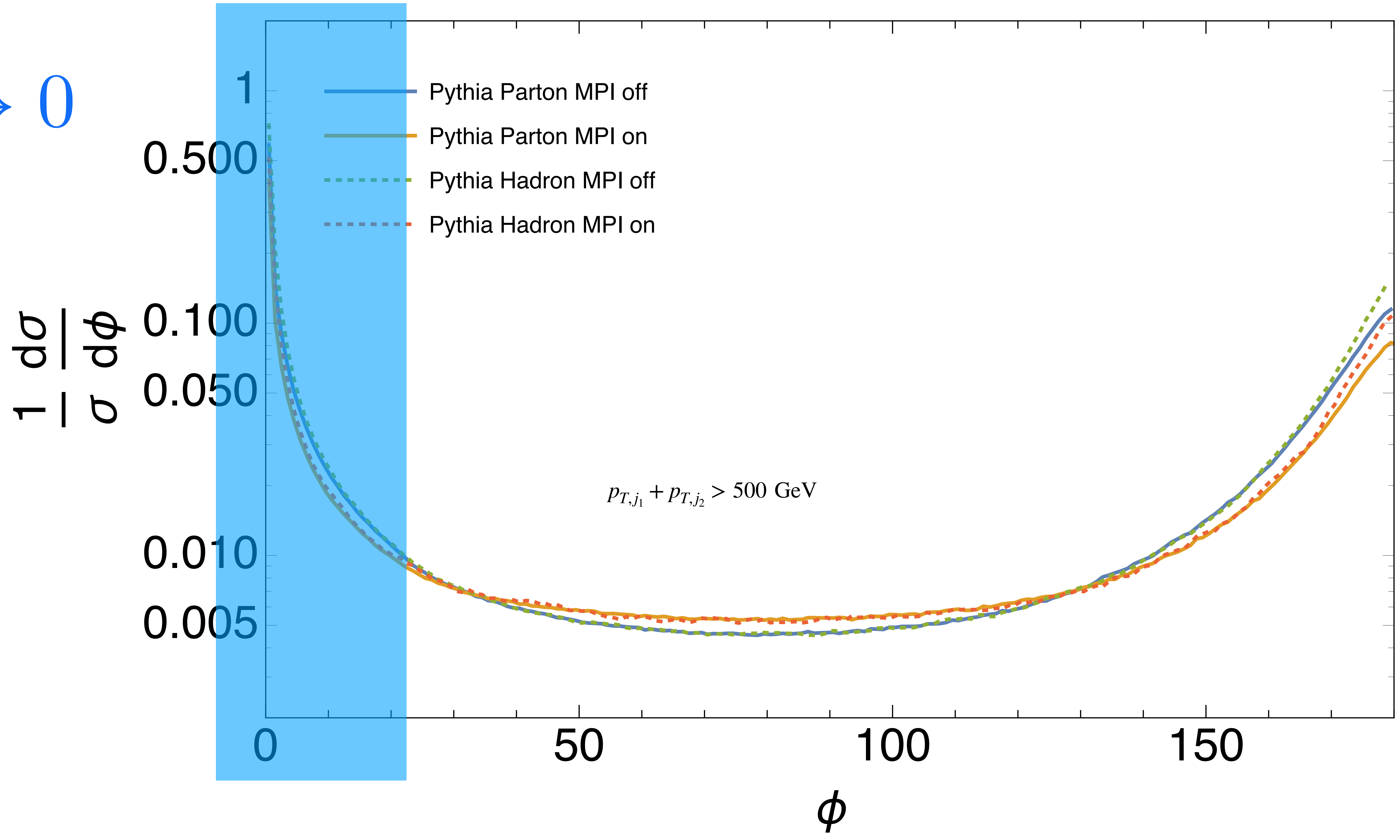
Collinear singularity

$$\cos\phi_{ab} \to 0$$



$\phi \sim 0$

$J \otimes H$



Dixon, Moult, Zhu, 2019
Kologlu, Kravchuk, et al 2019
Korchemsky 2019

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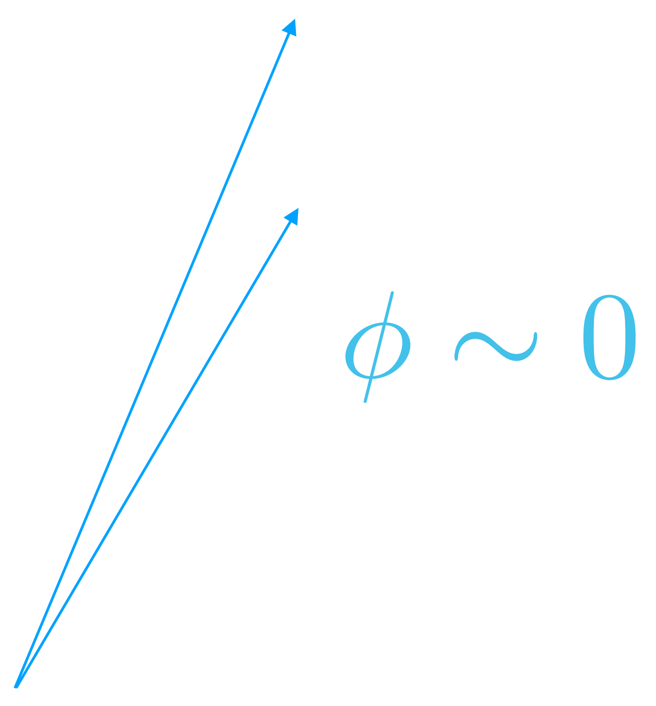

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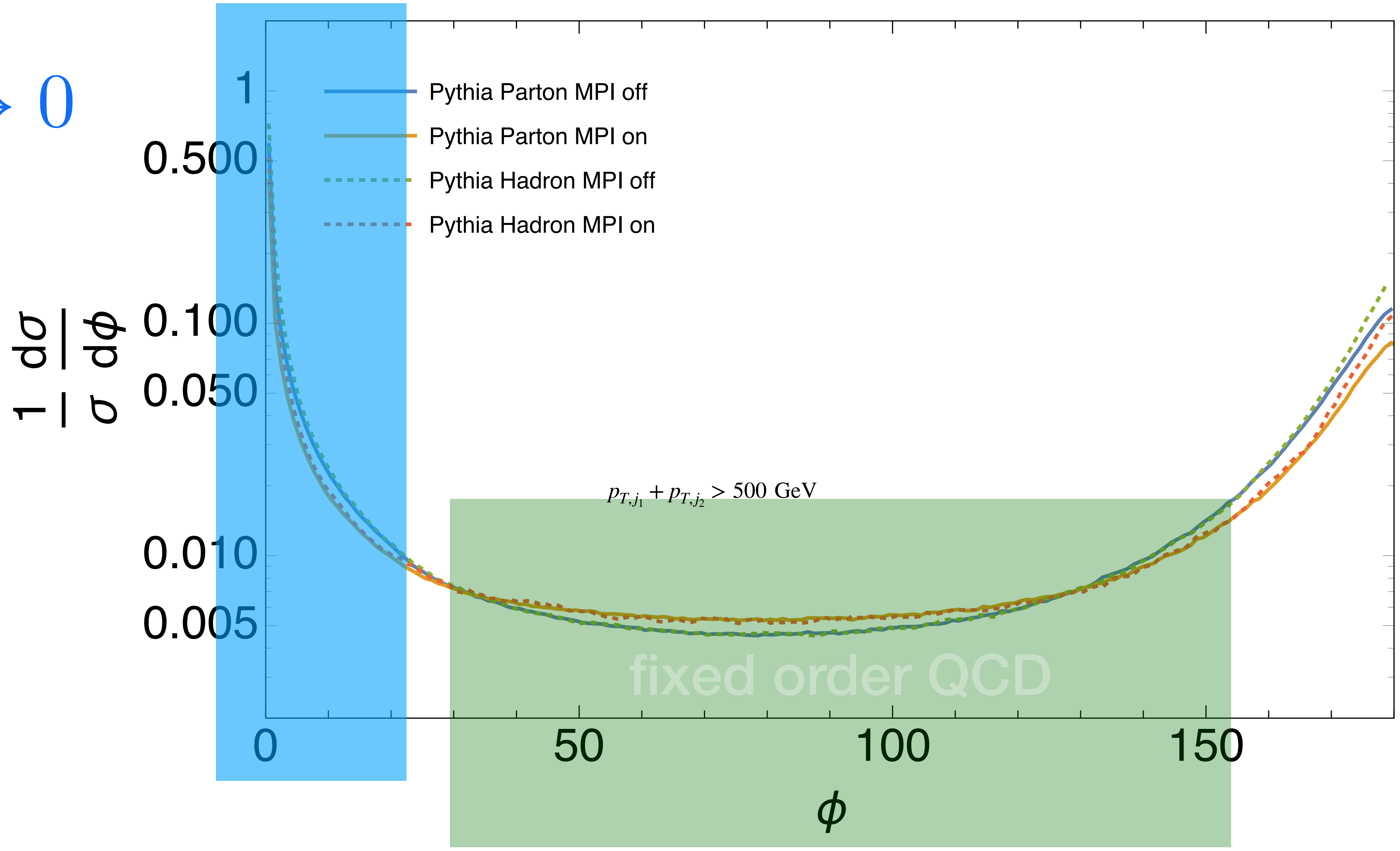

Dixon, Moult, Zhu, 2019
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Ali, Barreiro, Llorente, Wang, 2012
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Luo, Shtabovenko, Yang, Zhu, 2019
and many studied in $\mathcal{N}=4$ super-Yang-Mills theory
Alvarez, Cantero, Czakon, et al 2024

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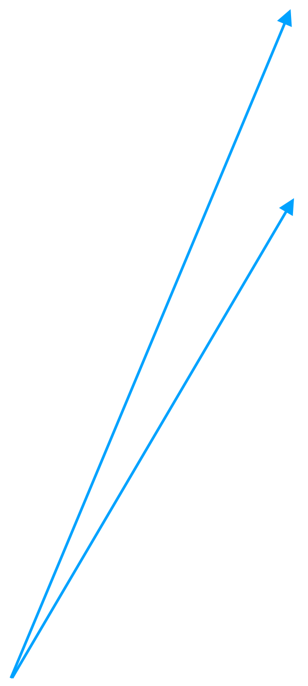

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Dixon, Moult, Zhu, 2019
Kologlu, Kravchuk, et al 2019
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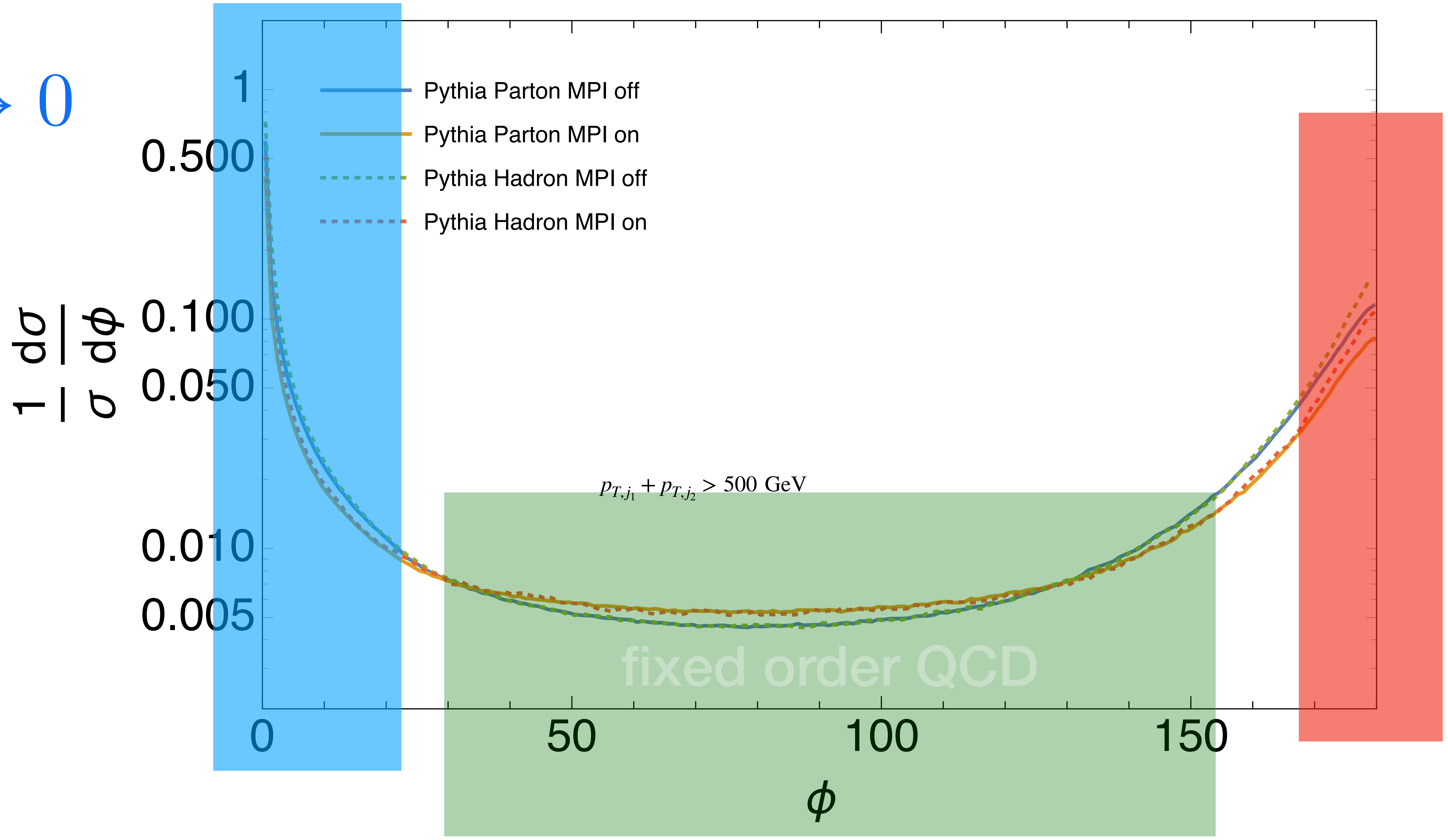

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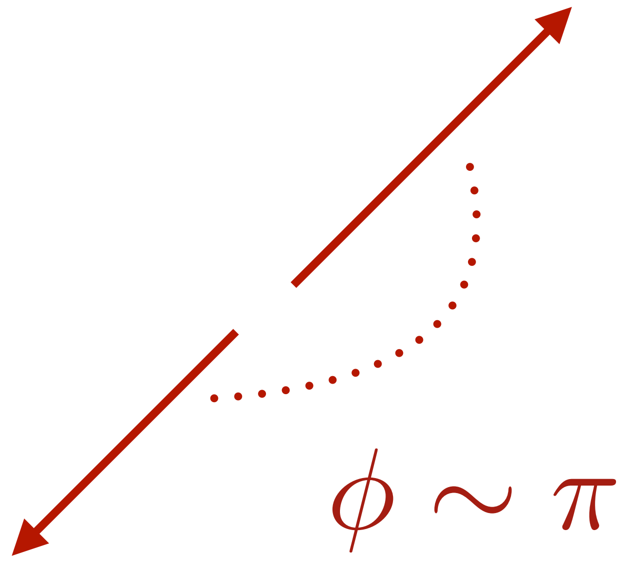
Luo, Shtabovenko, Yang, Zhu, 2019
and many studied in $\mathcal{N}=4$ super-Yang-Mills theory
Alvarez, Cantero, Czakon, et al 2024

Collinear and soft singularity

$\cos\phi_{ab} \to -1$



$\phi \sim \pi$

$B \otimes H \otimes J \otimes S$

Banfi, Salam and Zanderighi 2010
Gao, HTL, Moult, Zhu, 2019, 2023

EEC and TEEC

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where nonperturbative effect is supposed to be small

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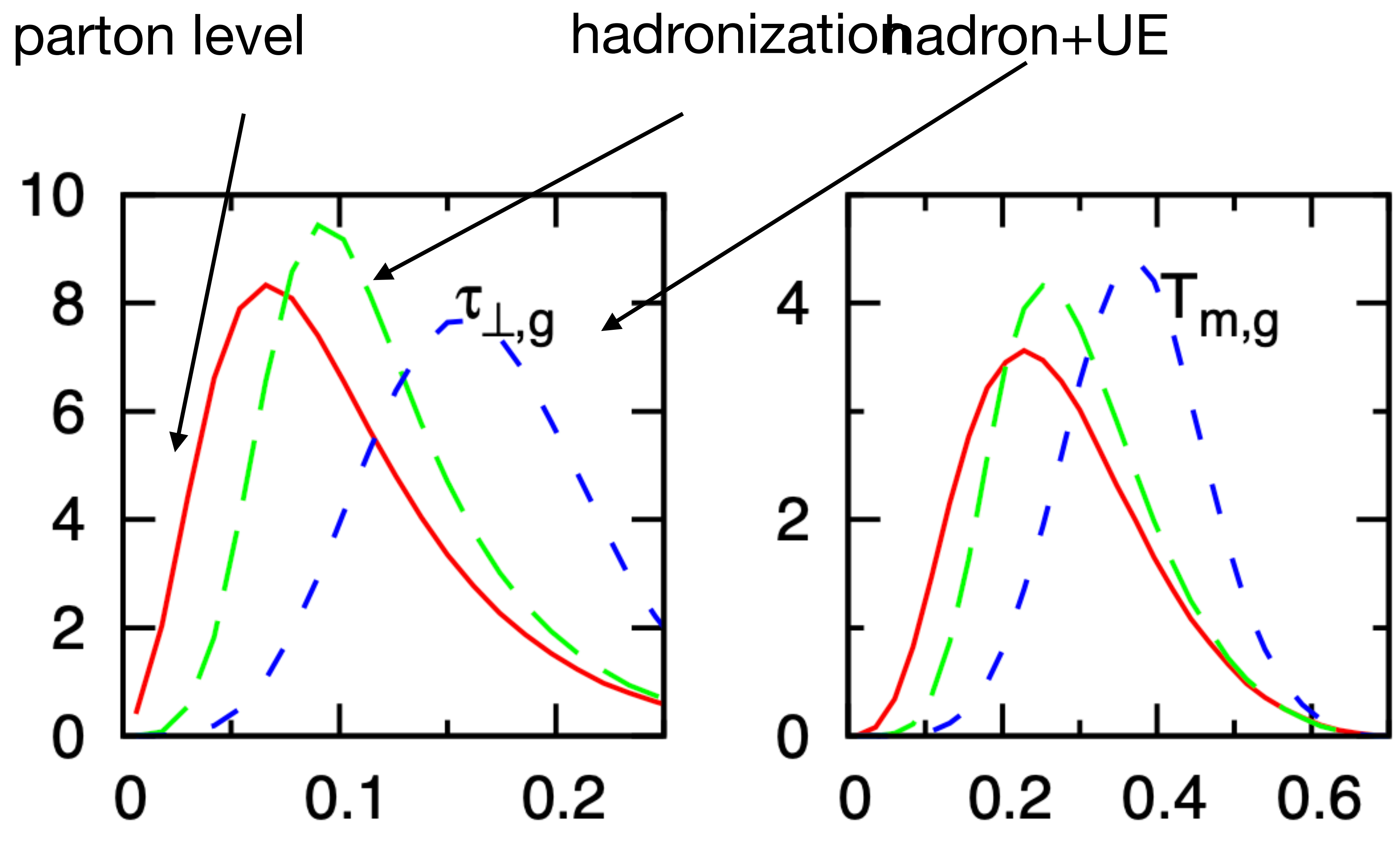

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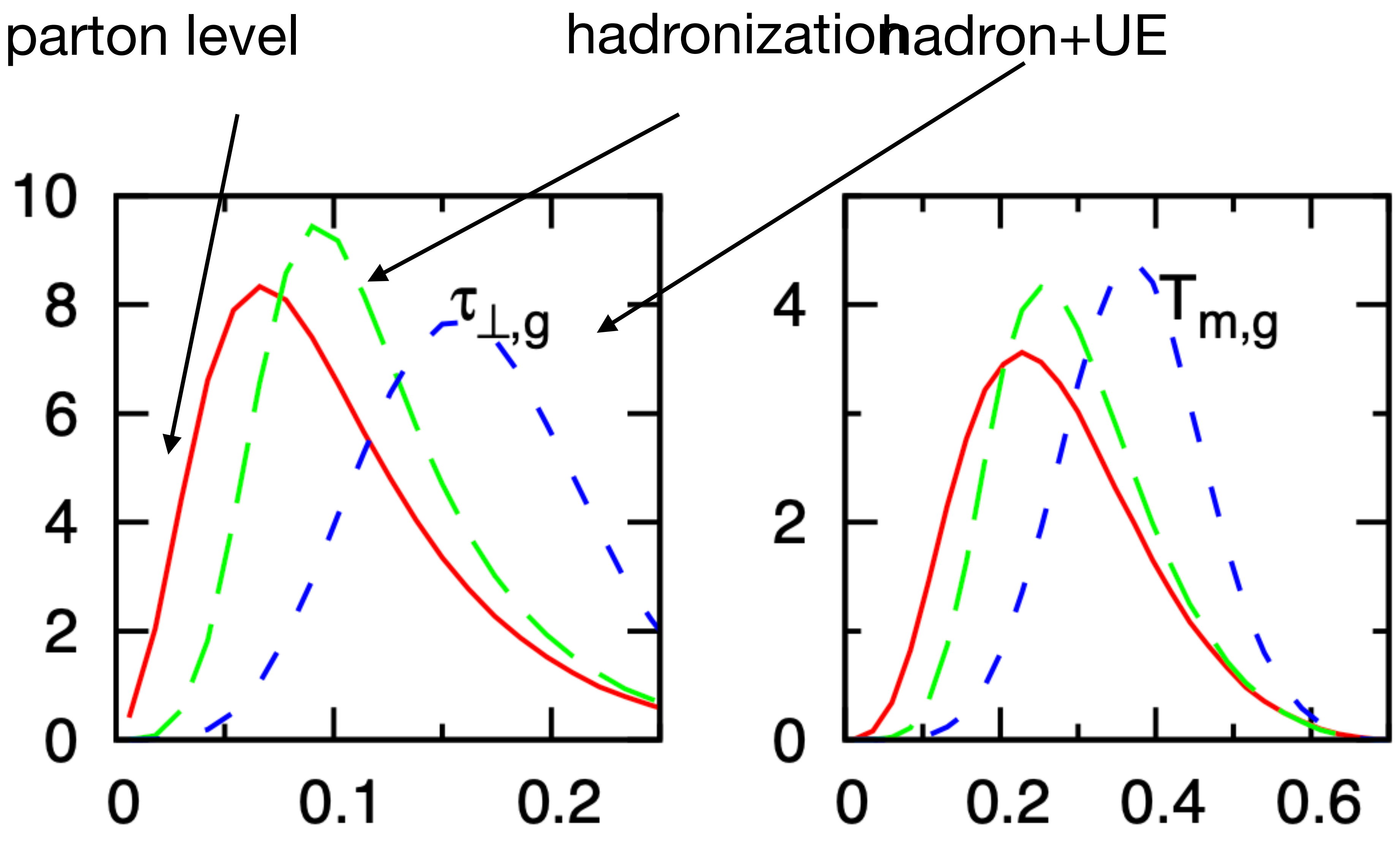
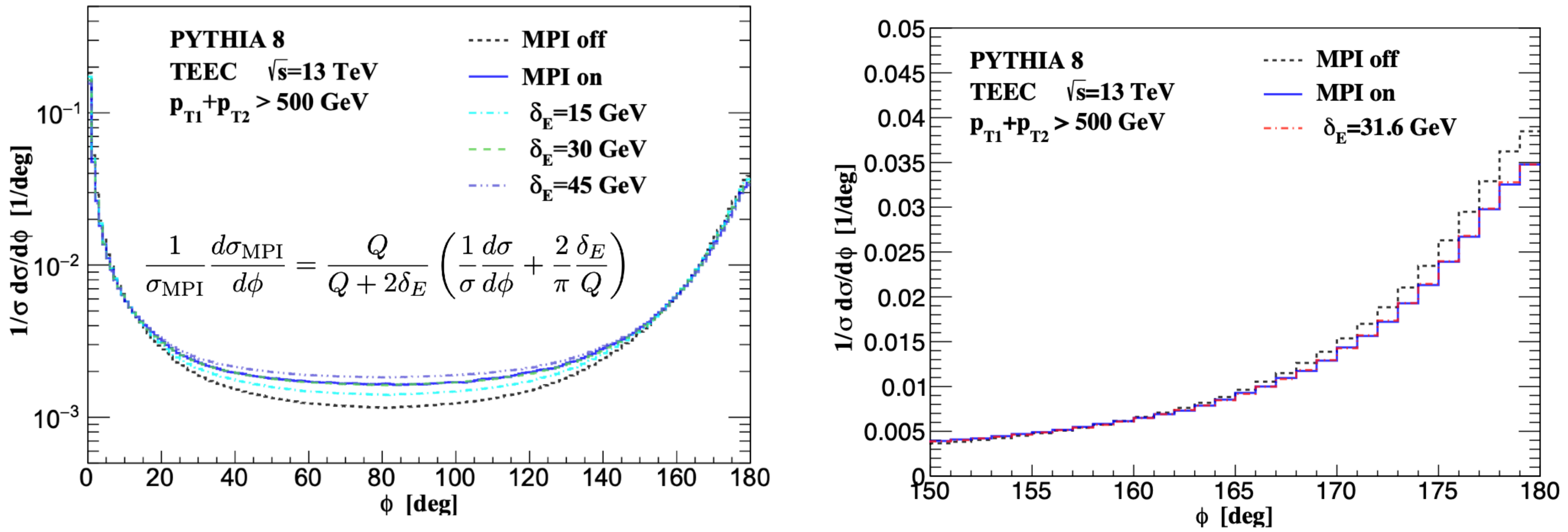


Banfi, Salam and Zanderighi 2010

test on sensitivity of TEEC to underlying events



Gao, HTL, Moult, Zhu, PRL, 2019, JHEP, 2024

EEC and TEEC

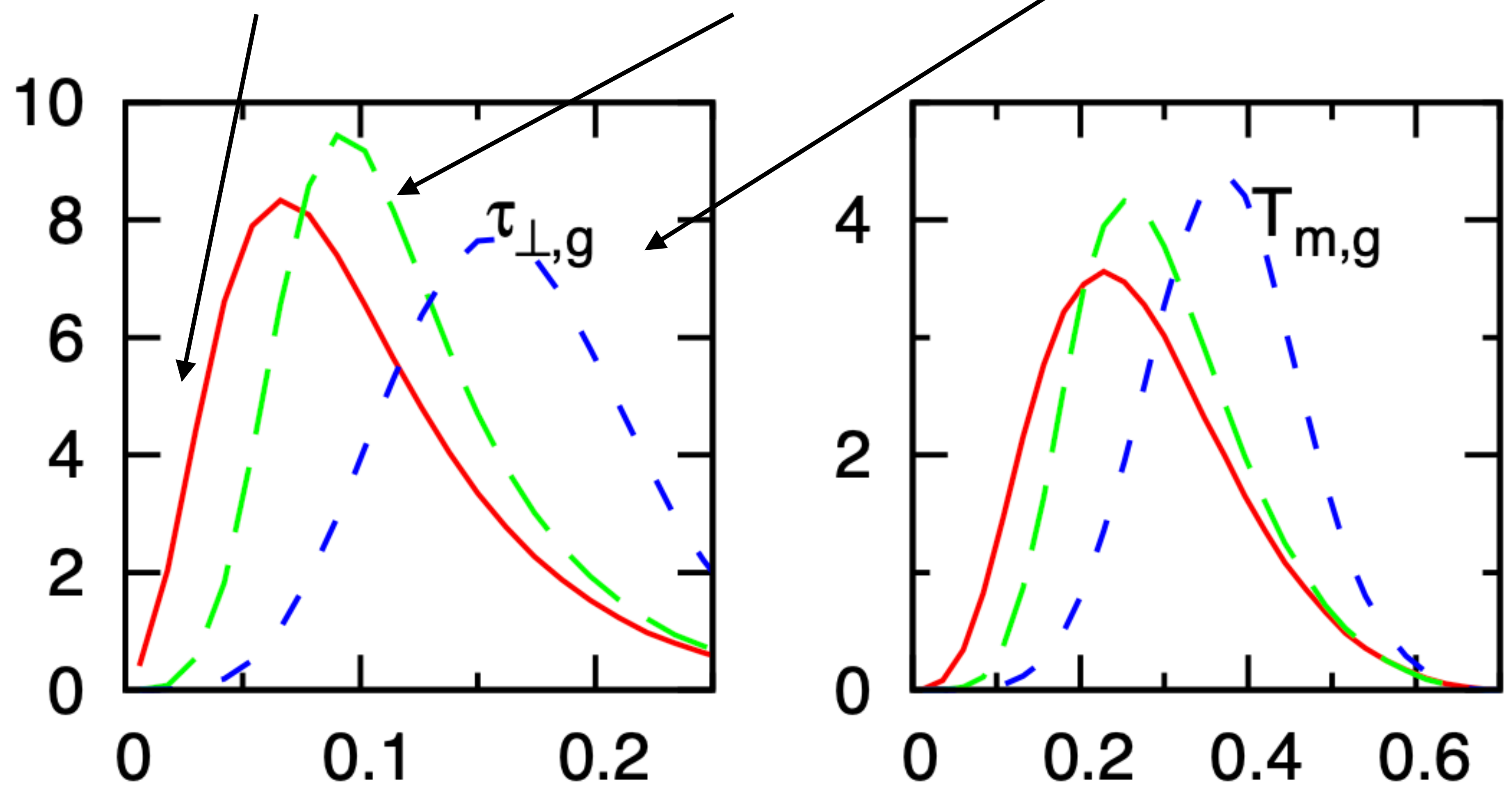
EEC/TEEC is a class of event shape variables

where nonperturbative effect is supposed to be small

parton level

hadronization

hadron+UE



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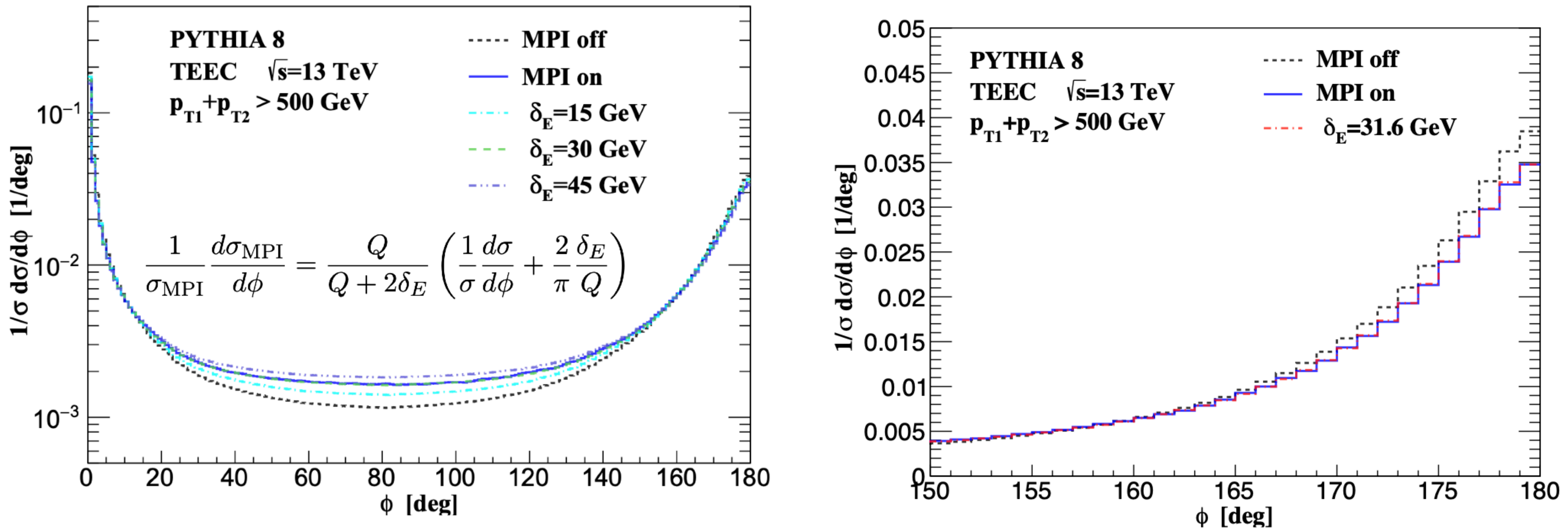


Gao, HTL, Moult, Zhu, PRL, 2019, JHEP, 2024

Recent developments (EEC/TEEC in DIS):

Nucleon Energy Correlators for Color Glass Condensate, Liu, Zhu et al, 2022,2023

Collins-type EEC jet in DIS, Kang et al, 2023; Imaging Cold Nuclear Matter with Energy Correlators, Devereaux et al 2023

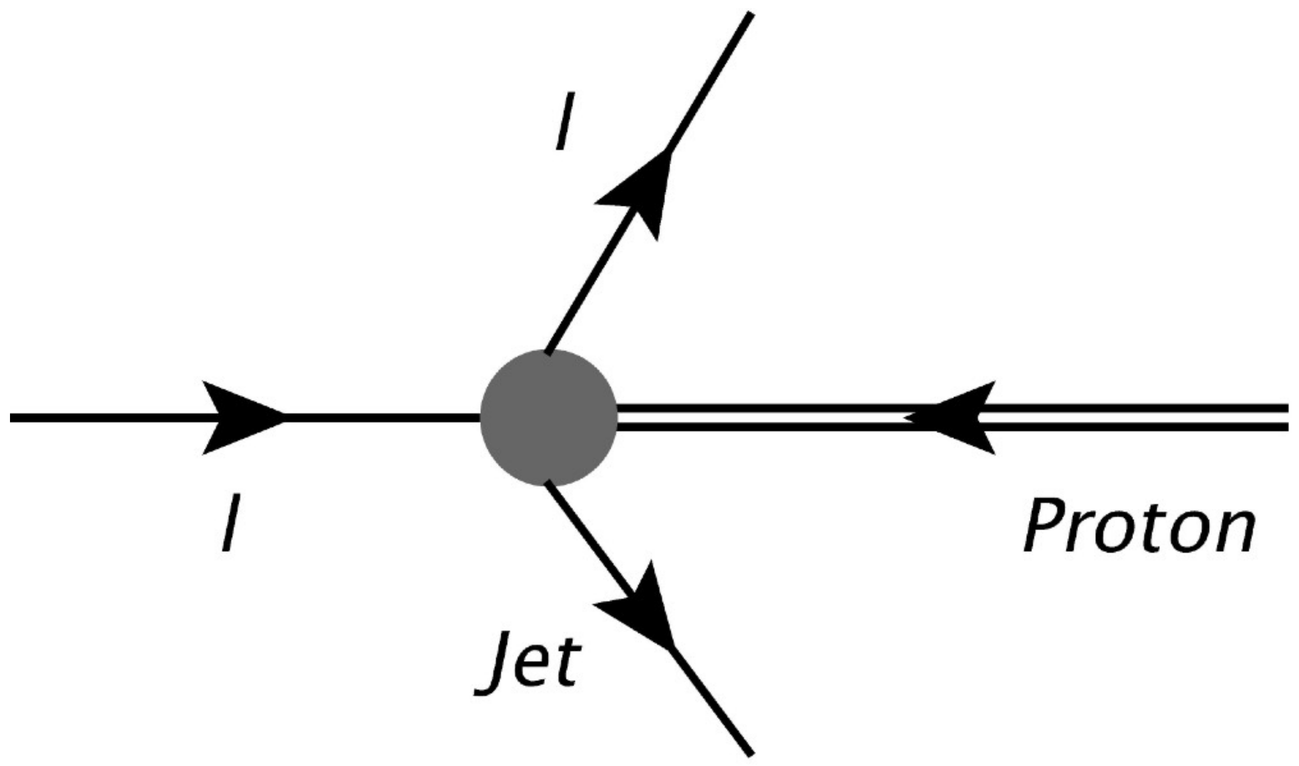
TEEC in the Color-Glass Condensate at the Electron-Ion Collider, Kang et al 2023;

and many other works

TMDs from Semi-inclusive Energy Correlators, Liu, Zhu, 2024; NEEC and frature function, Chen, Ma, Tong, 2024

EEC and TEEC in DIS

In Lab Frame *HTL, Vitev, Zhu, JHEP, 2020*



Definition

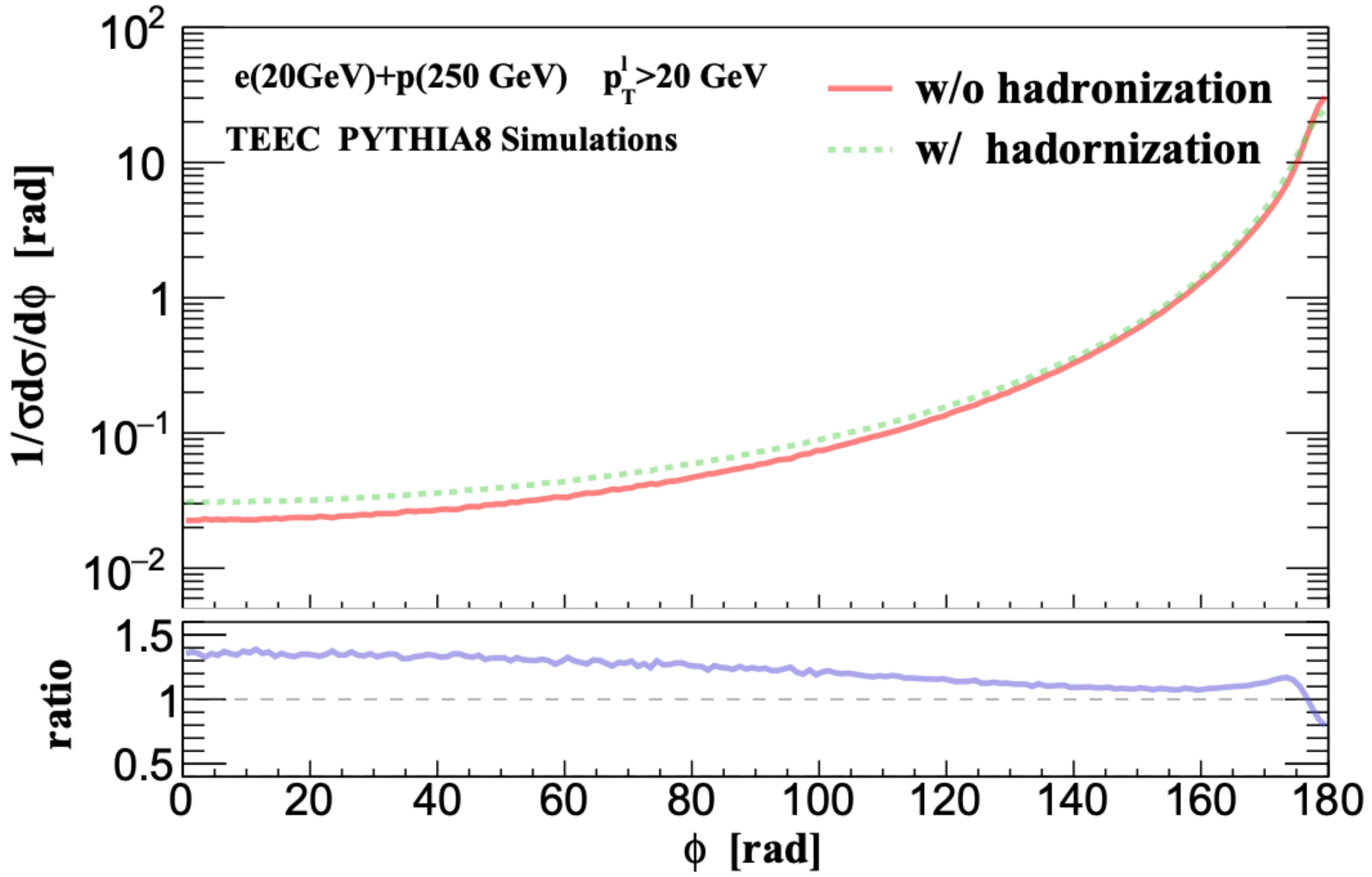
$$\text{TEEC} = \sum_a \int d\sigma_{lp\to l+a+X} \frac{E_{T,l}E_{T,a}}{E_{T,l}\sum_i E_{T,i}} \delta(\cos\phi_{la} - \cos\phi)$$

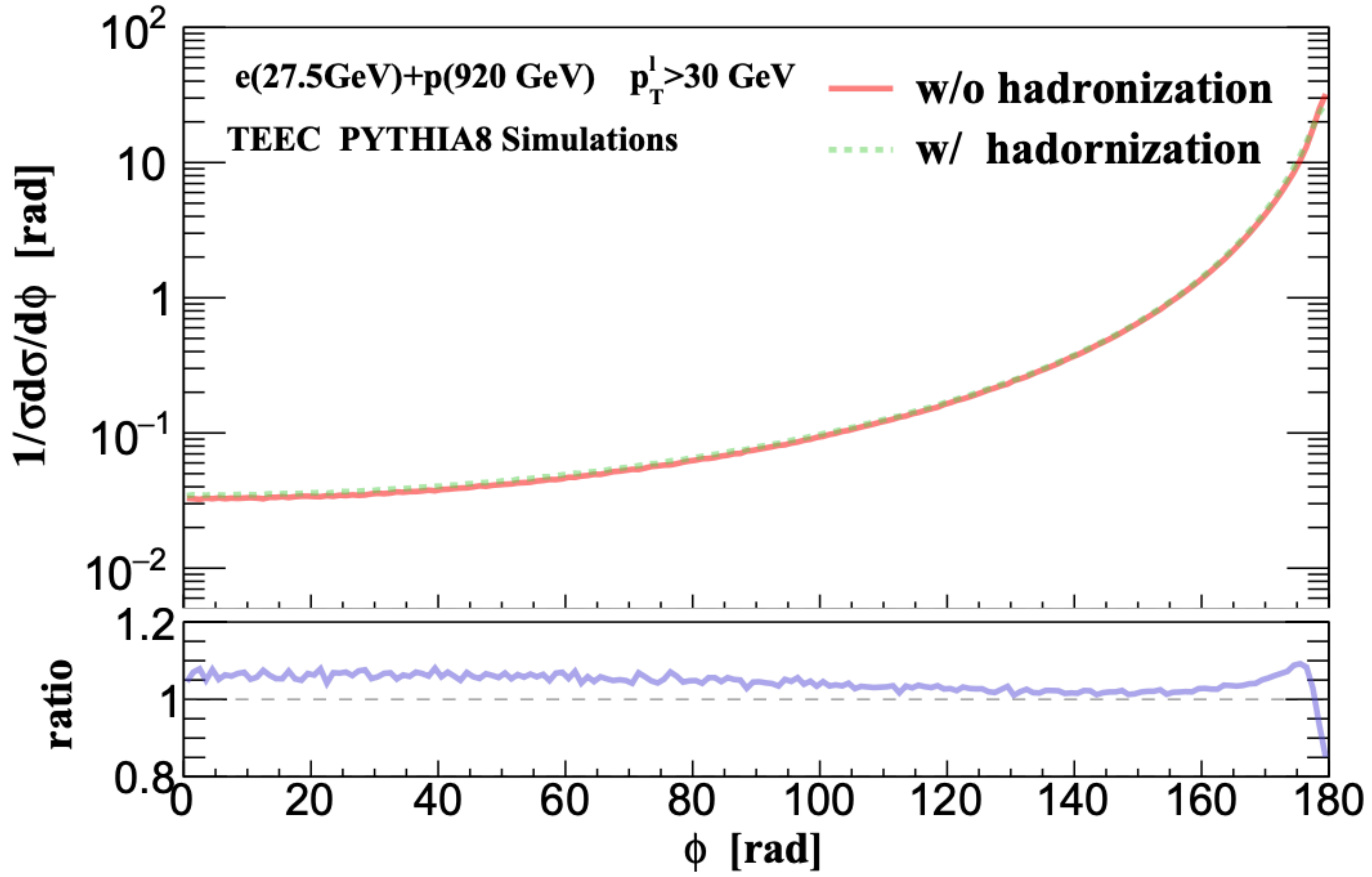
sum over all hadrons

energy weighted

measure azimuthal angle correlations

Simulation by Pythia





EEC and TEEC in DIS

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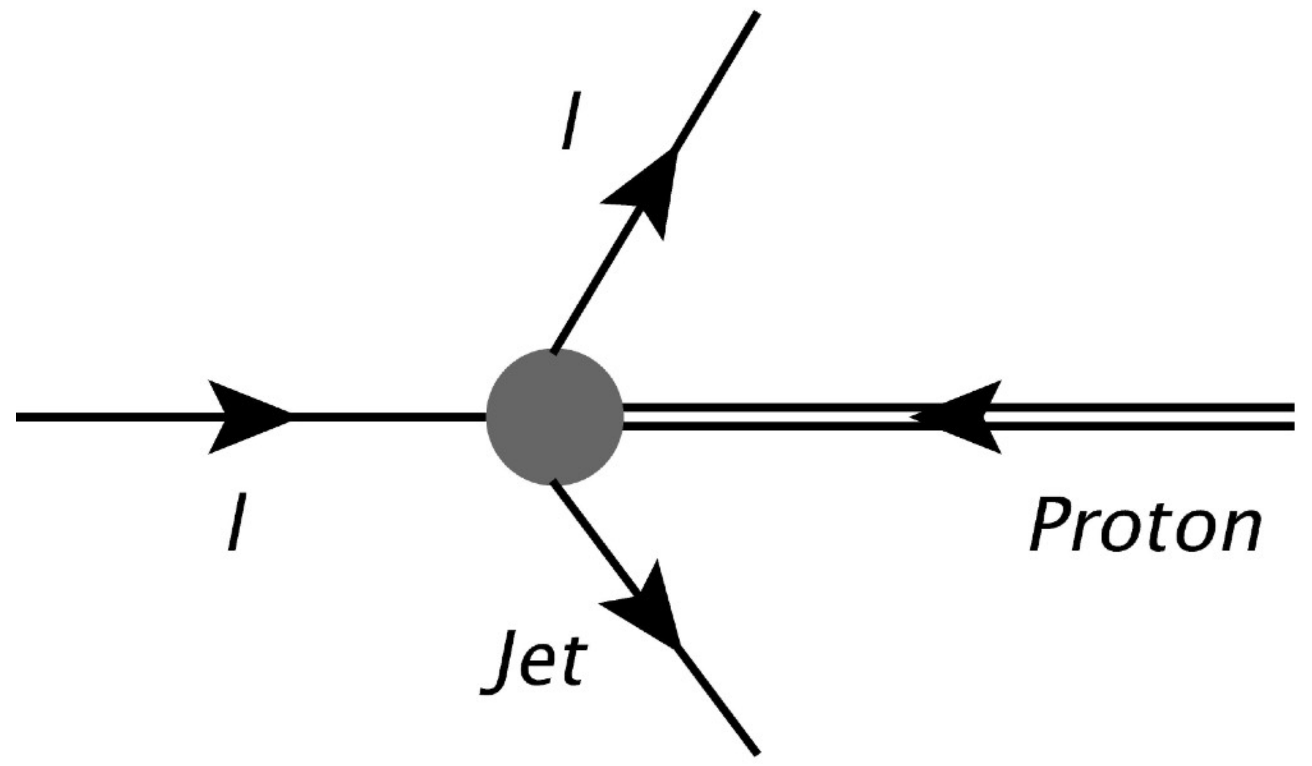
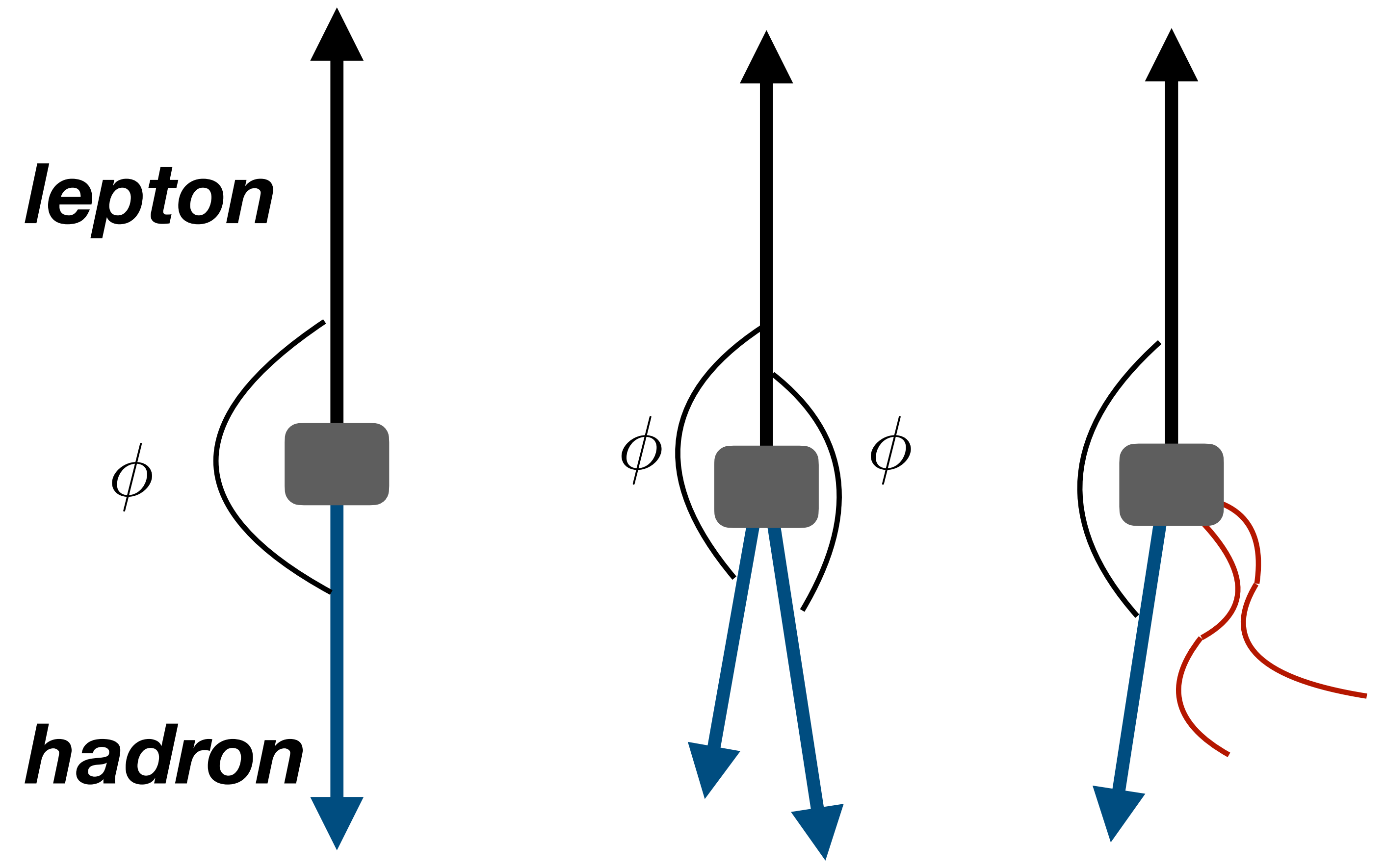


In transverse plane

$$\tau \approx \frac{\left| k_{2,y} - k_{s,y} + \frac{k_{4,y}}{\xi_4} \right|^2}{4p_T^2}$$

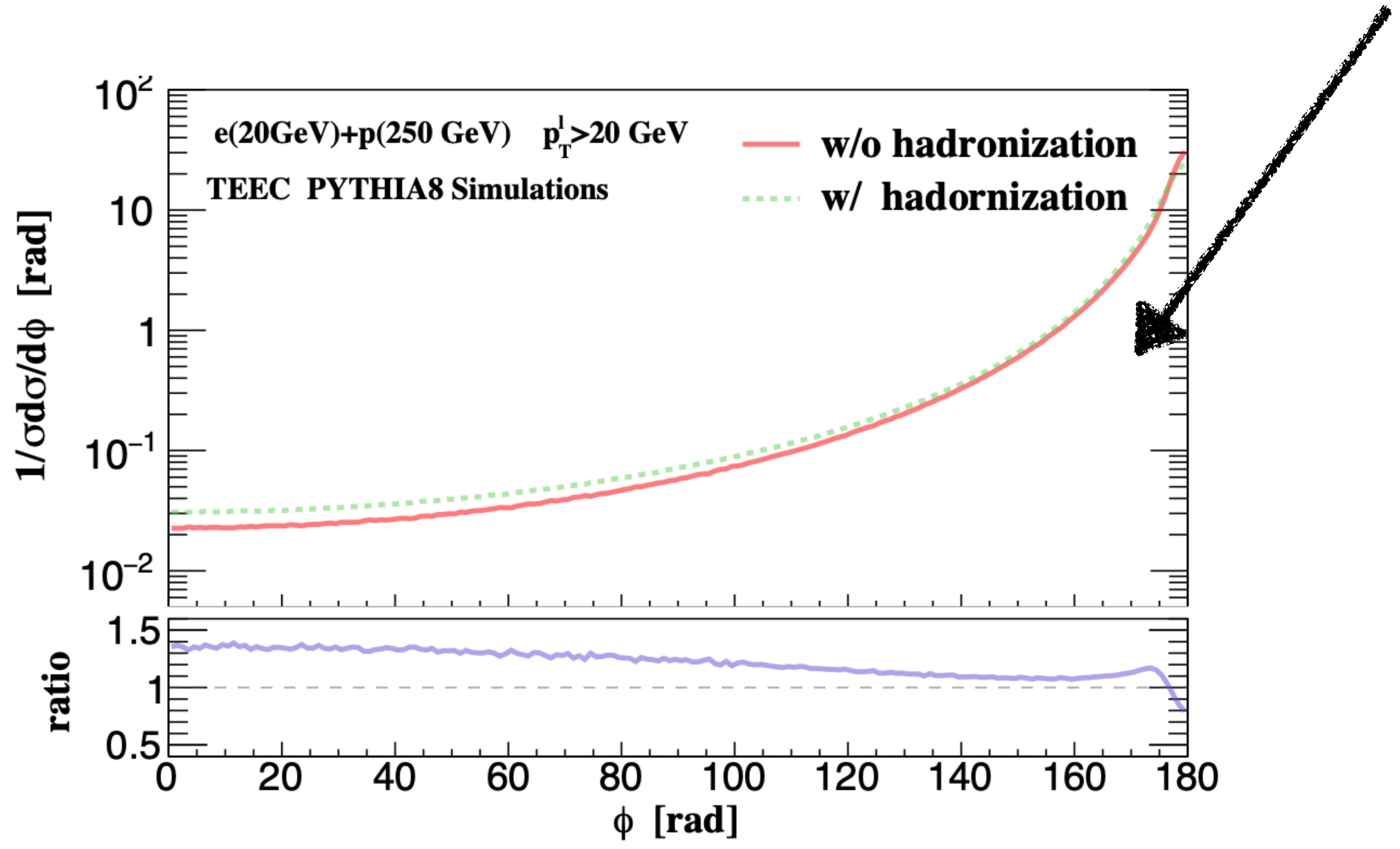


$\delta(\tau)$ $\quad A\delta(\tau) + B\frac{1}{\tau} + C\frac{\ln\tau}{\tau}\cdots$

$$\tau = \frac{1+\cos\phi}{2}$$

For $\tau \to 0$, small angle radiation, $\phi \to \pi$

For $\tau \to 1$, large angle radiation



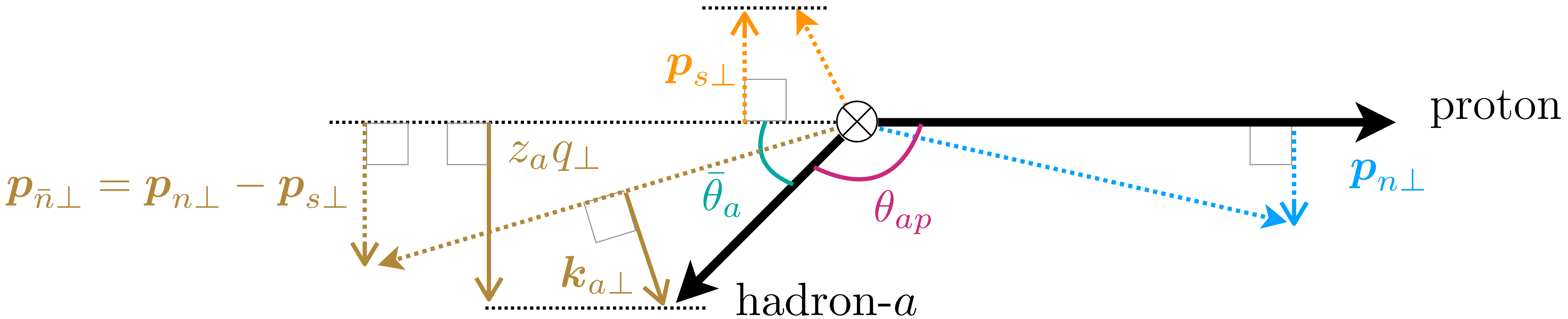
EEC and TEEC in DIS

In Breit Frame. *HTL, Makris, Vitev 2021*

From Lab frame to Breit Frame

- boost the system to proton rest frame
- rotate the system: virtual photon has zero $\vec{q}_T$
- boost along z direction: virtual photon has zero energy

$\gamma^* +$ proton $\rightarrow$ jet/hadron $+ X$



We proposed a new definition of EEC in DIS:
correlation between initial proton and final state hadron

$$\text{EEC} = \sum_a \int d\sigma_{lp\to l+a+X} \left(\frac{p\cdot p_a}{\sum_i p\cdot p_i}\right) \delta(\cos\chi - \cos\theta_{ap})$$

EEC and TEEC in DIS

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- weight function is Lorentz Invariant
- radiation close to the beam direction is suppressed
- soft radiation/hadronization effect is suppressed

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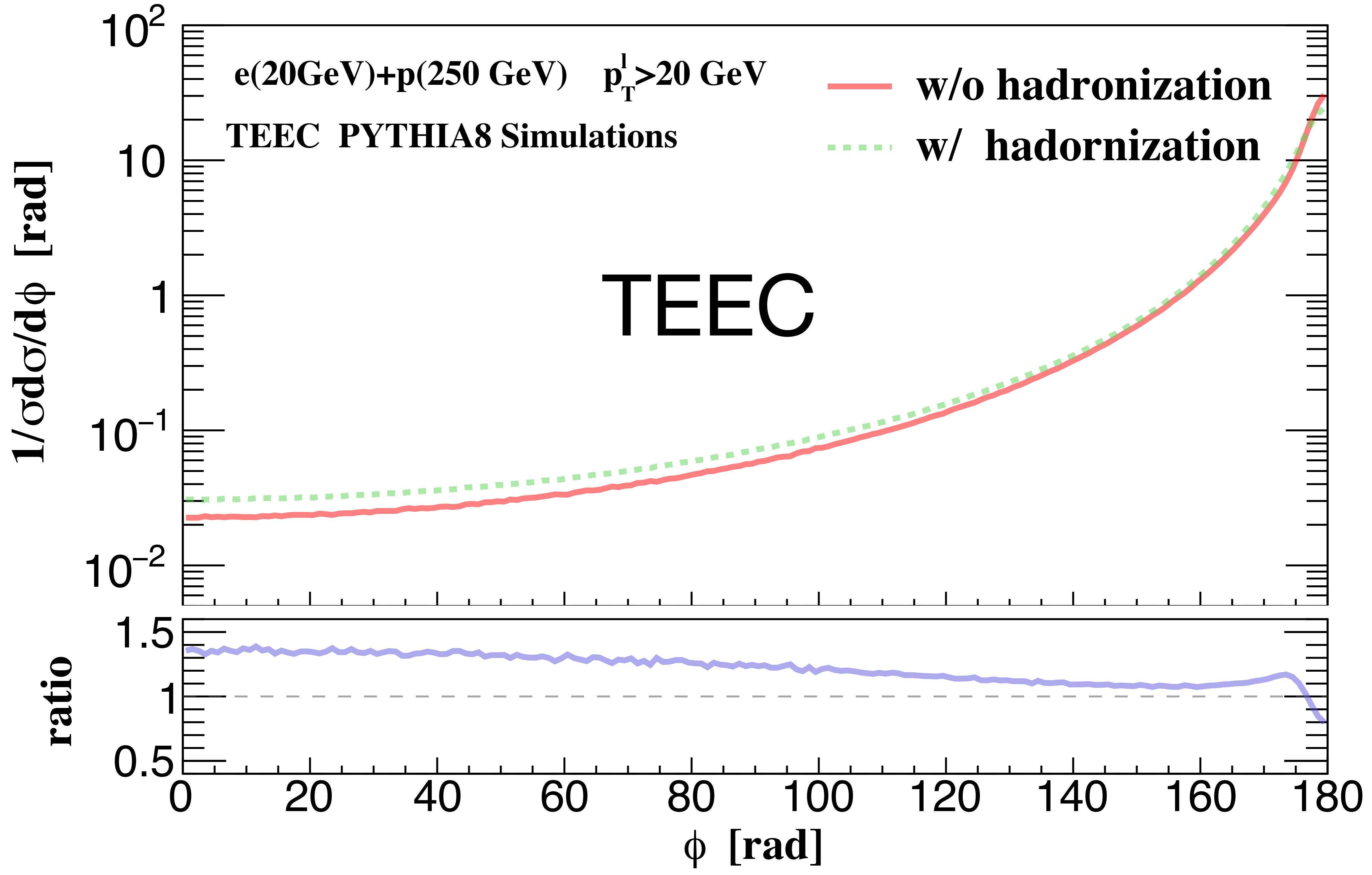

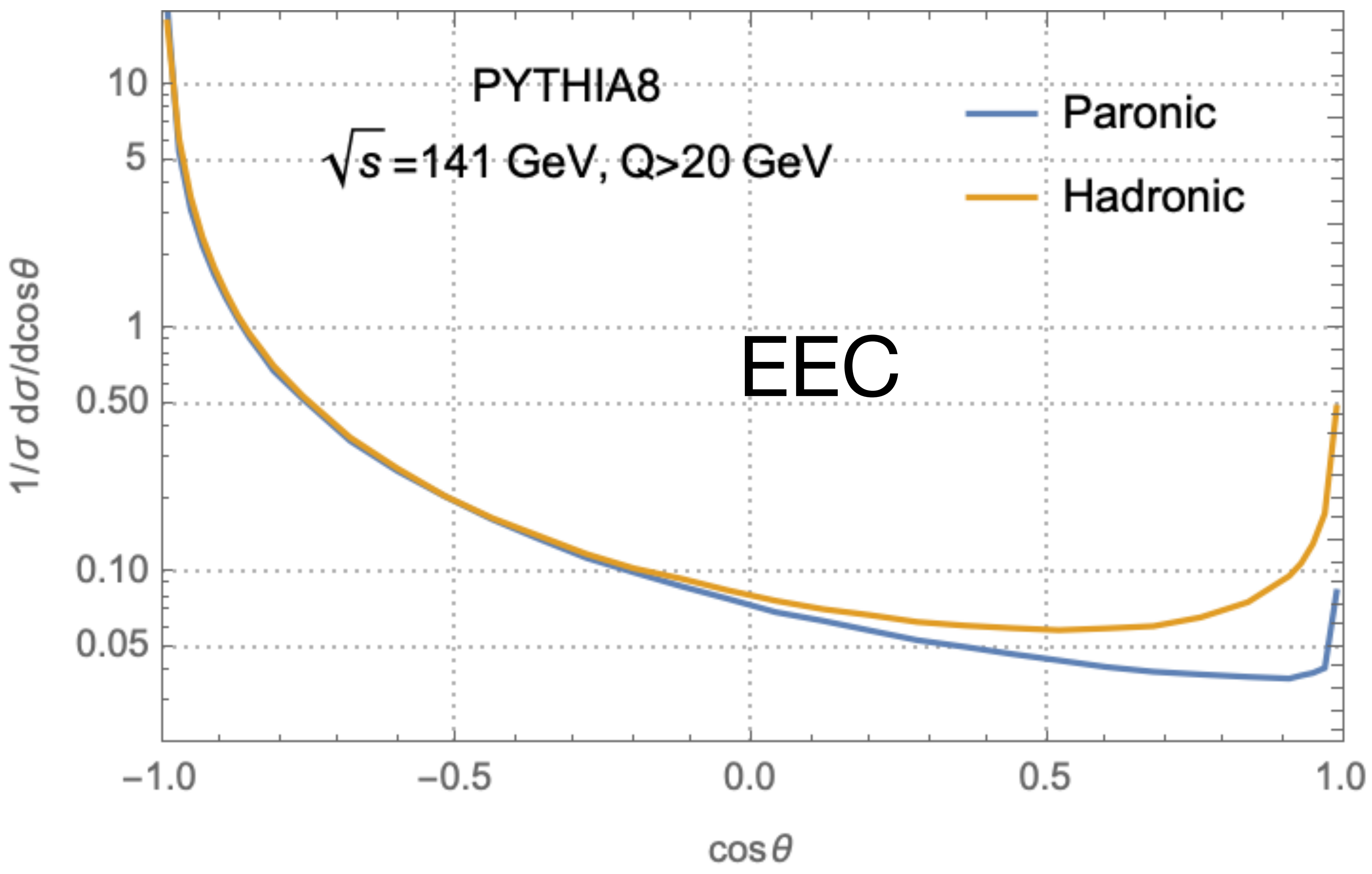

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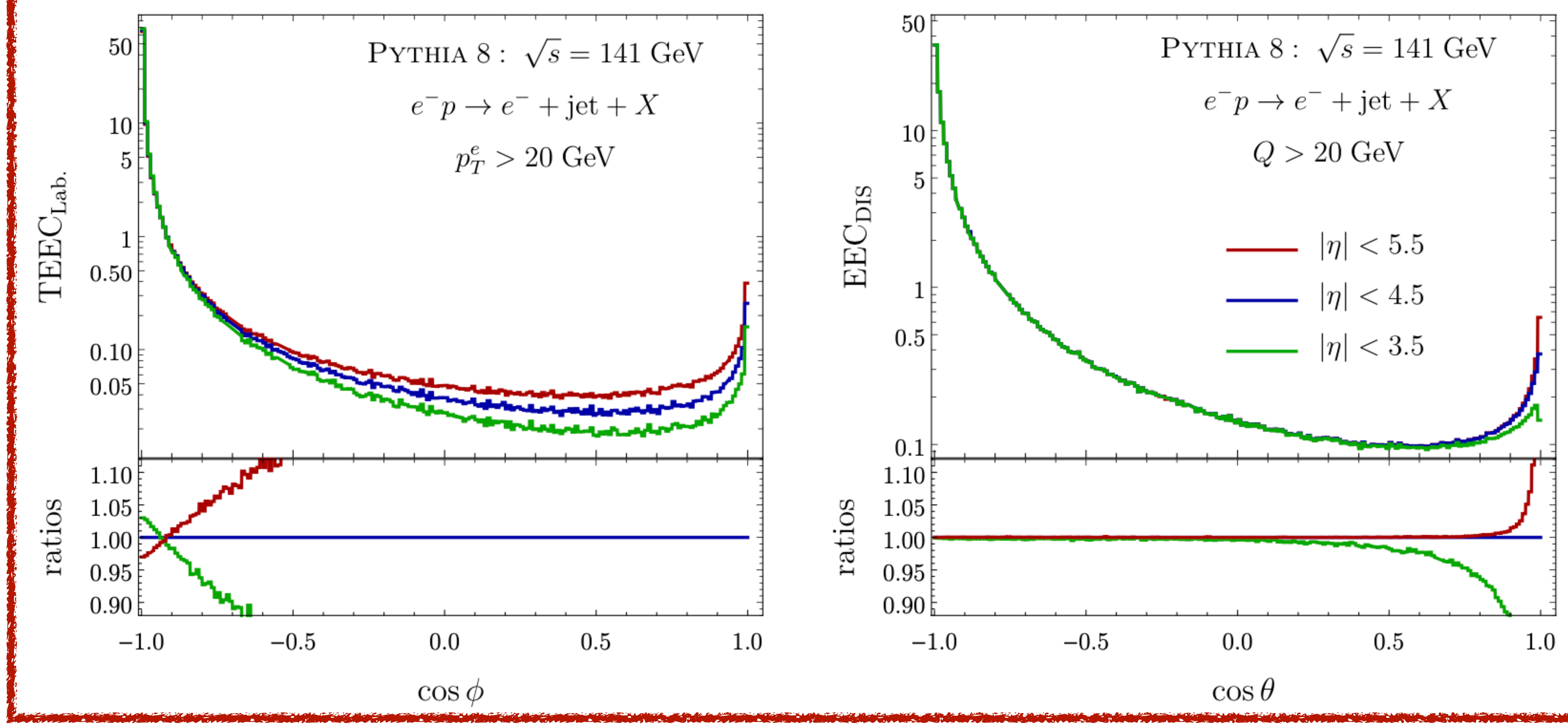

normalized to the cross section with cut $|\eta| < 5.5$

In Breit frame, rapidity cut only changes the cross section tail region

EEC and TEEC in DIS

In back-to-back limit, it is similar to 1-dimensional TMD factorization

hadron with small p_T

TMD PDF

$$\frac{d\sigma_h}{d^2p_\perp} = \sum_f \int \frac{d\xi dQ^2}{\xi Q^2} Q_f^2 H(Q,\mu) \int \frac{db}{2\pi} e^{ib_\perp \cdot p_\perp} \boxed{f_{f/N}\left(b,\xi,\mu,\nu\right)}$$
$$\boxed{S\left(b,\frac{n_2\cdot n_4}{2},\mu,\nu\right)} \boxed{\int dz F_{h/f}\left(z,b/z,E_4,\mu,\nu\right)}$$

TMD soft TMDFF

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$$S\left(b, \frac{n_2 \cdot n_4}{2}, \mu, \nu\right) \int dz F_{h/f}\left(z, b/z, E_4, \mu, \nu\right)$$

TMD soft

TMDFF

sum over all hadrons in the final state

$$\frac{d\sigma_h}{d\tau} = \sum_f \int \frac{d\xi dQ^2}{\xi Q^2} Q_f^2 H(Q,\mu) \int dk_y \int \frac{db}{2\pi} e^{-ib_y \cdot k_y} f_{f/N}\left(b,\xi,\mu,\nu\right)$$

$$S\left(b, \frac{n_2 \cdot n_4}{2}, \mu, \nu\right) \sum_h \int dz\ zF_{h/f}\left(z, b/z, E_4, \mu, \nu\right) \delta(\tau - \tau(k_y))$$

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Jet function

the second Mellin-Moment of the TMDFFs

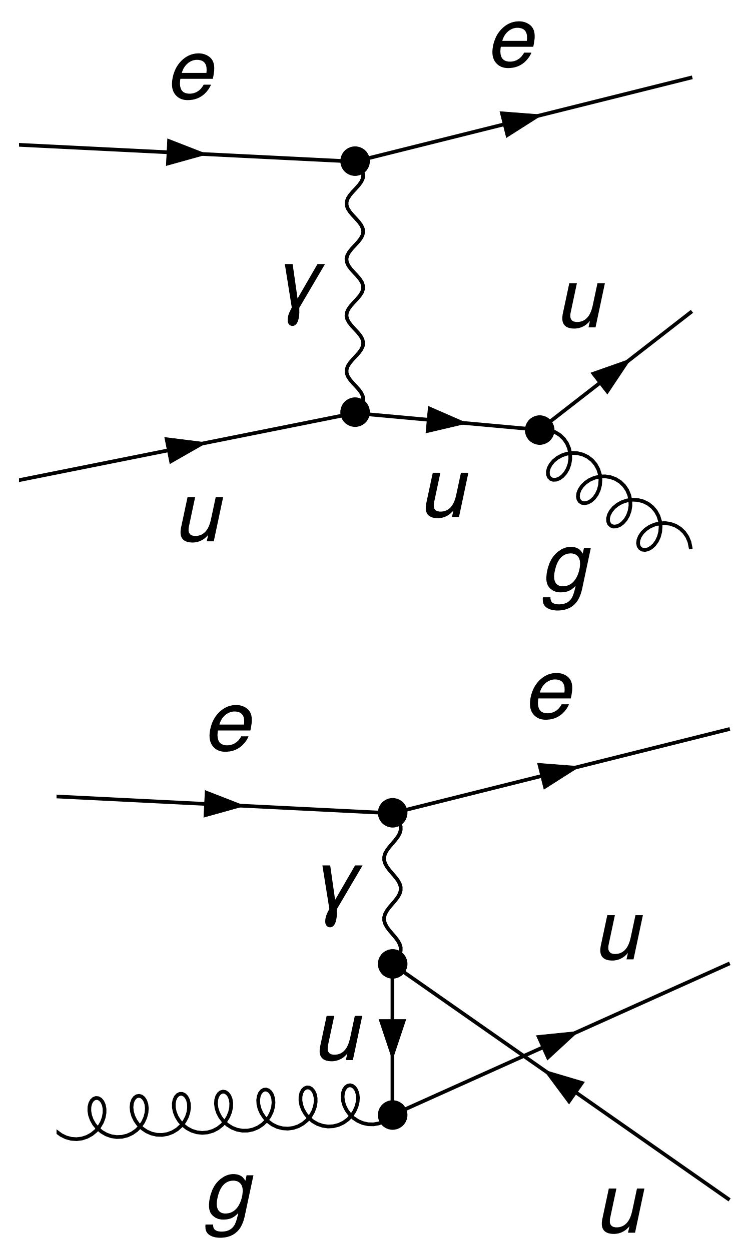
$$\sum_N \int_0^1 dzzF_{N/q}\left(z, b_\perp/z, \nu\right) = \sum_{i,N} \int_0^1 dzz \int_z^1 \frac{d\xi}{\xi} d_{N/i}(z/\xi) \mathscr{C}_{iq}\left(\xi, b_\perp/\xi, \nu\right) + \mathcal{O}\left(b_T^2 \Lambda_{\mathrm{QCD}}^2\right)$$

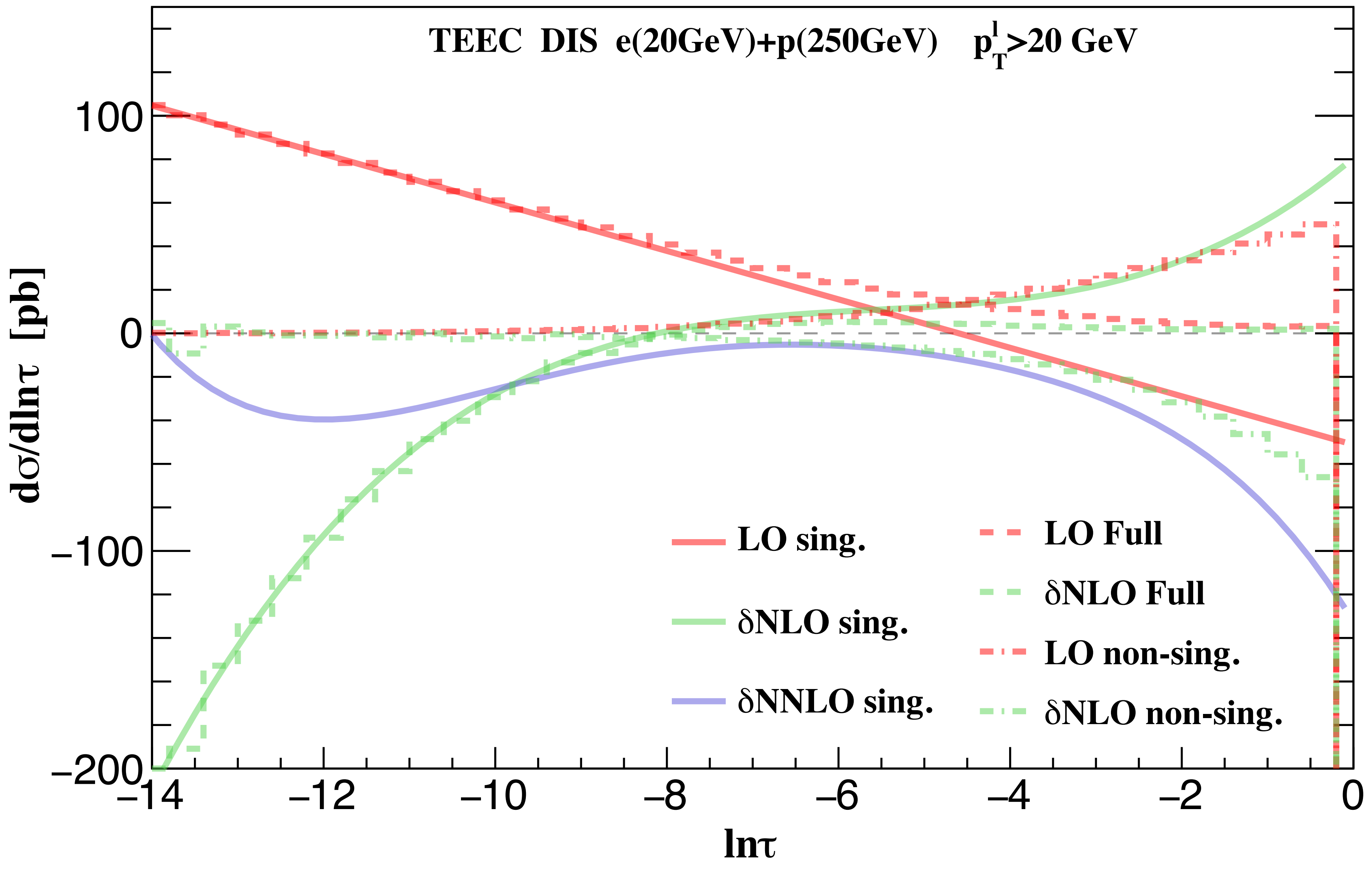
$$= \sum_{i,N} \int_0^1 dx x \mathscr{C}_{iq}\left(x, b_\perp/\xi, \nu\right) \int_0^1 d\xi \xi d_{N/i}(\xi) + \mathcal{O}\left(b_T^2 \Lambda_{\mathrm{QCD}}^2\right)$$

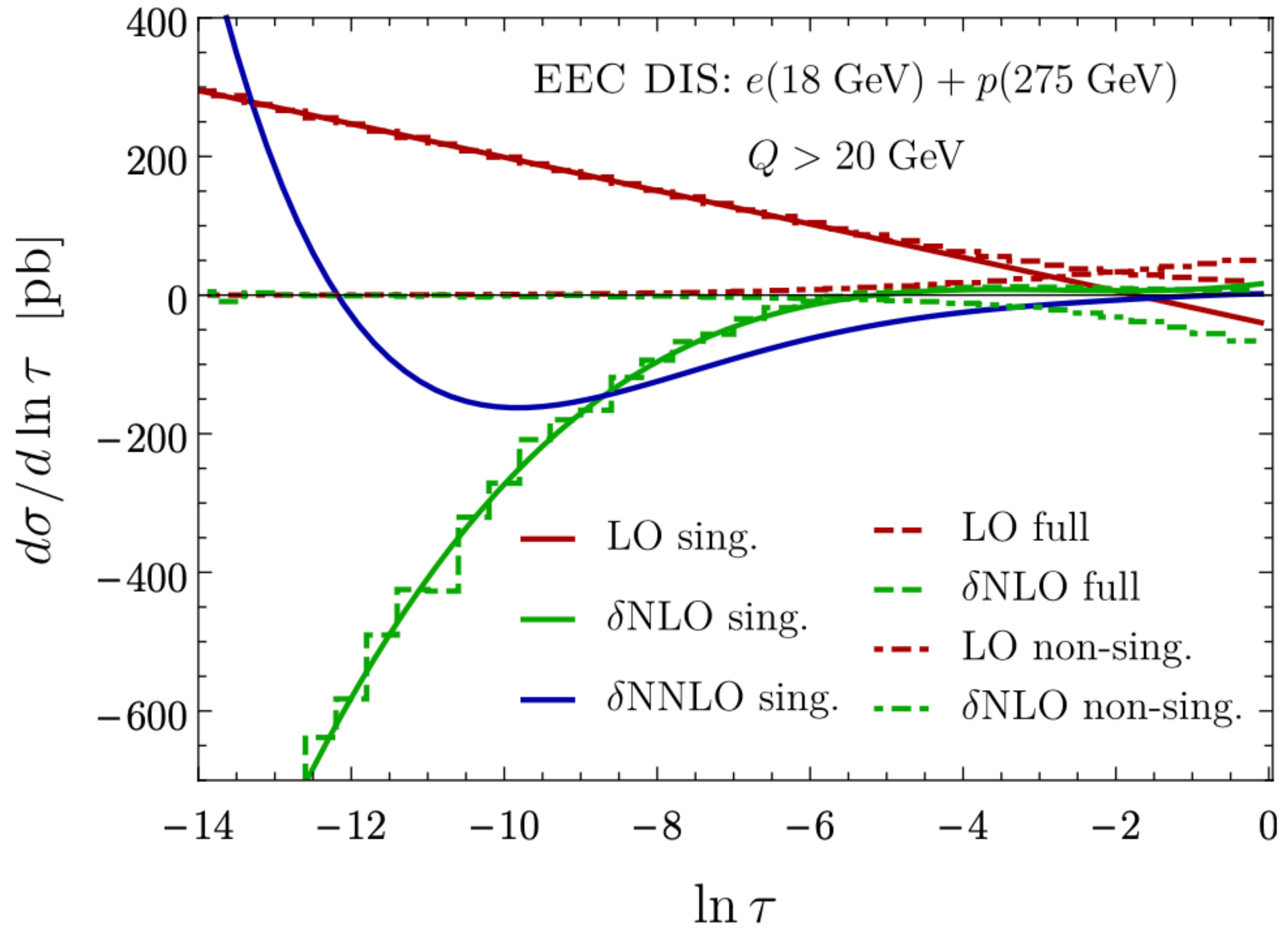
EEC and TEEC in DIS

The leading order process is

using NLOJET++



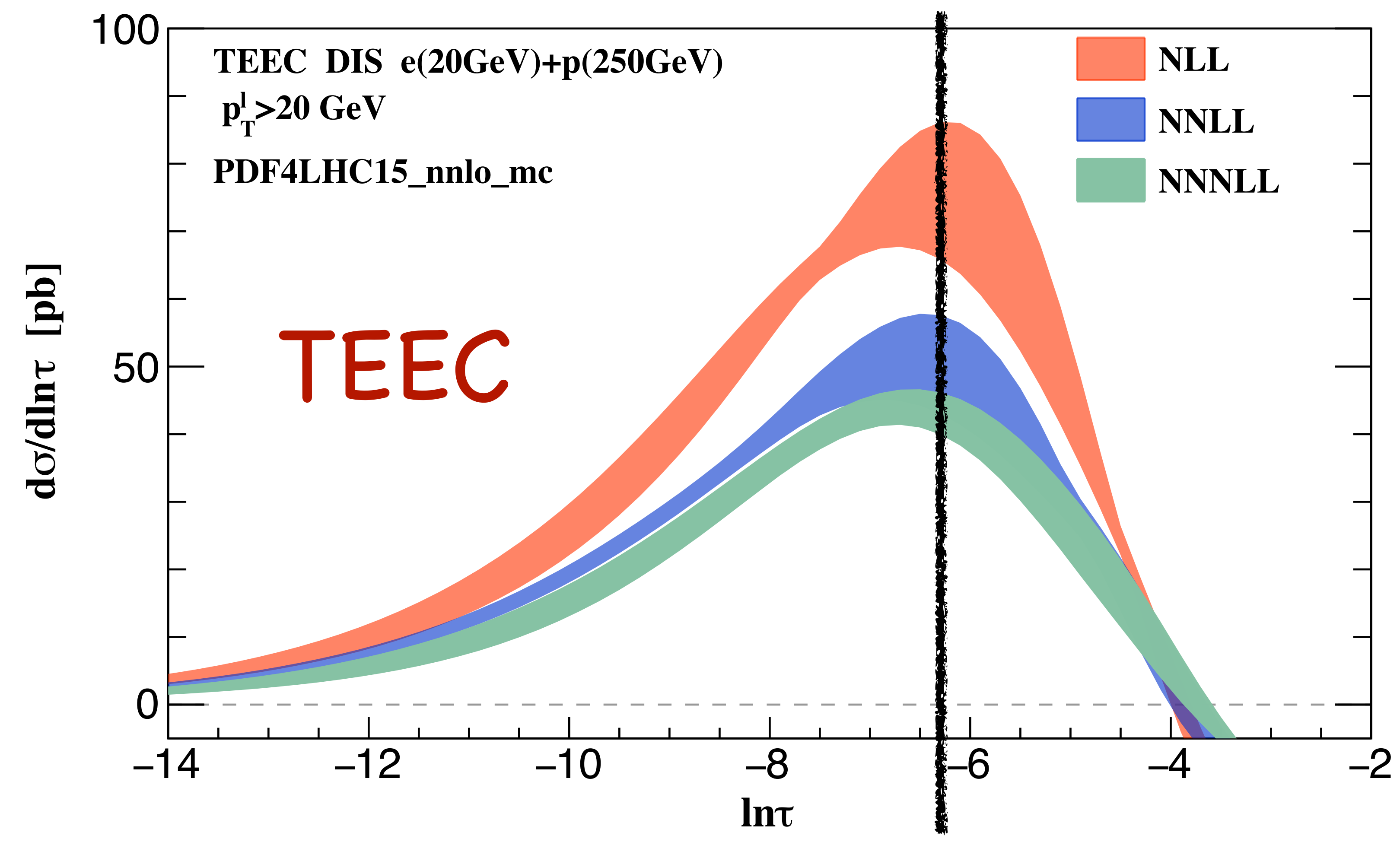


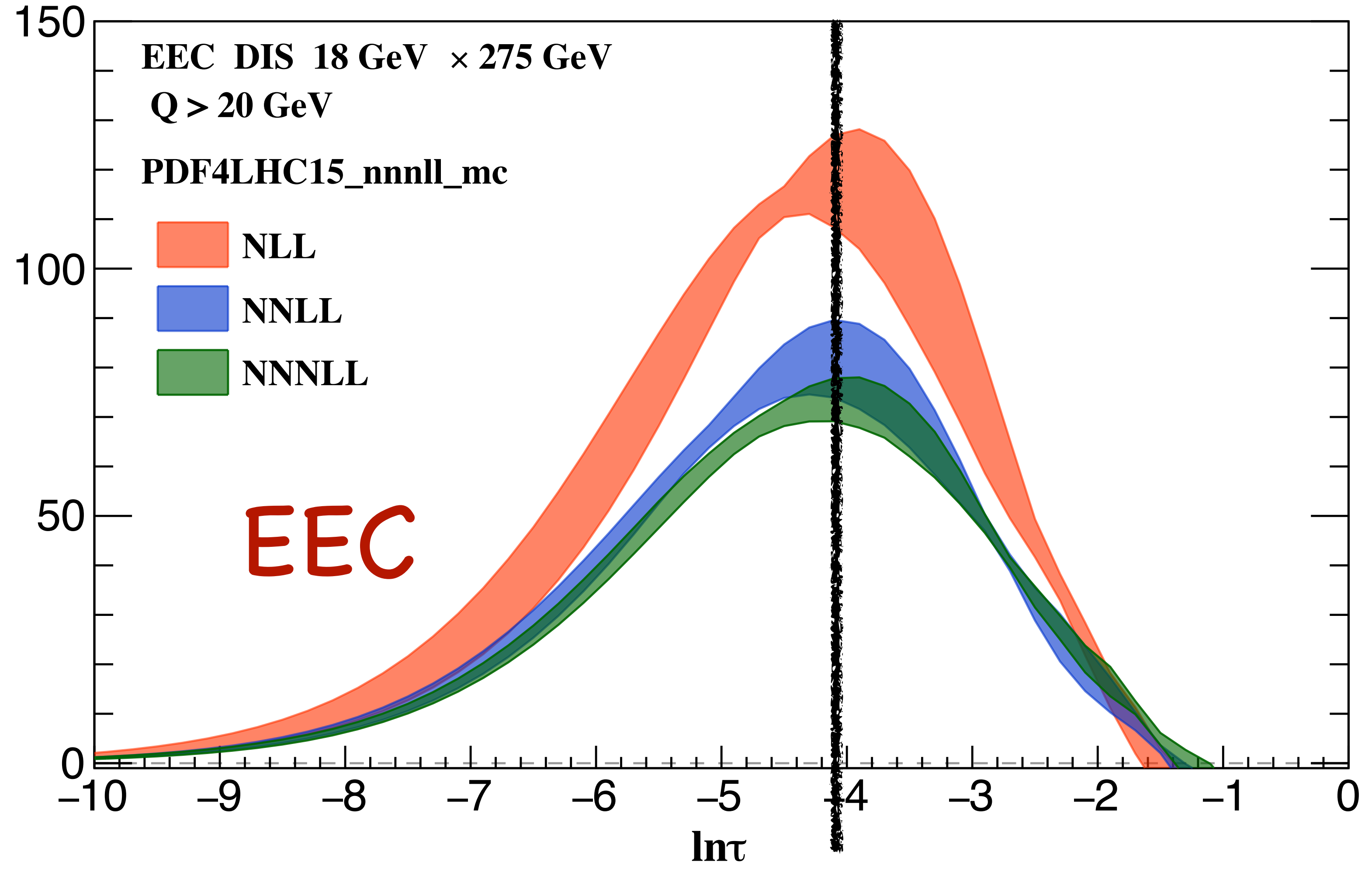


- Reproduced the singular behaviors
- Full control of the distributions in the back-back limit at LO and NLO.
- We obtained singular distribution up to NNLO (three loop anomalous

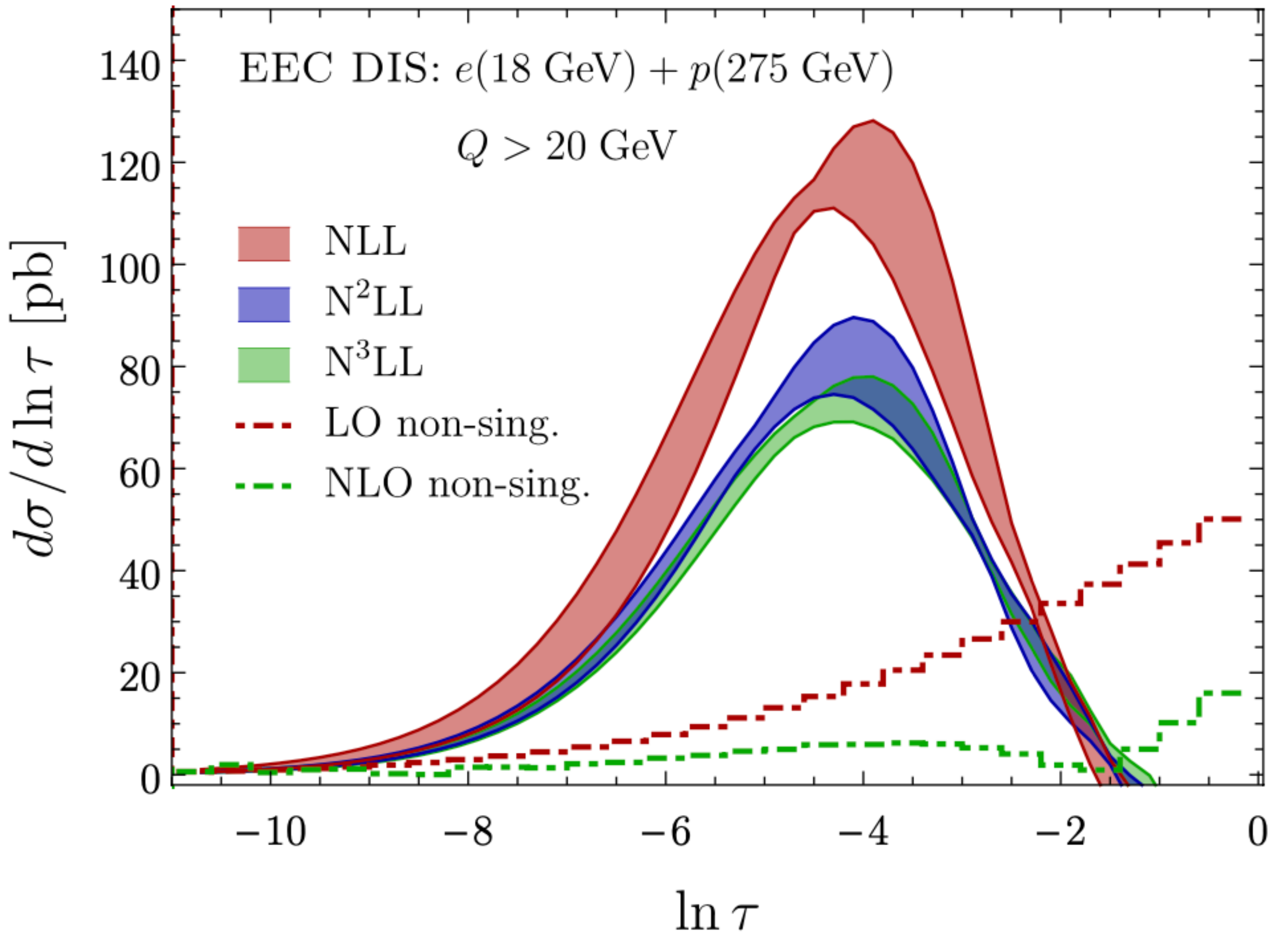
EEC and TEEC in DIS

resummation accuracy





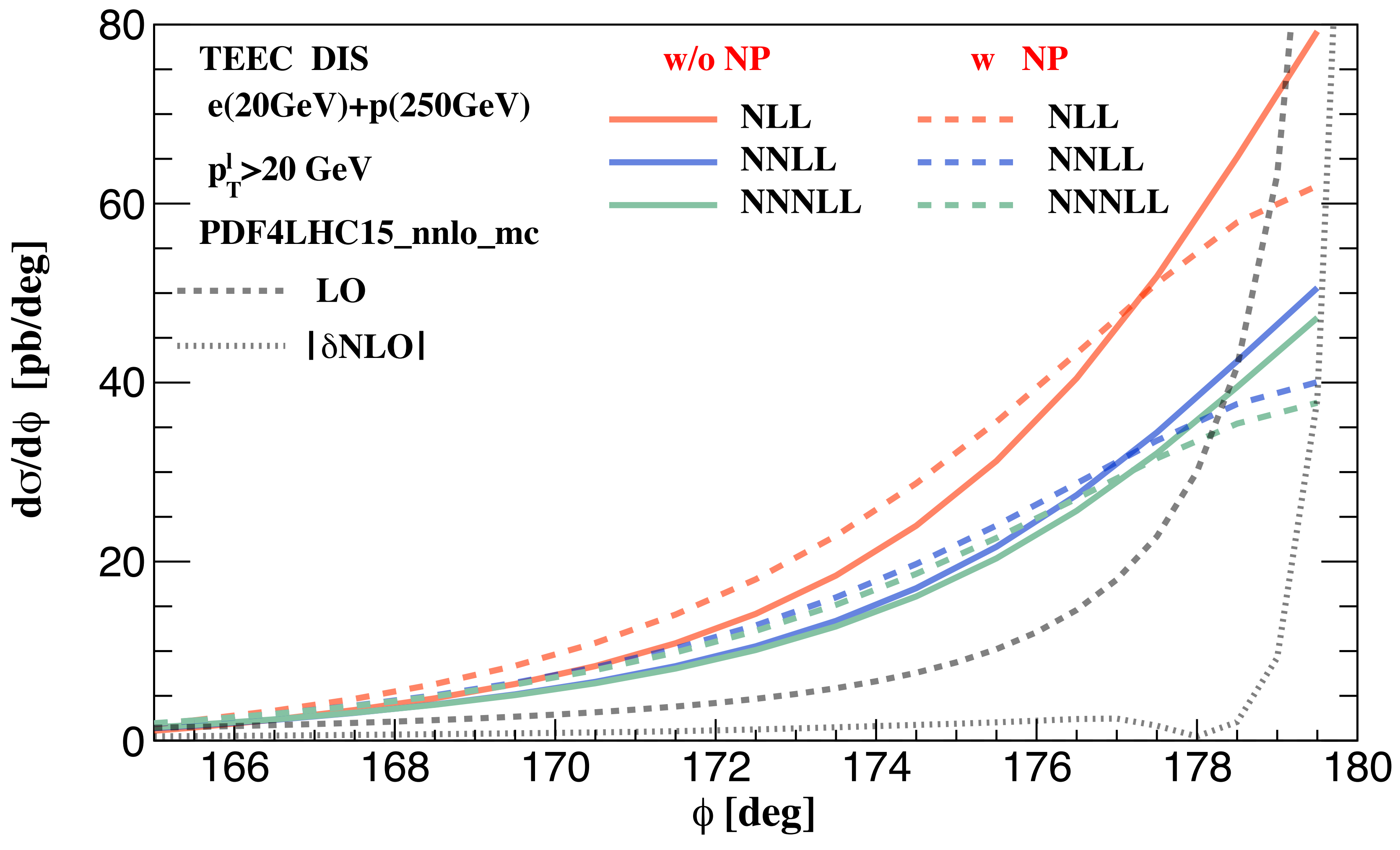
for EEC peak at larger τ, means small NP effects

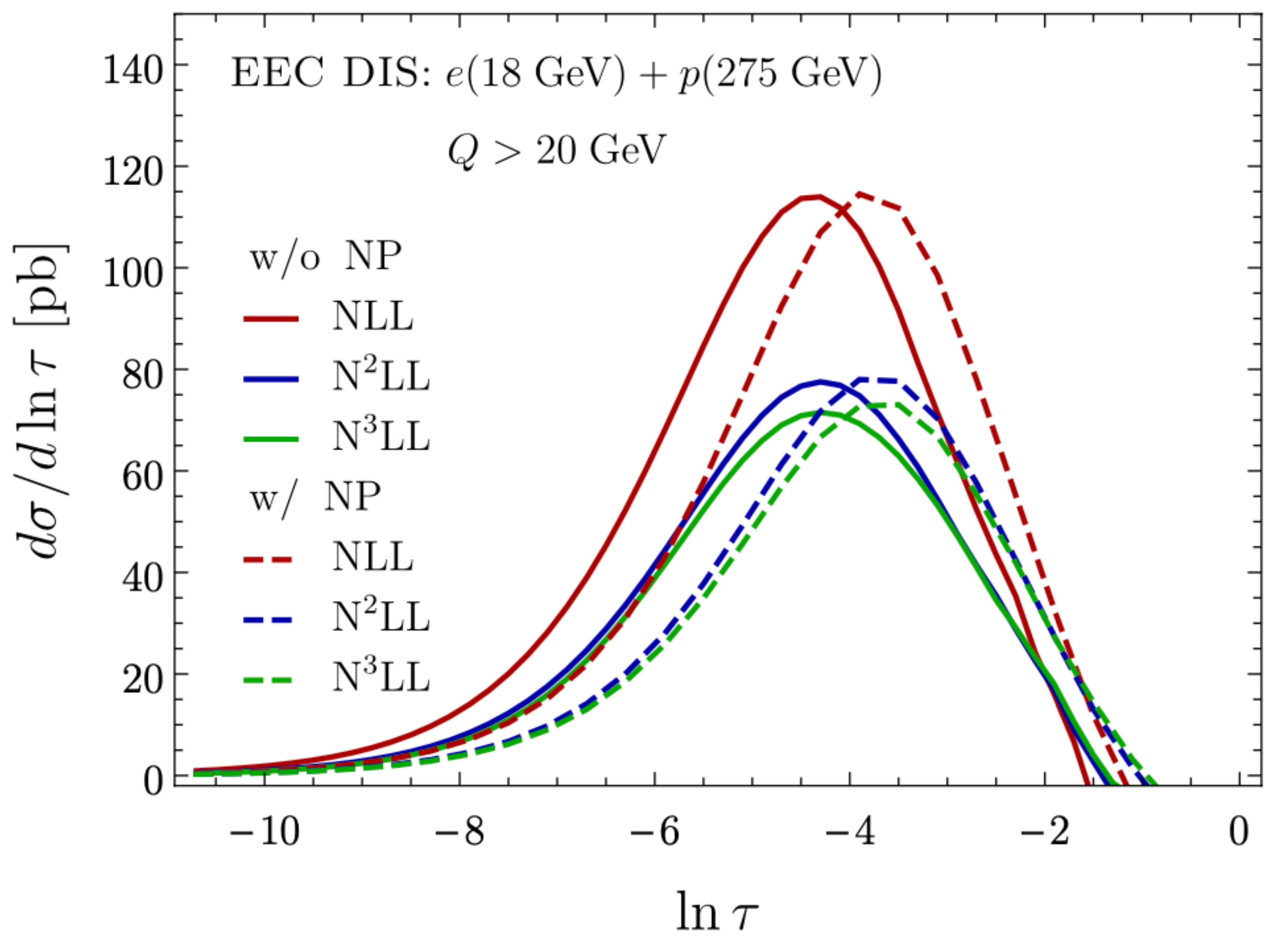


- Convergence in back-to-back limit after resummation
- Huge difference from NLL to NNLL and good perturbative convergence from NNLL to NNNLL
- Reduction of scale uncertainties order by order from NLL to NNNLL

Non-singular terms start to contribute which is less important for EEC

EEC and TEEC in DIS





☑ corrections to rapidity evolution

☑ corrections to the TMD matrix element

Non-perturbative form factors, which extracted from the semi-inclusive hadron production in DIS.

$$S_{\rm NP} = \exp\left[-0.106\ b^2 - 0.84 \ln Q/Q_0 \ln b/b*\right]$$

from TMD FFs

$$D_{i/a}^{\rm NP}(y,b) = \exp\left(-0.042\frac{b^2}{y^2}\right)$$

$$j_i(b) = \exp\left(-0.59b - 0.03b^2\right)$$

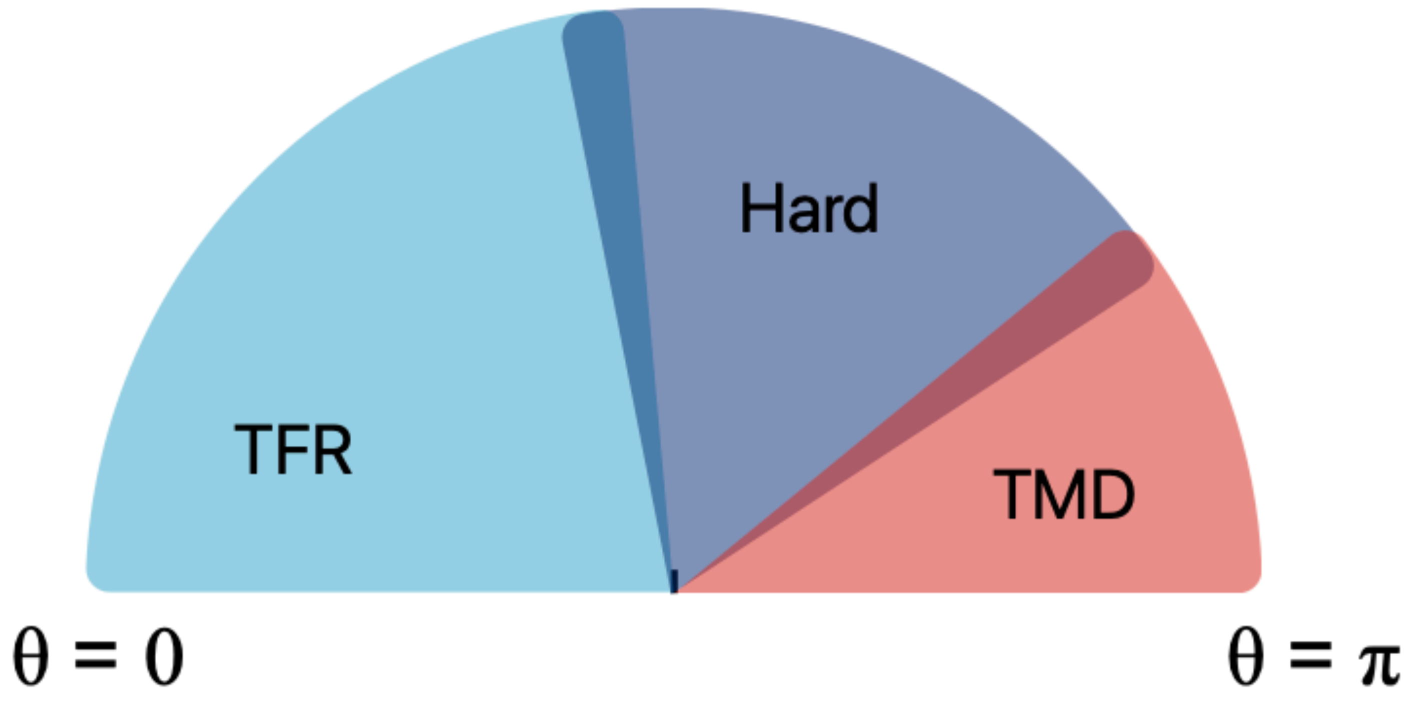
NP shifts the cross section

Sizable NP effects in back-to-back limit

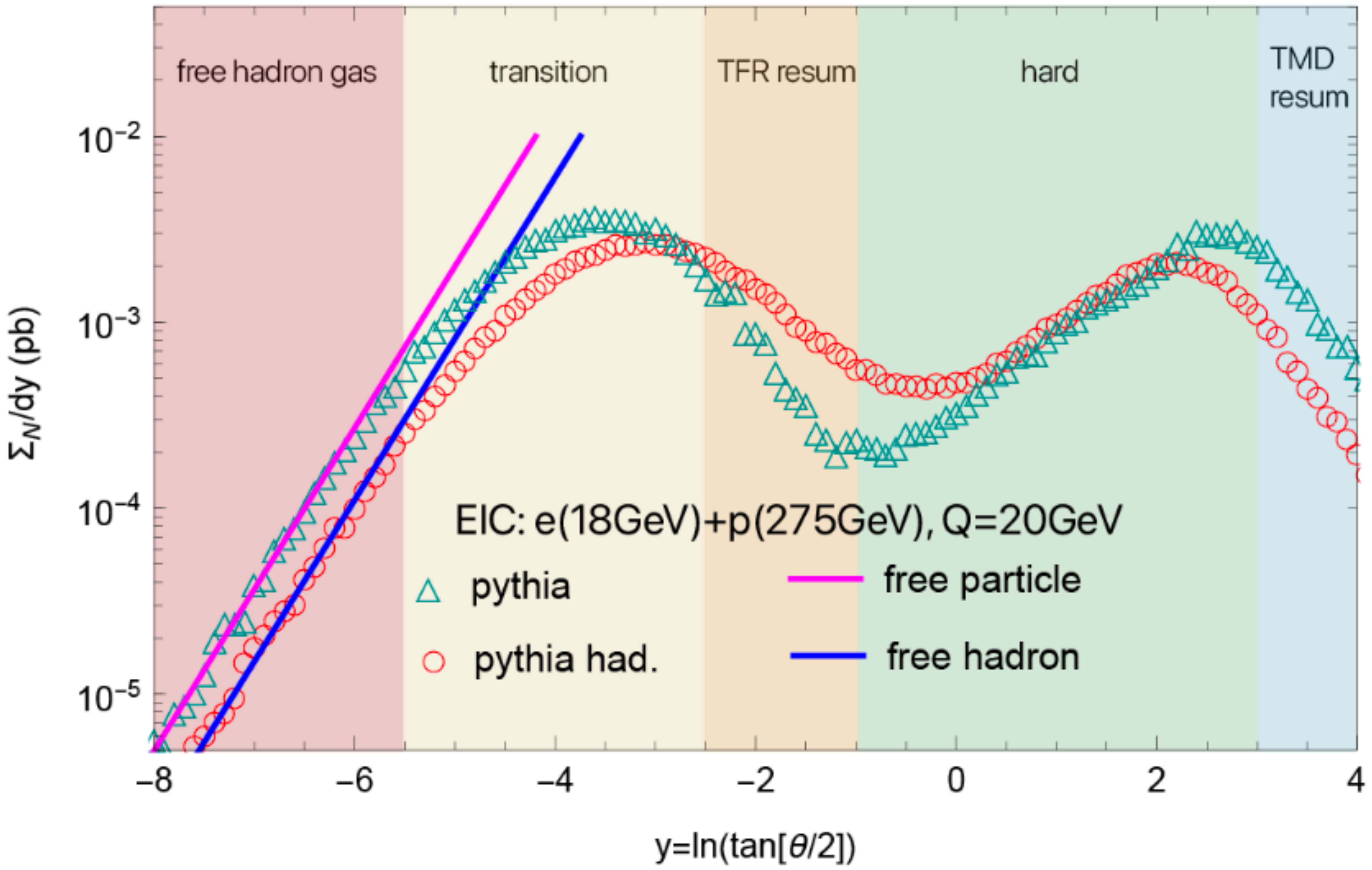
EEC and TEEC in DIS

$$\text{EEC} = \frac{1}{\sigma}\sum_a \int d\sigma(\ell + h \to \ell + a + X) x_B^{N-1} \frac{P \cdot p_a}{P \cdot q} \delta\left(\cos\theta_{ap} - \cos\theta\right)$$

NEEC, Liu, Zhu, arXiv:2209.02080; Cao, Liu, Zhu, arXiv:2303.01530.



Cao, HTL, Mi, arXiv:2312.07655



TFR: the correlation of the energy flows from the initial nucleon.

Hard: measures the perturbative behavior of QCD

TMD: measures perturbative and nonperturbative TMD physics

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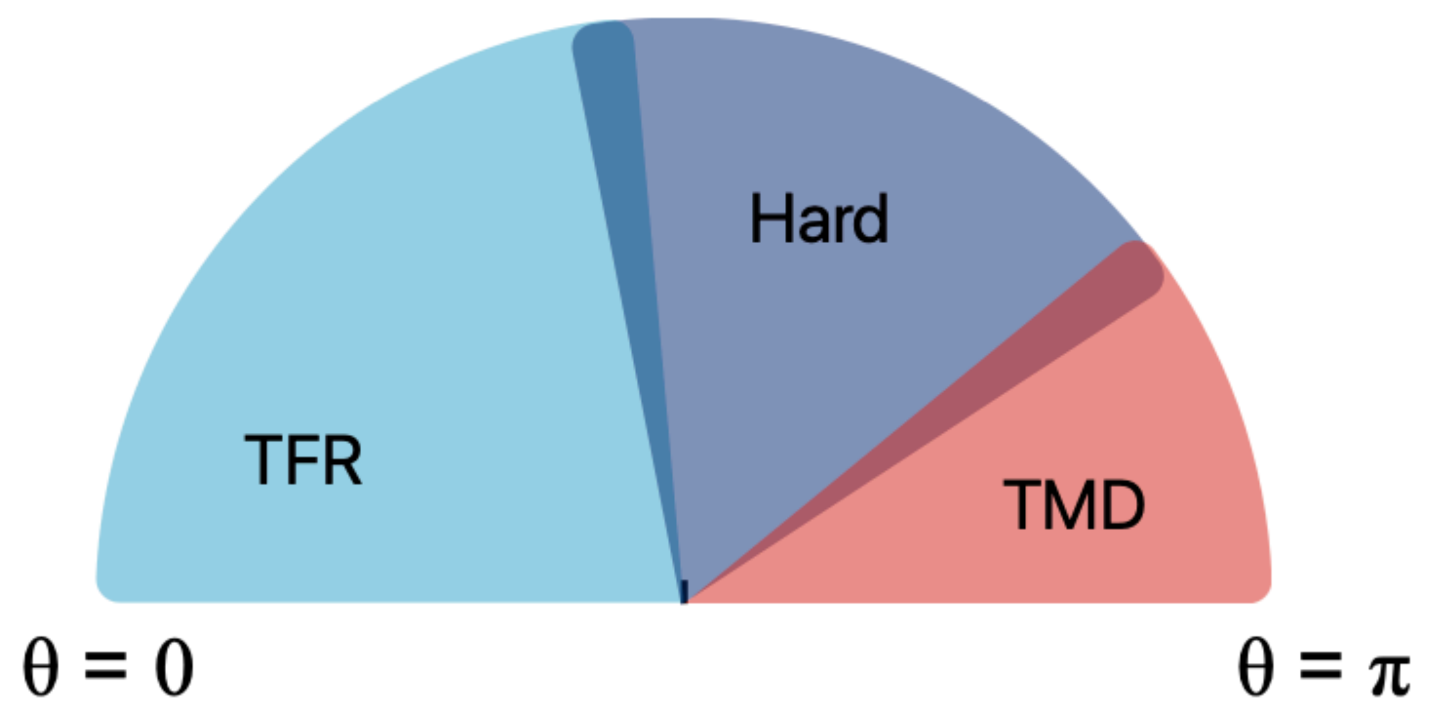

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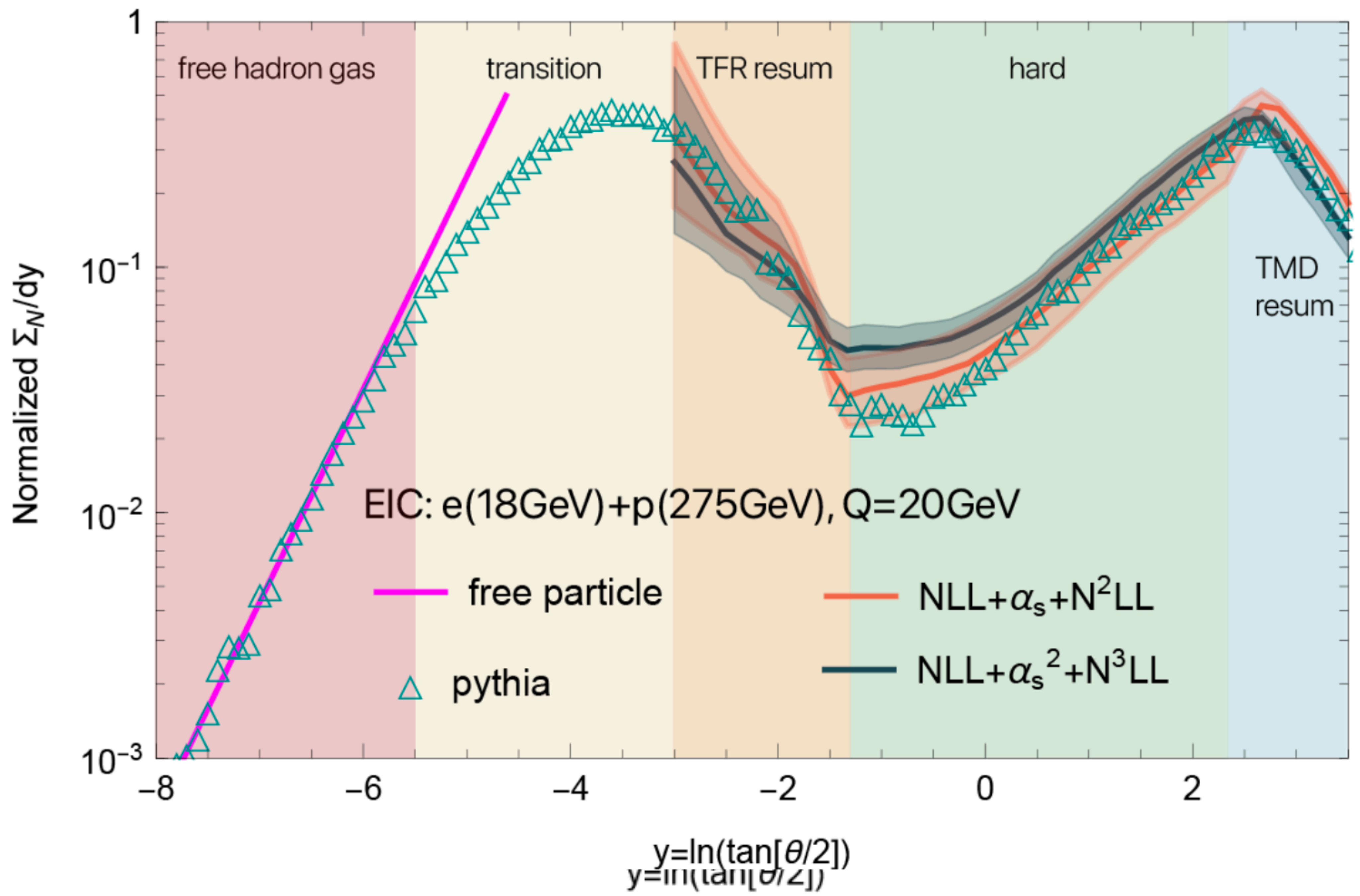

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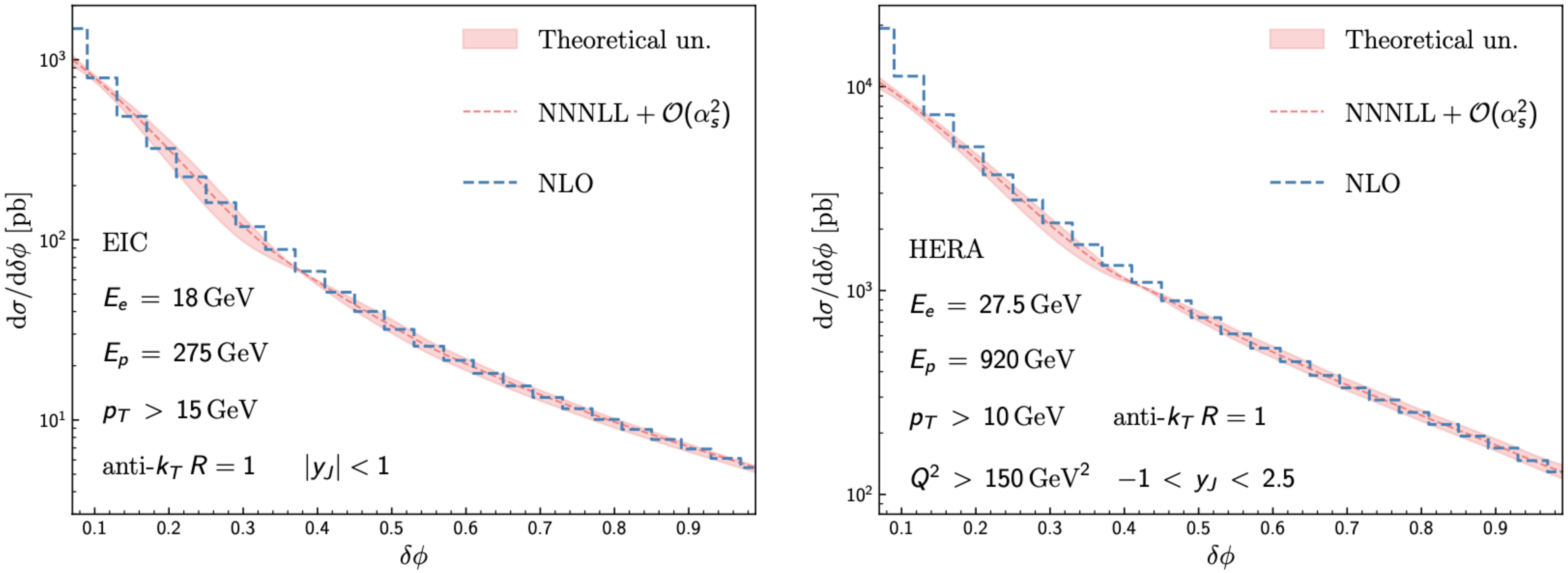
Lepton-Jet correlation in DIS

jets are defined by the anti-kTclustering algorithm and the winner-take-all recombination scheme.

$$\frac{d\sigma}{d^2 p_T\, dy_J\, d\lambda_x} = \sigma_0\, H(Q,\mu) \int_{-\infty}^{+\infty} \frac{db_x}{2\pi} e^{ib_x\lambda_x} \sum_q e_q^2\, \mathcal{B}_{q/p}\left(x_{\mathtt{bj}}, b_x, \mu, \zeta_B/\nu^2\right)$$

$$\times\, \mathcal{J}_q\left(b_x, \mu, \zeta_{\mathcal{J}}/\nu^2\right) \mathcal{S}\left(b_x, n\cdot n_J, \mu, \nu\right),$$

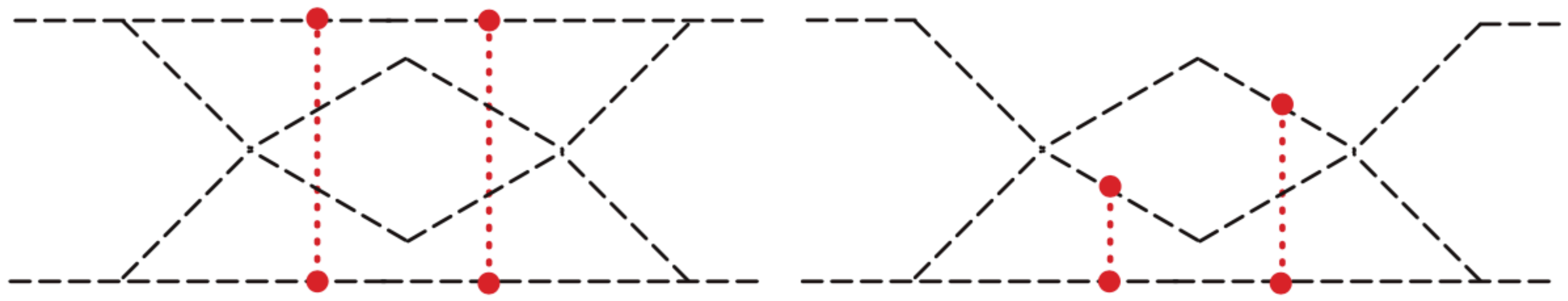


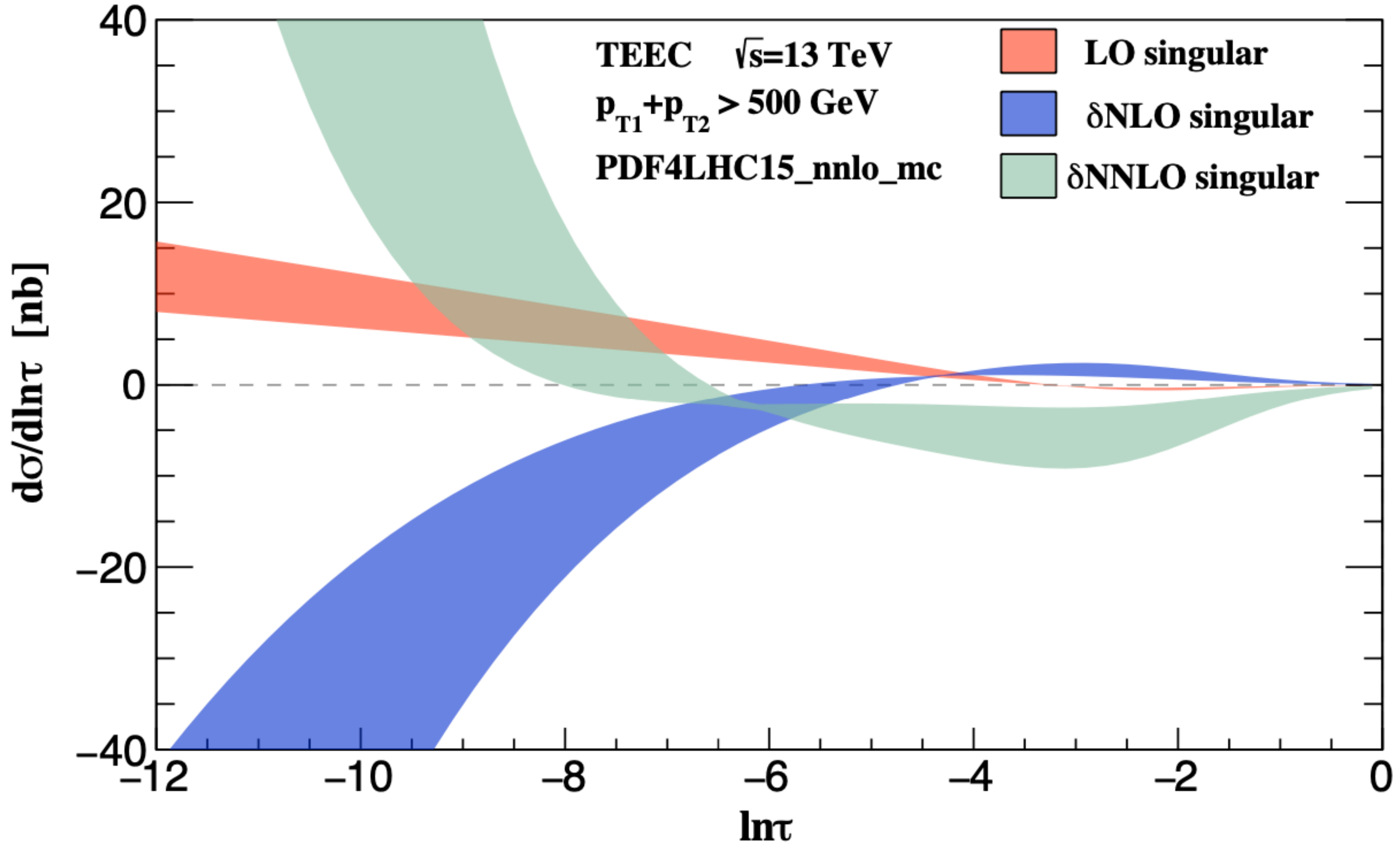
Feng, Gao, HTL, Shao, arXiv:2409.09248

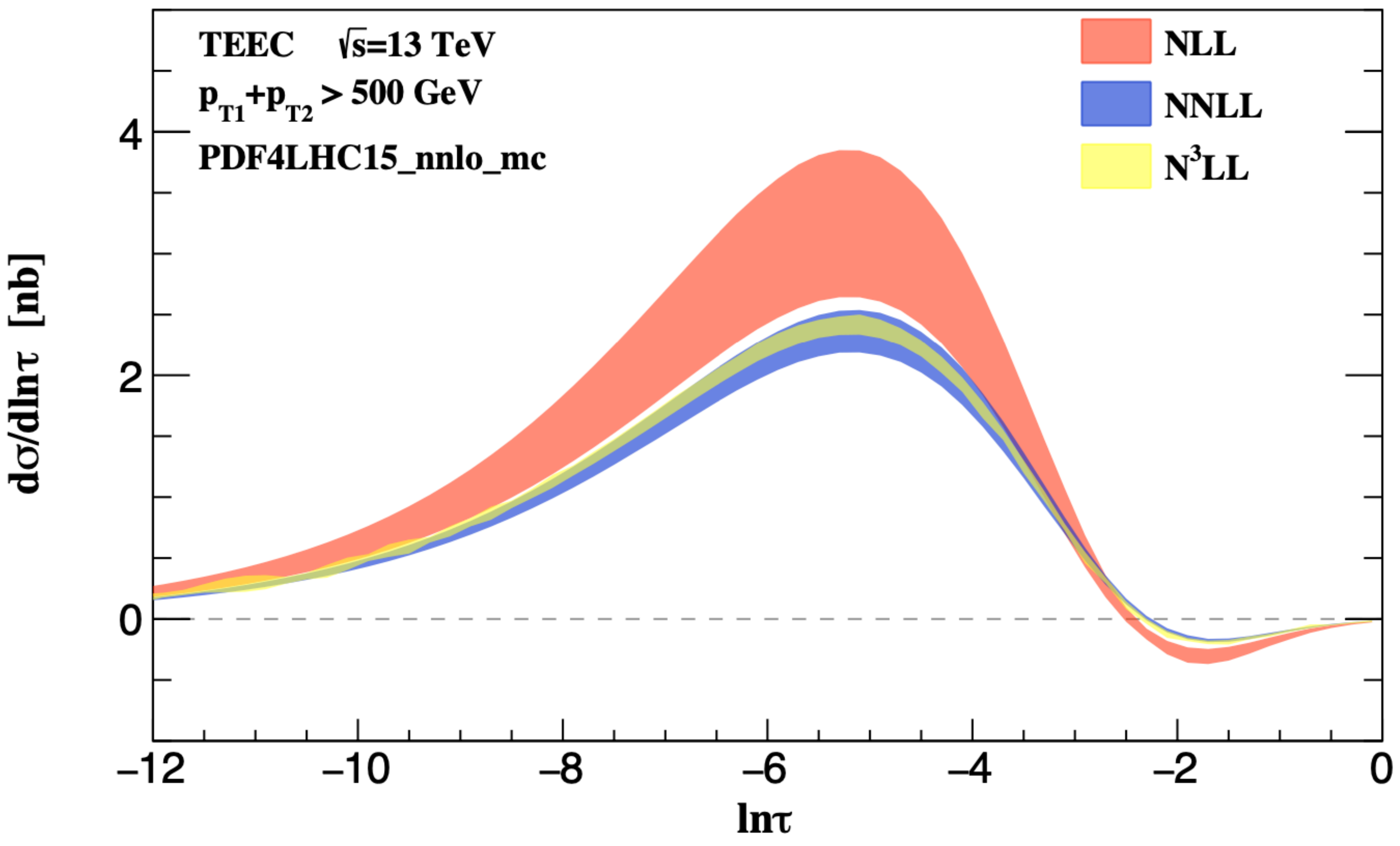
TEEC at the LHC

- Such as violation in collinear factorization
- Whether rapidity factorization is still valid
- RG invariance of the cross section

$$\mathscr{L}^{(0)} = \mathscr{L}^{(0)}_{B_1} + \mathscr{L}^{(0)}_{B_2} + \mathscr{L}^{(0)}_{J_1} + \mathscr{L}^{(0)}_{J_2} + \boxed{\mathscr{L}^{(0)}_{G}}$$







NNNLL accuracy for a hadron collider diet event shape for the first time.

Gao, HTL, Moult, Zhu, PRL, 2019 & JHEP 2024

TEEC at the LHC

EEC/TEEC is a class of observables which can be studied for various processes

In DIS

$$\text{TEEC} = \sum_a \int d\sigma_{lp\to l+a+X} \frac{E_{T,l} E_{T,a}}{E_{T,l} \sum_i E_{T,i}} \delta(\cos\phi_{la} - \cos\phi)$$

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For Drell-Yan

$$\frac{d\sigma}{d\cos\phi} = \int d\sigma_{pp\to l^+ + l^- + X} \delta\left(\cos\phi_{l^+l^-} - \cos\phi\right)$$

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Universality of QCD in the infrared regime

Summary

Motivation

- ❑ Event shapes serve as a QCD laboratory, a tool for QCD study
- ❑ EEC/TEEC can been studied for various processes

Observables

- ❑ TEEC and EEC in DIS

Application

- ❑ investigate QCD in low and high energy limits
- ❑ test and study TMD factorization
- ❑ extract TMD PDFs and TMD FFs

Thank you!