Emergent Symmetry from Entanglement Suppression

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Outline

1. It From (Qu)Bit & Quantum Entanglement
2. Emergent Symmetry
3. Maximal Symmetry in 2HDM
4. Conclusion

It from (qu)bit

J. A. Wheeler: Every it—every particle, every field of force, even the space-time continuum itself— derives its function, its meaning, its very existence entirely—even if in some contexts indirectly— from the apparatus-elicited answers to yes-or-no questions, binary choices, bits.

it	from	(qu)bit
Symmetry		**Entanglement**
Black Hole		Complexity
(OTOC) Correlation		Quantum Chaos
Holography		Error-Correcting Code
⋮		⋮

Entanglement Entropy

Subsystems cannot be individually described

$$|\Psi_{AB}\rangle \neq |\Psi_A\rangle \otimes |\Psi_B\rangle$$

How to quantify entanglement?

- general mixed state or bi-partite system $\rho_A = \mathrm{tr}_{\bar{A}} |\Psi\rangle\langle\Psi|$:

$$\text{Von Neumann Entropy: } E(\rho_A) = -\mathrm{tr}\left[\rho_A \log \rho_A\right]$$

$$\text{Linear Entropy: } E_L(\rho_A) \sim 1 - \mathrm{tr}\left[\rho_A^2\right]$$

- two-qubit system $|\Psi_{AB}\rangle = \sum_{a,b} c_{ab} |a\rangle_A \otimes |b\rangle_B$:

$$\text{Concurrence: } \Delta(\Psi_{AB}) = \sqrt{E_L(\rho_A)} = 2|c_{11}c_{22} - c_{12}c_{21}|$$

Generating Entanglement

How are entanglements generated?

- Entanglement Swapping (identical particles)
- Interaction $|\Psi_A\rangle \otimes |\Psi_B\rangle \to U|\Psi_A\rangle \otimes |\Psi_B\rangle$
    - QIS: U is a quantum gate.
    - HEP: U can be the S-matrix. (oberserved in $t\bar{t}$ production! *c.f. 2311.07288*)
- Local transformations (single-qubit gates) $V = U_A \otimes U_B$ do NOT change the entanglement!

$$\Delta(\Psi_{AB}) = \Delta(V\Psi_{AB}) \ .$$

The generators of two-qubit gates: $\{\underbrace{1 \otimes 1, \quad \sigma^i \otimes 1, \quad 1 \otimes \sigma^j}_{V \in U(1)\times SU(2)\times SU(2)}, \quad \sigma^i \otimes \sigma^j\}$

- Entanglement power: $\Delta(U) = \overline{\Delta(U\Psi_A \otimes \Psi_B)}$ averaging over Bloch spheres.

Minimal Entanglers

Equivalence among quantum gates

$$U \simeq U' \,, \qquad \text{if} \quad U = V_1 U' V_2$$

Using Cartan decomposition, for any two-qubit gates $U \in SU(4)$

$$U = V_1 e^{i\beta_i \sigma^i \otimes \sigma^i} V_2 \in \left[e^{i\beta_i \sigma^i \otimes \sigma^i} \right] \,, \qquad \begin{array}{l} V_{1,2} \in U(1) \times SU(2) \times SU(2) \\ i \in \{x, y, z\} \end{array} \,.$$

Which quantum gates *minimally* entangle the qubits? Low and Mehan [2104.10835]

$$\Delta(e^{i\beta_i \sigma^i \otimes \sigma^i}) = 0 \quad \Leftrightarrow \quad \beta_x = \beta_y = \beta_z = \begin{cases} 0 & \text{Identity gate } [\mathbf{1}] \\ \pi/4 & \text{SWAP gate } [\mathrm{SWAP}] \end{cases}$$

The only two "minimal entanglers"!

$$\mathbf{1} |a\rangle_A \otimes |b\rangle_B = |a\rangle_A \otimes |b\rangle_B \,, \qquad \mathrm{SWAP} |a\rangle_A \otimes |b\rangle_B = |b\rangle_A \otimes |a\rangle_B \,.$$

Outline

Simplest Example: np scattering

At non-relativistic regime, only the four-fermion contact interactions $(\bar{\psi}\Gamma\psi)^2$.

Only ${}^1S_0$ and ${}^3S_1$ channels are important:

$$\mathcal{S} = P_0 e^{2i\delta_0} + P_1 e^{2i\delta_1}$$

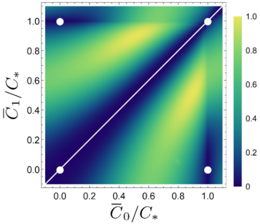
In the spin space $P_0 = \frac{1}{2}(1 - \mathrm{SWAP})$, $P_1 = \frac{1}{2}(1 + \mathrm{SWAP})$.

Condition for Entanglement Suppression

$$\begin{array}{lccl} \mathcal{S} \sim \mathbf{1} & \Leftrightarrow & \delta_0 = \delta_1 & \Rightarrow \text{ Spin-flavor} \\ \mathcal{S} \sim \mathrm{SWAP} & \Leftrightarrow & |\delta_0 - \delta_1| = \pi/2 & \Rightarrow \text{ Schrödinger} \end{array}$$

$\Downarrow$

Enhanced Symmetry



Kaplan, et.al. [1812.03138]

Baryon Scattering I

Now consider $n_f = 3$ Baryon octet $B \in \mathbf{8}$ of $SU(3)$

$$\mathbf{8} \times \mathbf{8} = \mathbf{1} + \mathbf{8}_S + \mathbf{8}_A + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{27}$$

The S-matrix has 6 channels and thus 6 independent scattering phases:

$$\mathcal{S} = P_0 \sum_{\mathbf{R} \in \{\mathbf{1}, \mathbf{8_S}, \mathbf{27}\}} P_{\mathbf{R}} e^{2i\delta_{0,\mathbf{R}}} + P_1 \sum_{\mathbf{R}' \in \{\mathbf{8_A}, \mathbf{10}, \overline{\mathbf{10}}\}} P_{\mathbf{R}'} e^{2i\delta_{1,\mathbf{R}'}}$$

Condition for Entanglement Suppression

$$\mathcal{S} \sim \mathbf{1} \quad \Leftrightarrow \quad \delta_{0,\mathbf{R}} = \delta_{1,\mathbf{R}'}$$

$$\mathcal{S} \sim \mathrm{SWAP} \quad \Leftrightarrow \quad |\delta_{0,\mathbf{R}} - \delta_{1,\mathbf{R}'}| = \pi/2$$

Liu, Low and Mehan [2210.12085]

Baryon Scattering II

We can enforce Entanglement Suppression in different (Q, S) sector, leading to different Enhanced Symmetry.

Flavor Subspace	Symmetry of Lagrangian
np $\Sigma^-\Xi^-$ $\Sigma^+\Xi^0$	$SU(6)$ spin-flavor symmetry or Schrödinger symmetry in $\mathbf{27}$ and $\overline{\mathbf{10}}$ irrep channels
$n\Sigma^-$ $p\,\Sigma^+$ $\Xi^-\Xi^0$	conjugate of $SU(6)$ spin-flavor symmetry or Schrödinger symmetry in $\mathbf{27}$ and $\mathbf{10}$ irrep channels
$(p\Lambda,\ p\Sigma^0,\ n\Sigma^+)$ $(n\Lambda,\ n\Sigma^0,\ p\Sigma^-)$ $(\Sigma^-\Lambda,\ \Sigma^-\Sigma^0,\ n\,\Xi^-)$ $(\Sigma^+\Lambda,\ \Sigma^+\Sigma^0,\ p\,\Xi^0)$ $(\Sigma^-\Xi^0, \Xi^-\Sigma^0, \Xi^-\Sigma^0)$ $(\Xi^-\Sigma^+, \Xi^0\Lambda, \Xi^0\Sigma^0)$	$SO(8)$ flavor symmetry or Schrödinger symmetry in $\mathbf{27}$, $\mathbf{8_S}$, $\mathbf{8_A}$, $\mathbf{10}$ and $\overline{\mathbf{10}}$ irrep channels
$(\Sigma^+\Sigma^-,\ \Sigma^0\Sigma^0,\ \Lambda\Sigma^0,\ \Xi^-p,\ \Xi^0 n,\ \Lambda\Lambda)$	$SU(16)$ symmetry or $SU(8)$ and Schrödinger symmetry

Outline

Two-Higgs-Doublet Model

Two flavors of $SU(2)_L$ doublet $\Phi_a = (\underbrace{\Phi^+_{a=1,2}}_{p_{\uparrow,\downarrow}}, \underbrace{\Phi^0_{a=1,2}}_{n_{\uparrow,\downarrow}})$.

$$\begin{aligned}\mathcal{V}(\Phi_1,\Phi_2) &= m_1^2\Phi_1^\dagger\Phi_1 + m_2^2\Phi_2^\dagger\Phi_2 - \left[m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right] \\ &+ \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &+ \left[\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 + \lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + h.c.\right] .\end{aligned}$$

Consider tree-level scattering $\mathcal{S}(\Phi_a^+, \Phi_b^0 \to \Phi_c^+, \Phi_d^0) \equiv 1 + iM_{ab,cd}\delta^{(4)}(p)$

Unbroken Phase: $$M_{ab,cd} = \begin{pmatrix} \lambda_1 & \lambda_6^* & \lambda_6^* & \lambda_5^* \\ \lambda_6 & \lambda_3 & \lambda_4 & \lambda_7^* \\ \lambda_6 & \lambda_4 & \lambda_3 & \lambda_7^* \\ \lambda_5 & \lambda_7 & \lambda_7 & \lambda_2 \end{pmatrix} \begin{matrix} 11 \\ 12 \\ 21 \\ 22 \end{matrix}$$
$$\begin{matrix} 11 & 12 & 21 & 22 \end{matrix}$$

Carena, Low, Wagner and **Xiao** [2307.08112]

Perturbative Entanglement Suppression

In the perturvative region, the S-matrix cannot reach [SWAP].

$$\mathcal{S}_{ab,cd} = \delta_{ab,cd} + \mathrm{i}\left(\alpha_i \sigma^i_{ac}\delta_{bd} + \beta_i \delta_{ac}\sigma^i_{bd} + \gamma\delta_{ac}\delta_{bd}\right) \ , \quad \forall\, \mathcal{S} \in [\mathbf{1}]$$

Therefore the perturbative amplitude that suppresses entanglement satisfies

$$M_{ab,cd} \sim \begin{pmatrix} \alpha_z + \beta_z + \gamma & \beta_x - \mathrm{i}\beta_y & \alpha_x - \mathrm{i}\alpha_y & 0 \\ \beta_x + \mathrm{i}\beta_y & \alpha_z - \beta_z + \gamma & 0 & \alpha_x - \mathrm{i}\alpha_y \\ \alpha_x + \mathrm{i}\alpha_y & 0 & -\alpha_z + \beta_z + \gamma & \beta_x - \mathrm{i}\beta_y \\ 0 & \alpha_x + \mathrm{i}\alpha_y & \beta_x + \mathrm{i}\beta_y & -\alpha_z - \beta_z + \gamma \end{pmatrix}$$

1. $M_{11,11} - M_{12,12} - M_{21,21} + M_{22,22} = 0\ . \qquad \Rightarrow\ \lambda_1 + \lambda_2 = 2\lambda_3\ .$
2. $M_{11,22} = M_{12,21} = M_{21,12} = M_{22,11} = 0\ . \qquad \Rightarrow\ \lambda_4 = \lambda_5 = 0\ .$
3. $M_{12,11} = M_{22,21}\ , \quad M_{21,11} = M_{22,12}\ . \qquad \Rightarrow\ \lambda_6 = \lambda_7\ .$

Enhanced Symmetry in the Unbroken Phase

The Bose symmetry for the s-wave amplitude imposes $\vec{\alpha} = \vec{\beta} = \vec{r}$

Enhanced Symmetry: $SO(2)$ rotation along $\vec{r}$

Redefine $\Phi'_a = {U_a}^b \Phi_b$ such that $U \in SU(2)$ brings $\vec{r} \parallel \hat{z}$.

$$\mathcal{V}(\Phi'_1, \Phi'_2) = \cdots + \frac{\lambda'_1}{2}({\Phi'_1}^\dagger \Phi'_1)^2 + \frac{\lambda'_2}{2}({\Phi'_2}^\dagger \Phi'_2)^2 + \lambda_3({\Phi'_1}^\dagger \Phi'_1)({\Phi'_2}^\dagger \Phi'_2)$$

- $\Phi'_{1,2}$ may have independent phase symmetries $e^{\mathrm{i}\phi_0}$ and $e^{\mathrm{i}\phi_z \sigma^z}$.
- In the original basis, $U^{-1} e^{\mathrm{i}\phi_z \sigma^z} U$ is the new $SO(2)$ rotation around $\vec{r}$.

The enhanced symmetry is typically broken by the Yukawa couplings

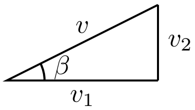
$$\mathcal{L} \supset Y_u \Phi_1 \bar{Q} u + Y_d \Phi_2^\dagger \bar{d} Q$$

Higgs Alignment

Now consider the Spontaneous Symmetry Breaking $\langle \Phi_a \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_a \end{pmatrix}$.

- Define Higgs basis: $\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$

  such that $\langle H_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$ and $\langle H_2 \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.



$$\mathcal{V}(\Phi_1, \Phi_2; \lambda_{1\sim 7}, m^2_{1,2,12}) = \mathcal{V}(H_1, H_2; Z_{1\sim 7}, Y_{1,2,3}) \ , \quad Y_{1,3} = -\frac{Z_{1,6}}{2} v^2 \ .$$

- Mass eigenstates in the neutral sector ($\tilde{\alpha} = \alpha - \beta$)

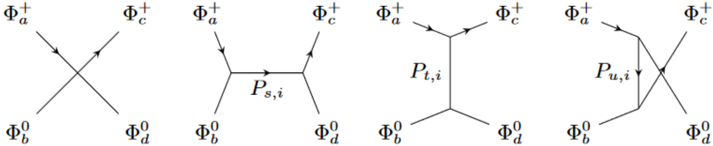
$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_{\tilde{\alpha}} & s_{\tilde{\alpha}} \\ -s_{\tilde{\alpha}} & c_{\tilde{\alpha}} \end{pmatrix} \begin{pmatrix} \mathrm{Re} H_1^0 \\ \mathrm{Re} H_2^0 \end{pmatrix}$$

- When $c_{\tilde{\alpha}} = 0$, **Higgs alignment**, we get a SM-like Higgs boson h.

Entanglement Suppression in the Broken Phase

New tree diagrams from 3-point vertices (gauge couplings turned off)

$$\mathrm{i}M_{ab,cd} = \mathrm{i}M^0_{ab,cd}(Z_i) - \frac{v^2}{2}\sum_{i=1}^{2}\sum_{r=s,t,u} M_i^r{}_{ab,cd}(Z_i,\tilde{\alpha}) \underbrace{\times P_{r,i}}_{\text{propagators}}$$



Demand all 7 of M^0 and the M_i^r satisfy:

1. $M_{11,11} - M_{12,12} - M_{21,21} + M_{22,22} = 0$.
2. $M_{11,22} = M_{12,21} = M_{21,12} = M_{22,11} = 0$.
3. $M_{12,11} = M_{22,21}\,, \quad M_{21,11} = M_{22,12}$.

No solution for all the flavor tensors!

Emergent Maximal Symmetry

A last chance: if the two charged scalars H_1^+ and H_2^+ are degenerate

$$P_1^s = P_2^s\ ,\ P_1^u = P_2^u \quad \Rightarrow \quad m_{H^+}^2 = 0,\ Y_2 = -\frac{Z_3}{2}v^2\ ,$$

we only need the combinations $M_1^s + M_2^s$ and $M_1^u + M_2^u$ to satisfy the entanglement suppression condition!

$$\begin{pmatrix} Z_1^2 + Z_6^2 & Z_1 Z_6 & (Z_1+Z_3)Z_6 & Z_6^2 \\ Z_1 Z_6 & Z_6^2 & Z_6^2 & 0 \\ (Z_1+Z_3)Z_6 & Z_6^2 & Z_6^2 + Z_3^2 & Z_3 Z_6 \\ Z_6^2 & 0 & Z_3 Z_6 & Z_6^2 \end{pmatrix} \Rightarrow Z_1 = Z_3 \text{ and } Z_6 = 0$$

The resulting potential: $\mathcal{V} = \frac{Z_1}{2}\left[H_1^\dagger H_1 + H_2^\dagger H_2 - \frac{v^2}{2}\right]^2$ has $SO(8)$ symmetry!

Outline

Conclusion & Outlook

- Information-theoretic properties may provide insights on the origin of physical principles.
- Entanglement suppression often leads to emergent enhanced symmetries.
  - Baryon scattering
  - Scalar scattering
  - Vector boson?
- Entanglement suppression in the broken phase impose constraints even on the spectrum, leading to the maximal symmetry in 2HDM.
- How does the entanglement behave when the maximal symmetry is softly broken in a realistic model?
- More general amplitudes and models need to be investigated!

Thank you for your attention!