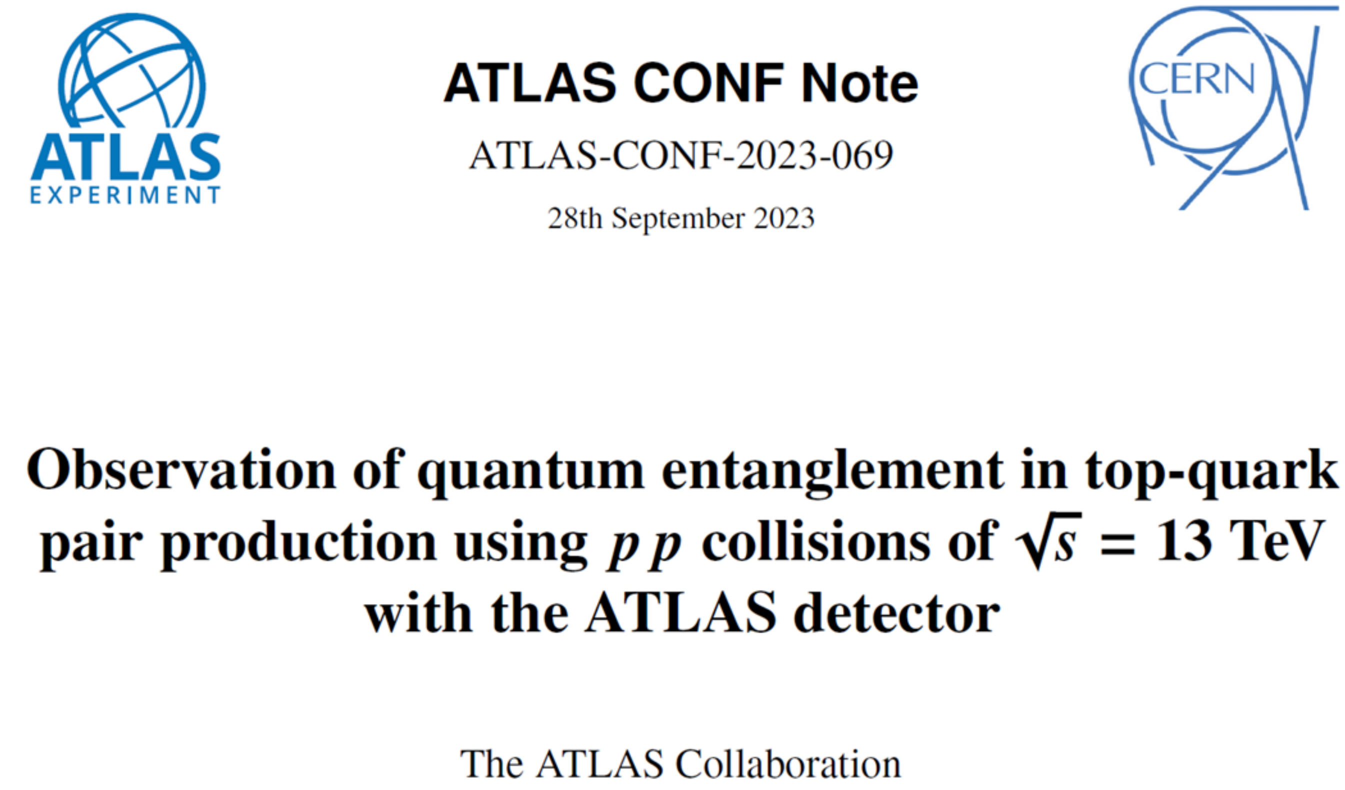
Quantum Entanglement in High Energy Physics

Yu Shi (施郁)

2024.7.9.

28th LHC MiniWorkshop



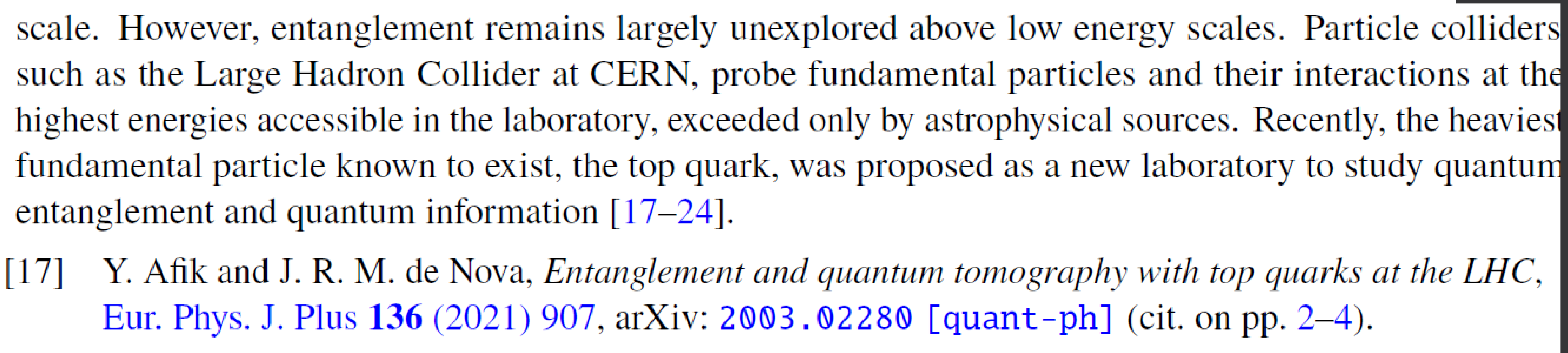
ATLAS CONF Note

ATLAS-CONF-2023-069

28th September 2023

Observation of quantum entanglement in top-quark pair production using pp collisions of $\sqrt{s} = 13$ TeV with the ATLAS detector

The ATLAS Collaboration



scale. However, entanglement remains largely unexplored above low energy scales. Particle colliders such as the Large Hadron Collider at CERN, probe fundamental particles and their interactions at the highest energies accessible in the laboratory, exceeded only by astrophysical sources. Recently, the heaviest fundamental particle known to exist, the top quark, was proposed as a new laboratory to study quantum entanglement and quantum information [17–24].

[17] Y. Afik and J. R. M. de Nova, *Entanglement and quantum tomography with top quarks at the LHC*, Eur. Phys. J. Plus **136** (2021) 907, arXiv: `2003.02280 [quant-ph]` (cit. on pp. 2–4).

CMS Physics Analysis Summary

Contact: cms-pag-conveners-top@cern.ch 2024/06/12

Measurements of polarization, spin correlations, and entanglement in top quark pairs using lepton+jets events from pp collisions at $\sqrt{s} = 13$ TeV

The CMS Collaboration

The top quark polarization and spin correlations measurement is interesting in its own right as a test of the standard model (SM), but it also provides new opportunities for testing quantum mechanics (QM) at high energies using the decay products of unstable particles as probes. This is not possible in experiments with stable particles, i.e., electrons and photons. One of the important predictions of QM is quantum entanglement which has been studied in connection with particle physics at high energies only in recent years [18–21].

[18] Y. Afik and J. R. M. de Nova, "Entanglement and quantum tomography with top quarks at the LHC", *Eur. Phys. J. Plus* **136** (2021) 907, `doi:10.1140/epjp/s13360-021-01902-1`, `arXiv:2003.02280`.

Eur. Phys. J. Plus (2021) 136:907
https://doi.org/10.1140/epjp/s13360-021-01902-1

THE EUROPEAN
PHYSICAL JOURNAL PLUS

Regular Article

Entanglement and quantum tomography with top quarks at the LHC

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The standard model (SM) of particle physics is a quantum field theory, based on special relativity and quantum mechanics. Therefore, it allows to test fundamental properties of quantum mechanics. For instance, entanglement has been studied in the context of particle physics [17–20] and is currently a hot topic of research in the field [21–25].

18. Yu. Shi, Entanglement in relativistic quantum field theory. Phys. Rev. D **70**, 105001 (2004)
21. Yu. Shi, J.-C. Yang, Entangled baryons: violation of inequalities based on local realism assuming dependence of decays on hidden variables. Eur. Phys. J. C **80**(2), 116 (2020)

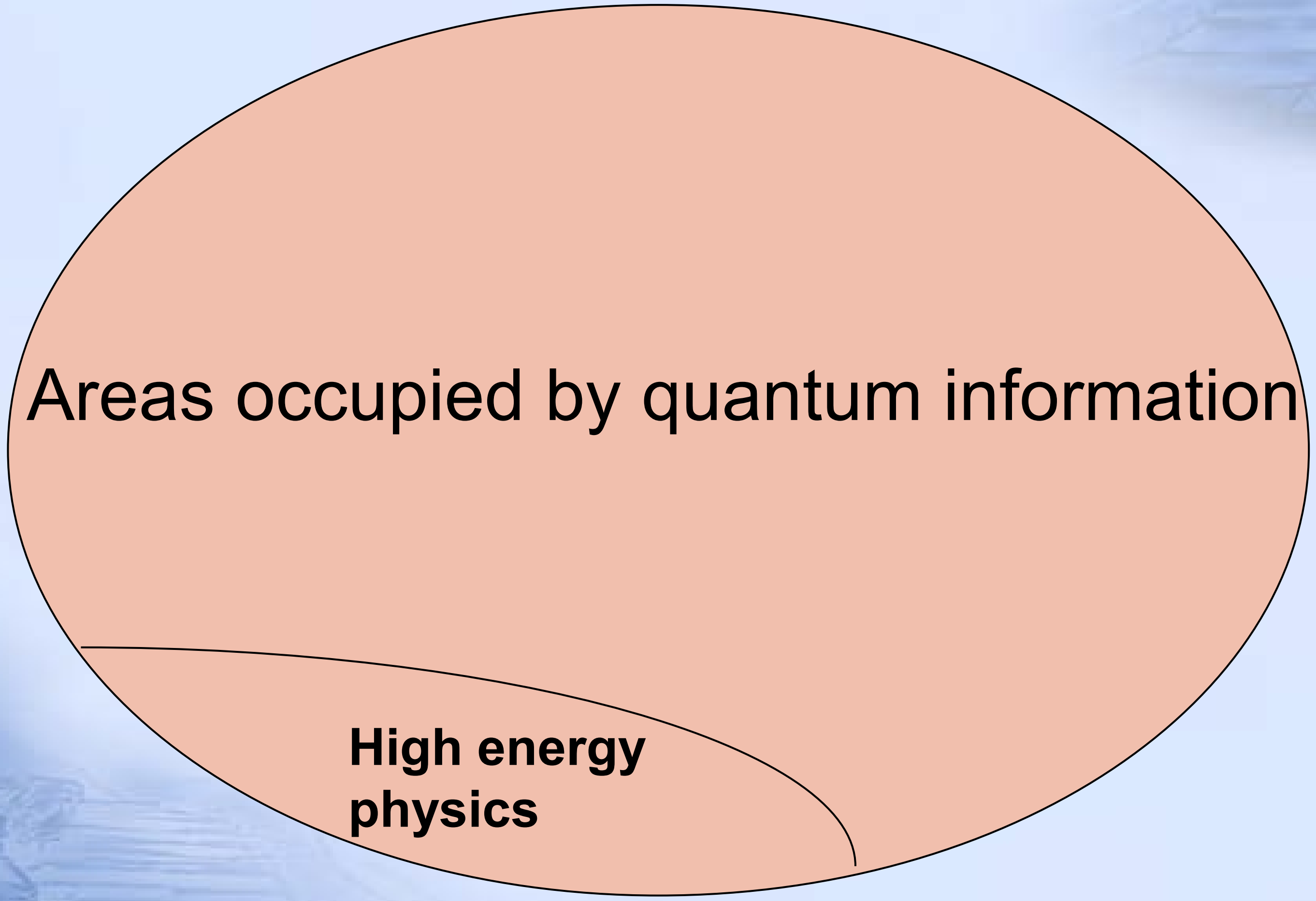
Towards high energy quantum information: quantum teleportation using neutral kaons

Yu Shi
(Fudan University)

Ref: quant-ph/0605070; appearing in Phys. Lett. B

International Conference on Quantum Foundation and Technology: Frontier and Future, Hangzhou, 27/8/2006

Map of Quantum Physics



Areas occupied by quantum information

High energy physics

New Map of Quantum Physics

Areas occupied by quantum information

高能量子信息初探：中性K介子的超距传态
Towards high energy quantum information: quantum teleportation using neutral kaons

施郁
(复旦大学)

Ref: quant-ph/0605070; Phys. Lett. B 641, 75 (2006)

中国物理学会2006年秋季会议，北京，9.16.

Is quantum information relevant to particle physics?

Yu Shi (施郁)
Fudan University

Institute of Theoretical Physics, Aug. 2007

Is quantum information relevant to particle physics?

Yu Shi (施郁)
Fudan University

Conference in honor of CN Yang's 85th Birthday
Singapore, 2 Nov 2007

PROF. C. N. YANG AND QUANTUM ENTANGLEMENT IN PARTICLE PHYSICS

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Department of Physics, Fudan University, Shanghai 200433, China

Is quantum information relevant to particle physics?

Yu Shi
Fudan University, Shanghai, China

Collaborator: Yue-Liang Wu (ITP, Beijing)

OSA workshop on entanglement and quantum decoherence,
Nara, 30 Jan 2008

粒子物理中的量子纠缠与量子信息

Quantum entanglement and quantum information in particle physics

施郁

复旦大学物理系

第十届全国粒子物理会议，南京，2008.4.28.

Conclusion

- Quantum information can be discussed in the setting of particle physics.
- Like that QI stimulates researches on quantum coherence in other areas, similar works could be done for particle physics.
- Such studies could reveal some new features unavailable in nonrelativistic regime, and deep connections between matter and information, provide new insights on particle physics!

Thank you for your attention!

粒子物理中的量子纠缠与量子信息
Quantum entanglement and quantum information in particle physics

施郁
复旦大学物理系

清华大学，2008.10.23

Connecting quantum information to particle physics

施郁

复旦大学物理系

上海大学，2008.12.22

粒子物理中的量子纠缠

施郁

复旦大学物理系

南京大学，2012.5.

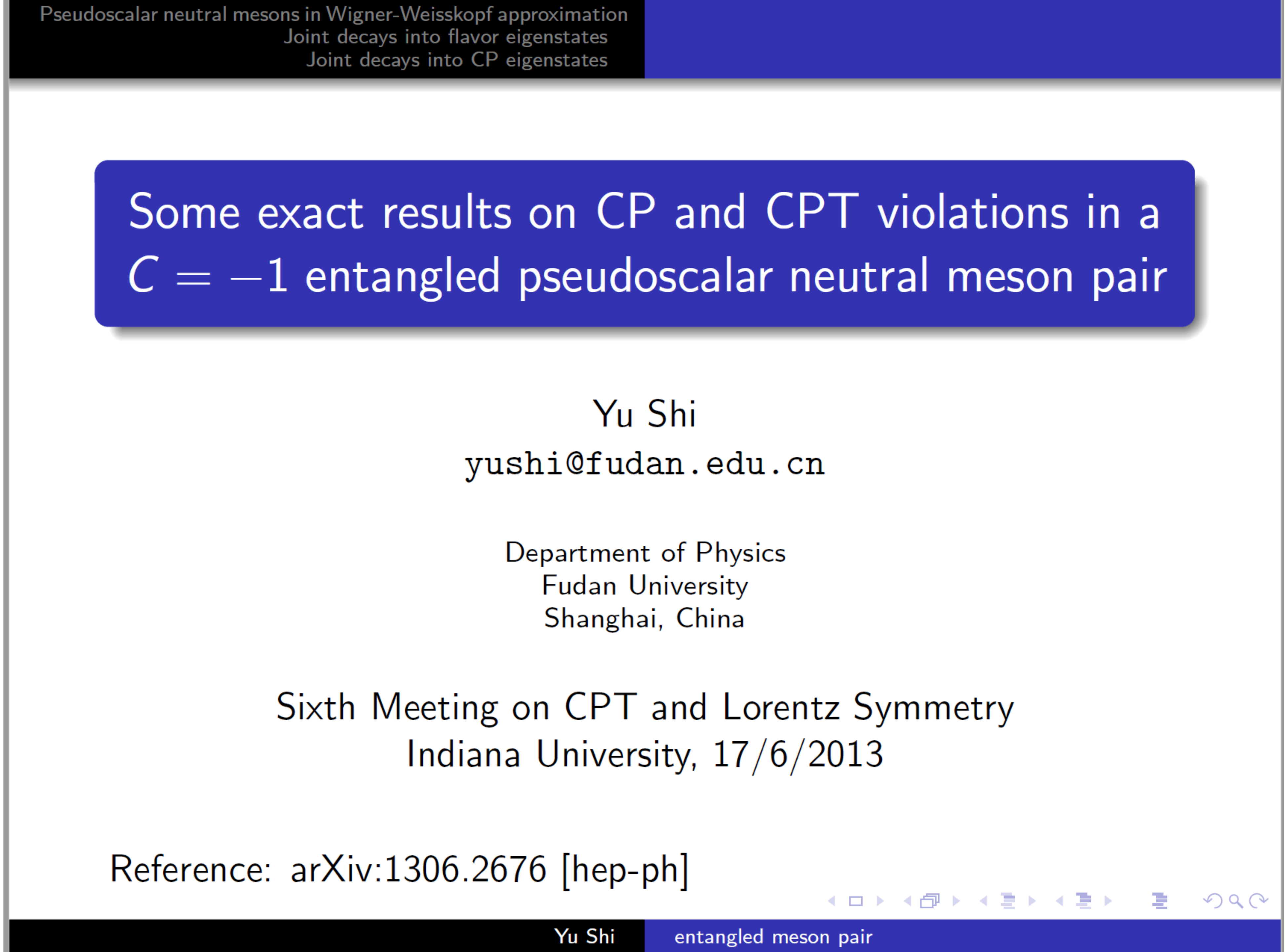
Some theoretical consideration of measuring CP and CPT violating parameters using quantum entanglement

Yu Shi

Fudan University, Shanghai, China

KLOE2, LNF, 26/6/2012

Collaborators:
Zhijie Huang
(Fudan),
Yueliang Wu
(ITP, CAS)



Some exact results on CP and CPT violations in a $C = -1$ entangled pseudoscalar neutral meson pair

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Department of Physics
Fudan University
Shanghai, China

Sixth Meeting on CPT and Lorentz Symmetry
Indiana University, 17/6/2013

Reference: arXiv:1306.2676 [hep-ph]

Some general results on CP and CPT violating parameters determined from C=-1 and C=+1 entangled states

Yu Shi
`yushi@fudan.edu.cn`
Fudan University, Shanghai, China

Workshop *Questioning Fundamental Physical Principles*, CERN, 6/5/2014

Refs: (1)YS, EPJC **73**, 2506 (2013); arXiv:1307:2676.
(2)Z. Huang & YS, PRD **89**, 016018 (2014); arXiv:1307:4459.
(3)Z. Huang & YS, EPJC **72**, 1900 (2012); arXiv:1112:3700.

Using Quantum Entanglement to Study CP and CPT Violations in Particle Physics

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Fudan University, Shanghai, China

Relativistic Quantum Information North 2014, University of Seoul, July 3 2014

References:

Z. Huang & YS, Euro. Phys. J. C **72**, 1900 (2012); arXiv:1112:3700.
YS, Euro. Phys. J. C **73**, 2506 (2013); arXiv:1307:2676.
Z. Huang & YS, Phys. Rev. D **89**, 016018 (2014); arXiv:1307:4459.

Quantum entanglement in high energy particle accelerators

Yu Shi

Fudan University

Ref: YS & J C Yang, arXiv:1612.07628
Huang & YS, PRD 89, 016018;
YS, EPJC 73, 2506; Huang & YS, EPJC 72, 1900.

Second Conference on Quantum Matter, Information, and Gravity
Zhang Jia Jie 2017.7.4.

用量子纠缠探索基本粒子、黑洞和宇宙

施郁
复旦大学物理学系

东华大学，2018．11．27．

Reference:
YS, J Yang, PRD 98, 075019 (18); Z Huang, YS, PRD 89, 016018; YS, EPJC 73, 2506; Huang, YS, EPJC 72, 1900; Y Li, Y Dai, YS, PRD 95, 036006; Y Li, Y Dai, YS, EPJC 77, 598; Y Dai, Z Shen, YS, PRD 94, 025012; Y Dai, Z Shen, YS, JHEP 02, 071.

量子力学与局域实在论冲突，非局域实在论呢？

施郁

(复旦大学)

量子力学年会，国科大，雁栖湖畔

2019.8.28

量子力学满足非局域实在论吗：从光子到高能粒子

施郁

复旦大学物理学系

2019.12.10.

南方科技大学

量子纠缠，量子计算与量子模拟

施郁

复旦大学物理学系

中科院高能物理研究所

2022.5.18.

量子纠缠之路：
从爱因斯坦到2022年诺贝尔物理学奖

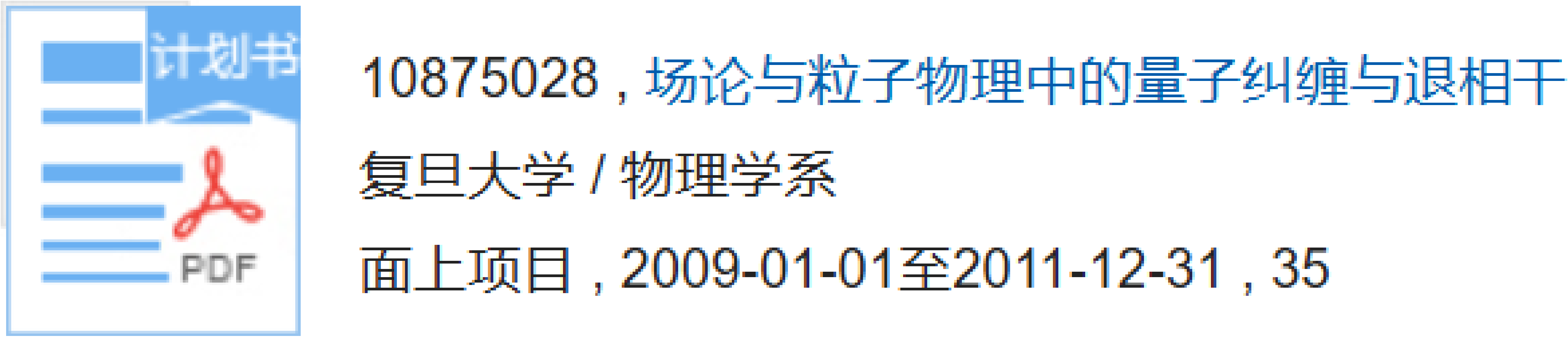
施郁

（复旦大学）

2022.11.19.

中国物理学会秋季会议

南方科技大学



10875028，场论与粒子物理中的量子纠缠与退相干

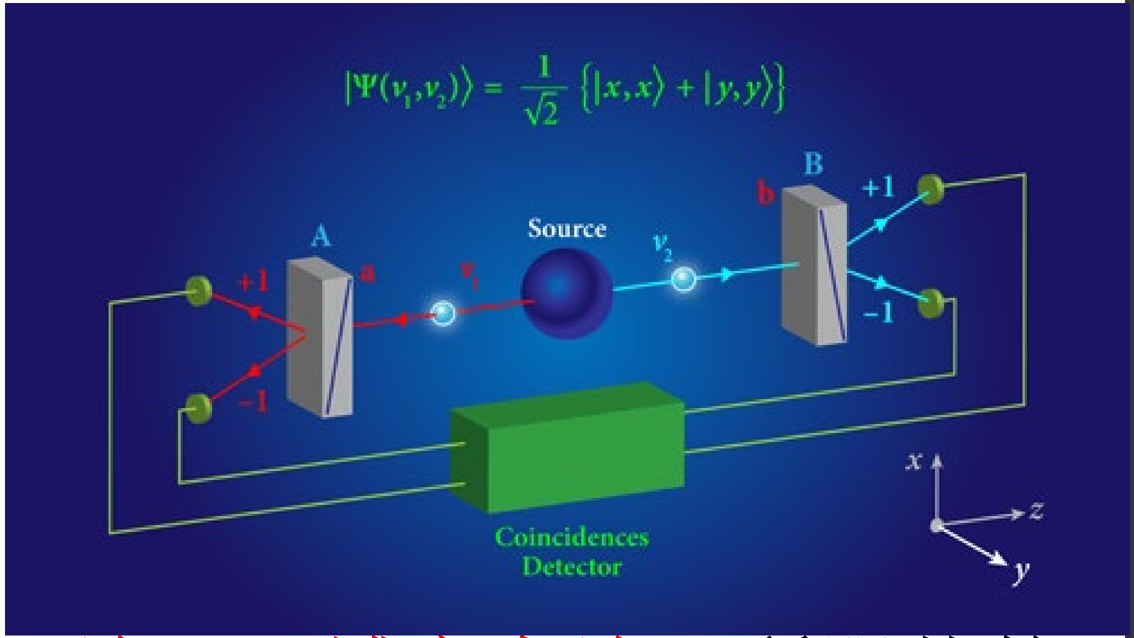
复旦大学 / 物理学系

面上项目，2009-01-01至2011-12-31，35

Historic background

- Ref:
- 施郁. 量子纠缠之路：从爱因斯坦到2022年诺贝尔物理学奖[J]. 自然杂志，2022，44(6): 455-465； Yu SHI. The road of quantum entanglement: from Einstein to 2022 Nobel Prize in Physics[J]. Chinese Journal of Nature, 2022, 44(6): 455-465.
- 施 郁 . 粒子物理中量子纠缠的历史起源 [J]. 物理学进展, 2023, 43(3): 57-67;
- Yu Shi. Historic Origin of Quantum Entanglement in Particle Physics[J]. Progress in Physics, 2023, 43(3): 57-67.

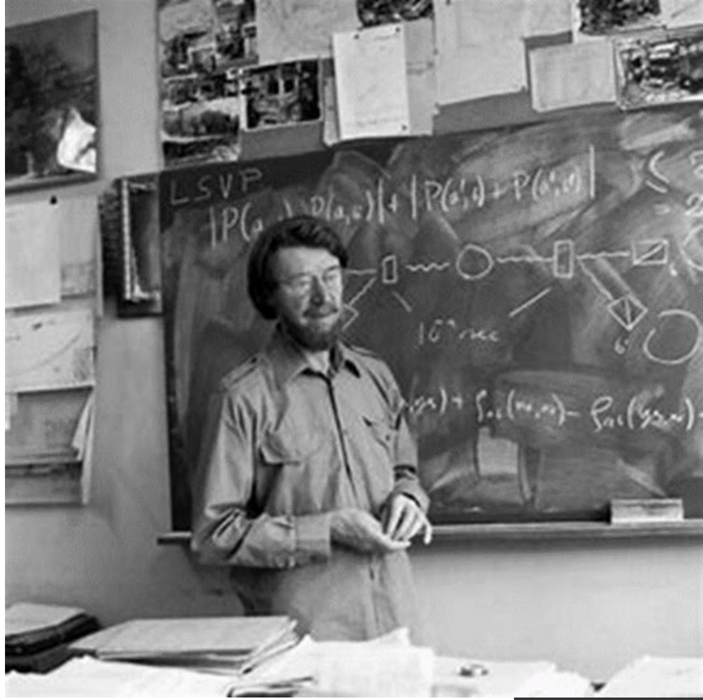
贝尔不等式



- 1964年，贝尔提出局域隐变量理论（局域实在论）所服从的不等式。
- $A(\mathbf{a},\lambda)=\pm 1$, $B(\mathbf{b},\lambda)=\pm 1$
- 局域实在论： $P(\mathbf{a},\mathbf{b})=\int d\lambda\rho(\lambda)A(\mathbf{a},\lambda)B(\mathbf{b},\lambda)$
- 假设完美反关联： $1+P(\mathbf{a},\mathbf{b})\geq|P(\mathbf{a},\mathbf{b})-P(\mathbf{a},\mathbf{c})|$
- 量子力学： $P(\mathbf{a},\mathbf{b})=\langle\boldsymbol{\sigma}\cdot\mathbf{a}\ \boldsymbol{\sigma}\cdot\mathbf{b}\rangle=-\mathbf{a}\cdot\mathbf{b}$
- 违反不等式。

CHSH-Bell不等式

- 更适合实际情况。
- S=P(**a**,**b**)+P(**a**,**b**’)+P(**a**’,**b**) – P(**a**’,**b**’)
- – 2≤S ≤2
- 量子纠缠态可以违反此式。
- 大量实验判定量子力学胜利，局域实在论失败。



贝尔测试中的“补洞”

- 局域性要求：每边的测量结果不应该依赖于另一边的仪器。
- 探测漏洞：当一边测量到光子时，另一边也探测到光子的概率需要大于2/3。
- 这些漏洞都已补上（Aspect, Zeilinger等人）。

自由选择（测量装置）漏洞

- 在所有以前的实验中，都是由仪器随机选择实验装置的安排（偏振片方向）。
- 这是不理想的，因为万一这些仪器所作的选择本身就是由隐变量决定的呢？
- 这叫做“自由选择漏洞”，也叫“测量装置漏洞”。
- 贝尔提出可以用人的自由选择来保证实验装置的安排的不可预测性。但是当时的技术做不到。
- 2018年5月9日发表的13个实验“用人的选择挑战局域实在论”填补了这个漏洞。

加密非局域实在论(Crypto-nonlocal realism)

- 矛盾来自哪里，局域论还是实在论？
- 为研究这个问题，Leggett考虑一种“加密非局域实在论”，导出Leggett不等式（被量子力学违反）：
  - 物理态是各种偏振方向的子系综的统计混合。
  - 给定隐变量，被测量量也依赖于另一边的偏振片方向（非局域）。
  - 对于每个子系综，被测量量对于隐参量的平均服从Malus Law（是局域的）。

$$\bar{A}(\mathbf{u}) = \int d\lambda\, \rho_{\mathbf{u},\mathbf{v}}(\lambda) A(\mathbf{a},\mathbf{b},\lambda) = \mathbf{u}\cdot\mathbf{a}$$

正负电子湮灭
与纠缠光子

施郁. 吴健雄的科学贡献和科学精神：从量子纠缠到宇称不守恒“纪念吴健雄先生诞辰110周年学术研讨会”（东南大学，2022.5.31）; 墨子沙龙，2023-6-2
.

施 郁 . 粒子物理中量子纠缠的历史起源 [J]. 物理学进展, 2023, 43(3): 57-67; Yu Shi. Historic Origin of Quantum Entanglement in Particle Physics[J]. Progress in Physics, 2023, 43(3): 57-67.


施郁，吴健雄的科学贡献与科学精神

Wu, Shaknov, 1950

- 验证量子电动力学，Wheeler建议研究Spin singlet的湮没，Ward-Pryce，Snyder-Pasternack-Hornbostel仔细计算。
- 正负电子湮灭产生2个光子，**偏振总是相反**，分别被电子散射。对于不同散射角，测量这2个光子运动方向垂直和平行两种情况下，符合概率之比（非对称性）。散射角=82度时，非对称性最大，2.85.（偏振方向决定角度分布）
- 吴健雄与Shaknov的**γ探测器敏感度是前人的10倍**。在哥伦比亚的回旋加速器上用氘撞击铜64，激发正电子源。
- 测到2.04∓0.08. 该装置的理论值=2.00.

The Angular Correlation of Scattered Annihilation Radiation*

C. S. WU AND I. SHAKNOV

Pupin Physics Laboratories, Columbia University, New York, New York

November 21, 1949

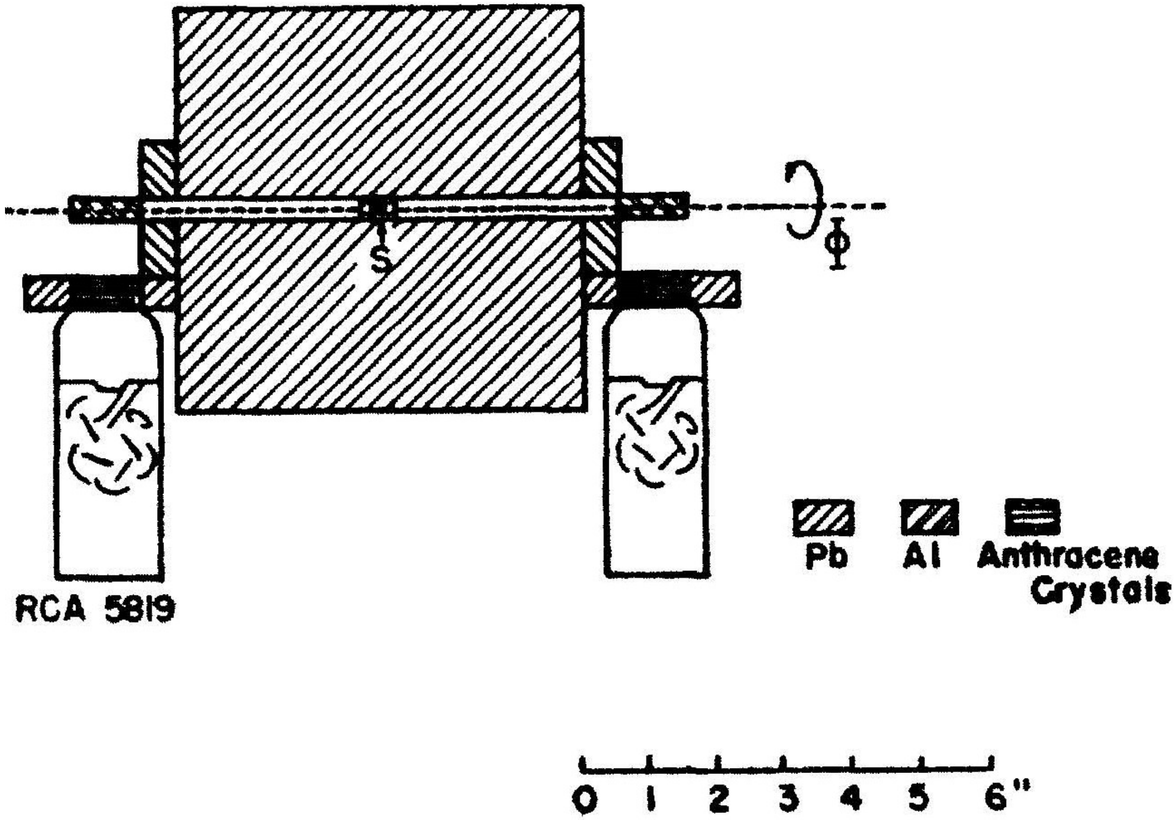


FIG. 1. Schematic diagram of experiment.

量子纠缠

- 1957，Bohm-Aharonov指出，Wu-Shaknov实现了光子偏振关联（Bohm-Aharonov不用“纠缠”一词）。Bohm-Aharonov证明非纠缠态不能给出Wu-Shaknov的实验结果。
- Wu-Shaknov第一次实验上实现了明确的、空间分离的量子纠缠态。今天的表达式：$\frac{1}{\sqrt{2}}(|h\rangle|v\rangle-|v\rangle|h\rangle)$

1975年，吴健雄小组尝试测试贝尔不等式

- 1964年，贝尔提出不等式。量子纠缠态违反贝尔不等式，但是需要沿既不平行也不垂直方向测量偏振。
- 1950年Wu-Shaknov实验中，两个光子被探测的方位角总是垂直。
- 1975年，吴健雄和学生Kasday、Ullman测量两个光子各自任意方位角的符合概率。(1975 | NUOVO CIMENTO B 25 (2) , pp.633-661).
- 与纠缠态的量子力学结果一致，与非纠缠态矛盾。
- 但是不能用来证明贝尔不等式违反，因为局域实在论也能计算康普顿散射，而且不能将在某个方位的光子探测与偏振方向明确锁定。
- 这里是高能光子，不好直接做偏振测量（可击穿偏振片）。低能光子的偏振可直接用偏振片测量。

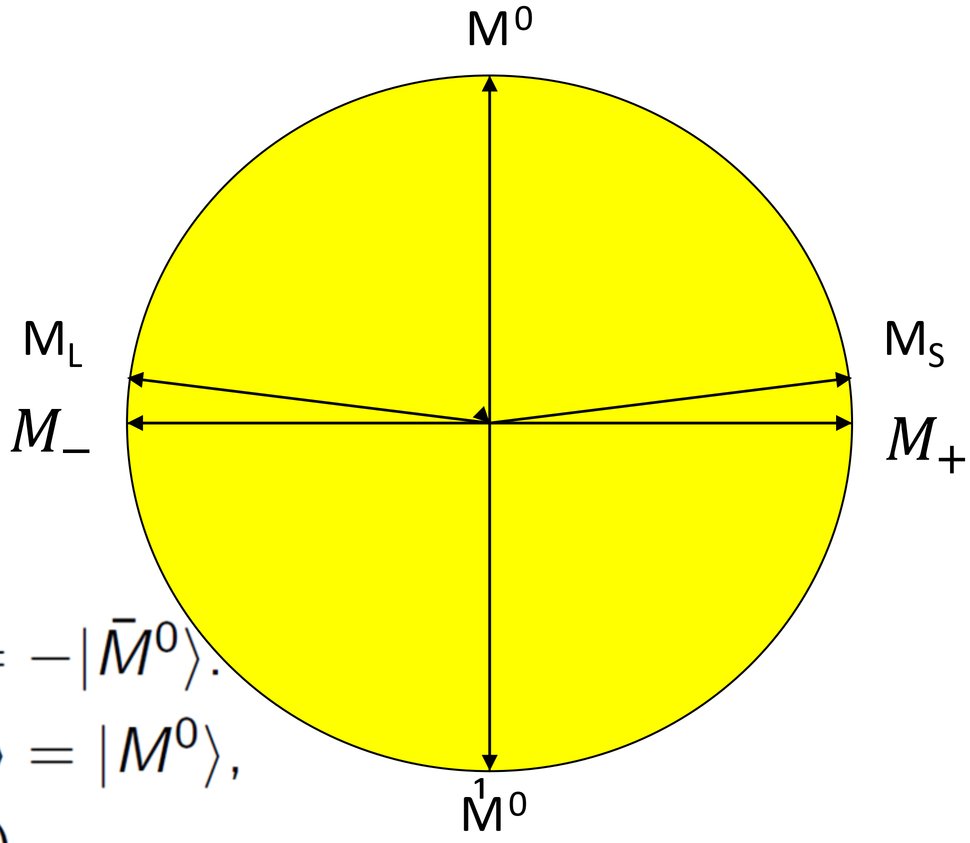
赝标量介子纠缠

- First noted by Lee and Yang in 1960.

- 施 郁 . 粒子物理中量子纠缠的历史起源 [J]. 物理学进展, 2023, 43(3): 57-67; Yu Shi. Historic Origin of Quantum Entanglement in Particle Physics[J]. Progress in Physics, 2023, 43(3): 57-67.

Two dimensional Hilbert space

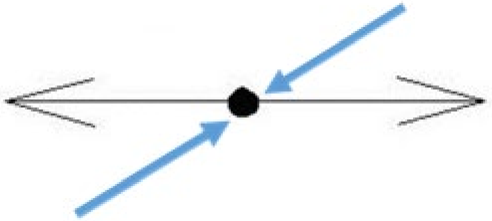
正反粒子态构成2维空间，类似自旋-1/2.
M=K,B,D.



- $\mathcal{F}|M^0\rangle = |M^0\rangle$, $\mathcal{F}|\bar{M}^0\rangle = -|\bar{M}^0\rangle$.
- $CP|M^0\rangle = |\bar{M}^0\rangle$, $CP|\bar{M}^0\rangle = |M^0\rangle$,
- $|M_\pm\rangle = \frac{1}{\sqrt{2}}(|M^0\rangle \pm |\bar{M}^0\rangle)$.
- $CP|M_\pm\rangle = \pm|M_\pm\rangle$.

$C = -1$ entangled state

正负电子碰撞，得到一个共振态，衰变得到纠缠正反粒子对。



$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}(|M^0\rangle_a|\bar{M}^0\rangle_b - |\bar{M}^0\rangle_a|M^0\rangle_b),$$

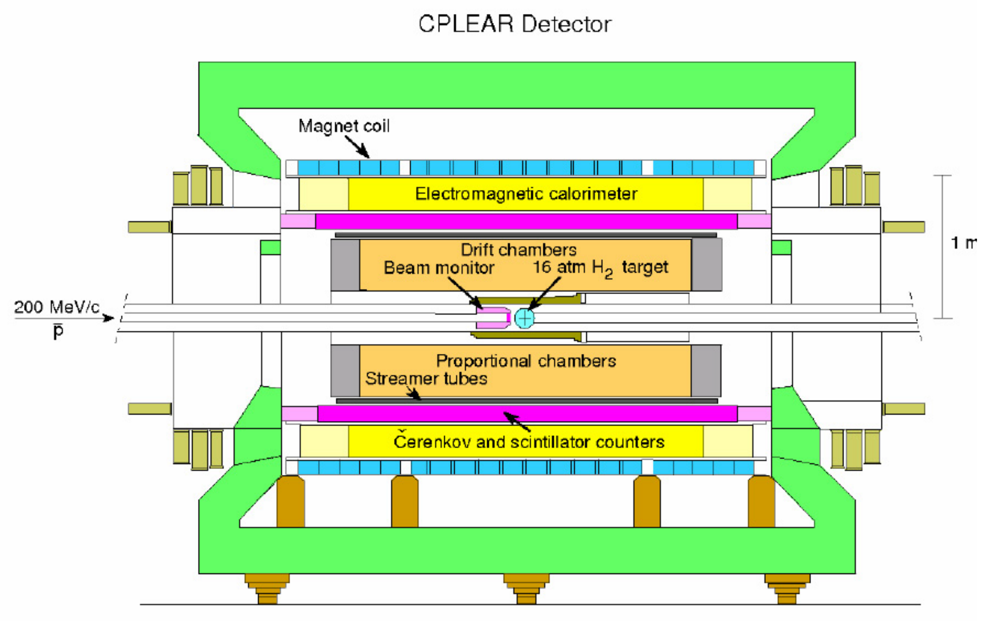
produced from a source of $J^{PC} = 1^{--}$, e.g. $K^0\bar{K}^0$ produced at ϕ decay in the KLOE, and $B_d^0\bar{B}_d^0$ produced at $\Upsilon(4s)$ in the BELLE detector.

- $|\Psi_-\rangle$ is also *exactly*

$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}(|M_-\rangle_a|M_+\rangle_b - |M_+\rangle_a|M_-\rangle_b).$$

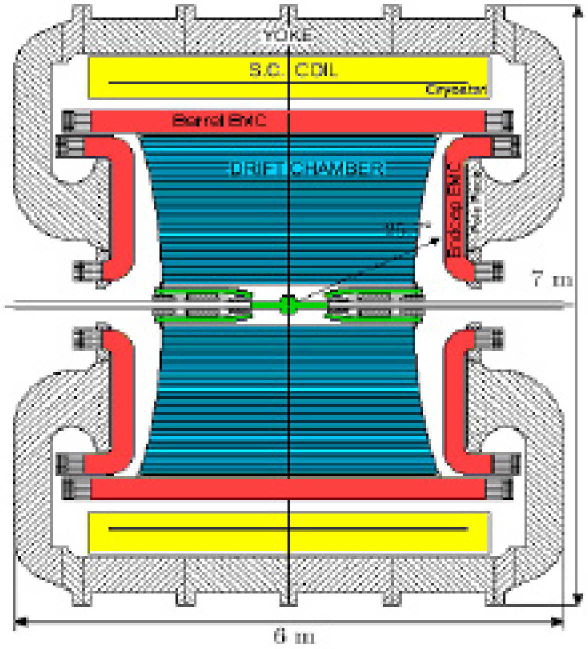
Experimental confirmation (1)

- $K^0\bar{K}^0$ produced in pp- annihilation in the CPLEAR detector in CERN (1998).



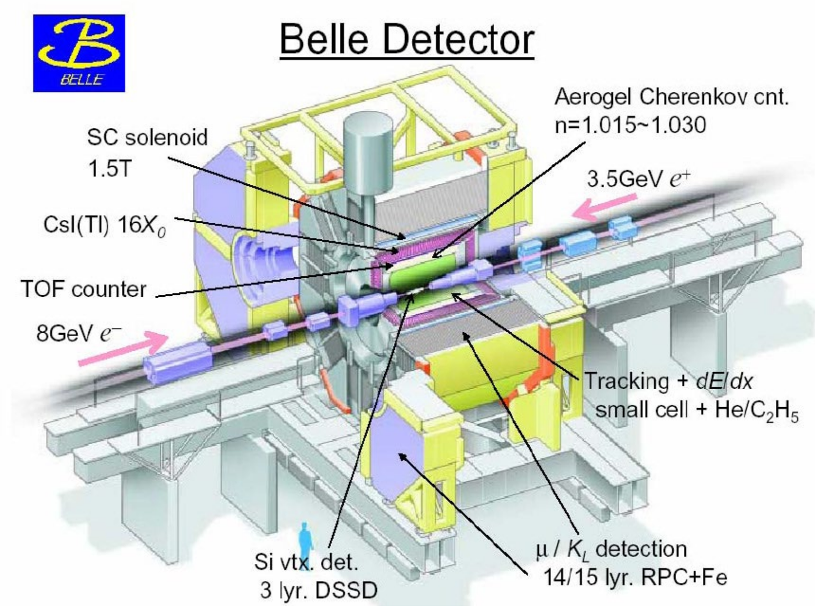
Experimental confirmation (2)

- $K^0\bar{K}^0$ produced in ϕ decay (1GeV) in the KLOE detector in DAΦNE (2003).



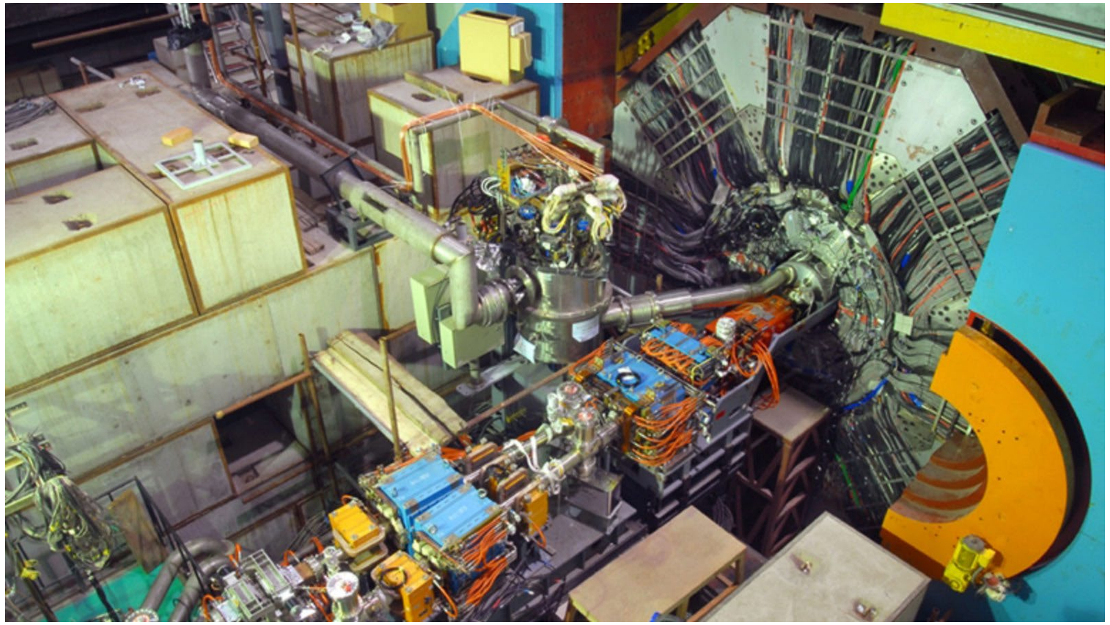
Experimental confirmation (3)

- $B^0\bar{B}^0$ produced in γ decay (10GeV) from ee+ annihilation in the BELLE detector in KEKB (2004).



Experimental confirmation (4)

- $D^0\overline{\mathrm{D}}^0$ produced in decay of ψ at 3.773GeV from ee+ annihilation in the BESIII detector (2015).



Our Work

1. 1st Proposal for a High Enegy Quantum Information Process: quantum teleportation in terms of entangled mesons

- YS, Phys. Lett. B 641, 75 (06); 641 (2006) 492.
- YS and Y. L. Wu, EPJC 55, 477 (08).

Generation of an EPR pair

- At t=0, a and b is created as

$$\begin{aligned}
|\Psi_{ab}(0)\rangle &= \frac{1}{\sqrt{2}}(|K^0\rangle_a|\bar{K}^0\rangle_b - |\bar{K}^0\rangle_a|K^0\rangle_b) \\
&= \frac{1}{\sqrt{2}}(|K_2\rangle|K_1\rangle - |K_1\rangle|K_2\rangle) \\
&= \frac{r}{\sqrt{2}}(|K_L\rangle_a|K_S\rangle_b - |K_S\rangle_a|K_L\rangle_b),
\end{aligned}$$

$$r = (|p|^2 + |q|^2)/2pq = (1 + |\epsilon|^2)/(1 - \epsilon^2)$$

- Decay under weak interaction:

$$|\Psi_{ab}(t)\rangle = M(t)|\Psi_-\rangle_{ab},$$

$$M(t) = \exp[-i(\lambda_S + \lambda_L)\Lambda_b t],\ \Lambda_b = 1/\sqrt{1 - v_b^2}$$

The third koan

- A third koan c is generated at t_z in an unknown state

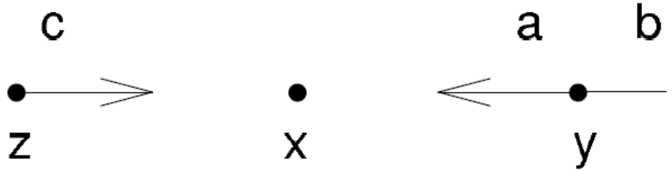
$$|\Psi_c(t_z)\rangle = \alpha|K^0\rangle_c + \beta|\bar{K}^0\rangle_c.$$

- Naturally decays

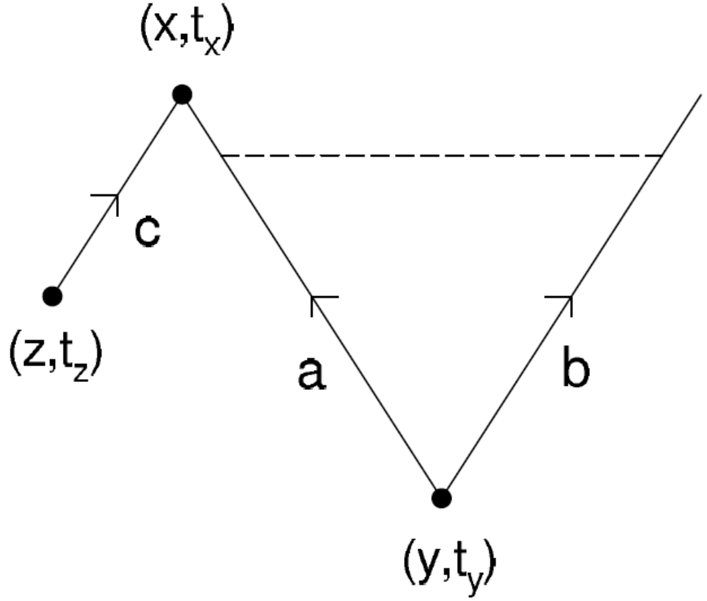
$$|\Psi_c(t)\rangle = F(t)|K^0\rangle_c + G(t)|\bar{K}^0\rangle_c,$$

$$\begin{aligned} F(t) &= [(\alpha + \beta p/q)e^{-i\lambda_S \Lambda_c (t-t_z)} + (\alpha - \beta p/q)e^{-i\lambda_L \Lambda_c (t-t_z)}]/2, \\ G(t) &= [(\alpha q/p + \beta)e^{-i\lambda_S \Lambda_c (t-t_z)} - (\alpha q/p - \beta)e^{-i\lambda_L \Lambda_c (t-t_z)}]/2. \end{aligned}$$

Let c and a collide!



Spacetime diagram:



Eigenstates of P, S and I of c+a

- P, S, I are conserved by strong interaction, which governs c-a collision.

$$|\phi_1\rangle_{ca} = |K^0 K^0\rangle \text{ with } P = 1,\ S = 2,\ I = 1;$$
$$|\phi_2\rangle_{ca} = |\bar{K}^0 \bar{K}^0\rangle \text{ with } P = 1,\ S = -2,\ I = 1;$$
$$|\phi_3\rangle_{ca} = |\Psi_+\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle|\bar{K}^0\rangle + |\bar{K}^0\rangle_c|K^0\rangle), \text{ with } P = 1,\ S = 0,\ I = 1;$$
$$|\phi_4\rangle_{ca} = |\Psi_-\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle|\bar{K}^0\rangle - |\bar{K}^0\rangle_c|K^0\rangle \text{ with } P = -1,\ S = 0,\ I = 0.$$

Three-kaon state before collision,decomposed in the strong interaction basis

$$\begin{aligned}
|\Psi_{cab}(t)\rangle &= |\Psi_c(t)\rangle \otimes |\Psi_{ab}(t)\rangle \\
&= \tfrac{M(t)}{2}\{\sqrt{2}F(t)|\phi_1\rangle_{ca}|\bar{K}^0\rangle_b \\
&\quad -\sqrt{2}G(t)|\phi_2\rangle_{ca}|K^0\rangle_b \\
&\quad -|\phi_3\rangle_{ca}[F(t)|K^0\rangle_b - G(t)|\bar{K}^0\rangle_b] \\
&\quad -|\phi_4\rangle_{ca}[F(t)|K^0\rangle_b + G(t)|\bar{K}^0\rangle_b]\}.
\end{aligned}$$

It is not a Bell basis. This basis is physical, while Bell basis is not.

Collision

- The collision effects a unitary transformation $\mathcal{S}$ in a negligible time interval

$$\begin{aligned}
|\Psi_{cab}(t_x)\rangle \quad = \quad & \frac{M(t_x)}{2}\{\sqrt{2}F(t_x)\mathcal{S}|\phi_1\rangle_{ca}|\bar{K}^0\rangle_b \\
& -\sqrt{2}G(t_x)\mathcal{S}|\phi_2\rangle_{ca}|K^0\rangle_b \\
& -\mathcal{S}|\phi_3\rangle_{ca}[F(t_x)|K^0\rangle_b - G(t_x)|\bar{K}^0\rangle_b] \\
& -\mathcal{S}|\phi_4\rangle_{ca}[F(t_x)|K^0\rangle_b + G(t_x)|\bar{K}^0\rangle_b]\}.
\end{aligned}$$

- As $\mathcal{S}$ is governed by strong interaction, $\mathcal{S}|\phi_i\rangle_{ca}$ $(i = 1, 2, 3, 4)$ is still an eigenstate of S, P and I, with the same eigenvalues as for $|\phi_i\rangle_{ca}$.

Detection of outgoing particles of c-a collision

- Using strong interaction with nuclear matter, the detection completes Alice's two-particle projection in the basis $\{\mathcal{S}|\phi_i\rangle_{ca}\}$.
- Probabilities: $|M(t_x)|^2|F(t_x)|^2/2,$
  $|M(t_x)|^2|G(t_x)|^2/2,$
  $|M(t_x)|^2[|F(t_x)|^2+|G(t_x)|^2]/4,$
  $|M(t_x)|^2[|F(t_x)|^2+|G(t_x)|^2]/4.$
- Corresponding projected state of b:
  $|\bar{K}^0\rangle_b,\ |K^0\rangle_b,\ [F(t_x)|K^0\rangle_b - G(t_x)|\bar{K}^0\rangle_b]/\sqrt{|F(t_x)|^2+|G(t_x)|^2},$
  $[F(t_x)|K^0\rangle_b + G(t_x)|\bar{K}^0\rangle_b]/\sqrt{|F(t_x)|^2+|G(t_x)|^2}.$
- Decay affects the probabilities.

Adopt a stochastic strategy

- This is because it is hard to implement subsequent unitary transformation on b.
- Bob chooses to retain or abandon b particle, based on the projection result of c-a.
- Teleportation of $F(t_x)|K^0\rangle_b + G(t_x)|\bar{K}^0\rangle_b$

  is made if projection result of c-a is

  $\mathcal{S}|\Psi_-\rangle_{ca}.$

Verification scheme (1)

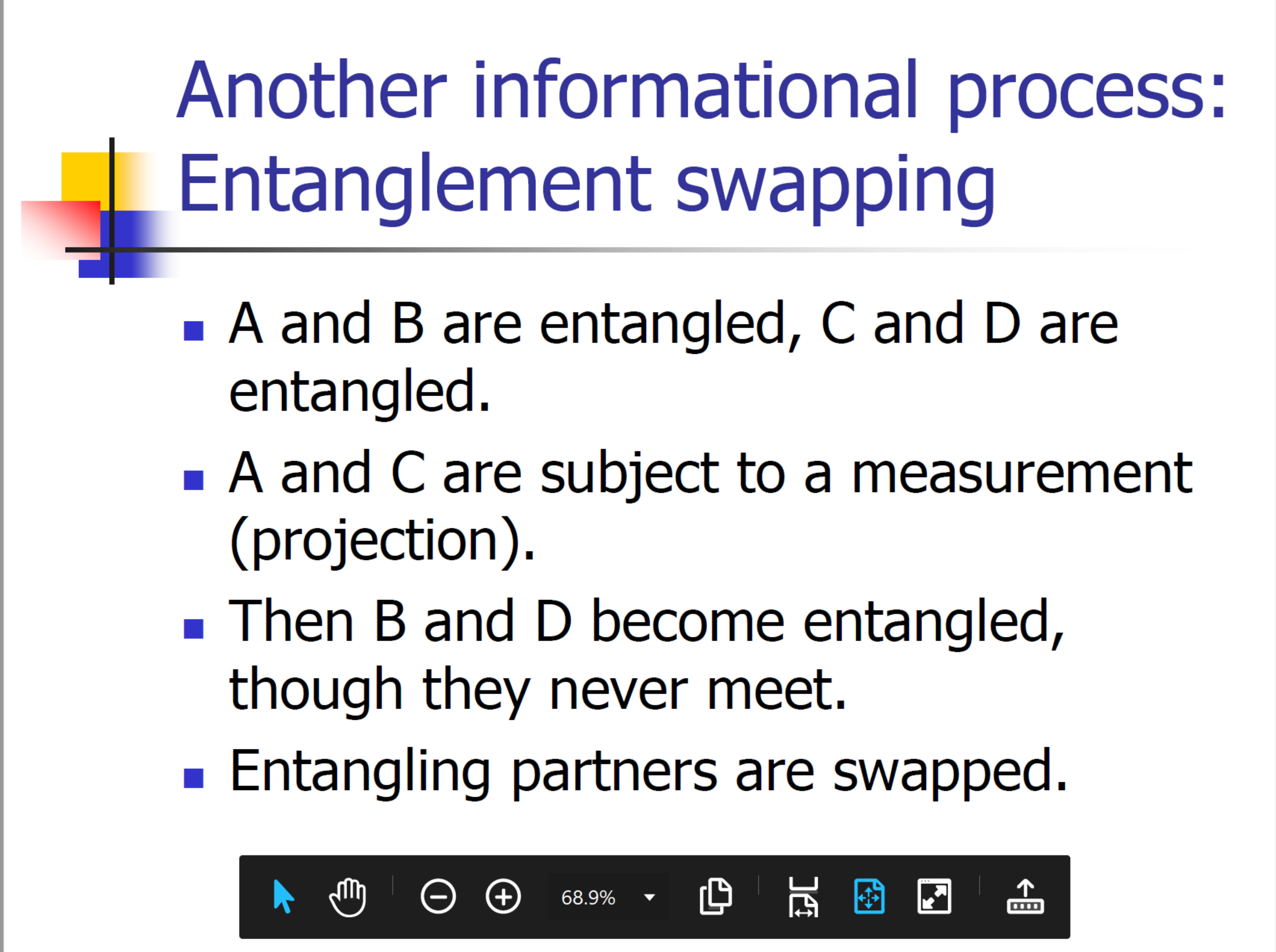
- Different projection results lead to different values of strangeness ratio ξ.
- For $|\Psi(t)\rangle_b = f(t)|K^0\rangle_b + g(t)|\bar{K}^0\rangle_b$, $\xi(t) = |f(t)|^2/g(t)|^2$.
- No projection/teleportation: $\xi(t) = 1.$
- Successful teleportation:

$$\xi(t) = \frac{|F(t_x)(e^{-\Gamma_S \tau/2}+e^{-\Gamma_L \tau/2})+G(t_x)(e^{-\Gamma_S \tau/2}-e^{-\Gamma_L \tau/2})|^2}{|F(t_x)(e^{-\Gamma_S \tau/2}-e^{-\Gamma_L \tau/2})+G(t_x)(e^{-\Gamma_S \tau/2}+e^{-\Gamma_L \tau/2})|^2},$$
$$\tau = \Lambda_b(t - t_x - 0).$$

very different from 1

Verification scheme (2)

- Different projection results lead to different values of CP ratio .
- For $|\Psi(t)\rangle_b = u_1(t)|K_1\rangle_b + u_2(t)|K_2\rangle_b$, $\zeta(t) = |u_1(t)|^2/u_2(t)|^2$.
- No projection/teleportation: $\zeta(t) = 1$.
- Successful teleportation: significantly different from 1.
- Advantage 1: valid no matter whether CP is violated.
- Advantage 2: easy experimental implementation, using non-leptonic decays (CP=1: decay to 2 pions; CP=-1: decay to 3 pions).



Another informational process: Entanglement swapping

- A and B are entangled, C and D are entangled.
- A and C are subject to a measurement (projection).
- Then B and D become entangled, though they never meet.
- Entangling partners are swapped.

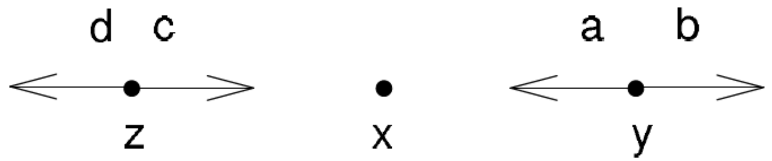
Preparation

In addition to $|\Psi_-\rangle_{ab}$ generated at $t=0$,
another kaon pair d and c is generated as $|\Psi_-\rangle_{dc}$ at time t_z.
$|\Psi_{dc}(t)\rangle = M'(t-t_z)|\Psi_-\rangle_{dc}$,
$M'(t-t_z) = \exp[-i(\lambda_S+\lambda_L)\Lambda_d(t-t_z)]$.

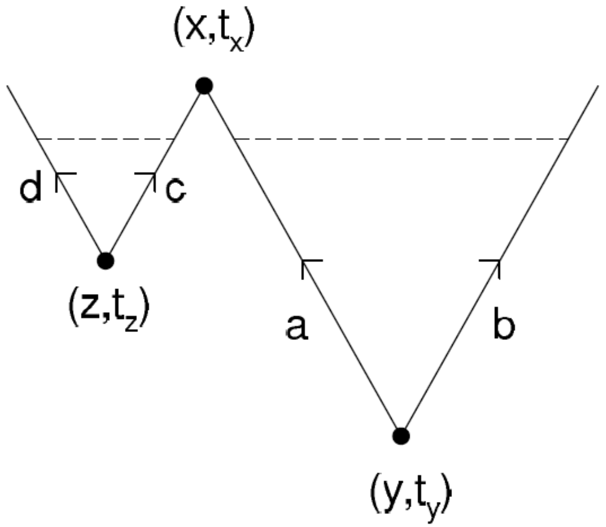
$$\begin{aligned} |\Psi_{dcab}(t)\rangle &= M'(t-t_z)M(t)|\Psi_-\rangle_{dc}|\Psi_-\rangle_{ab} \\ &= \tfrac{M'(t-t_z)M(t)}{2}(|\Psi_+\rangle_{ca}|\Psi_+\rangle_{db} - |\Psi_-\rangle_{ca}|\Psi_-\rangle_{db} \\ &\quad -|K^0K^0\rangle_{ca}|\bar{K}^0\bar{K}^0\rangle_{db} - |\bar{K}^0\bar{K}^0\rangle_{ca}|K^0K^0\rangle_{db}). \end{aligned}$$

Collision

Let c and a collide at t_x



Spacetime diagram:



Detection

Collision:

$$\begin{aligned}|\Psi_{dcab}(t_x+0)\rangle \quad = \quad & \tfrac{M'(t_x-t_z)M(t_x)}{2}(\mathcal{S}|\Psi_+\rangle_{ca}|\Psi_+\rangle_{db} \\ & -\mathcal{S}|\Psi_-\rangle_{ca}|\Psi_-\rangle_{db} \\ & -\mathcal{S}|K^0K^0\rangle_{ca}|\bar{K}^0\bar{K}^0\rangle_{db} \\ & -\mathcal{S}|\bar{K}^0\bar{K}^0\rangle_{ca}|K^0K^0\rangle_{db}).\end{aligned}$$

In detecting outgoing particles from the collision, c and a are projected to: $\mathcal{S}|\Psi_+\rangle_{ca}$, $\mathcal{S}|\Psi_-\rangle_{ca}$, $\mathcal{S}|K^0K^0\rangle_{ca}$ or $\mathcal{S}|\bar{K}^0\bar{K}^0\rangle_{ca}$. Correspondingly d and b are projected to: $|\Psi_+\rangle_{ca}$, $|\Psi_-\rangle_{ca}$, $|K^0K^0\rangle_{ca}$ and $|\bar{K}^0\bar{K}^0\rangle_{ca}$, respectively, each with probability $|M'(t_x-t_z)M(t_x)|^2/4$. .

The projection result is revealed by P, S and I of the outcomes of $c-a$ collision, according to which b and d are retained or abandoned.

Verification scheme (1)

- Measure the S asymmetry of b and d

$$A(t) = [p_{diff}(t) - p_{same}(t)]/[p_{diff}(t) + p_{same}(t)]$$

$p_{diff}(t)$ ($p_{same}(t)$):
probability to have different (same) strangeness values

- Many runs are needed.
- No entanglement swapping (all runs are considered): A(t)=0
- Entanglement swapping succeeds (consider those runs in which c-a project to $\mathcal{S}|\Psi_-\rangle_{ca}$: A(t)=1

Verification scheme (2)

- Measure the CP asymmetry of b and d

$$A(t) = [p_{diff}(t) - p_{same}(t)]/[p_{diff}(t) + p_{same}(t)]$$

$p_{diff}(t)$ $(p_{same}(t))$:
probability to have different (same) values of CP.

- Many runs (events) are needed.
- No entanglement swapping (all runs are considered): A(t)=-1
- Entanglement swapping succeeds (consider those runs in which c-a project to $\mathcal{S}|\Psi_-\rangle_{ca}$: A(t)=1

2. Using Quantum Entanglement to Study CP and CPT Violations

- Z. Huang, YS, Euro. Phys. J. C 72, 1900 (2012).
- YS, Euro. Phys. J. C 73, 2506 (2013.
- Z. Huang, YS, Phys. Rev. D 89, 016018 (2014).

Time evolution and joint decay of $|\Psi_\mp\rangle$

- $$|\Psi_\mp(t_a, t_b)\rangle = \frac{1}{\sqrt{2}}(|M^0(t_a)\rangle_a |\bar{M}^0(t_b)\rangle_b \mp |\bar{M}^0(t_a)\rangle_a |M^0(t_b)\rangle_b)$$
- For state $|\Psi_\mp(t_a, t_b)\rangle$, the joint rate that Alice decays to $|\psi_a\rangle$ at t_a while Bob decays to $|\psi_b\rangle$ at t_b is

$$I(\psi_a, t_a; \psi_b, t_b) = |\langle \psi_a, \psi_b | \mathcal{H}_a \mathcal{H}_b | \Psi_\mp(t_a, t_b)\rangle|^2.$$

$$\begin{aligned} &\langle \psi_a, \psi_b | \mathcal{H}_a \mathcal{H}_b | \Psi_\mp(t_a, t_b)\rangle \\ &\qquad = \frac{1}{\sqrt{2}}(\langle \psi_a | \mathcal{H}_a | M^0(t_a)\rangle \langle \psi_b | \mathcal{H}_b | \bar{M}_0(t_b)\rangle \\ &\qquad - \langle \psi_a | \mathcal{H}_a | \bar{M}_0(t_a)\rangle \langle \psi_b | \mathcal{H}_b | M^0(t_b)\rangle). \end{aligned}$$

Integrated rates and asymmetries

Integrated rate

$$I'(\psi^1, \psi^2, \Delta t) = \int_0^\infty I(\psi^1, t_a; \psi^2, t_a + \Delta t) dt_a,$$

which is simply $I(\psi^1, t_a; \psi^2, t_a + \Delta t)$ with $e^{-(\Gamma_S + \Gamma_L)t_a}$ replaced as $1/(\Gamma_S + \Gamma_L)$.

$$A(\psi^1\psi^2, \psi^3\psi^4, \Delta t) \equiv \frac{I'(\psi^1, \psi^2, \Delta t) - I'(\psi^3, \psi^4, \Delta t)}{I'(\psi^1, \psi^2, \Delta t) + I'(\psi^3, \psi^4, \Delta t)}.$$

We use notation $A_\mp$ for $|\Psi_\mp\rangle$.
Theorem 1: For $\Delta t = 0$, any unequal-state asymmetry $A_\mp(\psi^a\psi^b, \psi^b\psi^a; \Delta t = 0)$ always vanishes no matter whether there is CP or CPT violation.
Could be used to confirm the entanglement of the pair.

Semileptonic decays of $|\Psi_-\rangle$ into flavor eigenstates $|l^\pm\rangle$

- Examples for $|l^+\rangle$:
  $M^-\bar{l}\nu$, $D^-D_s^+$, D^-K^+, $\pi^-D_s^+$, π^-K^+ from B^0;
  $D_s^-\pi^+$, $D_s^-D^+$, $K^-\pi^+$, K^-D^+ from B_S^0.
- Examples for $|l^-\rangle$:
  $M^+l\bar{\nu}$, $D^+D_s^-$, D^+K^-, $\pi^+D_s^-$, π^+K^- from $\bar{B}^0$;
  $D_s^+\pi^-$, $D_s^+D^-$, $K^+\pi^-$, K^+D^- from $\bar{B}_s^0$.
- Define:
  $R^+ \equiv \langle l^+|\mathcal{H}|M^0\rangle = a + b,$
  $R^- \equiv \langle l^-|\mathcal{H}|\bar{M}^0\rangle = a^* - b^*,$
  $S^+ \equiv \langle l^+|\mathcal{H}|\bar{M}_0\rangle = c^* - d^*,$
  $S^- \equiv \langle l^-|\mathcal{H}|M_0\rangle = c + d.$
  a, b, c, d: quantities defined in literature.
  - Direct CP conservation $\Longrightarrow$ $R^+ = R^-$ and $S^+ = S^-$.
  - Direct CPT conservation $\Longrightarrow$ $(R^+)^* = R^-$ and $(S^+)^* = S^-$.
  - $\Delta\mathcal{F} = \Delta Q$ rule $\Longrightarrow$ $S^\pm = 0$.

Equal-flavor asymmetry for $|\Psi_-\rangle$

$$A_-(++,--,\Delta t)=$$
$$\frac{|x_S(S^+)^2-x_L^{-1}(R^+)^2+(1-\Omega)R^+S^+|^2-|x_S(R^-)^2-x_L^{-1}(S^-)^2+(1-\Omega)R^-S^-|^2}{|x_S(S^+)^2-x_L^{-1}(R^+)^2+(1-\Omega)R^+S^+|^2+|x_S(R^-)^2-x_L^{-1}(S^-)^2+(1-\Omega)R^-S^-|^2}.$$

- **Theorem 2:** $A_-(++,--,\Delta t)$ is always a constant independent of Δt.
- **Theorem 3:** *If* $A_-(++,--,\Delta t)\neq 0$*, then there exists one or two of the following violations: (1) CP is violated indirectly, (2) both CP and CPT are violated directly.*
- **Theorem 4:** *If* $A_-(++,--,\Delta t)\neq 0$ *while CPT is assumed to be conserved both directly* $((R^+)^*=R^-, (S^+)^*=S_-)$ *and indirectly* $(x_S=x_L=q/p, \Omega=1)$*, then in addition to indirect CP violation* $(q/p\neq 1)$*, we can draw the following conclusions:*
  (1) $|q/p|\neq 1$*, i.e. T must also be violated indirectly;*
  (2) $|R^\pm|\neq|S^\pm|$.

Unequal-flavor asymmetry for $|\Psi_-\rangle$

$$A_-(+-,-+,\Delta t) = \frac{(|U|^2-|V|^2)(e^{-\Gamma_L\Delta t}-e^{-\Gamma_S\Delta t})+4\Im(U^*V)\sin(\Delta m\Delta t)}{(|U|^2+|V|^2)(e^{-\Gamma_L\Delta t}+e^{-\Gamma_S\Delta t})+4\Re(U^*V)\cos(\Delta m\Delta t)}.$$

- **Theorem 5** $A_-(+-,-+,\Delta t = 0) = 0$.
- **Theorem 6** *If* $A_-(+-,-+,\Delta t) \neq 0$, *then CP must be violated, directly or indirectly or both.*
- **Theorem 7** *If* $A_-(+-,-+,\Delta t) \neq 0$ *for* $\Delta t \neq 0$ *while CPT is assumed to be conserved both directly and indirectly, then we can draw the following conclusions:*
  (1) $|R^+| = |R^-| \neq |S^+| = |S^-|$;
  (2) $S^\pm \neq 0$, *i.e.* $\Delta\mathcal{F} = \Delta Q$ *rule must be violated.*

Hadronic decays of $|\Psi_-\rangle$ into CP eigenstates $|h^\pm\rangle$

- Examples of $|h^+\rangle$: $\pi^+\pi^-$, $\pi^0\pi^0$.
- Examples of $|h^-\rangle$: $\pi^0\pi^0\pi^0$.
- Define:
  $Q^\pm \equiv \langle h^\pm|\mathcal{H}|M_\pm\rangle$,
  $X^\pm \equiv \langle h^\pm|\mathcal{H}|M_\mp\rangle$,
  $\xi_\pm \equiv \frac{X^\pm}{Q^\pm}$.
- They are related to $\eta's$ usually defined:

$$\begin{aligned} \eta_{h^+} &\equiv \frac{\langle h^+|\mathcal{H}|M_L\rangle}{\langle h^+|\mathcal{H}|M_S\rangle} = \frac{\xi^+ + \epsilon_L}{1+\epsilon_S\xi^+}, \\ \eta_{h^-} &\equiv \frac{\langle h^-|\mathcal{H}|M_S\rangle}{\langle h^-|\mathcal{H}|M_L\rangle} = \frac{\xi^- + \epsilon_S}{1+\epsilon_L\xi^-}. \end{aligned}$$

  - If CP is conserved directly $\Rightarrow X^\pm = 0$.
  - If CPT is conserved directly $\Rightarrow X^\pm$ is purely imaginary, i.e. $X^\pm = -X^{\pm *}$.
- One can, of course, calculate using basis $|M^0\rangle$ and $|\bar{M}^0\rangle$, rather than $M_\pm\rangle$.

Equal-CP asymmetry for $|\Psi_-\rangle$

Asymmetry:

$$B_-(h_a^x h_b^y, h_a^y h_b^x, \Delta t) = \frac{I[h_a^x,t_a;h_b^y,t_a+\Delta t]-I[h_a^y,t_a;h_b^x,t_a+\Delta t]}{I[h_a^x,t_a;h_b^y,t_a+\Delta t]+I[h_a^y,t_a;h_b^x,t_a+\Delta t]} = \frac{I'[h_a^x,h_b^y,\Delta t]-I'[h_a^y,h_b^x,\Delta t]}{I'[h_a^x,h_b^y,\Delta t]+I'[h_a^y,h_b^x,\Delta t]}.$$

Equal-CP asymmetry: $B_-(++,--,\Delta t) = \frac{P(Q^+,X^+)-P(X^-,Q^-)}{P(Q^+,X^+)+P(X^-,Q^-)}$,

$P(\beta,\gamma) \equiv$
$|-(1-x_L^{-1}+x_S-\Omega)\beta^2+2(x_L^{-1}+x_S)\beta\gamma+(1+x_L^{-1}-x_S-\Omega)\gamma^2|^2$.

Theorem 9: *$B_-(++,--,\Delta t)$ is always a constant independent of Δt.*

Simple results concerning joint decays of $|\Psi_+\rangle$

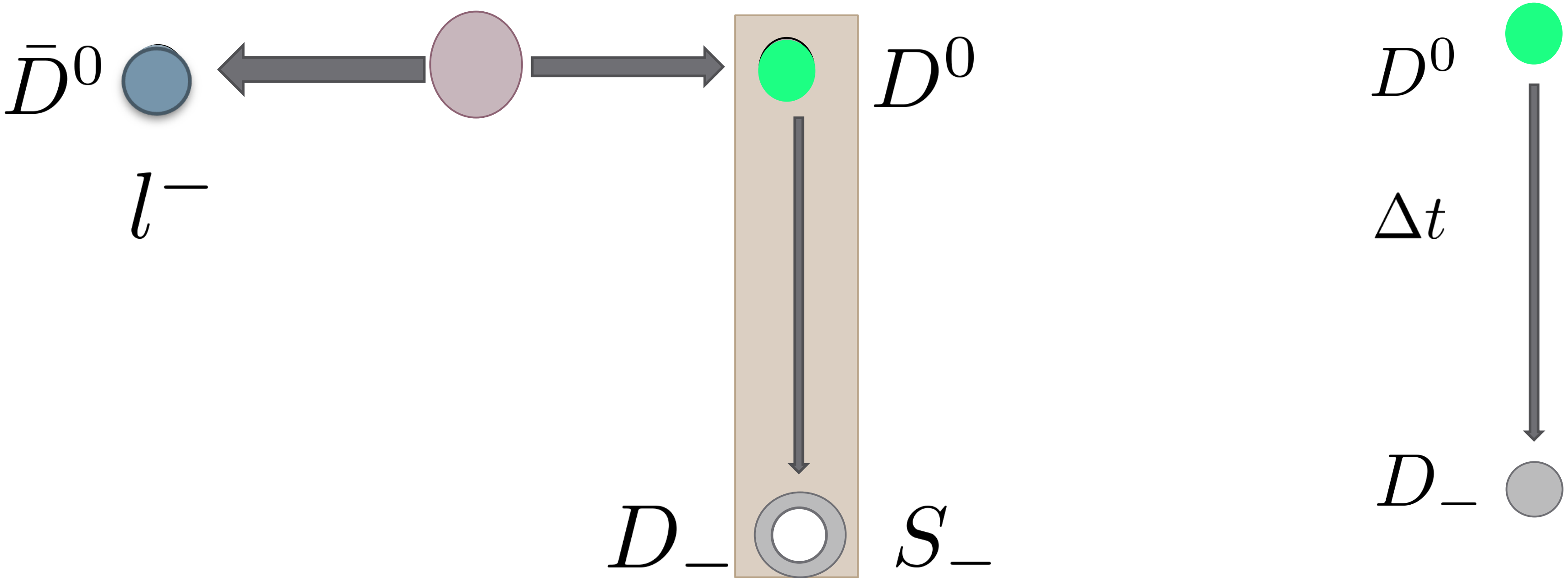
We have also calculated joint decays of $|\Psi_+\rangle$. Here are some simple results:

- **Theorem 10:** If $A_+(l^+l^-, l^-l^+; \Delta t)$ is nonzero, then CP must be violated indirectly.
- **Theorem 11:** If $A_+(l^+l^-, l^-l^+; \Delta t)$ depends on the first order of ϵ_M, then CP must be violated directly and the $\Delta\mathcal{F} = \Delta Q$ rule must be violated.
- **Theorem 12:** The deviation of $I_+(h^+, t_a; h^-, t_a)$ or $I_+(h^-, t_a; h^+, t_a)$ from zero implies CP violation, direct or indirect or both. [Immediately implied by $|\Psi_+\rangle = \frac{1}{\sqrt{2}}(|M_+\rangle|M_+\rangle - |M_-\rangle|M_-\rangle)$.]
- **Theorem 13:** If $I_+(h^+, t_a; h^-, t_a)$ or $I_+(h^-, t_a; h^+, t_a)$ or both are of the order of $O(\Delta_M)$ and $O(\epsilon_M)$, then CP is violated directly.

3. Genuine T Violation Signal in terms of entangled D mesons

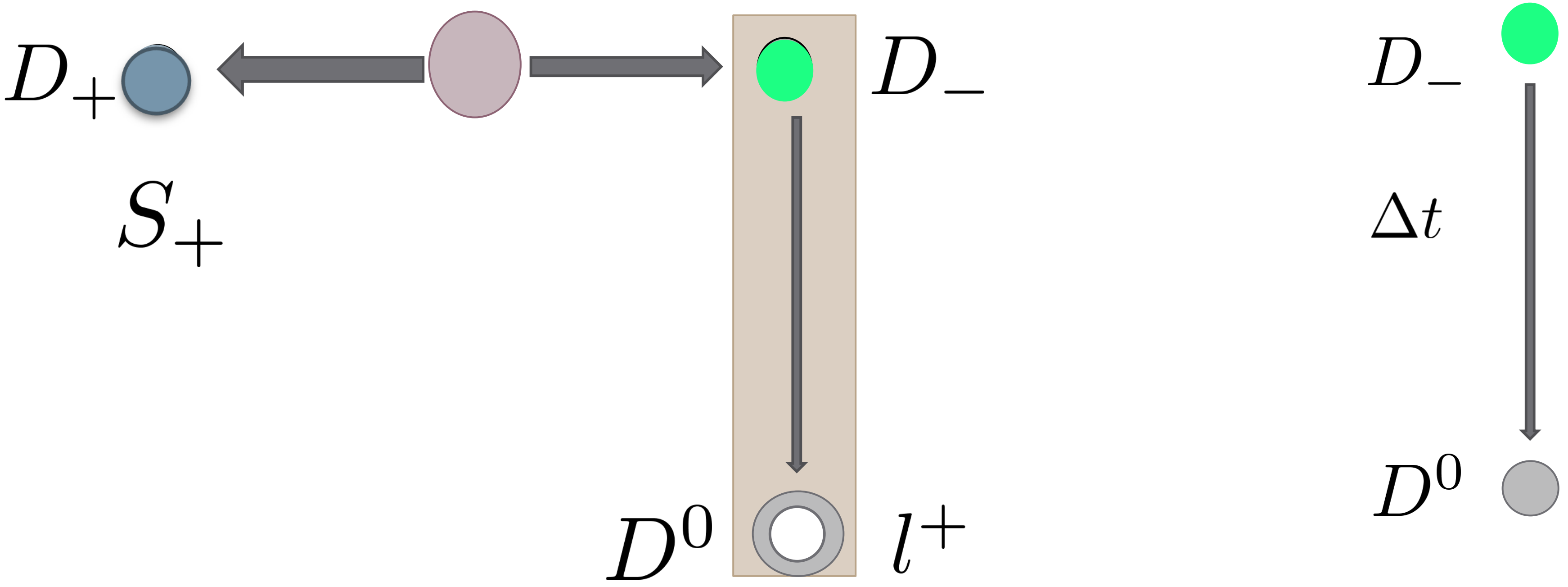
- YS, JC Yang, Time reversal symmetry violation in entangled pseudoscalar neutral charmed mesons, Phys. Rev. D 98, 075079 (2018).

T-conjugate processes from C=-1 entangled state (1)



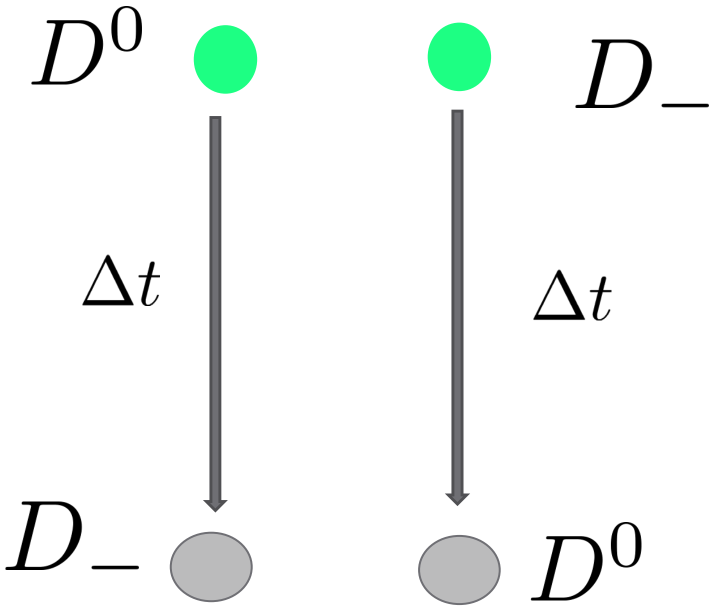
$$|\Psi_-\rangle = |M^0\rangle \otimes |\bar{M}^0\rangle - |\bar{M}^0\rangle \otimes |M^0\rangle$$

T-conjugate processes from C=-1 entangled state (2)



$$|\Psi_-\rangle = |M_+\rangle \otimes |M_-\rangle - |M_-\rangle \otimes |M_+\rangle$$

T-conjugate processes from C=-1 entangled state



Transition rates can be calculated from joint decay rates of the entangled state.

transition of meson b	final state of meson a	final state of meson b	T-conjugate transition	final state of meson a	final state of meson b
$D^0 \to D_-$	l^-	S_-	$D_- \to D^0$	S_+	l^+
$D^0 \to D_+$	l^-	S_+	$D_+ \to D^0$	S_-	l^+
$\bar{D}^0 \to D_-$	l^+	S_-	$D_- \to \bar{D}^0$	S_+	l^-
$\bar{D}^0 \to D_+$	l^+	S_+	$D_+ \to \bar{D}^0$	S_-	l^-

T-violation signals

$$A^1_-(\Delta t) = \frac{R_-(l^-, S_-, \Delta t) - R_-(S_+, l^+, \Delta t)}{R_-(l^-, S_-, \Delta t) + R_-(S_+, l^+, \Delta t)},$$

$$A^2_-(\Delta t) = \frac{R_-(l^-, S_+, \Delta t) - R_-(S_-, l^+, \Delta t)}{R_-(l^-, S_+, \Delta t) + R_-(S_-, l^+, \Delta t)},$$

$$A^3_-(\Delta t) = \frac{R_-(l^+, S_-, \Delta t) - R_-(S_+, l^-, \Delta t)}{R_-(l^+, S_-, \Delta t) + R_-(S_+, l^-, \Delta t)},$$

$$A^4_-(\Delta t) = \frac{R_-(l^+, S_+, \Delta t) - R_-(S_-, l^-, \Delta t)}{R_-(l^+, S_+, \Delta t) + R_-(S_-, l^-, \Delta t)}.$$

$$\hat{A}^1_- = \frac{R_-(l^-, S_-) - R_-(l^+, S_+)}{R_-(l^-, S_-) + R_-(l^+, S_+)}, \quad \hat{A}^2_- = \frac{R_-(l^-, S_+) - R_-(l^+, S_-)}{R_-(l^-, S_+) + R_-(l^+, S_-)}.$$

Results within Standard model

- Calculations come down to a few parameters:

$$x \equiv \frac{\Delta m}{\Gamma} = 0.0037, \quad y \equiv \frac{\Delta\Gamma}{2\Gamma} = 0.0066,$$

$$|q/p| = 0.91, \quad arg(q/p)/2 = 0.91.$$

- The time-dependent T-asymmetries for $\Delta t = 1/\Gamma$, and time-independent T-asymmetries are found to be of the order of

10^{-5}

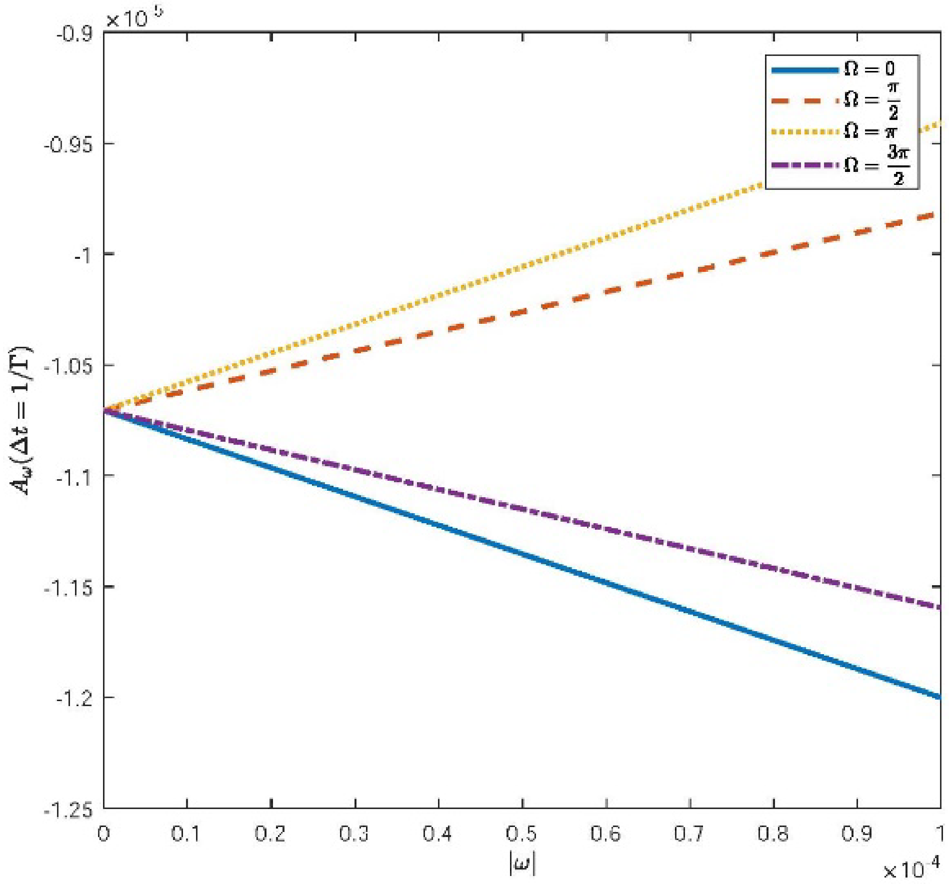
CPT violation

- Local quantum field theory (Lorentz invariance, local interaction,unitarity) → CPT.
- In some quantum gravity theories, because of objects unaccessible to low energy observers, CPT may be violated.

- Such CPTV leads to "omega effect". (Ellis et al).

$$|\Psi_\omega\rangle = |\Psi_-\rangle + \omega|\Psi_+\rangle$$

T-violating signals in presence of omega effect



$$\omega \equiv |\omega| \exp i\Omega.$$

For $|\omega| = 10^{-4}$, T-violation signals in D system are changed as large as 27%

我们对于用高能粒子检验贝尔/Leggett不等式的认识

- 相对论性，有质量，涉及电弱和强相互作用。
- 衰变的模式和产物类似于测量。
- 每个粒子的衰变模式、产物、时间都可能由粒子源的隐参数决定，从而两个粒子的衰变也可能有关联。
- 因此是一种测量设置漏洞。

我们的工作

- 在隐变数的前提中就假设测量设置也由隐参数决定；衰变由粒子源的隐参量决定。
- 推广加密非局域实在论、Leggett不等式。
- 推广局域实在论、贝尔不等式

4. Entangled mesons violate a generalized Leggett Inequality， in which decays also depend on hidden variables

- YS and JC Yang, Particle physics violating crypto-nonlocal realism, European Physical Journal C 80, 861 (2020).

测量设置（“方向”）也依赖于隐参数

对隐变量分布平均：

$$\int d\lambda \rho'_{\mathbf{u},\mathbf{v},\mathbf{a},\mathbf{b}}(\lambda) A(\mathbf{u},\mathbf{v},\tilde{\mathbf{a}}(\lambda),\tilde{\mathbf{b}}(\lambda),\lambda) = \bar{A}(\mathbf{u},\mathbf{a}) = \mathbf{u}\cdot\mathbf{a},$$

$$\int d\lambda \rho'_{\mathbf{u},\mathbf{v},\mathbf{a},\mathbf{b}}(\lambda) B(\mathbf{v},\mathbf{u},\tilde{\mathbf{b}}(\lambda),\tilde{\mathbf{a}}(\lambda),\lambda) = \bar{B}(\mathbf{v},\mathbf{b}) = \mathbf{v}\cdot\mathbf{b}.$$

$$\overline{AB}(\mathbf{u},\mathbf{v},\mathbf{a},\mathbf{b}) = \int d\lambda \rho'_{\mathbf{u},\mathbf{v},\mathbf{a},\mathbf{b}}(\lambda) A(\mathbf{u},\mathbf{v},\tilde{\mathbf{a}}(\lambda),\tilde{\mathbf{b}}(\lambda),\lambda) B(\mathbf{v},\mathbf{u},\tilde{\mathbf{b}}(\lambda),\tilde{\mathbf{a}}(\lambda),\lambda),$$

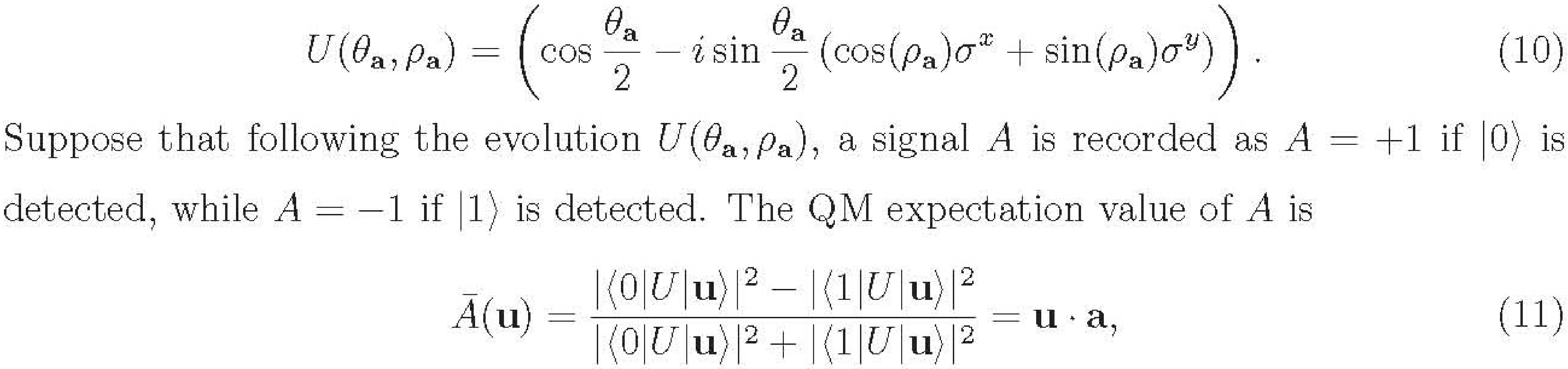
where $\rho'_{\mathbf{u},\mathbf{v},\mathbf{a},\mathbf{b}}(\lambda) \equiv \rho_{\mathbf{u},\mathbf{v}}(\lambda)\delta(\tilde{\mathbf{a}}(\lambda)-\mathbf{a})\delta(\tilde{\mathbf{b}}(\lambda)-\mathbf{b})$ is a shorthand.

不同的方向a,b 分别计算隐变量平均，得到不同的Malus Law.

对极化平均：

$$E(\mathbf{a},\mathbf{b}) = \int d\mathbf{u} d\mathbf{v} F(\mathbf{u},\mathbf{v}) \overline{AB}(\mathbf{u},\mathbf{v},\mathbf{a},\mathbf{b}),$$

演化后测量。等效于在一个含时基上测量，测量方向决定于演化算符。，相当于旋转的偏振片

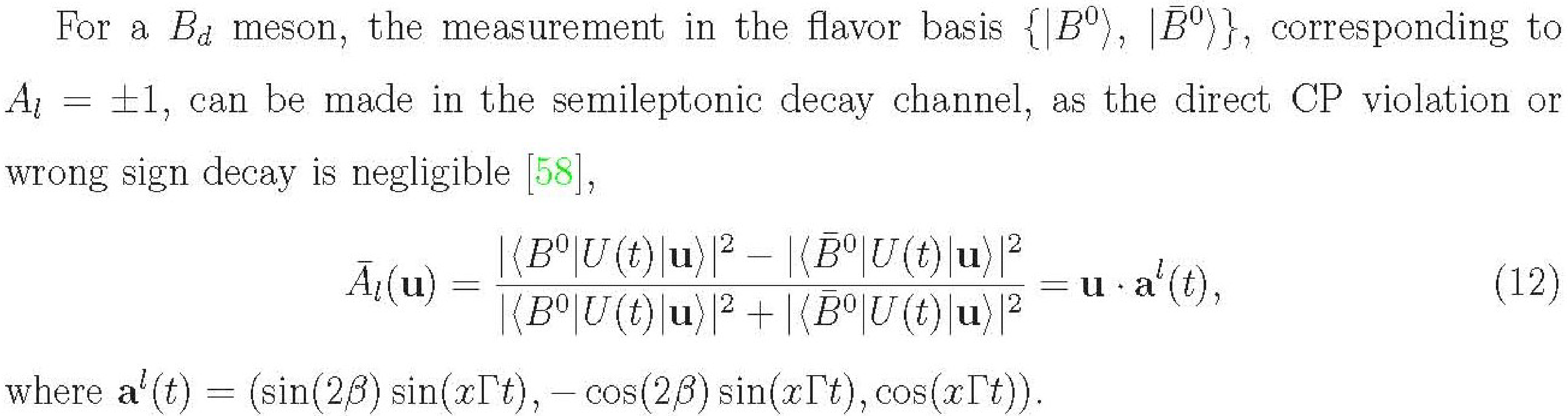


$$U(\theta_{\mathbf{a}},\rho_{\mathbf{a}}) = \left(\cos\frac{\theta_{\mathbf{a}}}{2} - i\sin\frac{\theta_{\mathbf{a}}}{2}\left(\cos(\rho_{\mathbf{a}})\sigma^x + \sin(\rho_{\mathbf{a}})\sigma^y\right)\right). \tag{10}$$

Suppose that following the evolution $U(\theta_{\mathbf{a}},\rho_{\mathbf{a}})$, a signal A is recorded as $A=+1$ if $|0\rangle$ is detected, while $A=-1$ if $|1\rangle$ is detected. The QM expectation value of A is

$$\bar{A}(\mathbf{u}) = \frac{|\langle 0|U|\mathbf{u}\rangle|^2 - |\langle 1|U|\mathbf{u}\rangle|^2}{|\langle 0|U|\mathbf{u}\rangle|^2 + |\langle 1|U|\mathbf{u}\rangle|^2} = \mathbf{u}\cdot\mathbf{a}, \tag{11}$$

B介子在味道基上的（半轻子）衰变，符合Malus Law，定出对应的“测量方向”



For a B_d meson, the measurement in the flavor basis $\{|B^0\rangle, |\bar{B}^0\rangle\}$, corresponding to $A_l = \pm 1$, can be made in the semileptonic decay channel, as the direct CP violation or wrong sign decay is negligible [58],

$$\bar{A}_l(\mathbf{u}) = \frac{|\langle B^0|U(t)|\mathbf{u}\rangle|^2 - |\langle \bar{B}^0|U(t)|\mathbf{u}\rangle|^2}{|\langle B^0|U(t)|\mathbf{u}\rangle|^2 + |\langle \bar{B}^0|U(t)|\mathbf{u}\rangle|^2} = \mathbf{u}\cdot\mathbf{a}^l(t), \tag{12}$$

where $\mathbf{a}^l(t) = (\sin(2\beta)\sin(x\Gamma t), -\cos(2\beta)\sin(x\Gamma t), \cos(x\Gamma t))$.

选择B介子，两种衰变宽度最接近，演化矩阵中的（间接）CP破坏是一个相位β。

xΓ是质量-衰变宽度两个本征值之差

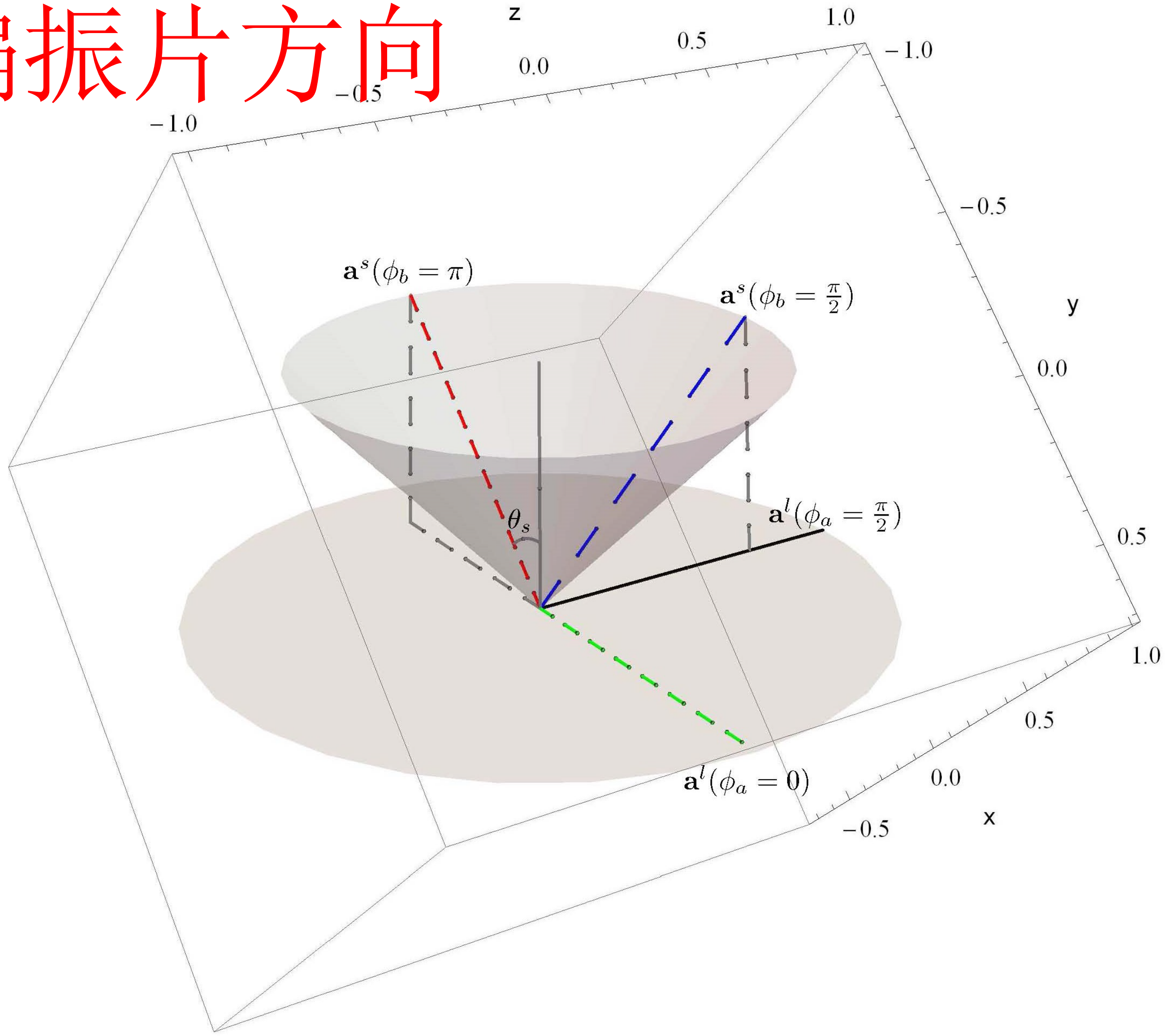
B介子在CP基上的衰变，也符合Malus Law，定出对应的“测量方向”

Likewise, observing the decay product to be CP eigenstates $S_\pm$ effectively measures the meson to be $|B_\pm\rangle \equiv \left(|B^0\rangle \pm |\bar{B}^0\rangle\right)/\sqrt{2}$, as the direct CP violation is negligible [58]. With $B_\pm$ corresponding to $A_s = \pm 1$,

$$\bar{A}_s(\mathbf{u}) = \frac{|\langle B_+|U(t)|\mathbf{u}\rangle|^2 - |\langle B_-|U(t)|\mathbf{u}\rangle|^2}{|\langle B_+|U(t)|\mathbf{u}\rangle|^2 + |\langle B_-|U(t)|\mathbf{u}\rangle|^2} = \mathbf{u}\cdot\mathbf{a}^s(t), \tag{13}$$

where $\mathbf{a}^s(t) = \left(\sin^2(2\beta)\cos(x\Gamma t) + \cos^2(2\beta), \sin(4\beta)\sin^2(x\Gamma t/2), -\sin(2\beta)\sin(x\Gamma t)\right)$.

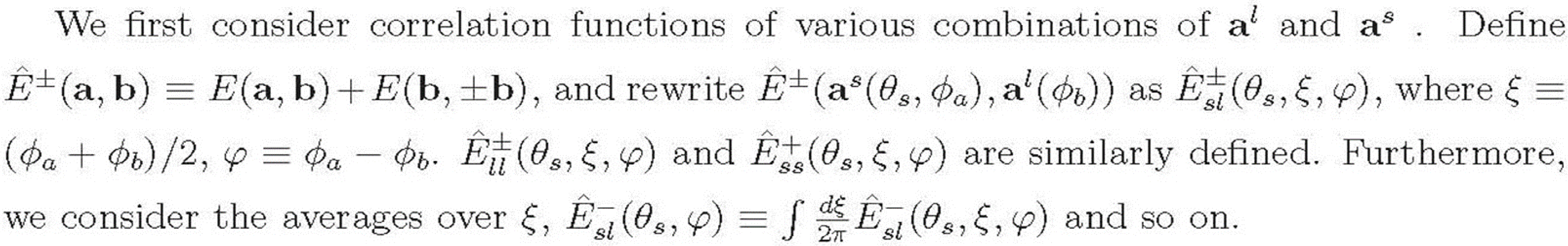
两种衰变对应的“偏振片方向



$\theta_s=2\beta$

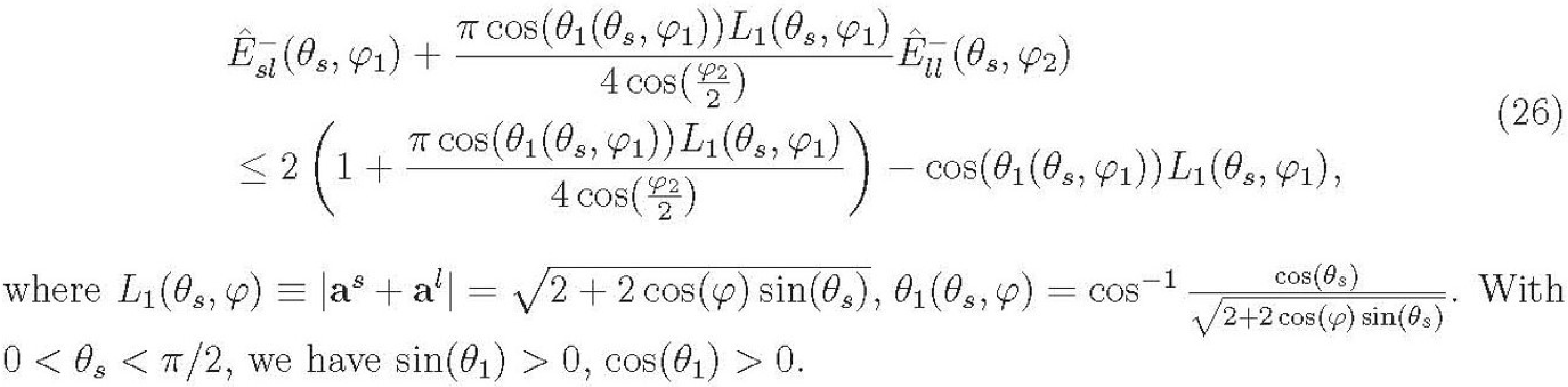
FIG. 1: The effective measuring directions $\mathbf{a}^l$ and $\mathbf{a}^s$. In a certain coordinate system, $\mathbf{a}^l(\phi_l)$ is on xy plane, $\mathbf{a}^s(\theta_s, \phi_s)$ is on a cone. For B_d mesons, $\phi_l = x\Gamma t$,

关联函数



We first consider correlation functions of various combinations of $\mathbf{a}^l$ and $\mathbf{a}^s$. Define $\hat{E}^{\pm}(\mathbf{a},\mathbf{b}) \equiv E(\mathbf{a},\mathbf{b}) + E(\mathbf{b},\pm\mathbf{b})$, and rewrite $\hat{E}^{\pm}(\mathbf{a}^s(\theta_s,\phi_a),\mathbf{a}^l(\phi_b))$ as $\hat{E}^{\pm}_{sl}(\theta_s,\xi,\varphi)$, where $\xi \equiv (\phi_a+\phi_b)/2$, $\varphi \equiv \phi_a - \phi_b$. $\hat{E}^{\pm}_{ll}(\theta_s,\xi,\varphi)$ and $\hat{E}^{+}_{ss}(\theta_s,\xi,\varphi)$ are similarly defined. Furthermore, we consider the averages over ξ, $\hat{E}^{-}_{sl}(\theta_s,\varphi) \equiv \int \frac{d\xi}{2\pi} \hat{E}^{-}_{sl}(\theta_s,\xi,\varphi)$ and so on.

上限：



$$\hat{E}^{-}_{sl}(\theta_s,\varphi_1)+\frac{\pi\cos(\theta_1(\theta_s,\varphi_1))L_1(\theta_s,\varphi_1)}{4\cos(\frac{\varphi_2}{2})}\hat{E}^{-}_{ll}(\theta_s,\varphi_2)$$
$$\leq 2\left(1+\frac{\pi\cos(\theta_1(\theta_s,\varphi_1))L_1(\theta_s,\varphi_1)}{4\cos(\frac{\varphi_2}{2})}\right)-\cos(\theta_1(\theta_s,\varphi_1))L_1(\theta_s,\varphi_1), \tag{26}$$

where $L_1(\theta_s,\varphi)\equiv|\mathbf{a}^s+\mathbf{a}^l|=\sqrt{2+2\cos(\varphi)\sin(\theta_s)}$, $\theta_1(\theta_s,\varphi)=\cos^{-1}\frac{\cos(\theta_s)}{\sqrt{2+2\cos(\varphi)\sin(\theta_s)}}$. With $0<\theta_s<\pi/2$, we have $\sin(\theta_1)>0$, $\cos(\theta_1)>0$.

下限

We find two lower bounds. The first is given as

$$
\begin{aligned}
&\hat{E}_{sl}^{+}(\theta_s,\varphi_1)+\frac{\pi\cos(\theta_2(\theta_s,\varphi_1))L_2(\theta_s,\varphi_1)}{4\left|\sin(\frac{\varphi_2}{2})\right|}\hat{E}_{ll}^{+}(\theta_s,\varphi_2)\\
&\geq -2\left(1+\frac{\pi\cos(\theta_2(\theta_s,\varphi_1))L_2(\theta_s,\varphi_1)}{4\left|\sin(\frac{\varphi_2}{2})\right|}\right)+\cos(\theta_2(\theta_s,\varphi_1))L_2(\theta_s,\varphi_1).
\end{aligned}
$$

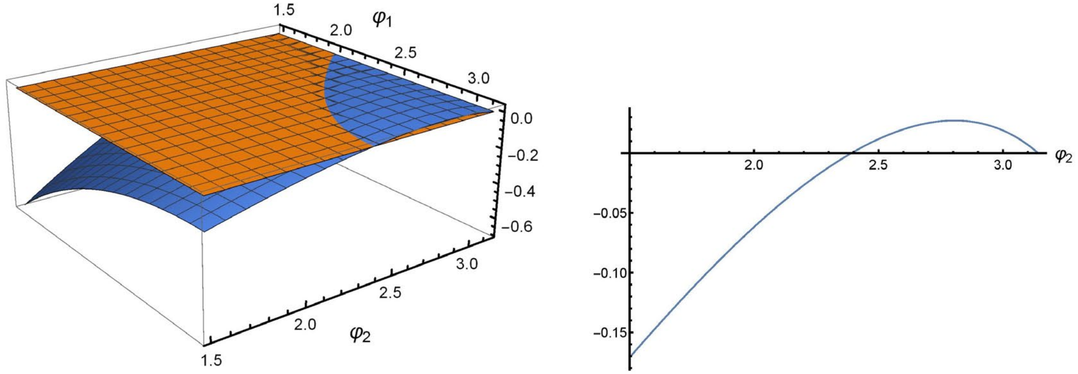
where $L_2(\theta_s,\varphi)=\sqrt{2-2\cos(\varphi)\sin(\theta_s)}$, $\theta_2(\theta_s,\varphi)=\cos^{-1}\frac{\cos(\theta_s)}{\sqrt{2-2\cos(\varphi)\sin(\theta_s)}}$.

The second lower bound is given as

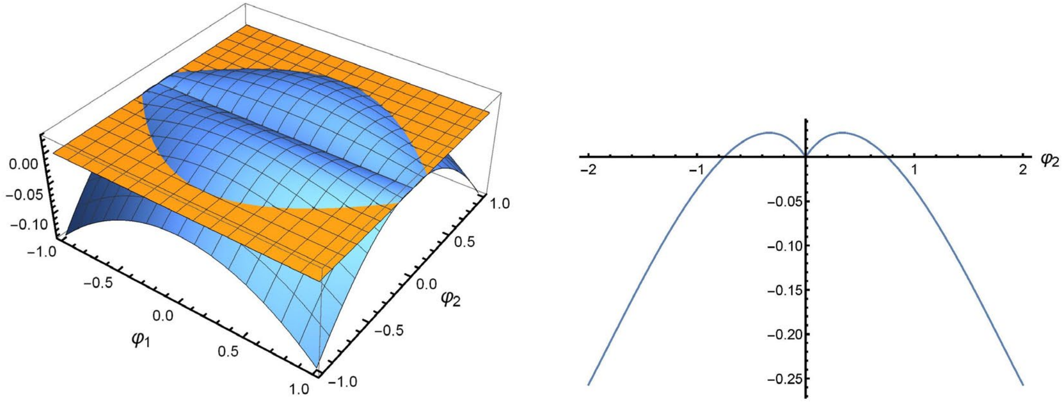
$$
\begin{aligned}
&\hat{E}_{sl}^{+}(\theta_s,\varphi_1)+\frac{\pi\cos(\theta_2(\theta_s,\varphi_1))L_2(\theta_s,\varphi_1)}{4\sin(\theta_s)\left|\sin(\frac{\varphi_2}{2})\right|}\hat{E}_{ss}^{+}(\varphi_2)\\
&\geq -2\left(1+\frac{\pi\cos(\theta_2(\theta_s,\varphi_1))L_2(\theta_s,\varphi_1)}{4\sin(\theta_s)\left|\sin(\frac{\varphi_2}{2})\right|}\right)+\cos(\theta_2)L_2(\theta_s,\varphi_1).
\end{aligned}
$$

LI被粒子物理标准模型违反

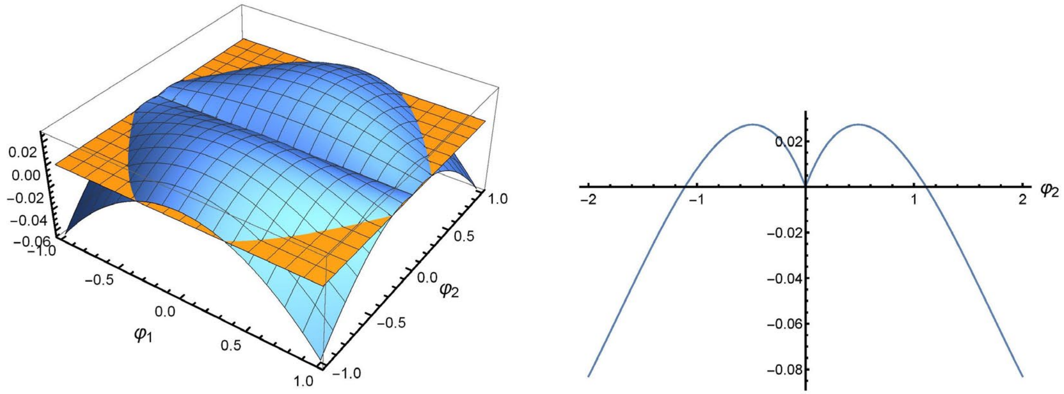
- g=（左−右）/|左|。
- 条件： θs≠0 （CP破坏）
- 事实上确实CP破坏！



(a) $g^u(\varphi_1, \varphi_2)$ (b) $g^u(\pi, \varphi_2)$



(c) $g^{d1}(\varphi_1, \varphi_2)$ (d) $g^{d1}(0, \varphi_2)$



(e) $g^{d2}(\varphi_1, \varphi_2)$ (f) $g^{d2}(0, \varphi_2)$

6. 超子自旋纠缠用于推广的Bell不等式（衰变也由隐变量决定）

YS and JC Yang, Entangled baryons: violation of inequalities based on local realism assuming dependence of decays on hidden variables, European Physical Journal C 80, 116 (2020).

新不等式（贝尔不等式的推广）

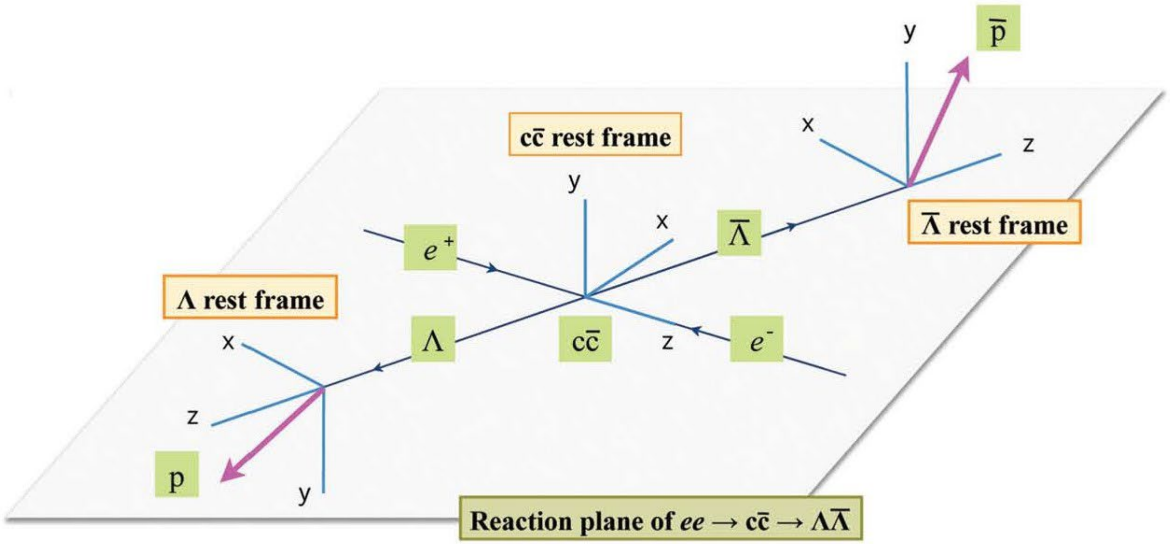
- 首次考虑极化值统计混合。
- 将类空间隔的局域隐变量分割。
- 在纠缠超子中实现。

- 超子自旋的纠缠。
- 超子衰变产生的质子的动量方向扮演测量设置的角色。
- 用角分布取代Malus Law

$$\frac{d\sigma(\Lambda\to p\pi^-)}{d\Omega_p}=\frac{1}{4\pi}(1+\alpha_\Lambda \mathbf{s}_\Lambda\cdot\mathbf{n}_p)\,,\quad \frac{d\sigma(\bar\Lambda\to \bar p\pi^+)}{d\Omega_{\bar p}}=\frac{1}{4\pi}(1-\alpha_\Lambda \mathbf{s}_{\bar\Lambda}\cdot\mathbf{n}_{\bar p})$$

$$\bar{\mathbf{A}}=\int d\lambda_A d\lambda_B \rho_A(\lambda_A)\rho_B(\lambda_B)\mathbf{A}(\lambda_A)=\frac{\alpha_\Lambda}{3}\mathbf{u},$$

$$\bar{\mathbf{B}}=\int d\lambda_A d\lambda_B \rho_A(\lambda_A)\rho_B(\lambda_B)\mathbf{B}(\lambda_B)=-\frac{\alpha_\Lambda}{3}\mathbf{v}$$



$$E(\mathbf{a},\mathbf{b})\equiv-\langle\overline{\mathbf{A}\cdot\mathbf{a}\mathbf{B}\cdot\mathbf{b}}\rangle=-\sum_{ij}a_ib_j\langle\overline{A_iB_j}\rangle$$

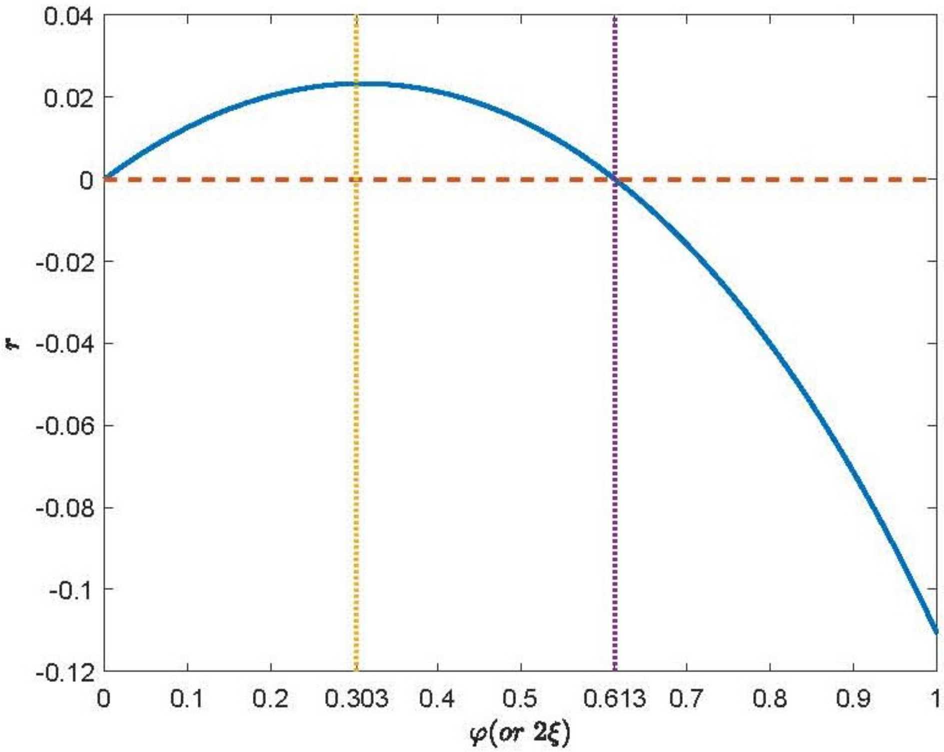
$$=-\int d\mathbf{u}d\mathbf{v}d\lambda_A d\lambda_B F(\mathbf{u},\mathbf{v})\rho_A(\lambda_A,\mathbf{u},\mathbf{v})\rho_B(\lambda_B,\mathbf{u},\mathbf{v})\mathbf{A}(\lambda_A,\mathbf{u},\mathbf{v})\cdot\mathbf{a}\mathbf{B}(\lambda_B,\mathbf{u},\mathbf{v})\cdot\mathbf{b}$$

$$=\frac{\alpha_\Lambda^2}{9}\int d\mathbf{u}d\mathbf{v}F(\mathbf{u},\mathbf{v})\mathbf{u}\cdot\mathbf{a}\mathbf{v}\cdot\mathbf{b},$$

$$\left|E_N^{\mathbf{ab}}(\varphi)+E_N^{\mathbf{ab}}(0)\right|+\left|E_N^{\mathbf{cd}}(\varphi)+E_N^{\mathbf{cd}}(0)\right|\leq\frac{\alpha_\Lambda^2}{9}\left(4-2u_N\left|\sin\frac{\varphi}{2}\right|\right)$$

$$\left|\hat{E}_N^{\mathbf{ab}}(\xi)+\hat{E}_N^{\mathbf{ab}}(0)\right|+\left|\hat{E}_N^{\mathbf{cd}}(\xi)+\hat{E}_N^{\mathbf{cd}}(0)\right|\leq\frac{\alpha_\Lambda^2}{9}\left(4-2u_N\left|\sin\xi\right|\right)$$

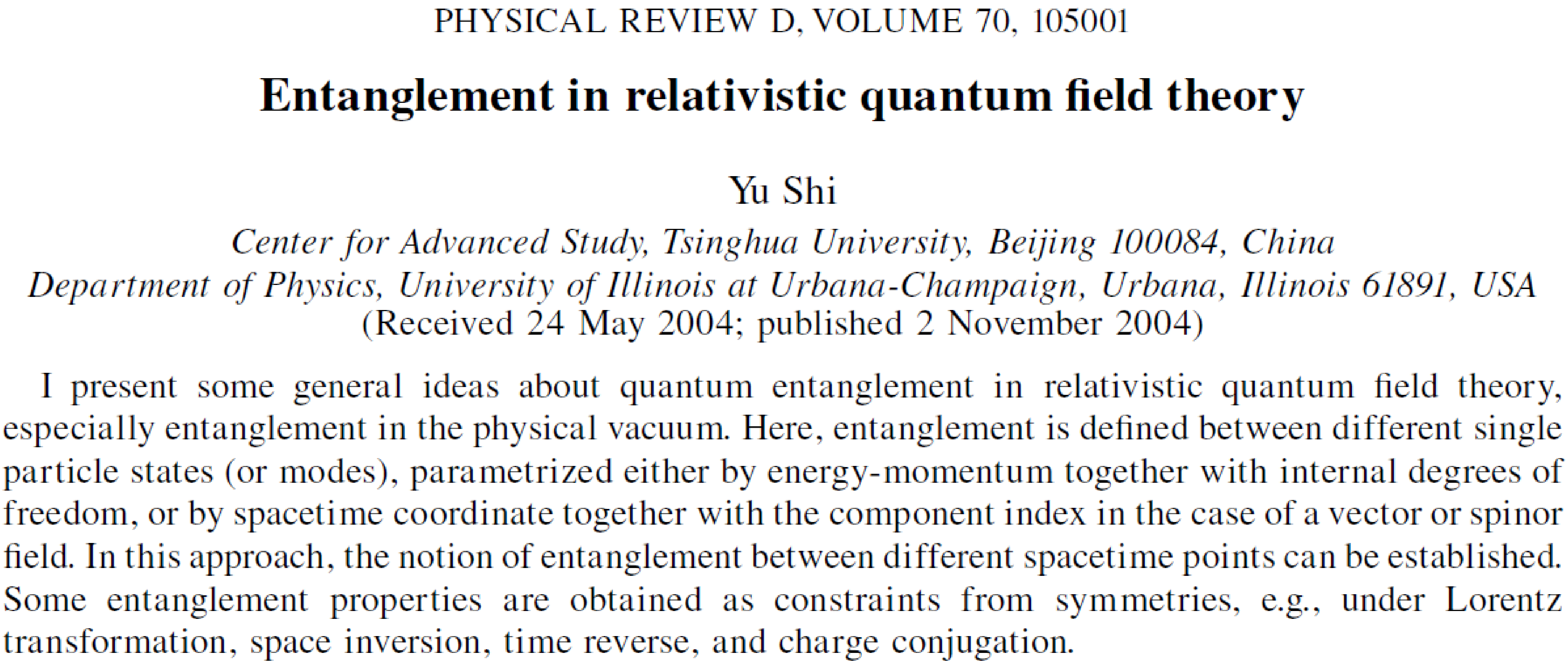
$$u_N=\cot\left(\pi/2N\right)/N$$



The violation ratio r for the first (second) inequality, as a function of φ (2ξ) for η_c (χ_{c_0}).

- BESIII已搜集的数据可能已经足够。

7. 量子场论中量子纠缠的一般性讨论



PHYSICAL REVIEW D, VOLUME 70, 105001

Entanglement in relativistic quantum field theory

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I present some general ideas about quantum entanglement in relativistic quantum field theory, especially entanglement in the physical vacuum. Here, entanglement is defined between different single particle states (or modes), parametrized either by energy-momentum together with internal degrees of freedom, or by spacetime coordinate together with the component index in the case of a vector or spinor field. In this approach, the notion of entanglement between different spacetime points can be established. Some entanglement properties are obtained as constraints from symmetries, e.g., under Lorentz transformation, space inversion, time reverse, and charge conjugation.

- This paper also contains an early discussion on entanglement in Unruh/Hawking radiation.

8. Later work on entanglement in Unruh/Hawking radiation

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2. Dawei Wu, Ji-chong Yang, Yu Shi, Conditions for anti-Unruh effect, European Physical Journal C 83, 1110 (2023).
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10. Yue Dai, Zhejun Shen and Yu Shi, Killing quantum entanglement by acceleration or a black hole, Journal of High Energy Physics 09, 071 (2015).

规范理论的量子模拟与赝量子模拟

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- Yiming Ding, Xiaopeng Cui and Yu Shi, Digital quantum simulation and pseudoquantum simulation of the Z2 gauge-Higgs model, Physical Review D 105, 054508 (2022).
- Xiaopeng Cui and Yu Shi, Correspondence between the Hamiltonian cycle problem and the quantum Z2 lattice gauge theory, Europhysics Letters 144, 48001 (2023).
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Summary

1. 1st Proposal for a High Enegy Quantum Information Process: quantum teleportation in terms of entangled mesons.
2. Using Quantum Entanglement to Study CP and CPT Violations.
3. Genuine T Violation Signal in terms of entangled D mesons.
4. Entangled mesons violate a generalized Leggett Inequality, in which decays also depend on hidden variables.
5. Entangled hyperons violated a generalized Bell inequality, in which decays also depend on hidden variables.
6. General methodology of entanglement in QFT.
7. Entanglement in Unruh/Hawking radiation.
8. Quantum simulation of gauge theories.