Quantum Entanglement at High-energy Colliders

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28th Mini-workshop on the frontier of LHC

Quantum information ⇔ Particle physics

Measuring Quantum Correlations at Colliders

Also Talk of Yu Shi

01. Quantum Review

02. Collider Review ($t\bar{t}$ at LHC)

03. Phenomenology

Quantum computing for high energy physics.

Talks of Ying-Ying Li, Xiaohui Liu and Ji-Chong Yang

Theory aspect:

- QI in QFT
- Entanglement and symmetry *Talk of Ming-Lei Xiao*
- ……

Quantum Review

State, entanglement and Bell inequality

Quantum Mechanics: State and Density matrix

- **Pure state** $\longrightarrow$ **Wave function** $|\psi\rangle$
- **Mixed state** $\longrightarrow$ **Generalize** $|\psi\rangle\langle\psi|$ **to density matrix** ρ
- **General density matrix (2x2) for 1 qubit, 3 parameters** B_i

$$\rho = \frac{I_2 + B_i\sigma_i}{2}, \quad \vec{B} = \mathrm{tr}(\vec{\sigma}\rho), \quad |\vec{B}| \leq 1$$

- **General density matrix (4x4) for 2 qubit, 15 parameters**

$$\rho = \frac{1}{4}\left(\mathbf{I}_4 + \underline{B_i^+\sigma_i \otimes \mathbf{I}_2} + \underline{B_i^-\mathbf{I}_2 \otimes \sigma_i} + \underline{C_{ij}\sigma_i \otimes \sigma_j}\right)$$

Example

$$|\Psi_0\rangle = \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}, \quad C = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

$$|\Psi_1\rangle = \frac{|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle}{\sqrt{2}}, \quad C = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$|\Psi_2\rangle = i\frac{|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle}{\sqrt{2}}, \quad C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$|\Psi_3\rangle = -\frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}, \quad C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

Quantum Mechanics: State and Density matrix

- **General density matrix (4x4) for 2 qubit, 15 parameters**

$$\rho = \frac{1}{4}\left(\mathbf{I}_4 + \underline{B_i^+\sigma_i \otimes \mathbf{I}_2} + \underline{B_i^-\mathbf{I}_2 \otimes \sigma_i} + \underline{C_{ij}\sigma_i \otimes \sigma_j}\right)$$

B_i^+ Polarization vector of qubit A

B_i^- Polarization vector of qubit B

C_{ij} Spin correlation matrix

- **Quantum State Tomography:**
  - **Reconstruction of quantum state from a complimentary set of measurements on a set of identically prepared states.**
  - **Two qubit quantum tomography: measure 15 parameters $B_i^{\pm}$ and C_{ij}**

Quantum Mechanics: Entanglement

separable

$$|\psi\rangle = |00\rangle = |0\rangle \otimes |0\rangle$$

entangled

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

A seperable state can be written as $|\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$. Entangled is non-separable.

[W. Wootters, quant-ph/9709029]

- **How to tell if a state is entangled?** **Concurrence:** $\mathscr{C}[\rho] = 0$ (seperable)
  $= 1$ (maximally entangled)

- **For unpolarized state** $\mathscr{C}[\rho] = \dfrac{-1 - 3D}{2}$ **, where**

  - $D = \min\{\frac{\mathrm{tr}(C)}{3}, \frac{c_1 - c_2 - c_3}{3}, \frac{c_2 - c_1 - c_3}{3}, \frac{c_3 - c_1 - c_2}{3}\}, \qquad c_i = \mathrm{eig}(C)$
  - D **is closely relatated to collider observables.** [Y. Afik and J. de Nova, 2003.02280]

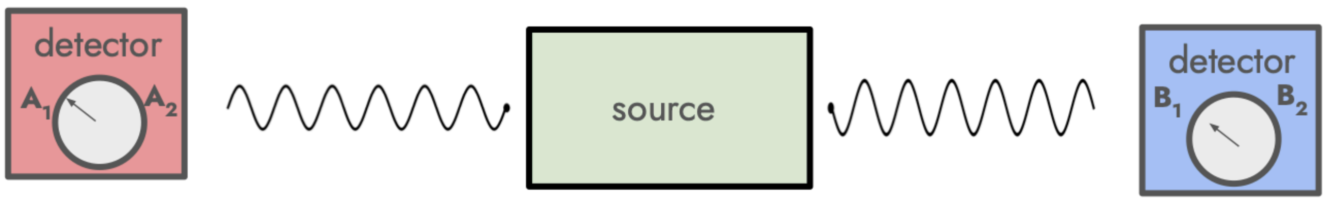
- $D < -1/3$ **(entangled),** $D > -1/3$ **(seperable)**

Quantum Mechanics: Bell (CHSH) inequality violation [J. Bell, *1964*]

Bell inequality: constructed from four two-outcome measurements $\hat{A}_{1,2}$ and $\hat{B}_{1,2}$

$$\left|\langle \hat{A}_1 \hat{B}_1 \rangle + \langle \hat{A}_1 \hat{B}_2 \rangle + \langle \hat{A}_2 \hat{B}_1 \rangle - \langle \hat{A}_2 \hat{B}_2 \rangle\right| \leq 2$$

[Clauser et al, *PRL* 23, 880 (1969)]



Depends on the choice of $A_{1,2}$ and $B_{1,2}$. Example:

$$A_1 = \sigma_x \qquad \langle \sigma_x \otimes \sigma_z \rangle = \mathrm{tr}[(\sigma_x \otimes \sigma_z)\rho]$$
$$B_2 = \sigma_z \qquad = C_{13}$$

Scanning all possible $A_{1,2}$ and $B_{1,2}$, the Bell inequality is violated *iff*

$$\mathcal{B}[\rho] = 2\sqrt{c_1^2 + c_2^2} > 2$$

[Horodecki et al, *PLA* 200, 340 (1995)]

c_1^2, c_2^2 **are the largest two eigenvalue of $C^T C$. Just measure spin correlation** [M. Fabbrichesi et al, 2102.11883]
[C. Severi et al, 2110.10112]

Stronger condition than entanglement

Quantum Mechanics: Hierarchy of quantum correlation

Quantum

Bell non-locality ⇒ $\mathcal{B}[\rho] = 2\sqrt{c_1^2 + c_2^2} > 2$

$\cap$

Quantum Steering

$\cap$

Quantum Entanglement ⇒ $\mathscr{C}[\rho] > 0, \quad (D < -\frac{1}{3})$

$\cap$

Quantum Discord

Classical

For discord and steering, see [Y. Afik and J. de Nova, 2209.03969]

Quantum State at Colliders

$t\bar{t}$ as an example

Collider View: Treating $t\bar{t}$ as quantum states

The underlying quantum state of $t\bar{t}$ produced at collider is defined in $\mathscr{H}_k \otimes \mathscr{H}_{\text{spin}} \otimes \mathscr{H}_{\text{color}}$, we can expand it in terms of $|\mathbf{k}, \alpha\bar{\alpha}\rangle$

$$|t\bar{t}\rangle \propto \int \mathrm{d}\mathbf{k} \sum_{\alpha\bar{\alpha}} |\mathbf{k}, \alpha\bar{\alpha}\rangle \langle \mathbf{k}, \alpha\bar{\alpha}| T |I, \lambda\rangle = \int \mathrm{d}\mathbf{k} \sum_{\alpha\bar{\alpha}} \mathcal{M}^{\lambda}_{\alpha\bar{\alpha}}(\mathbf{k}) |\mathbf{k}, \alpha\bar{\alpha}\rangle$$

At each scattering angle, the spin density matrix of $t\bar{t}$

$$R(\mathbf{k})_{\alpha\alpha',\bar{\alpha}\bar{\alpha}'} = \sum_{\lambda\lambda'} \mathcal{M}^{\lambda}_{\alpha\bar{\alpha}}(\mathbf{k}) \rho^{I}_{\lambda\lambda'} (\mathcal{M}^{\lambda'}_{\alpha'\bar{\alpha}'}(\mathbf{k}))^*$$

$$\rho(\mathbf{k})_{\alpha\alpha',\bar{\alpha}\bar{\alpha}'} = \frac{R(\mathbf{k})_{\alpha\alpha',\bar{\alpha}\bar{\alpha}'}}{\mathrm{Tr}\,(R(\mathbf{k}))}$$

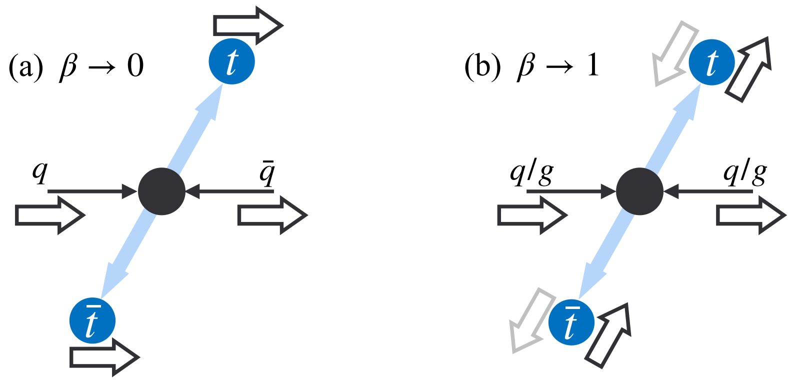
Different initial states

Different final state

→ **Different quantum information**

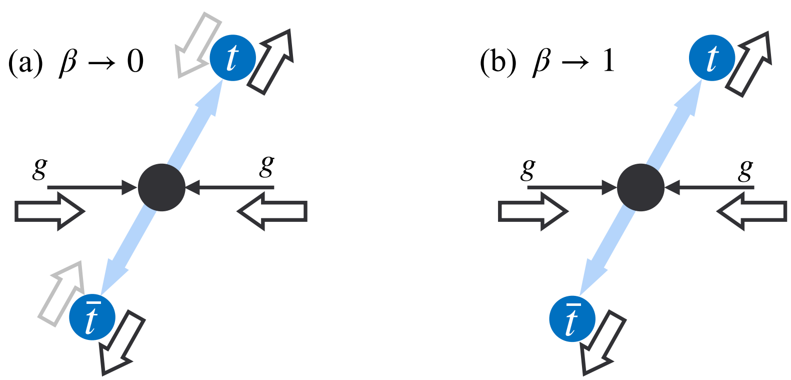
Colliders View: elementary QCD processes

Unlike-helicity initial state: $q\bar{q}$, $g_L g_R$



Positive spin correlation
(spin triplet at high-p_T region)

Like-helicity initial state: $g_L g_L$, $g_R g_R$



Negative spin correlation
(spin singlet near threshold)

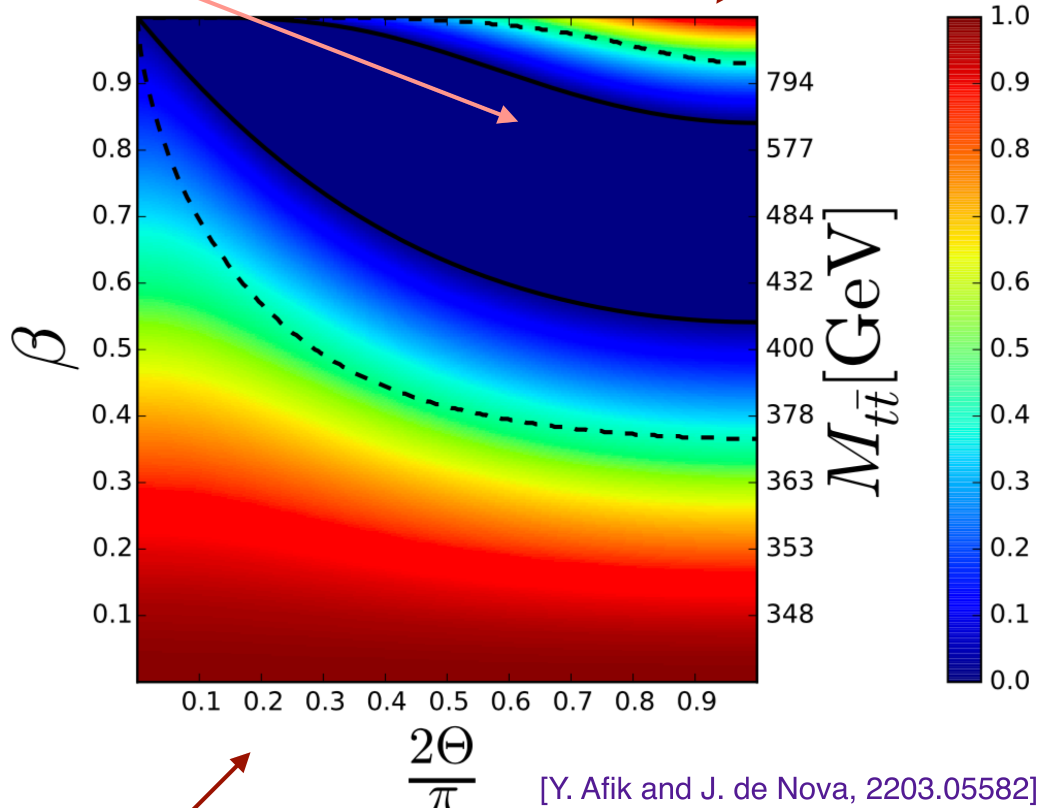
[G. Mahlon and S. Parke, 1001.3422]

Collider View: $gg \to t\bar{t}$ processes

$$\rho_{gg\to t\bar{t}} \propto |\mathcal{M}_{\text{unlike}}|^2 \rho_{\text{unlike}} + |\mathcal{M}_{\text{like}}|^2 \rho_{\text{like}}$$

high-p_T region: $g_L g_R$ dominant
spin triplet

Like and unlike-helicity gluons are comparable. Spin correlations are cancelled: seperable



[Y. Afik and J. de Nova, 2203.05582]

Quantum correlation hierachy:

Bell inequality violation (dashed line) is a strong condition than *entanglement* (solid line)

Threshold region: $g_L g_L / g_R g_R$ dominate
spin singlet

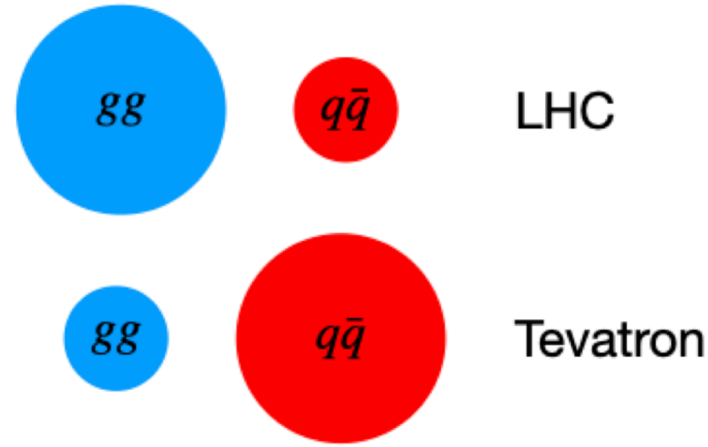
Hadron collision: $\rho^{t\bar{t}} = \omega^{q\bar{q}}\rho^{q\bar{q}\to t\bar{t}} + \omega^{gg}\rho^{gg\to t\bar{t}}$

$$\omega^I = \frac{L_I|\mathcal{M}_{I\to t\bar{t}}|^2}{L_{q\bar{q}}|\mathcal{M}_{q\bar{q}\to t\bar{t}}|^2 + L_{gg}|\mathcal{M}_{gg\to t\bar{t}}|^2}, \qquad I = q\bar{q},\ gg.$$

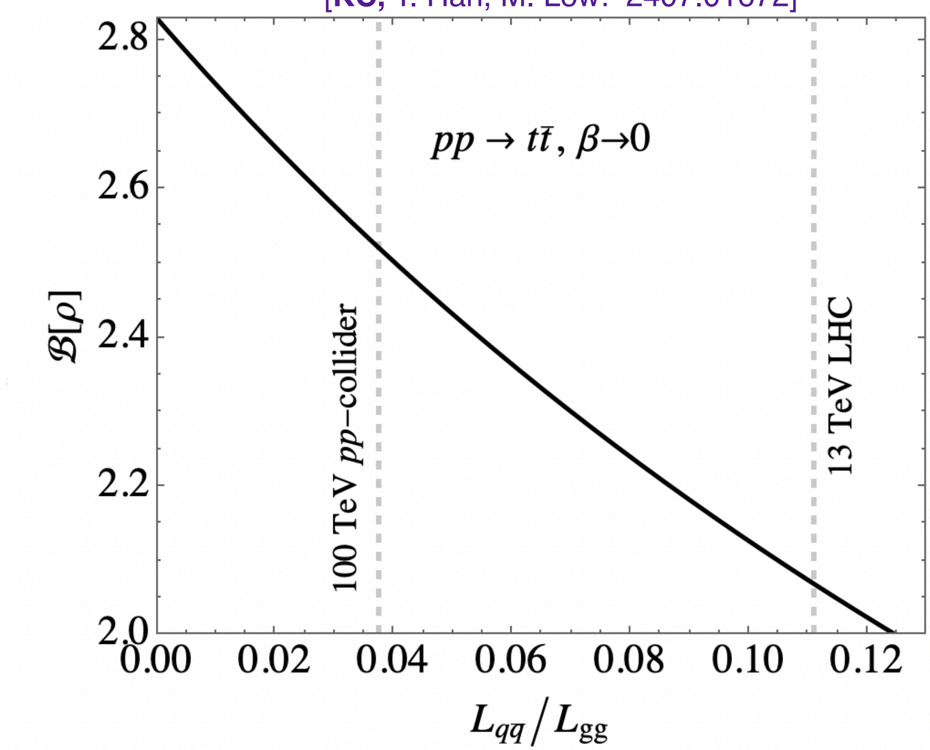
$g_L g_R \to t\bar{t}$ **and** $q\bar{q} \to t\bar{t}$ **: positive spin correlation.**
(high-p_T)

Cancellation

$g_L g_L / g_R g_R \to t\bar{t}$ **: negative spin correlation.**
(threshold)



[**KC**, T. Han, M. Low: 2407.01672]



Collider View: Phase space average and selection cuts

$$\rho_{t\bar{t}} = \frac{1}{4}\left(I_2 \otimes I_2 + B_i^+ \sigma_i \otimes I_2 + B_i^- I_2 \otimes \sigma_i + C_{ij}\,\sigma_i\,\sigma_j\right)$$

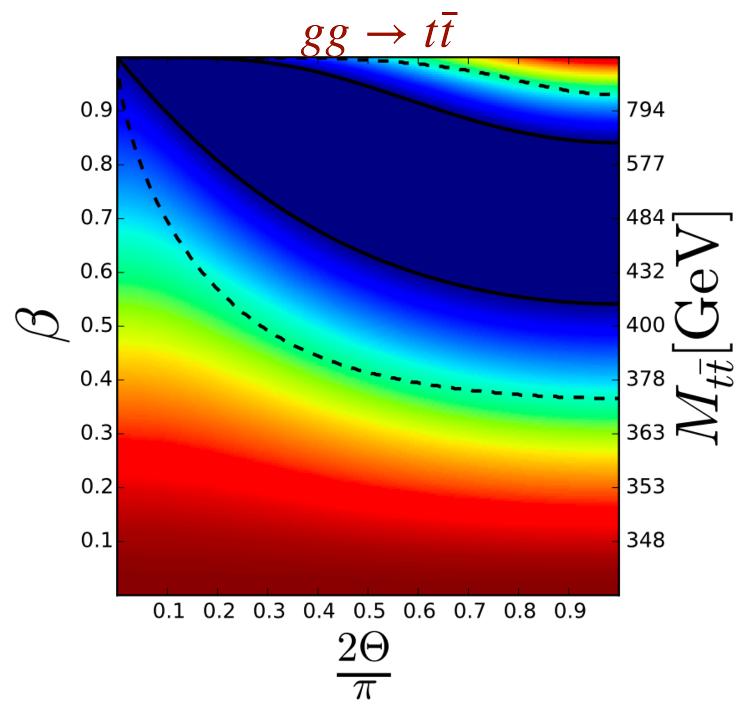
$$\bar{\rho} = \frac{1}{\sigma}\int \mathrm{d}\Omega\,\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\rho(\Omega)$$

$$\bar{C} = \frac{1}{\sigma}\int \mathrm{d}\Omega\,\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}C(\Omega)$$

Proper selection cut $\Longrightarrow$ Quantum state we want to study

***Example*:**

- $D = -0.237$, **no selection cuts on $t\bar{t}$** [CMS: 1907.03729]
- $D = -0.547$, **with $m_{t\bar{t}} < 380$ GeV** [ATLAS: 2311.07288]
- $D = -0.480$, **with $m_{t\bar{t}} < 400$ GeV** [CMS: 2406.03976]
- **A upper cut on the velocity of $t\bar{t}$ system can increase the spin singlet contribution** [J. Aguilar-Saavedra and J. Casas, 2205.00542]



Collider View: Phase space average and basis choice

[**KC**, T. Han and M. Low, 2311.07288, 2407.01672]

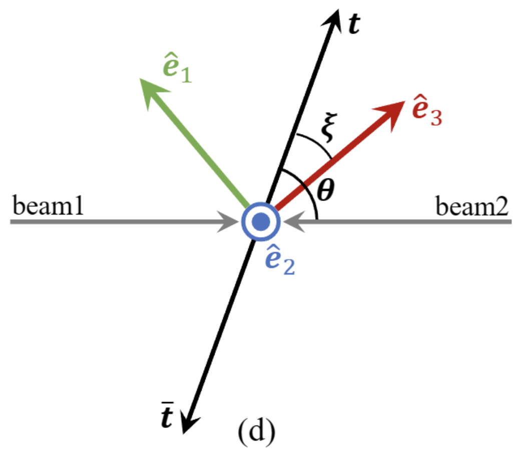
Event-dependent basis

⇩

$\bar{\rho}$ is basis-dependent

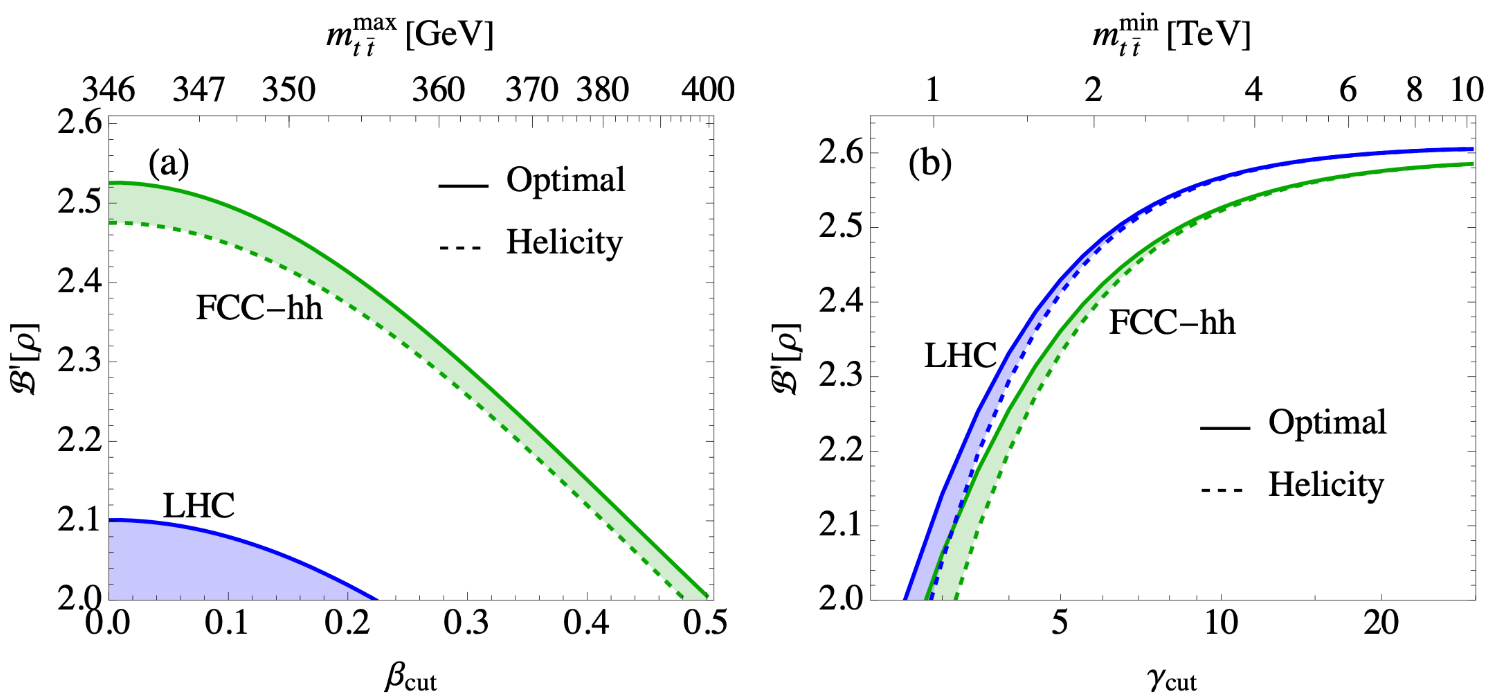
⇩

Optimal basis exits



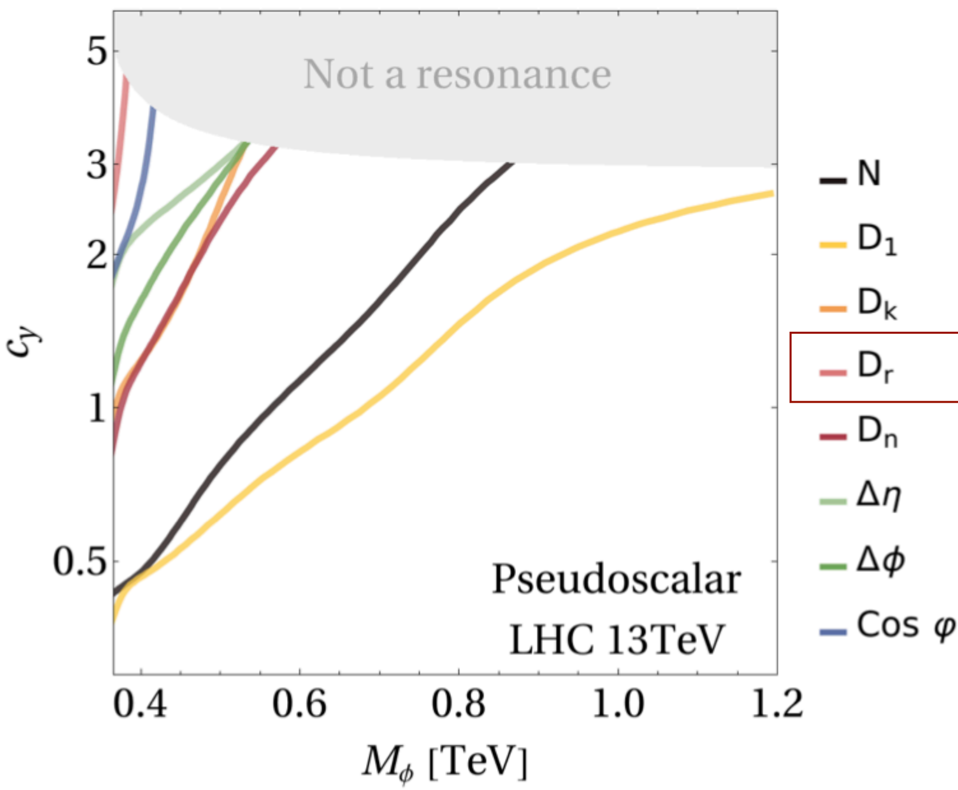
$$\bar{\rho} = \frac{1}{\sigma} \int \mathrm{d}\Omega \, \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \rho(\Omega)$$

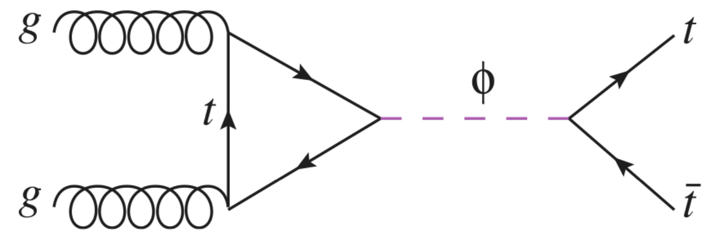
$$\bar{C} = \frac{1}{\sigma} \int \mathrm{d}\Omega \, \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} C(\Omega)$$



Collider View: NP contribution to quantum information

[F. Maltoni et al: 2401.08751]





Spin triplet is sensitive to this new physics contribution

SMEFT approachs:

$t\bar{t}$

[Severi and Vryonidou, 2210.09330]
[Fabbrichesi et al, 2208.11723]
[Aoude et al, 2203.05619]
…

WW, WZ, ZZ

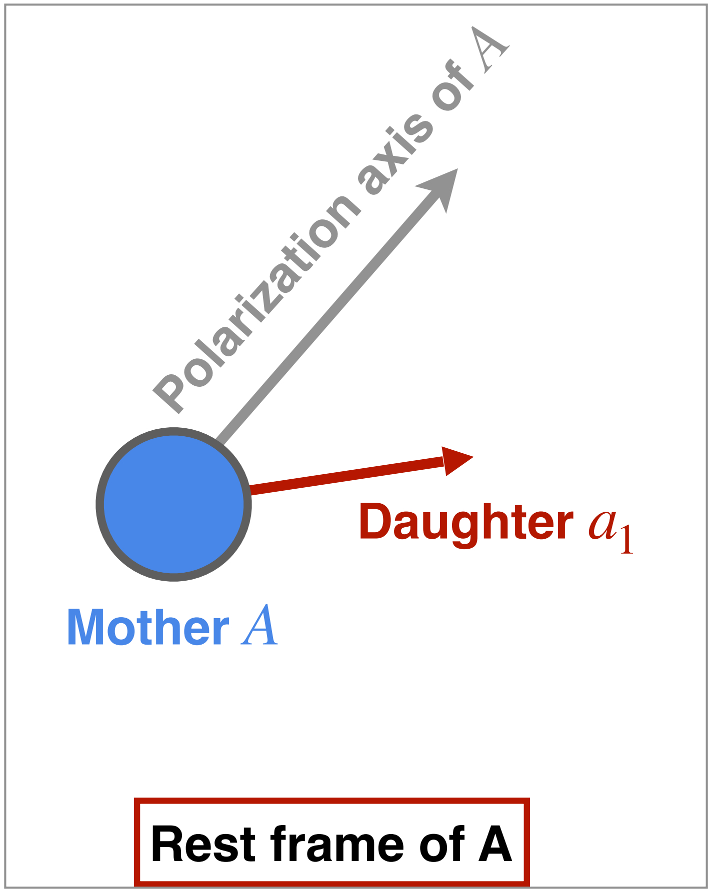
[Fabbrichesi et al, 2304.02403]
[Aoude et al, 2307.09675]
[Bernal et al, 2307.13496]
…

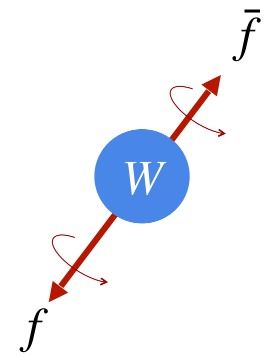
Phenomenology: Spin Measurements

Not real ***quantum*** measurements?

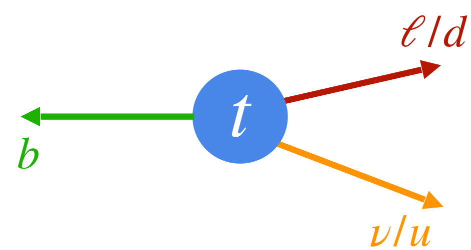
Phenomenology: Decay Products as Spin Analyzer

Decay angle distribution determined by spins of (mother) particles





Two body decay, choose either daughter particle is the same.



$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_v} = \frac{1}{2}\left(1 + |\vec{B}|\kappa_v \cos\theta_v\right)$$

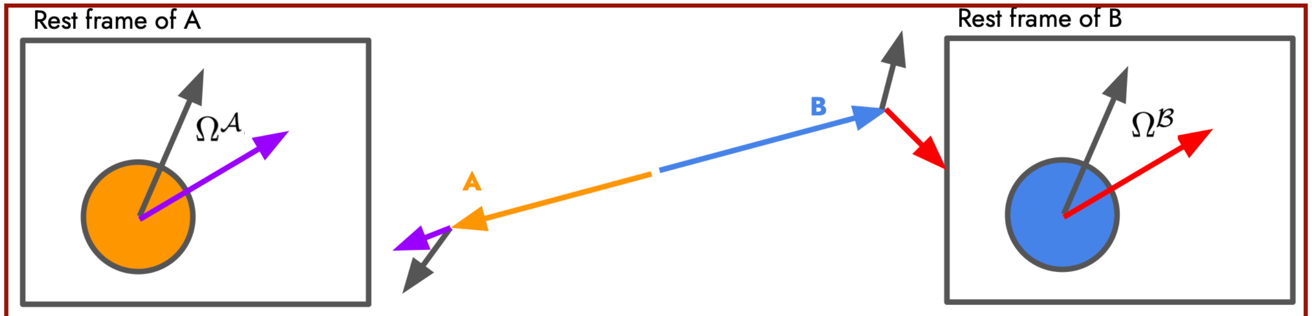
polarization ($|\vec{B}|$)

Spin analyzing power (κ_v)

Spin Analyzer	Power
lepton/down-quark	1.00
neutrino/up-quark	-0.34
b-quark or W	$\mp$0.40
soft-quark	0.50
optimal hadronic	0.64

[B. Tweedie, 1401.3021]

Phenomenology: Steps to measure entanglement at colliders



$$\frac{1}{\sigma}\frac{d^4\sigma}{d^2\Omega^{\mathcal{A}}d^2\Omega^{\mathcal{B}}} = \frac{1}{(4\pi)^2}\left(1+\sum_i\left(\kappa^{\mathcal{A}}B_i^{\mathcal{A}}\Omega_i^{\mathcal{A}}+\kappa^{\mathcal{B}}B_i^{\mathcal{B}}\Omega_i^{\mathcal{B}}\right)+\sum_{i,j}\kappa^{\mathcal{A}}\kappa^{\mathcal{B}}\,\Omega_i^{\mathcal{A}}C_{ij}\Omega_j^{\mathcal{B}}\right),$$

Use different decay mode (semi-leptonic is better)
[Z. Dong et al, 2305.07075]
[T. Han, M. Low, T. Wu, 2310.17696]

$$\rho_{\mathcal{AB}} = \frac{1}{4}\left(\mathbf{I}_4 + B_i^{\mathcal{A}}\sigma_i\otimes\mathbf{I}_2 + B_i^{\mathcal{B}}\mathbf{I}_2\otimes\sigma_i + C_{ij}\sigma_i\otimes\sigma_j\right)$$

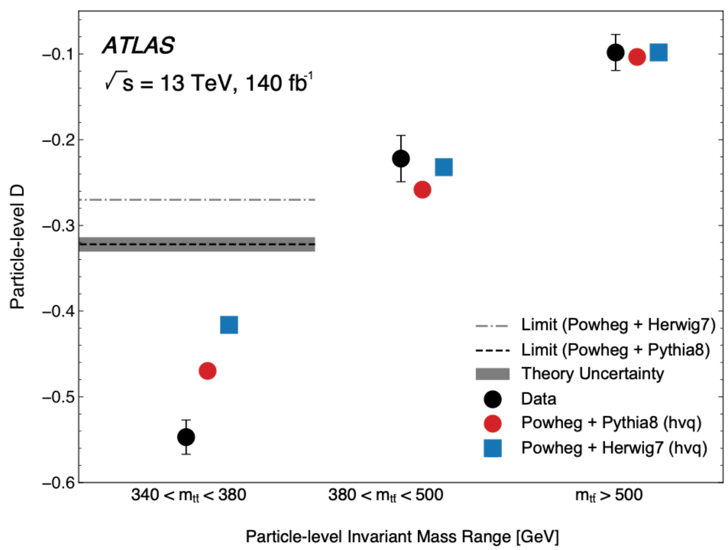
Proper observable can relax the experimental difficulties.
[J. Aguilar-Saavedra and J. Casas, 2205.00542]
[Q. Bi, Q.-H. Cao, **KC** and H. Zhang, 2307.14895]

Entanglement, Bell inequality……

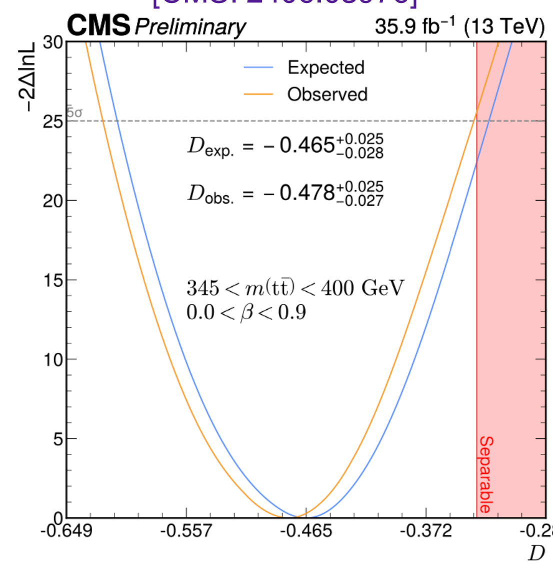
Conclusion: The study of entanglement in particle physics is gathering pace

- Quantum entanglement is measured at the highest energy we can achieve.
- Inspire both quantum community and high-energy community.

[ATLAS, 2311.07288]



[CMS: 2406.03976]



Outlook

- Measurement Bell inequality and other quantum observables at LHC.
- Other channels:
  - Spin space: WW, ZZ, $\tau^{\pm}$, even gg [YG, XL, FY and HXZ, 2406.05880]
  - Flavor space: $\bar{B}_0\bar{B}_0, K_0\bar{K}_0\cdots$ *See the talk of Yu Shi*
- Novel quantum experiments?

High-energy colliders are “quantum information laboratories”!