Two-loop electroweak correction to H→Z+γ

桑文龙 西南大学

In collaboration with Feng Feng and Yu Jia, based on arXiv:2405.03464

28th Mini-workshop on the frontier of LHC
2024/7/8—2024/7/11 通化师范学院

Outline：

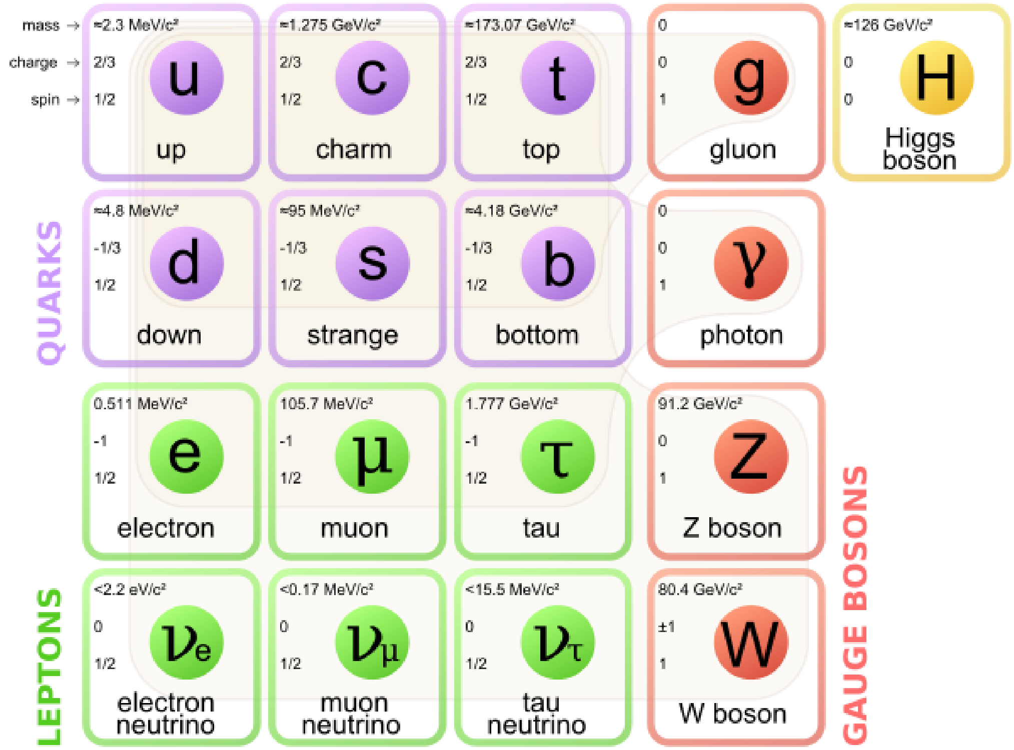
1. Introduction

2. Outline of calculation

3. Phenomenological discussion

4. Summary

1. Introduction

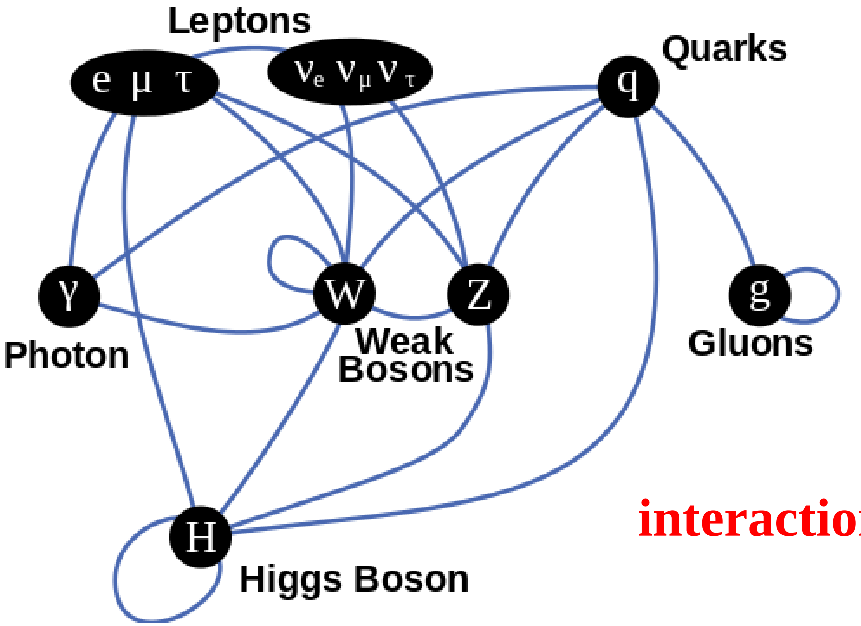


Elementary particles in the Standard Model

1) Quantum field theory

2) Gauge theory

3) Describing electromagnetic, weak, strong interactions

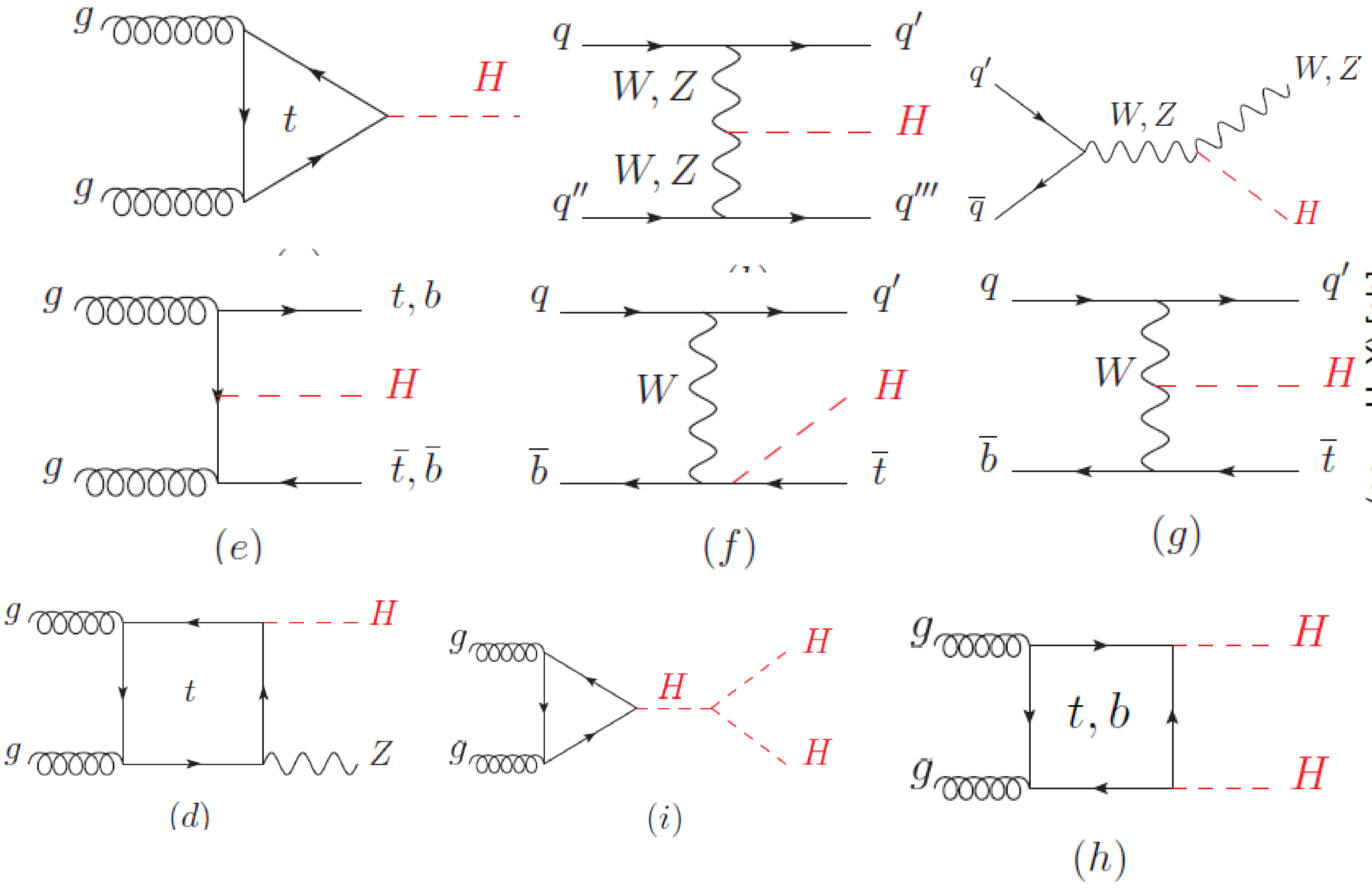


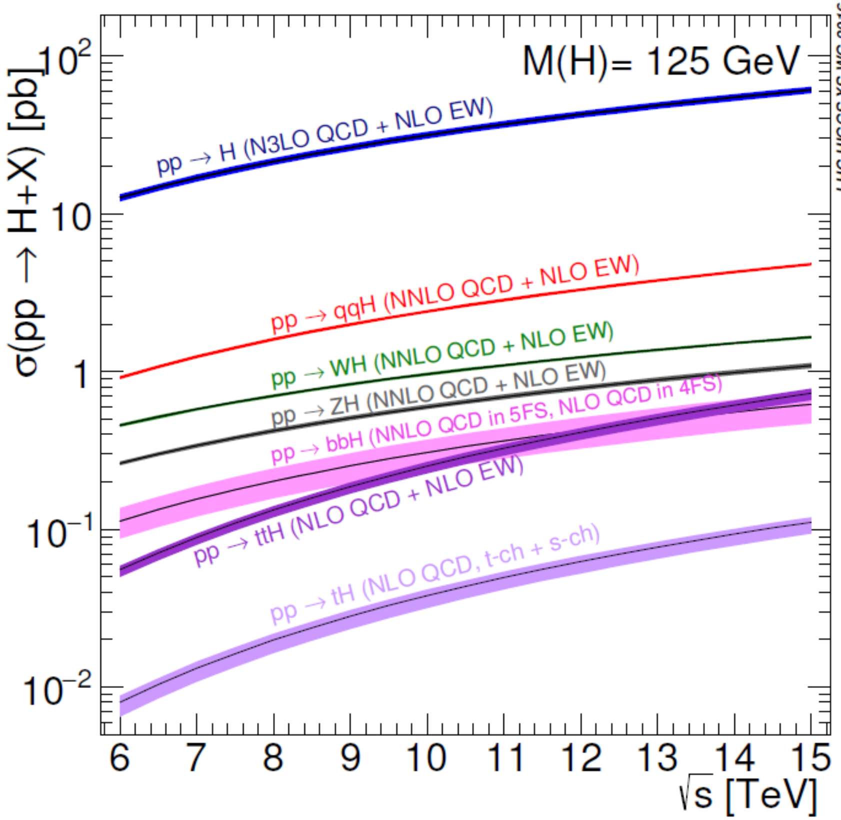
interactions between particles

The **Higgs boson** is the only fundamental scalar particle and plays a central and important role in the SM.

Higgs production at LHC

The main leading order Feynman diagrams for Higgs boson production at LHC

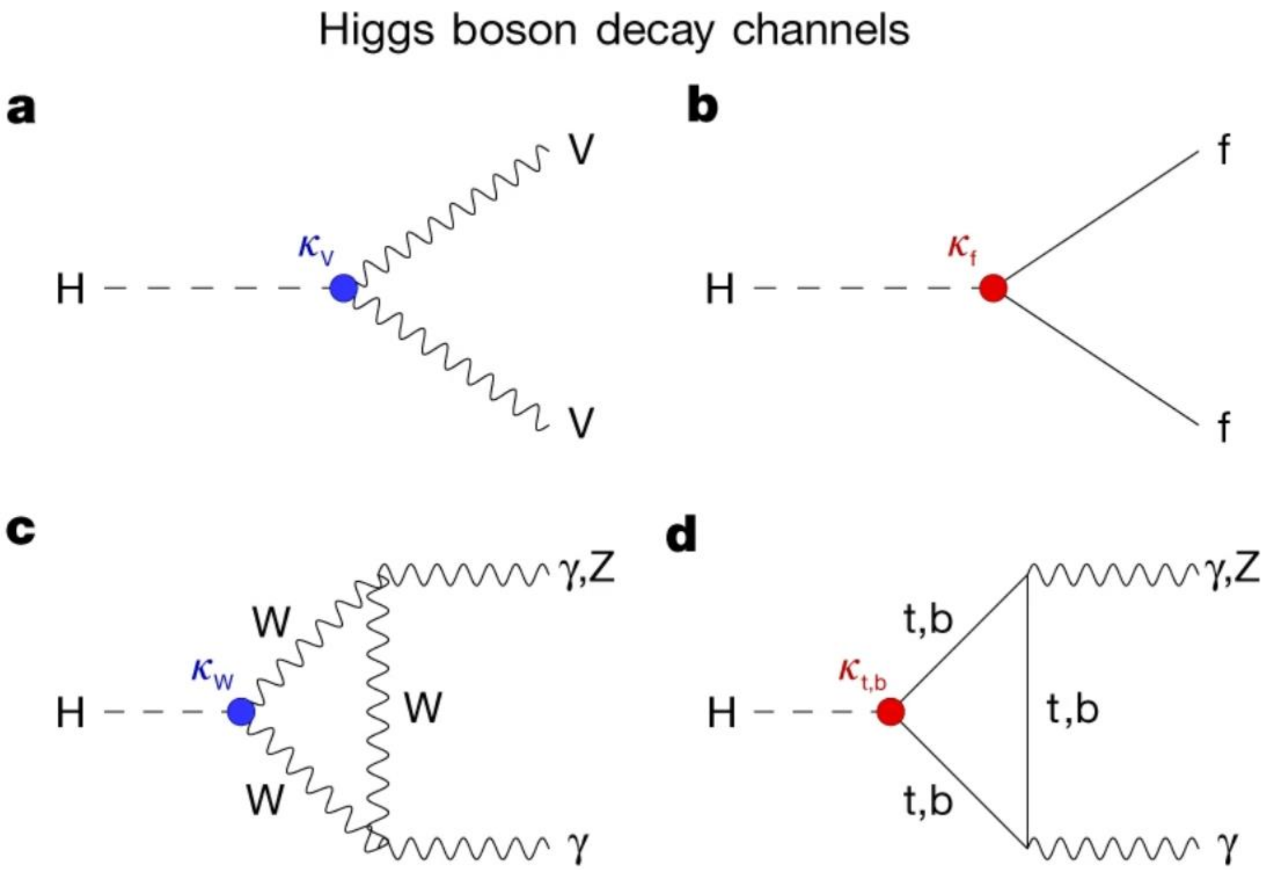


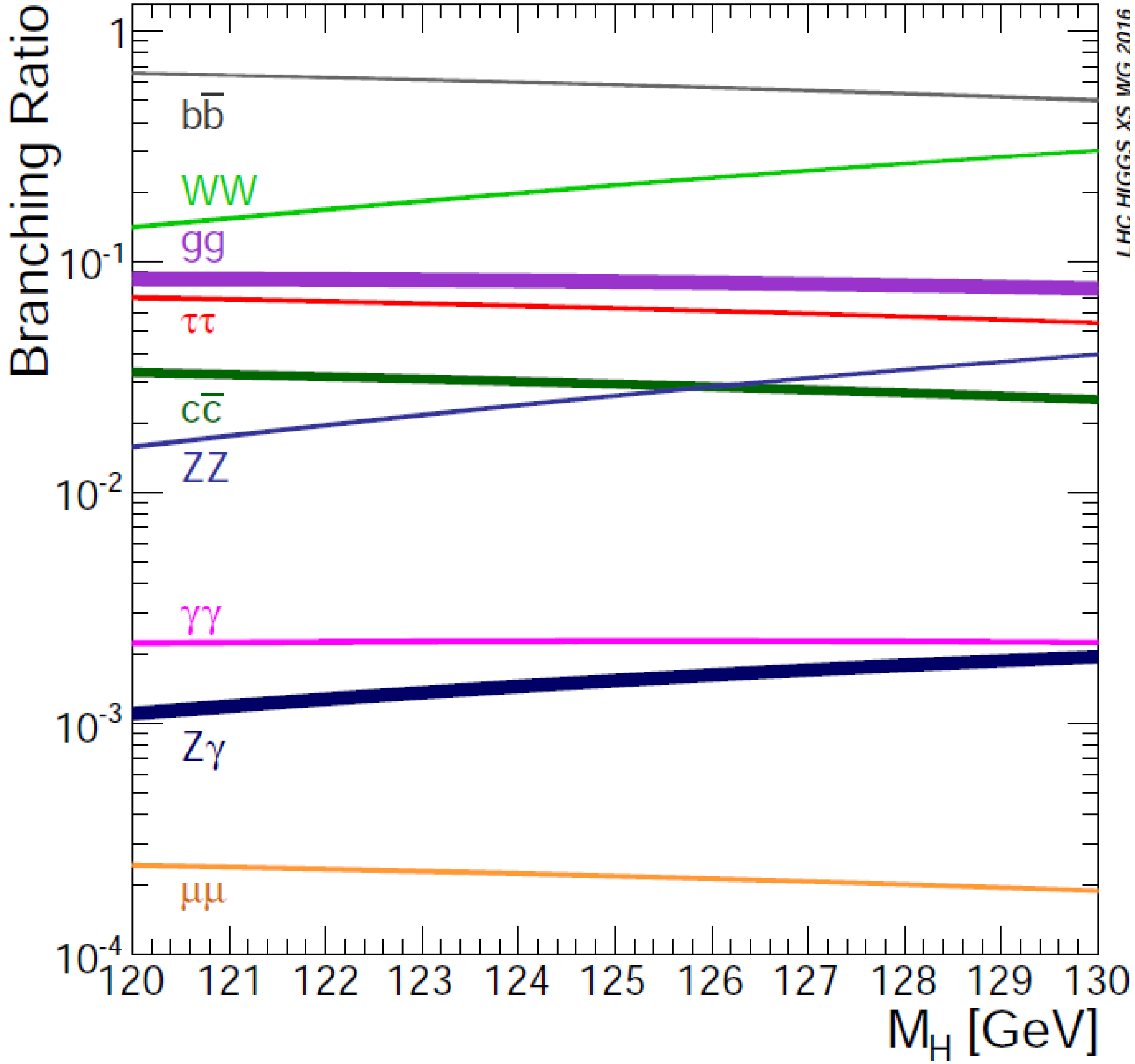


1. Introduction

LHC Higgs Cross Section Working Group
arXiv: 1610.07922

The main decay channels for Higgs boson

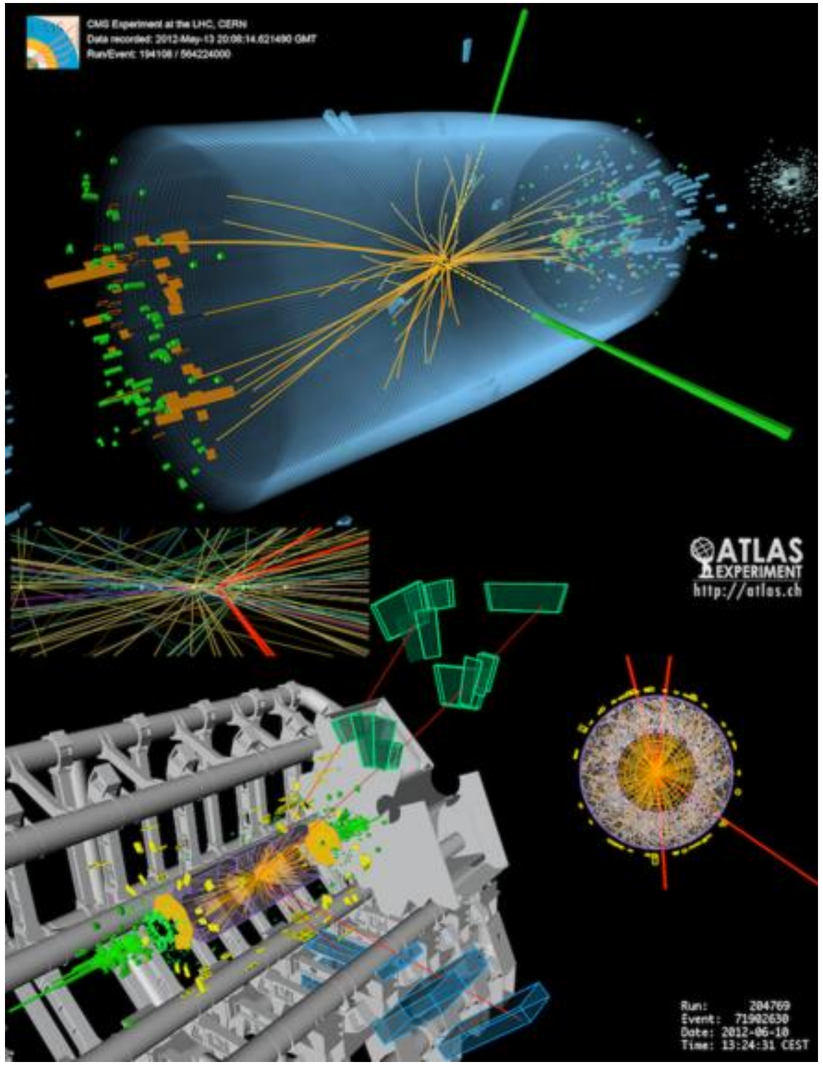


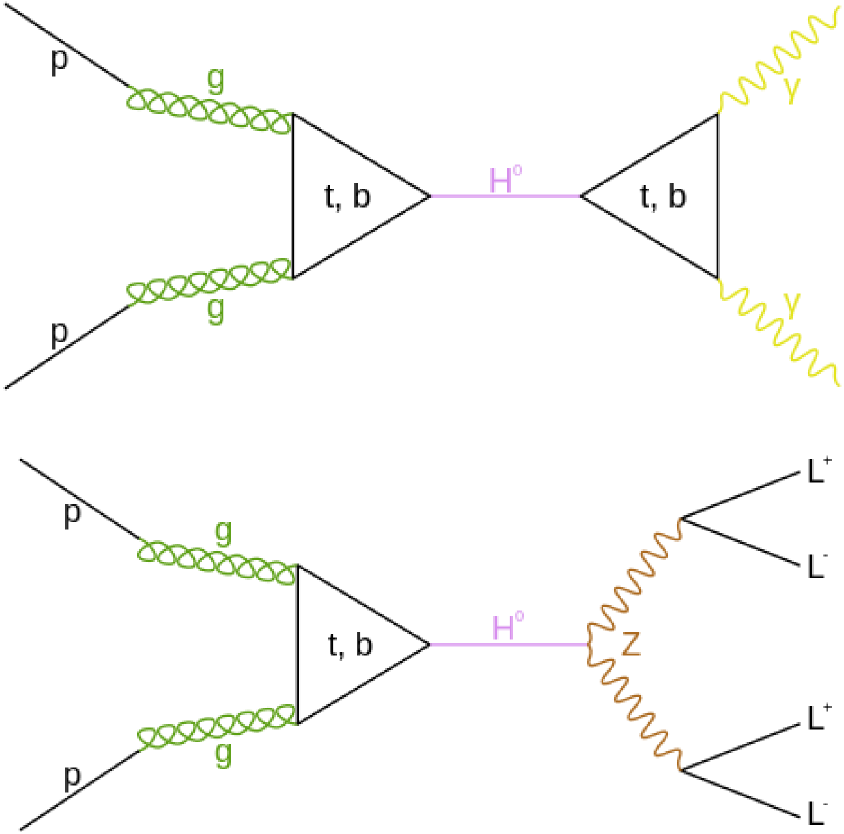


1. Introduction

Discovery a particle

On 4 July 2012, Higgs boson has been independently found by both CMS and ATLAS detectors at LHC.



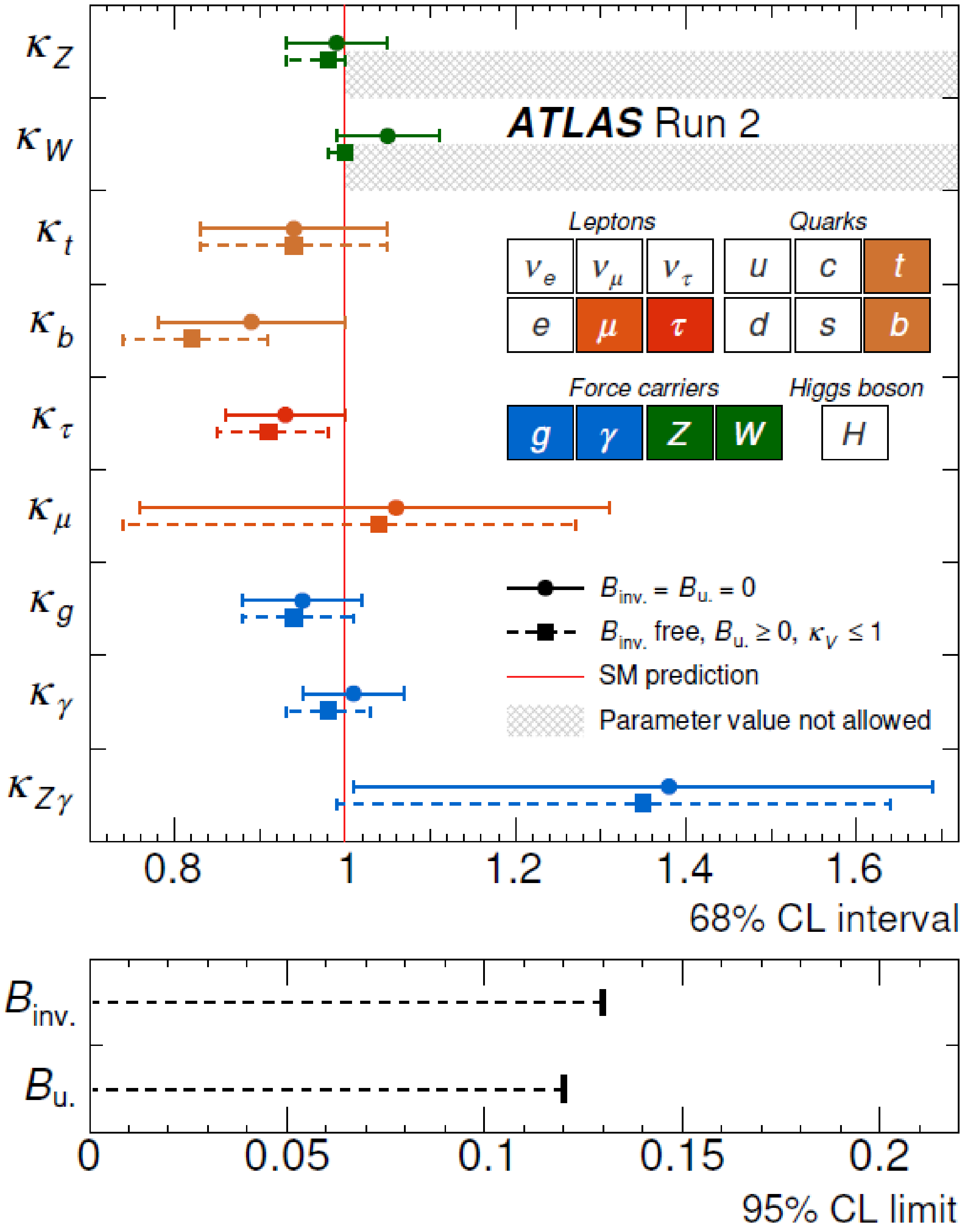


Feynman diagrams observed by ATLAS and CMS at the LHC

Is this new particle the Higgs boson predicted by SM?

1)Since the discovery, the particle has been shown to behave, interact, and decay in many of the ways predicted for Higgs particles by the SM

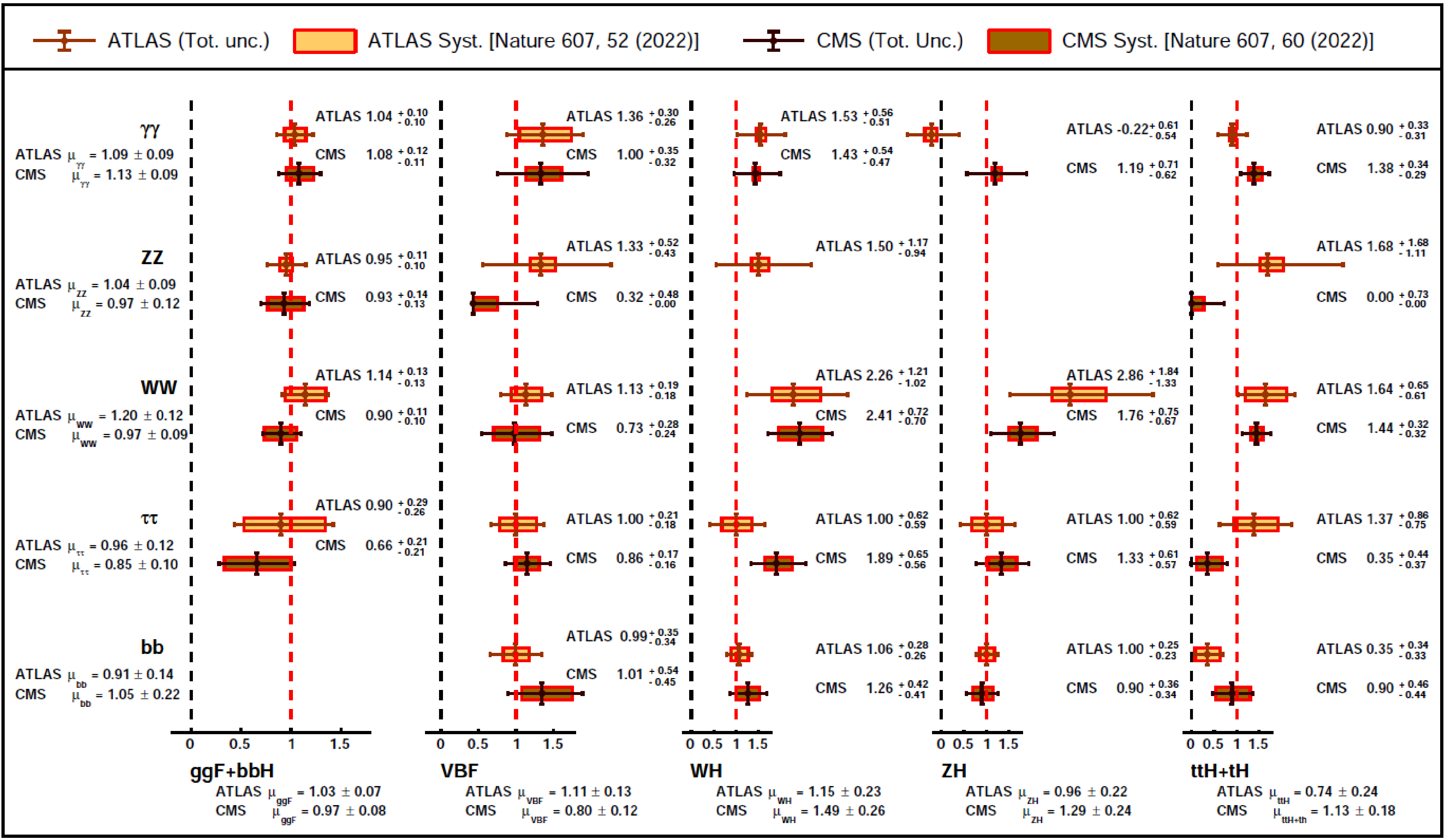
2) In-depth research shows the particle continuing to **behave in line with predictions for the SM Higgs boson**



From PDG2023

	LHC Run 1	ATLAS Run 2	CMS Run 2	HL-LHC (expected)
κ_γ	$0.87^{+0.14}_{-0.09}$	1.01 ± 0.06	1.10 ± 0.08	1.8%
κ_W	$0.87^{+0.13}_{-0.09}$	1.05 ± 0.06	1.02 ± 0.08	1.7%
κ_Z	-0.98 ± 0.10	0.99 ± 0.06	1.04 ± 0.07	1.5%
κ_g	$0.78^{+0.13}_{-0.10}$	0.95 ± 0.07	0.92 ± 0.08	2.5%
κ_t	$1.40^{+0.24}_{-0.21}$	0.94 ± 0.11	1.01 ± 0.11	3.4%
κ_b	$0.49^{+0.27}_{-0.15}$	0.89 ± 0.11	0.99 ± 0.16	3.7%
κ_τ	$0.84^{+0.15}_{-0.11}$	0.93 ± 0.07	0.92 ± 0.08	1.9%
κ_μ	–	$1.06^{+0.25}_{-0.30}$	1.12 ± 0.21	4.3%
$\kappa_{Z\gamma}$	–	$1.38^{+0.31}_{-0.36}$	1.65 ± 0.34	9.8%

κ_i denotes the ratios of the couplings of the Higgs bosons to **particle *i*** to their corresponding SM values



From PDG2023

Combined measurements by ATLAS and CMS of the

$$\frac{(\sigma \mathrm{Br})_{\mathrm{exp}}}{(\sigma \mathrm{Br})_{\mathrm{th}}}$$

signal strength Run I $\mu = 1.09 \pm 0.11 = 1.09 \pm 0.07\ (\mathrm{stat.})\ \pm 0.04\ (\mathrm{expt.})\ \pm 0.03\ (\mathrm{th.\ bkg.})\ \pm 0.07\ (\mathrm{th.\ sig.})$

ATLAS Run-2 $\mu = 1.05 \pm 0.06 = 1.05\ \pm 0.03\ (\mathrm{stat.}) \pm 0.03\ (\mathrm{exp.})\ \pm 0.02\ (\mathrm{th.\ bkg.})\ \pm 0.04\ (\mathrm{th.\ sig.})$

Despite the great success of the SM, it is widely believed that the SM is not a complete theory!
Some notable shortcomings include:

(a) Gravity
(b) Dark matter
(c) Matter-antimatter asymmetry
(d) Neutrino oscillations
……

More studies are needed to verify with higher precision that the discovered particle has all of the properties predicted or whether, as described by some theories, multiple Higgs bosons exist.

To more precisely test the SM, we need to improve the precision both in theory and experiment.

For Higgs boson decay

Decay channel	Branching ratio	Rel. uncertainty
$H \to \gamma\gamma$	2.27×10^{-3}	2.1%
$H \to ZZ$	2.62×10^{-2}	$\pm 1.5\%$
$H \to W^+W^-$	2.14×10^{-1}	$\pm 1.5\%$
$H \to \tau^+\tau^-$	6.27×10^{-2}	$\pm 1.6\%$
$H \to b\bar{b}$	5.82×10^{-1}	$^{+1.2\%}_{-1.3\%}$
$H \to c\bar{c}$	2.89×10^{-2}	$^{+5.5\%}_{-2.0\%}$
$\boxed{H \to Z\gamma}$	$\boxed{1.53 \times 10^{-3}}$	$\boxed{\pm 5.8\%}$
$H \to \mu^+\mu^-$	2.18×10^{-4}	$\pm 1.7\%$

The branching ratios and the relative uncertainty for a SM Higgs boson with M_H=125 GeV.

The uncertainties in the branching ratios include:

1) **the missing higher-order corrections** in the theoretical calculations

2) **the errors in the SM input parameters**, in particular fermion masses and the QCD gauge coupling, involved in the decay

The total width of a 125 GeV SM Higgs boson is $\Gamma_H = 4.07 \times 10^{-3}$ MeV, with a relative uncertainties of $^{4.0\%}_{-3.9\%}$.

1. Introduction

LHC Higgs Cross Section Working Group
arXiv: 1610.07922

Uncertainties from **missing higher orders**

Table 4: Estimated theoretical uncertainties from missing higher orders.

Partial width	QCD	electroweak	total
$H \to b\bar{b}/c\bar{c}$	$\sim 0.2\%$	$\sim 0.5\%$ for $M_H < 500$ GeV	$\sim 0.5\%$
$H \to \tau^+\tau^-/\mu^+\mu^-$		$\sim 0.5\%$ for $M_H < 500$ GeV	$\sim 0.5\%$
$H \to t\bar{t}$	$\lesssim 5\%$	$\sim 0.5\%$ for $M_H < 500$ GeV	$\sim 5\%$
$H \to gg$	$\sim 3\%$	$\sim 1\%$	$\sim 3.2\%$
$H \to \gamma\gamma$	$< 1\%$	$< 1\%$	$\sim 1\%$
$H \to Z\gamma$	$< 1\%$	$\sim 5\%$	$\sim 5\%$
$H \to WW/ZZ \to 4f$	$< 0.5\%$	$\sim 0.5\%$ for $M_H < 500$ GeV	$\sim 0.5\%$

1. Introduction

The current theoretical computation for $H \to Z\gamma$

Leading order evaluation (one loop):

Cahn, Chanowitz, Fleishon, PLB (1979); Bergstrom, Hulth, NPB (1985)

QCD correction (two loop):

Spira, Djouadi, Zerwas, PLB (1992)   Numerically computed

Gehrmann, Guns, Kara, JHEP (2015)   Analytically computed

Bonciani, Del Duca, Frellesvig, Henn, Moriello, Smirnov, JHEP(2015)   Analytically computed

Electroweak correction (two loop):

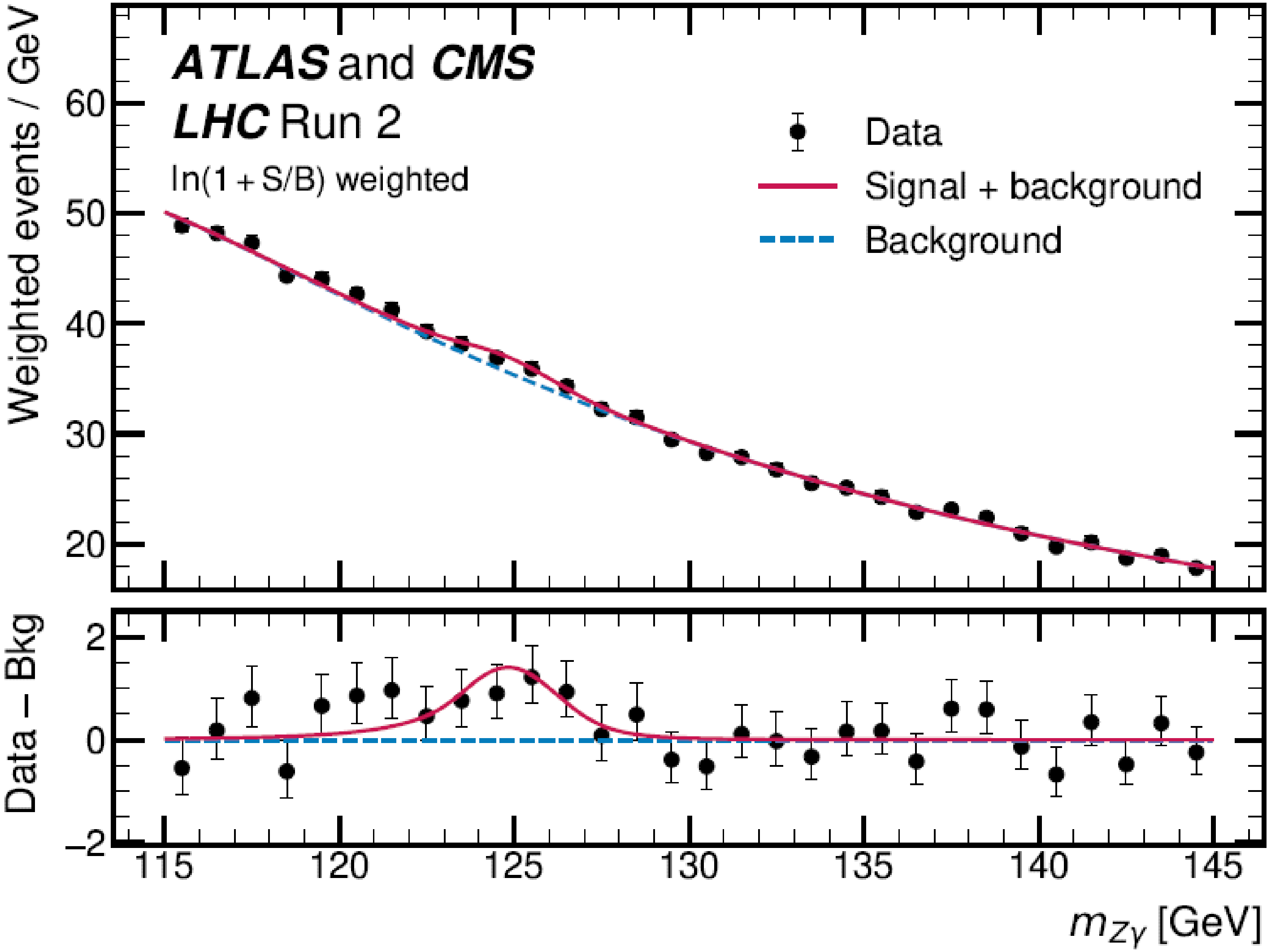
Currently, it is still absent.

Our aim is to compute this missing piece.

1. Introduction

ATLAS & CMS, PRL2024

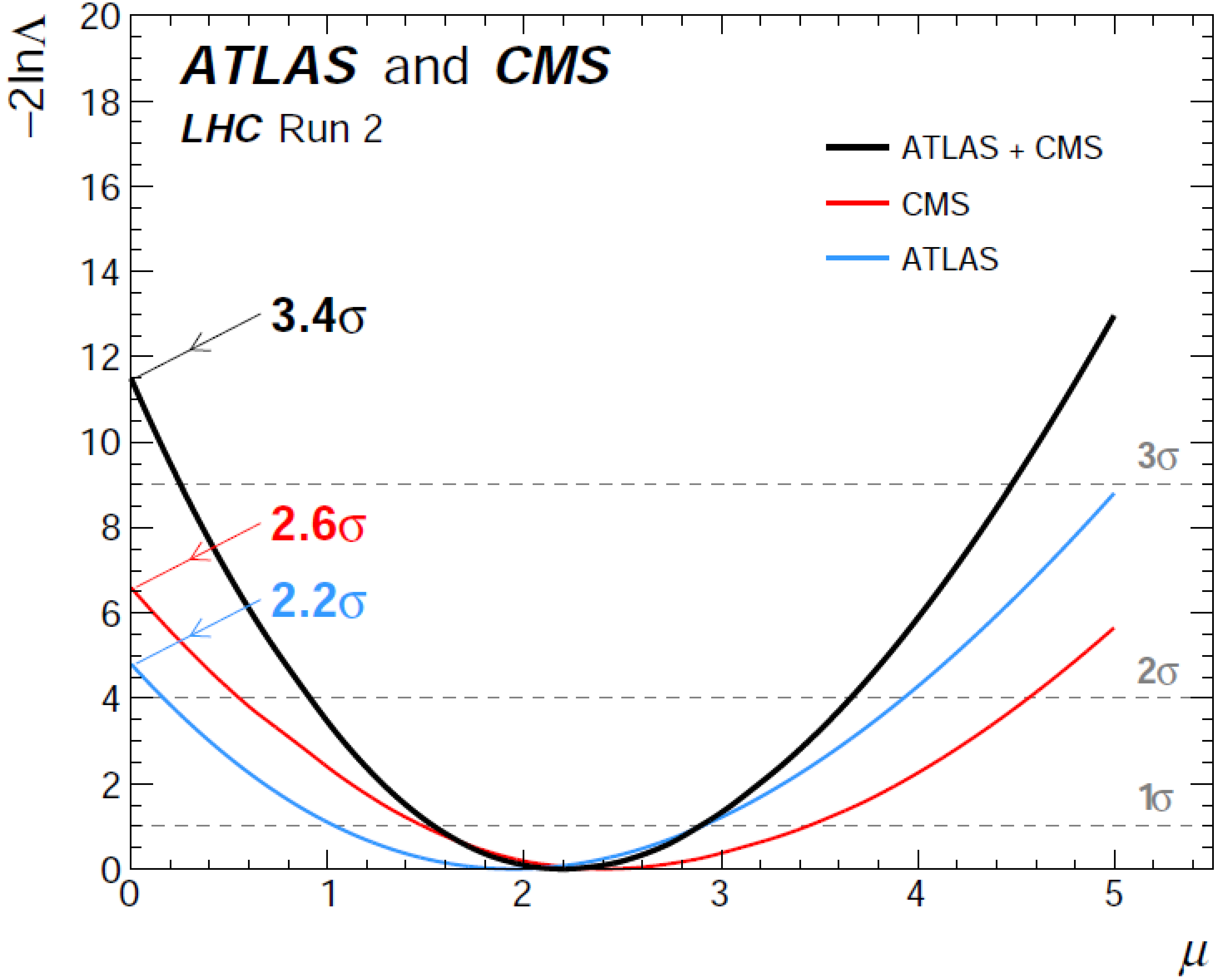
Combined analysis of the searches performed by the ATLAS and CMS Collaborations from 2015 to 2018, with integrated luminosity 140 fb^{-1}



Evidence for H→Zγ decay is established:

$$\boxed{Br_{\exp}[H \to Z\gamma] = (3.4 \pm 1.1) \times 10^{-3}}$$

with an observed significance of 3.4 standard deviations



$$Br_{\rm exp}[H \to Z\gamma] = (3.4 \pm 1.1) \times 10^{-3}$$

The SM prediction is

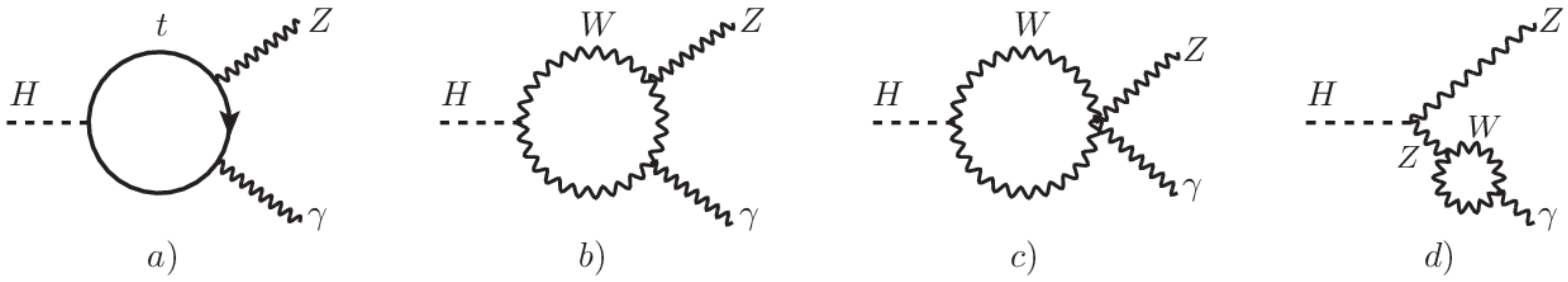
$$Br_{\rm th}[H \to Z\gamma] = (1.5 \pm 0.1) \times 10^{-3}$$

The observed signal yield is **2.2 ± 0.7** times the SM prediction

The result agrees with the SM prediction within 1.9 standard deviations

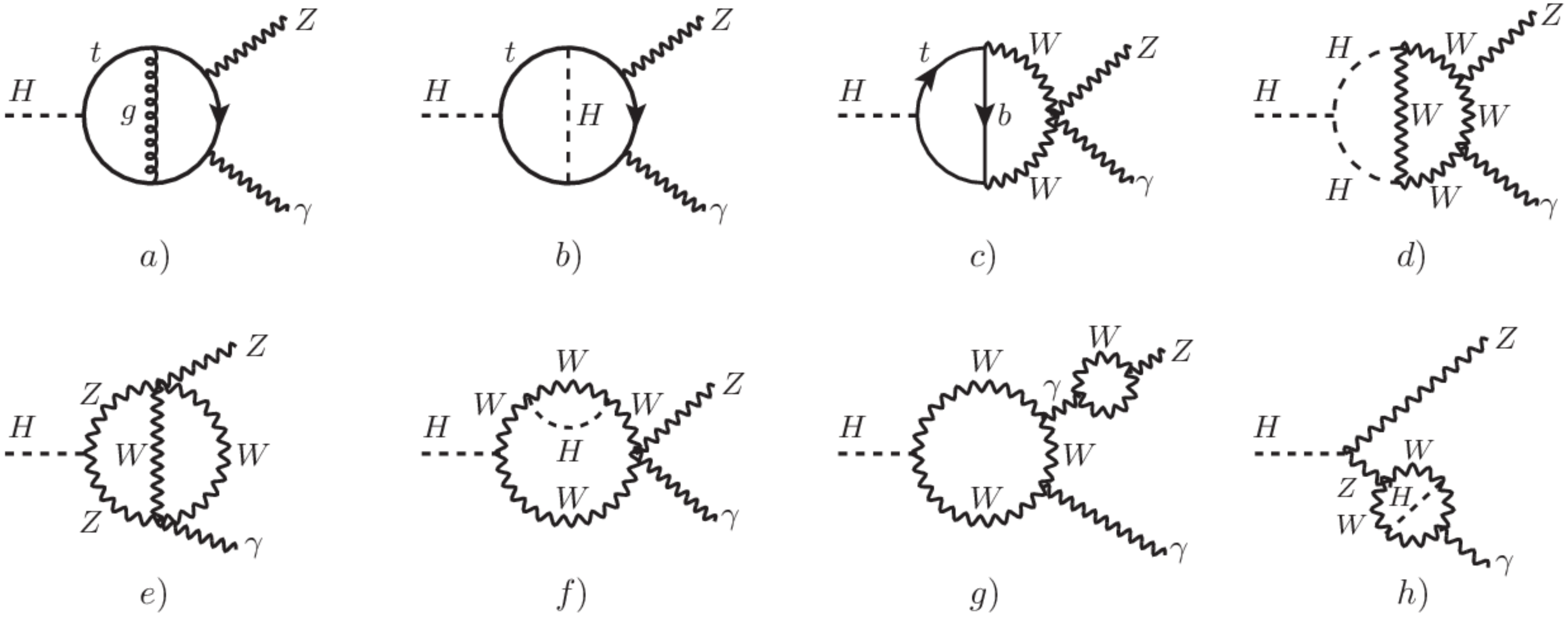
Improved theoretical prediction may not reduce the experimental deviation from the SM expectations.
Nonetheless, as experimental uncertainties are reduced, precise theoretical predictions become increasingly essential for discerning potential new physics beyond the SM.

2. Outline of calculation



About **50** diagrams at LO

Some representative **Leading Order** Feynman diagrams for $H \to Z\gamma$



About **10,000** diagrams at NLO

Some representative **Next-to-Leading Order** Feynman diagrams

Renormalization treatment

Denner, arXiv:0709.1075

In this work, we adopt **on-shell renormalization scheme**:

1) The **finite renormalized parameters are equal to the physical parameters** at all orders of perturbation theory.
2) So all parameters are endowed with **a clear physical interpretation** and can be **directly measured** in appropriate experimental setups.

Renormalization treatment

Denner, arXiv:0709.1075

on-shell renormalization scheme:

It is convenient to select **the masses of physical particles** as well as **the charge** of the electron ***e***, as our renormalization parameters.

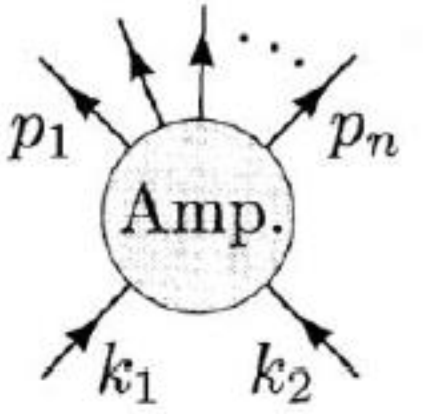
Specifically, we use the **multiplicative renormalization constants** to relate the bare quantities and the renormalized counterparts

$$M_{H,0} = Z_{M_H} M_H\,,\ M_{W,0} = Z_{M_W} M_W\,,\ M_{Z,0} = Z_{M_Z} M_Z\,,$$
$$M_{t,0} = Z_{M_t} M_t,\ e_0 = Z_e e.$$

Renormalization treatment

LSZ reduction formula

$$\langle \mathbf{p}_1 \ldots \mathbf{p}_n | \, S \, | \mathbf{k}_1 \mathbf{k}_2 \rangle = (\sqrt{Z})^{n+2}$$



Specifically, for the process $H \to Z + \gamma$, we have

$$\mathcal{A}_{\text{finite}} \quad = \quad Z_H^{1/2} Z_{ZZ}^{1/2} Z_{\gamma\gamma}^{1/2} \mathcal{A}_{\overline{\text{Amp}}}(M_{W,0}, M_{Z,0}, M_{t,0}, e_0)$$

where the Z_H, Z_{ZZ} and $Z_{\gamma\gamma}$ denote the field-strength renormalization constants associated with Higgs boson, Z boson and photon, respectively.

Renormalization treatment

Specifically, for the process $H \to Z + \gamma$, we have

$$\mathcal{A}_{\text{finite}} = Z_H^{1/2} Z_{ZZ}^{1/2} Z_{\gamma\gamma}^{1/2} \mathcal{A}_{\overline{\text{Amp}}}(M_{W,0}, M_{Z,0}, M_{t,0}, e_0)$$

Attention:

1) If we **use bare fields for computation**, it is not necessary to include counter terms.

2) When **using renormalized fields for computation**, the renormalization constants $Z_H = Z_{ZZ} = Z_{\gamma\gamma} = 1$. However, as a price, we must include counter terms, which is quite complicated in SM !

Three different charge renormalization (α-scheme)

$$\alpha_0 = Z\big|_{\alpha(0)}\alpha(0) = Z\big|_{\alpha(M_Z)}\alpha(M_Z) = Z\big|_{\alpha_{G_\mu}}\alpha_{G_\mu}$$

α(0) scheme **α(M_Z) scheme** **Gμ scheme**

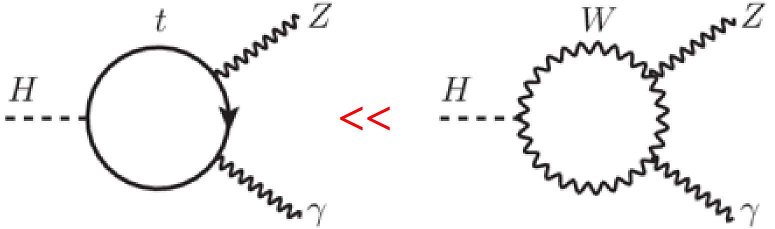
3. Phenomenlogical discussion

$$\mathcal{B}(H \to Z\gamma) = 1.55 \pm 0.06 \pm 0.02 \times 10^{-3}$$

Our key results

	α assignment at LO	Γ^{LO}	$\Gamma^{\mathcal{O}(\alpha)}$	$\Gamma^{\mathcal{O}(\alpha_s)}$	Γ^{Sum}	$\mathcal{B}(\times 10^{-3})$
$\alpha(0)$ scheme	$\alpha^3(0)$	5.921	0.439	0.019	6.379	1.57 ± 0.06
$\alpha(M_Z)$ scheme	$\alpha(0)\alpha^2(M_Z)$	6.689	-0.464	0.021	6.245	1.53 ± 0.06
G_μ scheme	$\alpha(0)\alpha^2_{G_\mu}$	6.365	-0.048	0.020	6.337	1.56 ± 0.06
Democratic scheme	$\alpha(0)\alpha(M_Z)\alpha_{G_\mu}$	6.525	-0.249	0.021	6.297	1.55 ± 0.06

TABLE I:



1) QCD correction is tiny, **0.3%** of the LO partial width

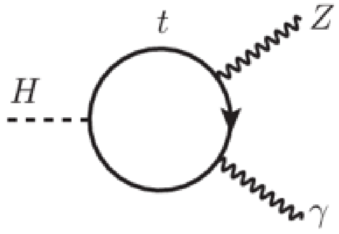
2) EW correction is sizable, may approach **7%** (depending on α scheme)

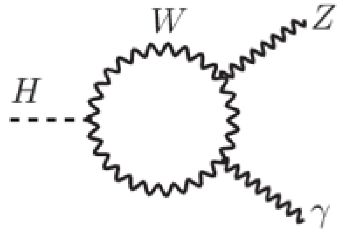
3) α-scheme dependence is significant at LO, **howerver becomes substantially reduced at NLO**!

4) The uncertainties **are primarily due to the uncertainty in the full width of the Higgs boson** and amount to only 2% among different α schemes

3. Phenomenlogical discussion

Potential sources of uncertainty





1) From the bottom quark mass

The contributions from the top quark loop constitute approximately -10% of the LO decay width. The Yukawa coupling strength of $Hb\bar{b}$ is suppressed with respect to $Ht\bar{t}$ by a factor of $m_b/m_t \approx 3\%$, and we estimate that **retaining the bottom quark mass** introduces a relative error of **several per mille**.

2) Uncertainty from $M_W = 80.377 \pm 0.012\ \mathrm{GeV}$ and $M_t = 172.69 \pm 0.30\ \mathrm{GeV}$
may also introduce uncertainty of several per mille

3. Phenomenlogical discussion

Compared with the results presented in **arXiv:2404.11441**, a similar work conducted by Qiao's group.

Two-loop Electroweak corrections to the Higgs boson rare decay process $H \to Z\gamma$

Zi-Qiang Chen, Long-Bin Chen, Cong-Feng Qiao, Ruilin Zhu (Apr 17, 2024)

e-Print: 2404.11441 [hep-ph]

scheme	input parameters	Γ^{LO}[keV]	$\Gamma^{\mathrm{NLO}}_{\mathrm{EW}}$[keV]	δ_{EW} (%)
$\alpha(0)$	$\alpha(0)$, m_f	5.920	6.234	5.3
$\alpha(m_Z^2)$	$\alpha(m_Z^2)$, m_f	7.273	6.303	-13
G_μ	G_μ, m_f	6.599	6.343	-3.9
mixed 1	$\alpha(0)$, $\alpha(m_Z^2)$	6.791	6.316	-7.0
mixed 2	$\alpha(0)$, G_μ	6.364	6.316	-0.75

$$\Gamma^{\mathrm{NLO}} = \Gamma^{\mathrm{NLO}}_{\mathrm{EW}} + \Gamma^{\mathrm{LO}}(1+\delta_b)(1+\delta_{\mathrm{QCD}}) - \Gamma^{\mathrm{LO}}$$
$$= 6.348^{+0.028}_{-0.085}\ \mathrm{keV}.$$

$$\mathrm{Br}(H \to Z\gamma) = 1.56^{+0.01}_{-0.02} \times 10^{-3}$$

1) Our results for the $\alpha(M_Z)$ and $\boldsymbol{G_\mu}$ schemes **are compatible with** their mixed 1 and mixed 2 schemes, provided that **the same input parameters** are used.
2) However, our result for the **$\alpha(0)$ scheme** slightly **differs from theirs**, probably due to the different treatment of the light quark contribution to Ze.

4. Summary

1) In on-shell renormalization scheme, we compute **the NLO EW correction** to the process $H \to Z+\gamma$

2) We **present the results using three different α schemes**, with the relative uncertainties among these schemes less than 2%

3) The EW correction turns out to **be substantial**, reaching up to 7% in the $\alpha(0)$ scheme, and is significantly larger than the QCD correction

4) Our most refined prediction is $\mathcal{B}(H \to Z\gamma) = 1.55 \pm 0.06 \pm 0.02 \times 10^{-3}$

5) Although the improved theoretical prediction may not reduce the experimental deviation from the SM expectations, precise theoretical predictions may become increasingly essential for discerning potential new physics beyond the SM

Thank you for your attention！

Backup slide

Current Status of Theoretical Computation

$H \to b\bar{b}/c\bar{c}$ massless QCD corrections up to N4LO + EW corrections up to NLO

$H \to gg$ QCD corrections up to N3LO in the limit of heavy top quarks + EW corrections up to NLO

$H \to \gamma\gamma$ QCD corrections up to N2LO + EW corrections up to NLO

$H \to Z\gamma$ QCD corrections up to NLO

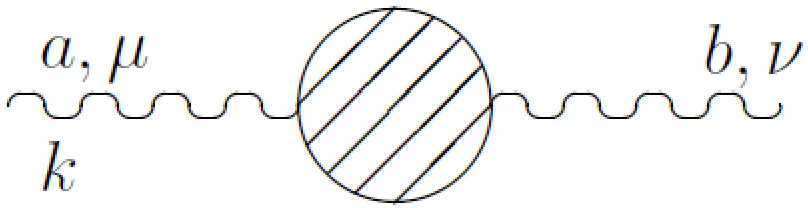
$H \to WW/ZZ \to 4f$ QCD corrections up to NLO + EW corrections up to NLO

Three different charge renormalization (α-scheme)

Implementing the charge renormalization in electroweak theory is not unique.

1) **α(0) scheme**

$$\delta Z_e|_{\alpha(0)} = \frac{1}{2}\Pi^{\gamma\gamma}(0) - \frac{s_W}{c_W}\frac{\Sigma_T^{\gamma Z}(0)}{M_Z^2}$$



The photon vacuum polarization is sensitive to the low-energy hadronic contribution thereby an intrinsically non-perturbative quantity.

$$\delta Z_e\Big|_{\alpha(0)} = \frac{1}{2}\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z) + \frac{1}{2}\mathrm{Re}\,\Pi^{\gamma\gamma(5)}(M_Z^2) + \frac{1}{2}\Pi^{\gamma\gamma}_{\mathrm{rem}}(0) - \frac{s_W}{c_W}\frac{\Sigma_T^{\gamma Z}(0)}{M_Z^2}$$

The first term can be determined from the measured ***R* values** in low-energy e^+e^- experiments

Three different charge renormalization (α-scheme)

2) $\alpha(M_Z)$ scheme

$$\delta Z_e\Big|_{\alpha(M_Z)} = \delta Z_e\Big|_{\alpha(0)} - \frac{1}{2}\Delta\alpha(M_Z)$$

$$\alpha\left(M_Z\right) = \frac{\alpha(0)}{1-\Delta\alpha\left(M_Z\right)},$$

3) G_μ scheme

$$\alpha_{G_\mu} = \frac{\sqrt{2}}{\pi} G_\mu M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right).$$

$$\delta Z_e\big|_{G_\mu} = \delta Z_e\big|_{\alpha(0)} - \frac{1}{2}\Delta r$$

Attention: **$\alpha(M_Z)$ and G_μ schemes are free of masses of the light fermions!**

Computing flowchart

8 one-loop MIs + about 700 two-loop MIs

Feynarts → FeynCalc/FormLink → Apart/FIRE

50 one-loop + about 10,000 two-loop Feynman diagrams

↓

AMFlow

↓

Renormalization

↓

In several α schemes

Finite form factor/ partial width

3. Phenomenlogical discussion

Our key results

	α assignment at LO	Γ^{LO}	$\Gamma^{\mathcal{O}(\alpha)}$	$\Gamma^{\mathcal{O}(\alpha_s)}$	Γ^{Sum}	$\mathcal{B}(\times 10^{-3})$
$\alpha(0)$ scheme	$\alpha^3(0)$	5.921	0.439	0.019	6.379	1.57 ± 0.06
$\alpha(M_Z)$ scheme	$\alpha(0)\alpha^2(M_Z)$	6.689	-0.464	0.021	6.245	1.53 ± 0.06
G_μ scheme	$\alpha(0)\alpha^2_{G_\mu}$	6.365	-0.048	0.020	6.337	1.56 ± 0.06
Democratic scheme	$\alpha(0)\alpha(M_Z)\alpha_{G_\mu}$	6.525	-0.249	0.021	6.297	1.55 ± 0.06

TABLE I: Predicted values of the partial width (in units of keV) for $H \to Z\gamma$ in various α schemes, at various levels of perturbative accuracy. Γ^{LO} represents the LO prediction to the decay width. The contributions from the NLO electroweak and QCD corrections are denoted by $\Gamma^{\mathcal{O}(\alpha)}$ and $\Gamma^{\mathcal{O}(\alpha_s)}$, respectively. We have taken the strong coupling constant $\alpha_s(M_H) = 0.115$.

1) The main parameters are obtained from the PDG, including various masses, the fine structure constant, etc;

2) $\alpha(M_Z)$ =1/128.932, taken from theoretical value(on-shell scheme);

3) Uncertainties of the Experimental measurement $\Gamma_H = 3.2^{+2.4}_{-1.7}$ MeV is large, we take the theoretical value $\Gamma_H = 4.07^{+4.0\%}_{-3.9\%}$ MeV