Wess-Zumino-Witten Interactions of Axions: A Consistent Chiral Lagrangian

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Outlines

- Axion effective and consistent ChPT Lagrangian
- WZW in QCD and counter-terms
- Full WZW interactions for axions
- Phenomenology at BESIII and STCF
- Summary

The QCD axion and the Strong CP problem

$$\mathscr{L} \supset -\frac{\theta g_s^2}{32\pi^2} G\tilde{G} - \left(\bar{u}_L M_u u_R + \bar{d}_L M_d d_R + \mathrm{h.c.}\right)$$

- The CKM matrix from $M_{u,d}$
  - CP violating phase θ_{CP} ~ 1.2 radian
- QCD induced CP violating phase, $\bar{\theta}$

$$\bar{\theta} = \theta + \arg\left[\det\left[M_u M_d\right]\right]$$

- $\bar{\theta}$ is invariant under quark chiral rotation

$$d^n_{\mathrm{EDM}} \sim \theta \times 10^{-16}\ \mathrm{e\ cm}$$

- According to neutron EDM experiment

$$d^n_{\exp} < 10^{-26}\ \mathrm{e\ cm}$$

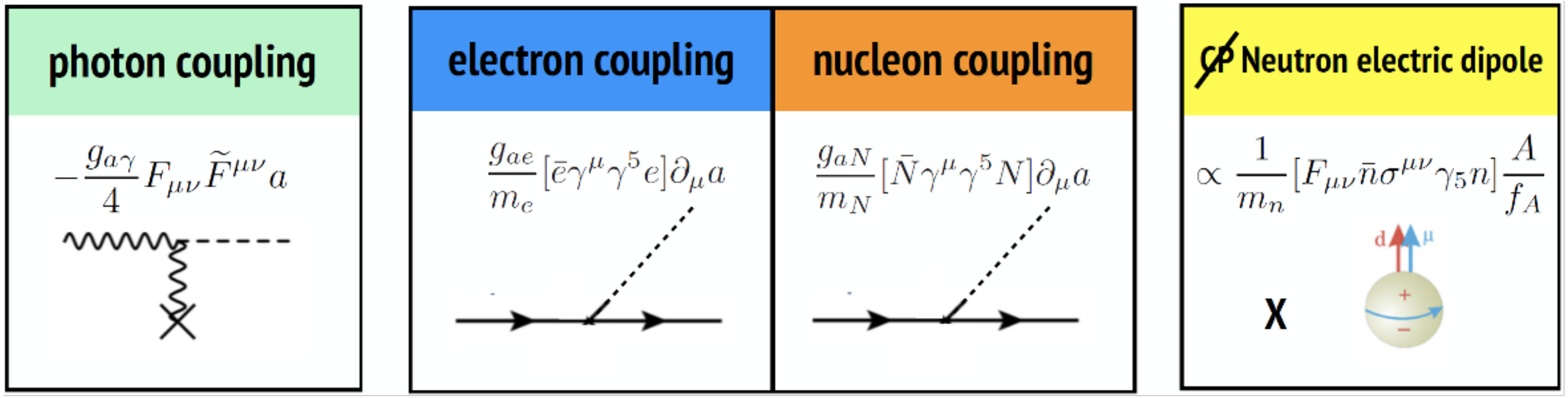
$$\bar{\theta} \lesssim 1.3 \times 10^{-10}\ \mathrm{radian}$$

The axion effective Lagrangian at quark-level

- Axion can couple to SM gauge bosons and fermions

$$\mathscr{L}_{\rm ALP} = g_{ag}\frac{a}{f_a}G\tilde{G} + g_{a\gamma}\frac{a}{f_a}F\tilde{F} + g_{af}\frac{\partial_\mu a}{2f_a}\bar{f}\gamma^\mu\gamma_5 f$$

- Detection of axion through various couplings



The axion effective Lagrangian at quark-level

- A more detailed effective Lagrangian

$$\mathscr{L}_{\text{eff},0} = \bar{q}_0(iD_\mu\gamma^\mu - \mathbf{m}_{q,0})q_0 + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2$$
$$+ g_{ag,0}\frac{a}{f_a}G\tilde{G} + g_{a\gamma,0}\frac{a}{f_a}F\tilde{F} + \frac{\partial_\mu a}{f_a}\left(\bar{q}_L\mathbf{k}_{L,0}\gamma^\mu q_L + \bar{q}_R\mathbf{k}_{R,0}\gamma^\mu q_R + \ldots\right)$$

Bauer et al, PRL 127 (2021), 081803

- Quark mass $\mathbf{m}_{q,0}$ diagonal and real

- Coupling to both left/right fermions $\mathbf{k}_{L,0}$ and $\mathbf{k}_{R,0}$

The axion-dependent chiral rotation

- Use an axion-dependent chiral rotation to eliminate $aG\tilde{G}$ term

Bauer et al, PRL 127 (2021), 081803

$$q_0(x) = \exp\left[-i(\boldsymbol{\delta}_{q,0} + \boldsymbol{\kappa}_{q,0}\gamma_5)c_{gg}\frac{a(x)}{f_a}\right] q(x) \qquad \mathrm{Tr}(\kappa_{q,0}) = 1$$

- New effective Lagrangian

$$\mathscr{L}_{\text{eff}} = \bar{q}(iD_\mu\gamma^\mu - \mathbf{m}_q(a))q + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2$$

$$+ g_{a\gamma}\frac{a}{f_a}F\tilde{F} + \frac{\partial_\mu a}{f_a}\left(\bar{q}_L\mathbf{k}_L(a)\gamma^\mu q_L + \bar{q}_R\mathbf{k}_R(a)\gamma^\mu q_R + \ldots\right)$$

The axion-dependent chiral rotation

- Define the chiral rotations (2-flavor for simplicity)

$$\boldsymbol{\theta}_L \equiv \boldsymbol{\delta}_{q,0} - \boldsymbol{\kappa}_{q,0} \qquad U_L \equiv \exp\left[-i\boldsymbol{\theta}_L a/f_a\right]$$

$$\boldsymbol{\theta}_R \equiv \boldsymbol{\delta}_{q,0} + \boldsymbol{\kappa}_{q,0} \qquad U_R \equiv \exp\left[-i\boldsymbol{\theta}_R a/f_a\right]$$

- The relations between parameters

$$\mathbf{m}_q(a) = U_L^\dagger \mathbf{m}_0 U_R \to \begin{pmatrix} m_{u,0}e^{-2i\kappa_{u,0}c_{gg}} & 0 \\ 0 & m_{d,0}e^{-2i\kappa_{d,0}c_{gg}} \end{pmatrix}$$

$$\mathbf{k}_L(a) = U_L^\dagger[\mathbf{k}_{L,0} + c_{gg}\boldsymbol{\theta}_{L,0}]U_L \to \mathbf{k}_{L,0} + c_{gg}\boldsymbol{\theta}_{L,0}$$

$$\mathbf{k}_R(a) = U_R^\dagger[\mathbf{k}_{R,0} + c_{gg}\boldsymbol{\theta}_{R,0}]U_R \to \mathbf{k}_{R,0} + c_{gg}\boldsymbol{\theta}_{R,0}$$

Anomalous axion contribution

$$g_{a\gamma} = g_{a\gamma_0} - 2N_c c_{gg}\mathrm{Tr}\left[\mathbf{Q}^2\boldsymbol{\kappa}_{q,0}\right]$$

The consistent ChPT axion Lagrangian

$$\mathscr{L}_{\text{eff}} = \bar{q}(iD_\mu\gamma^\mu - \mathbf{m}_q(a))q + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2$$

$$+g_{a\gamma}\frac{a}{f_a}F\tilde{F} + \frac{\partial_\mu a}{f_a}\left(\bar{q}_L\mathbf{k}_L(a)\gamma^\mu q_L + \bar{q}_R\mathbf{k}_R(a)\gamma^\mu q_R + \ldots\right)$$

- ChPT Lagrangian matching

$$U = \exp[(\sqrt{2}i/f_\pi)\pi^a\boldsymbol{\tau}^a]$$

$$\mathscr{L}_{\chi\text{PT}} = \frac{f_\pi^2}{8}\left[(D^\mu U)(D_\mu U)^\dagger\right] + \frac{f_\pi^2}{4}B_0\text{Tr}\left[\mathbf{m}_q(a)U^\dagger + h.c.\right] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 + g_{a\gamma}\frac{a}{f_a}F\tilde{F}$$

- The axion derivative coupling

Bauer et al, PRL 127 (2021), 081803

$$D^\mu U \to D^\mu U - i\frac{\partial^\mu a}{f_a}\left(\mathbf{k}_L U - U\mathbf{k}_R\right)$$

The importance of consistency

- The physical results should be independent of auxiliary parameters

$$q_0(x) = \exp\left[-i(\boldsymbol{\delta}_{q,0} + \boldsymbol{\kappa}_{q,0}\gamma_5)c_{gg}\frac{a(x)}{f_a}\right] q(x)$$

- The most important channel $K \to \pi a$ BR is wrong by 37 times

H. Georgi, D. B. Kaplan and L. Randall, Phys. Lett. B 169, 73-78 (1986)

- Model-independent expression for $K \to \pi a$ and $\pi^- \to e^- \bar{\nu}_e a$ have been obtained for all axion couplings

Bauer et al, PRL 127 (2021), 081803

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Wess-Zumino-Witten Interactions in QCD

- Describing anomalies in QCD
- Ensuring gauge invariance and completing chiral L
- Low-energy dynamics of mesons
  e.g. multiple mesons and photons interactions, $\pi_0 \rightarrow \gamma\gamma$

$$\begin{aligned}\Gamma_{WZW}(U,\mathcal{A}_L,\mathcal{A}_R) = \Gamma_0(U)\ &+ \mathcal{C}\int \mathrm{Tr}\Big\{(\mathcal{A}_L\alpha^3 + \mathcal{A}_R\beta^3) - \frac{i}{2}[(\mathcal{A}_L\alpha)^2 - (\mathcal{A}_R\beta)^2] \\ &+ i(\mathcal{A}_L U\mathcal{A}_R U^\dagger\alpha^2 - \mathcal{A}_R U^\dagger\mathcal{A}_L U\beta^2) + i(d\mathcal{A}_R dU^\dagger\mathcal{A}_L U - d\mathcal{A}_L dU\mathcal{A}_R U^\dagger) \\ &+ i\big[(d\mathcal{A}_L\mathcal{A}_L + \mathcal{A}_L d\mathcal{A}_L)\alpha + (d\mathcal{A}_R\mathcal{A}_R + \mathcal{A}_R d\mathcal{A}_R)\beta\big] + (\mathcal{A}_L^3\alpha + \mathcal{A}_R^3\beta) \\ &- (d\mathcal{A}_L\mathcal{A}_L + \mathcal{A}_L d\mathcal{A}_L)U\mathcal{A}_R U^\dagger + (d\mathcal{A}_R\mathcal{A}_R + \mathcal{A}_R d\mathcal{A}_R)U^\dagger\mathcal{A}_L U \\ &+ (\mathcal{A}_L U\mathcal{A}_R U^\dagger\mathcal{A}_L\alpha + \mathcal{A}_R U^\dagger\mathcal{A}_L U\mathcal{A}_R\beta) + i\left[\mathcal{A}_L^3 U\mathcal{A}_R U^\dagger - \mathcal{A}_R^3 U^\dagger\mathcal{A}_L U - \frac{1}{2}(U\mathcal{A}_R U^\dagger\mathcal{A}_L)^2\right]\Big\}.\end{aligned}$$

$$\alpha = dUU^\dagger$$

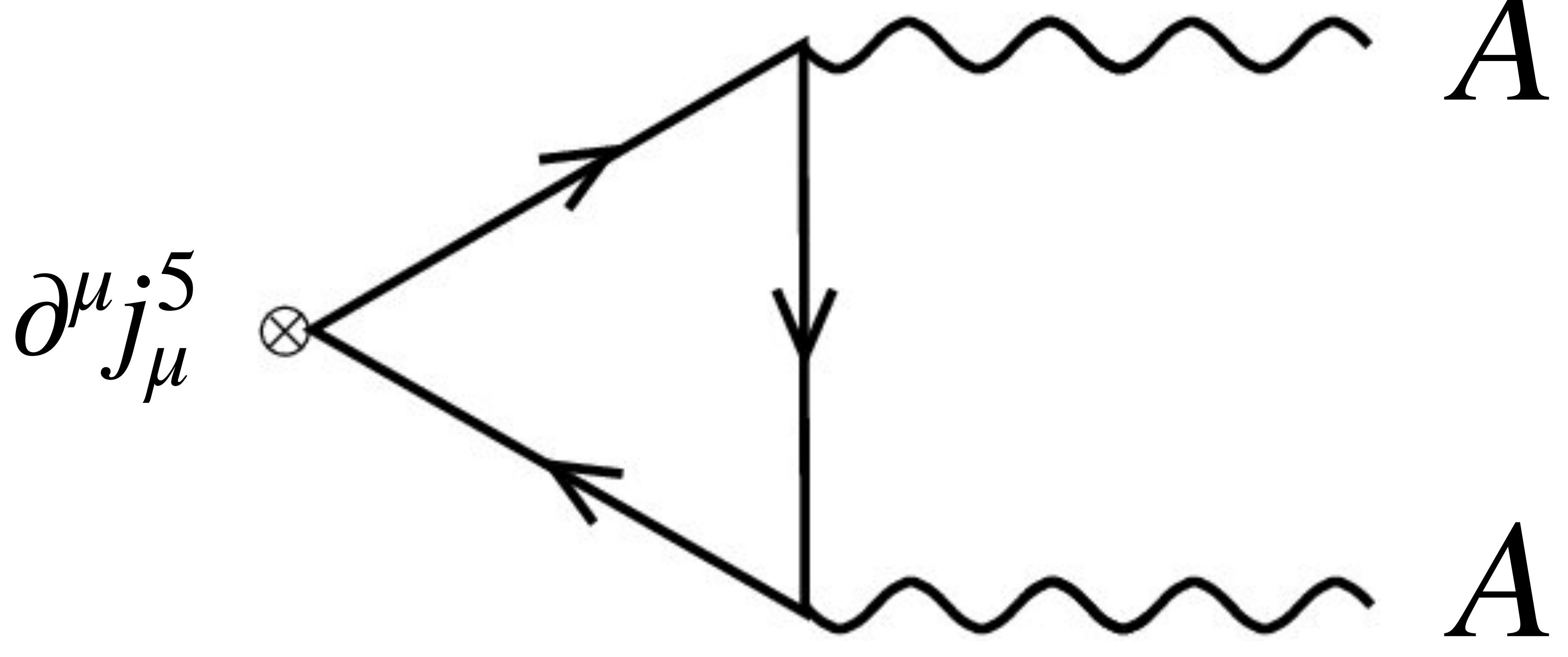
$$\beta = U^\dagger dU$$

$$\Gamma_0(U) = -\frac{i\mathcal{C}}{5}\int_{M^5}\mathrm{Tr}\left(\alpha^5\right) = \frac{iN_c}{240\pi^2}\int d^5x\,\epsilon^{ABCDE}\ \mathrm{Tr}\left(\alpha_A\alpha_B\alpha_C\alpha_D\alpha_E\right)\ ,$$

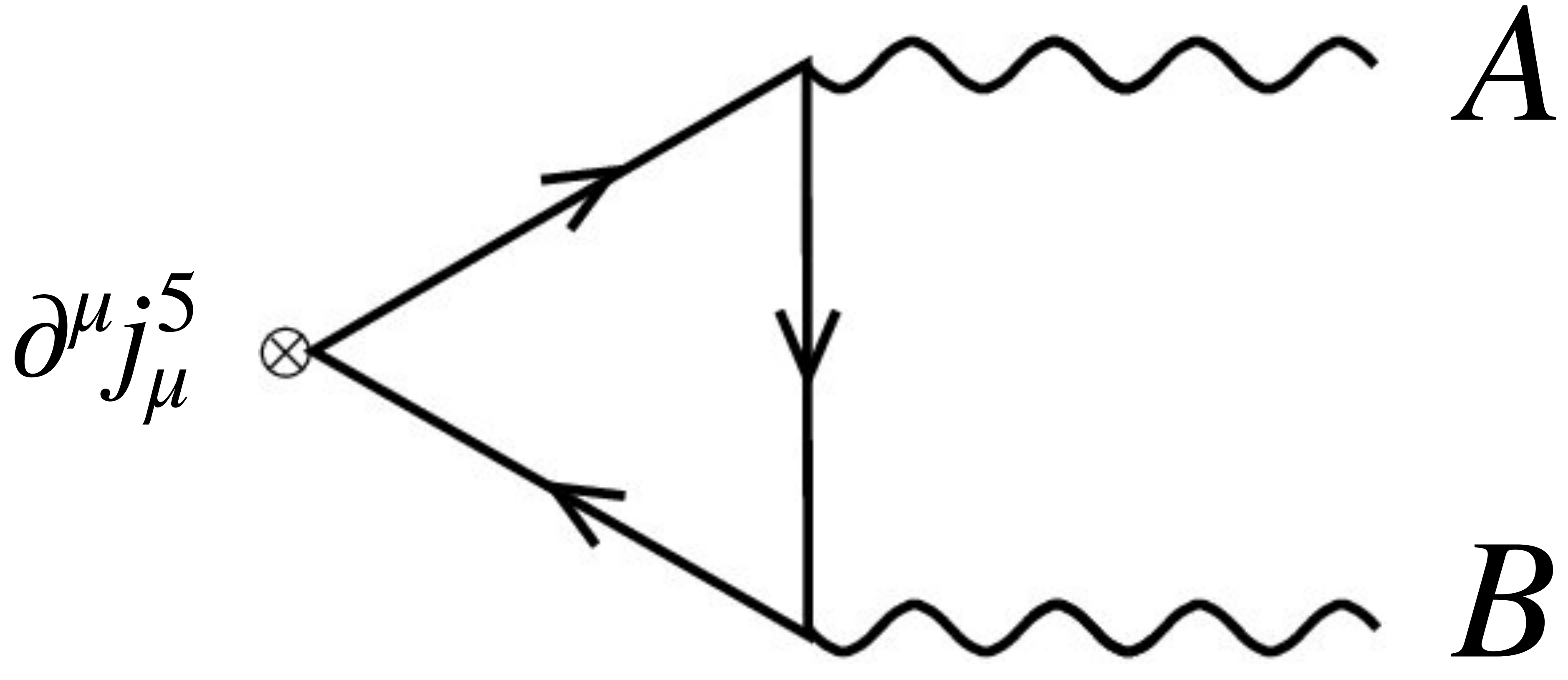
Global currents and background vector fields

- Background fields can couple to currents of $\mathscr{L}_{\chi\mathrm{PT}}$
  - Baryon currents U(1)$_\mathrm{B}$ in neutron star, ω meson
  - Z boson vector in neutrino dense environment
- SM gauge invariance needs counter terms

$$\partial^\mu j_\mu^5 = \frac{1}{48\pi^2}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$



$$\delta\partial^\mu j_\mu^5 \propto \epsilon_{\mu\nu\rho\sigma}A^{\mu\nu}B^{\rho\sigma}$$



WZW counter terms for global symmetry

J. A. Harvey, C. T. Hill, and R. J. Hill,
PRL 99 (2007) 261601,
PRD 77(2008) 085017

- Generic WZW interactions with counter terms
  - Vector fields in 1-form: $\mathscr{A}_{L/R} \equiv \mathbb{A}_{L/R} + \mathbb{B}_{L/R}$
    Similar to Hidden Local Symmetry

$$\mathscr{L}^{\mathrm{full}}_{\mathrm{WZW}}(U, \mathscr{A}_{L/R}) = \mathscr{L}_{\mathrm{WZW}}(U, \mathscr{A}_L, \mathscr{A}_R) + \mathscr{L}_c(\mathbb{A}_{L/R}, \mathbb{B}_{L/R})$$

- Counter terms ensures SM invariance

$$\Gamma_c = -2\mathscr{C} \int Tr\left[(\mathbb{A}_L d\mathbb{A}_L + d\mathbb{A}_L \mathbb{A}_L)\mathbb{B}_L + \frac{1}{2}\mathbb{A}_L(\mathbb{B}_L d\mathbb{B}_L + d\mathbb{B}_L \mathbb{B}_L) - \frac{3}{2} i \mathbb{A}_L^3 \mathbb{B}_L - \frac{3}{4} i \mathbb{A}_L \mathbb{B}_L \mathbb{A}_L \mathbb{B}_L - \frac{i}{2} \mathbb{A}_L \mathbb{B}_L^3\right] - (L \leftrightarrow R)$$

- Suitable for chiral gauge fields and background fields

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Axion treatment as a fictitious background field

$$\mathscr{L}_{\text{eff}} = \bar{q}(iD_\mu\gamma^\mu - \mathbf{m}_q(a))q + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2$$

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$$+g_{a\gamma}\frac{a}{f_a}F\tilde{F} + \frac{\partial_\mu a}{f_a}\left(\bar{q}_L\mathbf{k}_L(a)\gamma^\mu q_L + \bar{q}_R\mathbf{k}_R(a)\gamma^\mu q_R + \ldots\right)$$

- $D_\mu = \partial_\mu - ig(A_L P_L + A_R P_R)$
- Hints from quark-level L: $D_\mu \to D_\mu + i\frac{\partial_\mu a}{f_a}\left(\mathbf{k}_L P_L + \mathbf{k}_R P_R\right)$
- Hints from ChPT L: $D^\mu U \to D^\mu U - i\frac{\partial^\mu a}{f_a}\left(\mathbf{k}_L U - U\mathbf{k}_R\right)$

$$\mathscr{L}_{\chi\text{PT}} = \frac{f_\pi^2}{8}\left[(D^\mu U)(D_\mu U)^\dagger\right] + \frac{f_\pi^2}{4}B_0\text{Tr}\left[\mathbf{m}_q(a)U^\dagger + h.c.\right] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 + g_{a\gamma}\frac{a}{f_a}F\tilde{F}$$

Axion treatment as a fictitious background field

- Vector fields in 1-form: $\mathscr{A}_{L/R} \equiv \mathbb{A}_{L/R} + \mathbb{B}_{L/R}$
  Similar to Hidden Local Symmetry

- Axion 1-form field can be added into background fields:

$$\mathbb{B}_{L/R} \to \mathbb{B}_{L/R} + \mathbf{k}_{L/R,0}\frac{da}{f_a}$$

- 2-flavor ChPT with SM gauge bosons and background fields

$$\mathbb{A}_L = \frac{e}{s_{\rm W}} W^a \frac{\boldsymbol{\tau}^a}{2} + \frac{e}{c_{\rm W}} W^0 \mathbf{Y}_Q, \ \mathbb{A}_R = \frac{e}{c_{\rm W}} W^0 \mathbf{Y}_q$$

$$\mathbb{B}_V \equiv \mathbb{B}_L + \mathbb{B}_R = g\begin{pmatrix} \rho_0 & \sqrt{2}\rho^+ \\ \sqrt{2}\rho^- & -\rho_0 \end{pmatrix} + g'\begin{pmatrix} \omega & \\ & \omega \end{pmatrix} + (\mathbf{k}_{L,0} + \mathbf{k}_{R,0})\frac{da}{f}$$

$$\mathbb{B}_A \equiv \mathbb{B}_L - \mathbb{B}_R = g\begin{pmatrix} a_1 & \sqrt{2}a^+ \\ \sqrt{2}a^- & -a_1 \end{pmatrix} + g'\begin{pmatrix} f_1 & \\ & f_1 \end{pmatrix} + (\mathbf{k}_{L,0} - \mathbf{k}_{R,0})\frac{da}{f}$$

The consistent axion Lagrangian at low energy

- ChPT:

$$\mathscr{L}_{\chi\mathrm{PT}} = \frac{f_\pi^2}{8}\mathrm{Tr}\left[(D^\mu U)(D_\mu U)^\dagger\right] + \frac{f_\pi^2}{4}B_0\,\mathrm{Tr}\left[\mathbf{m}_q(a)U^\dagger + h.c.\right] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 + \frac{a}{f}\sum_{\mathscr{A}_{1,2}} c_{\mathscr{A}_1\mathscr{A}_2} F_{\mathscr{A}_1\,\mu\nu}\widetilde{F}^{\mu\nu}_{\mathscr{A}_2}$$

- Full WZW: $\mathscr{L}^{\mathrm{full}}_{\mathrm{WZW}}(U, \mathscr{A}_{L/R}) = \mathscr{L}_{\mathrm{WZW}}(U, \mathscr{A}_L, \mathscr{A}_R) + \mathscr{L}_c(\mathbb{A}_{L/R}, \mathbb{B}_{L/R})$

- Full $\mathscr{L}$: $\mathscr{L}^{\mathrm{full}}_{\mathrm{axion}} \equiv \left[\mathscr{L}_{\chi\mathrm{PT}} + \mathscr{L}^{\mathrm{full}}_{\mathrm{WZW}}\right]\left(U, \mathbf{m}_q(a), \mathscr{A}_{L/R} + \mathbf{k}_{L/R}(a)da/f_a\right)$

Matching between $\mathscr{L}_{\rm eff}$ and $\mathscr{L}^{\rm full}_{\rm axion}$

$$\begin{tikzcd}[column sep=huge, row sep=huge]
\mathscr{L}_{\rm eff,\,0}(q_0, \mathbf{m}_{q,0}, \mathbf{k}_{L,0}, \mathbf{k}_{R,0}) \arrow[d, "q_0 = \exp\left(- i\, c_{gg}\, \boldsymbol{\kappa}_{q,0}\, \gamma_5 \frac{a}{f}\right) q"] & \\
\mathscr{L}_{\rm eff}(q, \mathbf{m}_q, \mathbf{k}_L, \mathbf{k}_R) \arrow[r, "q' = \exp\left[i \left(\boldsymbol{\delta}_q + \boldsymbol{\kappa}_q \gamma_5 \right) \frac{a}{f} \right] q"'] \arrow[d, "\text{matching}"'] & \mathscr{L}_{\rm eff}(q', \mathbf{m}'_q, \mathbf{k}'_L, \mathbf{k}'_R) + \delta \mathscr{L}^{\rm ano}_{a} \arrow[d, "\text{matching}"] \\
\begin{array}{l} \mathscr{L}^{\rm full}_{\rm axion} \equiv \mathscr{L}_{\chi{\rm PT}}(U, \mathbf{m}_q, \mathscr{A}_{L/R} + \mathbf{k}_{L/R} da) \\ \quad + \mathscr{L}^{\rm full}_{\rm WZW}(U, \mathbf{m}_q, \mathscr{A}_{L/R} + \mathbf{k}_{L/R} da) \end{array} \arrow[r, "U' = U_L^\dagger U U_R"] & \begin{array}{l} \mathscr{L}_{\chi{\rm PT}}(U', \mathbf{m}'_q, \mathscr{A}_{L/R} + \mathbf{k}'_{L/R} da) \\ + \mathscr{L}^{\rm full}_{\rm WZW}(U', \mathbf{m}'_q, \mathscr{A}_{L/R} + \mathbf{k}'_{L/R} da) \\ + \delta \mathscr{L}^{\rm ano}_{\rm WZW} \end{array}
\end{tikzcd}$$

Effective Lagrangian for axions

- Initial effective Lagrangian:

$$\mathscr{L}_{\rm eff,0} = \mathscr{L}_{\rm SM} + \bar{q}_0 (i\not{D} - \mathbf{m}_{q,0}) q_0 + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2} a^2 + c_{gg} \frac{\alpha_s}{4\pi} \frac{a}{f} G_{\mu\nu} \widetilde{G}^{\mu\nu} + \frac{a}{f} \sum_{\mathscr{A}_{1,2}} c^0_{\mathscr{A}_1 \mathscr{A}_2} F_{\mathscr{A}_1\,\mu\nu} \widetilde{F}^{\mu\nu}_{\mathscr{A}_2} + \mathscr{L}_c$$

- Eliminating aGG term:

$$q_0(x) = \exp\left[-i(\boldsymbol{\delta}_{q,0} + \boldsymbol{\kappa}_{q,0} \gamma_5) c_{gg} \frac{a(x)}{f} \right] q(x), \quad \text{with } \mathrm{Tr}\big(\boldsymbol{\kappa}_{q,0}\big) = 1$$

- New effective Lagrangian:

$$\mathscr{L}_{\rm eff} = \mathscr{L}_{\rm SM} + \overline{q}\, i\not{D}\, q - [\overline{q}_L\, \mathbf{m}_q(a)\, q_R + h.c.] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2} a^2 + \frac{a}{f} \sum_{\mathscr{A}_{1,2}} c_{\mathscr{A}_1 \mathscr{A}_2} F_{\mathscr{A}_1\,\mu\nu} \widetilde{F}^{\mu\nu}_{\mathscr{A}_2} + \mathscr{L}_c$$

Auxiliary chiral rotation for effective Lagrangian

- Chiral rotation without regenerating aGG term $\mathrm{Tr}(\boldsymbol{\kappa}_q) = 0$

$$q' = \exp\left[i\left(\boldsymbol{\delta}_q + \boldsymbol{\kappa}_q \gamma_5\right) a/f\right] q$$

- Left/right rotation matrices

$$\boldsymbol{\theta}_{L/R} \equiv \boldsymbol{\delta}_q \mp \boldsymbol{\kappa}_q \qquad U_{L/R} \equiv \exp\left[-i\boldsymbol{\theta}_{L/R} a/f\right]$$

- Mass and coupling shifts

$$\mathbf{m}_q' = U_L^\dagger \mathbf{m}_q U_R, \quad \mathbf{k}_{L/R}' = U_{L/R}^\dagger(\mathbf{k}_{L/R} + \boldsymbol{\theta}_{L/R}) U_{L/R} = \mathbf{k}_{L/R} + \boldsymbol{\theta}_{L/R}$$

- Chiral basis change for effective Lagrangian

$$\mathscr{L}_{\rm eff}(q, \mathbf{m}_q, \mathbf{k}_L, \mathbf{k}_R) \to \mathscr{L}_{\rm eff}(q', \mathbf{m}_q', \mathbf{k}_L', \mathbf{k}_R') + \delta\mathscr{L}_a^{\rm ano}$$

The axion anomalous interactions

$$\delta \mathscr{L}_a^{\text{ano}} = -\,\delta\left[\mathscr{L}_{\text{WZW}} + \mathscr{L}_c\right](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R)$$

- The exact expressions

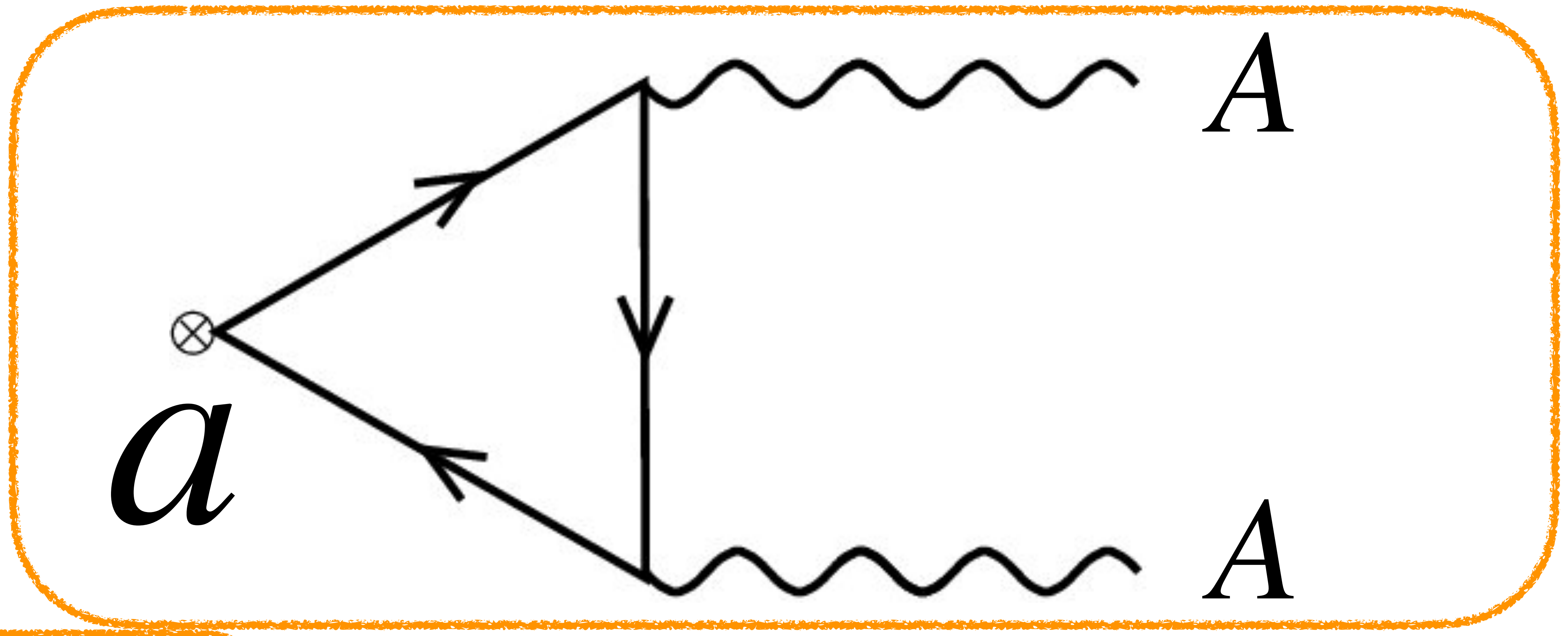
$$\delta\left[\Gamma_{\text{WZW}} + \Gamma_c\right](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R) = -2\mathscr{C}\boxed{\frac{a}{f}}\int \text{Tr}\Big\{\boxed{\boldsymbol{\theta}_L \Big[3(d\mathbb{A}_L - i\mathbb{A}_L^2)^2} + 3(d\mathbb{A}_L - i\mathbb{A}_L^2)(D\mathbb{B}_L) + D\mathbb{B}_L D\mathbb{B}_L - \frac{i}{2}D(\mathbb{B}_L^3)$$
$$+ i\mathbb{B}_L(d\mathbb{A}_L - i\mathbb{A}_L^2)\mathbb{B}_L - i(d\mathbb{A}_L - i\mathbb{A}_L^2)\mathbb{B}_L^2\Big]\Big\} - (L \leftrightarrow R)\ ,$$

- Covariant derivative $D\mathbb{B}_{L,R} = d\mathbb{B}_{L,R} - i\mathbb{A}_{L,R}\mathbb{B}_{L,R} - i\mathbb{B}_{L,R}\mathbb{A}_{L,R}$

- Covariant field strength: $F = d\mathbb{A}_L - i\mathbb{A}_L^2$

The axion anomalous interactions

$$\delta \mathscr{L}_a^{\text{ano}} = -\,\delta\left[\mathscr{L}_{\text{WZW}} + \mathscr{L}_c\right](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R)$$



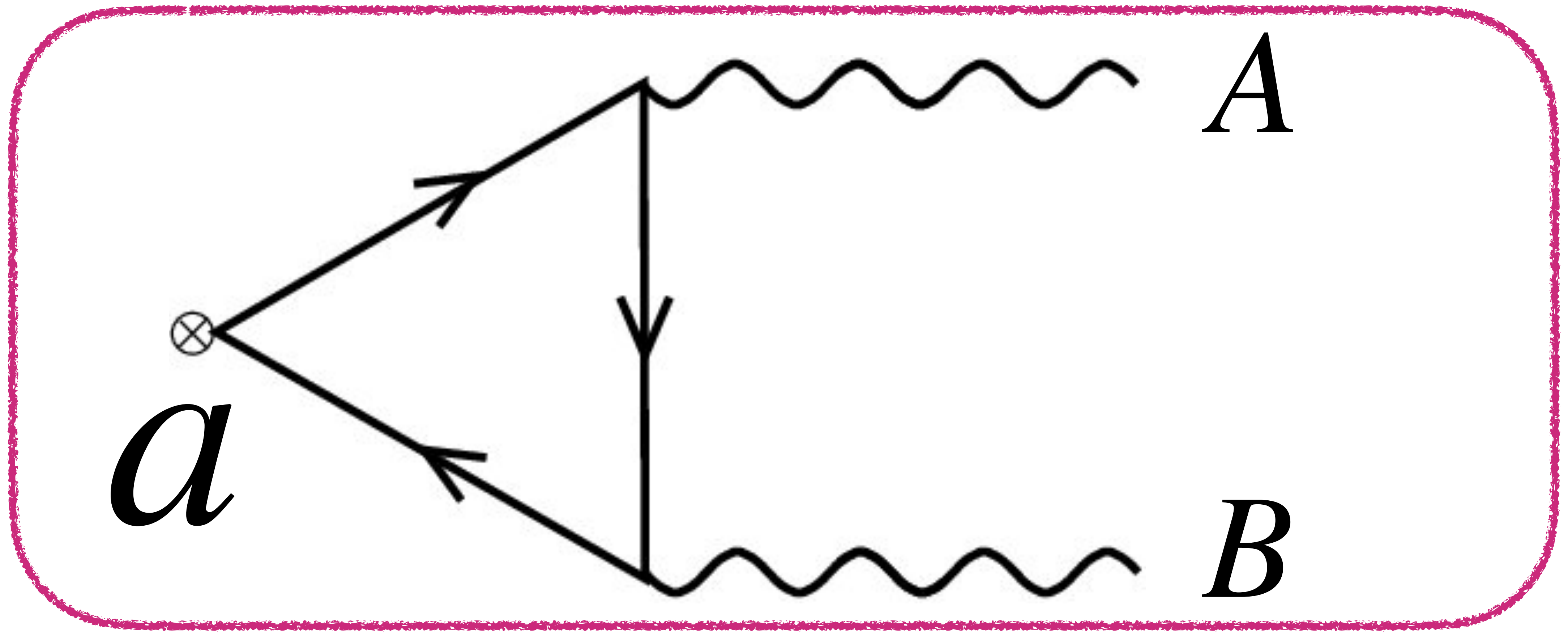
- The exact expressions

$$\delta\left[\Gamma_{\text{WZW}} + \Gamma_c\right](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R) = -2\mathscr{C}\boxed{\frac{a}{f}\int \mathrm{Tr}\Big\{\boldsymbol{\theta}_L\Big[3(d\mathbb{A}_L - i\mathbb{A}_L^2)^2} + 3(d\mathbb{A}_L - i\mathbb{A}_L^2)(D\mathbb{B}_L) + D\mathbb{B}_L D\mathbb{B}_L - \frac{i}{2}D(\mathbb{B}_L^3)$$
$$+ i\mathbb{B}_L(d\mathbb{A}_L - i\mathbb{A}_L^2)\mathbb{B}_L - i(d\mathbb{A}_L - i\mathbb{A}_L^2)\mathbb{B}_L^2\Big]\Big\} - (L \leftrightarrow R)\,,$$

- Covariant derivative $D\mathbb{B}_{L,R} = d\mathbb{B}_{L,R} - i\mathbb{A}_{L,R}\mathbb{B}_{L,R} - i\mathbb{B}_{L,R}\mathbb{A}_{L,R}$
- Covariant field strength: $F = d\mathbb{A}_L - i\mathbb{A}_L^2$

The axion anomalous interactions

$$\delta \mathscr{L}_a^{\rm ano} = -\,\delta\left[\mathscr{L}_{\rm WZW} + \mathscr{L}_c\right](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R)$$



- The exact expressions

$$\delta\left[\Gamma_{\rm WZW} + \Gamma_c\right](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R) = -2\mathscr{C}\frac{a}{f}\int {\rm Tr}\Big\{\boldsymbol{\theta}_L\Big[3(d\mathbb{A}_L - i\mathbb{A}_L^2)^2 + \boxed{3(d\mathbb{A}_L - i\mathbb{A}_L^2)(D\mathbb{B}_L)} + D\mathbb{B}_L D\mathbb{B}_L - \frac{i}{2}D(\mathbb{B}_L^3)$$
$$+ i\mathbb{B}_L(d\mathbb{A}_L - i\mathbb{A}_L^2)\mathbb{B}_L - i(d\mathbb{A}_L - i\mathbb{A}_L^2)\mathbb{B}_L^2\Big]\Big\} - (L \leftrightarrow R)\,,$$

- Covariant derivative $D\mathbb{B}_{L,R} = d\mathbb{B}_{L,R} - i\mathbb{A}_{L,R}\mathbb{B}_{L,R} - i\mathbb{B}_{L,R}\mathbb{A}_{L,R}$

- Covariant field strength: $F = d\mathbb{A}_L - i\mathbb{A}_L^2$

Effective and Chiral Lagrangian matching

$$\mathscr{L}_{\rm eff}(q, \mathbf{m}_q, \mathbf{k}_L, \mathbf{k}_R) \to \mathscr{L}_{\chi\rm PT}(q, \mathbf{m}_q, \mathbf{k}_L, \mathbf{k}_R)$$

- The correspondence

$$\mathscr{L}_{\rm eff} = \mathscr{L}_{\rm SM} + \bar{q} i \not{D} q - [\bar{q}_L \mathbf{m}_q(a) q_R + h.c.] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2} a^2 + \boxed{\frac{a}{f} \sum_{\mathscr{A}_{1,2}} c_{\mathscr{A}_1 \mathscr{A}_2} F_{\mathscr{A}_1 \mu\nu} \widetilde{F}^{\mu\nu}_{\mathscr{A}_2}} + \mathscr{L}_c$$

$$\mathscr{L}_{\chi\rm PT} = \frac{f_\pi^2}{8} \mathrm{Tr}\left[(D^\mu U)(D_\mu U)^\dagger\right] + \frac{f_\pi^2}{4} B_0 \mathrm{Tr}\left[\mathbf{m}_q(a) U^\dagger + h.c.\right] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2} a^2 + \boxed{\frac{a}{f} \sum_{\mathscr{A}_{1,2}} c_{\mathscr{A}_1 \mathscr{A}_2} F_{\mathscr{A}_1 \mu\nu} \widetilde{F}^{\mu\nu}_{\mathscr{A}_2}}$$

- The anomalous matching condition between UV and IR

$$\boxed{\mathscr{L}_{\chi\rm PT}^{\rm ano} \equiv \frac{a}{f_a} \sum_{\mathscr{A}_{1,2}} c_{\mathscr{A}_1 \mathscr{A}_2} F_{\mathscr{A}_1 \mu\nu} \widetilde{F}^{\mu\nu}_{\mathscr{A}_2}}$$

Effective and Chiral Lagrangian matching

$$\mathscr{L}_{\text{eff}}(q, \mathbf{m}_q, \mathbf{k}_L, \mathbf{k}_R) \to \mathscr{L}^{\text{full}}_{\text{axion}} = \mathscr{L}_{\chi\text{PT}}(q, \mathbf{m}_q, \mathbf{k}_L, \mathbf{k}_R) + \mathscr{L}^{\text{full}}_{\text{WZW}}(U, \mathscr{A}_{L/R})$$

- The correspondence

$$\mathscr{L}_{\text{eff}} = \mathscr{L}_{\text{SM}} + \bar{q} i \not{D} q - [\bar{q}_L \mathbf{m}_q(a) q_R + h.c.] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m^2_{a,0}}{2} a^2 + \frac{a}{f} \sum_{\mathscr{A}_{1,2}} c_{\mathscr{A}_1 \mathscr{A}_2} F_{\mathscr{A}_1\, \mu\nu} \widetilde{F}^{\mu\nu}_{\mathscr{A}_2} + \boxed{\mathscr{L}_c}$$

$$\mathscr{L}_{\chi\text{PT}} = \frac{f_\pi^2}{8} \text{Tr}\left[(D^\mu U)(D_\mu U)^\dagger\right] + \frac{f_\pi^2}{4} B_0 \text{Tr}\left[\mathbf{m}_q(a) U^\dagger + h.c.\right] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m^2_{a,0}}{2} a^2 + \frac{a}{f} \sum_{\mathscr{A}_{1,2}} c_{\mathscr{A}_1 \mathscr{A}_2} F_{\mathscr{A}_1\, \mu\nu} \widetilde{F}^{\mu\nu}_{\mathscr{A}_2}$$

- WZW term and counter terms

$$\boxed{\mathscr{L}^{\text{full}}_{\text{WZW}}(U, \mathscr{A}_{L/R}) = \mathscr{L}_{\text{WZW}}(U, \mathscr{A}_L, \mathscr{A}_R) + \mathscr{L}_c(\mathbb{A}_{L/R}, \mathbb{B}_{L/R})}$$

Consistent matching between $\mathscr{L}_{\text{eff}}$ and $\mathscr{L}_{\text{axion}}^{\text{full}}$

$$\delta\mathscr{L}_a^{\text{ano}} = -\,\delta\left[\mathscr{L}_{\text{WZW}} + \mathscr{L}_c\right](\boldsymbol{\theta}_L,\boldsymbol{\theta}_R) = \delta\mathscr{L}_{\text{WZW}}^{\text{ano}}$$

$$\begin{tikzcd}
\mathscr{L}_{\text{eff},\,0}(q_0,\mathbf{m}_{q,0},\mathbf{k}_{L,0},\mathbf{k}_{R,0}) \arrow[d, "q_0 = \exp\left(-\,i\,c_{gg}\,\boldsymbol{\kappa}_{q,0}\,\gamma_5\,\frac{a}{f}\right)q"] & \\
\mathscr{L}_{\text{eff}}(q,\mathbf{m}_q,\mathbf{k}_L,\mathbf{k}_R) \arrow[r, "q' = \exp\left[i\left(\boldsymbol{\delta}_q+\boldsymbol{\kappa}_q\gamma_5\right)\frac{a}{f}\right]q"'] \arrow[d, "\text{matching}"'] & \mathscr{L}_{\text{eff}}(q',\mathbf{m}'_q,\mathbf{k}'_L,\mathbf{k}'_R) + \boxed{\delta\mathscr{L}_a^{\text{ano}}} \arrow[d, "\text{matching}"] \\
\begin{array}{l} \mathscr{L}_{\text{axion}}^{\text{full}} \equiv \mathscr{L}_{\chi\text{PT}}(U,\mathbf{m}_q,\mathscr{A}_{L/R}+\mathbf{k}_{L/R}da) \\ \quad + \mathscr{L}_{\text{WZW}}^{\text{full}}(U,\mathbf{m}_q,\mathscr{A}_{L/R}+\mathbf{k}_{L/R}da) \end{array} \arrow[r, "U' = U_L^\dagger U U_R"] & \begin{array}{l} \mathscr{L}_{\chi\text{PT}}(U',\mathbf{m}'_q,\mathscr{A}_{L/R}+\mathbf{k}'_{L/R}da) \\ + \mathscr{L}_{\text{WZW}}^{\text{full}}(U',\mathbf{m}'_q,\mathscr{A}_{L/R}+\mathbf{k}'_{L/R}da) \\ + \boxed{\delta\mathscr{L}_{\text{WZW}}^{\text{ano}}} \end{array}
\end{tikzcd}$$

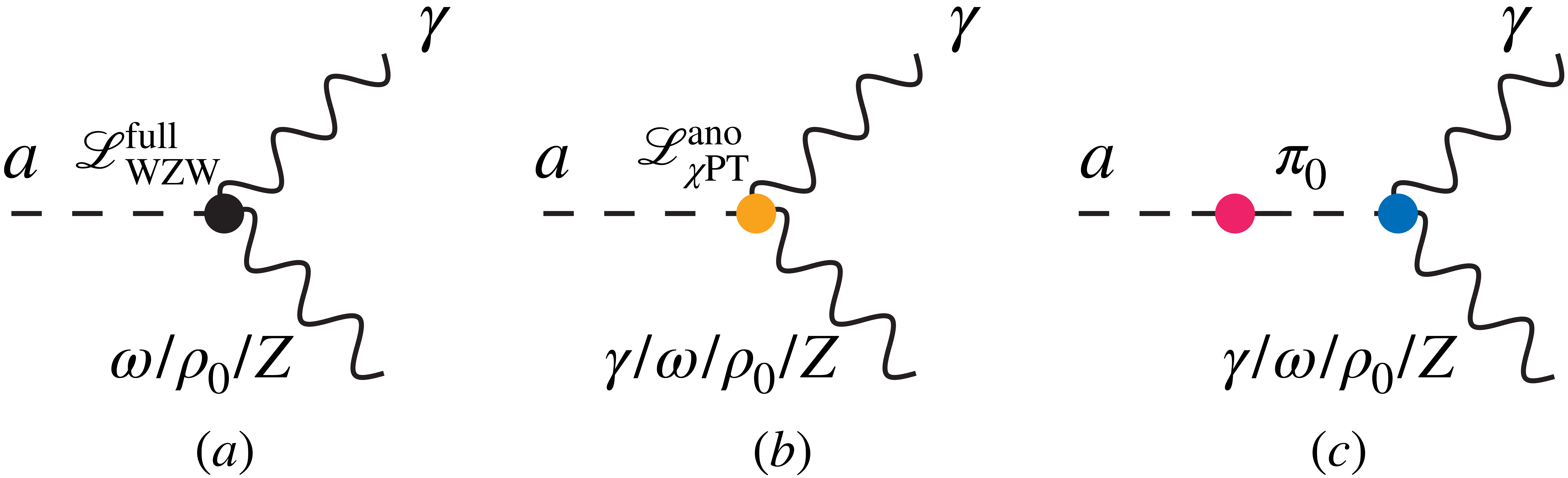
Outlines

- Axion effective and consistent ChPT Lagrangian
- WZW in QCD and counter-terms
- Full WZW interactions for axions
- Phenomenology at BESIII and STCF
- Summary

Consistent physical amplitudes

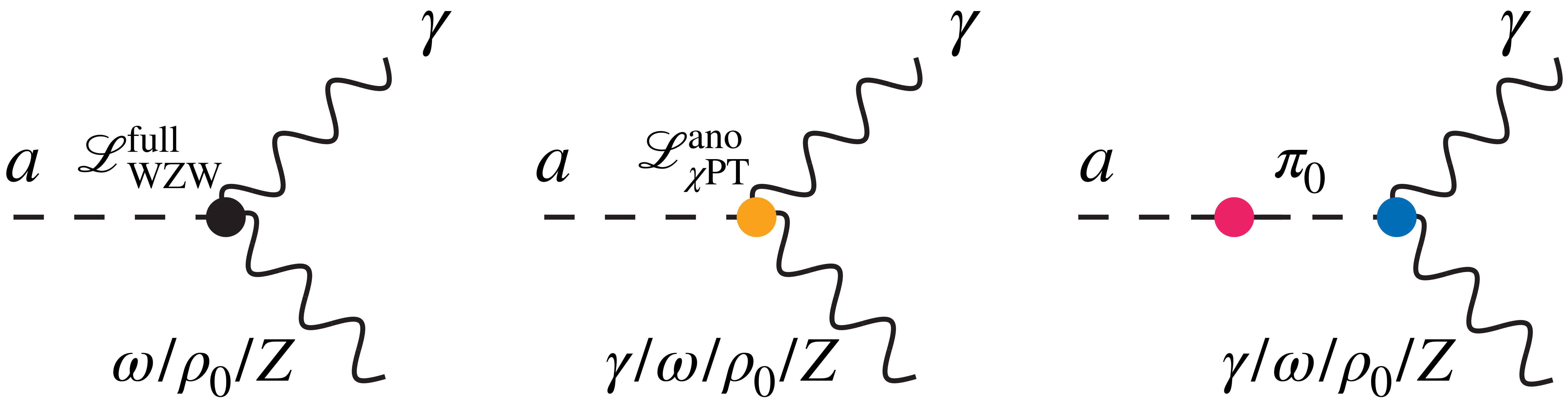
- A consistent Lagrangian will give physical amplitudes independent of auxiliary rotations

- Full WZW interactions are important for a-A-B amplitudes



Consistent physical amplitudes for $a-\gamma-\gamma$

- Auxiliary rotations are cancelled



(a) (b) (c)

$$ad\gamma d\gamma:\quad c_{\rm WZW}\equiv 0 \qquad ad\gamma d\gamma:\quad c_{\rm ano}\equiv -\frac{e^2N_c}{48\pi^2 f}12(Q_u^2\kappa_u+Q_d^2\kappa_d) \qquad \pi_0 d\gamma d\gamma:\quad c_{\pi_0}\equiv \frac{e^2N_c}{48\pi^2 f_\pi}6\sqrt{2}(Q_d^2-Q_u^2)$$

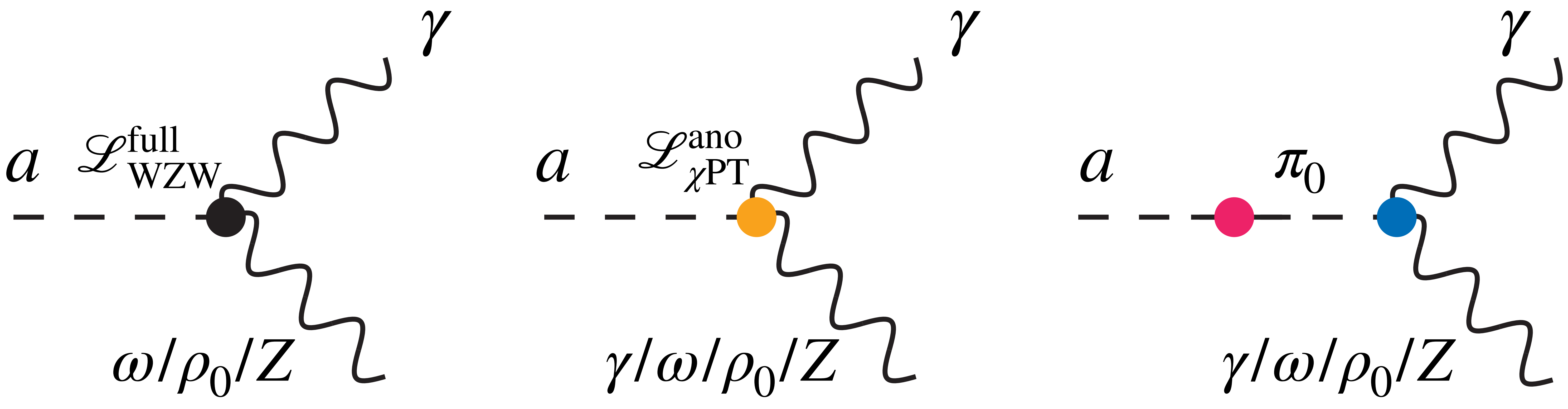
$$\mathscr{M}(a\to\gamma\gamma)\textbf{(auxiliary)} = CF\times\left(c_{\rm ano}+\theta'_{a-\pi_0}c_{\pi_0}+c_{\rm WZW}\right)$$

$$= CF\times e^2\left\{\frac{-N_c}{48\pi^2 f_a}12(Q_u^2\kappa_u+Q_d^2\kappa_d)+i\frac{f_\pi}{\sqrt{2}f}\left[(\kappa_u-\kappa_d)p_a^2-2\frac{m_u\kappa_u-m_d\kappa_d}{m_u+m_d}m_\pi^2\right]\frac{i}{p_a^2-m_\pi^2}\times\frac{N_c}{48\pi^2 f_\pi}6\sqrt{2}(Q_d^2-Q_u^2)\right\}$$

$$\underset{p_a^2=m_a^2}{\kappa_u+\kappa_d=0}\qquad \to 0$$

Consistent physical amplitudes for $a-\gamma-\omega$

- Auxiliary rotations are cancelled



(a) (b) (c)

$a d\omega d\gamma:\ \ c_{\rm wzw} = \frac{-eg'N_c}{48\pi^2 f}6(Q_u\kappa_u + Q_d\kappa_d)$

$a d\omega d\gamma:\ \ c_{\rm ano} = \frac{-eg'N_c}{48\pi^2 f}6(Q_u\kappa_u + Q_d\kappa_d)$

$\pi_0 d\omega d\gamma:\ \ c_{\pi_0} = \frac{eg'N_c}{48\pi^2 f_\pi}6\sqrt{2}(Q_d - Q_u)$

$\mathscr{M}(a \to \omega\gamma)$**(auxiliary)** $= CF \times (c_{\rm ano} + \theta'_{a-\pi_0}c_{\pi_0} + c_{\rm wzw})$

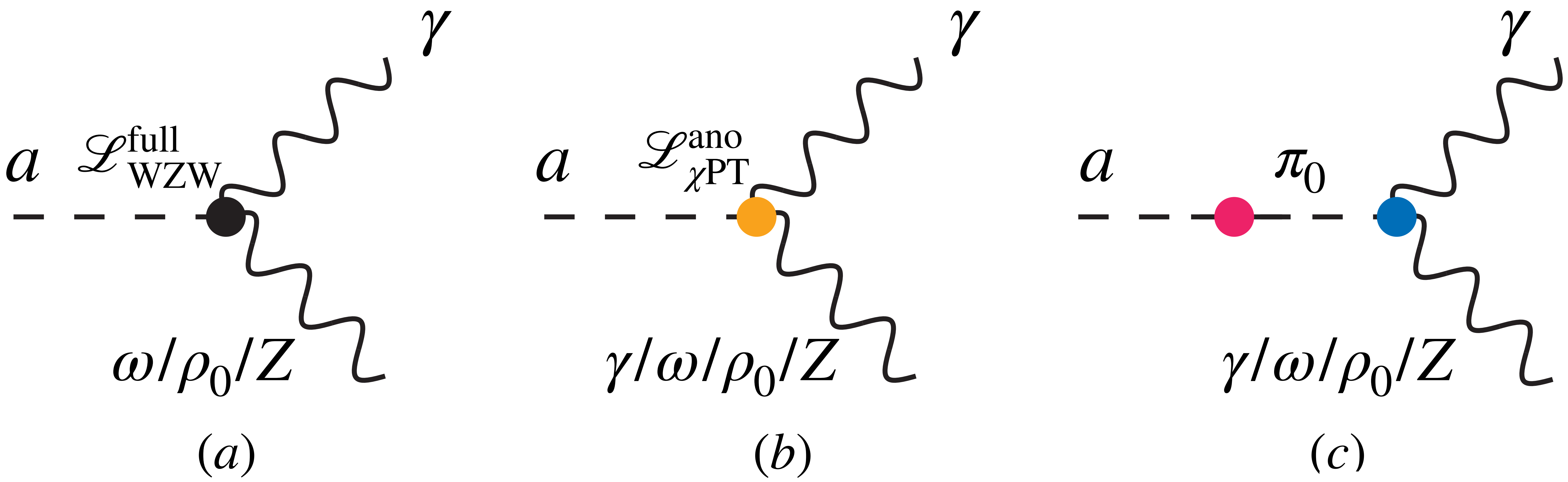
$$= CF \times eg'\left[\frac{-N_c}{48\pi^2 f}12(Q_u\kappa_u + Q_d\kappa_d) + i\frac{f_\pi}{\sqrt{2}f}((\kappa_u - \kappa_d)p_a^2 - 2\frac{m_u\kappa_u - m_d\kappa_d}{m_u + m_d}m_\pi^2)\frac{i}{p_a^2 - m_\pi^2} \times \frac{N_c}{48\pi^2 f_\pi}6\sqrt{2}(Q_d - Q_u)\right]$$

$\to 0$

Consistent physical amplitudes for $a - \gamma - Z$

- Auxiliary rotations are cancelled

$$ad\gamma dZ: \quad c_{\text{ano}} = \frac{N_c}{48\pi^2 f}\frac{2e^2}{3c_w s_w}\left[3\delta_d + 6\delta_u - 3\kappa_d - 6\kappa_u + 4s_w^2(\kappa_d + 4\kappa_u)\right]$$



$$ad\gamma dZ: \quad c_{\text{WZW}} = \frac{-2e^2 N_c}{48\pi^2 f s_w c_w}(\delta_d + 2\delta_u)$$

$$\pi_0 d\gamma dZ: \quad c_{\pi_0} = \frac{-e^2 N_c}{48\pi^2 f_\pi s_w c_w}\sqrt{2}(c_w^2 - 3s_w^2)$$

$$\mathscr{M}(a \to Z^*\gamma)\textbf{(auxiliary)} = CF \times (c_{\text{ano}} + \theta'_{a-\pi_0} c_{\pi_0} + c_{\text{wzw}})$$

$$= CF \times \left[c_{\text{wzw}} + c_{\text{ano}} + i\frac{f_\pi}{\sqrt{2}f}\left((\kappa_u - \kappa_d)p_a^2 - 2\frac{m_u\kappa_u - m_d\kappa_d}{m_u + m_d}m_\pi^2\right)\frac{i}{p_a^2 - m_\pi^2} \times c_{\pi_0}\right]$$

$$\to 0$$

Consistent amplitudes for three point vertex

$$c_{\gamma\gamma}^{\text{eff}} = c_{\gamma\gamma}^{0} + \boxed{\frac{e^2 c_{gg}}{16\pi^2 f}}\left(-\frac{10}{3} - 2\frac{m_u - m_d}{m_u + m_d}\frac{m_\pi^2}{m_a^2 - m_\pi^2}\right) - \frac{e^2}{16\pi^2 f}\frac{m_a^2}{m_\pi^2 - m_a^2}(c_u - c_d)$$

$$c_{\omega\gamma}^{\text{eff}} = \boxed{eg'}\left\{\frac{-c_{gg}}{8\pi^2 f} - \frac{3}{8\pi^2 f}\left[\frac{m_a^2}{m_\pi^2 - m_a^2}\left(\frac{c_u - c_d}{2}\right) + c_{gg}\frac{m_u - m_d}{m_u + m_d}\frac{m_\pi^2}{m_a^2 - m_\pi^2}\right] + \frac{1}{16\pi^2 f}(c_d + c_Q - 2c_u)\right\}$$

$$c_{\rho\gamma}^{\text{eff}} = eg\left\{\frac{-3c_{gg}}{8\pi^2 f} - \frac{1}{8\pi^2 f}\left[\frac{m_a^2}{m_\pi^2 - m_a^2}\left(\frac{c_u - c_d}{2}\right) + c_{gg}\frac{m_u - m_d}{m_u + m_d}\frac{m_\pi^2}{m_a^2 - m_\pi^2}\right] + \frac{1}{16\pi^2 f}\left(3c_Q - 2c_u - c_d\right)\right\}$$

$$c_{\gamma Z}^{\text{eff}} = c_{\gamma Z}^{0} + \frac{N_c c_{gg}}{48\pi^2 f}\frac{e^2}{s_w c_w}(-9 + 20 s_w^2) - c_{\pi_0}\frac{f_\pi}{\sqrt{2}f}\left(\frac{m_a^2}{m_\pi^2 - m_a^2}\frac{c_d - c_u}{2} - c_{gg}\frac{m_u - m_d}{m_u + m_d}\frac{m_\pi^2}{m_a^2 - m_\pi^2}\right) - \frac{N_c}{48\pi^2 f}\frac{2e^2}{s_{2w}}(c_d + 2c_u + 3c_Q)$$

- Vertex $\omega \to \gamma a$ benefit from large $g' \approx 5.7 \gg e$

Phenomenology at BESIII and STCF

- New channel $e^+e^- \to \gamma^*(J/\Psi) \to \omega a$

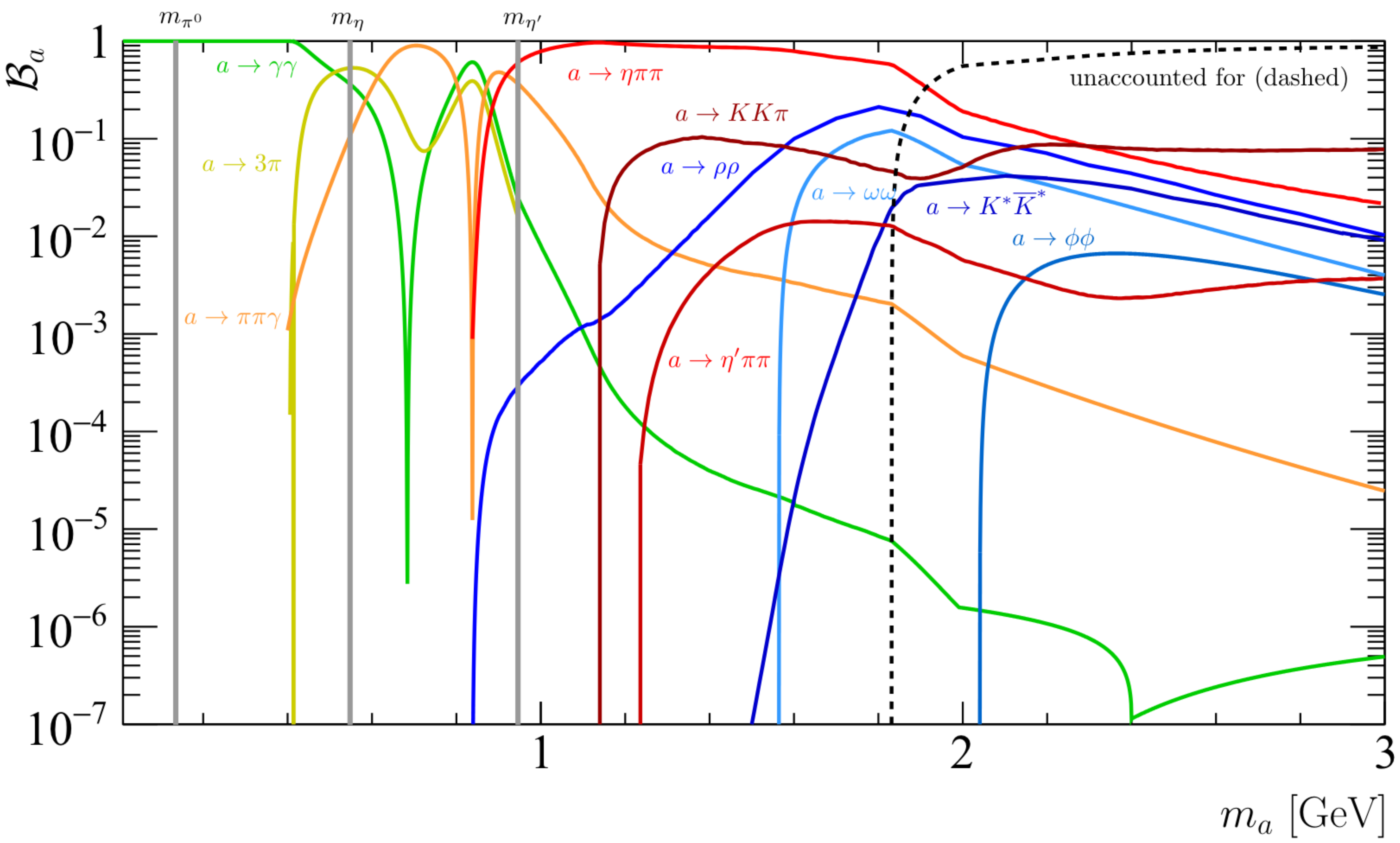
$$c_{\omega\gamma}^{\rm eff}(q^2) = -\frac{eg'c_{gg}}{8\pi^2 f}\frac{m_\omega^2}{m_\omega^2 - q^2 - i\sqrt{q^2}\Gamma_\omega} - \frac{3eg'c_{gg}}{8\pi^2 f}\frac{m_u - m_d}{m_u + m_d}\frac{m_\pi^2}{m_a^2 - m_\pi^2}\sum_{i=0}^{3}\frac{A_i\, M_i^2\, e^{i\phi_i}}{M_i^2 - q^2 - i\sqrt{q^2}\Gamma_i(\sqrt{q^2})}$$

- The model satisfies partial Vector Meson Dominance, therefore we can use form factor for $\gamma^* - \omega - a$
- The differential cross-section

$$\frac{d\sigma(e^+e^- \to \omega\ a)}{d\cos\theta} = \frac{\alpha\,|\,c_{\omega\gamma}^{\rm eff}(q^2)\,|^2\left[m_a^4 + (m_\omega^2 - s)^2 - 2m_a^2(m_\omega^2 + s)\right]}{64 f^2 s^2}(1 + \cos\theta^2)$$

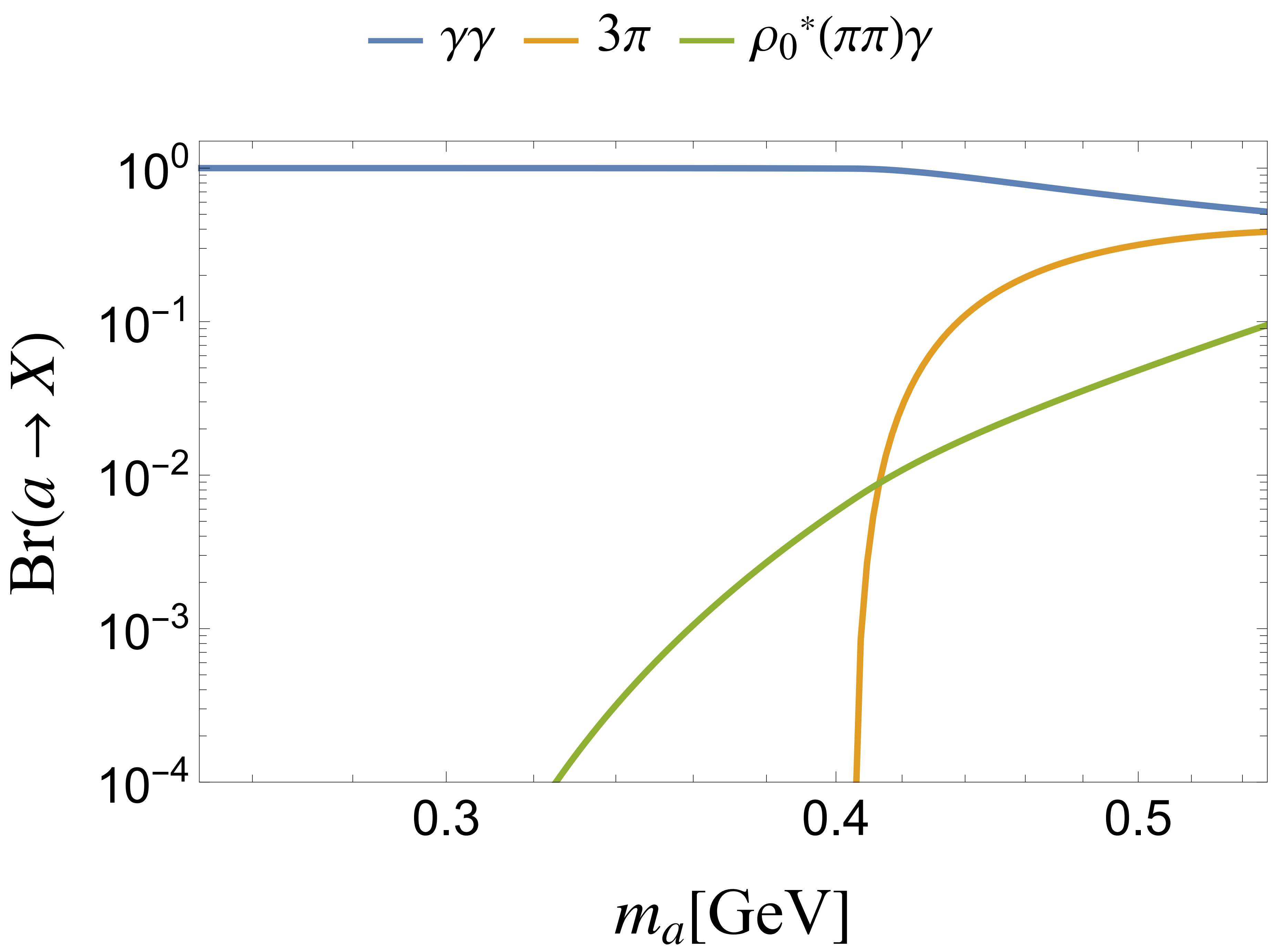
The decay of axion

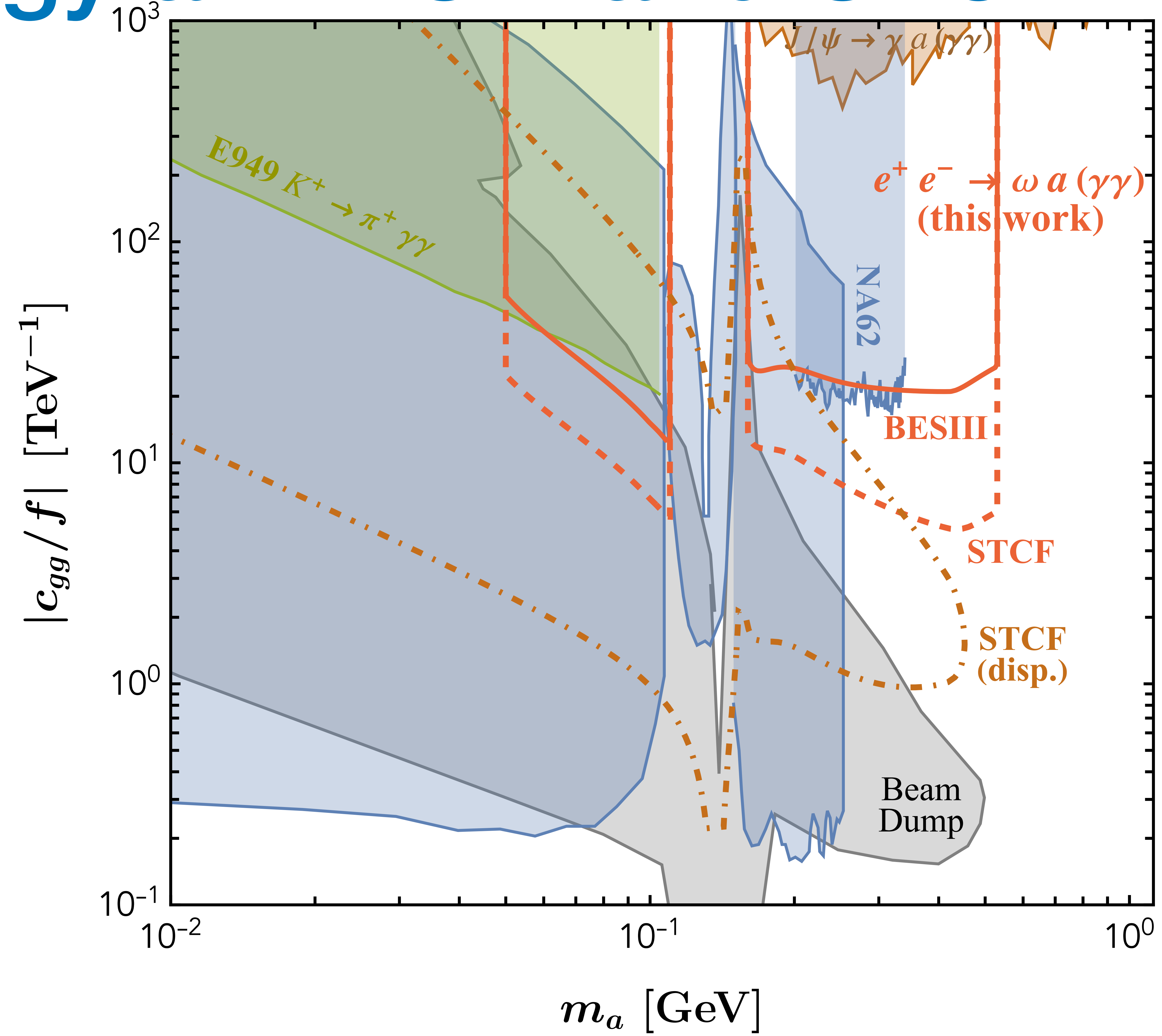
- Previous work (PRL 123 (2019) 031803) use Hidden Local Symmetry to describe pseudo scalar meson + vector meson interactions
- Assume axion mixes with π, η, η'
- Use data driven method to obtain form factor



- Lacks first chiral rotation contribution from $\mathscr{L}_{\chi\mathrm{PT}}^{\mathrm{ano}}$
- Lacks full WZW contribution from $\mathscr{L}_{\mathrm{WZW}}^{\mathrm{full}}$

Light axion phenomenology at BESIII and STCF





- Production $e^+e^- \to \gamma^*(J/\Psi) \to \omega a$

- Prompt decay: $a \to \gamma\gamma$

- Displaced decay of a

$$\frac{\mathrm{BR}(J/\psi\to\omega a)}{\mathrm{BR}(J/\psi\to ee)} = \frac{m_{J/\psi}^2}{32\pi\alpha}\left|c^{\mathrm{eff}}_{\omega\gamma}(q^2=m_{J/\psi}^2)\right|^2\left[\left(1-\frac{(m_a+m_\omega)^2}{m_{J/\psi}^2}\right)\left(1-\frac{(m_a-m_\omega)^2}{m_{J/\psi}^2}\right)\right]^{\frac{3}{2}}$$

Summary

- Global currents or background vector fields needs Wess-Zumino-Witten counter terms to be gauge invariant

- We found a way to add $U(1)_{\rm PQ}$ axion into full WZW interactions

  - $\mathbb{B}_{L/R} \to \mathbb{B}_{L/R} + \mathbf{k}_{L/R} da/f_a$

  - UV-IR anomaly matching is necessary

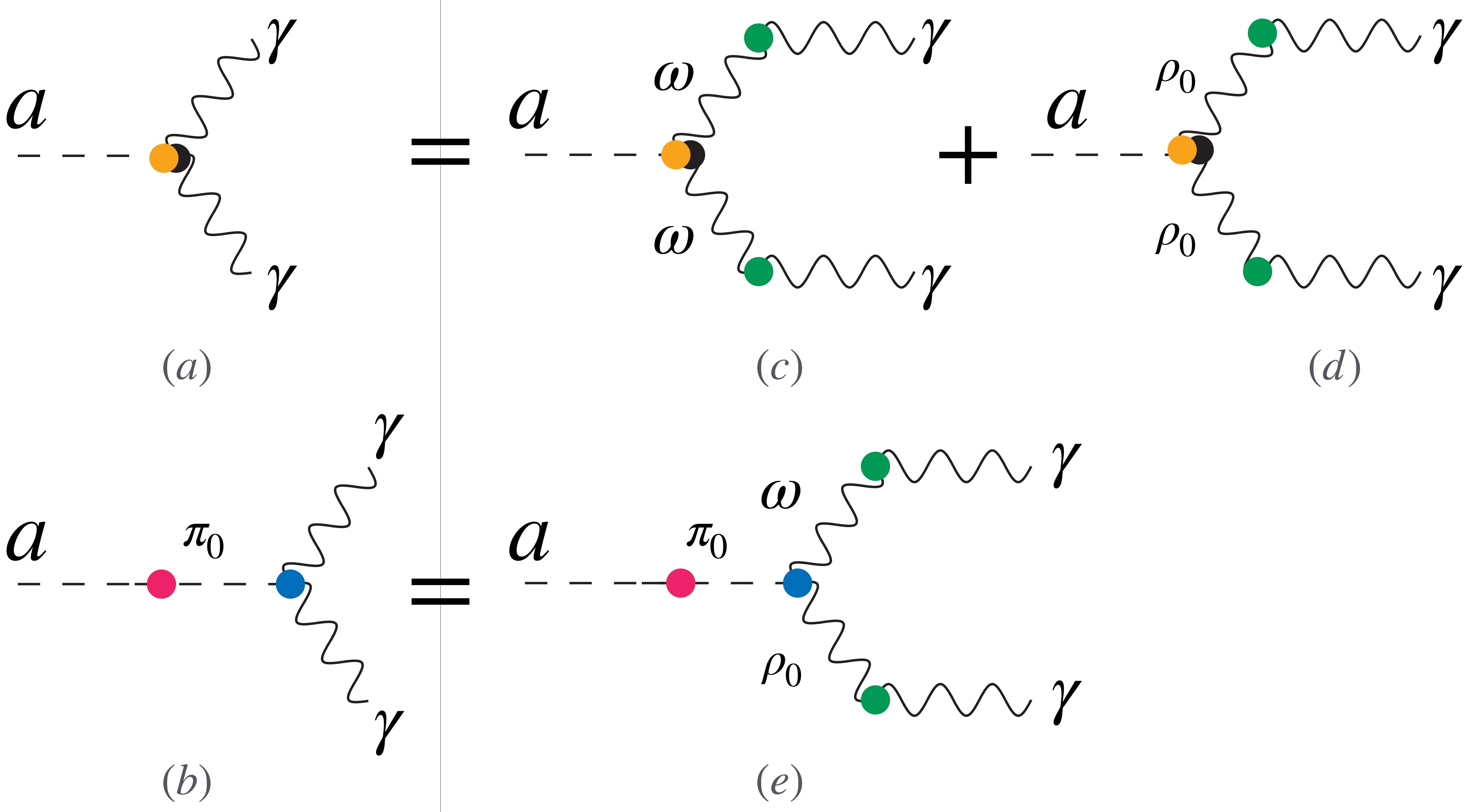
  - Obtain a full chiral axion Lagrangian for axion-pseudo-vector mesons

- Consistent physical amplitudes without auxiliary rotation parameters

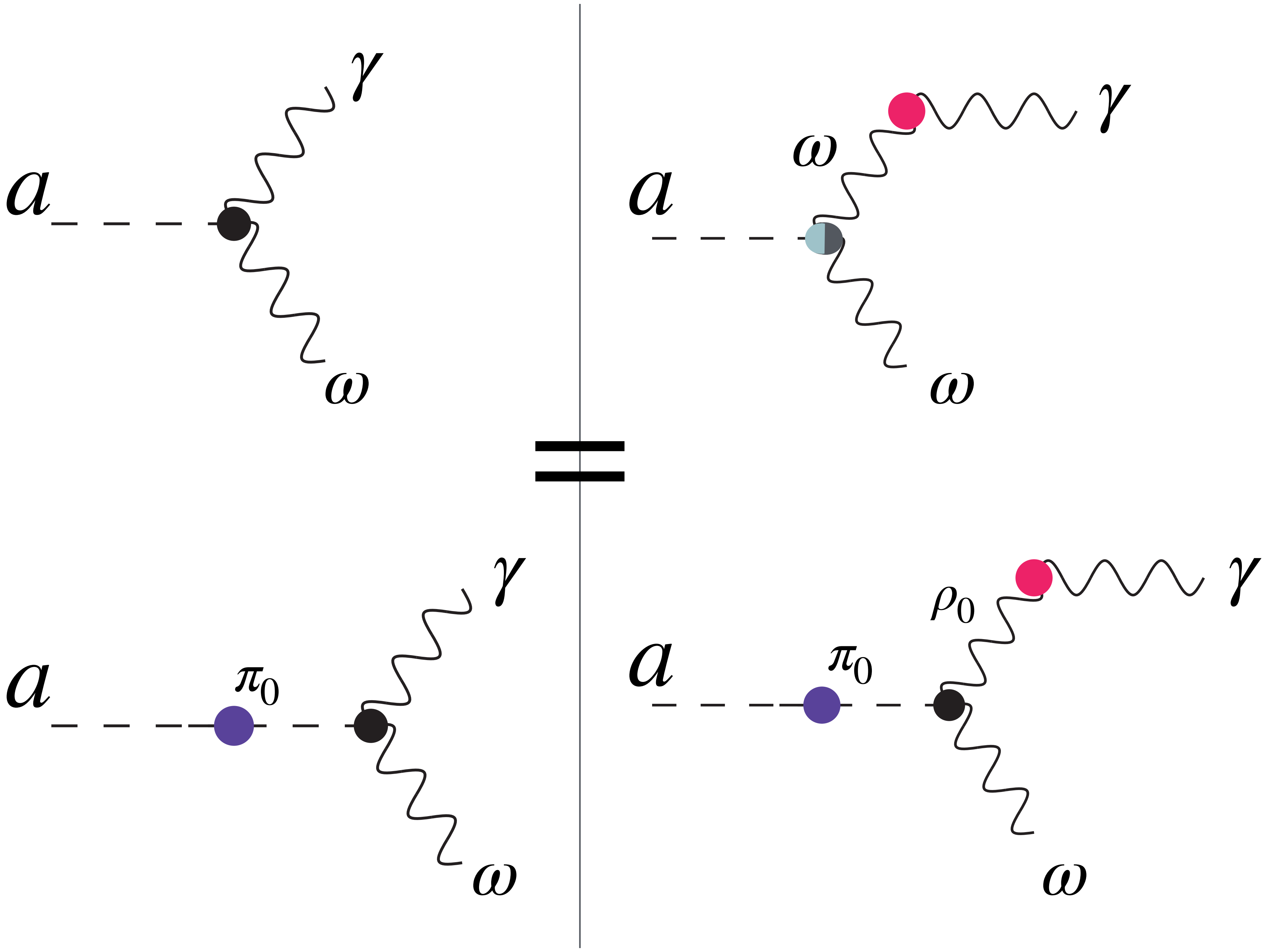
- New search channel involving $\omega \to \gamma a$ vertex

Backup: Vector Meson Dominance

$$a - \gamma - \gamma$$



$$a - \gamma - \omega$$



Backup: axion related WZW interactions

- Convention $\int d^4x \epsilon_{\mu\nu\rho\sigma} A^\mu B^\nu \partial^\rho C^\sigma \equiv \int ABdC$

$$
\begin{aligned}
\Gamma_{XdYda} = \frac{\mathscr{C}}{f} \int da \Bigg\{ & \frac{2e^2}{s_{2\mathrm{w}}}(k_d + 2k_u + 3k_Q)\gamma dZ + eg(k_d + 2k_u + 3k_Q)\gamma da_1 - eg'(k_d - k_Q - 2k_u)\gamma df_1 \\
& + eg(k_d - 3k_Q + 2k_u)\gamma d\rho_0 - eg'(k_d + k_Q - 2k_u)\gamma d\omega + \frac{2e^2}{s_{2\mathrm{w}}^2}\left[(k_d + 4k_Q + k_u) - 2s_\mathrm{w}^2(k_d + 3k_Q + 2k_u)\right] ZdZ \\
& + \frac{eg}{s_{2\mathrm{w}}}\left[(k_d + 4k_Q + k_u) - 2s_\mathrm{w}^2(k_d + 3k_Q + 2k_u)\right] Zda_1 - \frac{eg'}{s_{2\mathrm{w}}}\left[k_d - k_u + s_\mathrm{w}^2(-2k_d + 2k_Q + 4k_u)\right] Zdf_1 \\
& - \frac{eg}{s_{2\mathrm{w}}}\left[-3k_d - 3k_u + 2s_\mathrm{w}^2(k_d - 3k_Q + 2k_u)\right] Zd\rho_0 - \frac{eg'}{s_{2\mathrm{w}}}\left[3k_d - 3k_u - 2s_\mathrm{w}^2(k_d + k_Q - 2k_u)\right] Zd\omega \\
& + g^2(k_d + 2k_Q + k_u)a_1 d\rho_0 + gg'(k_u - k_d)a_1 d\omega + gg'(k_u - k_d)f_1 d\rho_0 + g'^2(k_d + 2k_Q + k_u)f_1 d\omega \\
& + g^2(k_d - 2k_Q + k_u)\rho_0 d\rho_0 + 2gg'(k_u - k_d)\rho_0 d\omega + g'^2(k_d - 2k_Q + k_u)\omega d\omega + \frac{3eg}{2s_\mathrm{w}}(k_u + k_d)W^\pm d\rho^\mp \\
& + \frac{eg}{2s_\mathrm{w}}(k_d + 4k_Q + k_u)a^\mp dW^\pm + g^2(k_d + 2k_Q + k_u)a^\mp d\rho^\pm + \frac{e^2}{s_\mathrm{w}^2}(k_d + 4k_Q + k_u)W^- dW^+ \Bigg\}
\end{aligned}
$$