G2HDM模型的线算子谱

薛迅

华东师范大学

合作者：杨睿，郭家明，李领海，阮自强

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Outline

- 线算子与广义对称性
- 标准模型的线算子谱
- G2HDM 模型的线算子谱
- 对称性自发破缺
- Theta 角
- 推广到相关标准模型拓展

广义对称性

- 在**微分形式语言中**， Noether 流可以认为是1-form的分量

$$J = J_\mu dx^\mu$$

- 它的Hodge dual

$$*J = \frac{1}{(D-1)!} J_\mu \epsilon^\mu_{\mu_1\cdots\mu_{D-1}} dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_{D-1}}$$

是一个(D-1)-form，且是一个闭合形式

- 守恒定律为

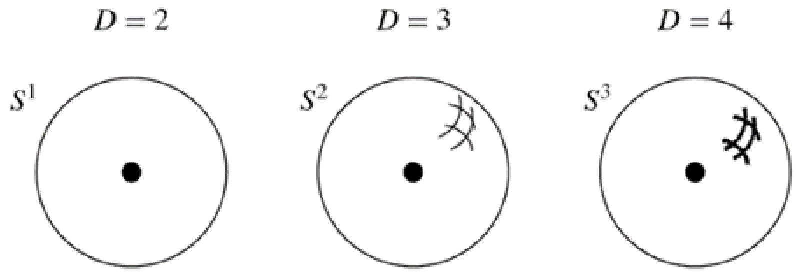
$$d * J = \partial_\mu J^\mu dx^0 \wedge \cdots \wedge dx^{D-1} = 0$$

- Noether 荷则在$(D-1)$维的子流形上积分

$$Q(\Sigma) = \int_\Sigma *J = \int_\Sigma \frac{1}{(D-1)!} J_\mu \epsilon^\mu_{\mu_1\cdots\mu_{D-1}} dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_{D-1}}$$

对称性的拓扑表述

- 在D维区域Ω_Σ进行积分，边界是 (D-1)维子流形Σ，$\partial\Omega_\Sigma = \Sigma$，得到
- $\int_{\Omega_\Sigma}\langle d\ *\ J\phi(y)\rangle = \int_\Sigma \langle * J\phi(y)\rangle = \langle Q(\Sigma)\phi(y)\rangle$
- Ward等式$\partial_{\mu x}\langle J_a^\mu(x)\varphi(y)\rangle = -i\delta(D)(x - y)\langle\delta_a\phi(y)\rangle$变为
- $\langle Q(\Sigma)\phi(y)\rangle = -i\int_{\Omega_\Sigma} d^D x\,\delta^{(D)}(x-y)\langle\delta_a\phi(y)\rangle$
- $\int_{\Omega_\Sigma} d^D x\,\delta^{(D)}(x-y)$可看成$\Omega_\Sigma$和$y$的相交数，等于$\Sigma$和$y$的环绕数（link number）
- $\mathrm{Link}(\Sigma, y) = \int_{\Omega_\Sigma} d^D x\,\delta^{(D)}(x-y)$
- $\langle Q(\Sigma)\phi(y)\rangle = -i\mathrm{Link}(\Sigma, y)\langle\delta\phi(y)\rangle$
- S^{D-1}维球和一个点的环绕



对称性的拓扑表述

- 考虑Ω_Σ到$\Omega_\Sigma{}' = \Omega_\Sigma \cup \Omega_0$的变形，使得$y$不属于$\Omega_0$

$$Q(\Sigma + \partial\Omega_0) = \int_{\Omega_\Sigma \cup \Omega_0} \langle d * J\phi(y)\rangle = \int_{\Omega_\Sigma} \langle d * J\phi(y)\rangle + \int_{\Omega_0} \langle d * J\phi(y)\rangle$$

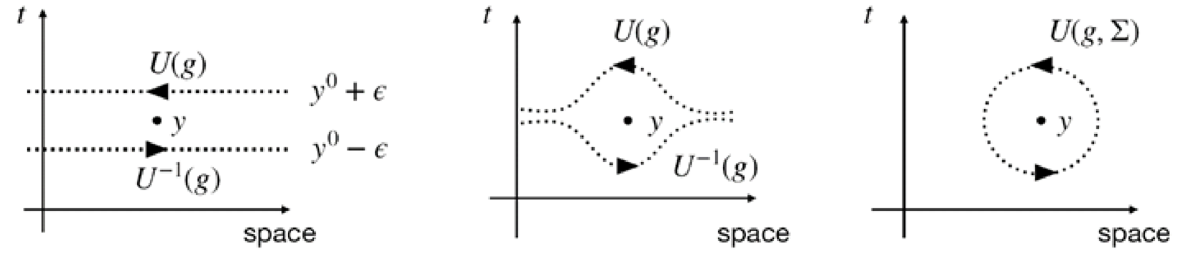
$$= \int_{\Omega_\Sigma} \langle d * J\phi(y)\rangle = Q(\Sigma)$$

- 守恒荷$Q(\Sigma)$是拓扑不变量
- 守恒定律$d * J = 0$表述成了算符$Q(\Sigma)$ 是拓扑不变量

对称性的拓扑表述

- $U(g)$ 对局域算符的作用，$\langle U(g)\phi(y)U^{-1}(g)\rangle = R(g)\langle\phi(y)\rangle$
- 如果y和Σ是环绕的，有
- $\langle U(g,\Sigma)\phi(y)\rangle = R(g)\langle\phi(y)\rangle$，其中$U(g,\Sigma)$是与对称群g相关联的幺正拓扑算符，R代表场变换相应的表示，

  $U(g,\Sigma) = e^{i\alpha_a Q_a}, R(g) = e^{\alpha_a t_a}$，其中$t_a$对应于$\phi$所属的表示的生成元
- 这称为0形式对称性，意味着在对称性下的带荷对象是在一个点，即在一个0维区域支撑的局域算符$\phi(y)$



广义对称性

- **从普通对称性到高形式对称性**
- 经典理论中

$$\delta S = \int d^D x J_a^\mu \partial_\mu \epsilon_a(x)$$

- 经典场的对称性

$$\partial_\mu J_a^\mu = 0,\quad Q_a = \int d^{D-1} x J_a^0 \;\Rightarrow\; \frac{d}{dt} Q_a = 0$$

- 经典场论对称变换正则形式

$$\delta \phi_a = \{\phi, Q_a\}$$

- 量子场论中对称变换的幺正算符可以通过Noether 荷构造

$$U = e^{i\epsilon_a Q_a}$$

- 量子场论对称变换正则形式

$$\delta_a \phi = i[Q_a, \phi]$$

高形式对称性

- **从0-form到1-form**
- 0-form的全局对称性：在对称性下的带荷对象是在一个点，参数ξ是闭合的0-form，$d\xi=0$；局域化$\delta S=\int_{\mathcal{M}^{(D)}} * J \wedge d\xi$，其中$\xi$变为局域参数，即d$\xi$不再是闭合的；守恒律$d * J=0$
- 1-form的全局对称性：参数$\xi_1=\xi_\mu dx^\mu$是闭合的1-form；变换的全局性质转化为参数的平坦条件$\partial_\mu \xi_\nu-\partial_\nu \xi_\mu=0$；作用量变分$\delta S=\int_{\mathcal{M}^{(D)}} * J \wedge d\xi_1$；$*J$是(D-2)-form，$J$是2-form；守恒律$d * J=0$，分量为$\partial_\mu J^{\mu\nu}=0, J^{\mu\nu}=-J^{\nu\mu}$；
- 在闭合$D-2$子流形上可定义守恒荷，$Q(\Sigma_{D-2}) \equiv \int_{\Sigma_{D-2}} * J$
- 这个对称性算符下的带荷对象，即与Σ_{D-2}具有非平凡环绕的对象

高形式对称性的带荷算符

- 为理解1形式对称性下带荷对象，对于0形式对称性变换参数ξ为常数的无穷小整体局部算符变换为

$$\phi(x) \rightarrow \phi'(x) = \phi(x) + \xi\delta\phi(x)$$

- 用庞加莱对偶将$(d-p)$-形式与p维流形关联起来（$D-1=d$）
- $\xi_{i_{p+1}\ldots i_d}(x) \equiv \frac{1}{p!}\int_{\Sigma_p} \epsilon_{i_1\ldots i_p i_{p+1}\ldots i_d}\,\delta^{(d)}(\vec{x}-\vec{y})dy^{i_1}\wedge\cdots\wedge dy^{i_p}$
- 整体对称性的参数为从p维子流形构造的$(d-p)$形式$\xi_{d-p}(\Sigma_p)$，自动满足$\mathrm{d}\xi_{d-p} = 0$

高形式对称性的带荷算符

- p = d得到$\xi(x)=\frac{1}{d!}\int_{\Sigma_d}\epsilon_{i_1\dots i_p}\delta^{(d)}(\vec{x}-\vec{y})dy^{i_1}\wedge\cdots\wedge dy^{i_d}=1$即普通全局对称性的参数是封闭的0形式，在流形的0维区域（点）上支撑，带荷对象为局域算符
- $p=d-1$的庞加莱对偶是一个1形式$\xi_1(\Sigma_{d-1})$，分量为

$$\xi_{i_d}(x)=\frac{1}{(d-1)!}\int_{\Sigma_{d-1}}\epsilon_{i_1\dots i_{d-1}i_d}\delta^{(d)}(\vec{x}-\vec{y})dy^{i_1}\wedge\cdots\wedge dy^{i_{d-1}}.$$

高形式对称性的带荷算符

- 1形式对称性下的带荷对象是沿线支撑的算符，即线算符。所以给定一个沿线$\mathcal{C}$支撑的算符，变换的参数是 $\int_{\mathcal{C}} \xi_i(\Sigma_{d-1}) = \int_{\mathcal{C}} \xi_i dx^i$
- 线算符的无穷小变换：$W[C] \to W[C]' = W[C] + \int_C \xi_i(\Sigma_{d-1})\, \delta W[C]$
- 一般对缺陷线的对称变换：$W[C] \to W[C]' = W[C] + \int_C \xi_1(\Sigma_{D-2})\, \delta W[C]$
- 在1-form对称下，带荷物体是线算子。

狄拉克量子条件

- 磁单极

$$B = \frac{g\boldsymbol{r}}{4\pi r^3} \Rightarrow \int d\boldsymbol{S} \cdot \boldsymbol{B} = g$$

- Dirac quantum condition

$$eg = 2\pi\hbar\mathbb{Z}，\ e_1 g_2 - e_2 g_1 = 2\pi\hbar\mathbb{Z}$$

- Theta term

$$S_\theta = \frac{\theta e^2}{4\pi^2\hbar} \int d^4x \frac{1}{4} * F^{\mu\nu} F_{\mu\nu} = -\frac{\theta e^2}{4\pi^2\hbar c} \int d^4x \boldsymbol{E} \cdot \boldsymbol{B}$$

- 全导数形式的Theta term

$$S_\theta = \frac{\theta e^2}{8\pi^2\hbar} \int d^4x \partial_\mu \left(\epsilon^{\mu\nu\rho\sigma} A_\nu \partial_\rho A_\sigma \right)$$

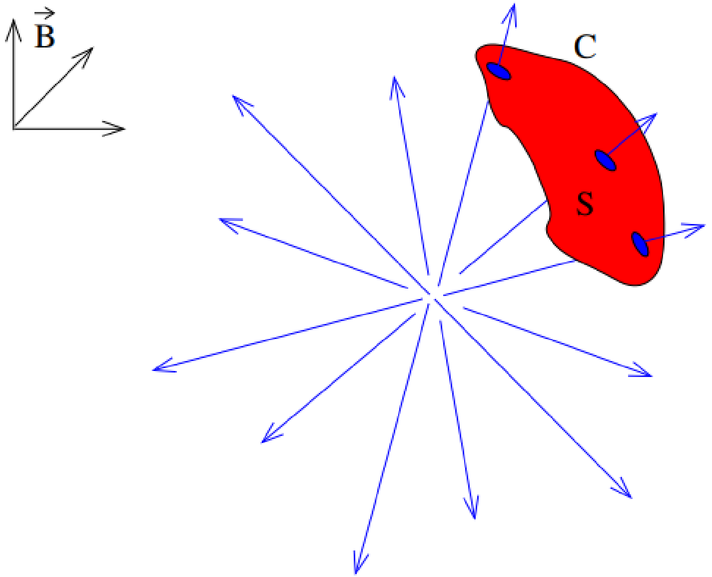
磁单极

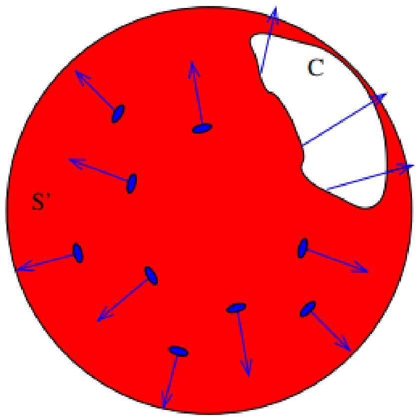
- 类似电荷，可以引入磁荷，或称磁单极。类似静电场，磁荷也可以得到磁荷的场

$$\boldsymbol{B} = \frac{g\hat{\boldsymbol{r}}}{4\pi r^3} \Rightarrow \int d\boldsymbol{S}\cdot\boldsymbol{B} = g$$

- 如果有电荷为e的带电粒子，在磁荷为g的磁单极背景场中，保持磁单极固定，让带电粒子沿着磁单极外一闭合路径走一圈。如图，走过的路径是包围面积为S的边界，立体角为Ω，那么带电粒子获得相位

$$e^{ie\alpha/\hbar}, \quad \alpha = \int_S d\boldsymbol{S}\cdot\boldsymbol{B} = \frac{\Omega g}{4\pi}$$





狄拉克量子条件

- 如右图，路径也是S'的边界，立体角$\Omega' = 4\pi - \Omega$，则获得相位变成

$$\alpha' = -\frac{(4\pi - \Omega)g}{4\pi}$$

- 两者获得相位必须相等，即$e^{ie\alpha/\hbar} = e^{ie\alpha\prime/\hbar}$。这就给出Dirac量子条件

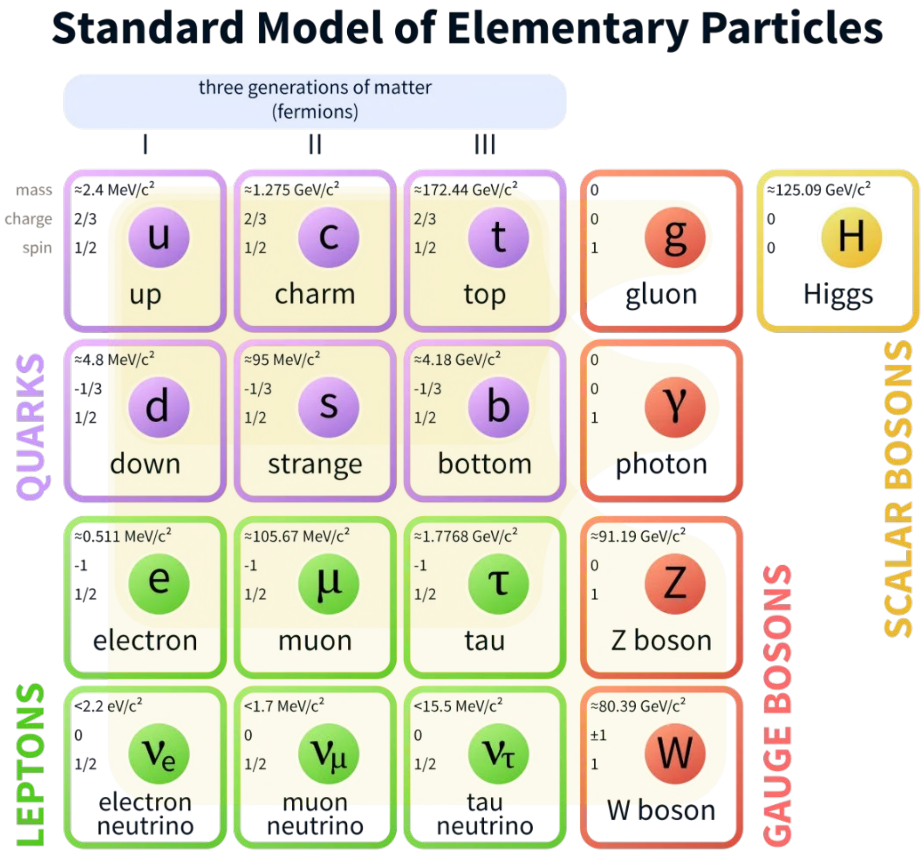
$$eg = 2\pi\hbar n，\ n \in \boldsymbol{Z}$$

- 相似地，对于两个既携带电荷又带有磁荷的dyon粒子，带荷分别为(e_1, g_1)、(e_2, g_2)，Dirac量子化条件推广为

$$e_1 g_2 - e_2 g_1 \in 2\pi\hbar\boldsymbol{Z}$$

标准模型

- 标准模型有三代粒子，规范群为$\tilde{G} = SU(3)_C \times SU(2)_L \times U(1)_Y$



标准模型粒子谱

标准模型

- 标准模型规范群为$\tilde{G} = SU(3)_C \times SU(2)_L \times U(1)_Y$

Matter Fields	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Q_L	3	2	1
U_R	3	1	4
D_R	3	1	-2
L_L	1	2	-3
E_R	1	1	-6
H	1	2	3

标准模型粒子谱

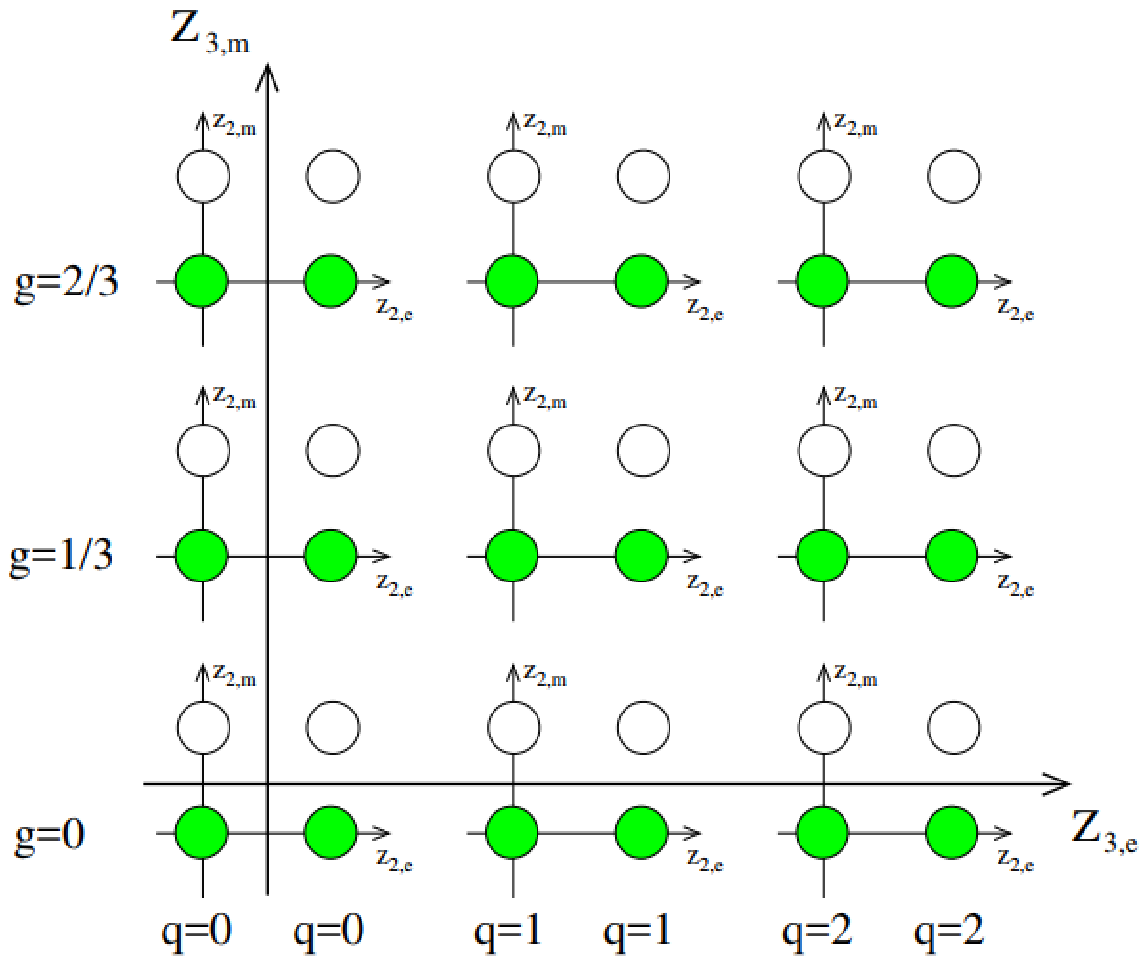
标准模型的规范群

- 标准模型规范群可以取为$G=\tilde{G}/\Gamma$，其中$\Gamma=\mathbf{1},\boldsymbol{Z}_2,\boldsymbol{Z}_3,\boldsymbol{Z}_6$，是$G$的中心或中心的子群，其规范动力学完全相同
- Γ的生成元可以写为：$e^{2\mathrm{i}\pi\frac{q}{6}}e^{\mathrm{i}\pi z_2^e}e^{\frac{2}{3}\mathrm{i}\pi z_3^e}$
- 不同的Γ选择，规范理论整体性质不同，其线算子（Wilson 线， ‘t Hooft 线）谱不同
- 线算子Wilson line：$W[C]=Tr_{R_e}Pe^{\mathrm{i}\oint a}$，与表示有关，可以用权格子空间的点$z^e$来标记
- 线算子 ‘t Hooft line：$T[C]=Tr_{R_m}Pe^{\mathrm{i}\oint \tilde{a}}$，用根格子空间的点$z^m$标记

标准模型的线算子

- 一般的线算子既含有电荷又有磁荷，可以用(z^e, z^m)来标记
- 标准模型的Wilson line可以用(z_2^e, z_3^e, q)标记， ‘t Hooft line用(z_2^m, z_3^m, g)标记
- 它们之间满足Dirac量子化条件：$3z_2^e z_2^m + 2z_3^e z_3^m - 6gq \in 6\boldsymbol{Z}$
- 对$G = \tilde{G}/\Gamma$，Wilson line在Γ群元变换下不变（比如：对于$\Gamma = \boldsymbol{Z}_3$，生成元为$e^{\frac{2}{3}\mathrm{i}\pi q} e^{\frac{2}{3}\mathrm{i}\pi z_3^e} = e^{\frac{2}{3}\mathrm{i}\pi(q+z_3^e)}$，则有$e^{\frac{2}{3}i\pi(q+z_3^e)} = e^{\frac{4}{3}i\pi(q+z_3^e)} = 1$。）可以给出对电荷的约束：$q = z_3^e \mathrm{mod} 3$
- 通过电荷的约束，求解Dirac量子化条件，可以得到对磁荷的约束：$3g = z_3^m \mathrm{mod} 3$

标准模型的线算子谱



标准模型的Theta项

- 规范不变的拉氏量还容许一个表面项，即θ项。对$U(1)\times SU(N)$

$$S_\theta=\frac{\theta_N}{16\pi^2}\int tr(\ ^*f_N f_N)+\frac{\tilde{\theta}}{16\pi^2}\int\ ^*ff$$

- $\Gamma=1$时$\theta\in[0,2\pi)$，$\theta=0,\pi$理论时间反演不变。
- $\Gamma=\boldsymbol{Z}_N$，θ项变为

$$S_\theta=\frac{\theta_N}{16\pi^2}\int tr(\ ^*gg)+\frac{\tilde{\theta}-N\theta_N}{16\pi^2N^2}\int\ ^*(trg)(trg)$$

- θ的取值范围会随着Γ而改变；时间反演不变的θ角会因Γ不同而不同
- 例如：$\Gamma=\boldsymbol{Z}_2$，$\tilde{\theta}\in[0,8\pi)$，$\tilde{\theta}=0,4\pi$理论时间反演不变

G2HDM模型

- 把暗物质纳入规范理论框架，探索新物理需要对标准模型进行拓展
- 规范双希格斯二重态模型 (G2HDM)，规范群：$\tilde{G} = SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(2)_H \times U(1)_X$

Matter Fields	$\mathrm{SU}(3)_C$	$\mathrm{SU}(2)_L$	$\mathrm{SU}(2)_H$	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_X$	h-parity
$Q_L = (u_L \;\; d_L)^{\mathrm{T}}$	3	2	1	1/6	0	$+\,+$
$U_R = \left(u_R \;\; u_R^H\right)^{\mathrm{T}}$	3	1	2	2/3	1	$+\,-$
$D_R = \left(d_R^H \;\; d_R\right)^{\mathrm{T}}$	3	1	2	$-1/3$	-1	$-\,+$
u_L^H	3	1	1	2/3	0	$-$
d_L^H	3	1	1	$-1/3$	0	$-$
$L_L = (\nu_L \;\; e_L)^{\mathrm{T}}$	1	2	1	$-1/2$	0	$+\,+$
$N_R = \left(\nu_R \;\; \nu_R^H\right)^{\mathrm{T}}$	1	1	2	0	1	$+\,-$
$E_R = \left(e_R^H \;\; e_R\right)^{\mathrm{T}}$	1	1	2	-1	-1	$-\,+$
ν_L^H	1	1	1	0	0	$-$
e_L^H	1	1	1	-1	0	$-$
$H = (H_1 \;\; H_2)^{\mathrm{T}}$	1	2	2	1/2	1	$+\,-$
$\Phi_H = (\Phi_1 \;\; \Phi_2)^{\mathrm{T}}$	1	1	2	0	1	$-\,+$
$\mathcal{S}$	1	1	1	0	0	$+$

G2HDM的粒子谱

G2HDM的规范群取法

$$G=\frac{U(1)_Y\times SU(2)_L\times SU(3)_C}{\mathbf{Z}_p}\times\frac{SU(2)_H\times U(1)_X}{\mathbf{Z}_m}(1)=\frac{U(1)_Y\times SU(2)_H\times SU(3)_C}{\mathbf{Z}_p}\times\frac{SU(2)_L\times U(1)_X}{\mathbf{Z}_m}(2)$$

$$=\frac{U(1)_Y\times SU(2)_L}{\mathbf{Z}_p}\times\frac{SU(3)_C\times SU(2)_H\times U(1)_X}{\mathbf{Z}_m}(3)=\frac{U(1)_Y\times SU(2)_H}{\mathbf{Z}_p}\times\frac{SU(2)_L\times SU(3)_C\times U(1)_X}{\mathbf{Z}_m}(4)$$

$$=\frac{U(1)_Y\times SU(2)_L\times U(1)_X}{\mathbf{Z}_p}\times\frac{SU(3)_C\times SU(2)_H}{\mathbf{Z}_m}(5)=\frac{U(1)_Y\times SU(2)_H\times U(1)_X}{\mathbf{Z}_p}\times\frac{SU(3)_C\times SU(2)_L}{\mathbf{Z}_m}(6)$$

$$=\frac{U(1)_Y\times SU(3)_C\times U(1)_X}{\mathbf{Z}_N}\times\frac{SU(2)_L}{\mathbf{Z}_M}\times\frac{SU(2)_H}{\mathbf{Z}_P}(7)=\frac{U(1)_Y\times SU(3)_C\times SU(2)_L\times U(1)_X}{\mathbf{Z}_p}\times\frac{SU(2)_H}{\mathbf{Z}_m}(8)$$

$$=\frac{U(1)_Y\times SU(3)_C\times SU(2)_H\times U(1)_X}{\mathbf{Z}_p}\times\frac{SU(2)_L}{\mathbf{Z}_m}(9)$$

G2HDM的Dirac量子条件

- Dirac量子条件：$-6gq+3z_2^e z_2^m+2z_3^e z_3^m+3x_2^e x_2^m-6kh=0\mathrm{mod}6$
- 以$G=\frac{U(1)_Y\times SU(2)_L\times SU(3)_C}{\boldsymbol{Z}_p}\times\frac{SU(2)_H\times U(1)_X}{\boldsymbol{Z}_m}$为例
- 可取的中心有$\Gamma=\mathbf{1},\boldsymbol{Z}_{2L},\boldsymbol{Z}_{2H},\boldsymbol{Z}_3,\boldsymbol{Z}_{2L}\times\boldsymbol{Z}_{2H},\boldsymbol{Z}_{6L},\boldsymbol{Z}_3\times\boldsymbol{Z}_{2H},\boldsymbol{Z}_{6L}\times\boldsymbol{Z}_{2H}$共8种
- 对应生成元有两个$e^{2\mathrm{i}\pi\frac{q}{6}}e^{\mathrm{i}\pi z_2^e}e^{\frac{2}{3}\mathrm{i}\pi z_3^e}$，$e^{2\mathrm{i}\pi\frac{h}{2}}e^{\mathrm{i}\pi x_2^e}$
- 局域动力学在任一中心下不变，可以给出对电荷的约束
- 通过电荷的约束，再解Dirac量子化条件，可以得到对磁荷的约束

G2HDM的线算子谱

- 以$\Gamma = \boldsymbol{Z}_3 \times \boldsymbol{Z}_{2H}$为例，对应两个生成元分别为

$$e^{\frac{2}{3}\mathrm{i}\pi q} e^{\frac{2}{3}\mathrm{i}\pi z_3^e} = e^{\frac{2}{3}\mathrm{i}\pi(q+z_3^e)}, \quad e^{\mathrm{i}\pi x_2^e} e^{\mathrm{i}\pi h} = e^{\frac{1}{3}\mathrm{i}\pi(3x_2^e+3h)}$$

- 给出对电荷的限制

$$q = z_3^e \mathrm{mod} 3, \quad h = x_2^e \mathrm{mod} 2$$

- 给出对磁荷的限制

$$3g = z_3^m \mathrm{mod} 3, \quad 2k = x_2^m \mathrm{mod} 2$$

- 以$\Gamma = \boldsymbol{Z}_3$为例，$\tilde{\theta} \in [0,18\pi)$，$\tilde{\theta} = 0,9\pi$理论时间反演不变

G2HDM的线算子谱

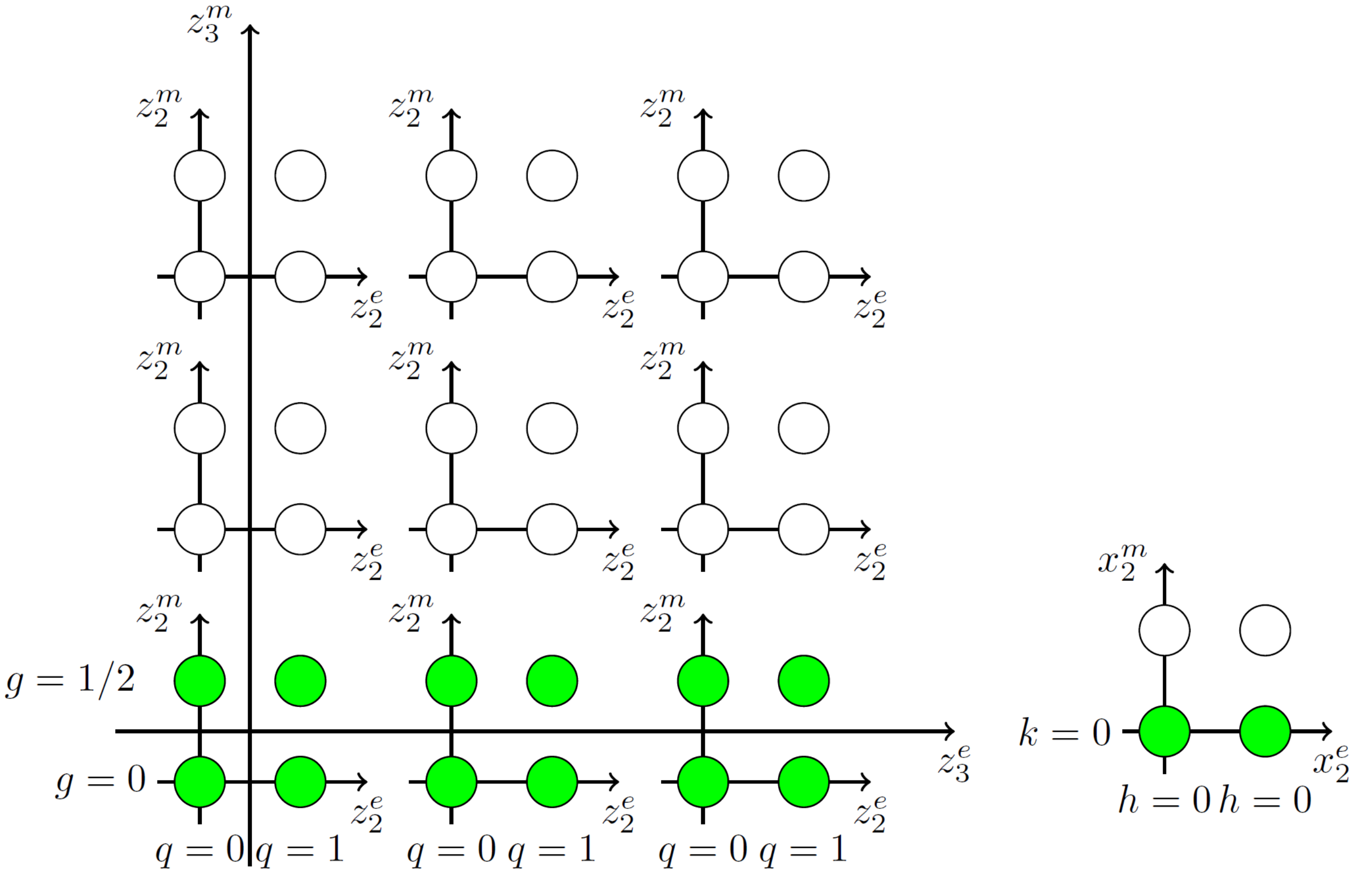


Figure 2: $\Gamma = \boldsymbol{Z}_{2L}$

G2HDM的线算子谱

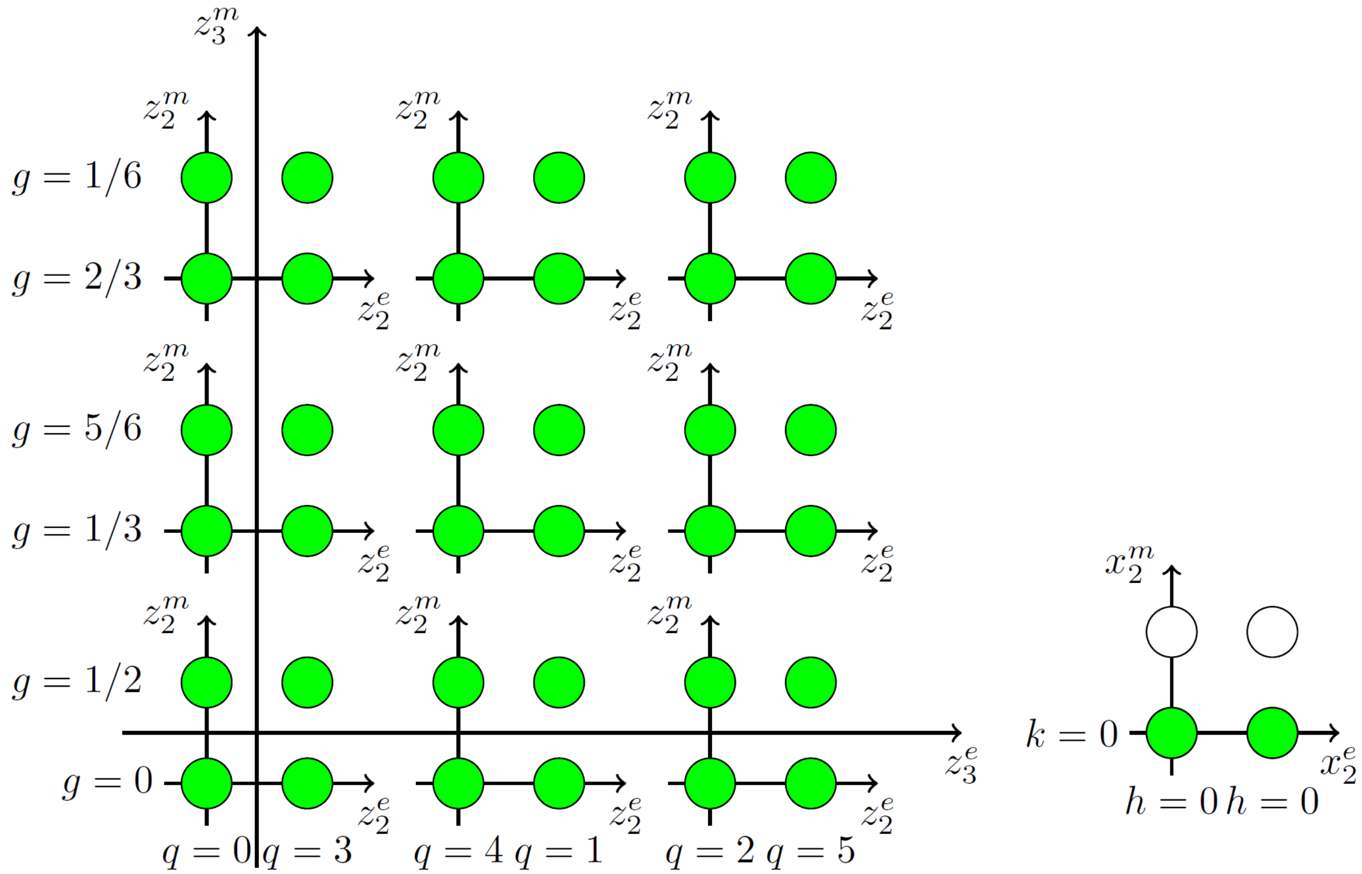
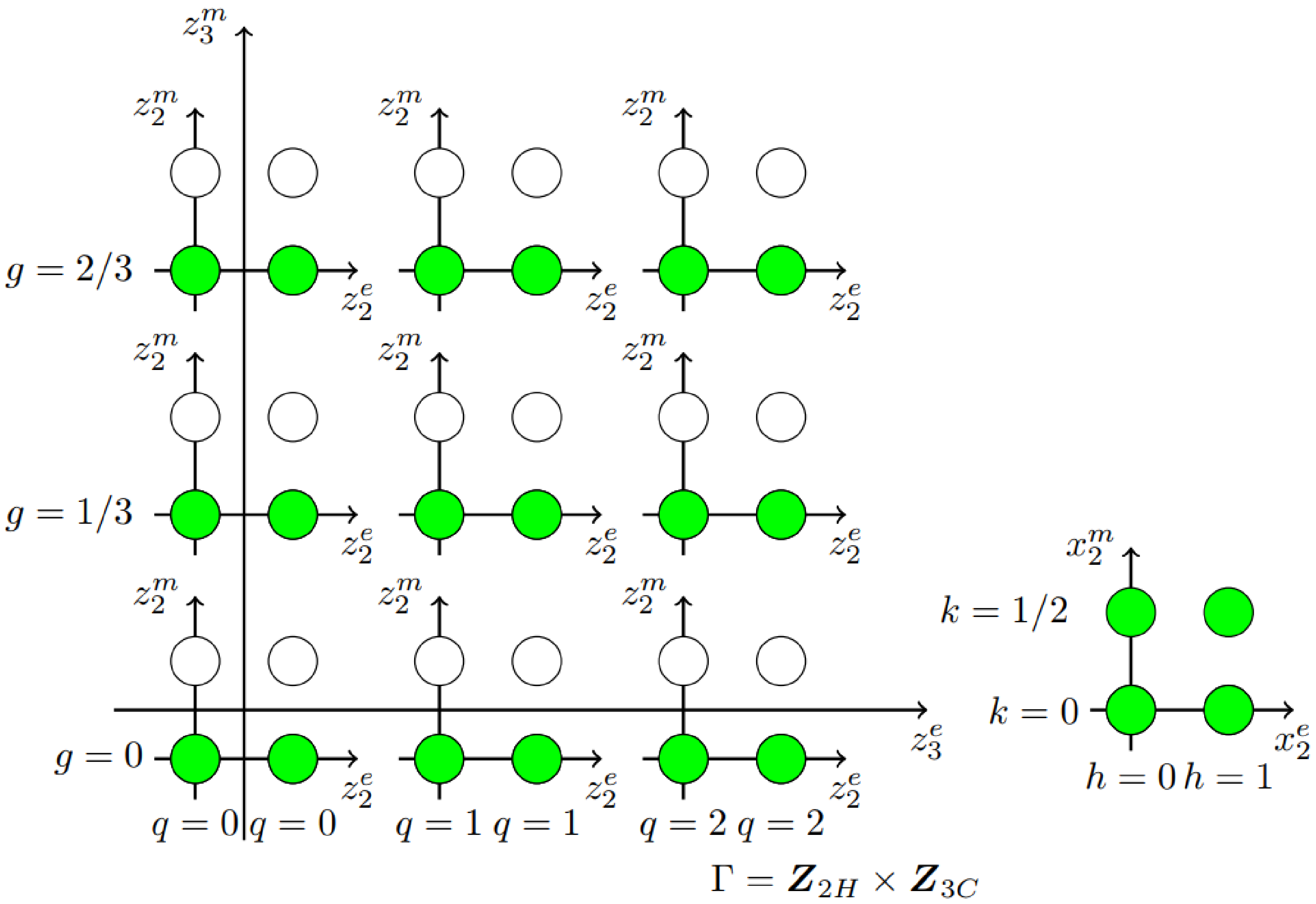


Figure 5: $\Gamma = \boldsymbol{Z}_{6L}$

G2HDM的线算子谱



z_3^m
z_2^m
$g = 2/3$
z_2^e
$g = 1/3$
$g = 0$
z_3^e
$q = 0$ $q = 0$
$q = 1$ $q = 1$
$q = 2$ $q = 2$
x_2^m
$k = 1/2$
$k = 0$
x_2^e
$h = 0$ $h = 1$
$\Gamma = \boldsymbol{Z}_{2H} \times \boldsymbol{Z}_{3C}$

G2HDM模型的对称性破缺

- 盖尔曼-西岛关系：$Q = \frac{q}{6} + \frac{1}{2} z_2^e$，$Q_X = \frac{h}{2} + \frac{1}{2} x_2^e$
- Higgs$(2,2,1)_{3,1}$和$(1,2,1)_{0,1}$确定未破缺的‘t Hooft line：$6g = z_2^m mod2$， $2k = x_2^m mod2$
- 未破缺的‘t Hooft line的磁荷：$G = 6g$，$G_X = 2k$

G2HDM模型的对称性自发破缺

Gellmann Nishijima formula

$$Q=\frac{q}{6}+\frac{1}{2}\lambda_2^e \qquad Q_X=\frac{h}{2}+\frac{1}{2}\rho_2^e$$

$$\textit{Dirac quantization condition}\ -6qg+3z_2^e z_2^m+2z_3^e z_3^m+3x_2^e x_2^m-6hk=0\bmod 6$$

$$\left(SU(2)_L, SU(2)_H, SU(3)\right)_{Y,X}\Rightarrow\left(z_2^e, x_2^e, z_3^e\right)_{q,h}$$

$$higgs=(2,2,1)_{3,1}\Rightarrow(1,1,0)_{3,1}=\left(z_2^e, x_2^e, z_3^e\right)_{q,h}$$

$$-6qg+3z_2^e z_2^m+3x_2^e x_2^m-6hk=-6\times 3g+3z_2^m+3x_2^m-6k=0\bmod 6\Rightarrow -6g+z_2^m+x_2^m-2k=0\bmod 2\Rightarrow 6g+2k=\left(z_2^m+x_2^m\right)\bmod 2$$

$$higgs=(1,2,1)_{0,1}\Rightarrow(0,1,0)_{0,1}=\left(z_2^e, x_2^e, z_3^e\right)_{q,h}$$

$$-6qg+3z_2^e z_2^m+3x_2^e x_2^m-6hk=3x_2^m-6k=0\bmod 6\Rightarrow 2k=x_2^m\bmod 2$$

$$G=6g=z_2^m\bmod 2=\lambda_2^m$$

$$G_X=2k=x_2^m\bmod 2=\rho_2^m$$

$$-6qg+3z_2^e z_2^m+3x_2^e x_2^m-6hk=0\bmod 6\Rightarrow g\left(-q+\frac{1}{2g}z_2^e z_2^m\right)+k\left(-h+\frac{1}{2k}x_2^e x_2^m\right)=6g\left(-\frac{q}{6}+\frac{1}{6}\frac{1}{2g}z_2^e 6g\right)+2k\left(-\frac{h}{2}+\frac{1}{2}\frac{1}{2k}x_2^e 2k\right)$$

$$=6g\left(-\frac{q}{6}+\frac{1}{2}z_2^e\right)+2k\left(-\frac{h}{2}+\frac{1}{2}x_2^e\right)=QG+Q_X G_X$$

G2HDM模型的对称性自发破缺

$$G=\frac{U(1)_Y\times SU(2)_L\times SU(3)_C}{\boldsymbol{Z}_N}\times\frac{SU(2)_H\times U(1)_X}{\boldsymbol{Z}_M}$$

$$Q=\frac{q}{6}+\frac{1}{2}\lambda_2^e \qquad Q_X=\frac{h}{2}+\frac{1}{2}\rho_2^e$$

$$G=6g \qquad G_X=2k$$

$$\left(SU(2)_L,SU(2)_H,SU(3)\right)_{Y,X}\Leftarrow\left(z_2^e,x_2^e,z_3^e\right)_{q,h}$$

$$\boldsymbol{1}:\begin{cases}q\in\boldsymbol{Z}\Rightarrow Q=\frac{1}{6}+0=\frac{1}{6}\Rightarrow(1,1,1)_{1,0}\Rightarrow(Q_X=0);g\in\boldsymbol{Z}\Rightarrow G=6;\lambda_2^m=6\\ h\in\boldsymbol{Z}\Rightarrow Q_X=\frac{1}{2}+0=\frac{1}{2}\Rightarrow(1,1,1)_{0,1}\Rightarrow(Q=0);k\in\boldsymbol{Z}\Rightarrow G_X=2;\rho_2^m=2\end{cases}$$

$$\boldsymbol{Z}_{2L}:\begin{cases}q=z_2^e\bmod 2;z_2^e=0,1\Rightarrow Q=\frac{2}{6}+0=\frac{1}{3}\Rightarrow(1,1,1)_{2,0}\Rightarrow(Q_X=0);g-2z_2^m=0\bmod 2;z_2^m=0,1\bmod 2\Rightarrow G=3;\lambda_2^m=3\\ Q_X=\frac{1}{2}\Rightarrow(1,1,1)_{0,1}\Rightarrow(Q=0);G_X=2;\rho_2^m=2\end{cases}$$

$$\boldsymbol{Z}_{2H}:\begin{cases}Q=\frac{1}{6}\Rightarrow(1,1,1)_{1,0}\Rightarrow(Q_X=0);G=6;\lambda_2^m=6\\ h=x_2^e\bmod 2;x_2^e=0,1\Rightarrow Q_X=\frac{2}{2}+0=1\Rightarrow(1,1,1)_{0,2}\Rightarrow(Q=0);k-2x_2^m=0\bmod 2;x_2^m=0,1\bmod 2\Rightarrow G_X=1;\rho_2^m=1\end{cases}$$

G2HDM模型的对称性自发破缺

$$\boldsymbol{Z}_{3C}:\begin{cases} q = z_3^e \bmod 3; z_3^e = 0,1,2 \Rightarrow Q = \frac{1}{6} + 0 = \frac{1}{6} \Rightarrow (1,1,3)_{1,0} \Rightarrow (Q_X = 0); g - 3z_3^m = 0 \bmod 3; z_3^m = 0,1,2 \bmod 3 \Rightarrow G = 2; \lambda_2^m = 2 \\ Q_X = 1 \Rightarrow (1,1,1)_{0,1} \Rightarrow (Q = 0); G_X = 2; \rho_2^m = 6 \end{cases}$$

$$\boldsymbol{Z}_{2L} \times \boldsymbol{Z}_{2H}:\begin{cases} Q = \frac{1}{3} \Rightarrow (1,1,1)_{2,0} \Rightarrow (Q_X = 0); G = 3, \lambda_2^m = 3 \\ Q_X = 1 \Rightarrow (1,1,1)_{0,2} \Rightarrow (Q = 0); G_X = 1, \rho_2^m = 1 \end{cases}$$

$$\boldsymbol{Z}_6:\begin{cases} q = (3z_2^e - 2z_3^e) \bmod 6; z_2^e = 0,1; z_3^e = 0,1,2 \Rightarrow Q = \frac{2}{6} + 0 = \frac{1}{3} \Rightarrow (1,1,3)_{2,0} \Rightarrow (Q_X = 0); 6g = (3z_2^m + 2z_3^m) \bmod 6; z_2^m = 0,1 \bmod 2; z_3^m = 0,1,2 \bmod 3 \Rightarrow G = 1; \lambda_2^m = 1 \\ Q_X = \frac{1}{2} \Rightarrow (1,1,1)_{0,1} \Rightarrow (Q = 0); G_X = 2; \rho_2^m = 2 \end{cases}$$

$$\boldsymbol{Z}_{2H} \times \boldsymbol{Z}_{3C}:\begin{cases} Q = \frac{1}{6} \Rightarrow (1,1,3)_{1,0} \Rightarrow (Q_X = 0); G = 2, \lambda_2^m = 2 \\ Q_X = 1 \Rightarrow (1,1,1)_{0,2} \Rightarrow (Q = 0); G_X = 1, \rho_2^m = 1 \end{cases}$$

$$\boldsymbol{Z}_{2H} \times \boldsymbol{Z}_6:\begin{cases} Q = \frac{1}{6} \Rightarrow (1,1,3)_{2,0} \Rightarrow (Q_X = 0); G = 1, \lambda_2^m = 1 \\ Q_X = 1 \Rightarrow (1,1,1)_{0,2} \Rightarrow (Q = 0); G_X = 1, \rho_2^m = 1 \end{cases}$$

G2HDM模型的Theta角

- how the line operators vary under the θ angle. There are 5 such angles, calling $\tilde{\theta}$,θ_2,θ_3,$\widetilde{\omega}$ and ω_2, for $U(1)_Y$,$SU(2)_L$,$SU(3)_C$,$U(1)_X$ and $SU(2)_H$, respectively.
- the quotient gauge group of the G2HDM could be divided into two categories. $\frac{U(1)\times\cdots}{Z_N}\times\frac{U(1)\times\cdots}{Z_M}\times\cdots$ or $\frac{U(1)\times U(1)\times\cdots}{Z_N}\times\cdots$
- Theta terms of the G2HDM
- $S_\theta=\frac{\widetilde{\theta}}{16\pi^2}\int dx^4\,\widetilde{{}^{\star}f}\tilde{f}+\frac{3\theta_2}{16\pi^2}\int dx^4 tr\left({}^{\star}f_2 f_2\right)+\frac{2\theta_3}{16\pi^2}\int dx^4 tr\left({}^{\star}f_3 f_3\right)+\frac{\widetilde{\omega}}{16\pi^2}\int dx^4\,\widetilde{{}^{\star}g}\tilde{g}+\frac{\omega_2}{16\pi^2}\int dx^4 tr\left({}^{\star}gg\right)$
- Where$\tilde{f}$,f_2,$f_3$are the strength of $U(1)_Y$,$SU(2)_L$,$SU(3)_C$field, respectively. And $\tilde{g}$,g for $U(1)_X$ and $SU(2)_H$, respectively.

G2HDM模型的Theta角

- Theta terms of $\frac{U(1)\times SU(N)}{Z_N}$theory are
- $S_\theta = \frac{\theta_N}{16\pi^2}\int d^4x\mathrm{tr}(\,^\star FF) + \frac{\tilde{\theta}-N\theta_N}{16\pi^2 N^2}\int d^4x^\star(\mathrm{tr}F)(\mathrm{tr}F)$
- Strength$F$derived from field $a+\tilde{a}1_N$. a and $\tilde{a}$ are $U(1)$ and $SU(N)$ gauge fields, respectively.$\theta_N \in [0,2\pi)$,$\tilde{\theta} \in [0,2\pi N^2\big)$, now.
- Consider $\frac{U(1)\times SU(N)\times SU(M)}{Z_{N\times M}}$. Theta terms of $U(1)\times SU(N)\times SU(M)$ theory are
- $S_\theta = \frac{M\theta_N}{16\pi^2}\int dx^4 tr\big(\,^\star f_N 1_M f_N 1_M\big) + \frac{N\theta_M}{16\pi^2}\int dx^4 tr\big(\,^\star f_M 1_N f_M 1_N\big) + \frac{\widetilde{\theta}}{16\pi^2}\int dx^4\,\widetilde{^\star f}\tilde{f}$

G2HDM模型的Theta角

- To describe this new $U(N \times M)$ gauge theory, we introduce two gauge fields $a_N + \tilde{a}1_N$ and $a_M + \tilde{a}1_M$. Then, we could obtain the strength of these two fields $F_N = f_N + \tilde{f}1_N$a nd $F_M = f_M + \tilde{f}1_M$. Theta terms for $\frac{U(1)\times SU(N)\times SU(M)}{Z_{N\times M}}$ theory are
- $S_\theta = \frac{M\theta_N}{16\pi^2}\int dx^4 tr\left({}^\star F_N 1_M F_N 1_M\right) + \frac{N\theta_M}{16\pi^2}\int dx^4 tr\left({}^\star F_M 1_N F_M 1_N\right) + \frac{\tilde{\theta} - NM^2\theta_N - MN^2\theta_M}{16\pi^2(N\times M)^2}\int dx^4 \ {}^\star tr\left(\tilde{f}1_{N\times M}\right) tr\left(\tilde{f}1_{N\times M}\right)$
- $tr\left(\tilde{f}1_{N\times M}\right)$ the last term equals to $tr(F_N 1_M)$ or $tr(F_M 1_N)$.We can find that $\theta_N \in [0, 2\pi)$,$\theta_M \in [0, 2\pi)$ while $\tilde{\theta} \in \left[0, 2\pi N^2 M^2\right)$

G2HDM模型的Theta角

- What if there are two $U(1)$ gauge group in the denominator of the quotient group? Theta terms for$U(1)_Y \times U(1)_X \times SU(N)$ gauge theory are
- $$S_\theta = \frac{\theta_N}{16\pi^2}\int dx^4 tr\left({}^\star f_N f_N\right) + \frac{\tilde{\theta}}{16\pi^2}\int dx^4 \widetilde{{}^\star f}\tilde{f} + \frac{\tilde{\omega}}{16\pi^2}\int dx^4 \widetilde{{}^\star g}\tilde{g}$$
- We introduce two gauge fields, and$a_N + \tilde{b}1_N$. Where $\tilde{b}$ is $U(1)_X$gauge field whose strength is$\tilde{g}$, and$a_N$is$SU(N)$gauge field whose strength is f_N. Then, we can obtain the strength of these two fields
- $F = f_N + \tilde{f}1_N$ and $G = f_N + \tilde{g}1_N$

G2HDM模型的Theta角

- So, the theta terms for $\frac{U(1)_Y \times U(1)_X \times SU(N)}{Z_N}$ can then be written as
- $S_\theta = \frac{\theta_N}{16\pi^2}\frac{1}{2}\int dx^4\left[tr\left(\,^\star FF\right) + tr\left(\,^\star GG\right)\right] + \frac{\tilde{\theta}-\frac{N\theta_N}{2}}{16\pi^2 N^2}\int dx^4\,^\star tr\left(\tilde{f}1_N\right)tr\left(\tilde{f}1_N\right) + \frac{\widetilde{\omega}-\frac{N\theta_N}{2}}{16\pi^2 N^2}\int dx^4\,^\star tr(\tilde{g}1_N)tr(\tilde{g}1_N)$
- Where $tr\left(\tilde{f}1_N\right)$equals to trF,and $tr(\tilde{g}1_N)$ equals to trG. From this equation, we get $\theta \in [0, 2\pi)$ while $\tilde{\theta} \in \left[0, 2\pi N^2\right)$ and $\widetilde{\omega} \in \left[0, 2\pi N^2\right)$.

G2HDM模型的Theta角

- for $\frac{U(1)_Y \times U(1)_X \times SU(N) \times SU(M)}{Z_{N\times M}}$ gauge theory. Theta terms for $U(1)_Y \times U(1)_X \times SU(N) \times SU(M)$ are
- $S_\theta = \frac{M\theta_N}{16\pi^2}\int dx^4 tr\left({}^\star f_N 1_M f_N 1_M\right) + \frac{N\theta_M}{16\pi^2}\int dx^4 tr\left({}^\star f_M 1_N f_M 1_N\right) + \frac{\tilde{\theta}}{16\pi^2}\int dx^4 \widetilde{{}^\star f}\tilde{f} + \frac{\tilde{\omega}}{16\pi^2}\int dx^4 \widetilde{{}^\star g}\tilde{g}$
- There are four canonically normalized gauge fields that introduced, that $a_N + \tilde{a}1_N$,$a_N + \tilde{b}1_N$,$a_M + \tilde{a}1_M$and $a_M + \tilde{b}1_M$. The strength of these fields are
- $F_N = f_N + \tilde{f}1_N$,$G_N = f_N + \tilde{g}1_N$,$F_M = f_M + \tilde{f}1_M$and$G_M = g_M + \tilde{g}1_M$
- respectively.

G2HDM模型的Theta角

- the theta terms become $S_\theta = \frac{M\theta_N}{16\pi^2}\frac{1}{2}\int dx^4[\ ^\star tr(F_N 1_M F_N 1_M) + tr(\ ^\star G_N 1_M G_N 1_M)] + \frac{\theta_M}{16\pi^2}\frac{1}{2}\int[tr(\ ^\star F_M 1_N F_M 1_N) + tr(\ ^\star G_M 1_N G_M 1_N)] +$

$$\frac{\frac{\tilde{\theta}}{N^2M^2} - \frac{1}{2}\frac{\theta_N}{N} - \frac{1}{2}\frac{\theta_M}{M}}{16\pi^2}\int dx^4\ ^\star tr(\tilde{f}1_{N\times M})tr(\tilde{f}1_{N\times M})$$
$$+\frac{\frac{\widetilde{\omega}}{N^2M^2} - \frac{1}{2}\frac{\theta_N}{N} - \frac{1}{2}\frac{\theta_M}{M}}{16\pi^2}\int dx^4\ ^\star tr(\tilde{g}1_{N\times M})tr(\tilde{g}1_{N\times M})$$

- Where $tr(\tilde{f}1_{N\times M})$ equals to $tr(F_N 1_M)$ and $tr(F_M 1_N)$, $tr(\tilde{g}1_{N\times M})$ equals to $tr(G_N 1_M)$and $tr(G_M 1_N)$. We can tell that $\theta_N, \theta_M \in [0, 2\pi)$ while $\tilde{\theta} \in [0, 2\pi N^2M^2)$ and $\widetilde{\omega} \in [0, 2\pi N^2 M^2)$

G2HDM模型CP不变Theta角

- For different quotients, there are different kind of theta terms.
- $G = \frac{U(1)_Y \times SU(2)_L \times SU(3)_C}{Z_N} \times \frac{SU(2)_H \times U(1)_X}{Z_M}$
- Take $\Gamma = Z_{2L} \times Z_{2H}$ for example. There are two $U(2)$ θ angles, say ,$\tilde{\theta}$ and $\widetilde{\omega}$. In which,$\theta_3 = 0, \pi$. Here, when $\theta_2 = 0$, it has $\tilde{\theta} = 0{,}4\pi$; when$\theta_2 = \pi$, it has $\tilde{\theta} = 2\pi, 6\pi$. Meanwhile, it has $\widetilde{\omega} = 0{,}4\pi$ when $\omega_2 = 0$ and $\widetilde{\omega} = 2\pi, 6\pi$ when $\omega_2 = \pi$. All of these are CP invariant, like the theories of $\Gamma = Z_{2L}$and $\Gamma = Z_{2H}$

Conclusion and outlook

- The spectrum of line operator in G2HDM model is obtained for all the possible cases
- The variation of Theta angles under different center group is presented for all the possible cases
- The spectrum of line operator under spontaneously symmetry breaking is discussed for all the possible cases
- The similar treatment to the ledt-right symmetric model of elctro-weak interaction with $\tilde{G} = (SU(3) \times SU(2)_L \times U(1)_{B-L}) \times SU(2)_R$ is in progress
- The inclusion of axion couplings can result the non-invertible symmetry

- THANKS