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Berry Phase in Axion Physics

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Introduction

- **Studies on Berry phase in the axion physics**

Cosmological birefringence	Hoseini et al. 2019, PLB Jain et al. 2021, JCAP
Axion-photon mixing	Capolupo et al. 2015
Formal theory research	Baggio et al. 2017, JHEP

- **We study the Berry phase in the axion physics and show that**
  - **the equivalence of Berry phase induced by axion-fermion and axion-photon interactions**
  - **the connection between the Berry phase and generalized symmetry**

Berry Phase

$$i\frac{\partial}{\partial t}\left|\psi\right\rangle = H(t)\left|\psi\right\rangle$$

Time dependent system

$$\begin{cases} \xi_{\mathrm{dym}} = \int E(t)\mathrm{d}t \\ \xi_{\mathrm{Berry}} = i\oint_C A_\mu \mathrm{d}R^\mu \end{cases}$$

Dynamical Phase

Berry Phase

closed loop

potential

parameter

Example :

$$H(t) = \mathbf{B}(t)\cdot\mathbf{j}$$

Magnetic Field

Spin Operator

Can axions induce the Berry phase?

Berry Phase in Axion Physics

- **The Lagrangian in axion physics**

$$\mathscr{L}_{a\gamma} = \frac{1}{4}\frac{g_\gamma}{f_a} a F^{\mu\nu}\tilde{F}_{\mu\nu} \qquad \mathscr{L}_{af} = -\frac{1}{2}\frac{g_f}{f_a}\partial_\mu a \bar{f}\gamma^\mu\gamma^5 f$$

	a	$F^{\mu\nu}\tilde{F}_{\mu\nu}$	$\partial_\mu \bar{f}\gamma^\mu\gamma^5 f$
CP Parity	**-1**	**-1**	**-1**
T Parity	**-1**	**-1**	**-1**

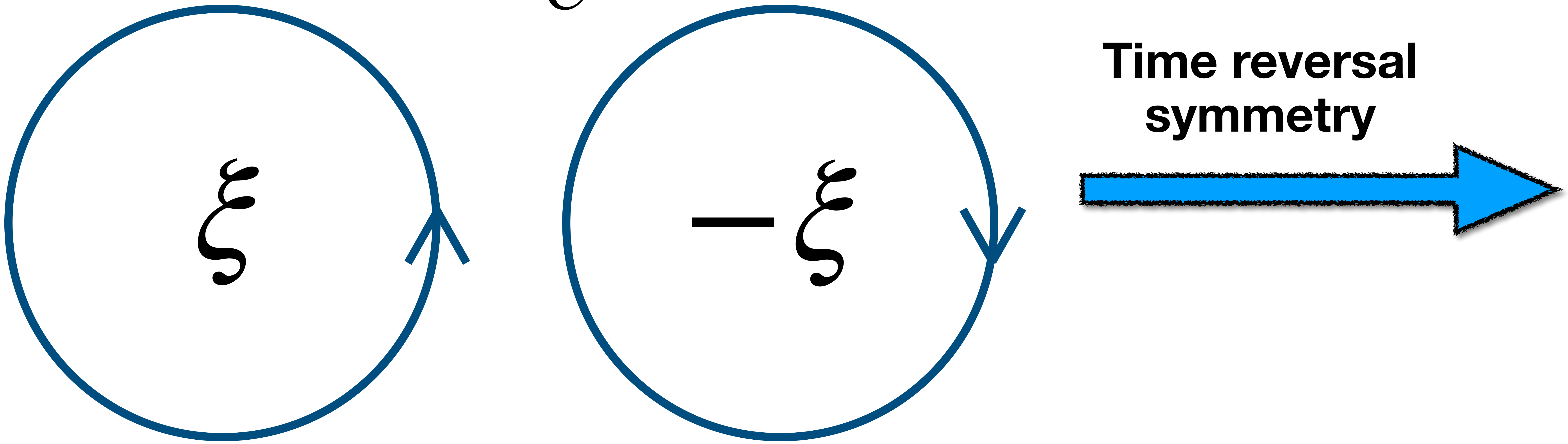
The key property to induce the Berry phase

Berry Phase in Axion Physics

For a non-degenerate quantum system with time reversal symmetry, the Berry phase must be zero.

Baggio et al. 2017, JHEP

$$\xi_{\mathrm{Berry}} = i\oint_C A_\mu \mathrm{d}R^\mu$$



Time reversal symmetry

$$\xi = -\xi$$

No Berry phase

Berry Phase in Axion Physics

$$\mathcal{L}_{af} = -\frac{1}{2}\frac{g_f}{f_a}\partial_\mu a\bar{f}\gamma^\mu\gamma^5 f$$

$$\mathcal{L}_{a\gamma} = \frac{1}{4}\frac{g_\gamma}{f_a}aF^{\mu\nu}\tilde{F}_{\mu\nu}$$

$$\eta_a = \frac{\mathrm{d}a}{\mathrm{d}t}$$

$$H_f = \frac{g_f}{2f_a}\eta_a\frac{\mathbf{p}\cdot\boldsymbol{\sigma}}{m_f}$$

Same Form !

$$H_\gamma = \frac{g_\gamma}{2f_a}\eta_a\frac{\mathbf{k}\cdot\mathbf{S}}{|\mathbf{k}|}$$

$$H(t) = \mathbf{V}(t)\cdot\mathbf{j}$$

Berry Phase in Axion Physics

$$H(t) = \mathbf{V}(t) \cdot \mathbf{j} \quad \begin{cases} \text{Scenario I: vector's direction changes with time} \\ \text{Scenario II: vector's magnitude changes with time} \end{cases}$$

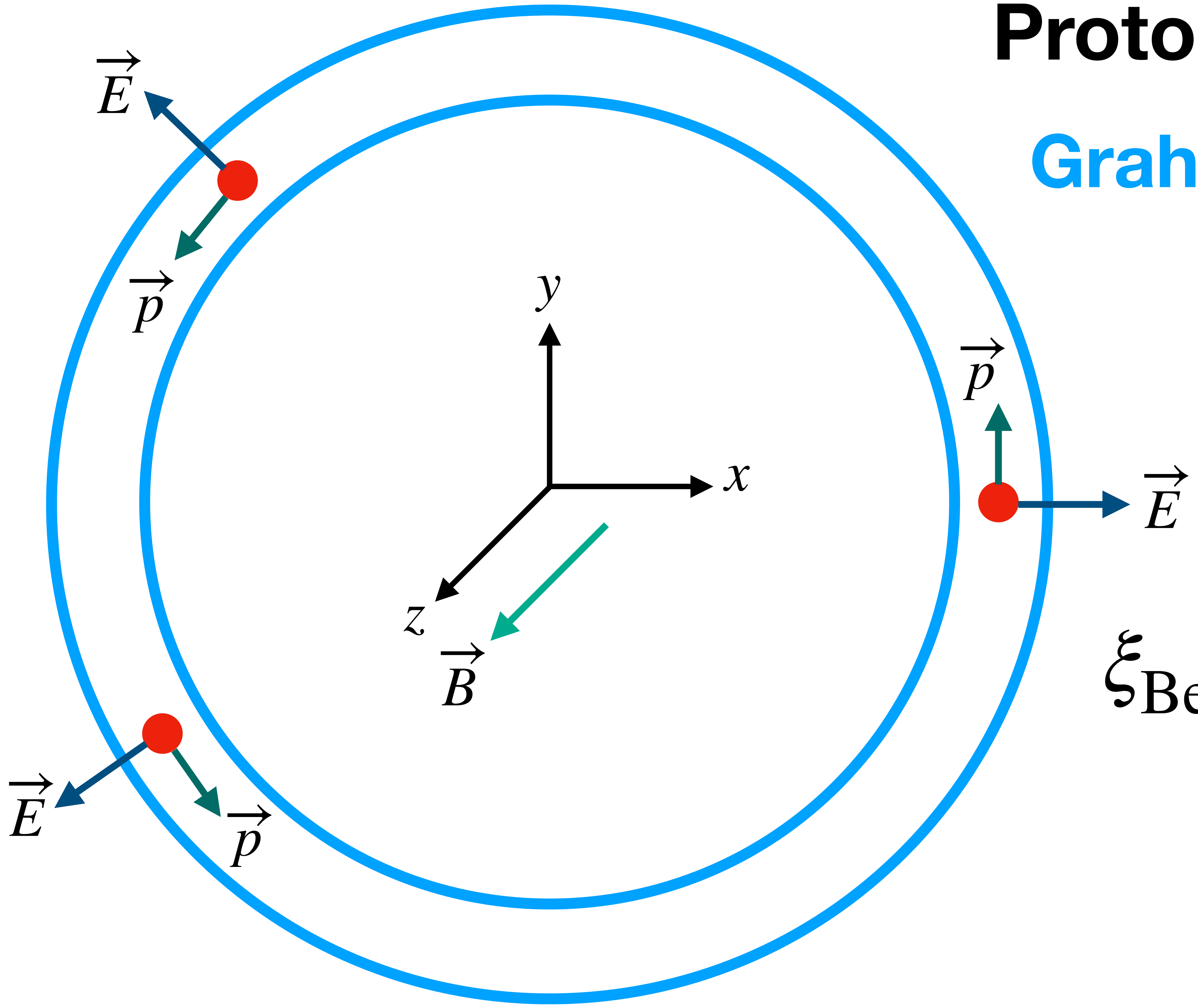
Scenario I: Take the axion-fermion system as an example

Two scenarios are applicable for both systems

Scenario II: Take the axion-photon system as an example

Scenario One: Direction

$$H_f = \frac{g_f}{f_a}\eta_a \frac{\mathbf{p}\cdot\sigma}{m_f} - \frac{gq}{2m_f}\mathbf{B}\cdot\sigma - \frac{gq}{2m_f^2}(\mathbf{E}\times\mathbf{p})\cdot\sigma + (\gamma-1)\frac{\mathbf{a}\times\mathbf{v}}{v^2}\cdot\sigma$$



Proton Ring Experiment

Graham et al. 2017, PRD

Assume η_a is a constant

⇕

Axion mass is very small

$$\xi_{\mathrm{Berry}} \sim \mathcal{O}\left(\frac{g_f^2/f_a^2}{|B-\omega|}\right) \sim 10^{-36}$$

→ **Very small value**

Scenario One: Direction

Q: Why the Berry phase is so small

A: Very large Standard Model background

Rotation Effect

$$\xi_{\text{Berry}} \sim \mathcal{O}\left(\frac{g_f^2/f_a^2}{|B-\omega|}\right)$$

Electromagnetism Effect

Resonance Condition

$$GB + vE\left(G - \frac{1}{\gamma^2 - 1}\right) = 0$$

$$\gamma = \frac{1}{1 - v^2} \qquad G = \frac{g-2}{2}$$

$$\xi_{\text{Berry}} = -2\pi m$$

$$m = \pm\frac{1}{2}$$

Spin Precession Experiment

$$H_f = \frac{g_f}{2f_a}\eta_a \frac{\mathbf{p}\cdot\boldsymbol{\sigma}}{m_f}$$

$$H_\gamma = \frac{g_\gamma}{2f_a}\eta_a \frac{\mathbf{k}\cdot\mathbf{S}}{|\mathbf{k}|}$$

Proton ring experiment

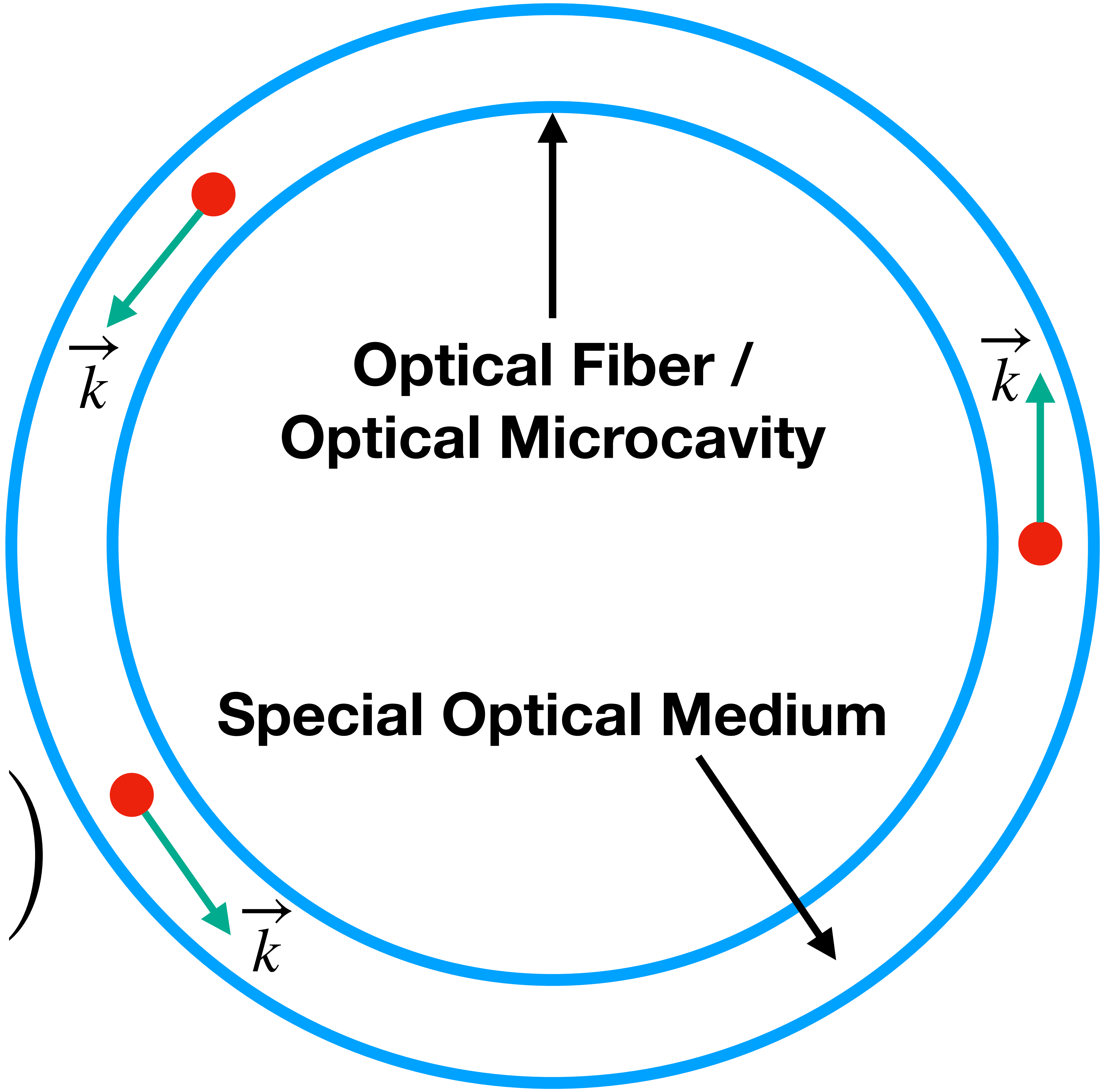
Photon ring experiment

Photon Ring Experiment

The resonance condition can be realized by the birefringent material.

$$\alpha_{\text{photon}} = 1.63 \times 10^{-9}\ \text{rad} \times \left(\frac{g_\gamma/f_a}{10^{-12}\ \text{GeV}^{-1}}\right)\left(\frac{\sqrt{\rho_{\text{DM}}}}{\sqrt{0.3\ \text{GeV}\cdot\text{cm}^{-3}}}\right)\left(\frac{t}{1\ \text{s}}\right)$$

It is promising to probe the axion.



Scenario Two: Magnitude

- **Focus on the situation where η_a changes with time**

- **Assume photons propagate along the z direction**

$$H_\gamma = \frac{g_\gamma}{2f_a}\eta_a(t)\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \Rightarrow U_\gamma(t) = \begin{pmatrix} \cos\left(\frac{g_\gamma}{2f_a}\Delta a\right) & -\sin\left(\frac{g_\gamma}{2f_a}\Delta a\right) \\ \sin\left(\frac{g_\gamma}{2f_a}\Delta a\right) & \cos\left(\frac{g_\gamma}{2f_a}\Delta a\right) \end{pmatrix}$$

where $\Delta a(t) = \tilde{a}(t) + At$

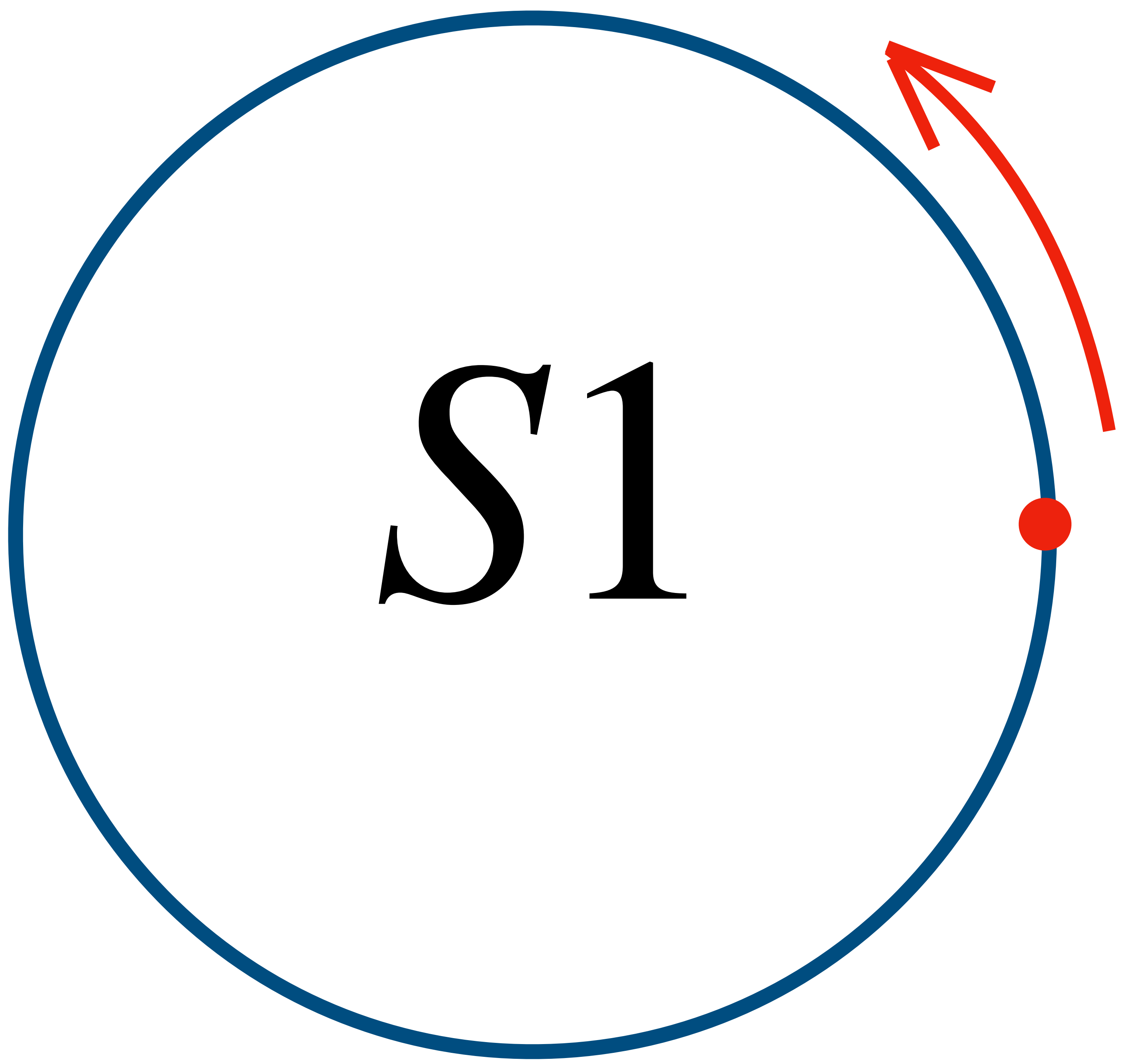
$$\Rightarrow \xi_{\text{Berry}} = m\frac{g_\gamma}{2f_a}\left[\tilde{a}(T) - \tilde{a}(0)\right],\ m = \pm 1$$

which means the Berry phase must be zero for a closed loop.

Scenario Two: Magnitude

- **Quantization of the axion: shift symmetry** Choi et al. 2024, PRL

$$a \sim a + 2\pi f_a$$



$$\tilde{a}(t+T) = \tilde{a}(t) + 2\pi N_w f_a$$

$$\xi_{\text{Berry}} = m\frac{g_\gamma}{2f_a}\left[\tilde{a}(T) - \tilde{a}(0)\right]$$

$$\xi_{\text{Berry}} = m\pi g_\gamma N_w, \ m = \pm 1$$

Winding number

The non-zero winding number can be realized by the axion string, axion domain wall, etc.

Jain et al. 2021, JCAP

Application of The Berry Phase

- **Berry phase is dependent only on g_γ , not f_a**
- **g_γ is related to the generalized symmetry** Choi et al. 2024, PRL

$$\xi_{\text{Berry}} = m\pi g_\gamma N_w, \ m = \pm 1$$

$$g_\gamma = \frac{\alpha}{2\pi}\left(\frac{E}{N} - 1.92\right)$$

Berry phase measurement

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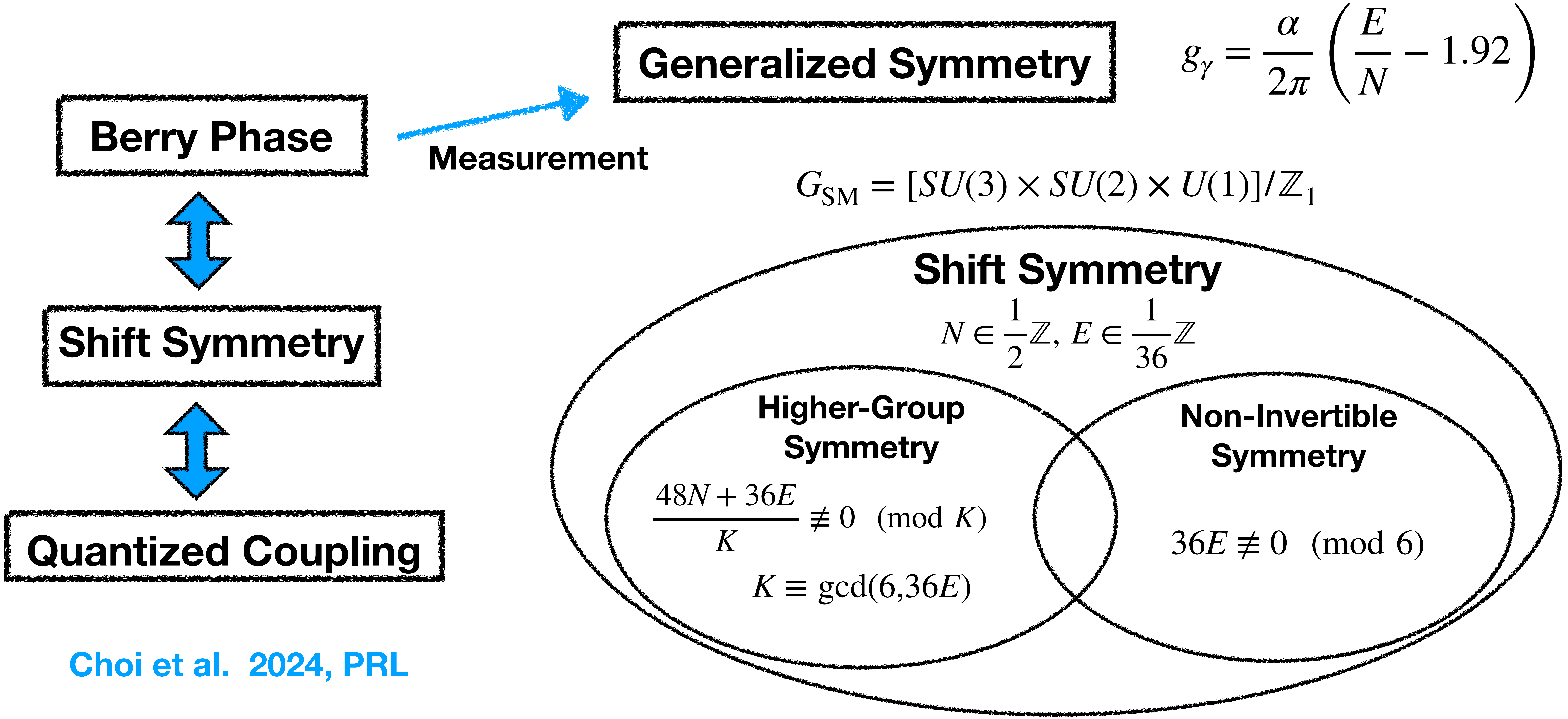
Generalized symmetry research

Application of The Berry Phase



Berry Phase

Measurement

Generalized Symmetry

$$g_\gamma = \frac{\alpha}{2\pi}\left(\frac{E}{N} - 1.92\right)$$

Shift Symmetry

Quantized Coupling

$$G_{\text{SM}} = [SU(3) \times SU(2) \times U(1)]/\mathbb{Z}_1$$

Shift Symmetry

$$N \in \frac{1}{2}\mathbb{Z},\ E \in \frac{1}{36}\mathbb{Z}$$

Higher-Group Symmetry

$$\frac{48N + 36E}{K} \not\equiv 0 \pmod{K}$$

$$K \equiv \gcd(6{,}36E)$$

Non-Invertible Symmetry

$$36E \not\equiv 0 \pmod{6}$$

Choi et al. 2024, PRL

Conclusion

- **We perform a systematical study on the Berry phase in the axion physics.**
- **We find the unified form of axion-fermion and axion-photon Hamiltonian and research two different scenarios**
- **The photon ring experiment is a new way to probe the axion.**
- **The birefringence can be used to measure the Berry phase which can help us understand the topology property (generalized symmetry) of the axion.**

Thank You!

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