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# Primordial Black Holes and Gravitational waves from Axion

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# Why we need QCD axion?

Strong CP problem:  $\mathcal{L}_{\text{QCD}} \supset \bar{\theta} \frac{g_s^2}{32\pi^2} \tilde{G}^{a\mu\nu} G_{\mu\nu}^a$  (CP-odd term)

$$\bar{\theta} = \theta_{\text{vac}} + \arg \det(Y_u Y_d)$$

SU(3)<sub>color</sub> vacuum topology

quark sector

→ theory:  $\bar{\theta} \in (0, 2\pi)$

→ experiments:  $\bar{\theta} \lesssim 10^{-10}$

(neutron electric dipole moment)

Why  $\bar{\theta}$  is so unnaturally small (Fine-tuning)?

# Solution: Peccei-Quinn (PQ) mechanism

Global symmetry  $U(1)_{\text{PQ}}$ :  $\Phi = \rho \exp(ia/f_a)$

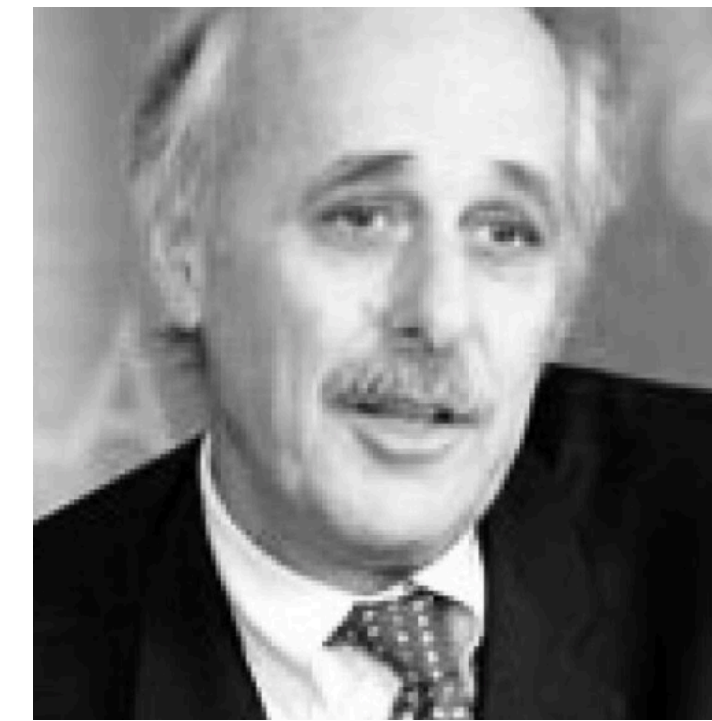
$$\Rightarrow \frac{a}{f_a} \frac{g_s^2}{32\pi^2} \tilde{G}^{a\mu\nu} G_{\mu\nu}^a, \quad [SU(3)_{\text{color}}^2 \times U(1)_{\text{PQ}} \text{ anomaly}]$$

$$\mathcal{L}_{\text{QCD}} \supset \left( \bar{\theta} + \frac{a}{f_a} \right) \frac{g_s^2}{32\pi^2} \tilde{G}^{a\mu\nu} G_{\mu\nu}^a$$

$\bar{\theta}$  is absorbed into  $a$ :

$\bar{\theta} + a/f_a \rightarrow a/f_a$ , dynamical field.

$a$  is called axion.



(Roberto Peccei)



(Hellen Quinn)



Bigbox Asia · 缺货  
Axion Lime Dishwashing P...



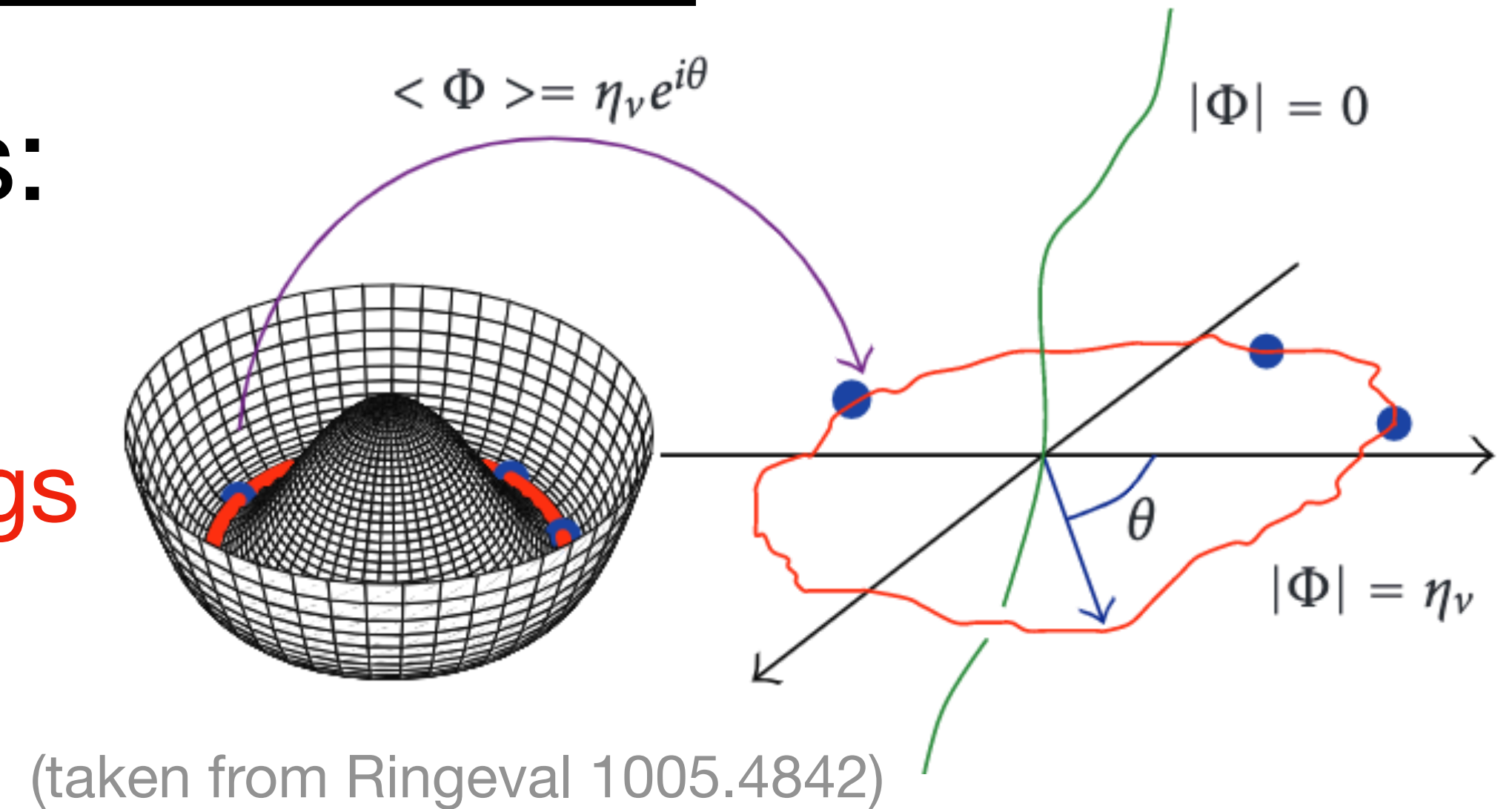
# Axion Topological Defects

1. PQ symmetry spontaneous breaks:

$$\mathcal{L}_\phi \supset -\frac{\lambda}{4}(\Phi^\dagger\Phi - f_a^2)^2 \quad \longrightarrow \quad \text{Axion Strings}$$

$$\phi = \rho \exp(ia/f_a)$$

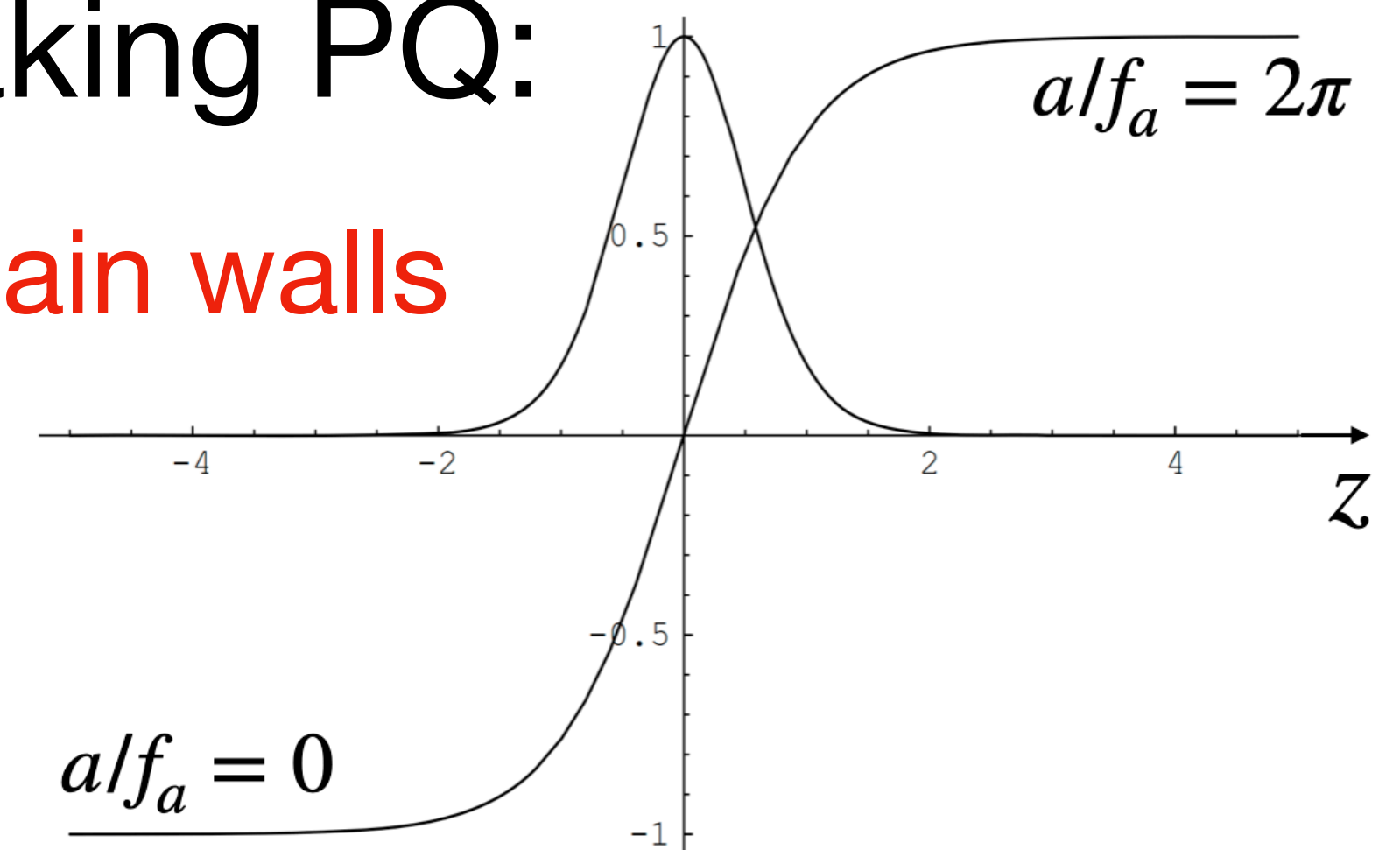
$$T \sim f_a \sim 10^{12} \text{ GeV}$$



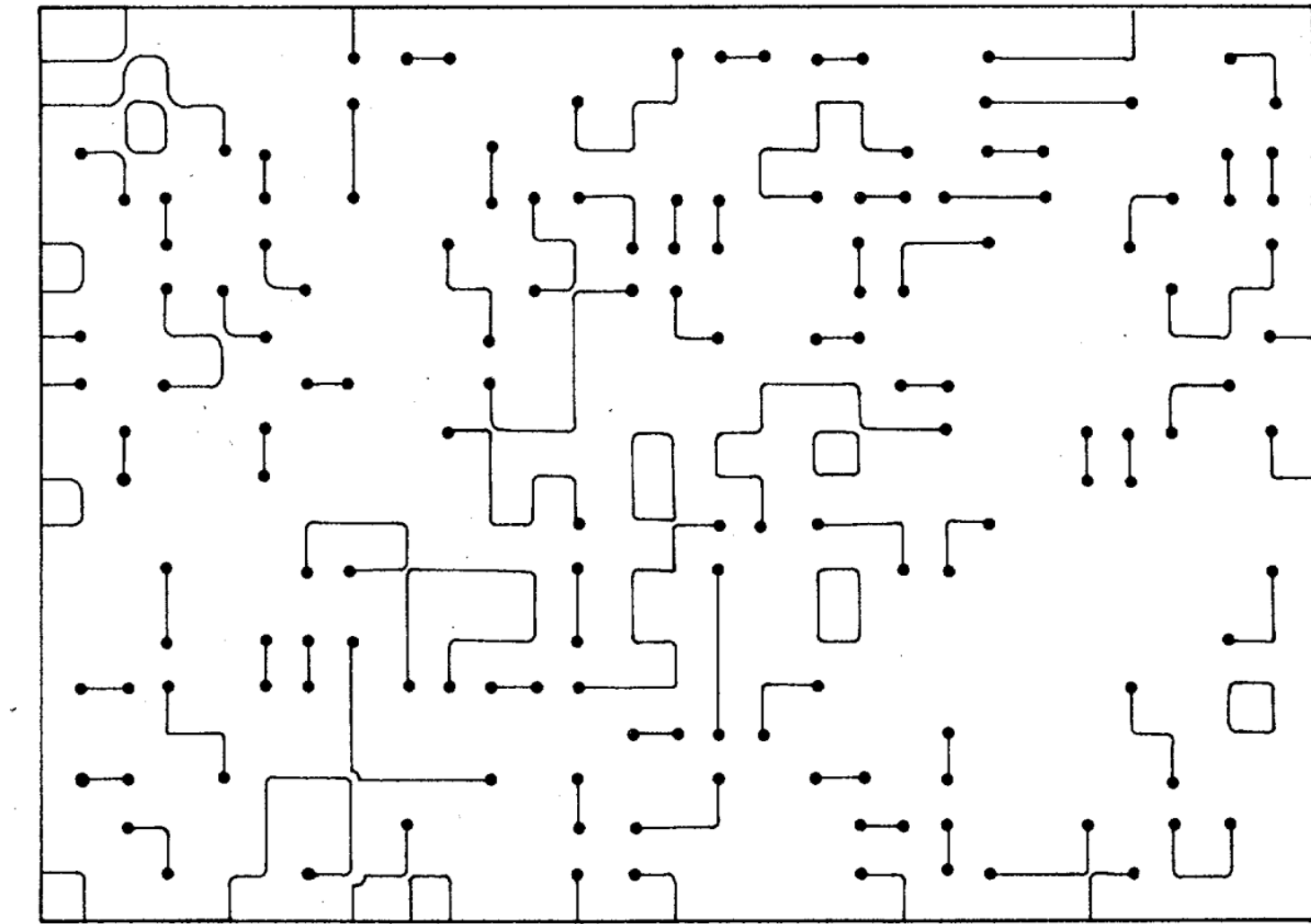
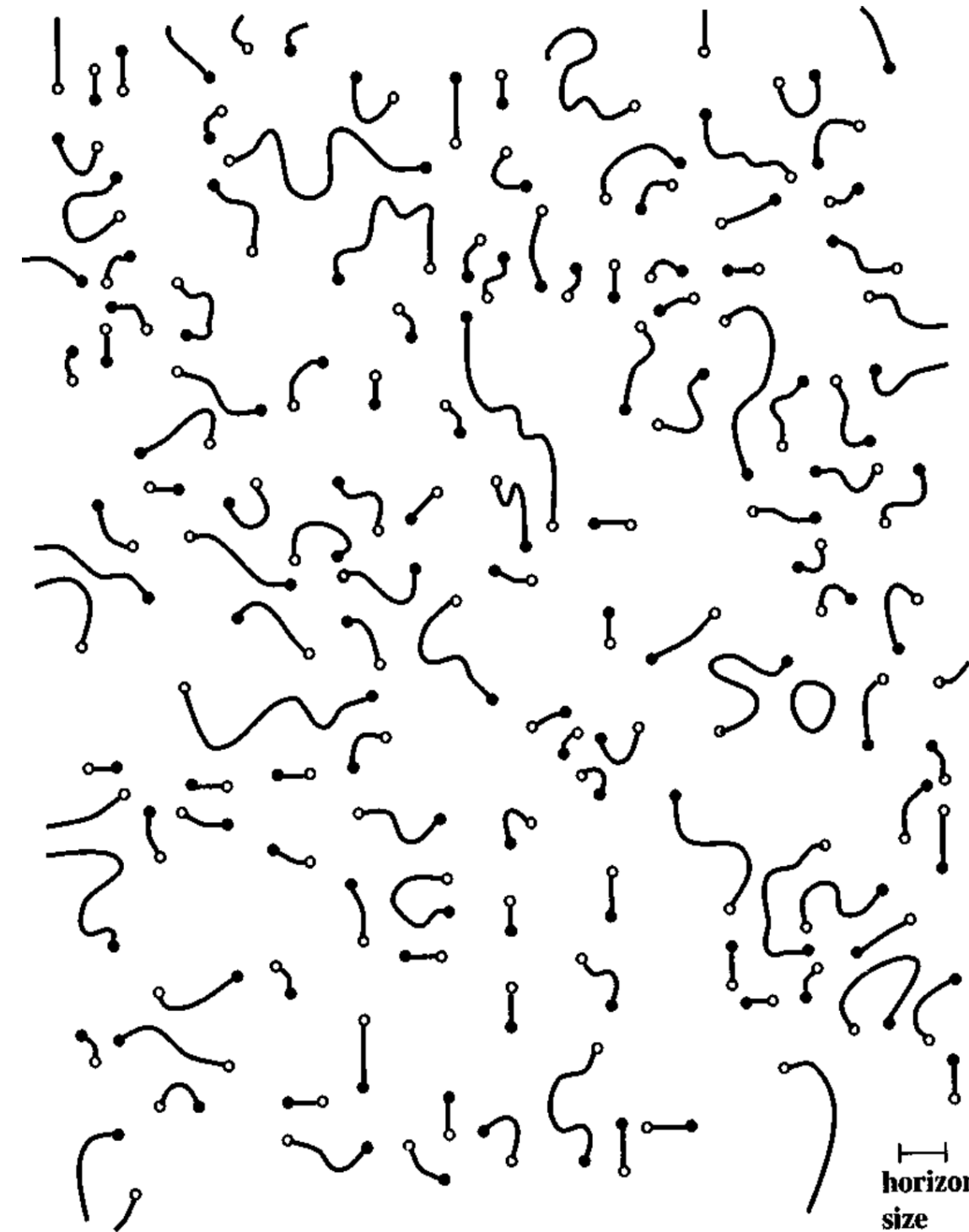
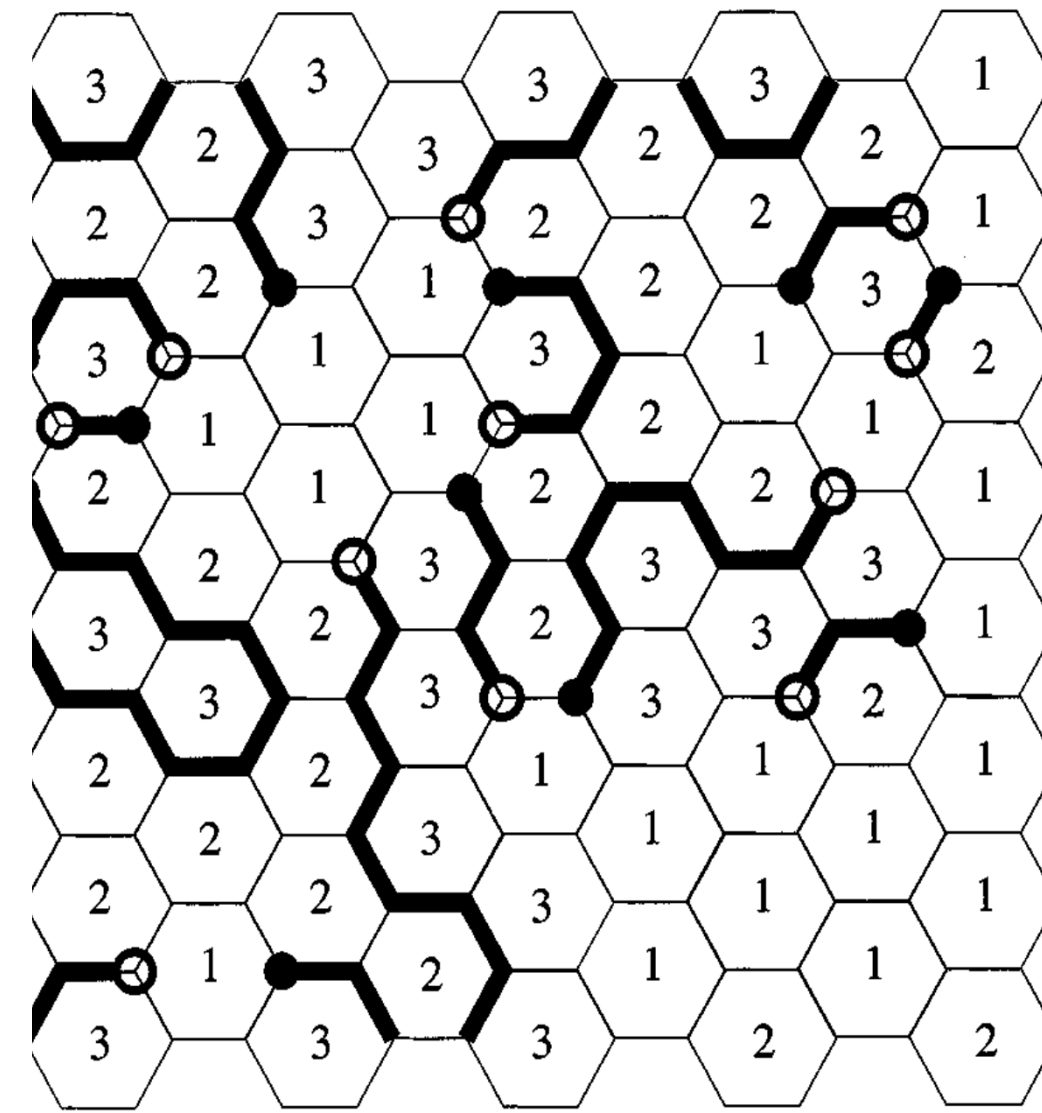
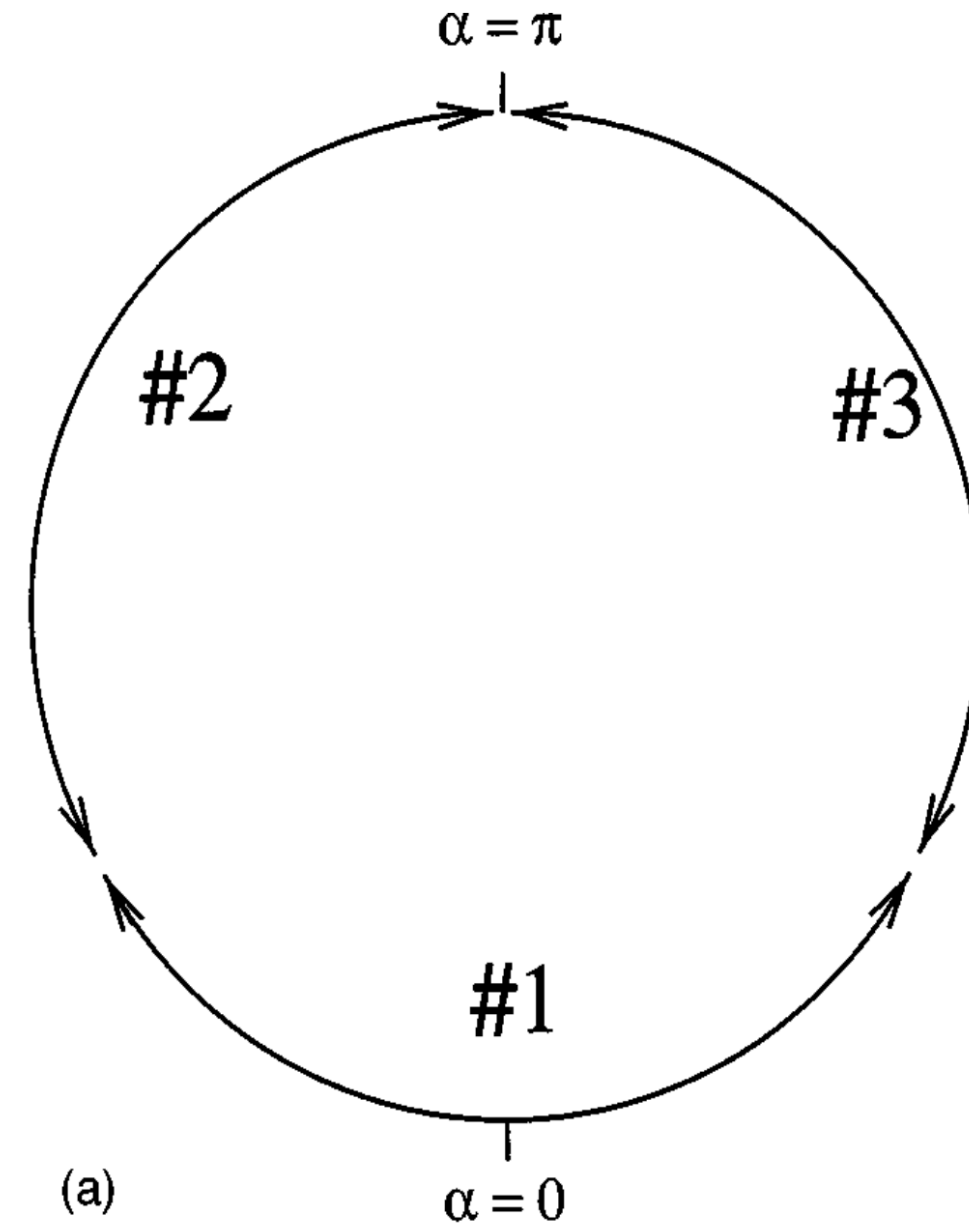
2. Non-perturbative effects explicitly breaking PQ:

$$V_a \simeq m_a^2 f_a^2 [1 - \cos(a/f_a)] \quad \longrightarrow \quad \text{Axion domain walls}$$

$$T \sim 1 \text{ GeV}$$



# Axion Topological Defects



*(Chang, Hagmann & Sikivie, 1998)*

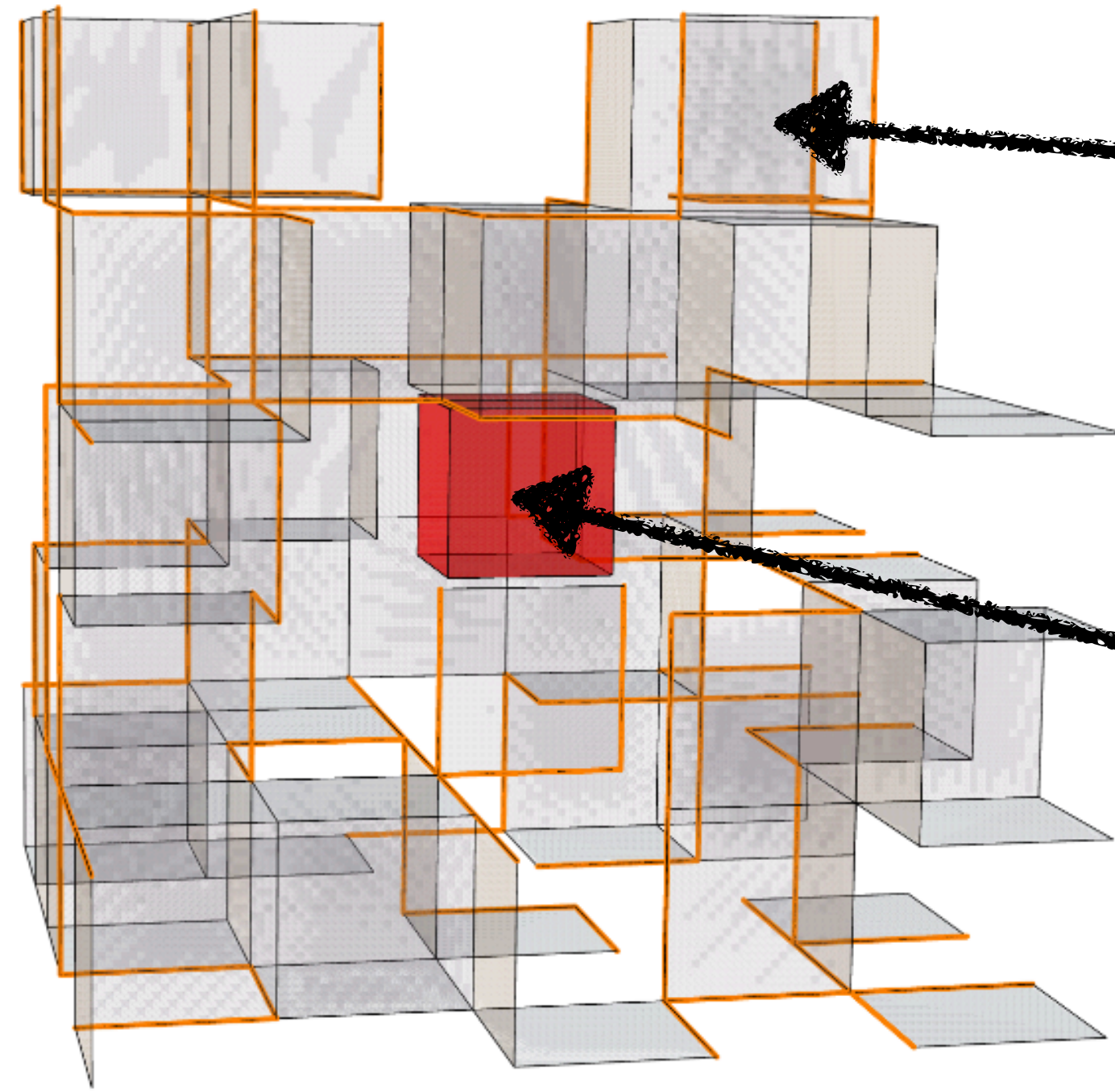
*(Vachaspati & Vilenkin, 1984)*

Computer Simulations

# Simulation of the string-wall network

(SG, Jinhui Guo, Jia Liu,  
2023)

$(N_{\text{DW}} = 1)$



open walls bounded by strings

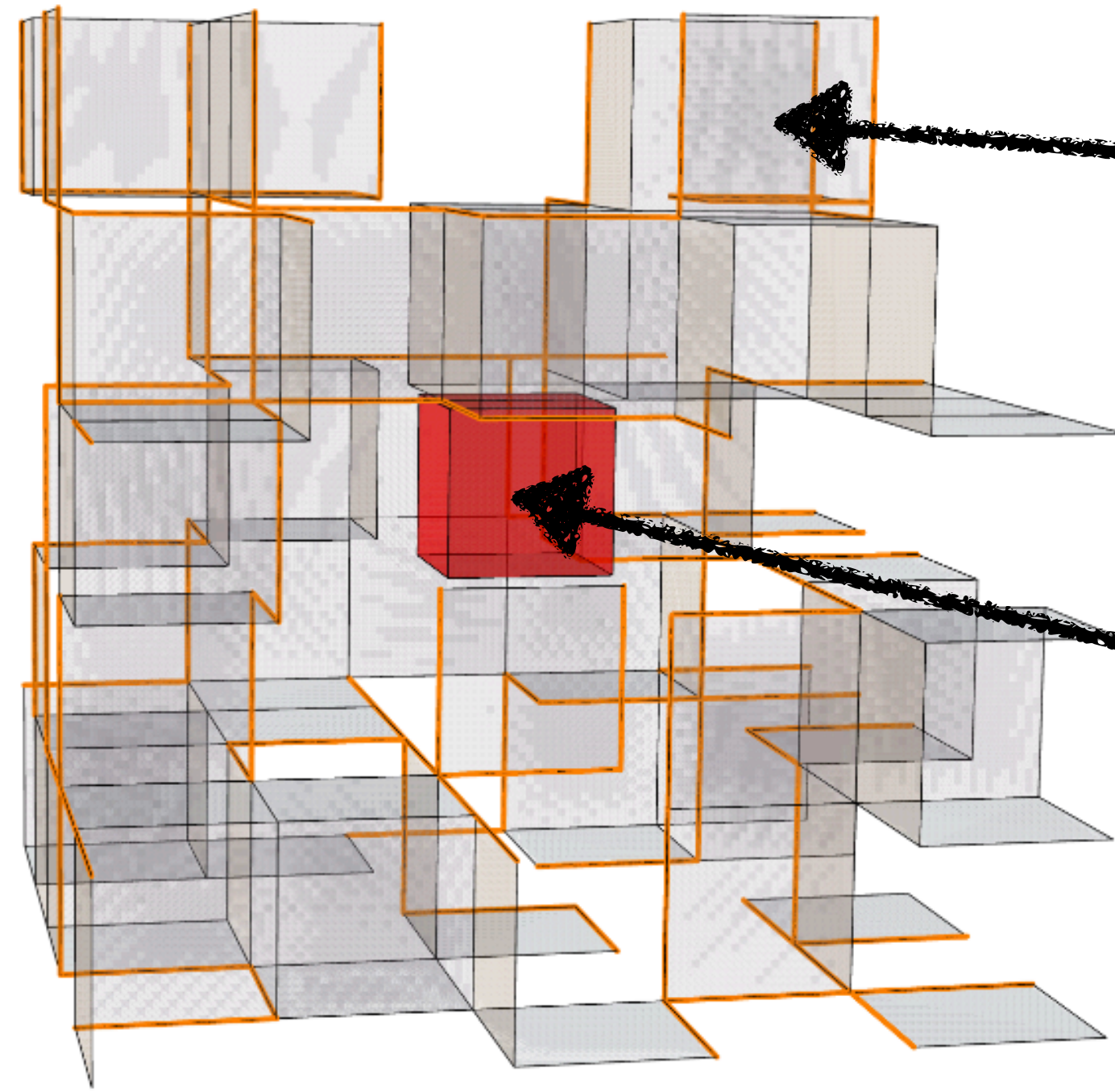
Closed domain wall



# Simulation of the string-wall network

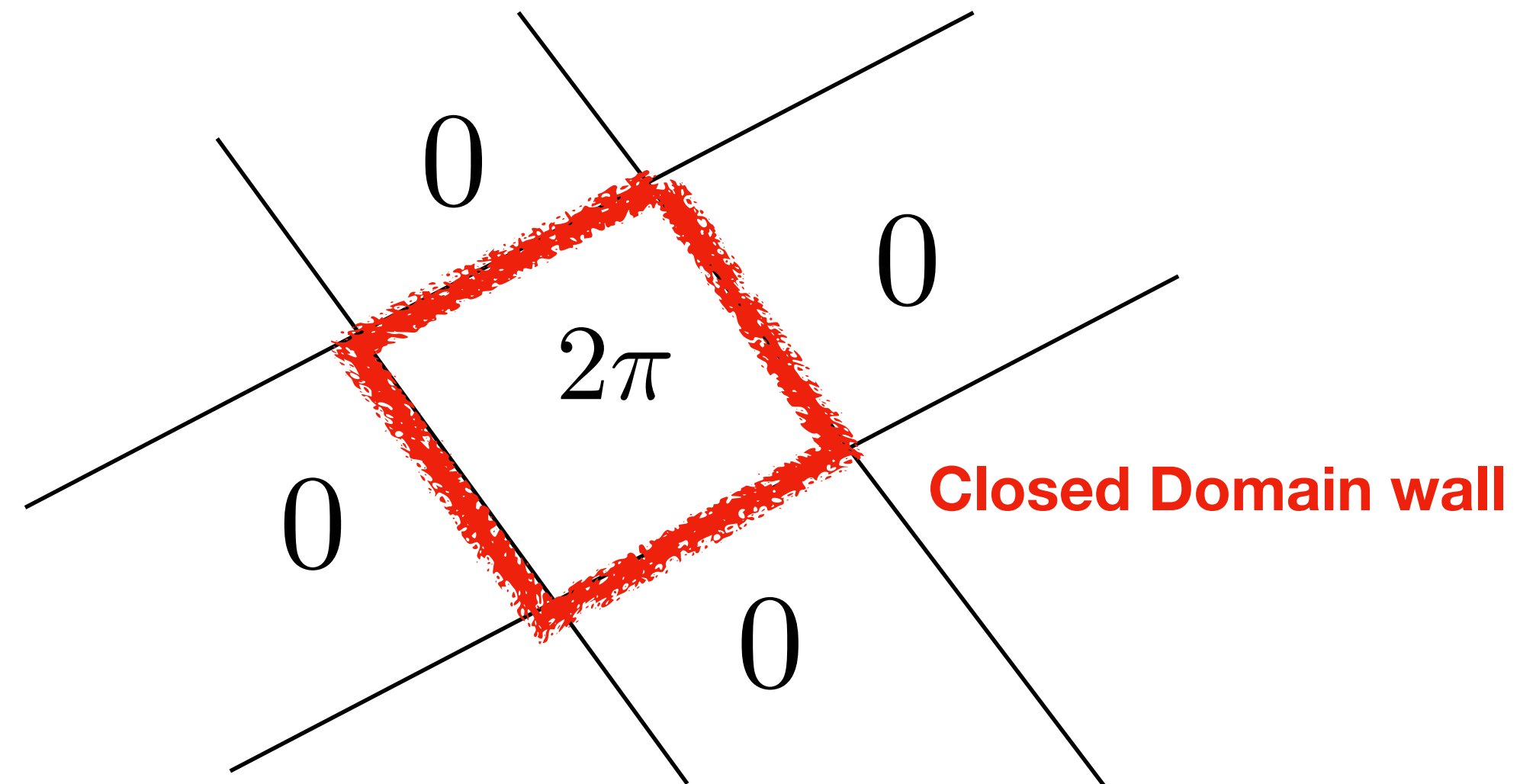
(SG, Jinhui Guo, Jia Liu,  
2023)

$(N_{\text{DW}} = 1)$



open walls bounded by **strings**

**Closed domain wall**



# Fate of closed domain walls?

>> Collapses and oscillates.

Energy released mainly in the form of free axions.

Nothing interesting.

>> Schwartzchild radius  $R_S = 2GM$ .

If the wall radius  $R < R_S(R, t)$ , a Black Hole is formed!

$M = 4\pi R^2 \cdot \sigma$  where  $\sigma$  is the domain wall surface energy (i.e., tension)  
so the criterion becomes:

$$R > 1/(8\pi G\sigma)$$

*(Widrow, 1989)*

*(Vachaspati, 2017)*

**(SG, 2019)**



# Different Scenarios of Axion Cosmology

>> **Pre-inflationary scenario:** PQ symmetry breaks **before** inflation.

The formed strings will be blown away by inflation;

$$f_a \gtrsim H_I/2\pi$$

Axion field gets homogenized;

No axion domains walls can form.

>> **Post-inflationary scenario:** PQ symmetry breaks **after** inflation.

String-wall network will form.

$$f_a \lesssim H_I/2\pi$$

>> **During-inflation scenario:** PQ symmetry breaks **during** inflation.

String-wall network will form but re-enters horizon much later.

$$f_a \sim H_I/2\pi$$

# During-inflationary Scenario

>> **During-inflation scenario:** PQ symmetry breaks **during** inflation. String-wall network will form but re-enters horizon much later.

$$f_a \sim H_I/2\pi : \quad \text{fine-tuning? NO!}$$

**This relation is not necessary**

This scenario can be naturally realized if the PQ symmetry breaking driven by inflation, for example:

*see e.g.,*

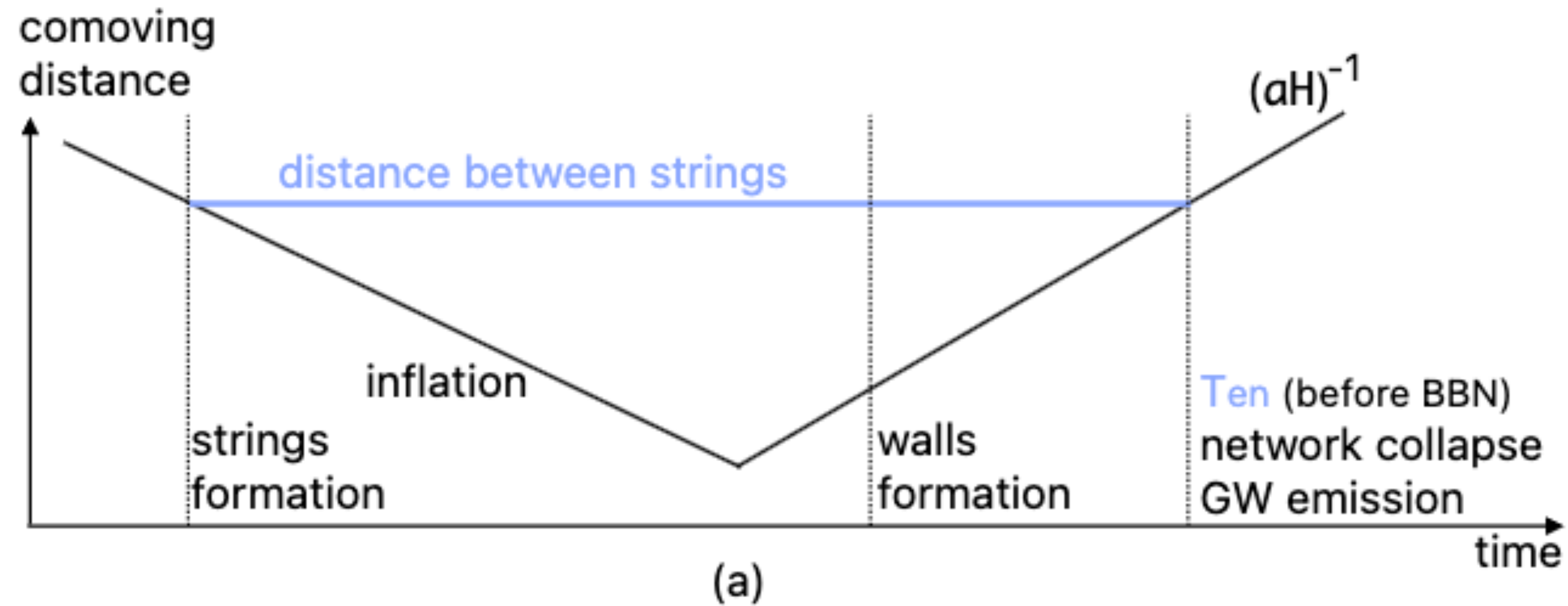
couple the PQ field  $\Phi$  to the inflaton field  $\phi$  via  $c\phi^2\Phi^\dagger\Phi$  *(Keisuke Harigaya, Lian-Tao Wang, 2022),*

*(Michele Redi, Andrea Tesi, 2022)*

*(Haipeng An, Chen Yang, 2023)*

**PQ symmetry breaks when**  $\phi$  rolls down to  $\phi = \sqrt{\lambda/2c}v_a$ .

# During-inflationary Scenario



$T_{\text{en}}$ : the temperature of re-entering horizon

Size of closed axion domain walls:  $R \sim H^{-1}(T_{\text{en}})$



# Criterion for PBH formation

Numerical calculation:

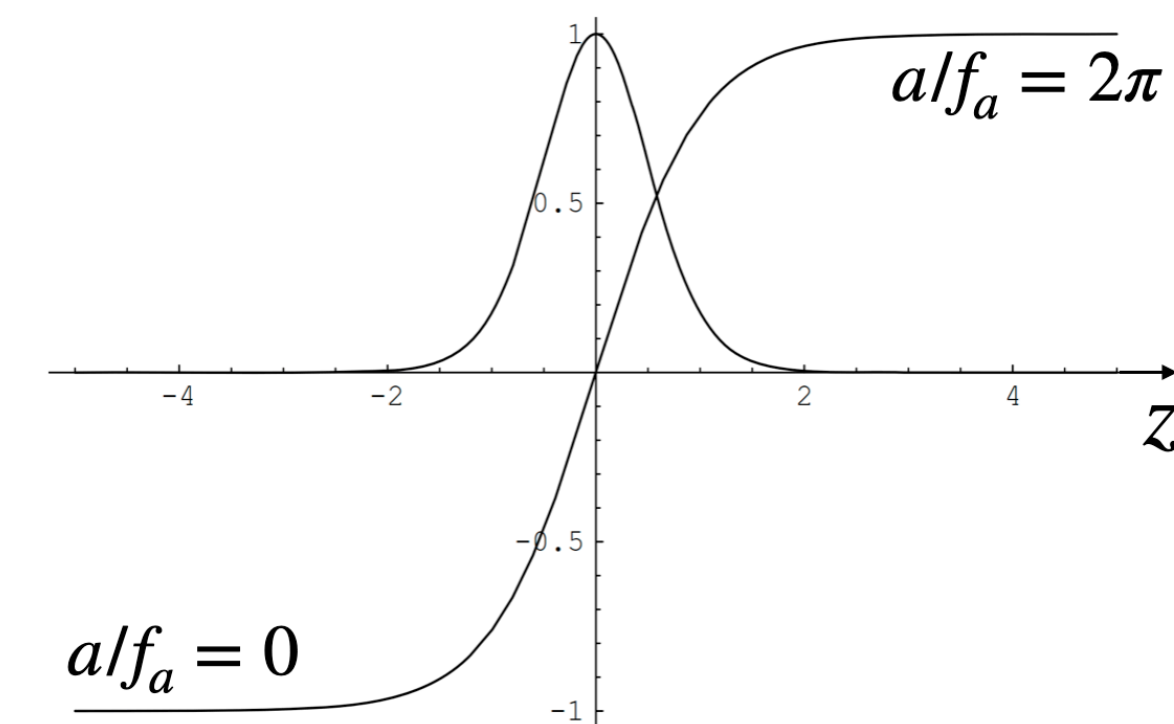
$$\mathcal{L} = 1/2(\partial_\mu\phi)^2 - V_a \quad V_a = m_a^2(T)f_a^2[1 - \cos(\phi/f_a)]$$

Equation of motion: 
$$\left[ \partial_t^2 + \frac{3\partial_t}{2t} - \frac{\partial_{\mathcal{R}}^2}{a^2(t)} - \frac{2\partial_{\mathcal{R}}}{a^2(t)\mathcal{R}} \right] \tilde{\phi} + m_a^2(t) \sin \tilde{\phi} = 0$$

$\mathcal{R} = R/a(t)$  co-moving distance

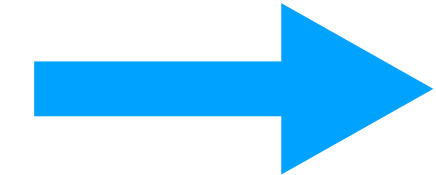
Initial condition:

$$\tilde{\phi}(t = t_2, \mathcal{R}) = 4 \left\{ \tan^{-1} [e^{m_a(t_2)(\mathcal{R}-R_2)}] + \tan^{-1} [e^{m_a(t_2)(-\mathcal{R}-R_2)}] \right\}$$



# Criterion for PBH formation

Criterion:  $R < R_s(R, t)$

  $S(t, R) \gtrsim m_{\text{P}}^2$

$$R_s = 2GE(t, R)$$

$$S(t, R) \equiv 2E(t, R)/R$$

The maximum value of  $S(\tilde{t}, \tilde{r})$  during the collapse is

$$S_{\text{max}} = \max_{(\tilde{t}, \tilde{r})} S(\tilde{t}, \tilde{r})$$

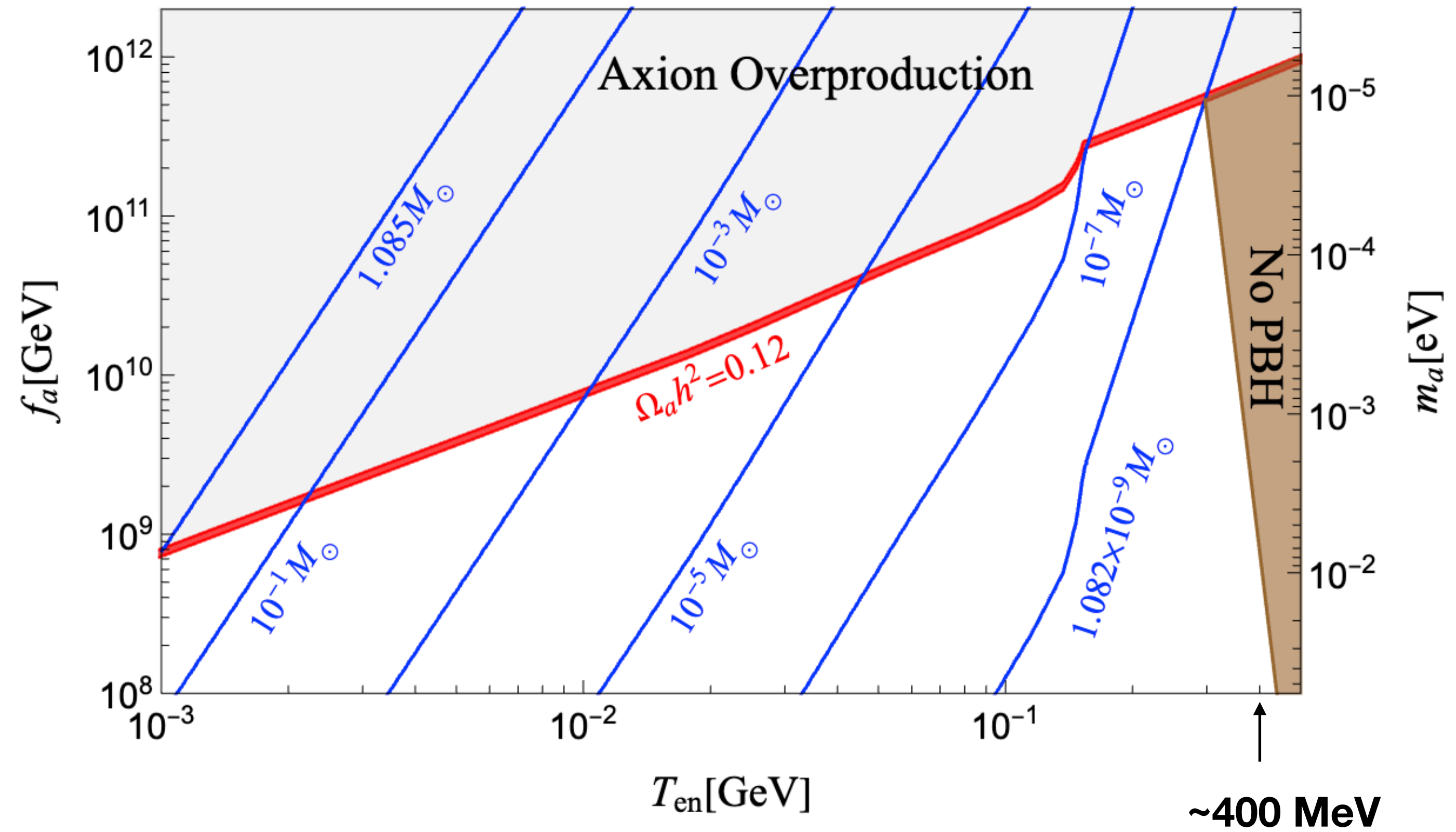
*(Vachaspati, 2017)*

**(SG, 2019)**

We get,

$$\frac{S_{\text{max}}}{f_a^2} = \begin{cases} 19.66(m_a R_0)^{2.74}, & T_{\text{en}} \lesssim T_c \\ 3.1 \times 10^3 [m_a(T_{\text{en}}) R_0]^{2.76}, & T_{\text{en}} \gtrsim T_c \end{cases}.$$

# Criterion for PBH formation



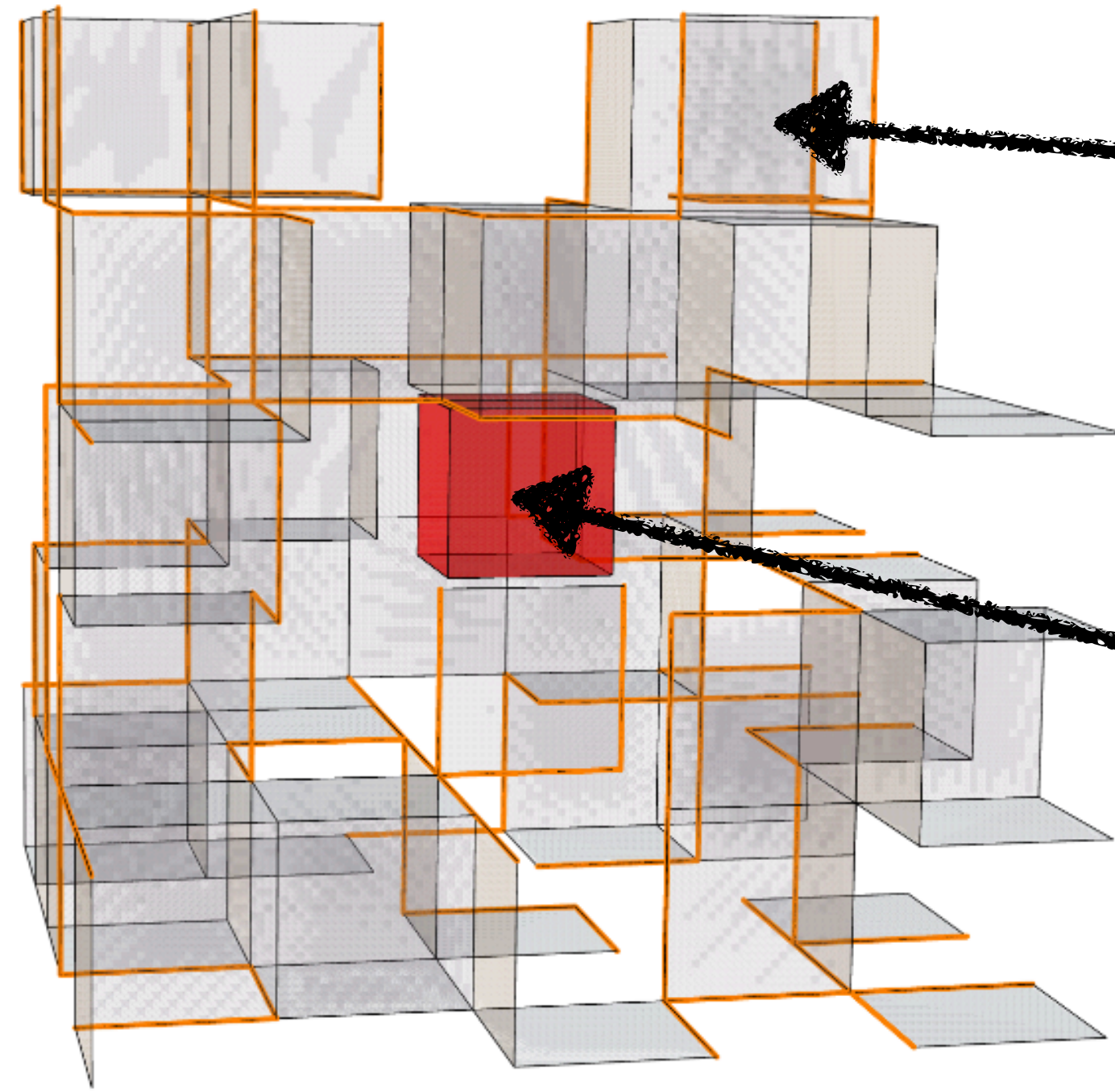
$$10^{-9} M_\odot \lesssim M_{\text{PBH}} \lesssim 1 M_\odot.$$



# Abundance

(SG, Jinhui Guo, Jia Liu,  
2023)

$(N_{\text{DW}} = 1)$



open walls bounded by strings

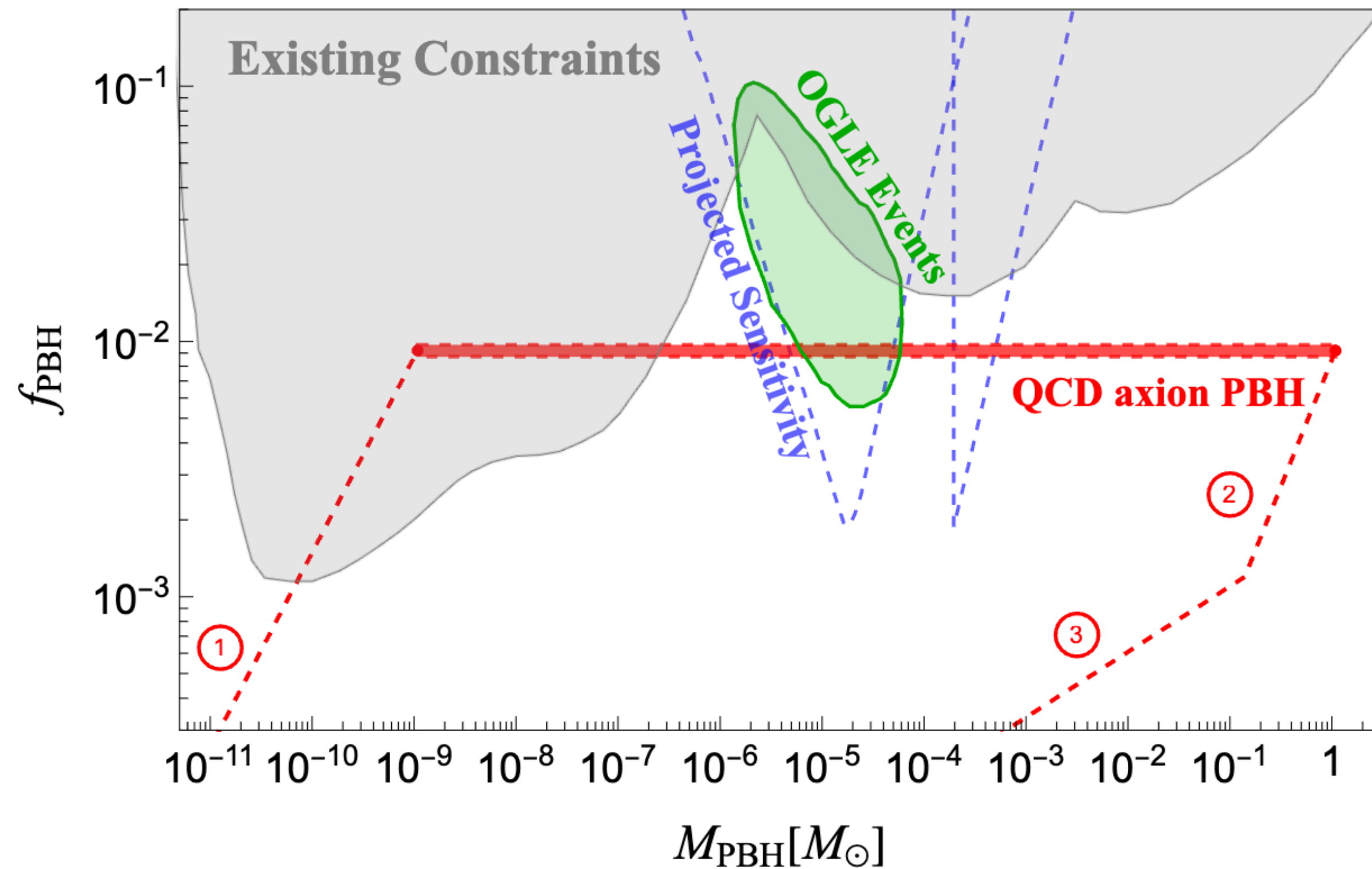
Closed domain wall

$\mathcal{A} \approx 37\%$  fraction of lattice cells occupied by a piece of wall.

$$\gamma = 0.285\% \quad \gamma \equiv \frac{\text{total closed wall area}}{\text{total DW area}}$$

The simulation results are independent of axion parameters or  $T_{\text{en}}$

# PBH abundance



Intriguing astronomical implications:

>> Anomalous microlensing events observed by the OGLE collaboration.

*(Mroz et al 2017; Niikura et al, 2019),*

>> Explain the Planet 9 in our solar system if it is a PBH.

*(Scholt and Unwin, 2020; Witten, 2020)*



# PBH abundance

PHYSICAL REVIEW LETTERS **125**, 051103 (2020)

Editors' Suggestion

Featured in Physics

## What If Planet 9 Is a Primordial Black Hole?

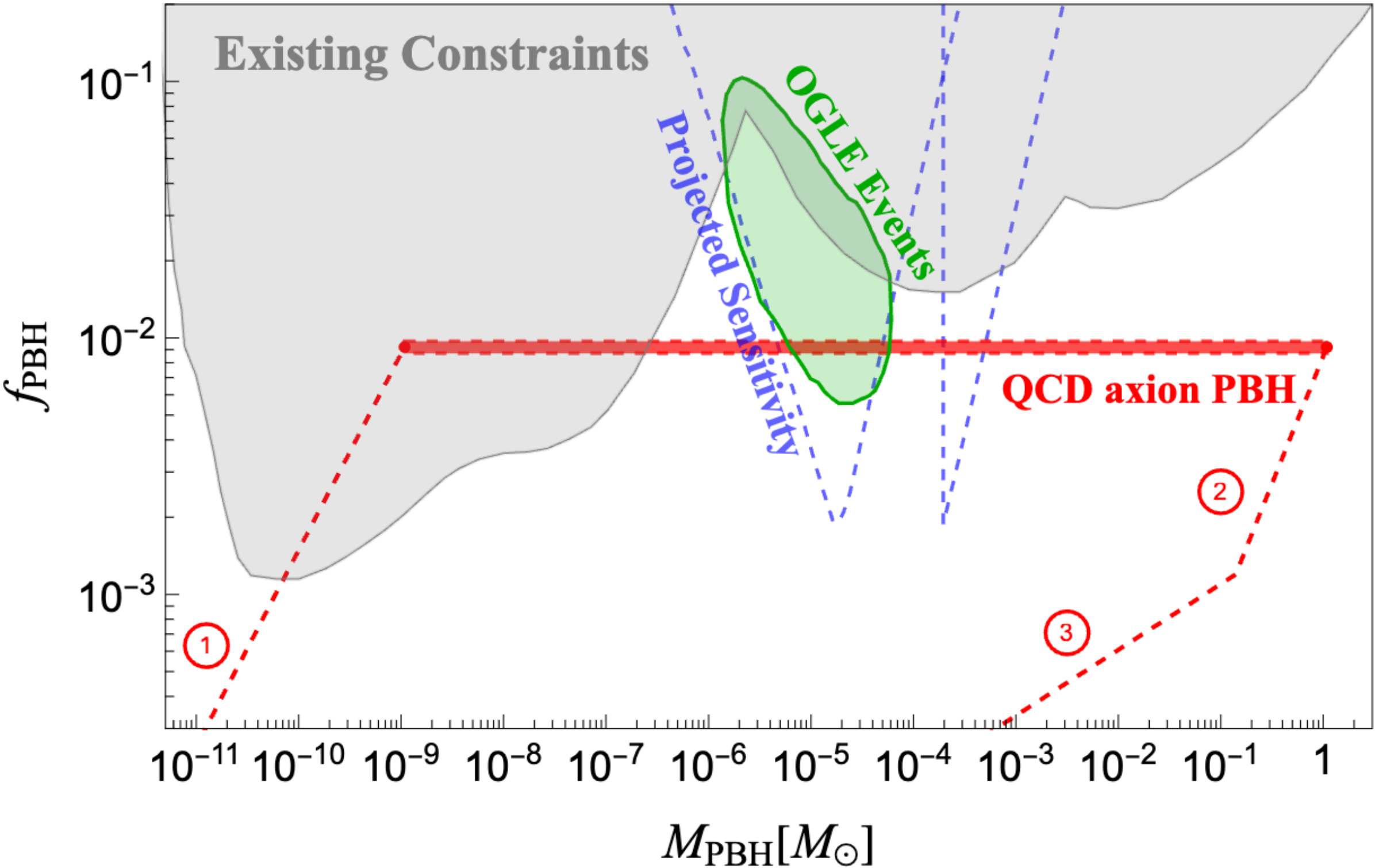
Jakub Scholtz<sup>1</sup> and James Unwin<sup>2</sup>

<sup>1</sup>*Institute for Particle Physics Phenomenology, Durham University, Durham DH1 3LE, United Kingdom*

<sup>2</sup>*Department of Physics, University of Illinois at Chicago, Chicago, Illinois 60607, USA  
and Department of Physics, University of California, Berkeley and Theoretical Physics Group,  
LBNL and Mathematics Sciences Research Institute, Berkeley, California 94720, USA*

(Received 13 November 2019; revised 10 February 2020; accepted 26 June 2020; published 29 July 2020)

We highlight that the anomalous orbits of trans-Neptunian objects (TNOs) and an excess in microlensing events in the 5-year Optical Gravitational Lensing Experiment data set can be simultaneously explained by a new population of astrophysical bodies with mass several times that of the Earth ( $M_{\oplus}$ ). We take these objects to be primordial black holes (PBHs) and point out the orbits of TNOs would be altered if one of these PBHs was captured by the Solar System, inline with the Planet 9 hypothesis. Capture of a free floating planet is a leading explanation for the origin of Planet 9, and we show that the probability of capturing a PBH instead is comparable. The observational constraints on a PBH in the outer Solar System significantly differ from the case of a new ninth planet. This scenario could be confirmed through annihilation signals from the dark matter microhalo around the PBH.





# PBH abundance

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## What If Planet 9 Is a Primordial Black Hole?

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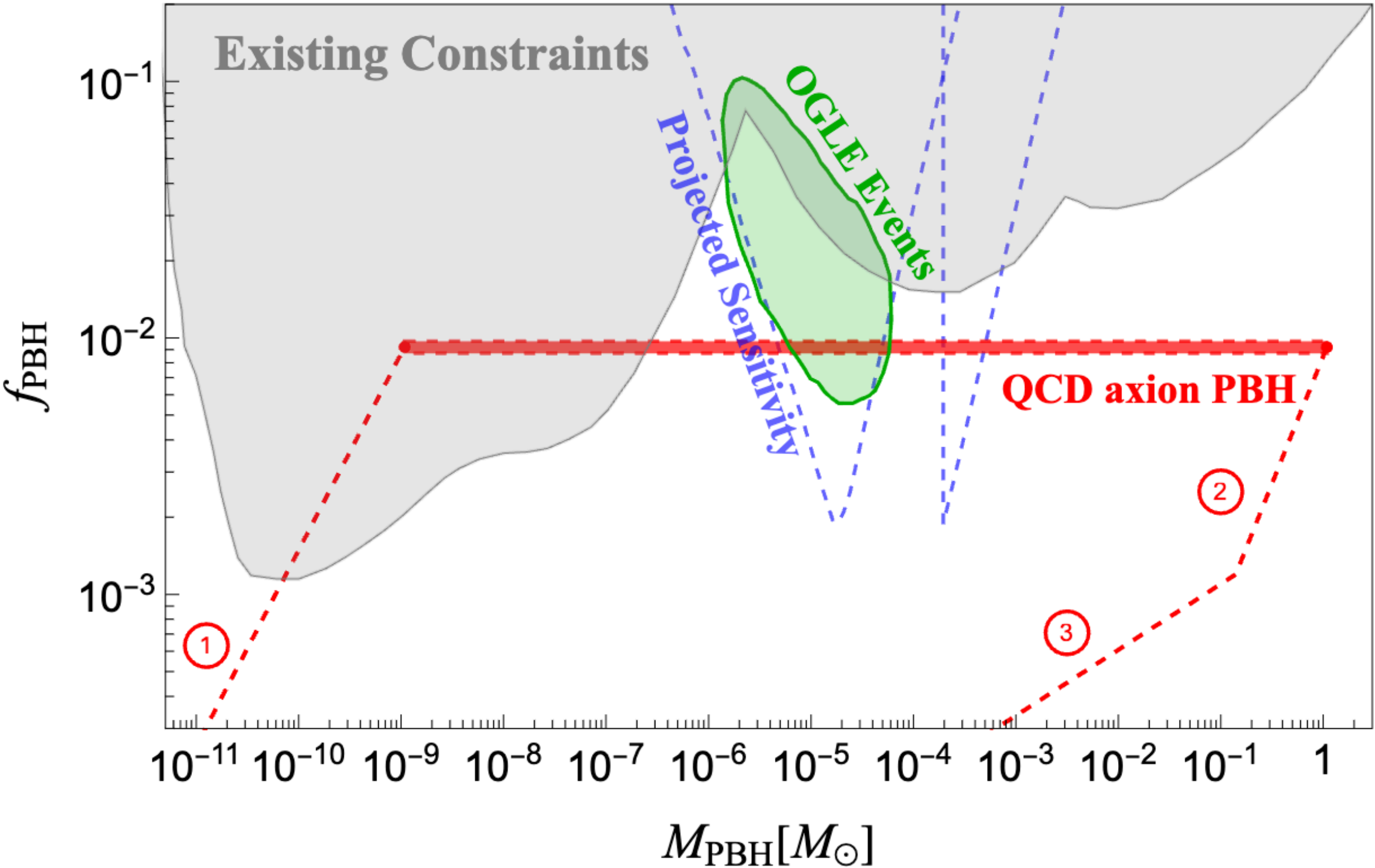
## Searching for a Black Hole in the Outer Solar System

Edward Witten

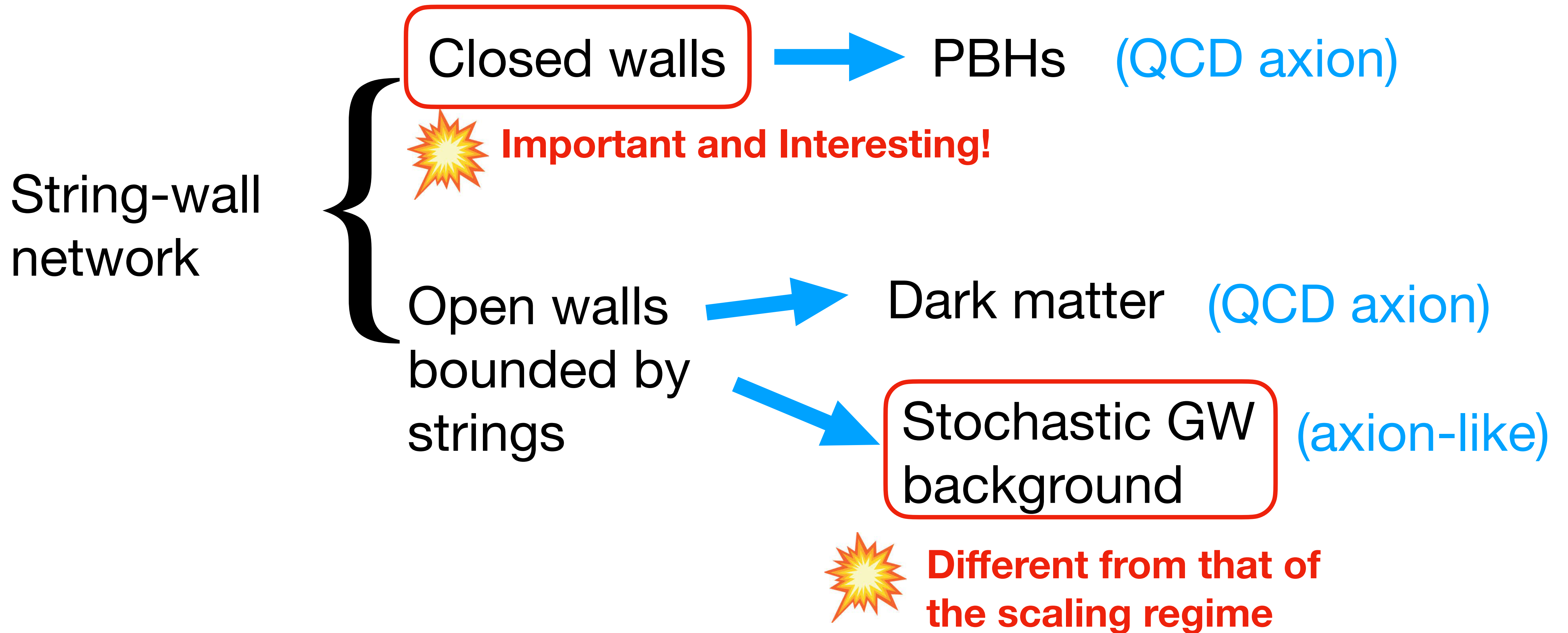
*Institute for Advanced Study  
Einstein Drive, Princeton, NJ 08540 USA*

### Abstract

There are hints of a novel object (“Planet 9”) with a mass  $5 - 10 M_{\oplus}$  in the outer Solar System, at a distance of order 500 AU. If it is a relatively conventional planet, it can be found in telescopic searches. Alternatively, it has been suggested that this body might be a primordial black hole (PBH). In that case, conventional searches will fail. A possible alternative is to probe the gravitational field of this object using small, laser-launched spacecraft, like the ones envisioned in the Breakthrough Starshot project. With a velocity of order  $.001 c$ , such spacecraft can reach Planet 9 roughly a decade after launch and can discover it if they can report timing measurements accurate to  $10^{-5}$  seconds back to Earth.



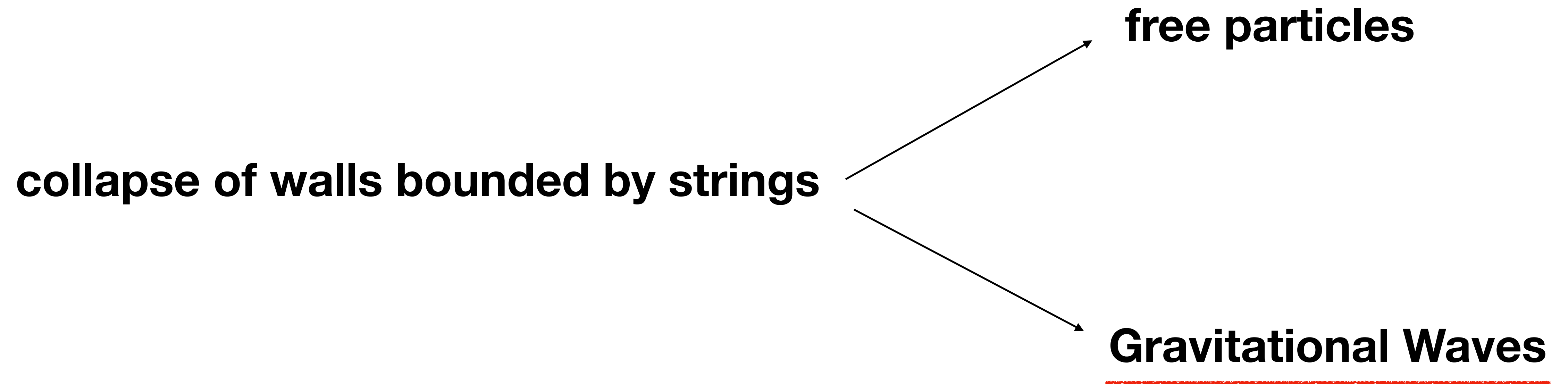
# Summary



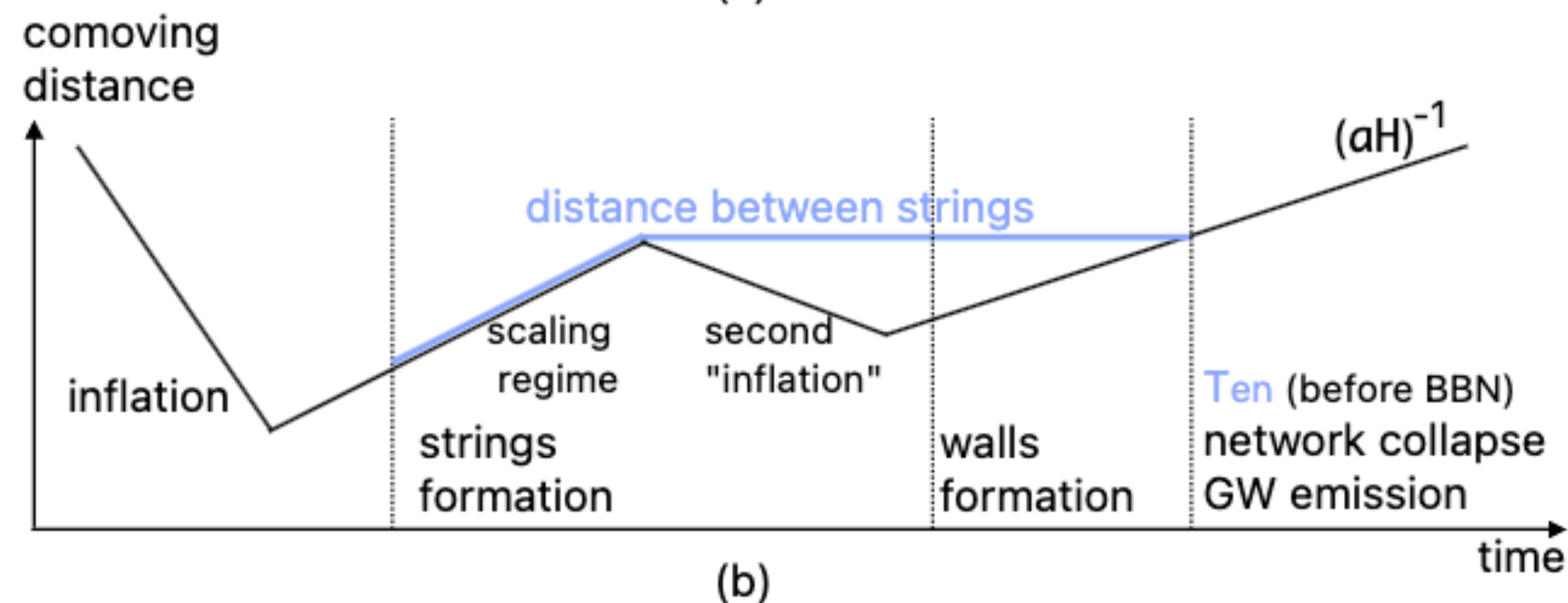
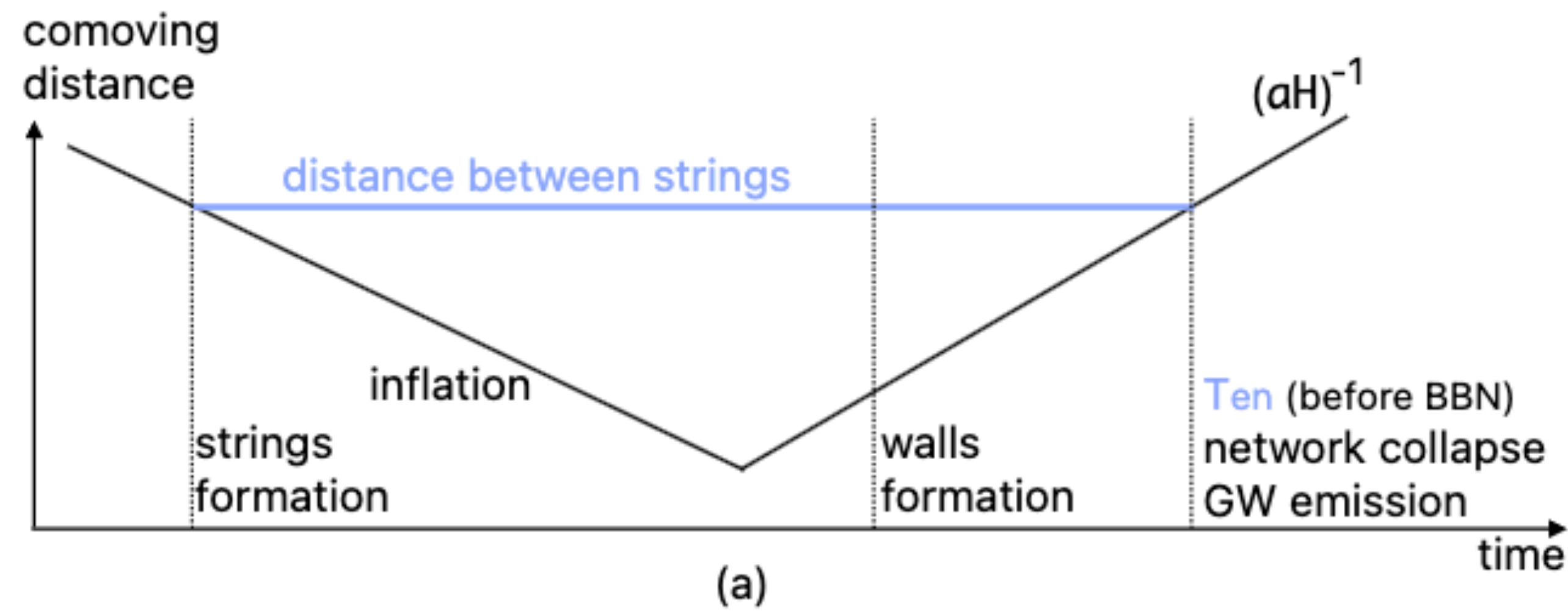
# **Thank you for watching**



# Part II: Gravitational Waves



In our during-inflationary scenario, there is **no scaling-regime dynamics**.  
 The **network collapse** at  $T_{\text{en}}$  becomes the **dominant GW source**.



**The dynamics of collapse of walls bounded by strings can be parameterized as:**

$$R(t) \simeq R_0 \underbrace{e^{-c_R \cdot \frac{\omega_R}{\pi} (t - t_{\text{en}})}}_{\text{exponential decrease of the amplitude due to energy loss into free particles}} \underbrace{\cos[\omega_R (t - t_{\text{en}})]}_{\text{oscillation}}.$$

exponential decrease of  
the amplitude due to energy  
loss into free particles

oscillation

$c_R \sim \mathcal{O}(0.1)$  inferred from the numerical result of (S. Chang, C. Hagmann,  
and P. Sikivie hep-ph/9807374)

Lorentz factor of the resultant free axions  $\gamma_a \approx 3.2$ , (see e.g., T. Hiramatsu et al, 2012;  
M. Kawasaki et al, 2015),

Approximately,  $\omega_R \sim \pi/2 \cdot \langle v_a \rangle / R_0$

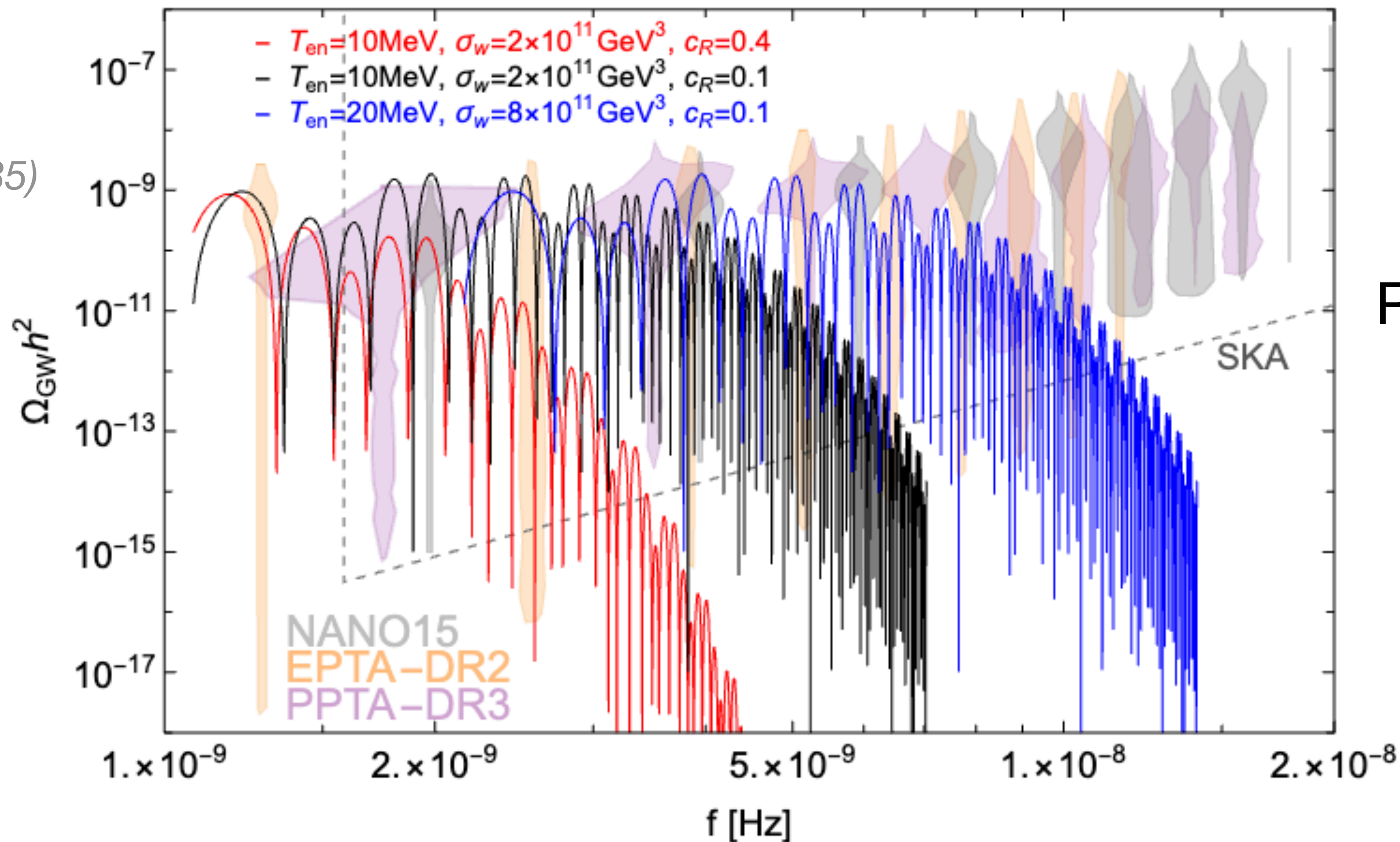


# GW spectra

$$P_{\text{GW}}(t) \sim G \ddot{Q}_{ij} \ddot{Q}_{ij}, \quad Q \sim \mathcal{A} M(t) R^2(t) \sim \mathcal{A} \sigma_w R^4(t) \quad \frac{d\rho_{\text{GW}}(t)}{dt} \sim \frac{P_{\text{GW}}(t)}{H^{-3}(t_{\text{en}})} \frac{a^3(t_{\text{en}})}{a^3(t)} \sim P_{\text{GW}}(t) t_{\text{en}}^{-3/2} t^{-3/2}.$$

$$\Omega_{\text{GW}}(t_0) \equiv \frac{1}{\rho_{\text{cr}}(t_0)} \frac{d\rho_{\text{GW}}(t_0)}{d \ln f(t_0)} \simeq \frac{1}{\rho_{\text{cr}}(t_0)} \frac{a^4(t)}{a^4(t_0)} \frac{d\rho_{\text{GW}}(t)}{dt} \frac{1}{H(t)}.$$

(SG, 2307.08185)



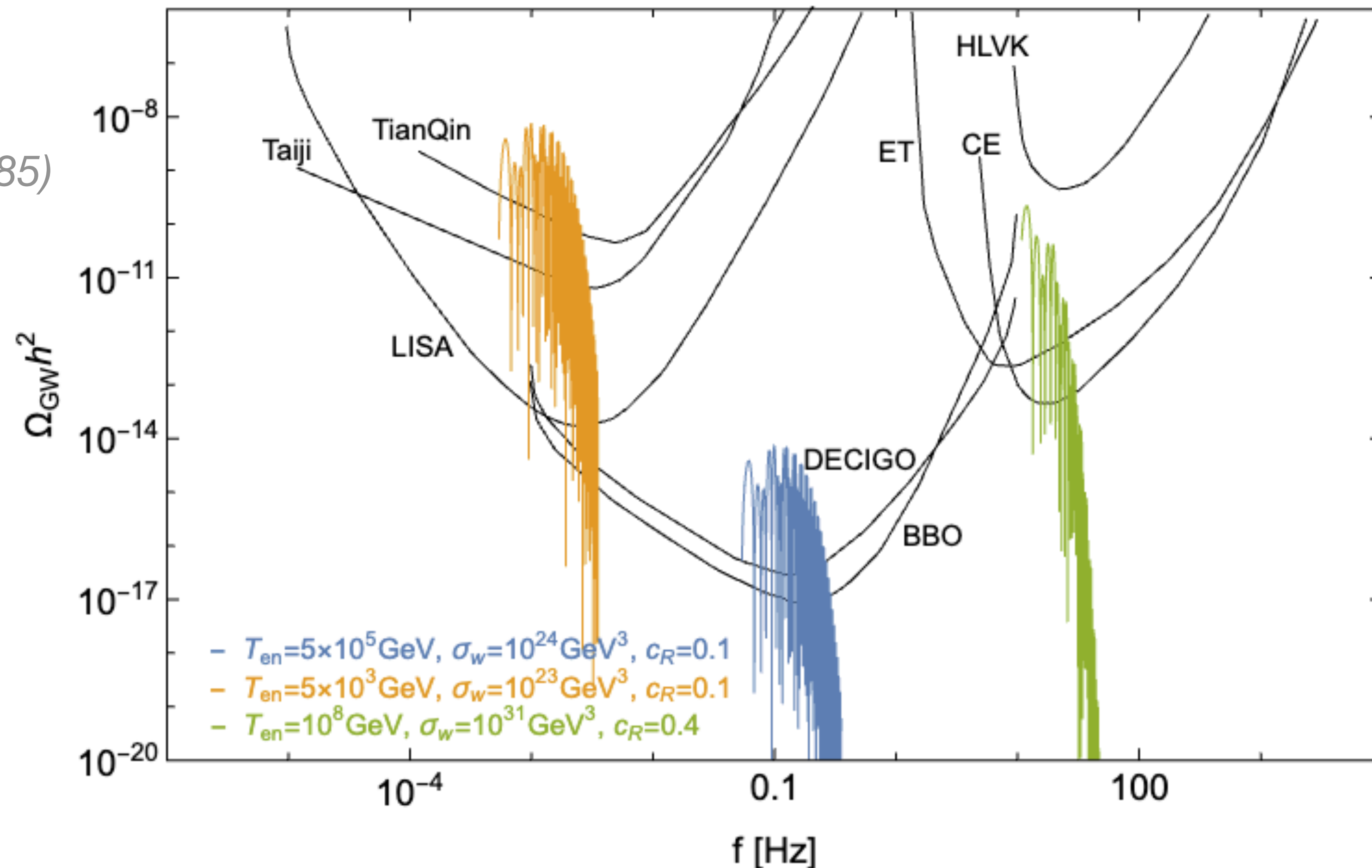
PTA observations

# GW spectra

$$P_{\text{GW}}(t) \sim G \ddot{Q}_{ij} \ddot{Q}_{ij}, \quad Q \sim \mathcal{A} M(t) R^2(t) \sim \mathcal{A} \sigma_w R^4(t) \quad \frac{d\rho_{\text{GW}}(t)}{dt} \sim \frac{P_{\text{GW}}(t)}{H^{-3}(t_{\text{en}})} \frac{a^3(t_{\text{en}})}{a^3(t)} \sim P_{\text{GW}}(t) t_{\text{en}}^{-3/2} t^{-3/2}.$$

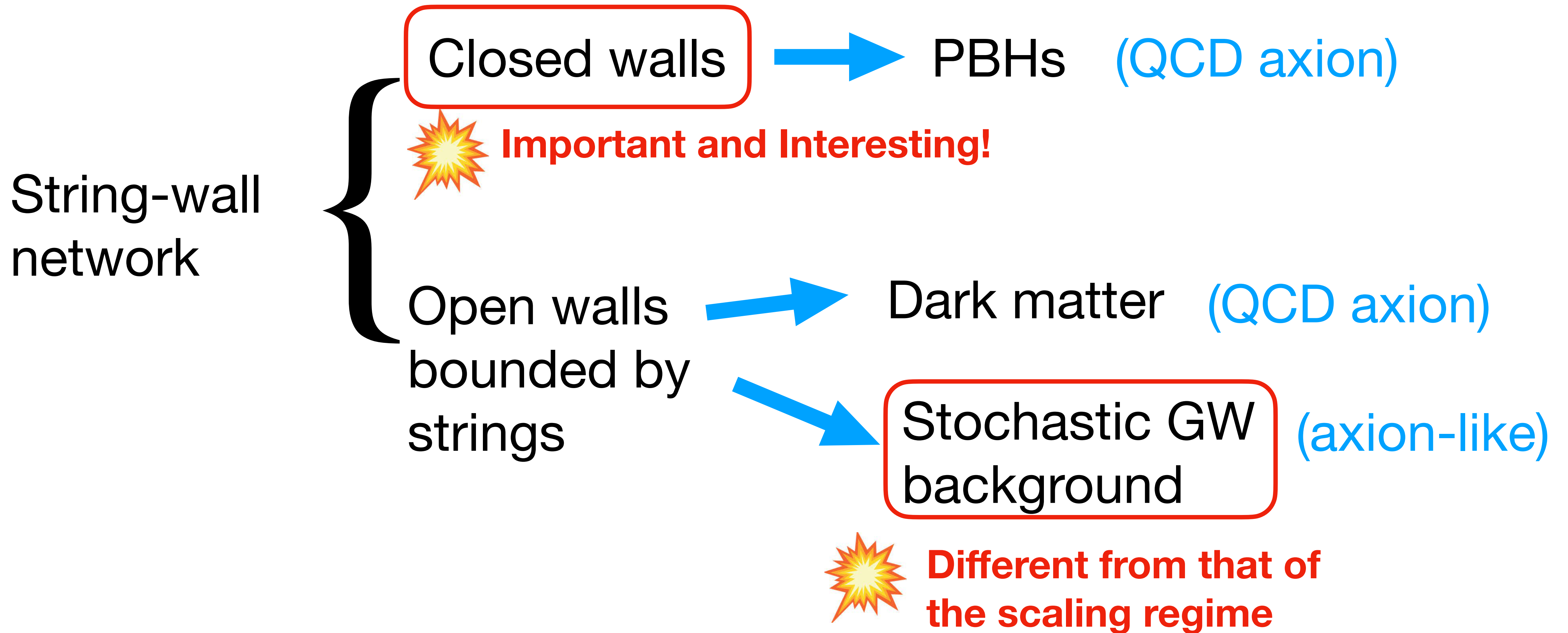
$$\Omega_{\text{GW}}(t_0) \equiv \frac{1}{\rho_{\text{cr}}(t_0)} \frac{d\rho_{\text{GW}}(t_0)}{d \ln f(t_0)} \simeq \frac{1}{\rho_{\text{cr}}(t_0)} \frac{a^4(t)}{a^4(t_0)} \frac{d\rho_{\text{GW}}(t)}{dt} \frac{1}{H(t)}.$$

(SG, 2307.08185)



Sensitivities of  
GW interferometry  
experiments

# Summary



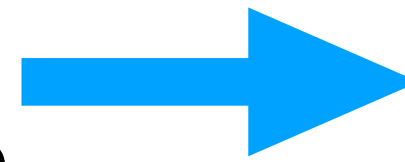


# **Thank you for watching**

# Backup Slides

# Abundance

open walls  
bounded by strings



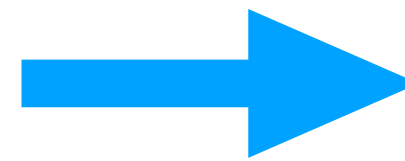
free axions that  
explain **dark matter**

$$\Omega_a h^2 \simeq 0.068 \frac{\mathcal{A}}{\gamma_a} \left[ \frac{10.75}{g_*(T_{\text{en}})} \right]^{\frac{1}{2}} \left( \frac{f_a}{10^9 \text{ GeV}} \right) \left( \frac{20 \text{ MeV}}{T_{\text{en}}} \right) \quad (\text{Keisuke Harigaya, Lian-Tao Wang, 2022}),$$

(SG, Jinhui Guo, Jia Liu, 2023)

Lorentz factor  $\gamma_a \approx 3.2$ , (see e.g., T. Hiramatsu et al, 2012; M. Kawasaki et al, 2015),

closed walls



PBHs

$$f_{\text{PBH}} \equiv \frac{\Omega_{\text{PBH}}}{\Omega_{\text{DM}}} = \gamma \cdot \gamma_a \frac{\Omega_a}{\Omega_{\text{DM}}}. \quad (\text{SG, Jinhui Guo, Jia Liu, 2023})$$

If  $\Omega_a = \Omega_{\text{DM}}$ , we have  $f_{\text{PBH}} = \gamma \cdot \gamma_a = 2.56 \%$ ,  
independent of axion parameters and  $T_{\text{en}}$  !



Gravitational waves are mainly generated during the **scaling regime** of the string wall-network

**scaling regime: roughly one piece of wall per horizon volume**

$$\rho_{\text{DW}}(t) \propto H(t) \quad \rho_{\text{DW}} = \mathcal{A} \sigma_{\text{DW}} H \quad \mathbf{A \text{ is an } \mathcal{O}(1) \text{ number}}$$

DW networks radiate GWs with the power

$$P_{\text{GW}} \sim G \ddot{Q}_{ij} \ddot{Q}_{ij} \text{ where } Q_{ij} \sim \mathcal{A} \sigma_{\text{DW}} H^{-4} \quad Q_{ij} \sim \mathcal{A} \sigma_{\text{DW}} H^{-4}$$

$$\rho_{\text{GW}} = \epsilon P_{\text{GW}} t / H^{-3} = \epsilon G \mathcal{A}^2 \sigma_{\text{DW}}^2$$

Define the GW spectrum as,  $\Omega_{\text{GW}}(f, T) \equiv \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}(f, T)}{d \ln f}$ ,

At the peak frequency,

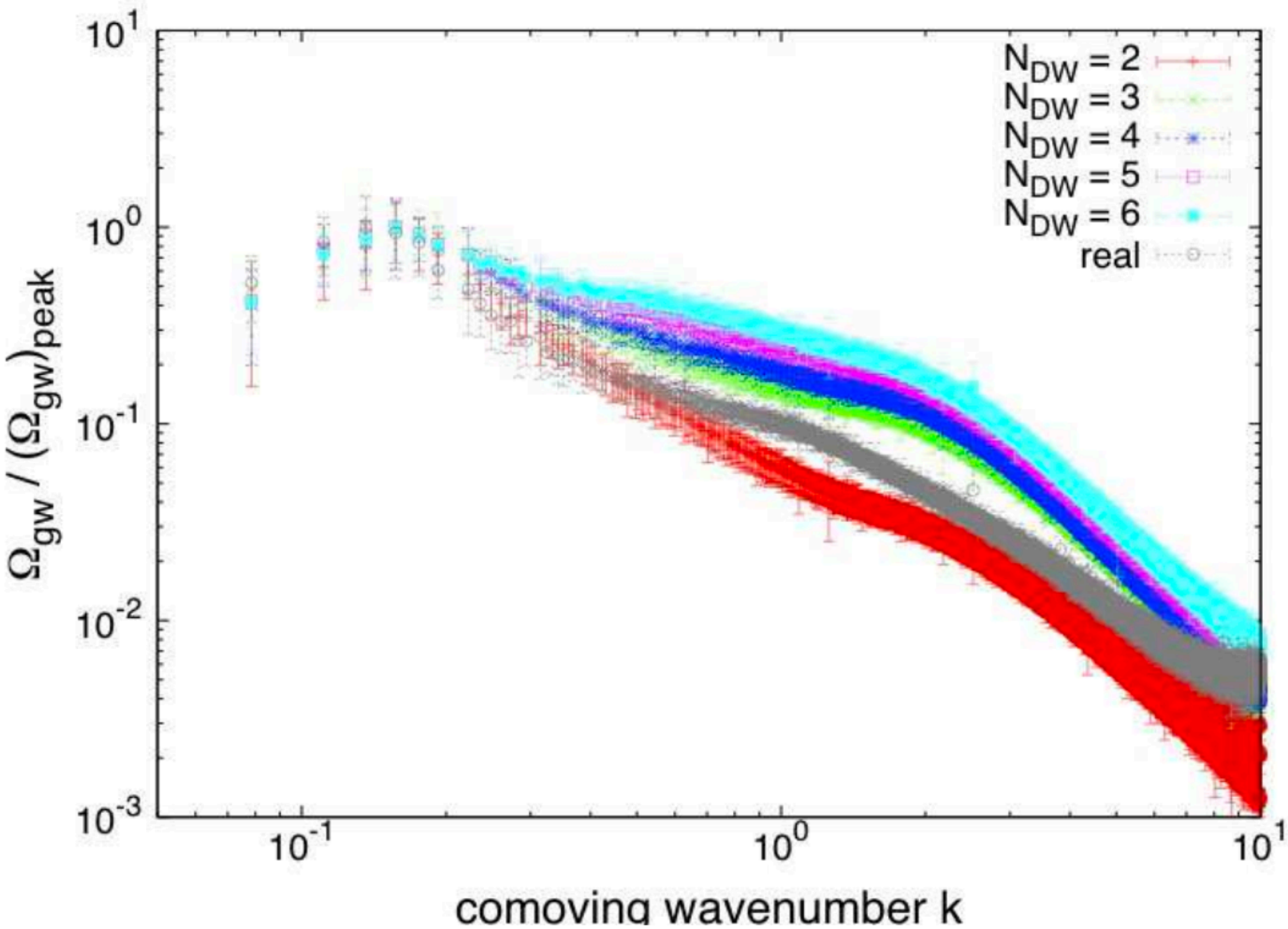
see eg (T. Hiramatsu et al 1207.3166);

(Ligong Bian, **SG**, Changhong Li, Jing Shu, Junchao Zong  
2212.07871)

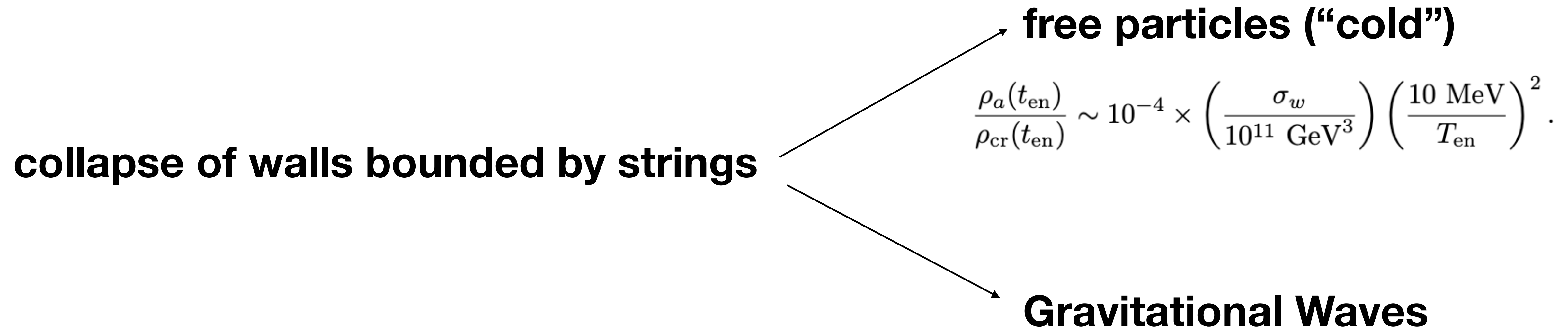
$$\Omega_{\text{GW}}(f_p, T_0) h^2 \simeq 6.5 \times 10^{-10} \mathcal{A}^2 \tilde{\epsilon} \cdot \left[ \frac{10.75}{g_*(T_d)} \right]^{4/3} \left( \frac{\sigma_{\text{DW}}}{10^6 \text{ TeV}^3} \right)^2 \left( \frac{100 \text{ MeV}}{T_d} \right)^4$$

Simulation result of the  
corresponding GW spectra

(T. Hiramatsu et al 1207.3166)



# Overproduction of free particles





# Overproduction of free particles

To avoid the overproduction, we require free particles to further decay into relativistic species. (not disturbing BBN)

1. Decay into SM particles:  $\mathcal{L}_{a\gamma\gamma \text{ or } agg} = \frac{1}{4} \frac{\beta_\gamma}{f_a} a F_{\mu\nu} \tilde{F}^{\mu\nu}, \text{ or } \frac{1}{4} \frac{\beta_g}{f_a} a G_{\mu\nu} \tilde{G}^{\mu\nu}.$

$$\Gamma_{a \rightarrow \gamma\gamma, gg} = \frac{\beta_{\gamma, g}^2}{64\pi} \frac{m_a^3}{f_a^2}.$$

2. Decay into dark photons:

$$\mathcal{L}_{a\gamma'\gamma'} = \frac{1}{4} \frac{\beta_{\gamma'}}{f_a} a F'_{\mu\nu} \tilde{F}'^{\mu\nu},$$

The resultant dark photons may help alleviate the Hubble tension, see (Ligong Bian, **SG**, Changhong Li, Jing Shu, Junchao Zong, 2212.07871)

# Future directions

1. A detailed solution of the cosmic evolution of the multiple components (axions decaying into dark photons) during the BBN epoch. Related to Hubble tension and dark matter abundance.
2. The GW spectra of gauge strings (i.e., cosmic strings) will also be significantly altered by the scenario of re-entering horizon.

....