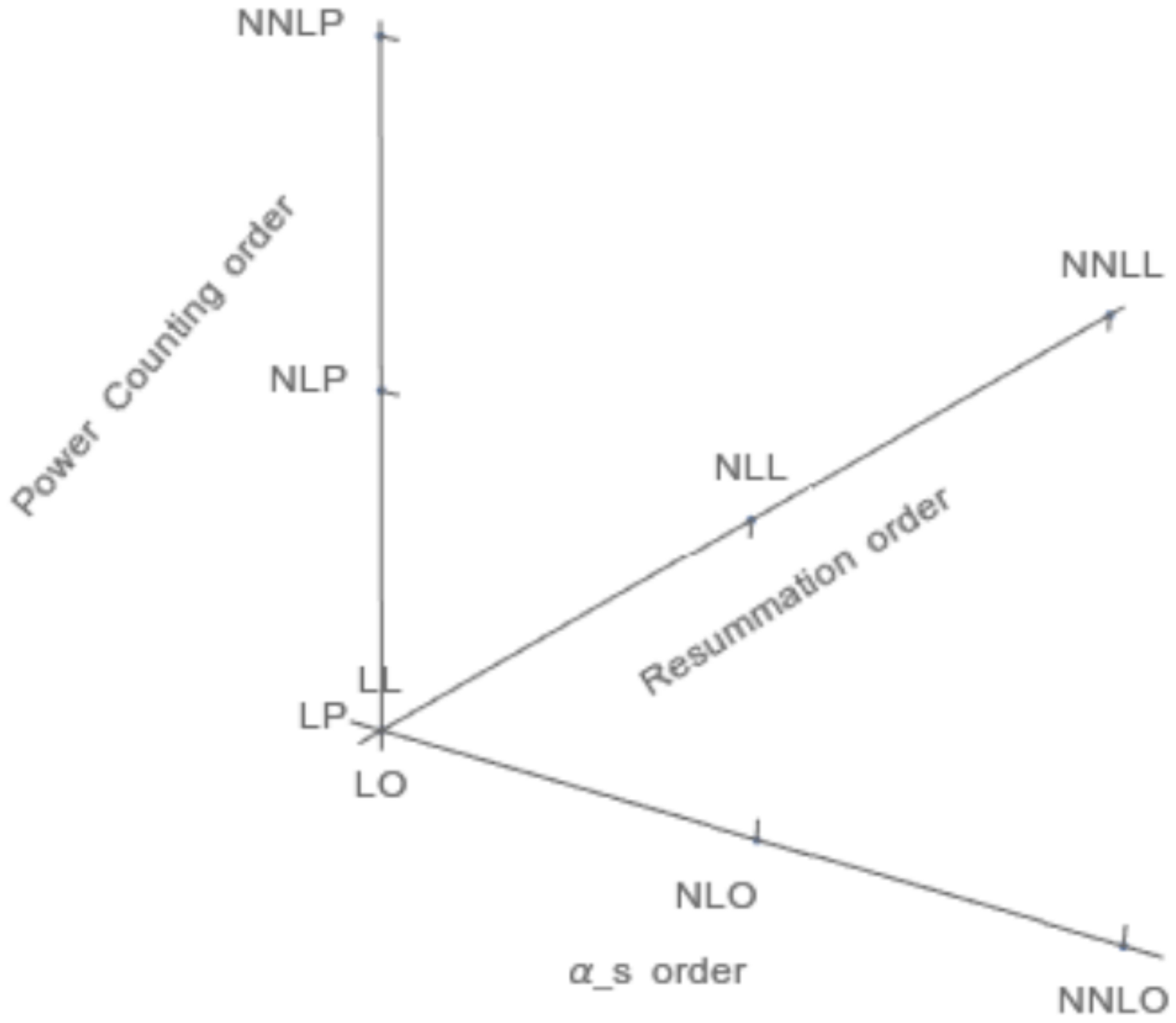
Multi-scale factorization and joint resummation

邵鼎煜

复旦大学

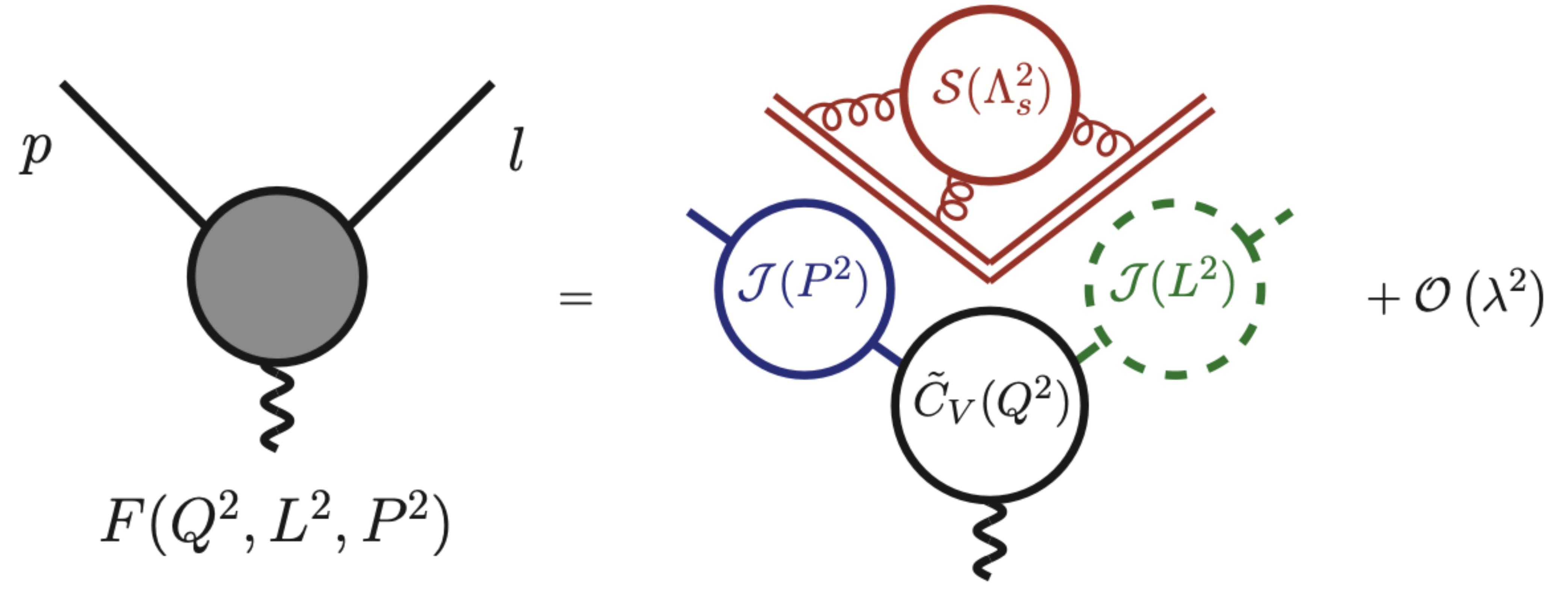
28th Mini-workshop on the frontier of LHC　通化师范学院　Jul 10th, 2024

高能散射截面的微扰级数展开



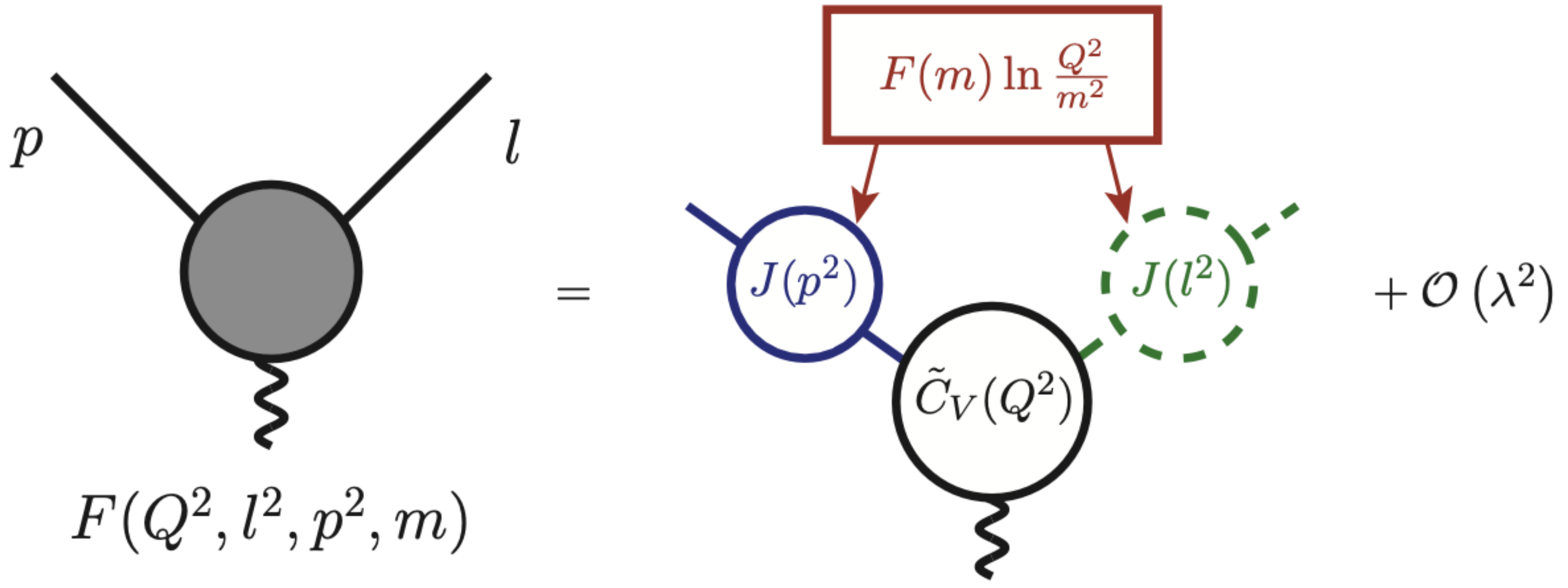
Factorization of Sudakov form factor

$$L^2 \sim P^2 \ll Q^2$$



$$C_V(s,t)\bar{\chi}_c^{(0)}(x_+ + x_\perp + s\bar{n})\, S_n^\dagger(0)\, S_{\bar{n}}(0)\, \gamma_\perp^\mu \chi_{\bar{c}}^{(0)}(x_- + x_\perp + tn)$$

$$L^2 \sim P^2 \sim m^2 \ll Q^2$$



$$e^{-F(m^2,\mu)\ln\frac{Q^2}{m^2}} J_c(p^2, m^2, \mu)\, J_{\bar{c}}(l^2, m^2, \mu)$$

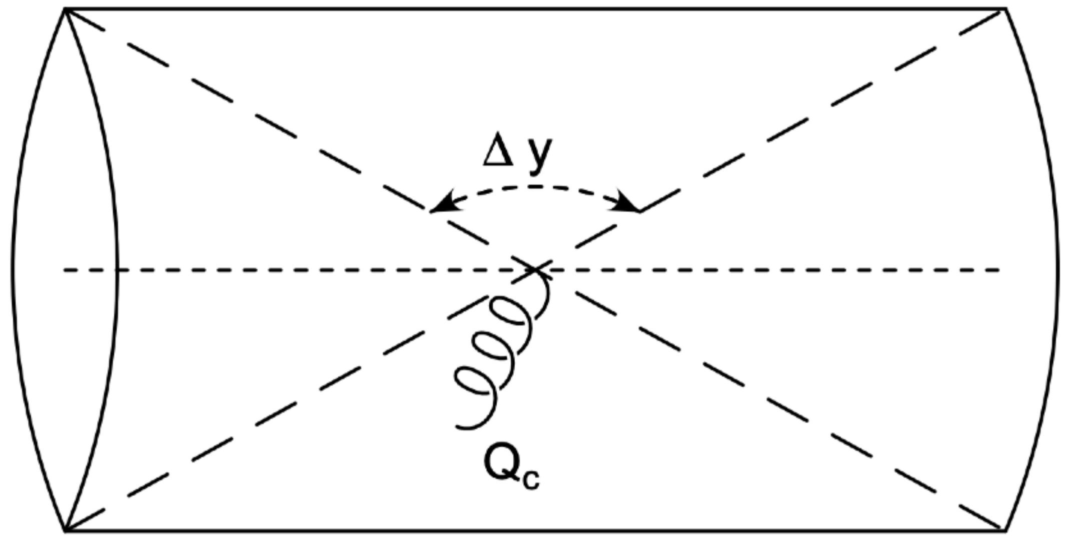
All-order results of Super-Leading Logs for jet veto

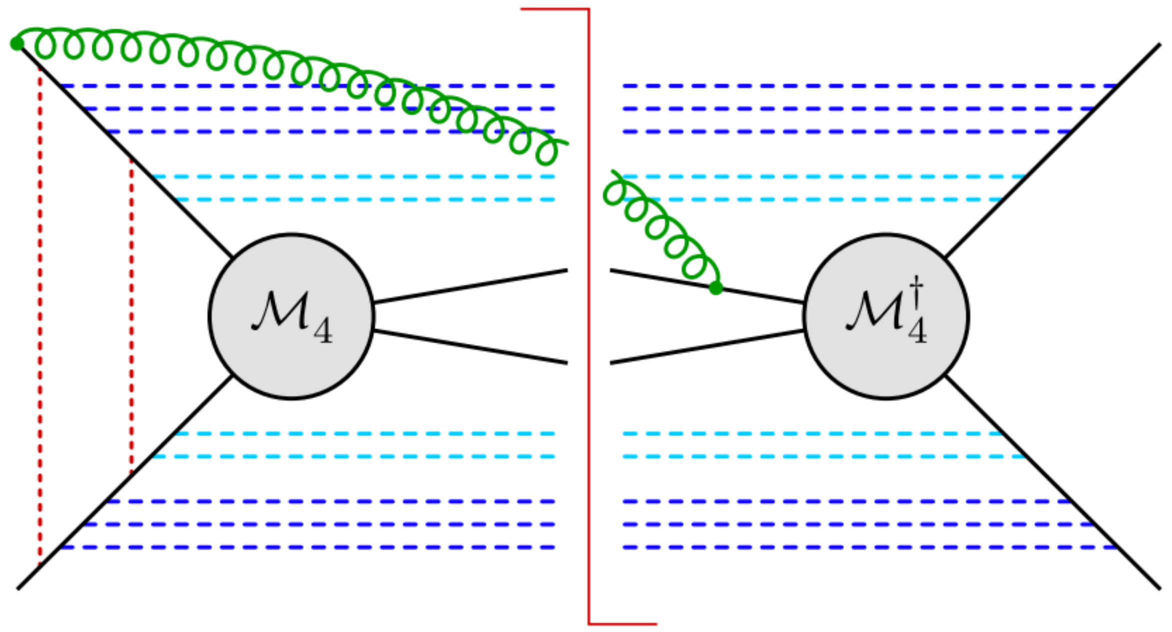
(Becher, Neubert, DYS ’21 PRL + Stillger ’23 JHEP)

All-order structure: Kampe de Feriet function (a two-variable generalization of the generalized hypergeometric series)

$$\Sigma(v,w) = \sum_{m=0}^{\infty}\sum_{r=0}^{\infty} \frac{(1)_{m+r}(1)_m\left(\frac{1}{2}\right)_r}{(2)_{m+r}\left(\frac{5}{2}\right)_{m+r}} \frac{(-w)^m\,(-vw)^r}{m!\,r!}$$

$$= {}^{1+1}F_{2+0}\left(\begin{matrix} 1 : 1, \frac{1}{2} ; \\ 2, \frac{5}{2} : \quad ; \end{matrix}, -w, -vw\right)$$



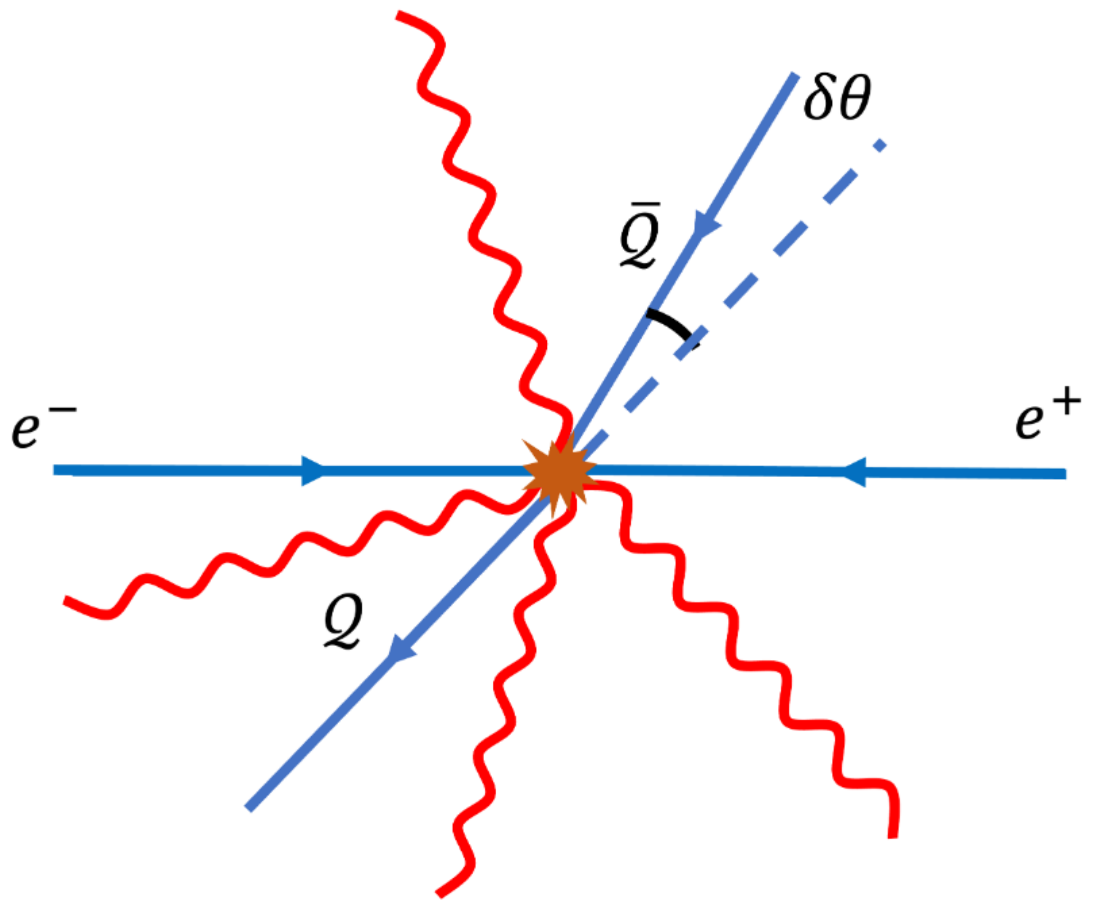


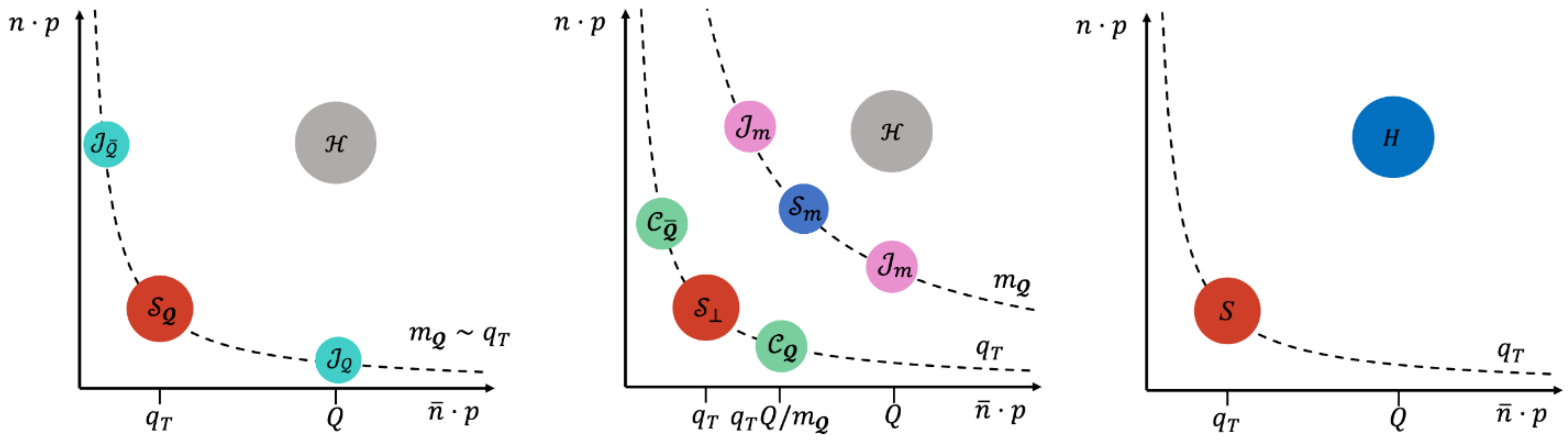
Sudakov suppression of the superleading logarithms is weaker than the one present for global observables

Global logs $\xrightarrow{\omega \to \infty}$ $e^{-\omega}$ **Superleading logs** $\xrightarrow{\omega \to \infty}$ $\frac{1}{\omega}$

Factorization of heavy quark pair angular correlations

Dai, Jiang, DYS in progress





$$\text{Region 1}: \; Q \gg m_Q \sim q_T,$$

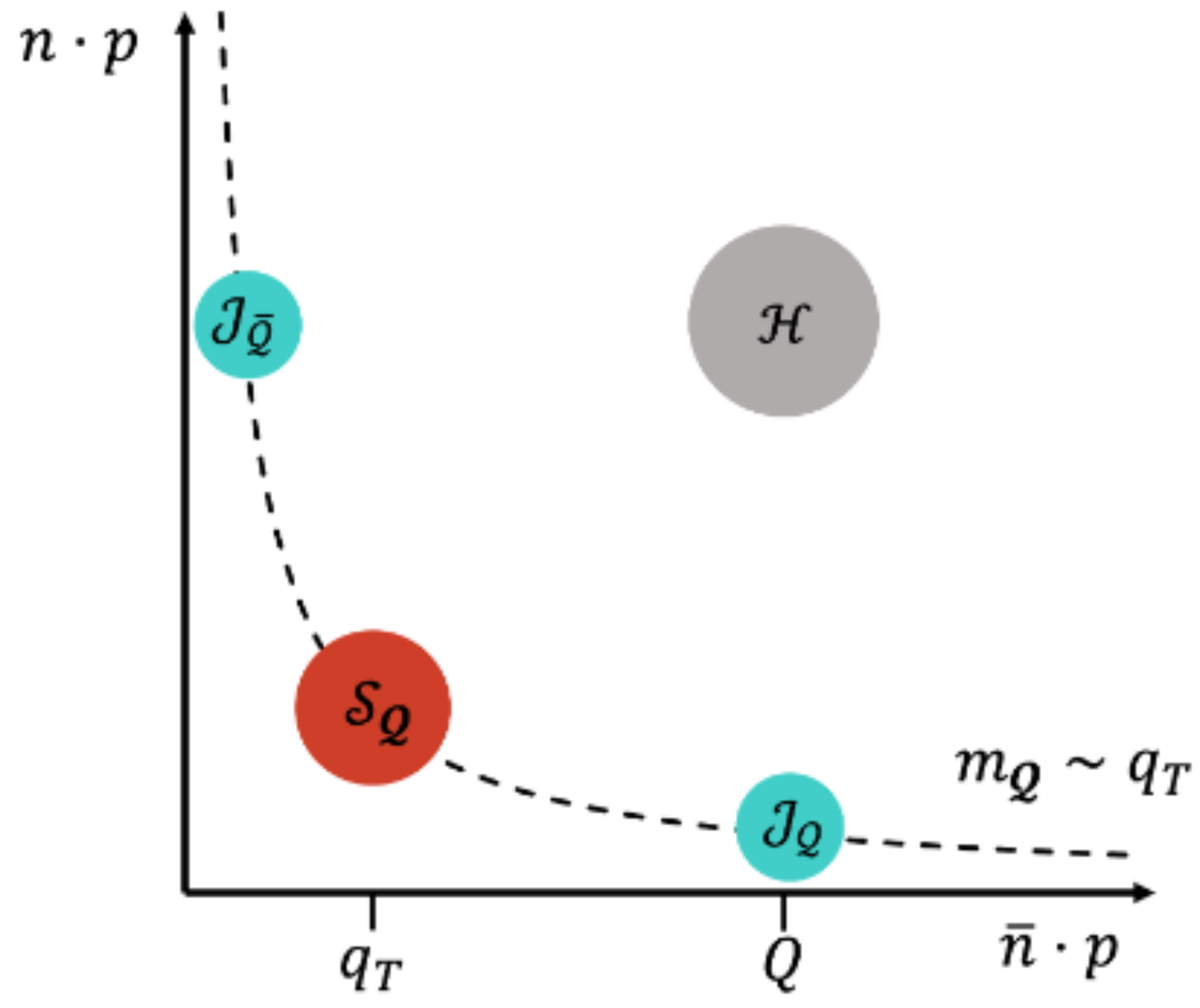
$$\text{Region 2}: \; Q \gg m_Q \gg q_T,$$

$$\text{Region 3}: \; Q \sim m_Q \gg q_T.$$

Factorization in region 1 $Q \gg m_Q \sim q_T$

Standard TMD factorization: two scales

$$\begin{aligned} \text{hard:} \quad & p_h^\mu \sim Q\,(1,1,1) \\ \text{collinear:} \quad & p_c^\mu \sim Q\left(1,\lambda^2,\lambda\right) \\ \text{soft:} \quad & p_s^\mu \sim q_T\,(1,1,1) \end{aligned}$$



Match the quark current operator onto the SCET operator

$$\bar{\psi}\gamma^\mu\psi \to \mathcal{J}_{\mathrm{SCET}} = \bar{\chi}_{\bar{n}} S_{\bar{n}}^\dagger \gamma_\perp^\mu S_n \chi_n$$

Factorization formula

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{q}_T} = \sigma_0 \mathcal{H}(Q,\mu) \int \frac{\mathrm{d}^2\boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{b}_T\cdot\boldsymbol{q}_T} \sum_f e_f^2 \mathcal{J}_{Q/f}\left(b_T, m_Q, \mu, \zeta/\nu^2\right) \mathcal{J}_{\overline{Q}/\bar{f}}\left(b_T, m_Q, \mu, \zeta/\nu^2\right) \mathcal{S}_Q\left(b_T, m_Q, \mu, \nu\right)$$

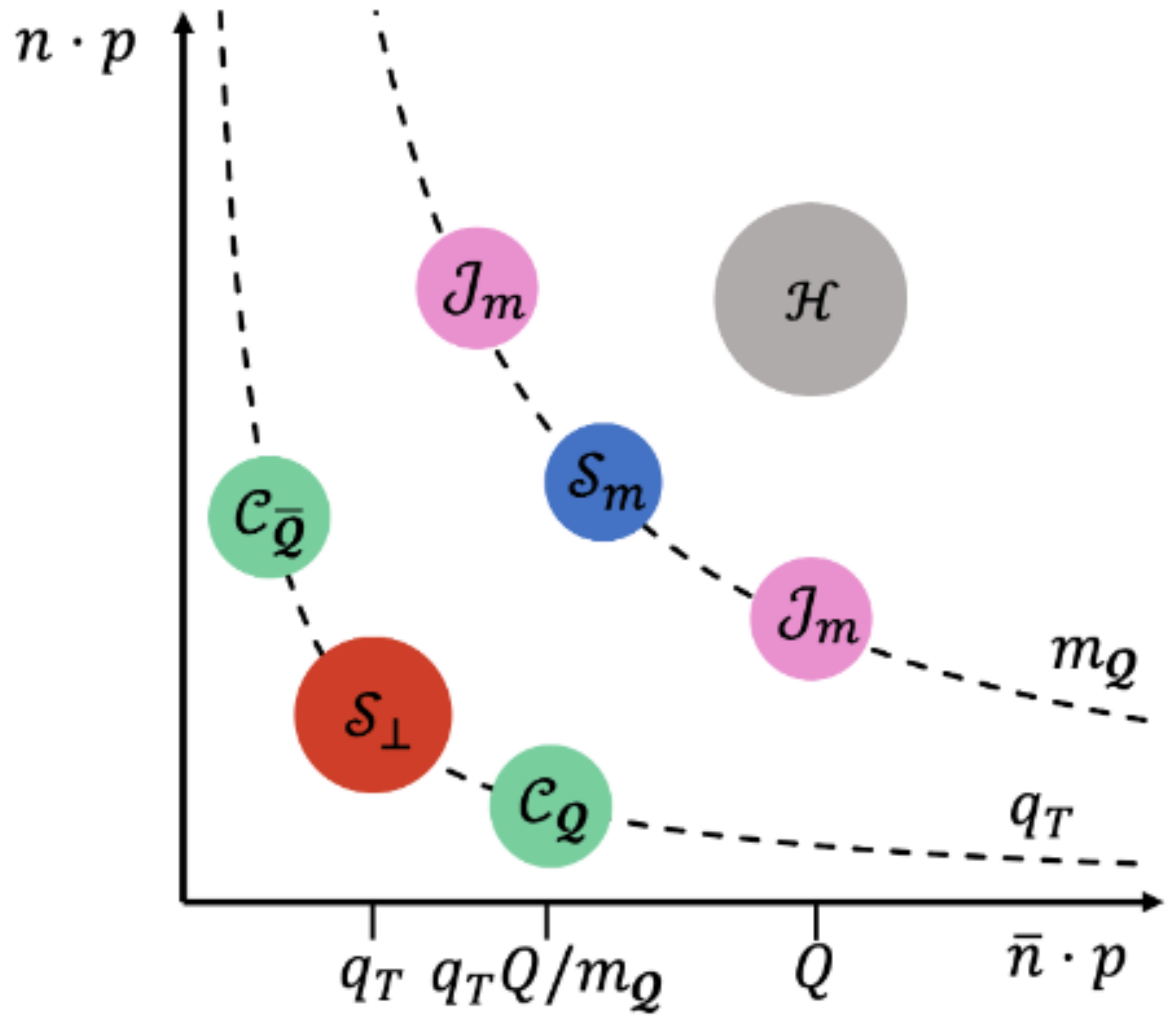
Jet and soft function depend on two scales: *q_T* and *m_Q*

Factorization in region 2 $Q \gg m_Q \gg q_T$

- In the limit $m_Q \gg q_T$, jet function in region 1 contains $\alpha_s^n \log^m(m_Q/q_T)$
- The factorization formula in region 1 with active flavors $n_f = n_l + 1$ should be matched onto a theory with n_l active flavors

  E.g. matching relation of α_s from region 1 to 2

$$\alpha_s^{(n_f)}(\mu) = \alpha_s^{(n_l)}(\mu)\left[1 + \frac{4}{3}T_F \frac{\alpha_s^{(n_l)}(\mu)}{4\pi}\ln\frac{\mu^2}{m_Q^2} + \mathcal{O}(\alpha_s^2)\right]$$



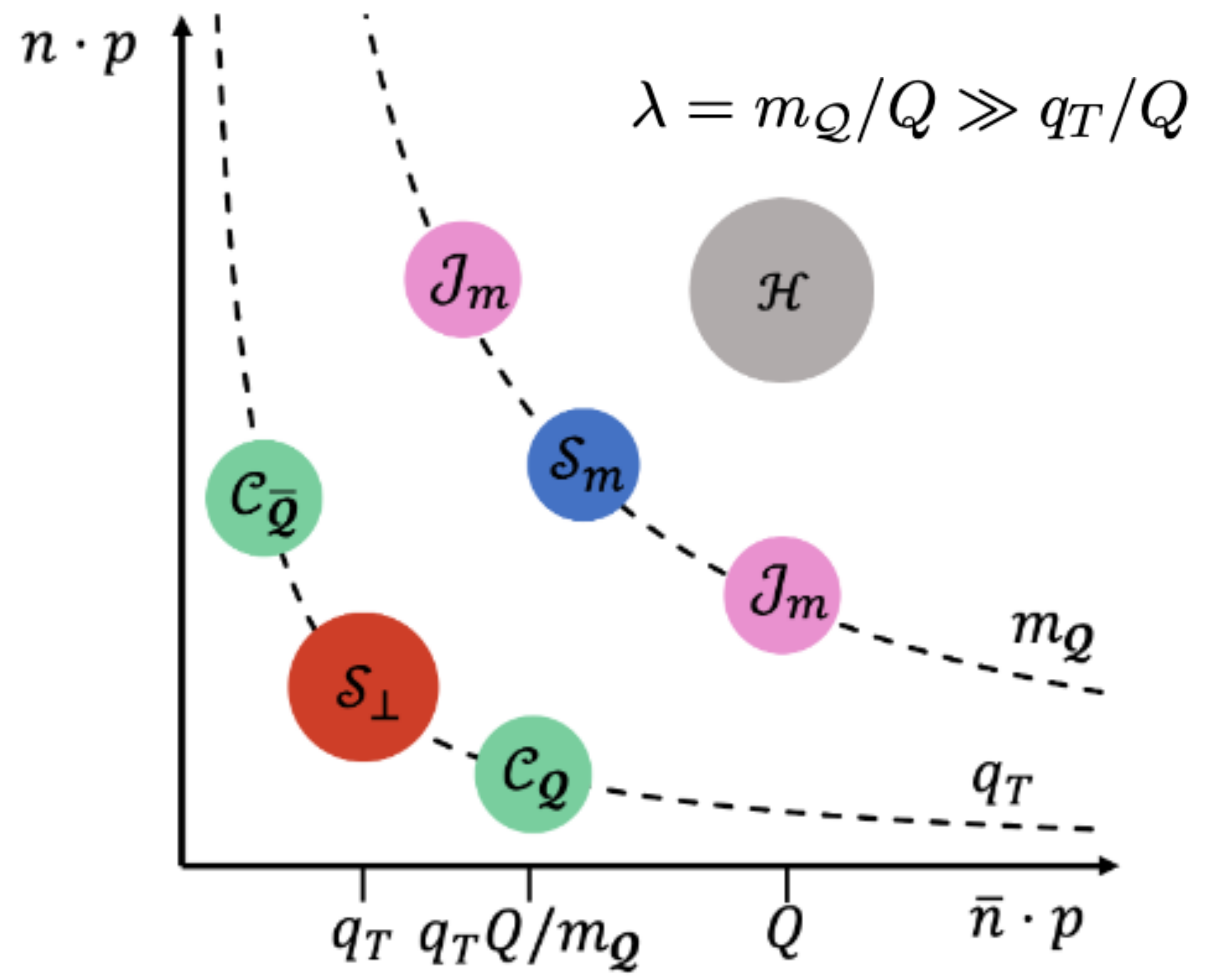
- Heavy quark momenta $P_Q^\mu = m_Q v_+^\mu + p^\mu$

$$v_+^\mu = \left(\frac{Q}{m_Q}, \frac{m_Q}{Q}, 0_\perp\right), \quad p_{uc}^\mu \sim q_T\left(\frac{Q}{m_Q}, \frac{m_Q}{Q}, 1\right)$$

- Decouple the interaction between heavy quark and ultra-collinear modes; Match SCET onto bHQET Fleming, Hoang, Mantry & Stewart '07

$$h_{v_\pm} \to Y_{v_\pm} h_{v_\pm} \qquad \mathcal{J}_{\text{SCET}} \to \mathcal{J}_{\text{bHQET}} = \bar{h}_{v_-} W_{\bar{n}}^\dagger Y_{v_-}^\dagger \gamma_\perp^\mu Y_{v_+} W_n h_{v_+}$$

Factorization in region 2 $Q \gg m_Q \gg q_T$



hard: $p_h^\mu \sim Q\,(1,1,1)$

collinear: $p_c^\mu \sim Q\,(1,\lambda^2,\lambda)$

massive-soft: $p_{ms}^\mu \sim Q\,(\lambda,\lambda,\lambda)$

soft: $p_s^\mu \sim q_T\,(1,1,1)$

ultra-collinear: $p_{uc}^\mu \sim q_T/\lambda\,(1,\lambda^2,\lambda)$

Factorization formula

Matching coefficients form SCET to bHQET

$$\frac{d\sigma}{d^2\boldsymbol{q}_T} = \sigma_Q \mathcal{H}(Q,\mu)\, \mathcal{J}_m^2(m_Q,\mu,\zeta_{\mathcal{J}}/\nu^2)\, \mathcal{S}_m(m_Q,\mu,\nu)$$

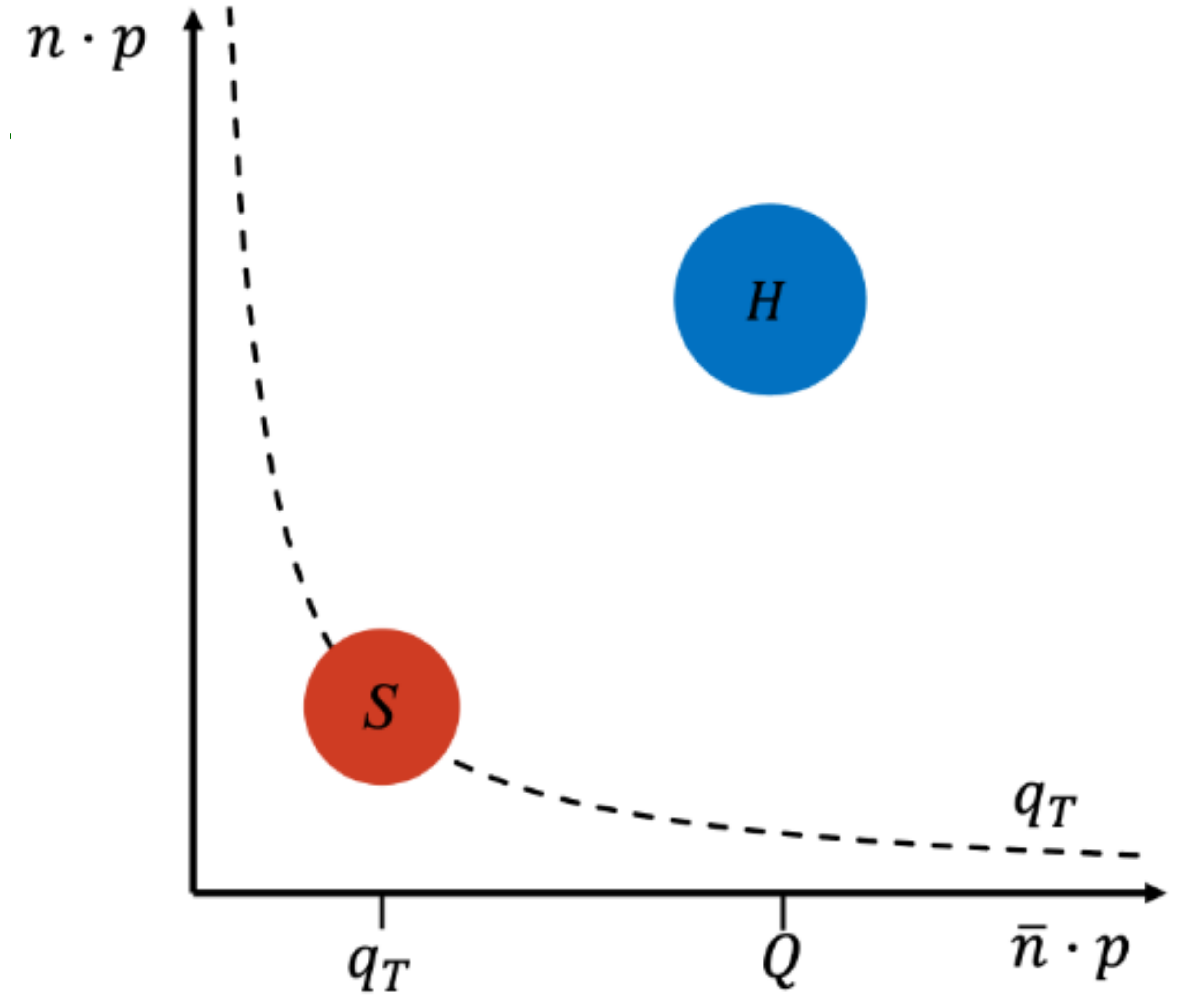
$$\times \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2}\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{q}_T}\, \mathcal{C}_Q(b_T,\mu,\zeta_{\mathcal{C}}/\nu^2)\, \mathcal{C}_{\bar{Q}}(b_T,\mu,\zeta_{\mathcal{C}}/\nu^2)\, \mathcal{S}_\perp(b_T,\mu,\nu)$$

Hoang, Pathak, Pietrulewicz &Stewart, '15

Standard TMD soft function

Factorization in region 3 $Q \sim m_Q \gg q_T$

- **In region 3, TMD factorization of heavy quark pair production are well studied**
  - **Factorization and resummation** (Li, Li, DYS, Yang, Zhu '12 '13 & Catani, Grazzini, Torre '14 & Catani, Grazinni & Sargsyan '18; Ju, Schönher '22
  - **Two-loop soft function** Angeles-Martinez, M. Czakon, and S. Sapeta'18; Catan Mazzitelli '23



- **Factorization formula**

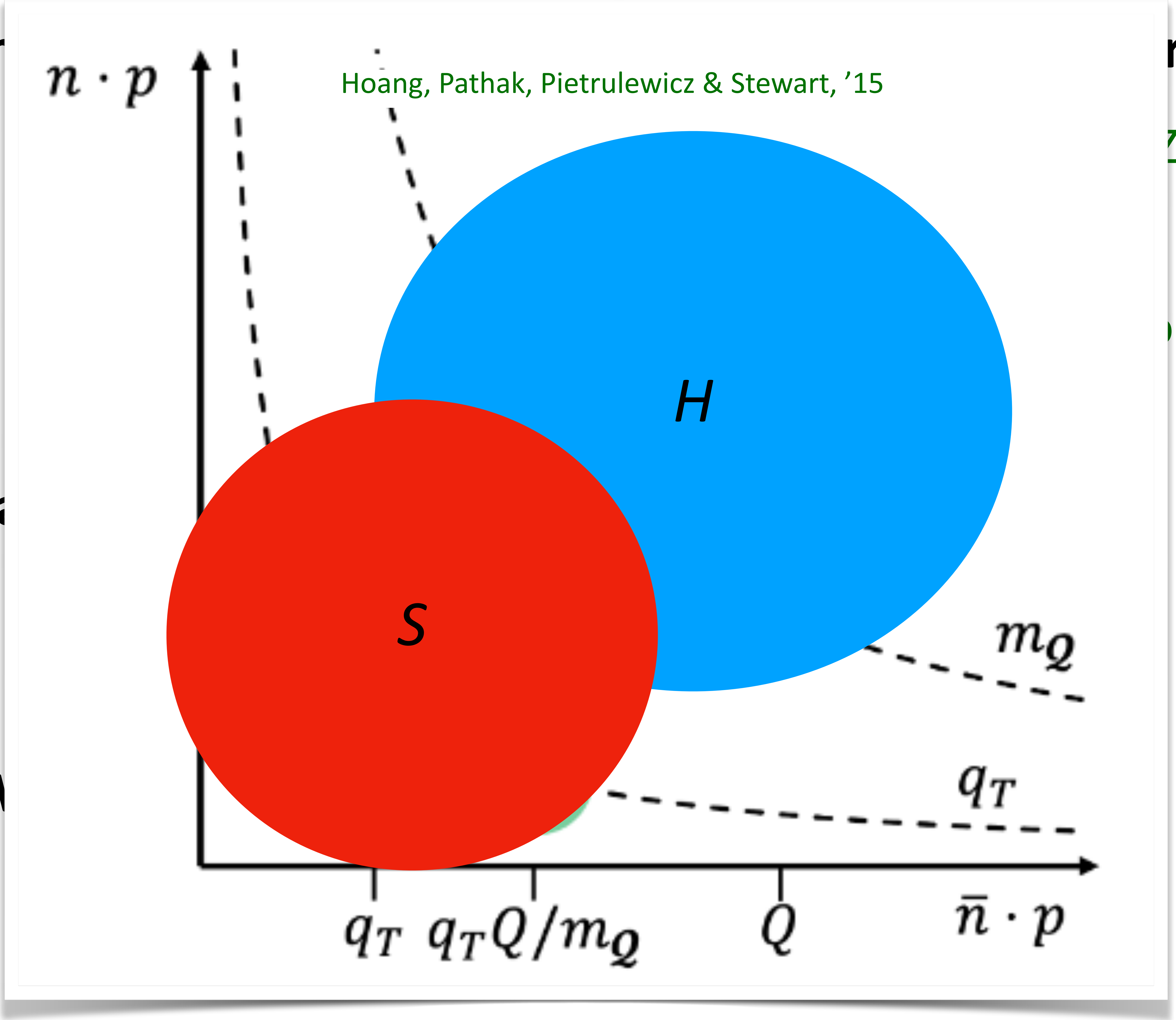
$$\frac{d\sigma}{d^2\boldsymbol{q}_T} = \sigma_Q H(Q, m_Q, \mu) \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{b}_T \cdot \boldsymbol{q}_T} S(b_T, \beta_Q, \mu)$$

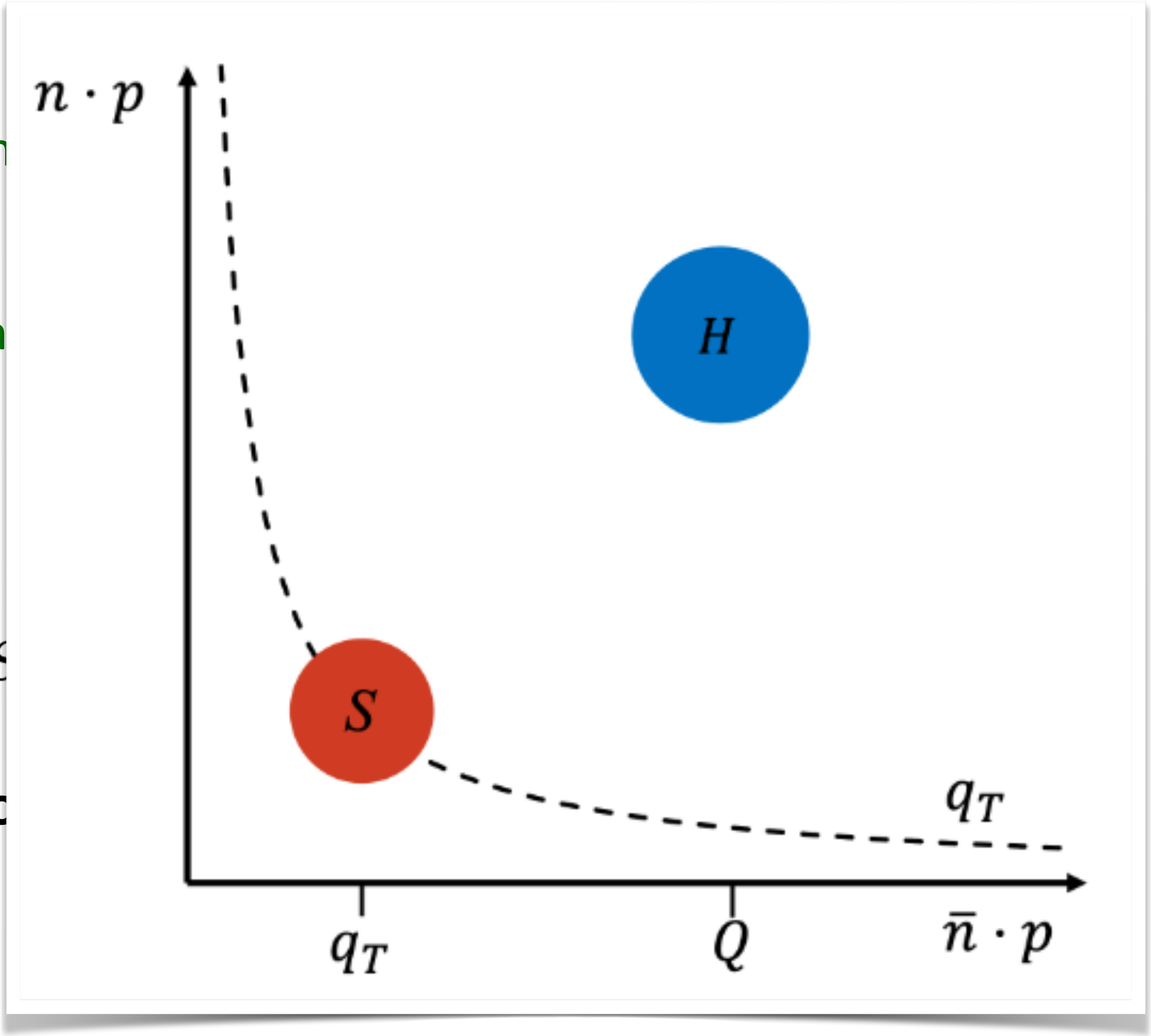
- **We use their expressions to verify refactorization of soft function at two loop**

$$\gamma_\mu^S \xrightarrow{Q \gg m_Q} 2\gamma_\mu^{\mathcal{C}_Q} + \gamma_\mu^{\mathcal{S}}$$

- **Two loop ultra-collinear function can be determined based on** $S \xrightarrow{Q \gg m_Q} \mathcal{C}_Q^2 \mathcal{S}_\perp$

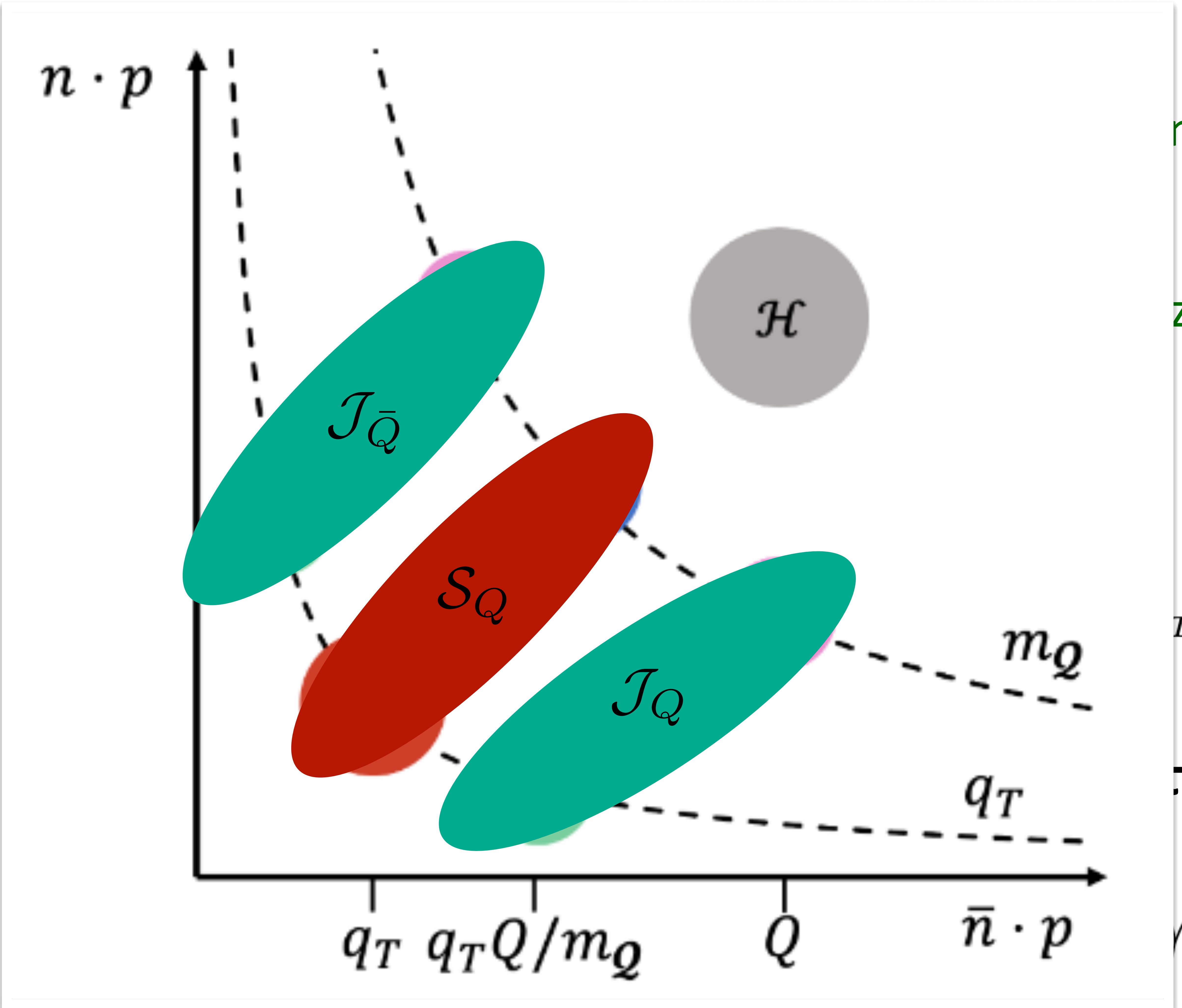
Factorization in region 3 $Q \sim m_Q \gg q_T$

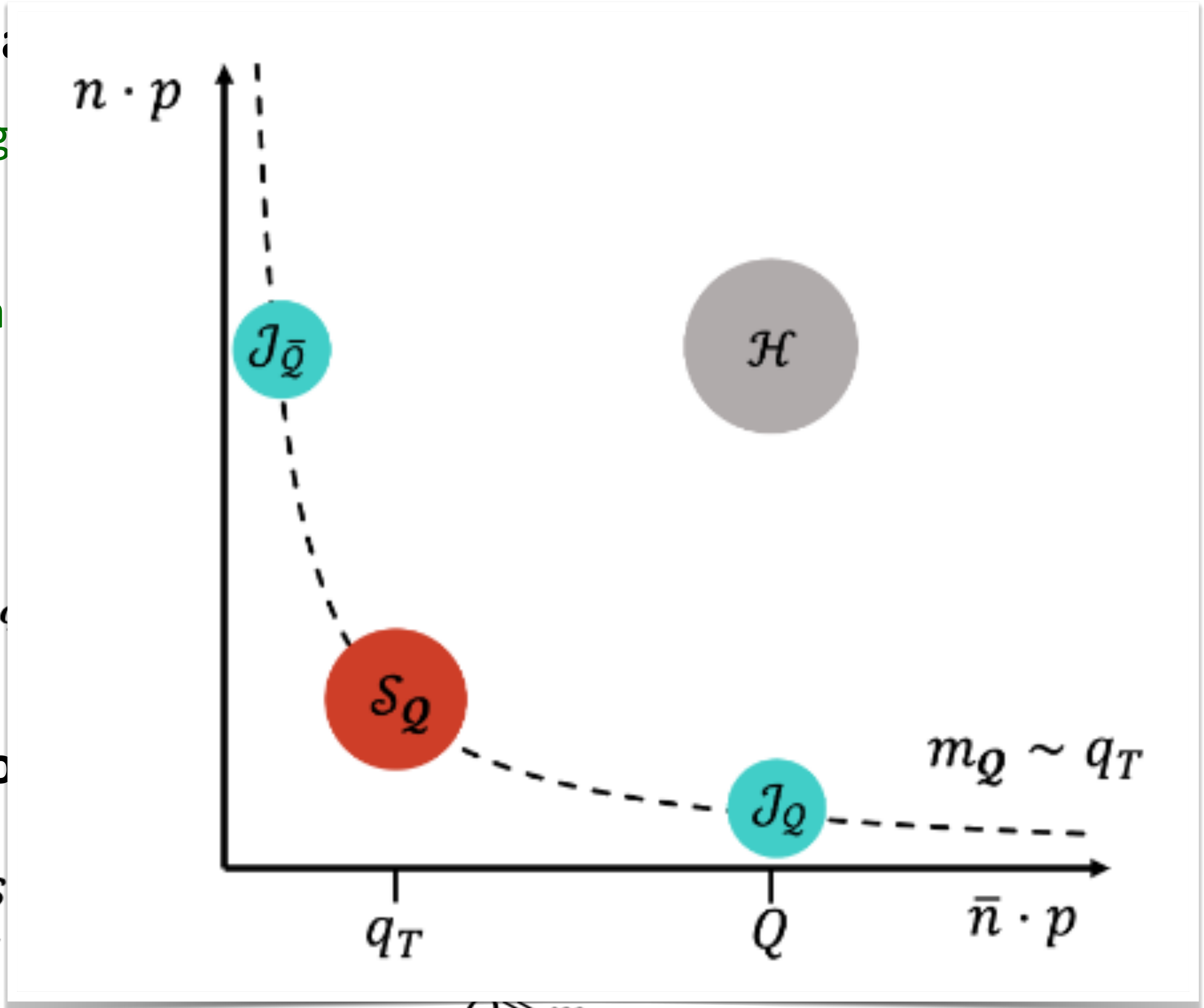




- **Two loop ultra-collinear function can be determined based on** $S \xrightarrow{Q \gg m_Q} \mathcal{C}_Q^2 \mathcal{S}_\perp$

Factorization in region 3 $Q \sim m_Q \gg q_T$

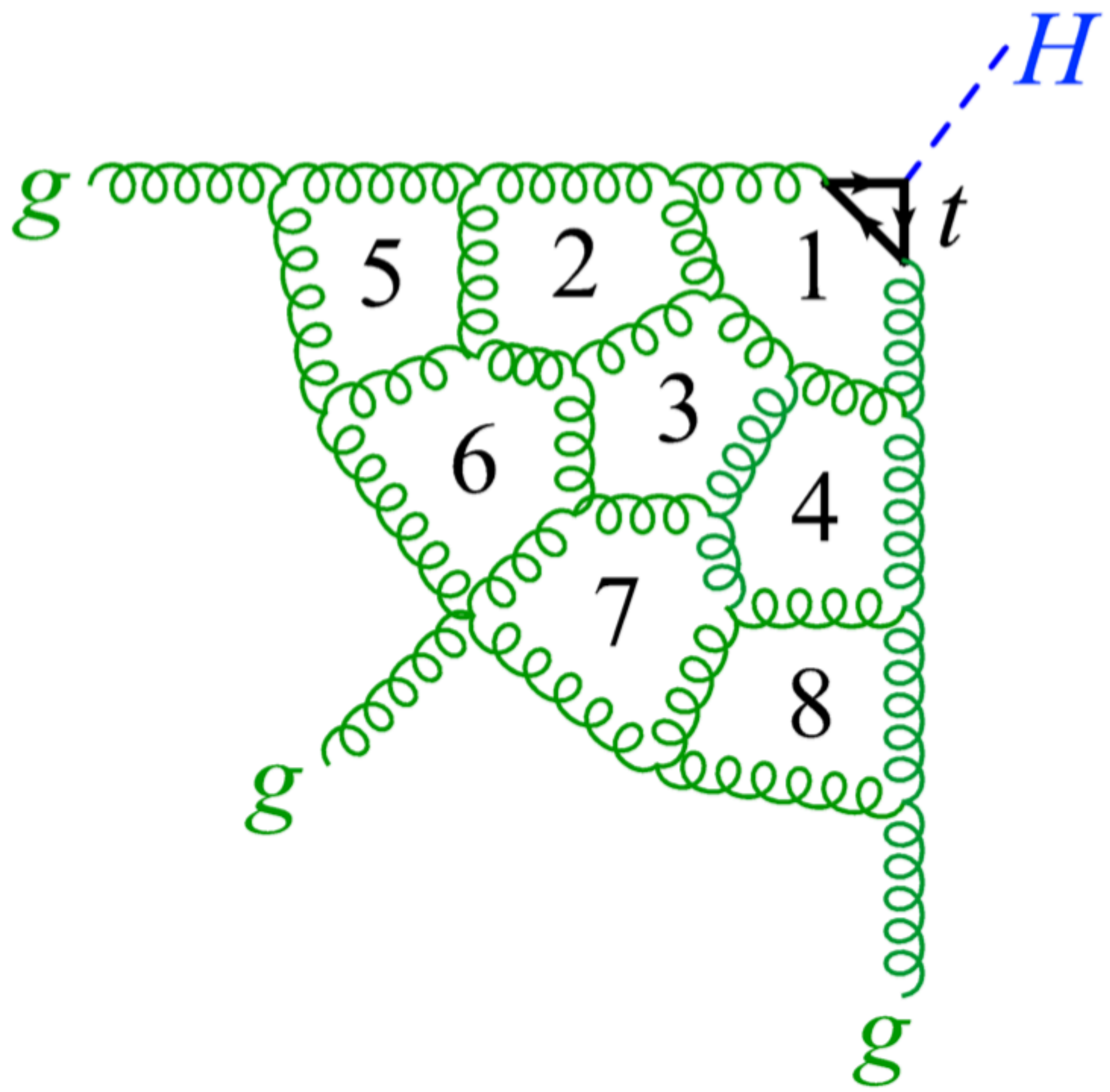




- **Two loop ultra-collinear function can be determined based on** $S \xrightarrow{Q \gg m_Q} \mathcal{C}_Q^2 \mathcal{S}_\perp$

Transformer meets amplitudes

Cai, Merz, Charton, Nolte, Wilhelm, Crammer, Dixon '24



$$H \to ggg, \quad L = 8$$

Alphabet: $\mathcal{L}_{\mathbf{3gFF}} = \{a, b, c, d, e, f\}$

Key idea: Symbol as Language

- Symbol to Sequence

$$8\, b \otimes b \otimes c \otimes d \to \{'+', '8', 'b, b, c, d'\}$$

- Sequence to Transformer

Solution at L = 5~8 provide enough samples to learn

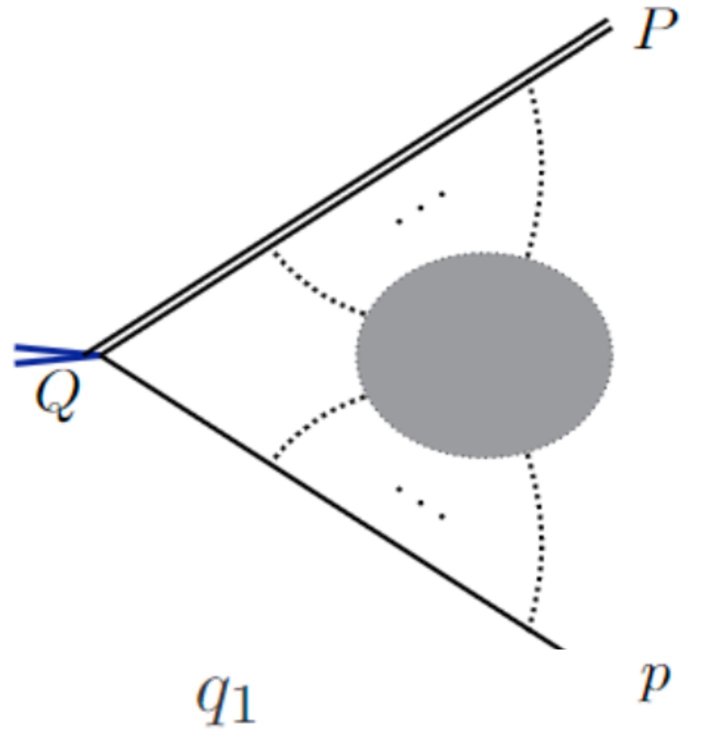
- Test and prediction

$$c_i \underbrace{b \otimes \cdots \otimes d}_{2L} \to \text{Transformer} \to c_i = ?$$

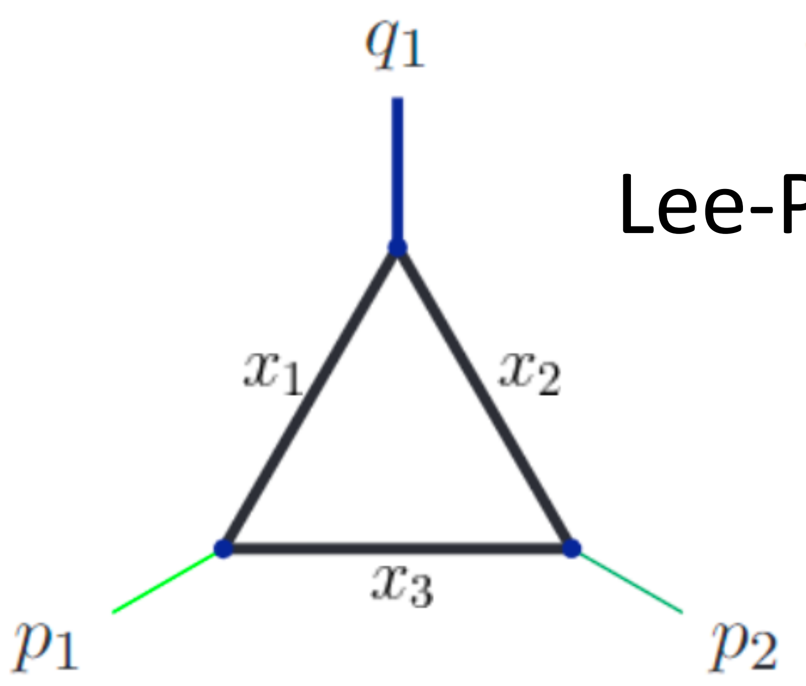
help calculate high loops
discover new symmetries

Heavy-to-light form factor

$$P^2 = M^2 \sim Q^2, \quad p^2 = m^2 \sim \lambda Q^2, \quad P\cdot p \sim Q^2$$



full phase space



Lee-Pomeransky representation ’13

Asy2, ASPIRE, pySecDec, ...

$$\int_0^\infty dx_1 dx_2 dx_3 x_1^{\nu_1-1} x_2^{\nu_2-1} x_3^{\nu_3-1} \over (x_1+x_2+x_3-p_1^2x_1x_3-p_2^2x_2x_3-q_1^2x_1x_2)^{D/2}$$

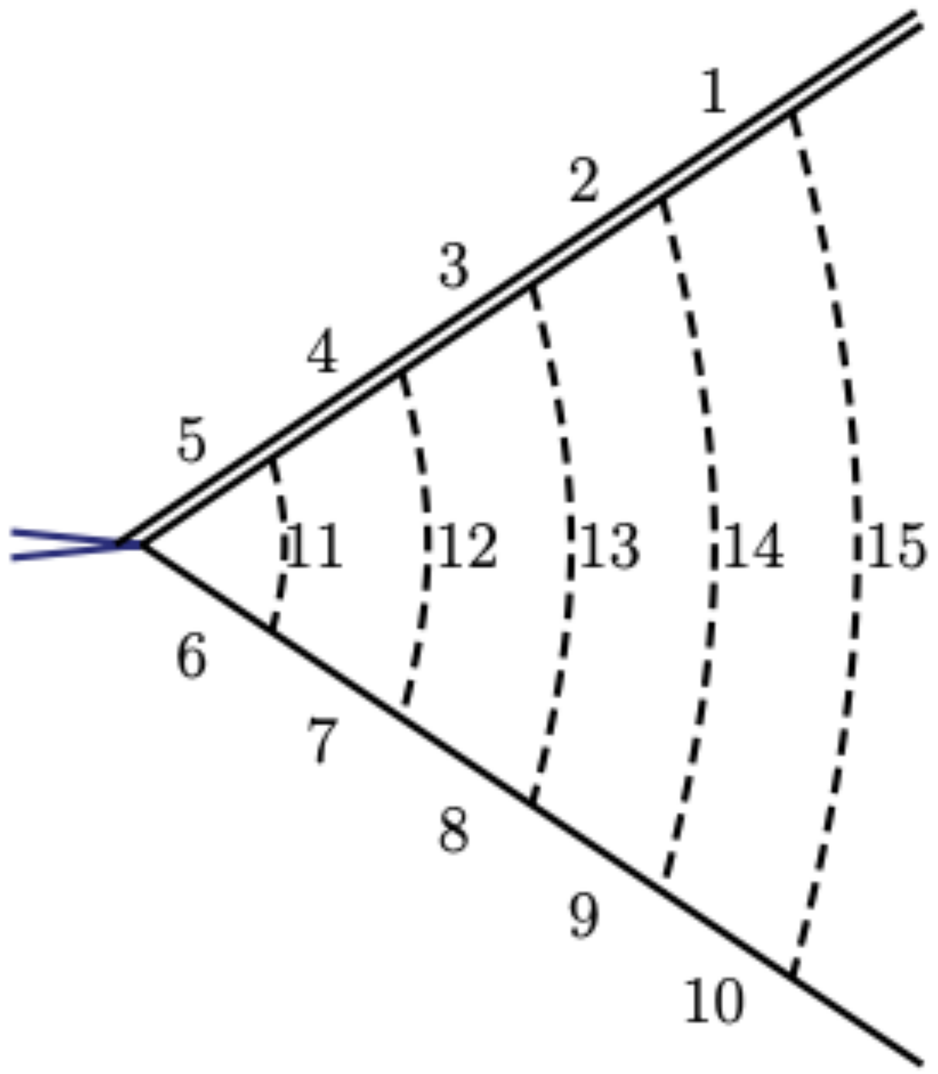
$$s x_1^{v_1} x_2^{v_2}\cdots x_n^{v_n} \to (v_1, v_2, \ldots, v_n; a) \quad \text{if} \quad s\sim \lambda^a Q^2$$

Regions in the mass expansion

• More modes are included:	Starting from
hard mode Q(1,1,1),	1 loop
collinear mode Q(1,λ,λ½),	1 loop
soft mode Q(λ,λ,λ),	2 loops
soft·collinear mode $Q(\lambda,\lambda^2,\lambda^{3/2})$,	3 loops
soft² mode $Q(\lambda^2,\lambda^2,\lambda^2)$, ...	4 loops
semihard mode Q(λ½,λ½,λ½),	2 loops
semihard·collinear, semihard·soft, ...,	3 loops, nonplanar
semicollinear mode $Q(1,\lambda^{1/2},\lambda^{1/4})$,	3 loops, nonplanar
semihard·semicollinear, ...	4 loops, nonplanar

马尧 SCET2024 Ref:arXiv:2312.14012

Heavy-to-light form factor



64 region vectors

Region vector as Language?

$(-1,-1,-1,0,0,-1,-1,-1,-2,-2,-1,-1,-2,-3,-3;1)_{\mathrm{I}}$,
$(-1,-1,-1,0,0,-1,-1,-1,-1,-1,-1,-1,-2,-2,-2;1)_{\mathrm{I}}$,
$(-1,-1,-1,0,0,0,-1,-1,-1,-2,0,-1,-2,-2,-3;1)_{\mathrm{I}}$,
$(-1,-1,0,0,0,-1,-1,-1,-1,-2,-1,-1,-1,-2,-3;1)_{\mathrm{I}}$,
$(-1,-1,0,0,0,-1,0,-1,-1,-2,0,0,-1,-2,-3;1)_{\mathrm{IIA}}$,
$(-1,-1,0,0,0,0,-1,-1,-1,-2,0,-1,-1,-2,-3;1)_{\mathrm{I}}$,
$(-1,-1,0,0,0,0,0,-1,-1,-2,0,0,-1,-2,-3;1)_{\mathrm{I}}$,
$(-1,0,0,0,0,-1,-1,-1,-1,-1,-1,-1,-1,-1,-2;1)_{\mathrm{I}}$,
$(-1,0,0,0,0,-1,0,-1,-1,-1,0,0,-1,-1,-2;1)_{\mathrm{IIA}}$,
$(-1,0,0,0,0,0,-1,-1,-1,-1,0,-1,-1,-1,-2;1)_{\mathrm{I}}$,
$(-1,0,0,0,0,0,0,-1,-1,-1,0,0,-1,-1,-2;1)_{\mathrm{I}}$,
$(0,0,0,0,0,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1;1)_{\mathrm{I}}$,
$(0,0,0,0,0,-1,-1,-1,0,-1,-1,-1,0,0,-1;1)_{\mathrm{IIA}}$,
$(0,0,0,0,0,-1,-1,0,-1,-1,-1,0,0,-1,-1;1)_{\mathrm{IIA}}$,
$(0,0,0,0,0,-1,-1,0,0,-1,-1,0,0,0,-1;1)_{\mathrm{IIA}}$,
$(0,0,0,0,0,-1,0,-1,-1,-1,0,0,-1,-1,-1;1)_{\mathrm{IIA}}$,
$(0,0,0,0,0,-1,0,-1,0,-1,0,0,0,0,-1;1)_{\mathrm{IIA}}$,
$(0,0,0,0,0,-1,0,0,-1,-1,0,0,0,-1,-1;1)_{\mathrm{IIA}}$,
$(0,0,0,0,0,-1,0,0,0,-1,0,0,0,0,-1;1)_{\mathrm{IIA}}$,
$(0,0,0,0,0,0,-1,-1,-1,-1,0,-1,-1,-1,-1;1)_{\mathrm{I}}$,

$(-1,-1,-1,0,0,-1,-1,-1,-1,-2,-1,-1,-2,-2,-3;1)_{\mathrm{I}}$
$(-1,-1,-1,0,0,0,-1,-1,-2,-2,0,-1,-2,-3,-3;1)_{\mathrm{I}}$,
$(-1,-1,-1,0,0,0,-1,-1,-1,-1,0,-1,-2,-2,-2;1)_{\mathrm{I}}$,
$(-1,-1,0,0,0,-1,-1,-1,-1,-1,-1,-1,-1,-2,-2;1)_{\mathrm{I}}$,
$(-1,-1,0,0,0,-1,0,-1,-1,-1,0,0,-1,-2,-2;1)_{\mathrm{IIA}}$,
$(-1,-1,0,0,0,0,-1,-1,-1,-1,0,-1,-1,-2,-2;1)_{\mathrm{I}}$,
$(-1,-1,0,0,0,0,0,-1,-1,-1,0,0,-1,-2,-2;1)_{\mathrm{I}}$,
$(-1,0,0,0,0,-1,-1,0,-1,-1,-1,0,0,-1,-2;1)_{\mathrm{IIA}}$,
$(-1,0,0,0,0,-1,0,0,-1,-1,0,0,0,-1,-2;1)_{\mathrm{IIA}}$,
$(-1,0,0,0,0,0,-1,0,-1,-1,0,0,0,-1,-2;1)_{\mathrm{IIA}}$,
$(-1,0,0,0,0,0,0,0,-1,-1,0,0,0,-1,-2;1)_{\mathrm{I}}$,
$(0,0,0,0,0,-1,-1,-1,-1,0,-1,-1,-1,0,0;1)_{\mathrm{IIA}}$,
$(0,0,0,0,0,-1,-1,-1,0,0,-1,-1,0,0,0;1)_{\mathrm{IIA}}$,
$(0,0,0,0,0,-1,-1,0,-1,0,-1,0,0,0,0;1)_{\mathrm{IIA}}$,
$(0,0,0,0,0,-1,-1,0,0,0,-1,0,0,0,0;1)_{\mathrm{IIA}}$,
$(0,0,0,0,0,-1,0,-1,-1,0,0,0,-1,0,0;1)_{\mathrm{IIA}}$,
$(0,0,0,0,0,-1,0,-1,0,0,0,0,0,0,0;1)_{\mathrm{IIA}}$,
$(0,0,0,0,0,-1,0,0,-1,0,0,0,0,0,0;1)_{\mathrm{IIA}}$,
$(0,0,0,0,0,-1,0,0,0,0,0,0,0,0,0;1)_{\mathrm{IIA}}$,
$(0,0,0,0,0,0,-1,-1,-1,0,0,-1,-1,0,0;1)_{\mathrm{IIA}}$,

Conclusion

- We investigate the factorization and resummation formula for heavy quark pair production in back to back limit
- We analyze factorization in three distinct scale hierarchies within SCET, bHQET, HQET

$$Q \gg m_Q \sim q_T \qquad Q \gg m_Q \gg q_T \qquad Q \sim m_Q \gg q_T$$

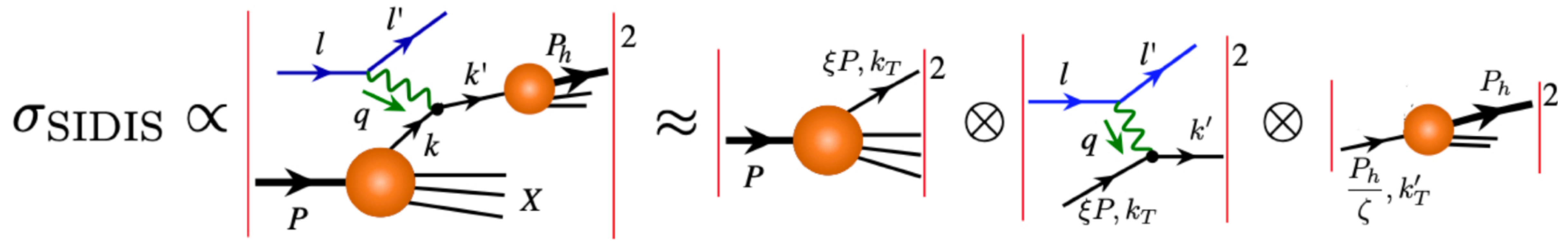
- Different modes and regions up to two loop orders are identified; RG consistency are checked
- All two-loop anomalous dimensions are obtained
- Apply AI to identify different modes in scattering processes: full phase space and phase space with measurement function

Thank you

Backup slides

TMDs in the large-x limit

- **The usual TMD factorization is defined at moderate x value, i.e the x is not too low or too high**



- **TMDs at small x is important for gluon saturation in the Regge asymptotic of QCD, which was investigated in** Balitsky, Tarasov ’15, Zhou ’16, Xiao, Yuan, Zhou ’17

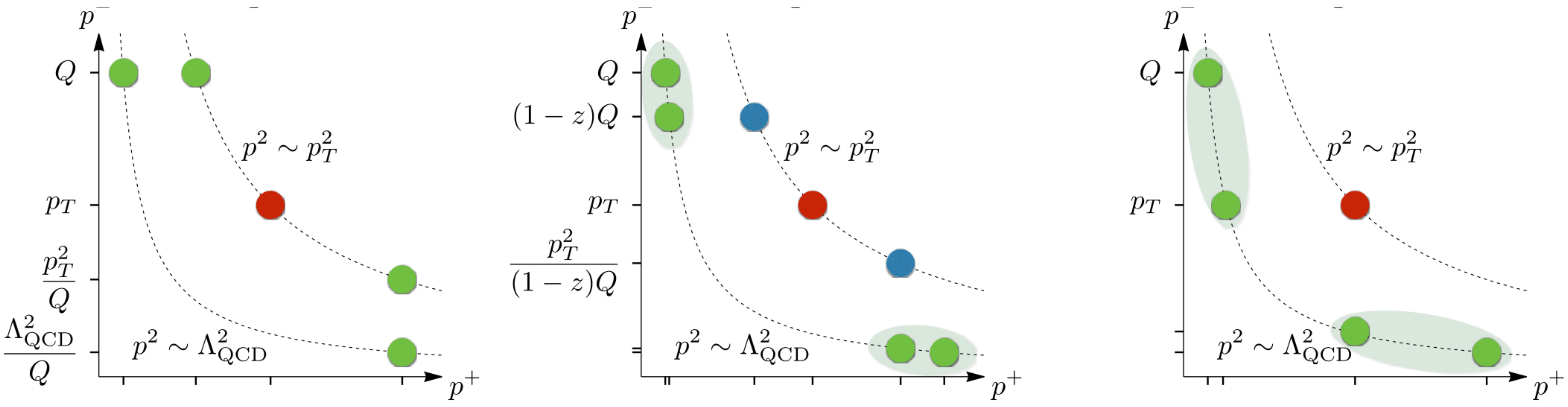
$$xG_{WW}(x, k_\perp, \zeta_c = \mu_F = Q) \;=\; -\frac{2}{\alpha_S}\int \frac{d^2v_\perp d^2v'_\perp}{(2\pi)^4} e^{ik_\perp\cdot r_\perp}\mathcal{H}^{WW}(\alpha_s(Q))e^{-\mathcal{S}_{sud}(Q^2,r_\perp^2)}\,\mathcal{F}^{WW}_{Y=\ln 1/x}(v_\perp, v'_\perp)$$

- **In the limit x->1 (threshold), the phase space of real radiations is restricted**
  - **The NLL joint resummation framework of threshold and TMD logarithms was first developed by** Laenen, Sterman & Vogelsang ’00
  - **A factorization formula based on SCET then was given by** Lustermans, Waalewijn, Zeune ’16 . Y. Li, D. Neill, H. X. Zhu ’16
  - **We apply the joint threshold and TMD factorization theorem to introduce new threshold-TMDs — TTMDs** Kang, Samanta, Shao, Zeng ‘23

Effective theory in the joint limit

In the threshold limit, the new scale hierarchy introduces an additional degrees of freedom known as collinear-soft degrees of freedom

$$k_{cs}^{\mu} \equiv (\bar{n}\cdot k_{cs},\, n\cdot k_{cs},\, k_{cs,\perp}) \sim \left(Q(1-\hat{\tau}),\, \frac{q_T^2}{Q(1-\hat{\tau})},\, q_T\right)$$



$Q \sim (1-z)Q \gg p_T$ $\quad$ $Q \gg (1-z)Q \gg p_T$ $\quad$ $Q \gg (1-z)Q \sim p_T$

Collinear-soft function

Kang, Samanta, Shao, Zeng ‘23

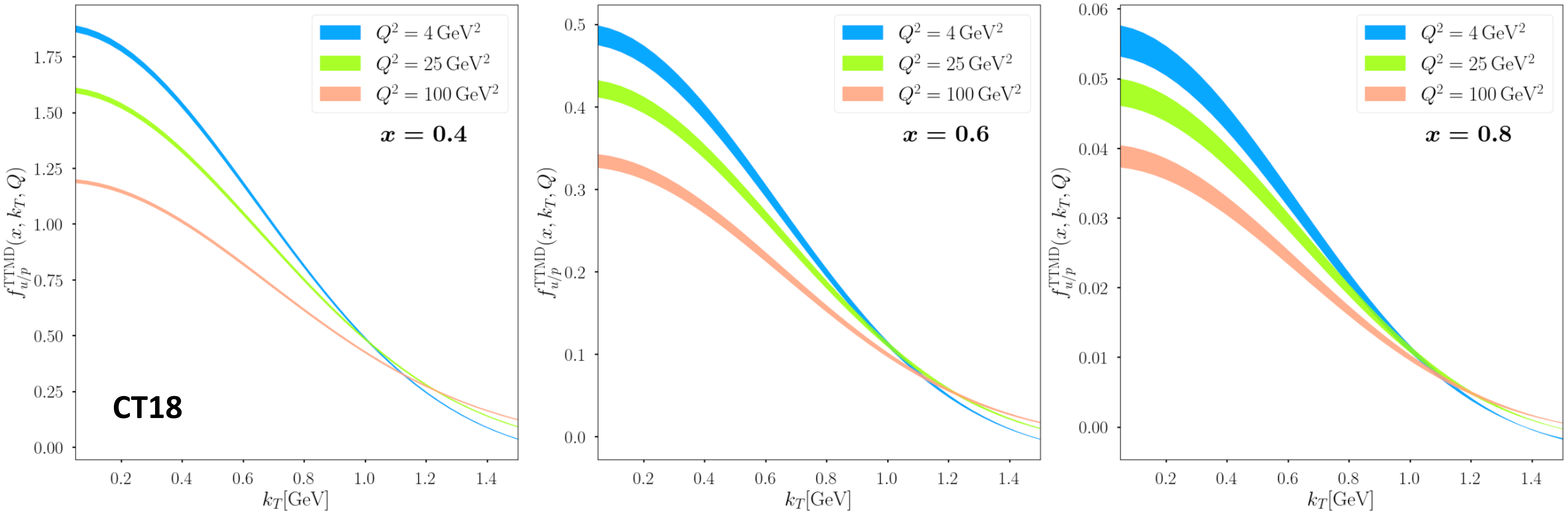
In the threshold limit, the new scale hierarchy introduces an additional degrees of freedom known as collinear-soft degrees of freedom

To validate the factorization theorem, we employ the threshold expressions of the perturbative matching coefficients to ascertain the three-loop collinear-soft function.

$$
\begin{aligned}
\tilde{S}_c^{(1)} &= 2C_F L_b L_\zeta , \\
\tilde{S}_c^{(2)} &= 2C_F^2 L_b^2 L_\zeta^2 + C_F C_A \left[\frac{11}{3} L_b^2 + \left(\frac{134}{9} - 4\zeta_2\right) L_b + \left(\frac{404}{27} - 14\zeta_3\right)\right] L_\zeta \\
&\quad + C_F T_F n_f \left(-\frac{4}{3} L_b^2 - \frac{40}{9} L_b - \frac{112}{27}\right) L_\zeta , \\
\tilde{S}_c^{(3)} &= \frac{4}{3} C_F^3 L_b^3 L_\zeta^3 + C_F^2 C_A \left[\frac{22}{3} L_b^3 + \left(\frac{268}{9} - 8\zeta_2\right) L_b^2 + \left(\frac{808}{27} - 28\zeta_3\right) L_b\right] L_\zeta^2 \\
&\quad + C_F C_A^2 \left[\frac{242}{27} L_b^3 + \left(\frac{1780}{27} - \frac{44}{3}\zeta_2\right) L_b^2 + \left(\frac{15503}{81} - \frac{536}{9}\zeta_2 - 88\zeta_3 + 44\zeta_4\right) L_b \right. \\
&\quad \left. + \left(\frac{297029}{1458} - \frac{3196}{81}\zeta_2 - \frac{6164}{27}\zeta_3 + \frac{88}{3}\zeta_2\zeta_3 - \frac{77}{3}\zeta_4 + 96\zeta_5\right)\right] L_\zeta \\
&\quad + C_F C_A T_F n_f \left[-\frac{176}{27} L_b^3 + \left(-\frac{1156}{27} + \frac{16}{3}\zeta_2\right) L_b^2 + \left(-\frac{8204}{81} + \frac{160}{9}\zeta_2\right) L_b \right. \\
&\quad \left. + \left(-\frac{62626}{729} + \frac{824}{81}\zeta_2 + \frac{904}{27}\zeta_3 - \frac{20}{3}\zeta_4\right)\right] L_\zeta \\
&\quad + C_F T_F^2 n_f^2 \left(\frac{32}{27} L_b^3 + \frac{160}{27} L_b^2 + \frac{800}{81} L_b + \frac{3712}{729} + \frac{64}{9}\zeta_3\right) L_\zeta \\
&\quad + C_F^2 T_F n_f \left[-\frac{8}{3} L_b^3 L_\zeta^2 + \left(-4 - \frac{80}{9} L_\zeta\right) L_b^2 L_\zeta + \left(-\frac{224}{27} - \frac{110}{3} + 32\zeta_3\right) L_b L_\zeta^2 \right. \\
&\quad \left. + \left(-\frac{1711}{27} + \frac{304}{9}\zeta_3 + 16\zeta_4\right) L_\zeta\right] , \qquad (14)
\end{aligned}
$$

Numerical results for threshold-TMDPDFs

Kang, Samanta, Shao, Zeng '23



The uncertainty bands correspond to the 1-σ variation from CT18 PDFs using the Hessian method

We are working on NNNLL joint resummation + NNNLL TMD resummation