Heavy quark physics from CLQCD ensembles

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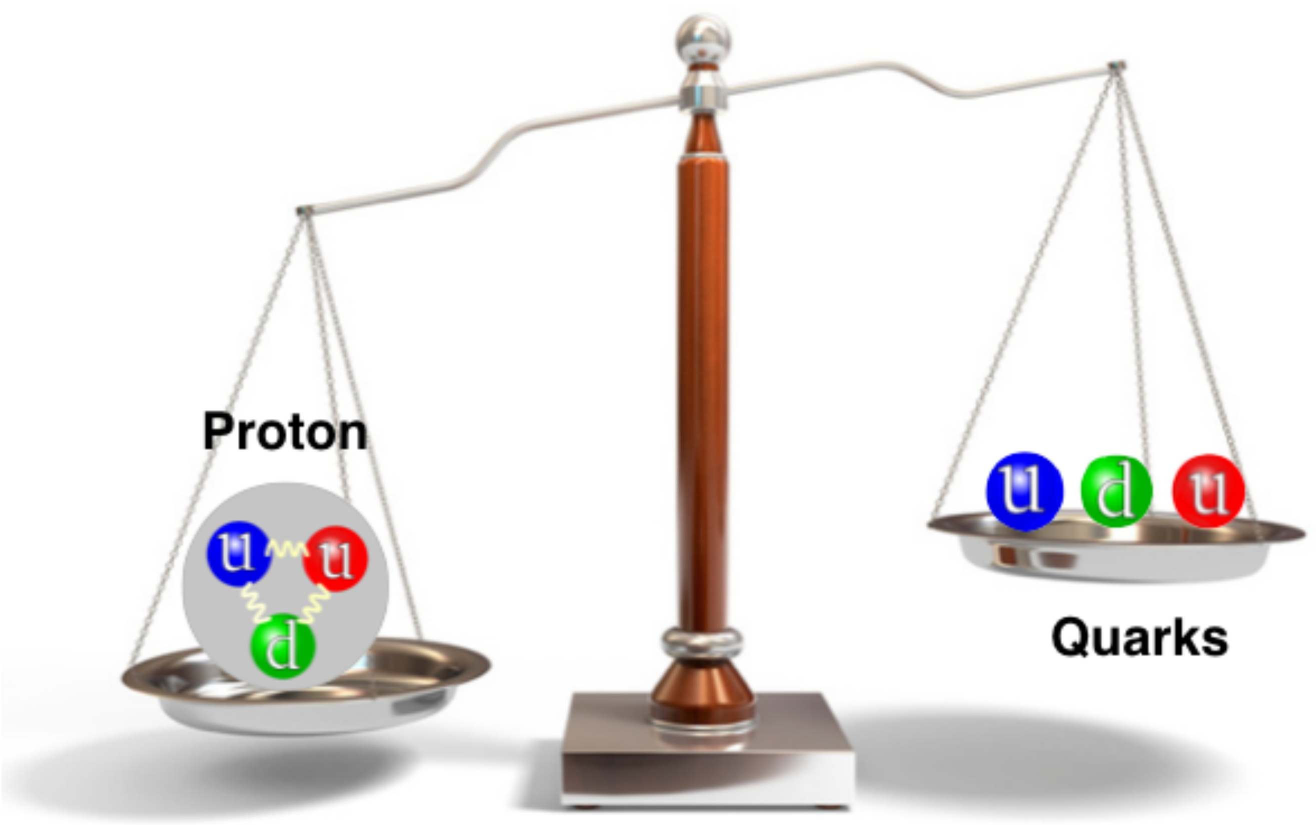
With Hai-Yang Du, Bo-Lun Hu, Peng Sun, et.al.,

For CLQCD collaboration

CLQCD

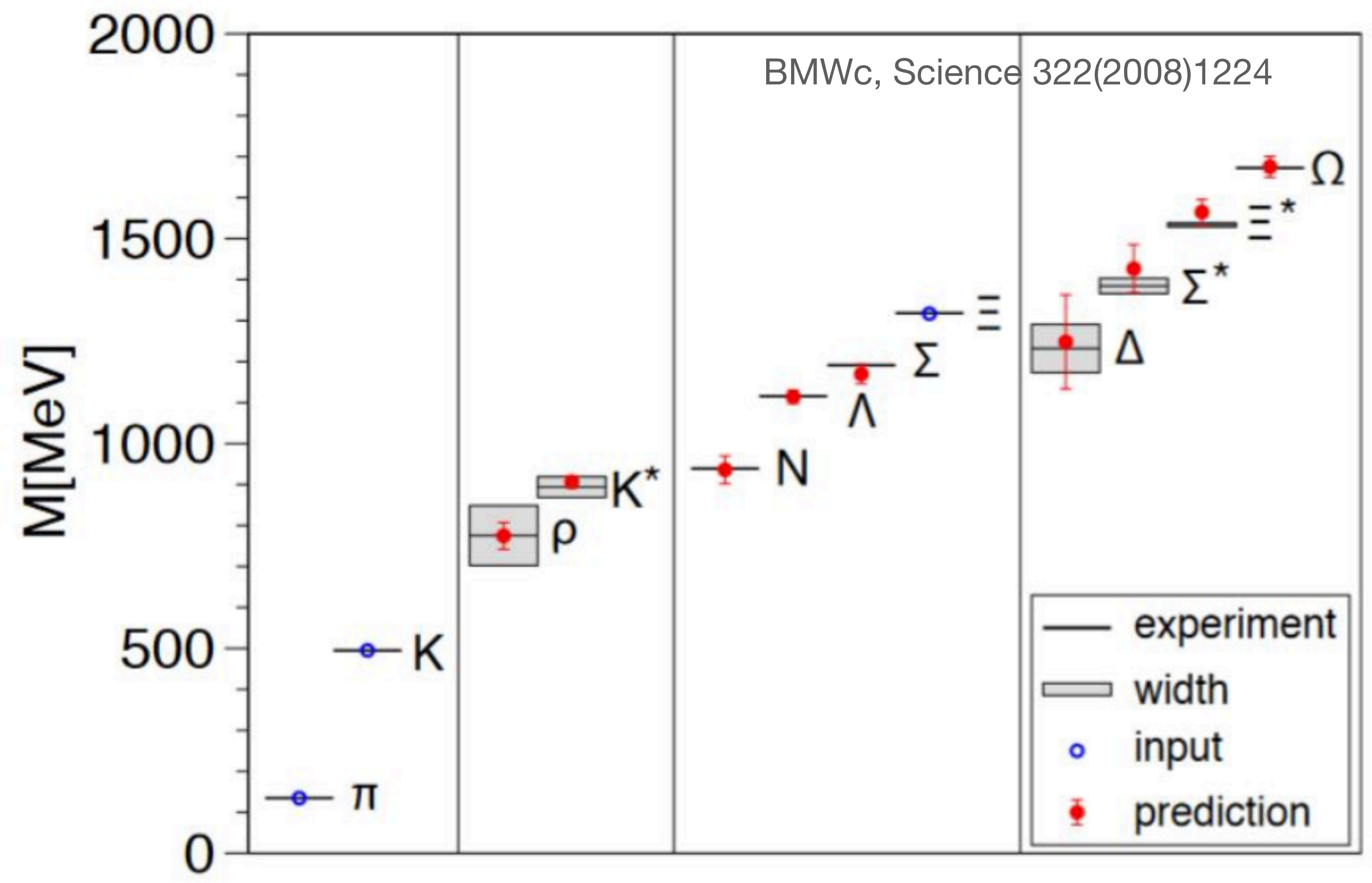
Hadron mass

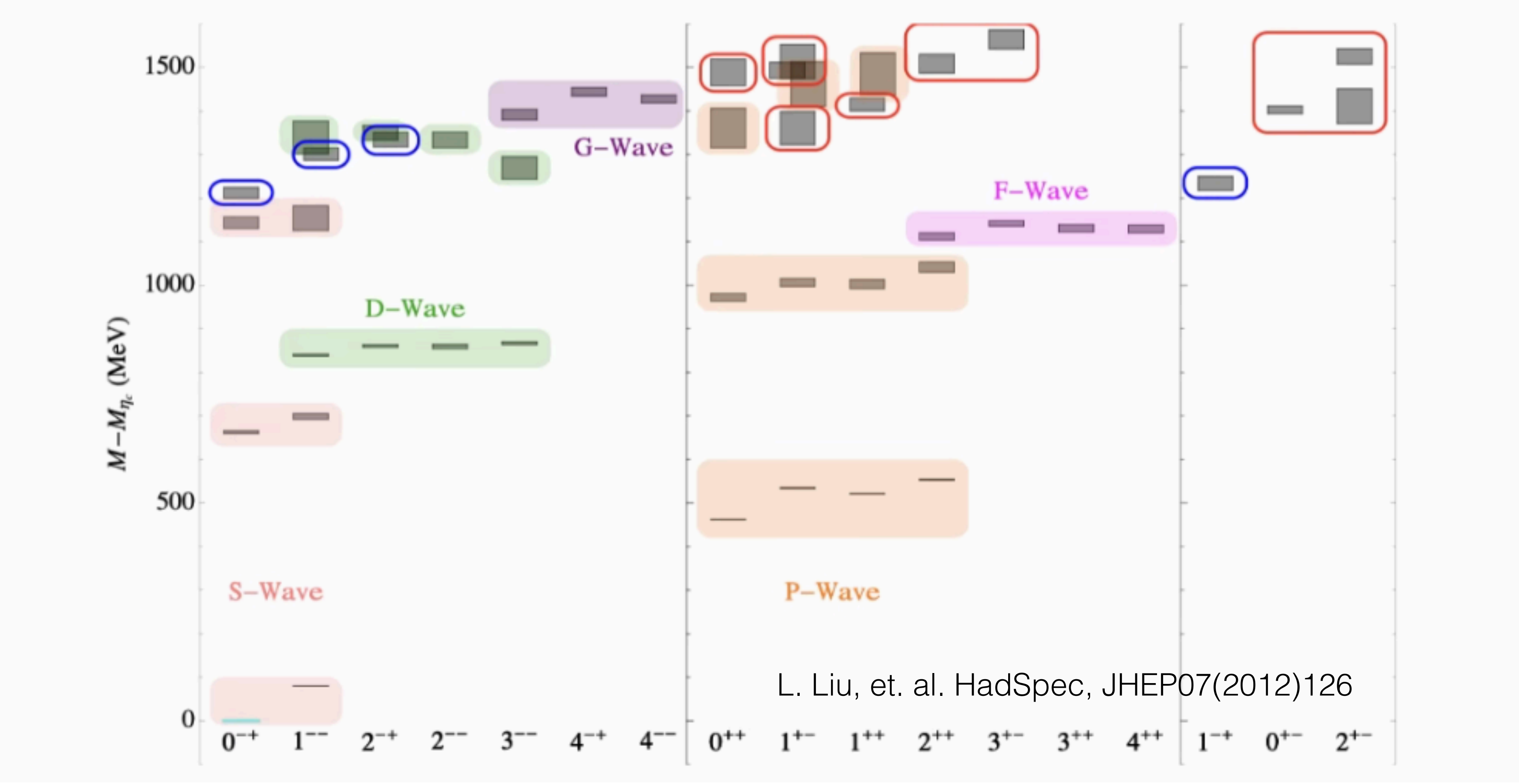
and its origin



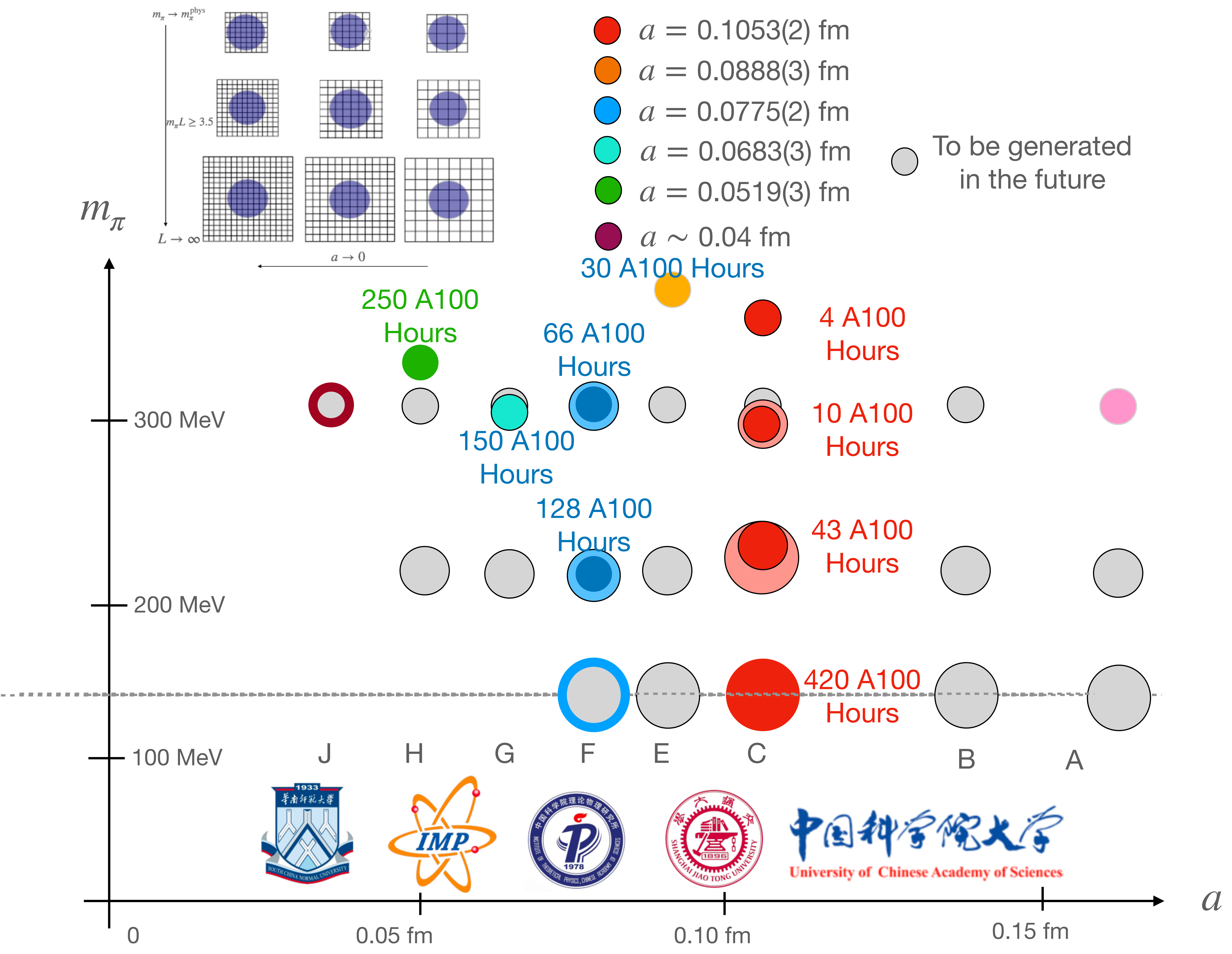
Nucleon mass is much larger than the contribution of quark masses which come from the Higgs boson:

- How about the hadron with heavier quark flavors?
- How about the exotic states?





CLQCD ensembles



Informations

14 ensembles with more than 5,000 configurations in total:

- 5 lattice spacings from 0.05 fm to 0.11 fm;
- 7 pion masses from 130 MeV to 350 MeV;
- 4 Volumes from 2.5 fm to 5.0fm.
- More ensembles are in production;

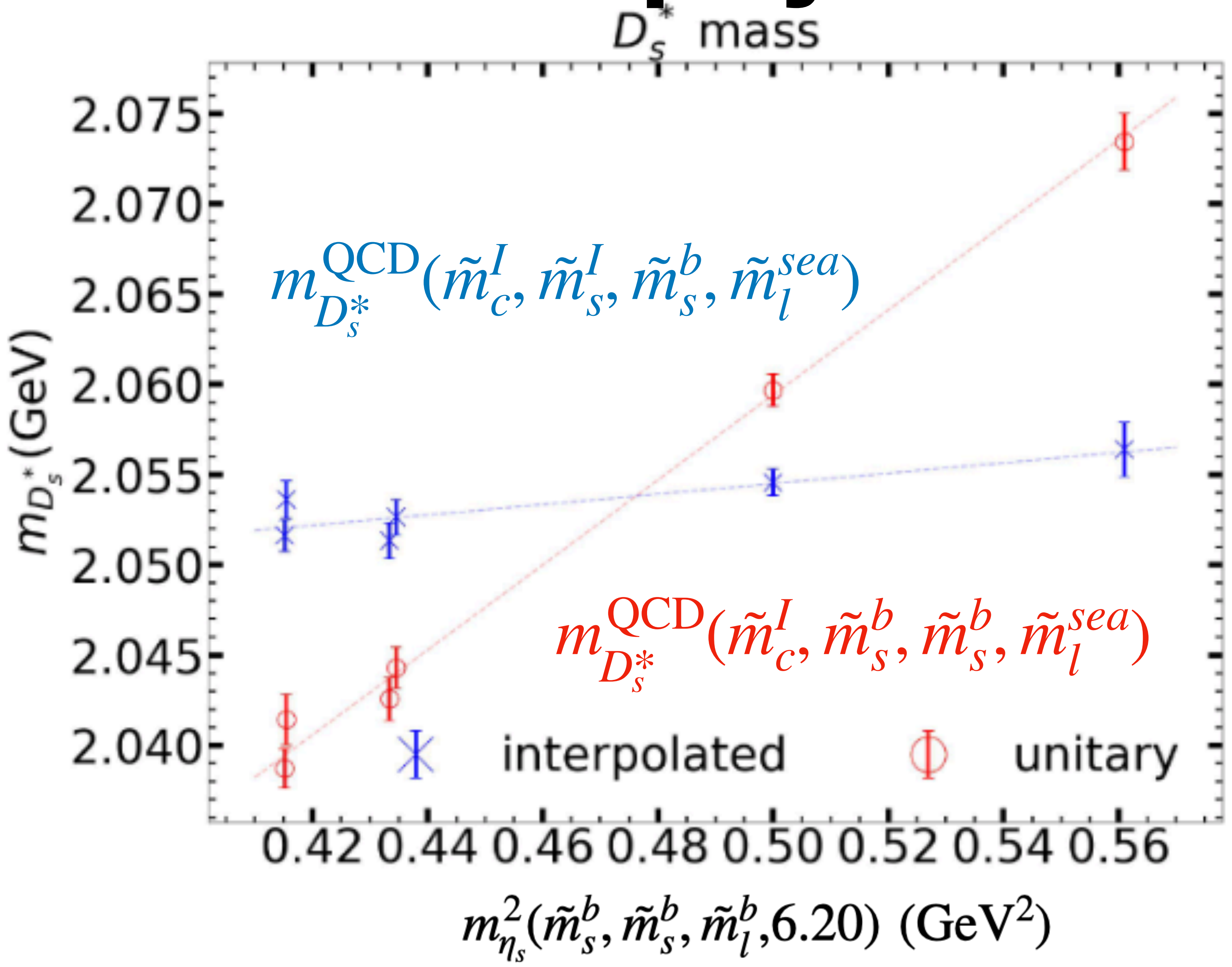
Ensembles with another setup are preparing for better control on systematic uncertainties.

Z.-H. Hu, B.-L. Hu, J.-H. Wang, et. al., CLQCD, PRD109(2024) 054507

Quark mass

Toward the charm physics

Symbol	$\hat{\beta}$	a (fm)	u_0	v_0	$\tilde{m}_l^b$	$\tilde{m}_s^b$	$\tilde{L}^3 \times \tilde{T}$	m_π (MeV)	m_{η_s} (MeV)	$\tilde{m}_s^{\mathrm{I}}$	$\tilde{m}_c^{\mathrm{I}}$	$n_{\mathrm{cfg}} \times n_{\mathrm{src}}$
C24P34	6.200	0.10530(18)	0.855453	0.951479	-0.2770	-0.2310	$24^3 \times 64$	340.5(1.7)	748.99(73)	-0.2396(2)	0.4080(26)	200×32
C24P29			0.855453	0.951479	-0.2770	-0.2400	$24^3 \times 72$	292.7(1.2)	658.29(65)	-0.2357(2)	0.4168(26)	760×3
C32P29			0.855453	0.951479	-0.2770	-0.2400	$32^3 \times 64$	292.4(1.1)	659.22(41)	-0.2358(2)	0.4158(26)	489×3
C32P23			0.855520	0.951545	-0.2790	-0.2400	$32^3 \times 64$	228.0(1.2)	644.36(45)	-0.2338(2)	0.4198(26)	400×3
C48P23			0.855520	0.951545	-0.2790	-0.2400	$48^3 \times 96$	225.6(0.9)	644.58(62)	-0.2338(2)	0.4214(26)	62×3
C48P14			0.855548	0.951570	-0.2825	-0.2310	$48^3 \times 96$	135.5(1.6)	707.06(44)	-0.2335(2)	0.4212(26)	188×3
E28P35	6.308	0.08877(30)	0.859646	0.954385	-0.2490	-0.2170	$28^3 \times 64$	352.1(1.2)	720.31(94)	-0.2204(3)	0.2707(37)	147×4
F32P30	6.410	0.07750(18)	0.863437	0.956942	-0.2295	-0.2050	$32^3 \times 96$	303.2(1.3)	677.6(1.0)	-0.2039(2)	0.1968(21)	250×3
F48P30			0.863473	0.956984	-0.2295	-0.2050	$48^3 \times 96$	303.4(0.9)	676.32(62)	-0.2038(2)	0.1949(21)	99×3
F32P21			0.863488	0.957017	-0.2320	-0.2050	$32^3 \times 64$	210.9(2.2)	660.27(94)	-0.2024(2)	0.1989(21)	194×3
F48P21			0.863499	0.957006	-0.2320	-0.2050	$48^3 \times 96$	207.2(1.1)	663.39(65)	-0.2026(2)	0.1991(21)	82×12
G36P29	6.498	0.06826(27)	0.866476	0.958910	-0.2150	-0.1926	$36^3 \times 108$	295.1(1.2)	693.2(1.0)	-0.1929(2)	0.1378(28)	68×4
H48P32	6.720	0.05187(26)	0.873378	0.963137	-0.1850	-0.1700	$48^3 \times 144$	317.2(0.9)	695.9(1.3)	-0.1703(2)	0.0533(24)	157×12



- $m_{\eta_s}(\tilde{m}_s^v = \tilde{m}_s^I, \tilde{m}_s^{sea} = \tilde{m}_s^b, \tilde{m}_l^{sea} = \tilde{m}_l^b, \hat{\beta}) = 689.89(49)\ \mathrm{MeV} \neq m_{\eta_s}(\tilde{m}_s^b, \tilde{m}_s^b, \tilde{m}_l^b, \hat{\beta})$

$m_{\eta_s} = 687.4(2.2)$ MeV

BMWc, Nature 593(2021)51

Z.C. Hu, B.L. Hu, J.H. Wang, et. al., CLQCD, Phys.Rev.D109 (2024) 054507

- $m^{\mathrm{QCD}}_{D_s}(\tilde{m}_c^v = \tilde{m}_c^I, \tilde{m}_s^v = \tilde{m}_s^I, \tilde{m}_s^{sea} = \tilde{m}_s^b, \tilde{m}_l^{sea} = \tilde{m}_l^b, \hat{\beta}) = m^{\mathrm{phys}}_{D_s} - \Delta^{\mathrm{QED}} m_{D_s} = 1966.7(1.5)\ \mathrm{MeV}$.

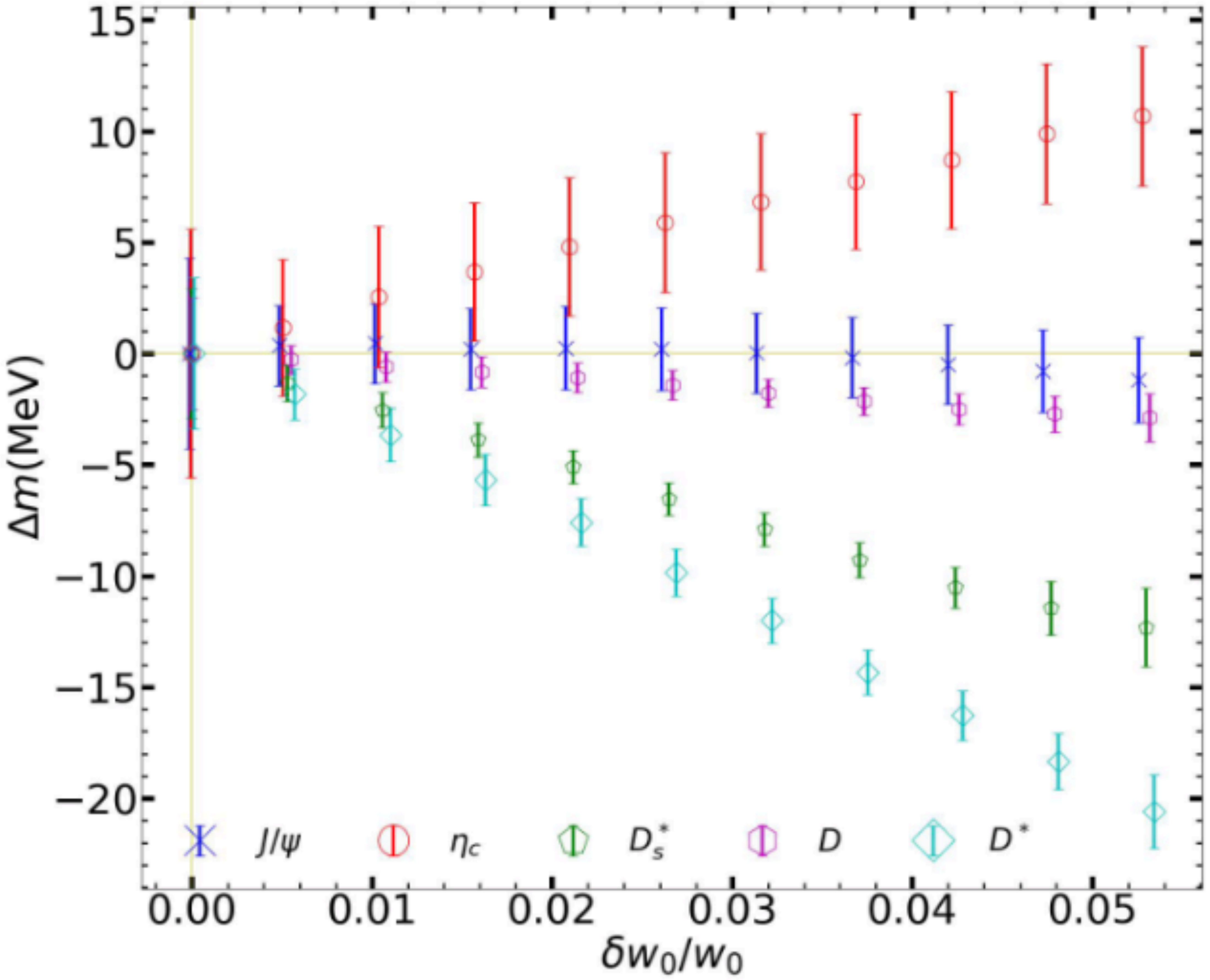
$\Delta^{\mathrm{QED}} m_{D_s} = 2.4$ MeV under the $m^{\overline{\mathrm{MS}}}_{q,\mathrm{QCD+QED}}(2\mathrm{GeV}) = m^{\overline{\mathrm{MS}}}_{q,\mathrm{QCD}}(2\mathrm{GeV})$ scheme

RM123, Phys.Rev.D100 (2019) 1904.08731

- $X(m_\pi, m_{\eta_s}, a) = X(m_\pi^{\mathrm{phys}}, m_{\eta_s}^{\mathrm{phys}}, 0) + d_1^X(m_\pi^2 - (m_\pi^{\mathrm{phys}})^2) + d_2^X(m^2_{\eta_s,\mathrm{sea}} - (m_{\eta_s}^{\mathrm{phys}})^2) + d_3^X a^2 + d_4^X a^4$

Dependence of scale setting

for the hadron masses



All the dimensional quantities depend on the QCD scale setting parameter w_0. When $\delta w_0/w_0 \sim 0.5\,\%$:

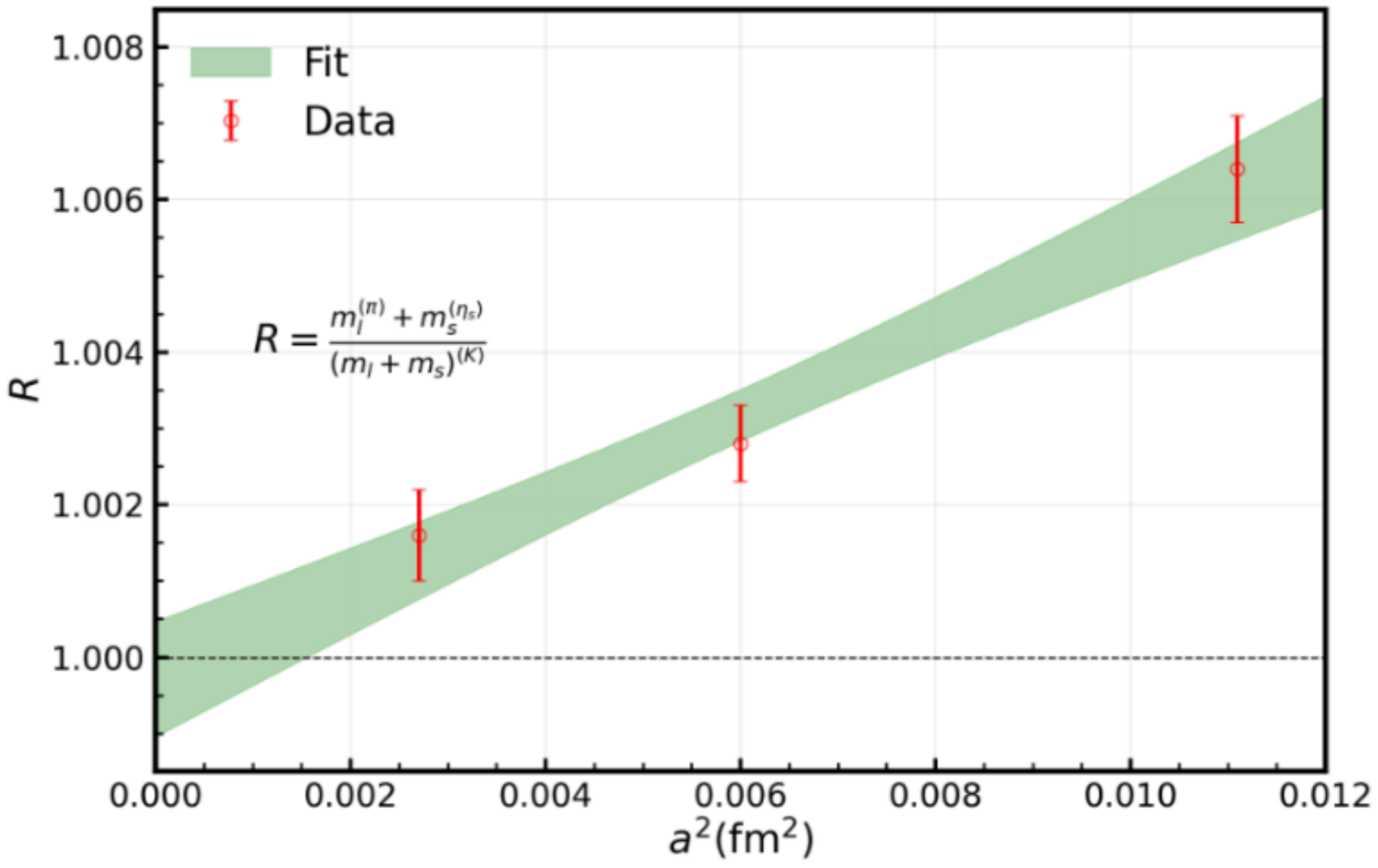
- Naive scale setting:
  - $\delta m_D \sim 10$ MeV, $\delta m_{J/\psi} \sim 15$ MeV;
  - $\delta m_B \sim 30$ MeV, $\delta m_\Upsilon \sim 50$ MeV.
- Determine $\tilde{m}_c^{\mathrm{I}}$ using $m_{D_s}^{\mathrm{QCD}}$ and keep the correlation between different charmed hadron:
  - $\delta m_D \sim 0.3$ MeV, $\delta m_{J/\psi} \sim 0.1$ MeV.

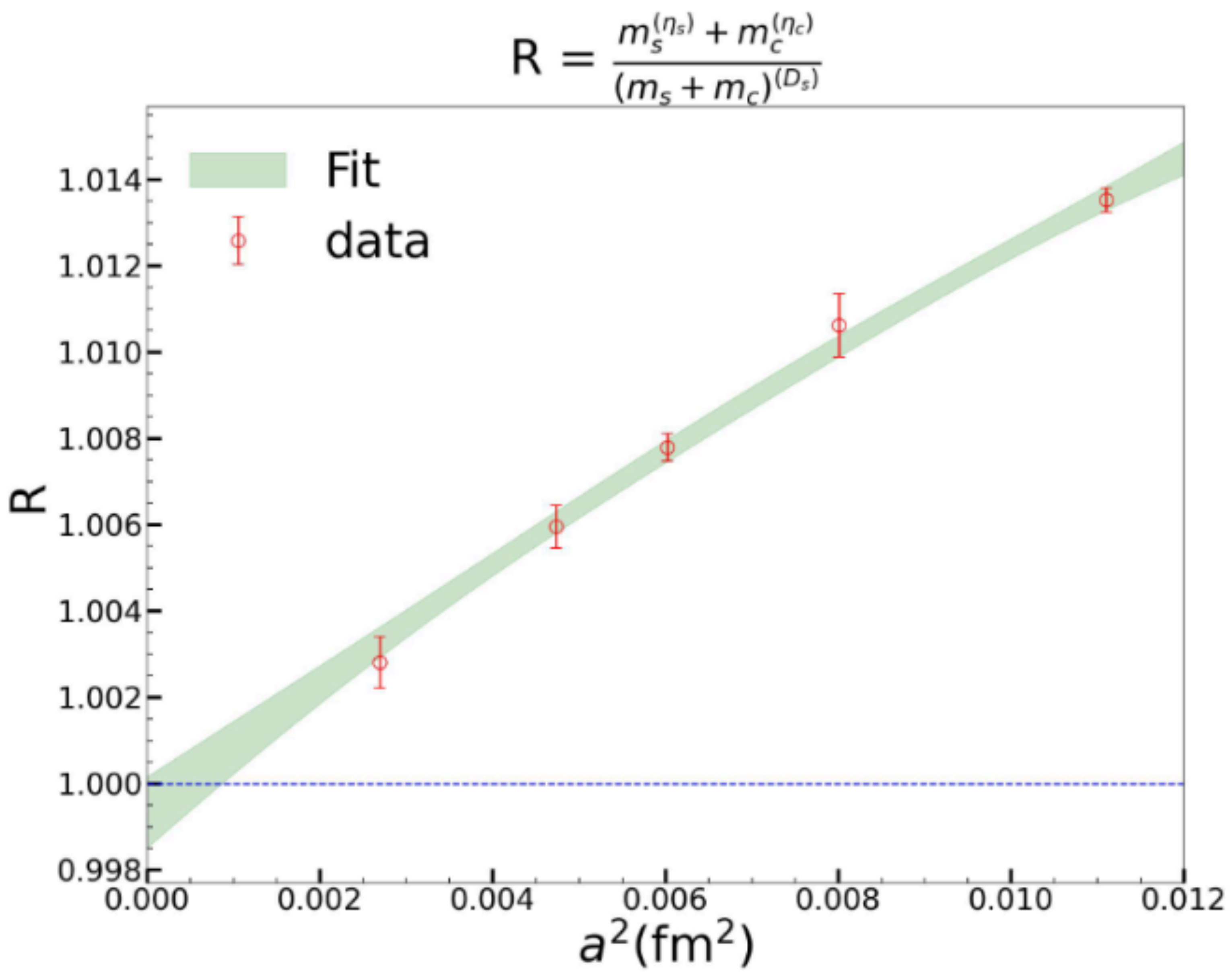
Essential to obtain the precise prediction for the hadron spectrum with heavy flavors.

PCAC quark masses

Definition

$$m_q^{\rm PC} = \frac{\langle 0|\partial_4 A_4|{\rm PS}\rangle}{2\langle 0|P|{\rm PS}\rangle} = \frac{m_{\rm PS}\sum_{\vec{x}}\langle A_4(\vec{x},t)P^\dagger(\vec{0},0)\rangle}{2\sum_{\vec{x}}\langle P(\vec{x},t)P^\dagger(\vec{0},0)\rangle}\Big|_{t\to\infty}$$





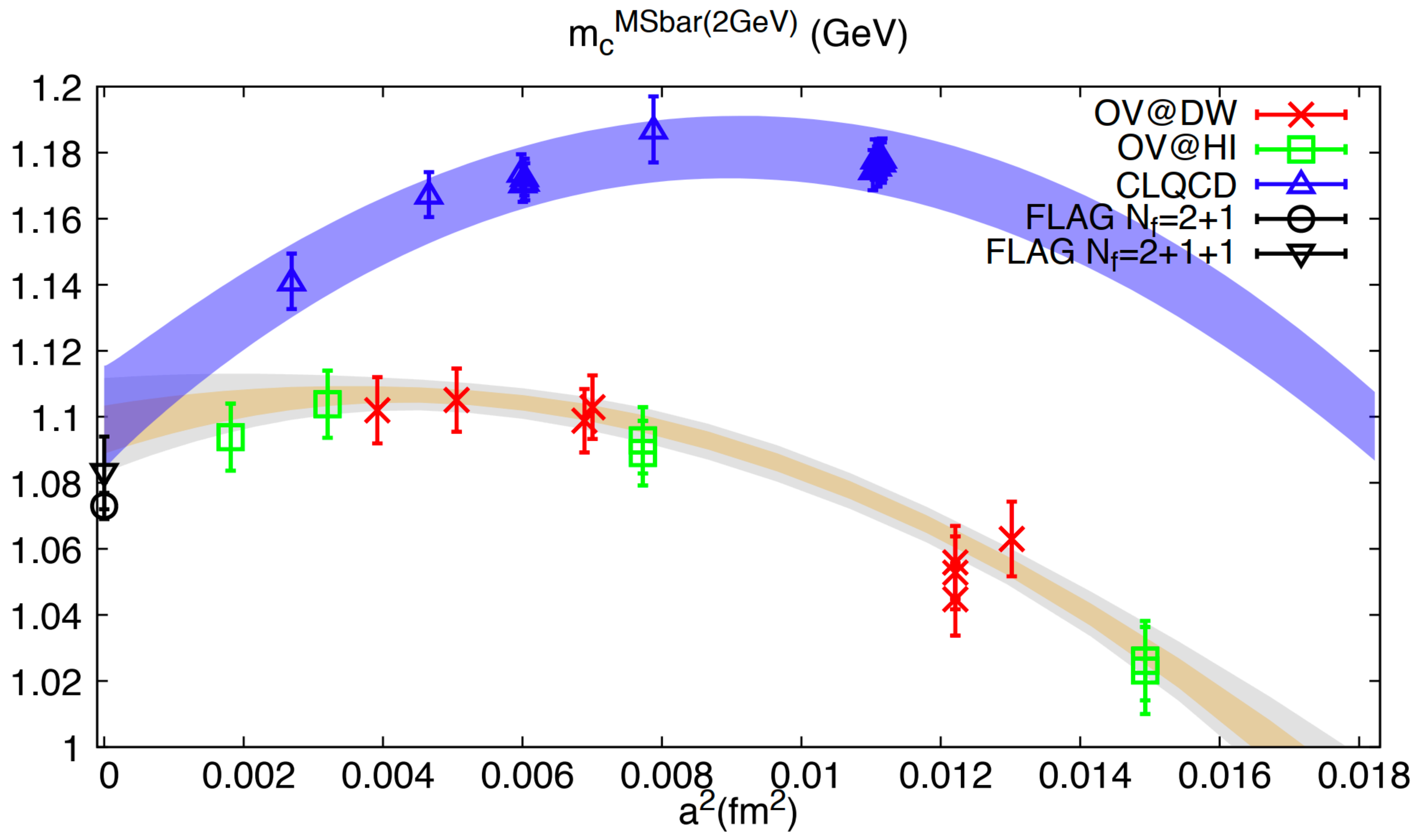
Defining quark mass from the PCAC relation can avoid the additive renormalization of the clover fermion action:

- PCAC quark mass defined from different PS hadron can differ by ~1% at the coarsest lattice spacing ~0.11 fm;
- And becomes consistent with each other at 0.1% level after the continuum extrapolation.

Also see the talk of Dian-Jun Zhao

Renormalized quark masses

Charm quark mass



Hai-Yang Du, B.L. Hu, et. al., CLQCD, 2408.03548

D.J. Zhao, et. al., χQCD, in preparation

Based on the $a^2 + a^4$ extrapolation:

- The impact of unphysical light and strange quark masses have been corrected based on the global fit.
- Such a value is similar to the current lattice averages within ~2%.

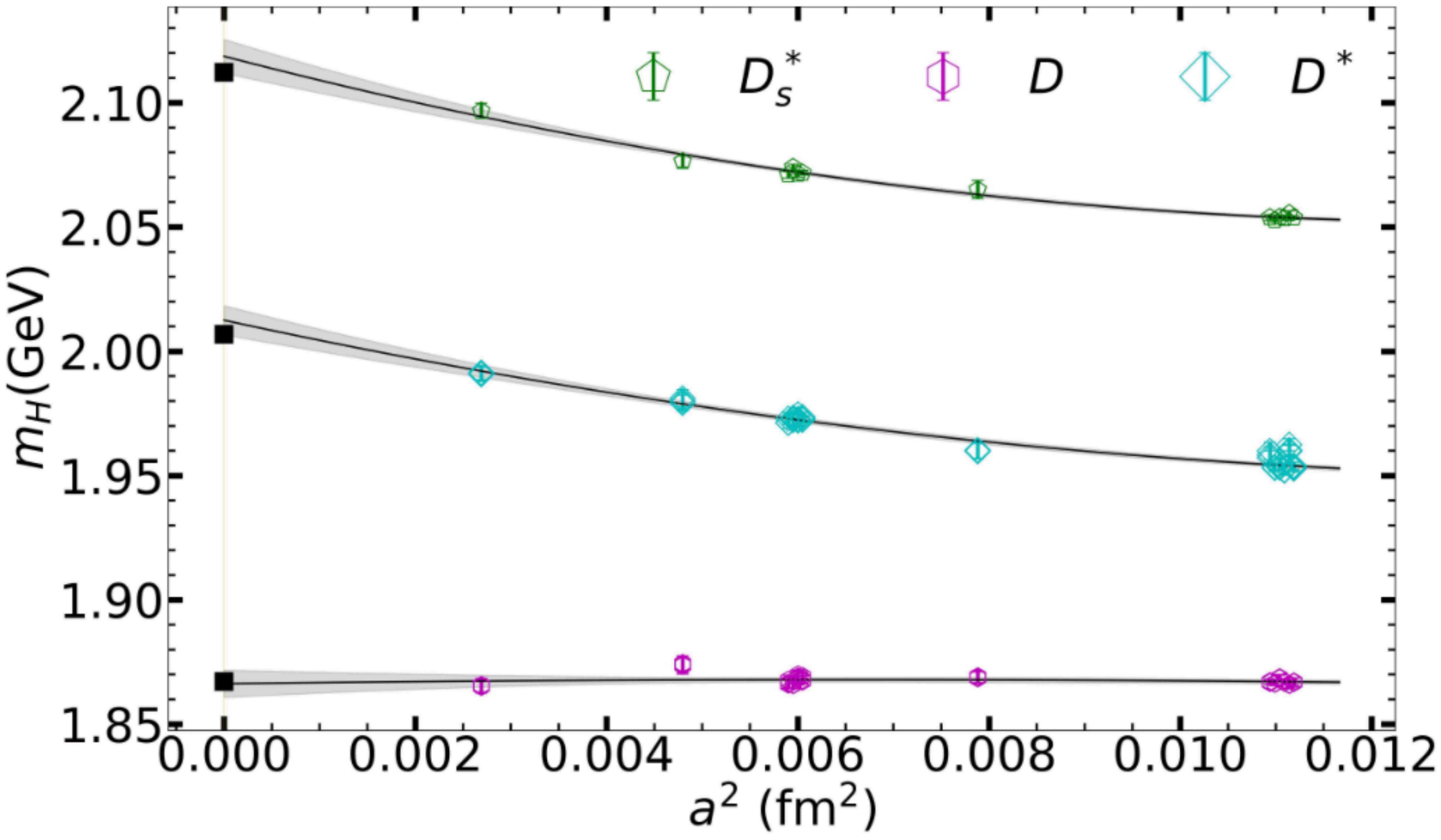
Charmed meson spectrum

Open charm cases

$$m_{D_s}^{\mathrm{QCD}} = m_{D_s}^{\mathrm{phys}} - \Delta^{\mathrm{QED}} m_{D_s} = 1966.7(1.5)\ \mathrm{MeV}\,.$$

RM123, Phys.Rev.D100 (2019) 034514

Input to determine the charm quark mass



- m_D is almost constant at different lattice spacing, with $m_D^{\pm} - m_D^0 = 2.9(2)_{\mathrm{QCD}} + 2.4(5)_{\mathrm{QED}} = 5.3(2)(5)\ \mathrm{MeV}$;

  RM123, Phys.Rev.D95(2017) 114504

- Agree with the PDG value 4.8(1) MeV well.

- Both m_D^* and $m_{D_s}^*$ have obvious lattice spacing dependence and the continuum extrapolated values agree with PDG well.

Hai-Yang Du, B.L. Hu, et. al., CLQCD, 2408.03548

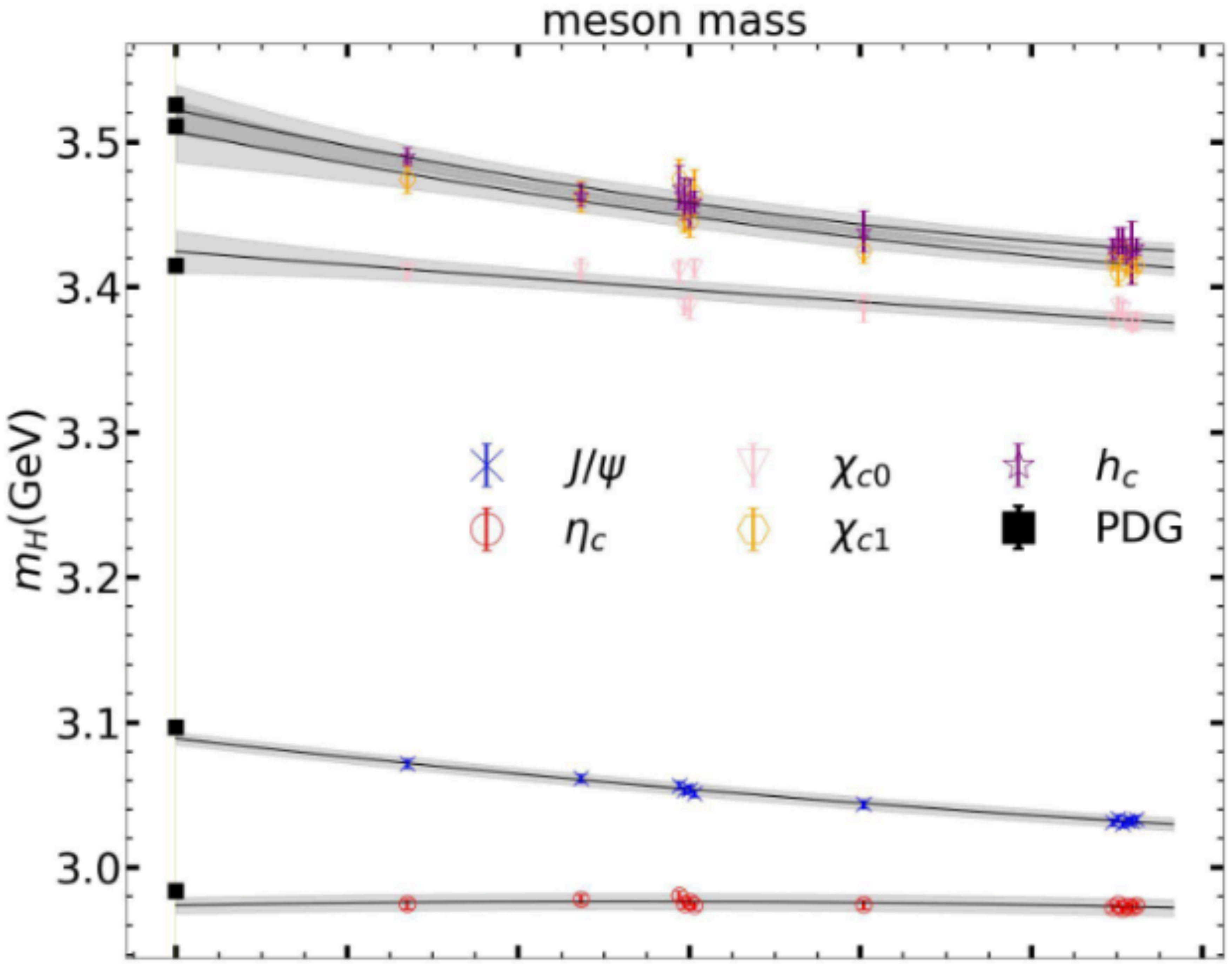
Charmed meson spectrum

charmonium cases

$$m_{D_s}^{\mathrm{QCD}} = m_{D_s}^{\mathrm{phys}} - \Delta^{\mathrm{QED}} m_{D_s} = 1966.7(1.5)\ \mathrm{MeV}\,.$$

RM123, Phys.Rev.D100 (2019) 034514

Input to determine the charm quark mass



- $m_{J/\psi}$ agrees with PDG well but m_{η_c} is a few MeV lower;
- $m_{J/\psi} - m_{\eta_c} = 116(3)\ \mathrm{MeV}$ agree with previous HPQCD pure QCD prediction 119(1) MeV.
- P-wave charmonium masses also agree with PDG well, with $m_{1P} - m_{1S} = 461(19)\ \mathrm{MeV}$.

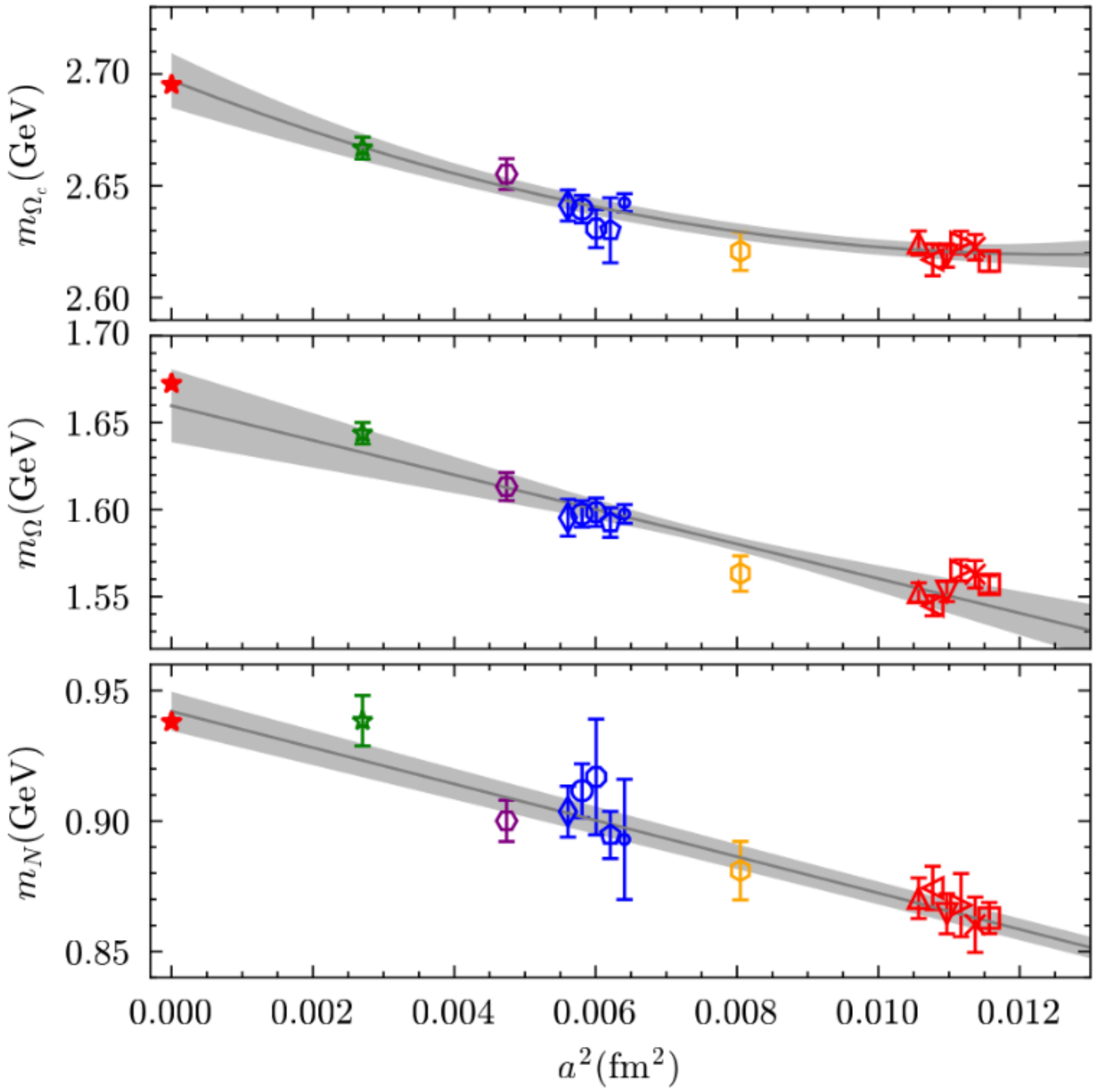
Hai-Yang Du, B.L. Hu, et. al., CLQCD, 2408.03548

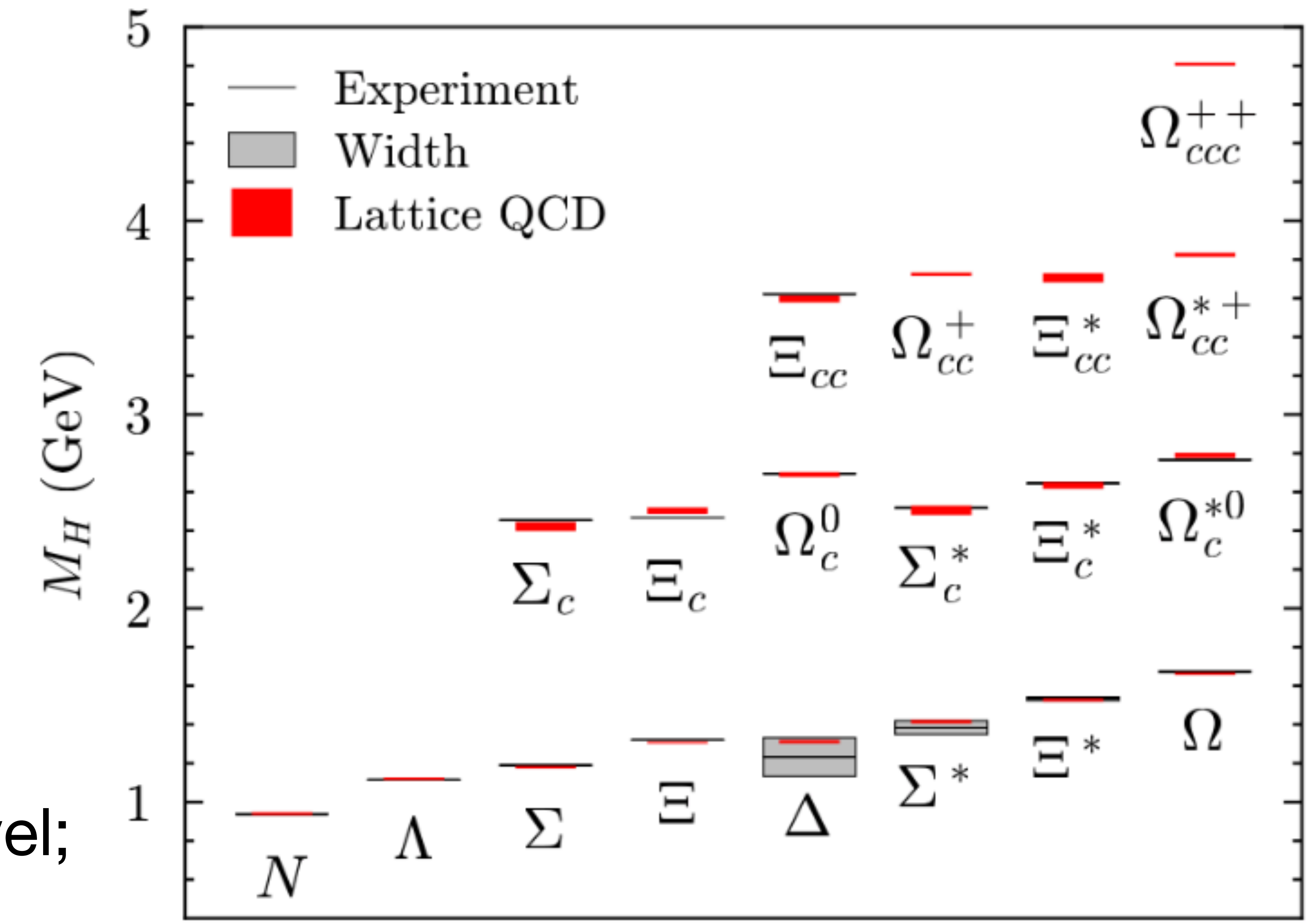
Also see the poster of Bo-Lun Hu

Baryon spectrum

of four light flavors

$$X(m_\pi, m_{\eta_s}, a) = X(m_\pi^{\rm phys}, m_{\eta_s}^{\rm phys}, 0) + d_1^X(m_\pi^2 - (m_\pi^{\rm phys})^2) + d_2^X(m_{\eta_s,{\rm sea}}^2 - (m_{\eta_s}^{\rm phys})^2) + d_3^X a^2 + d_4^X a^4 \,.$$





- Generally agree with the PDG values at 1% level;
- The missing QED effect will be investigated in the near future.

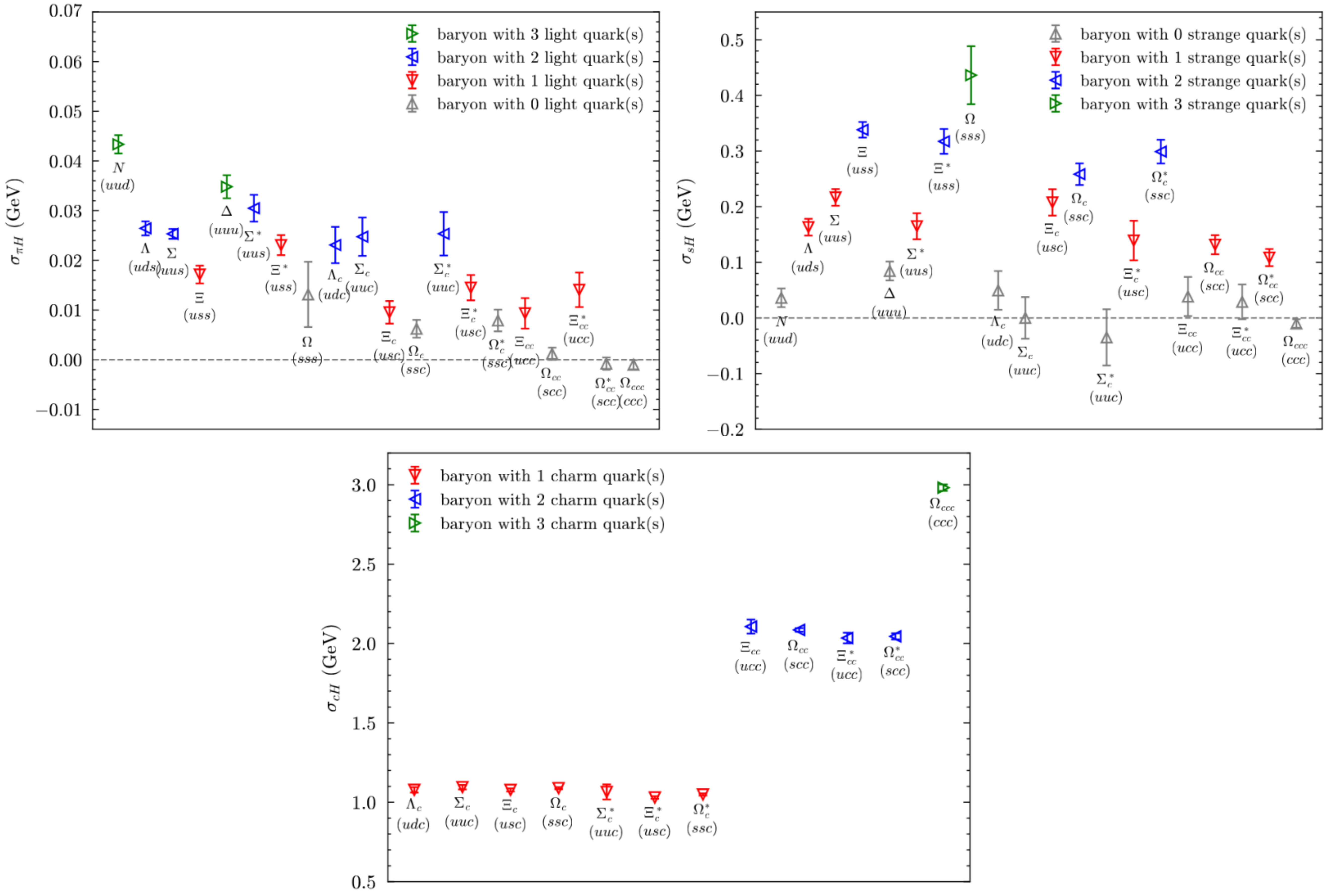
B.-L. Hu, et. al., CLQCD, in preparation

Also see the poster of Bo-Lun Hu

Baryon spectrum

Quark mass contribution

$$m_H = m\langle\bar{\psi}\psi\rangle_H + [\frac{2}{\pi}\alpha_s + \mathcal{O}(\alpha_s^2)]m\langle\bar{\psi}\psi\rangle_H + [(-\frac{11}{8\pi} + \frac{N_f}{12\pi})\alpha_s + \mathcal{O}(\alpha_s^2)]\langle F^2\rangle_H .$$



- Light quark contribution is 10-20 MeV per quark;
- Strange quark contribution is ~150 MeV per quark;
- Charm quark contribution is ~1 GeV per quark.

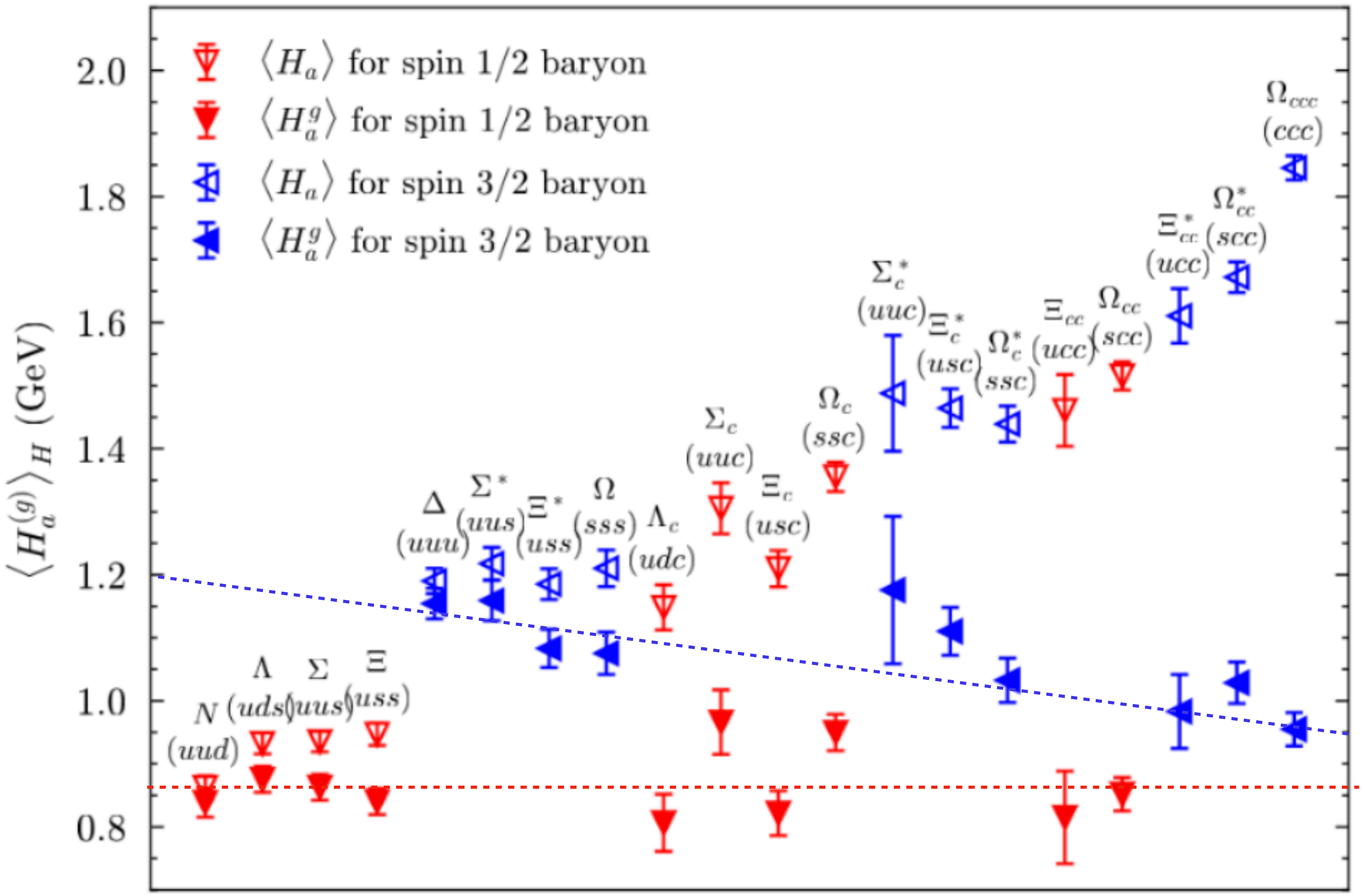
B.-L. Hu, et. al., CLQCD, in preparation

Also see the poster of Bo-Lun Hu

Baryon spectrum

Trace anomaly contribution

$$m_H = m\langle\bar{\psi}\psi\rangle_H + [\frac{2}{\pi}\alpha_s + \mathcal{O}(\alpha_s^2)]m\langle\bar{\psi}\psi\rangle_H + [(-\frac{11}{8\pi} + \frac{N_f}{12\pi})\alpha_s + \mathcal{O}(\alpha_s^2)]\langle F^2\rangle_H .$$



- Total trace anomaly can be obtained through the difference between baryon mass and its quark mass contributions;
- Gluon trace anomaly $\langle H_a^g \rangle_H = \langle H_a \rangle_H - \gamma_m \sigma_H$ can be more insensitive to flavor if we use $\gamma_m \sim 0.3$;
- That in the J=3/2 baryon is larger than that of J=1/2 baryon, while the difference becomes smaller with more heavy flavors.

B.-L. Hu, et. al., CLQCD, in preparation

Also see the poster of Hai-Yang Du

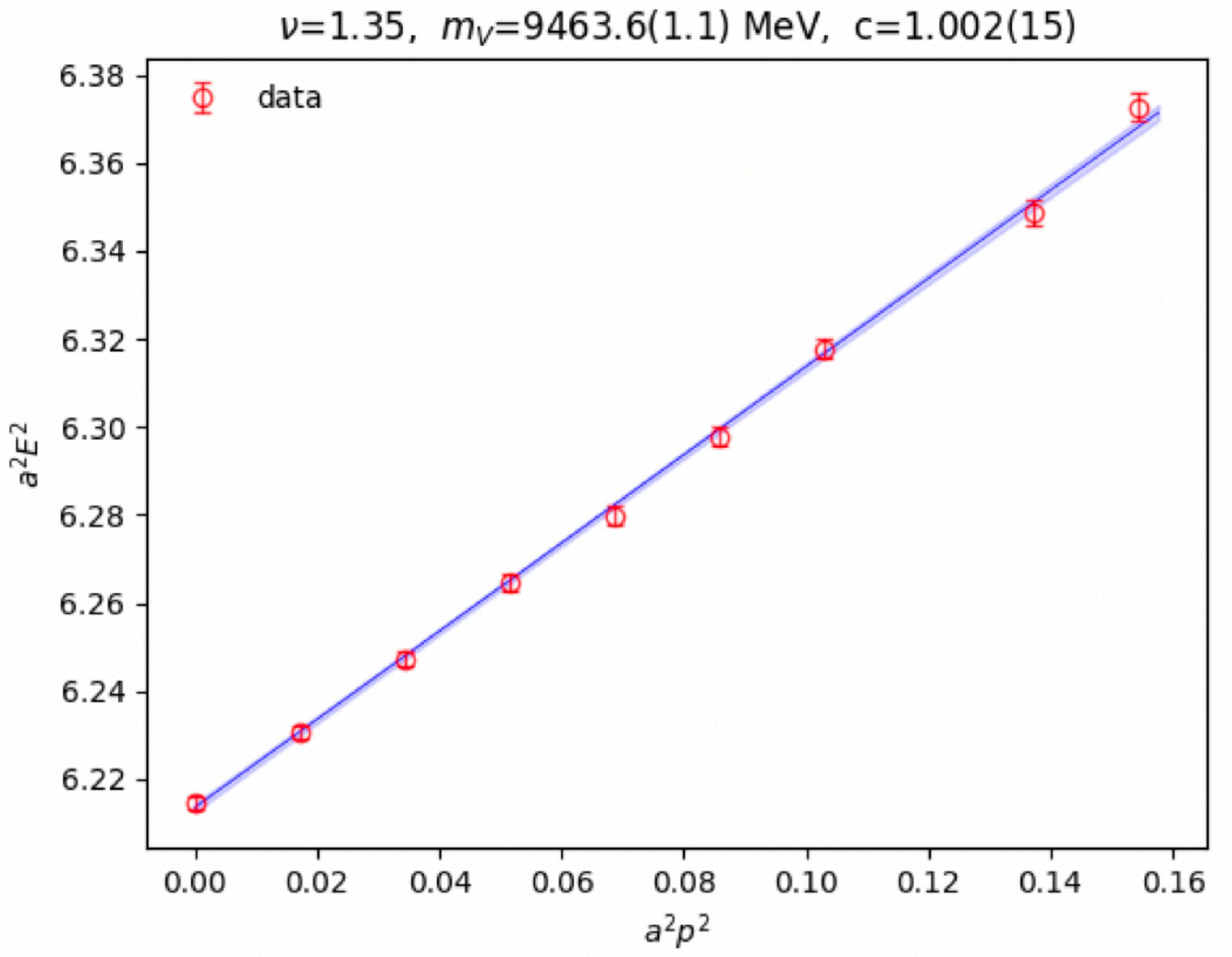
Toward the bottom physics

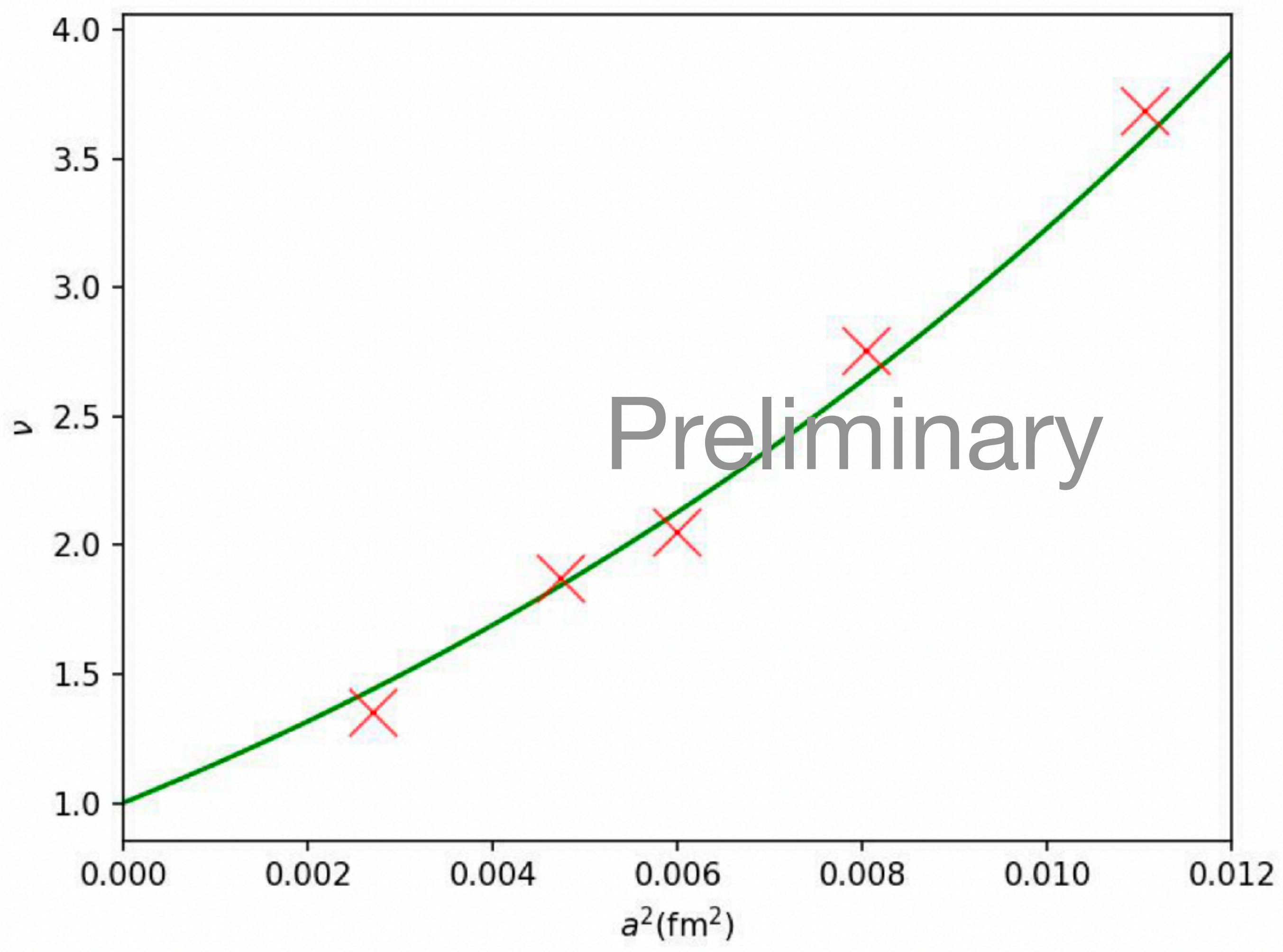
anisotropic action

$$S_Q = a^4 \sum_x \bar{Q}\mathcal{M}Q,\ \mathcal{M} = \left[m_Q + \gamma_4\nabla_4 - \frac{a}{2}\nabla_4^2 + \nu\sum_{i=1}^{3}(\gamma_i\nabla_i - \frac{a}{2}\nabla_i^2) - \frac{1+\nu}{4u_0^3}a\sum_{i=1}^{3}\sigma_{i4}F_{i4} - \frac{\nu}{4u_0^3}a\sum_{i,j=1}^{3}\sigma_{ij}F_{ij}\right]$$

L. Liu, et. al., PRD81(2010)094505
Z. S. Brown, et. al., PRD90(2014)094507

Ensemble	a(fm)	$\tilde{L}^3 \times \tilde{T}$	m_π(MeV)	$N_{\rm cfg} \times N_{src}$	ν	m_Q	m_Υ(MeV)	c
C24P29	0.10521(11)	$24^3 \times 72$	292.3(1.0)	25×3	3.68	7.42	9460.8(1.6)	1.008(24)
E28P35	0.08970(26)	$28^3 \times 64$	351.4(1.4)	24×4	2.75	4.87	9466.2(1.9)	1.002(26)
F32P30	0.07751(14)	$32^3 \times 96$	300.4(1.2)	24×3	2.05	3.48	9462.7(1.5)	0.998(19)
G36P29	0.06884(18)	$36^3 \times 108$	297.2(0.9)	25×4	1.87	2.64	9457.2(1.5)	1.006(18)
H48P32	0.05198(20)	$48^3 \times 144$	316.6(1.0)	25×3	1.35	1.52	9463.6(1.1)	1.002(15)





- Determine the bare bottom quark mass using the physical Υ mass using the anisotropic action;
- The anisotropic rate ν is determined by requiring the dispersion relation of Υ to be the same as that in the continuum;
- ν approaches 1 in the continuum limit with $\mathcal{O}(a^2)$ corrections, while careful estimate of its uncertainty is in progress.

H.-Y. Du, et. al., CLQCD, in preparation

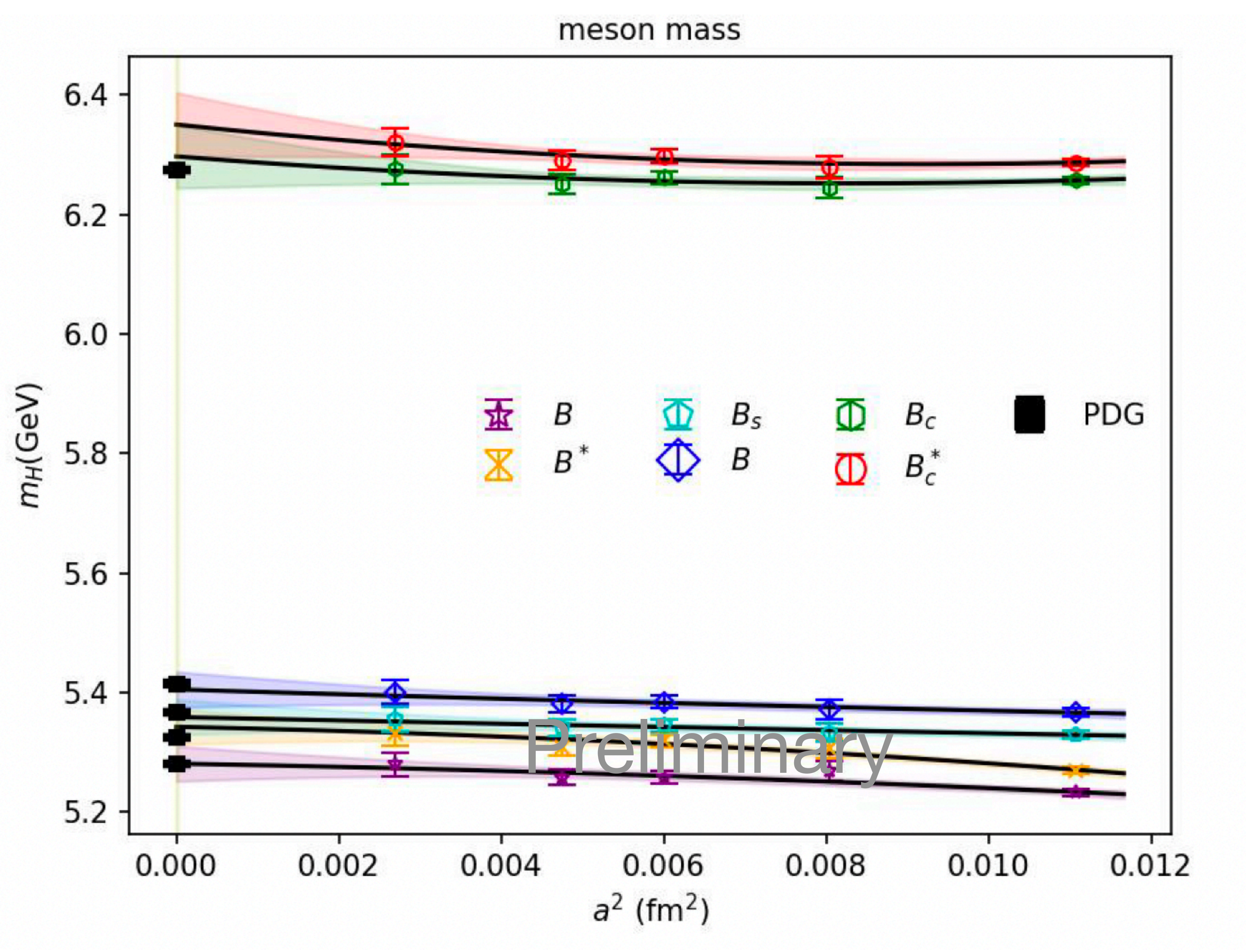
Also see the poster of Hai-Yang Du

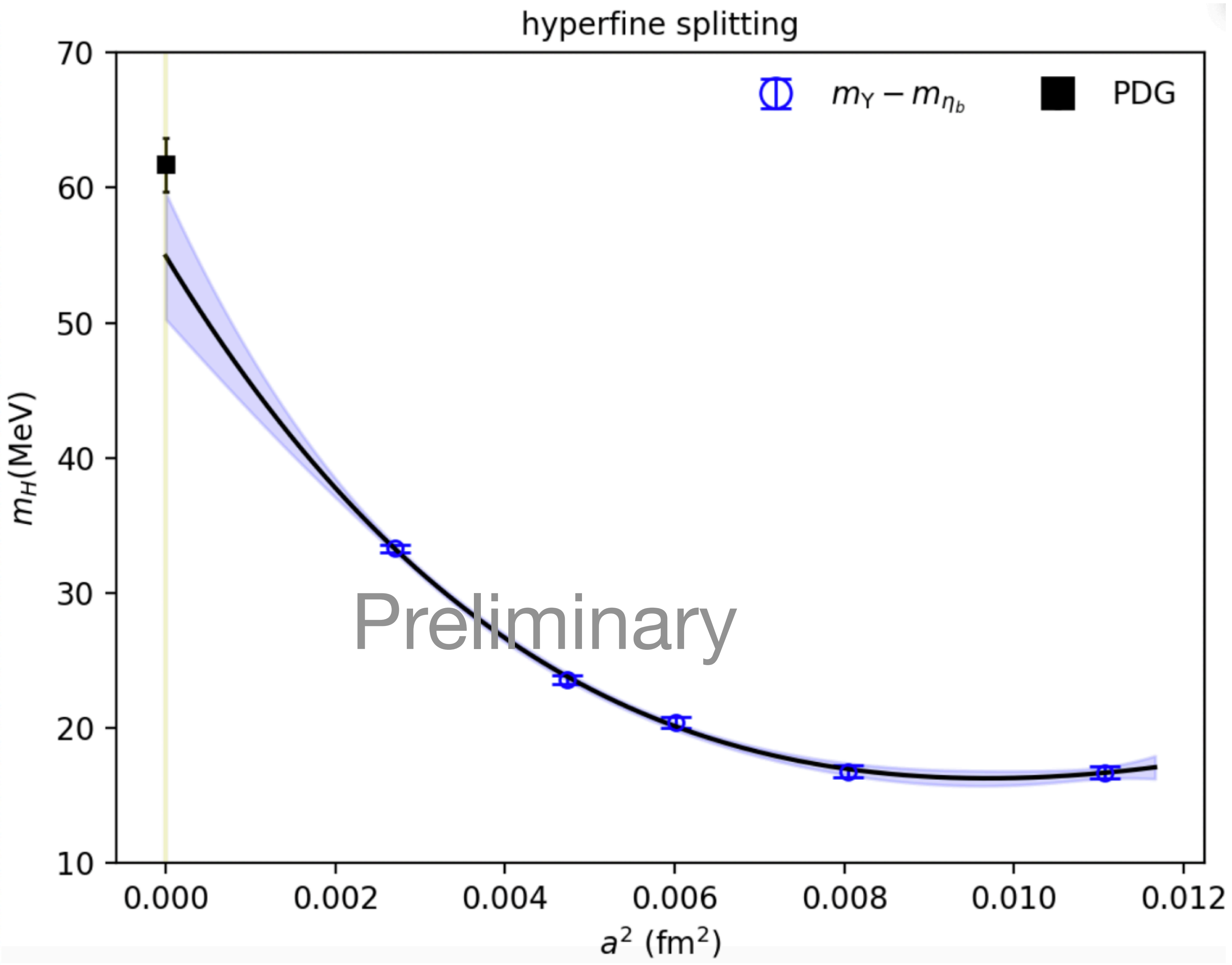
Toward the bottom physics

Hadron spectrum

$$S_Q = a^4 \sum_x \bar{Q}\mathcal{M}Q,\ \mathcal{M} = \left[m_Q + \gamma_4\nabla_4 - \frac{a}{2}\nabla_4^2 + \nu\sum_{i=1}^{3}(\gamma_i\nabla_i - \frac{a}{2}\nabla_i^2) - \frac{1+\nu}{4u_0^3}a\sum_{i=1}^{3}\sigma_{i4}F_{i4} - \frac{\nu}{4u_0^3}a\sum_{i,j=1}^{3}\sigma_{ij}F_{ij}\right]$$

L. Liu, et. al., PRD81(2010)094505
Z. S. Brown, et. al., PRD90(2014)094507





H.-Y. Du, et. al., CLQCD, in preparation

- Based on this action, the $B_{(s/c)}^{(*)}$ masses agree with experiment within sub-percent statistical uncertainty;
- The hyperfine splitting $m_\Upsilon - m_{\eta_b}$ suffers from sizable discretization error and requires input from smaller lattice spacing.

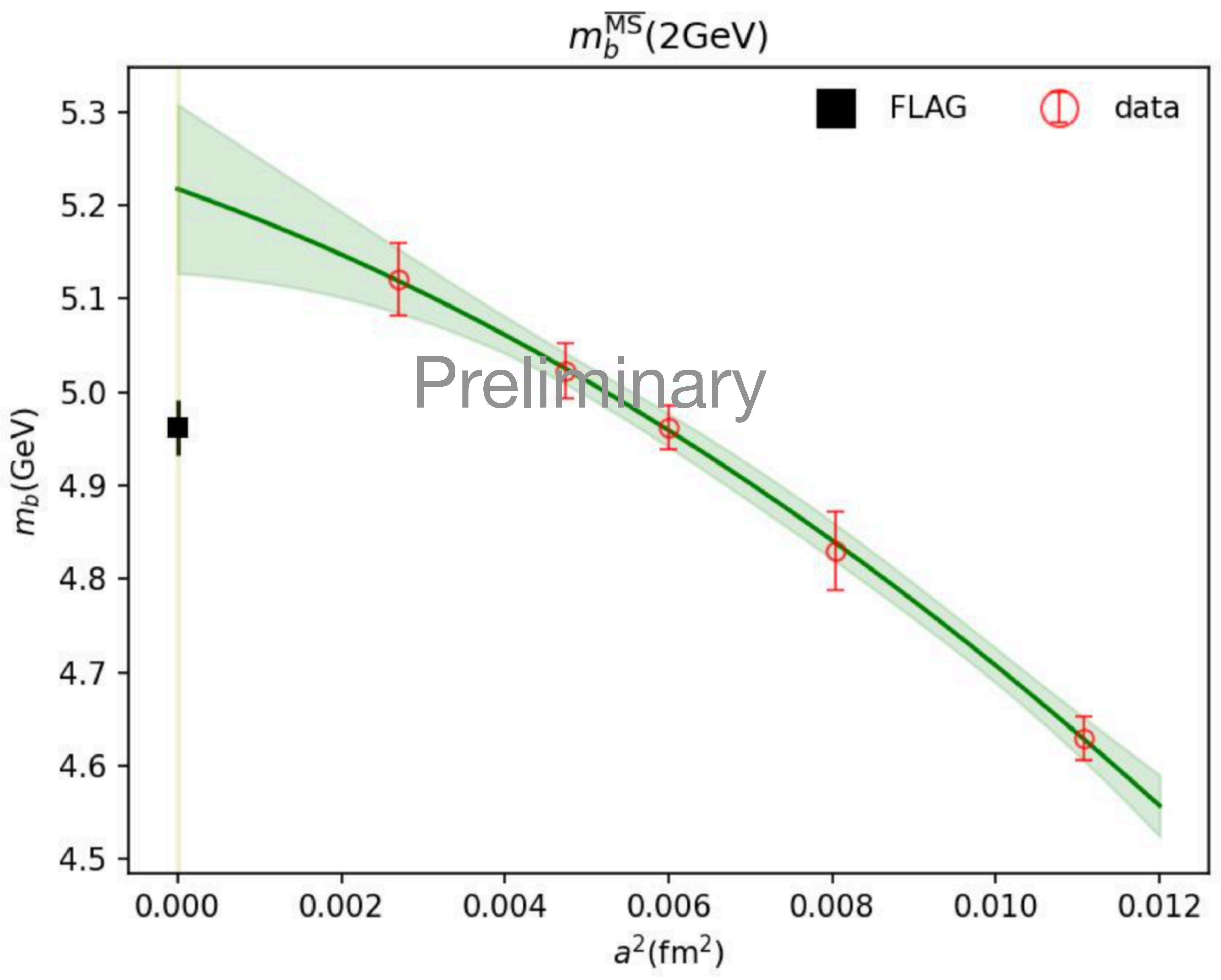
Also see the poster of Hai-Yang Du

Toward the bottom physics

$$S_Q = a^4 \sum_x \bar{Q}\mathcal{M}Q,\ \mathcal{M} = \left[m_Q + \gamma_4\nabla_4 - \frac{a}{2}\nabla_4^2 + \nu\sum_{i=1}^{3}(\gamma_i\nabla_i - \frac{a}{2}\nabla_i^2) - \frac{1+\nu}{4u_0^3}a\sum_{i=1}^{3}\sigma_{i4}F_{i4} - \frac{\nu}{4u_0^3}a\sum_{i,j=1}^{3}\sigma_{ij}F_{ij}\right]$$

L. Liu, et. al., PRD81(2010)094505
Z. S. Brown, et. al., PRD90(2014)094507

$$m_q^{\rm PC} = \frac{m_{\rm PS}\sum_{\vec{x}}\langle A_4(\vec{x},t)P^\dagger(\vec{0},0)\rangle}{2\sum_{\vec{x}}\langle P(\vec{x},t)P^\dagger(\vec{0},0)\rangle}\Big|_{t\to\infty}$$



H.-Y. Du, et. al., CLQCD, in preparation

Bottom quark mass

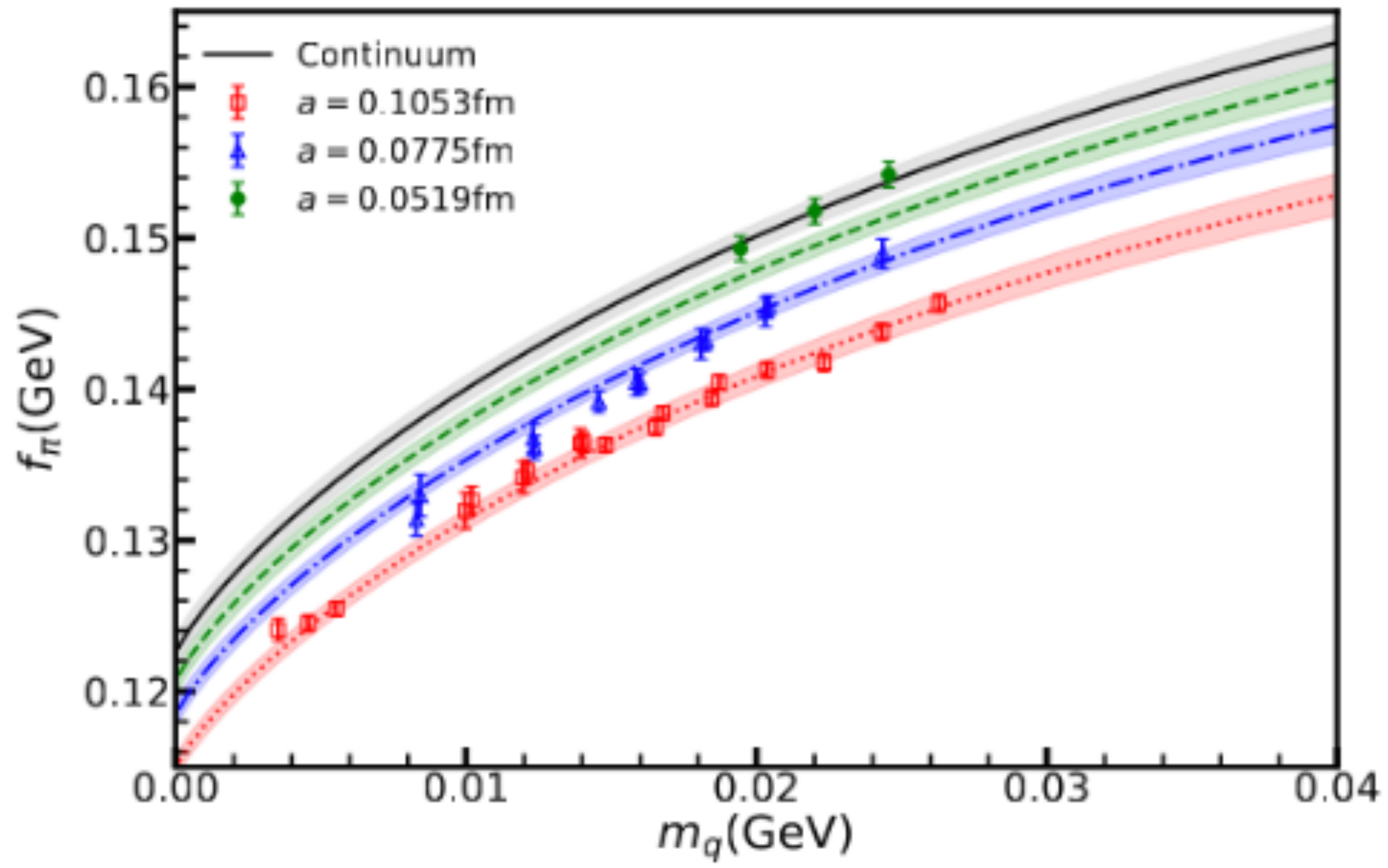
If we define the bottom quark mass though PCAC relation and use the Z_P/Z_A in the chiral limit:

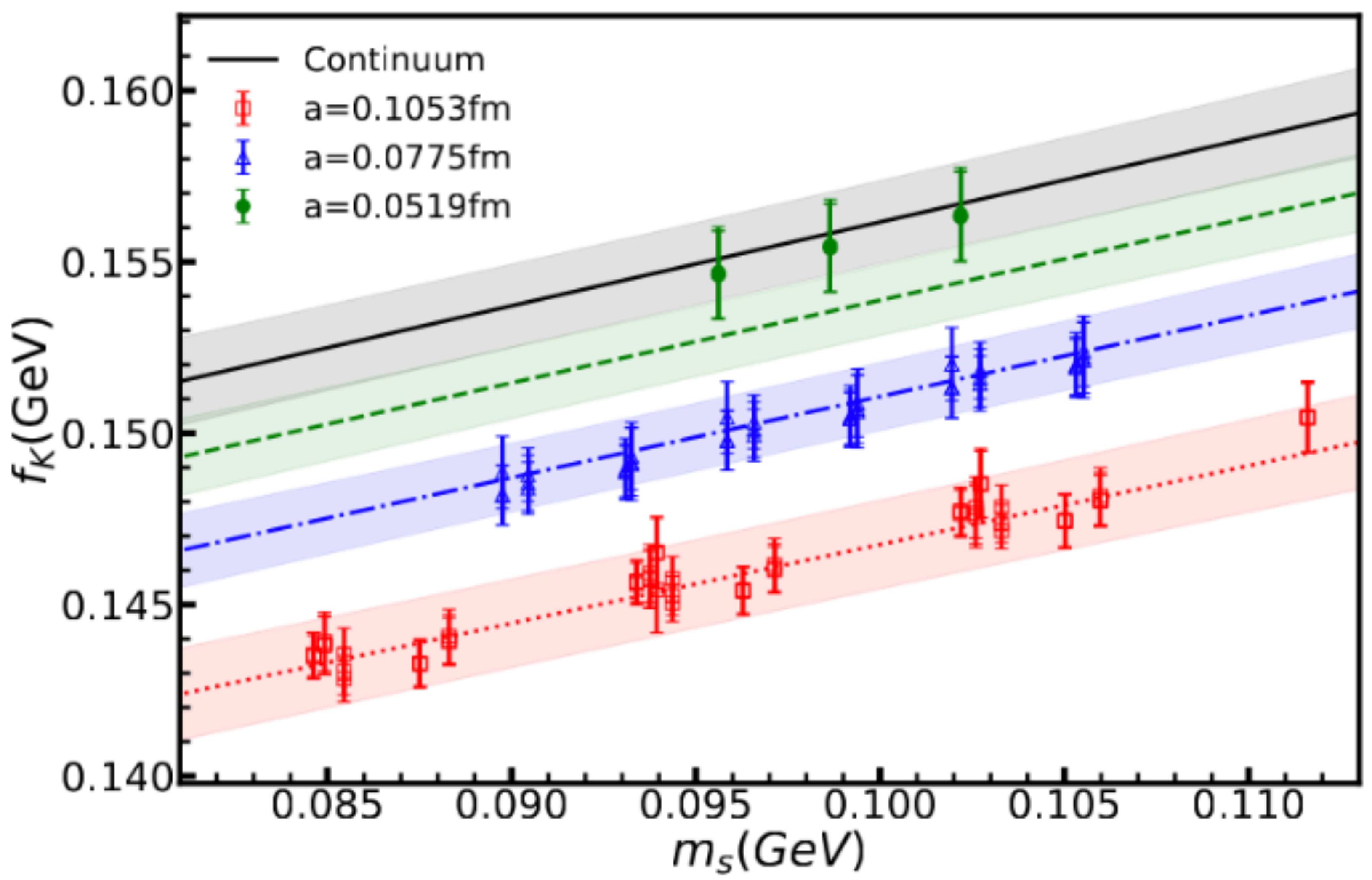
- Renormalized bottom quark mass will be ~5(2)% higher than the FLAG value.

But:

- Current statistics is very limited (25 cfgs).
- Systematic uncertainties from unphysical light quark masses are not included yet;
- Systematic uncertainty from the impact of ν on Z_P/Z_A is not considered yet.

Decay constants





Pion and Kaon cases

$$\frac{f_K}{f_\pi} = 1.1907(76)(03)$$

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = 0.27683(29)_{\text{exp}}(20)_{\text{th}} \longrightarrow$$

$$|V_{ud}| = 0.9740(03)_{\text{lat}}(01)_{\text{ph}}$$

$$|V_{us}| = 0.2265(14)_{\text{lat}}(03)_{\text{ph}}$$

$$|V_{us}| = 0.2243(8)_{\text{PDG}}$$

$$1 = |V_{ud}|^2 + |V_{us}^2| + |V_{ub}|^2 = |V_{ud}|^2 + |V_{us}^2| + 0.0035^2$$

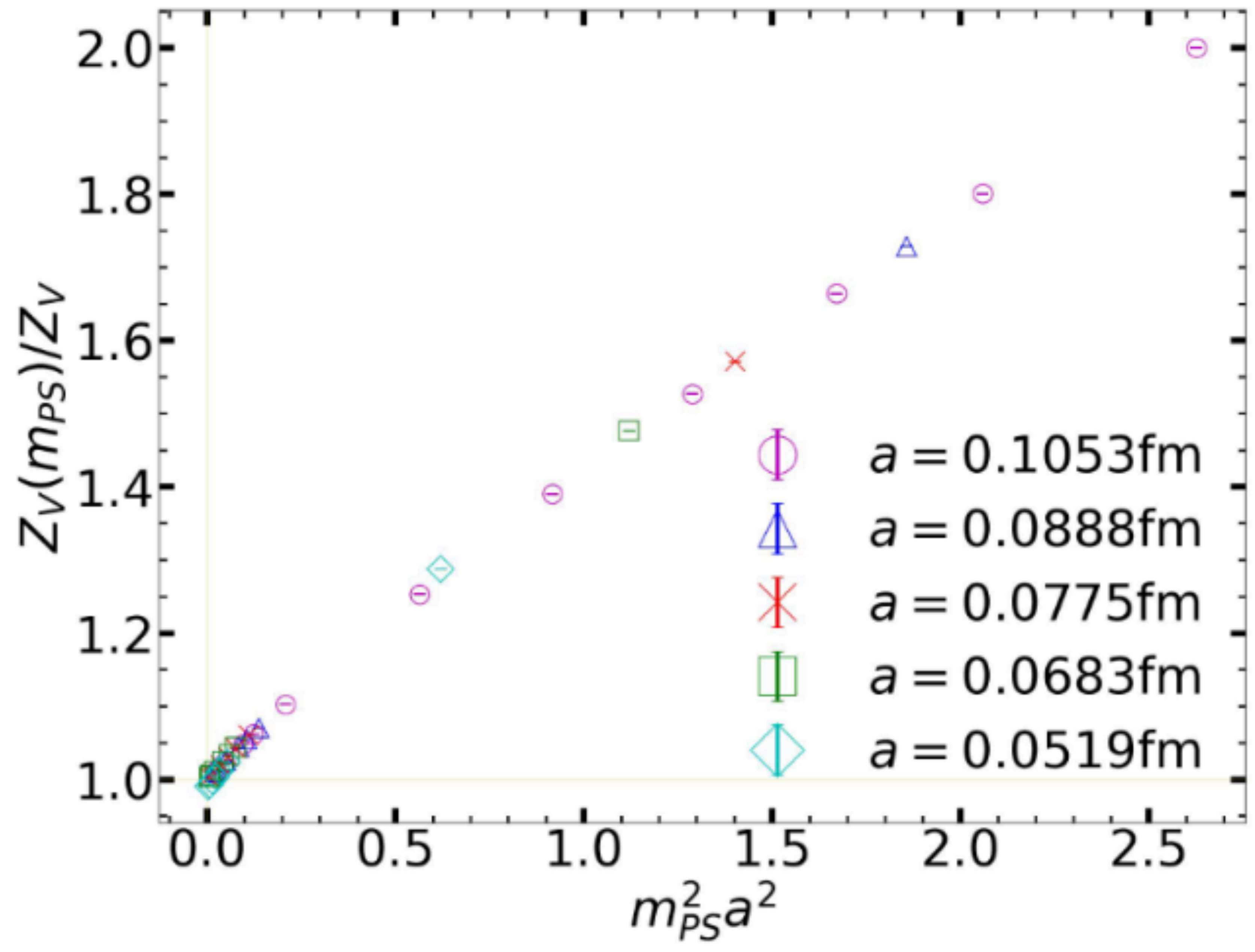
- Additional input likes the form factor of the semileptonic decay $K^0 \to \pi^- l \nu$ is required to determine $|V_{ud(s)}|$ directly and verify the unitarity of CKM.

Z.-H. Hu, B.-L. Hu, J.-H. Wang, et. al., CLQCD, PRD109(2024) 054507

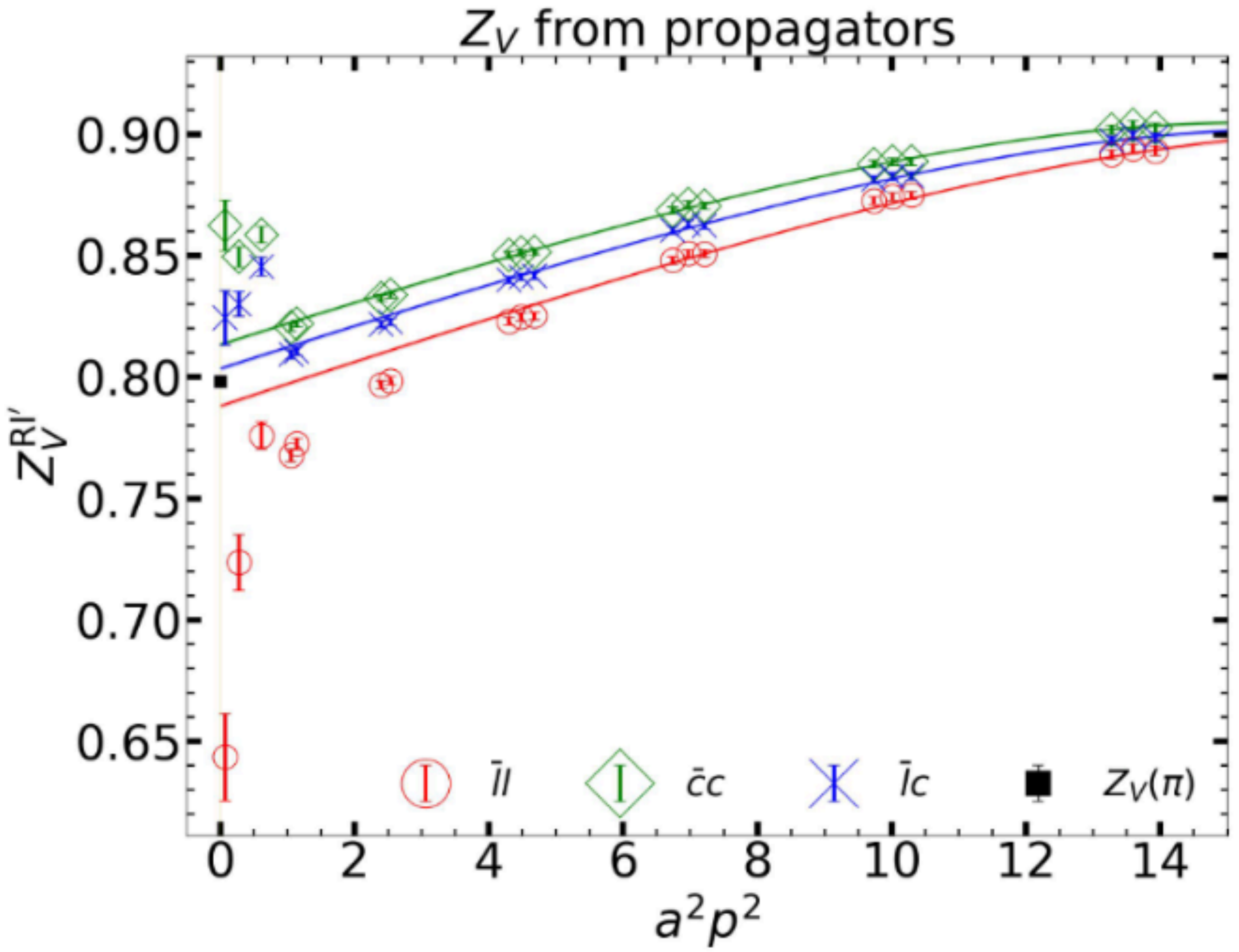
Matrix elements

Heavy quark improved normalization

$$Z_V(H)\frac{\langle H|V_4|H\rangle}{\langle H|H\rangle} = 1, \qquad Z_V = Z_V(\pi)|_{m_\pi \to 0}$$



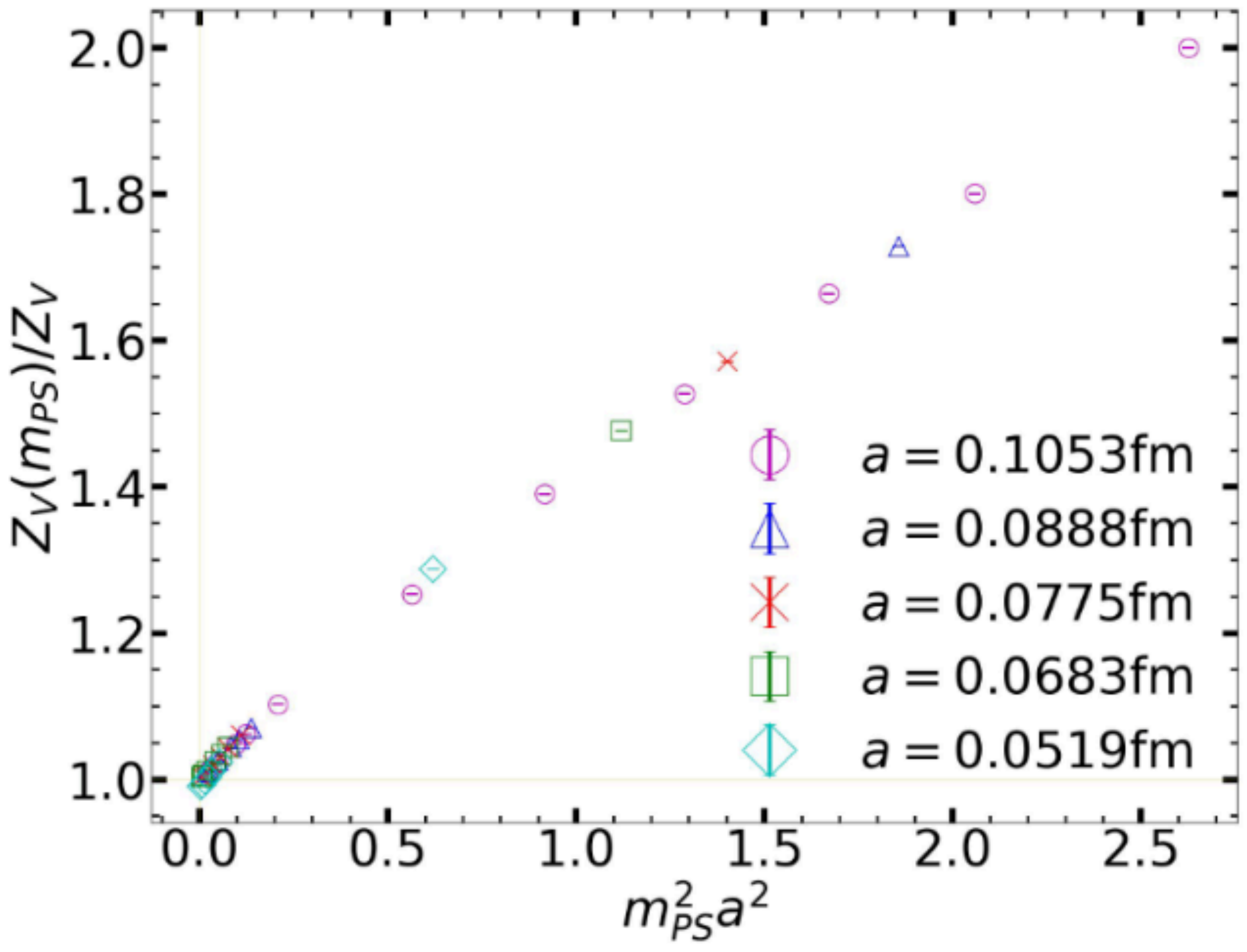
- The vector current normalization Z_V constant can have sizable $m_{\rm PS}^2 a^2$ error;
- Such an effect does not exist if we calculate Z_V using the off-shell quarks;
- Should be a hadronic effect while the origin is still unclear.



Matrix elements

Heavy quark improved normalization

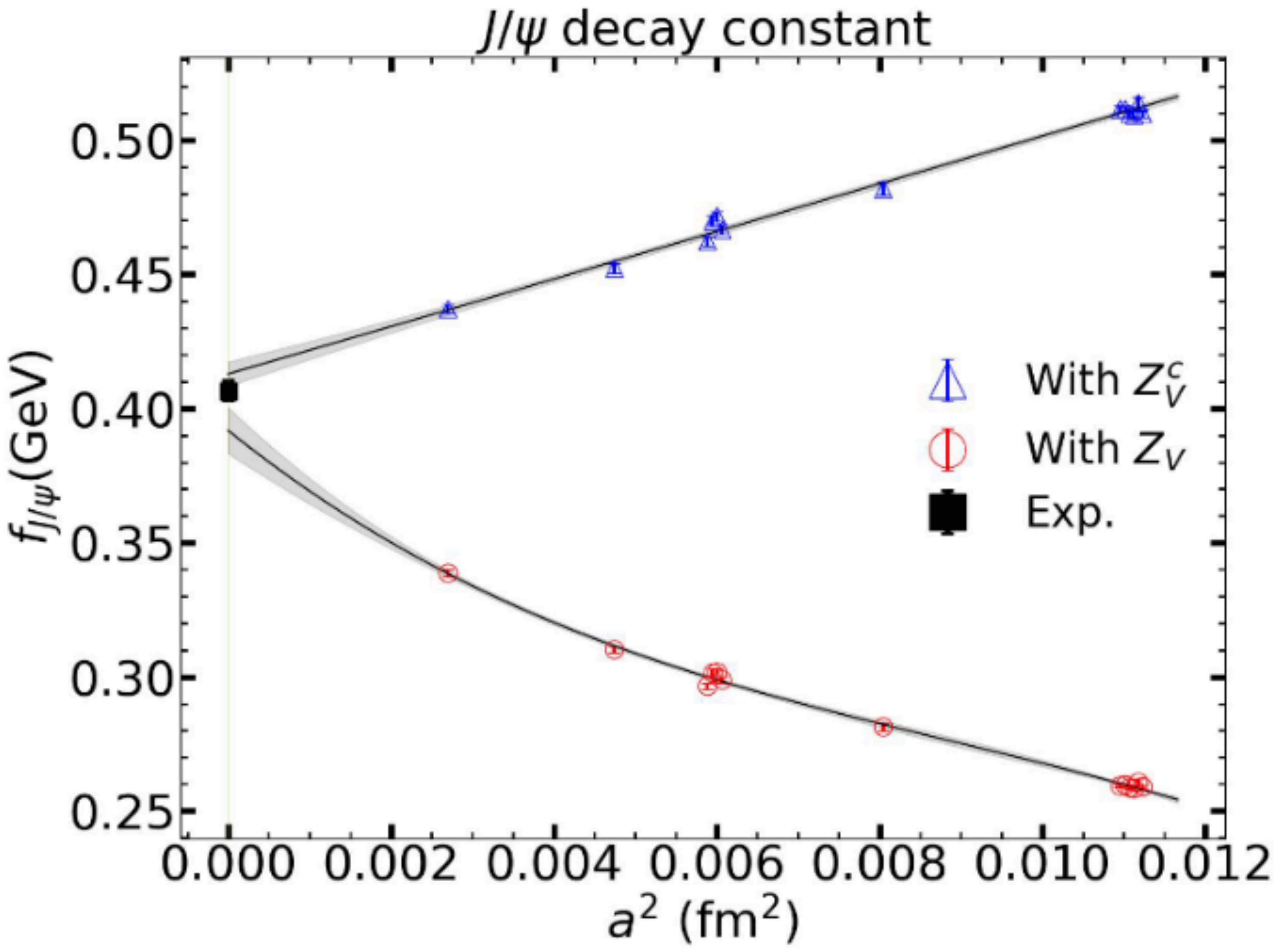
$$Z_V(H)\frac{\langle H|V_4|H\rangle}{\langle H|H\rangle} = 1, \qquad Z_V = Z_V(\pi)|_{m_\pi\to 0}$$



One can define $Z_V^c = Z_V(\eta_c)$ to suppress the discretization error of the ME of charm quark bi-linear operator:

- $f_{J/\Psi}$ with Z_V^c is linear on a^2 and the continuum extrapolated value agree with experiment well;
- $f_{J/\Psi}$ with Z_V has much larger discretization error and approaches to the correct limit with $\mathcal{O}(a^6)$ correction.

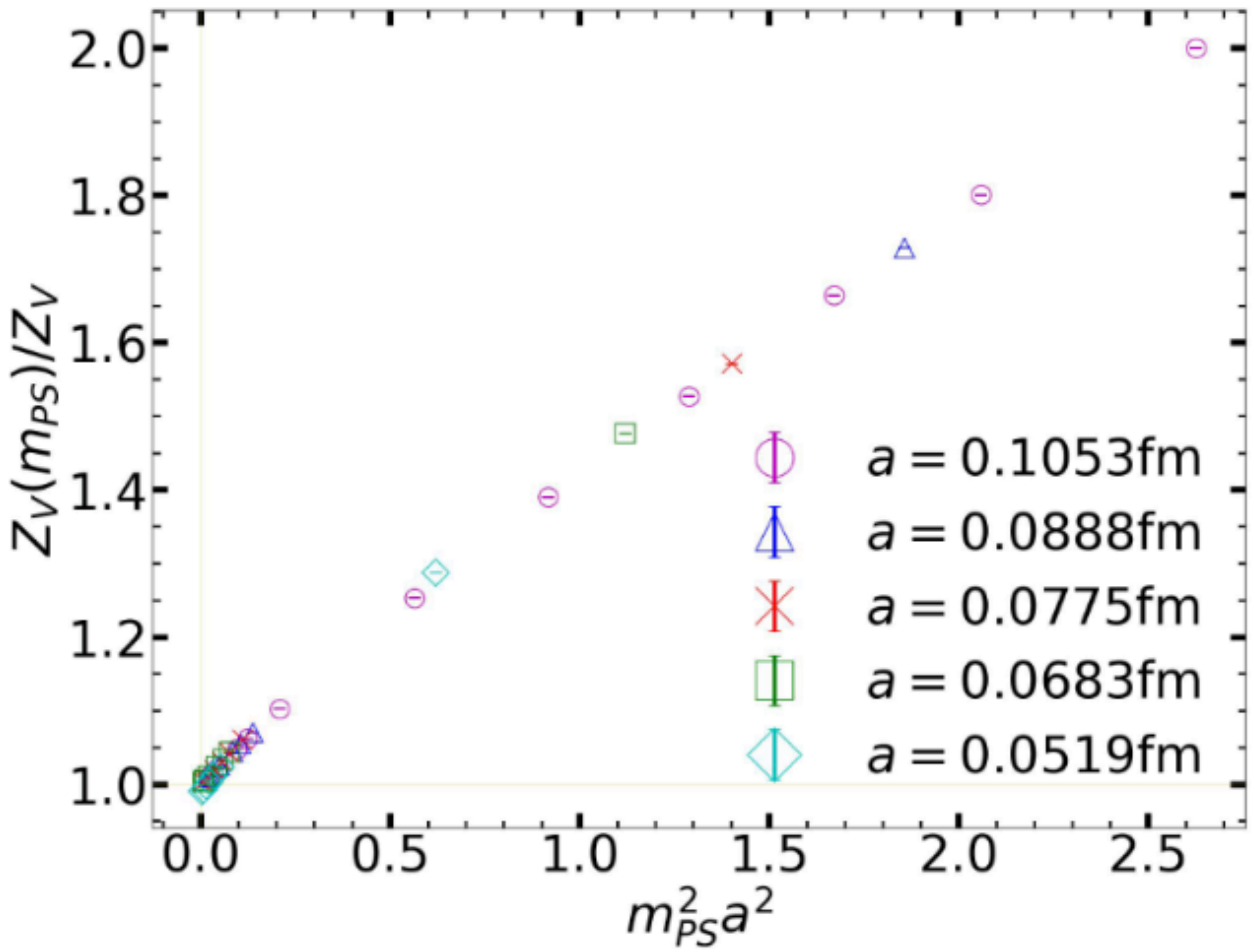
$$\langle \bar{c}\gamma_\mu c|J/\psi\rangle = \epsilon_\mu m_{J/\Psi} f_{J/\Psi}$$



Matrix elements

Heavy quark improved normalization

$$Z_V(H)\frac{\langle H|V_4|H\rangle}{\langle H|H\rangle} = 1, \qquad Z_V = Z_V(\pi)|_{m_\pi\to 0}$$

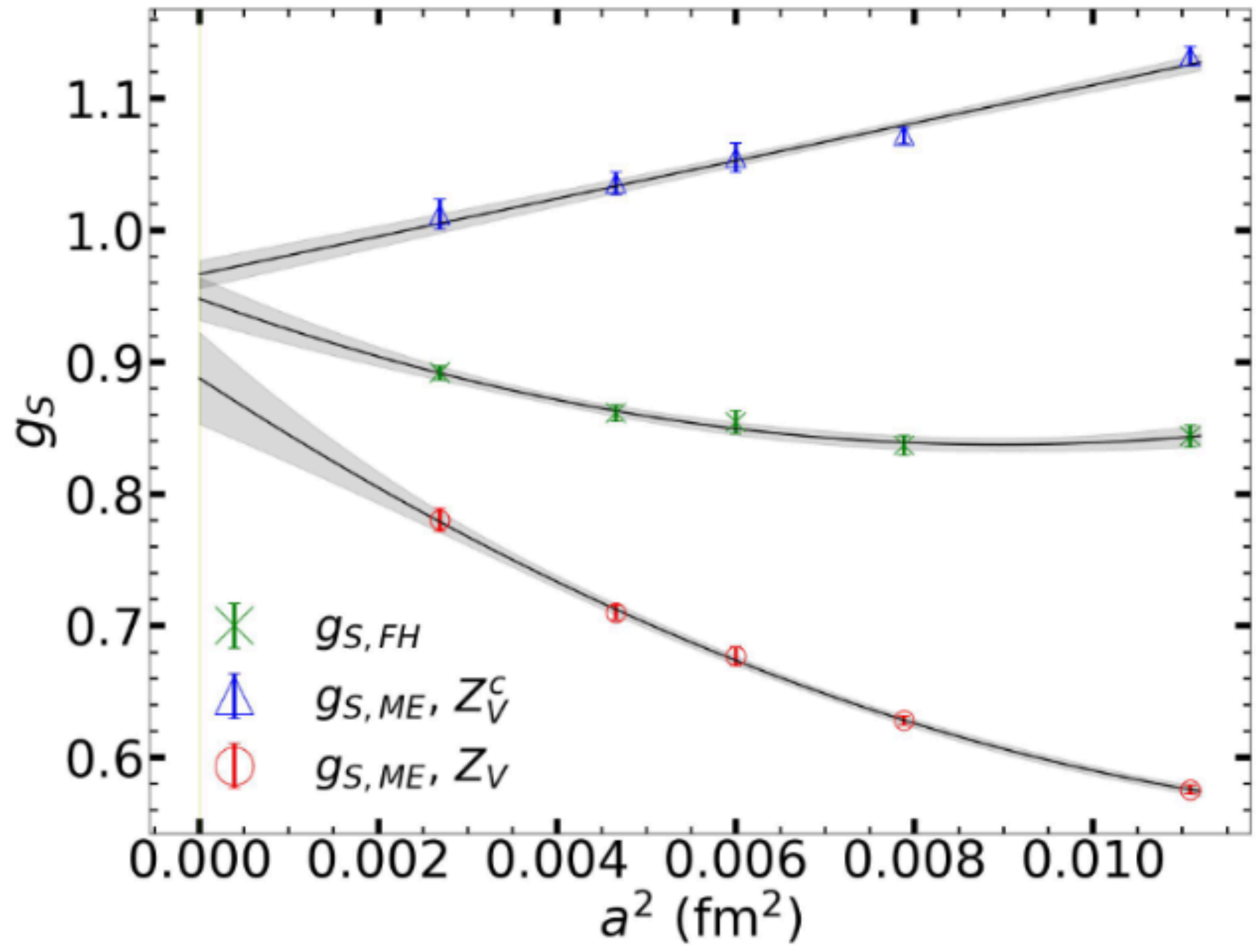


One can define $Z_V^c = Z_V(\eta_c)$ to suppress the discretization error of the ME of charm quark bi-linear operator:

- $g_{S,ME}$ with $Z_V^c\frac{Z_S}{Z_V}$ is linear on a^2 and the continuum extrapolated value agree with that of $g_{S,FH} \equiv \frac{Z_P}{Z_A}\frac{\partial m_{\eta_c}}{\partial m_c^{\rm PCAC}}$ well;

- $g_{S,ME}$ with $Z_S = Z_V\frac{Z_S}{Z_V}$ has much larger discretization error and approaches to the correct limit with $\mathcal{O}(a^6)$ correction.

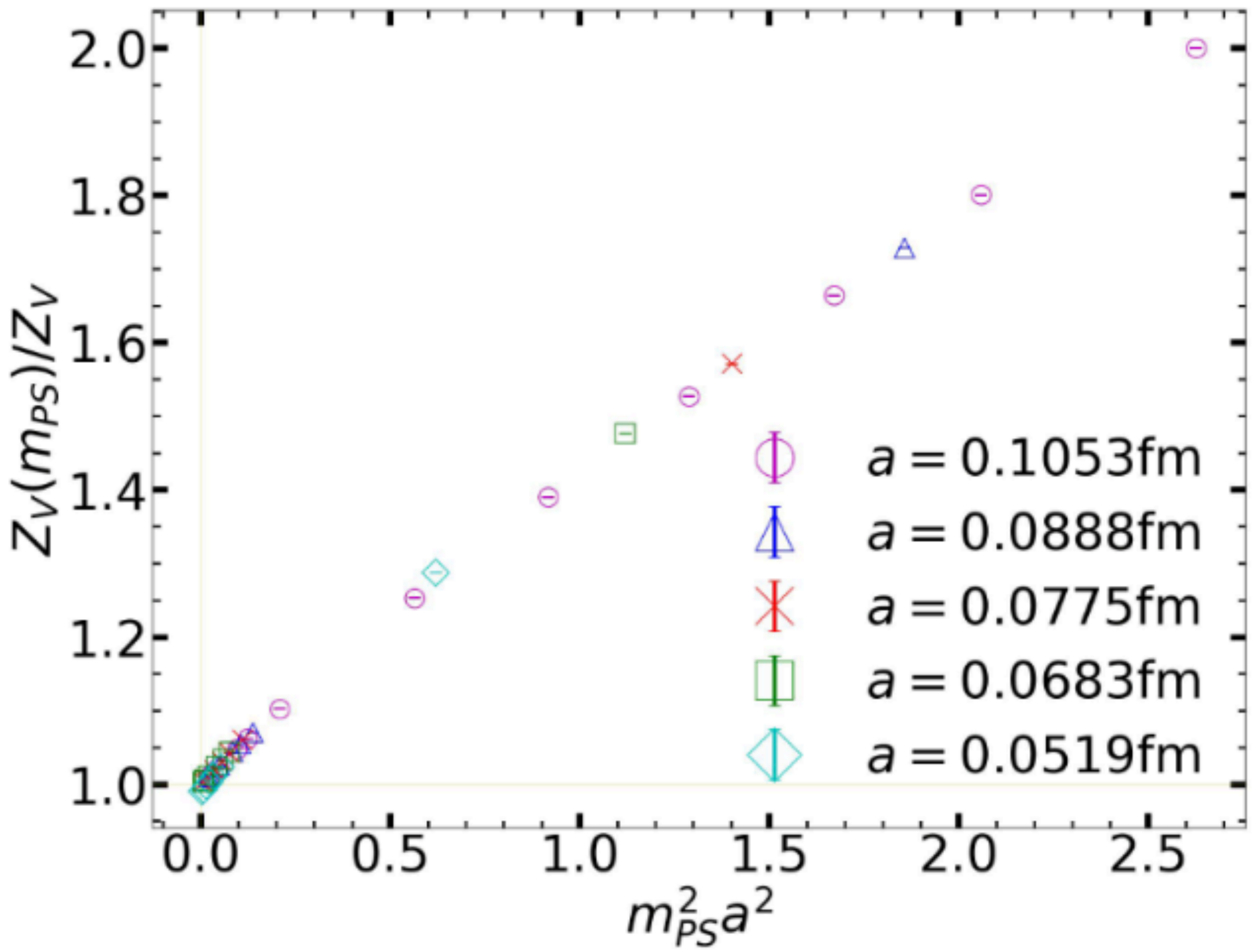
$$g_S = \frac{\langle \eta_c|\bar{c}c|\eta_c\rangle}{\langle \eta_c|\eta_c\rangle}$$



Matrix elements

Heavy quark improved normalization

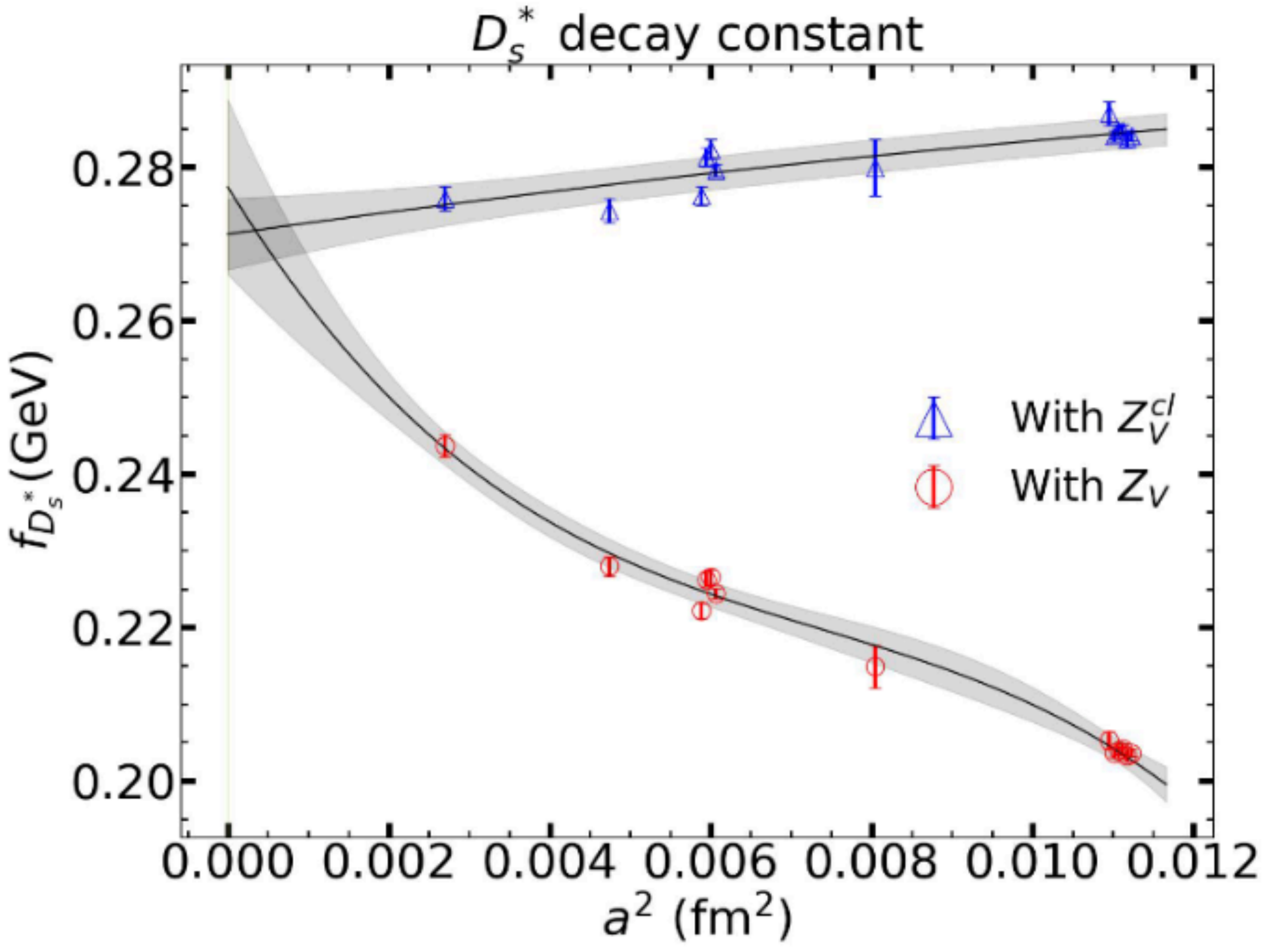
$$Z_V(H)\frac{\langle H|V_4|H\rangle}{\langle H|H\rangle}=1, \qquad Z_V = Z_V(\pi)|_{m_\pi\to 0}$$



One can define $Z_V^{cl}=\sqrt{Z_V(\eta_c)Z_V}$ to suppress the discretization error of the ME of charm-light quark bi-linear operator:

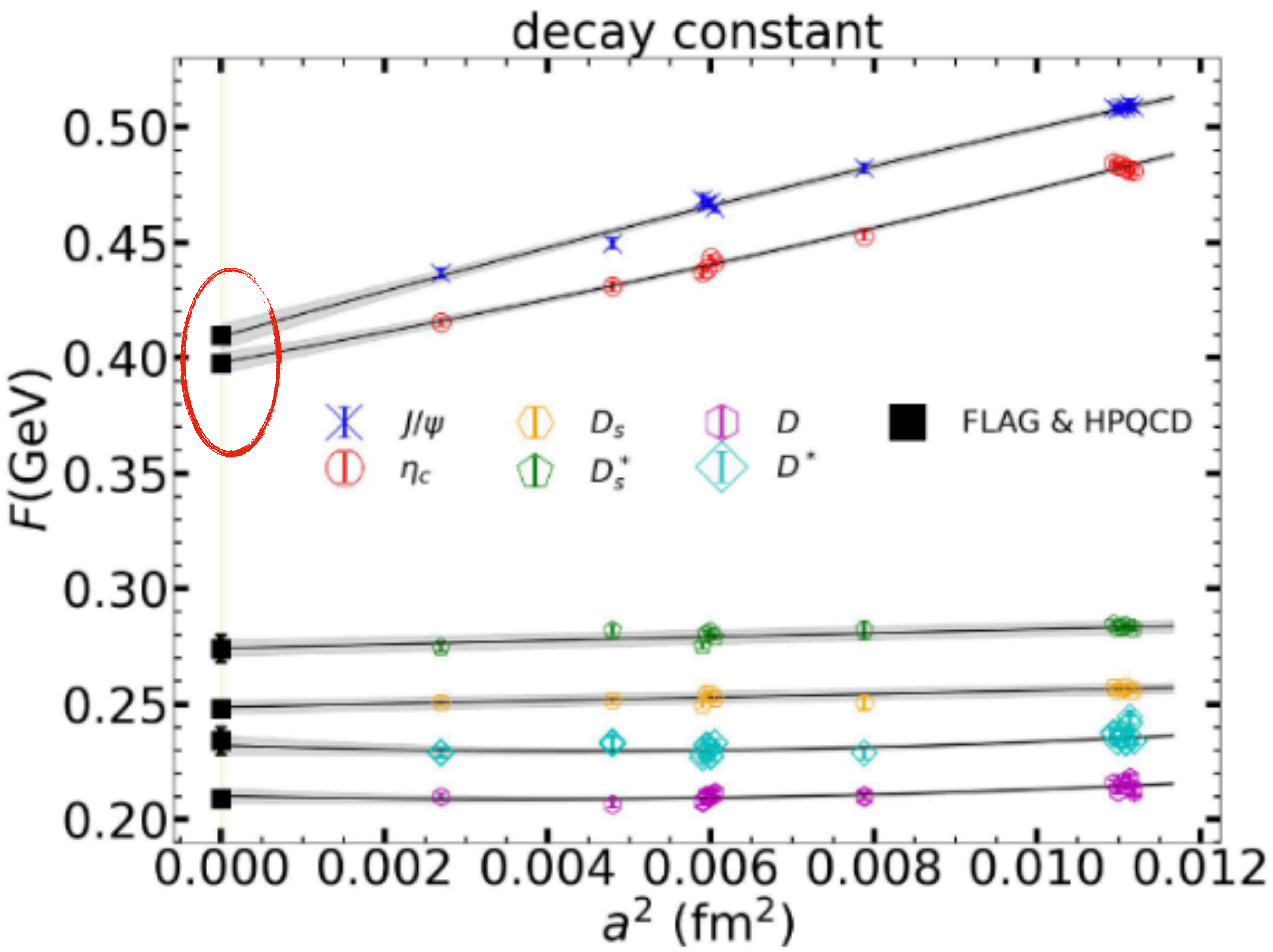
- $f_{D_s^*}$ with Z_V^{cl} is linear on a^2;
- $f_{D_s^*}$ with Z_V has much larger discretization error but agrees with $f_{D_s^*}$ with Z_V^{cl} after the continuum extrapolation with $\mathcal{O}(a^6)$ correction.

$$\langle \bar{c}\gamma_\mu s|D_s^*\rangle = \epsilon_\mu m_{D_s^*} f_{D_s^*}$$



Decay constants

S-wave charmonium

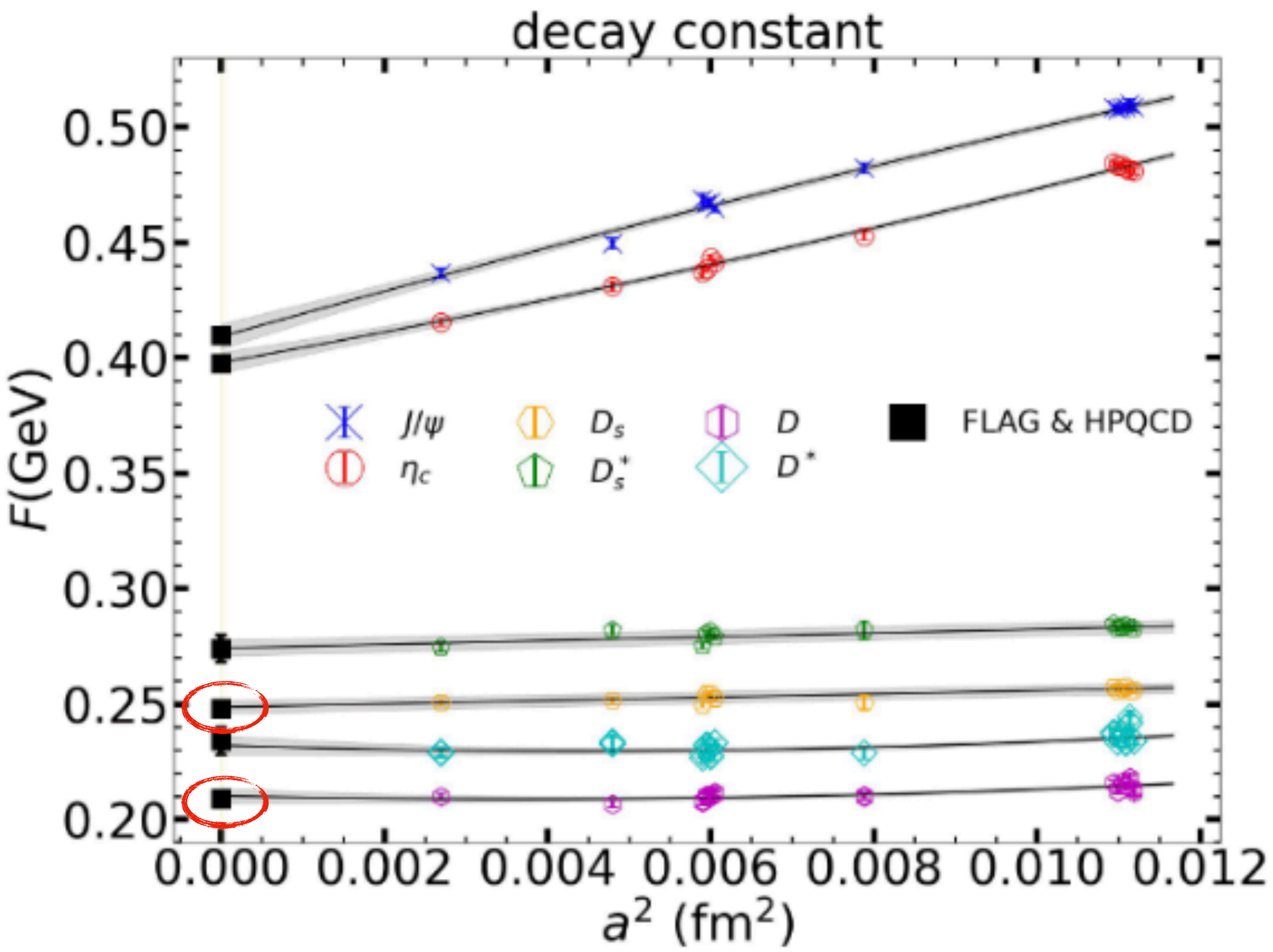


- Our prediction $f_{J/\psi} = 413.1(4.6)(2.2)$ MeV is consistent with the experimental value 406.5(3.7)(0.5) MeV and also HPQCD prediction 409.6(1.6) MeV;
- We also predict $f_{\eta_c} = 397.4(3.9)(1.8)$ MeV which is consistent with the HPQCD prediction 397.5(1.0) MeV.

Hai-Yang Du, B.L. Hu, et. al., CLQCD, 2408.03548

Decay constants

Open charm cases



$f_{D^+} = 0.2113(33)_{\rm lat}$ MeV

$f_{D^+}|V_{cd}| = 45.8(1.1)_{\rm exp}$ MeV $\longrightarrow |V_{cd}| = 0.2168(33)_{\rm lat}(52)_{\rm exp}$

$f_{D_s^+}|V_{cs}| = 243.5(2.7)_{\rm exp}$ MeV $\longrightarrow |V_{cs}| = 0.975(13)_{\rm lat}(11)_{\rm exp}$

$f_{D_s^+} = 0.2498(33)_{\rm lat}$ MeV

- Verified the unitarity of CKM matrix elements involving the charm quark: $|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 0.999(25)(22)$.
- Also provide the most precise f_{D^*} and $f_{D_s^*}$ so far.

Hai-Yang Du, B.L. Hu, et. al., CLQCD, 2408.03548

Summary on CLQCD determinations of Stand model parameters

$m_{\pi^\pm}(m_u, m_d)$

$m_{\pi^0} = 134.98$ MeV

$w_0 = 0.1736(9)$ fm

$(m_u + m_d)^{\overline{\rm MS}(2\text{ GeV})}/2 = 3.60(19)$ MeV

$m_{K^0}^{\rm QCD} = 497.44(02)$ MeV

$m_{K^\pm}^{\rm QCD} = 491.44(15)$ MeV

$m_u^{\overline{\rm MS}(2\text{ GeV})} = 2.45(30)$ MeV

$m_d^{\overline{\rm MS}(2\text{ GeV})} = 4.74(14)$ MeV

$m_s^{\overline{\rm MS}(2\text{ GeV})} = 98.8(5.5)$ MeV

$m_K(m_u^{val}, m_d^{val}, m_l^{sea}, m_s^{sea})$

$m_{D_s}^{\rm QCD} = 1966.7(1.5)$ MeV

$m_c^{\overline{\rm MS}(m_c)} = 1293(13)(11)$ MeV

$m_{D_s}(m_c^{val}, m_s^{val}, m_l^{sea}, m_s^{sea})$

$\frac{f_K}{f_\pi} = 1.1907(76)_{\rm lat}$

$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = 0.27683(29)_{\rm exp}(20)_{\rm th}$

$|V_{ud}| = 0.9740(03)_{\rm lat}(01)_{\rm ph}$

$|V_{us}| = 0.2265(14)_{\rm lat}(03)_{\rm ph}$

$1 = |V_{ud}|^2 + |V_{us}^2| + |V_{ub}|^2 = |V_{ud}|^2 + |V_{us}^2| + 0.0035^2$

$f_{D^+} = 0.2113(33)_{\rm lat}$ MeV

$f_{D^+}|V_{cd}| = 45.8(1.1)_{\rm exp}$ MeV

$|V_{cd}| = 0.2168(33)_{\rm lat}(52)_{\rm exp}$

$f_{D_s^+}|V_{cs}| = 243.5(2.7)_{\rm exp}$ MeV

$|V_{cs}| = 0.975(13)_{\rm lat}(11)_{\rm exp}$

$f_{D_s^+} = 0.2498(33)_{\rm lat}$ MeV

Summary

- The state-of-the-arts Lattice QCD ensemble should have enough ensembles to approach the continuum, infinite volume and physical quark masses reliably; and the present CLQCD ensembles have been close to this goal.
- Up, down, strange and charm quark masses have been determined at a few percent level;
- The charmed meson and baryon masses are predicted at ~0.3% uncertainty and agree with the experimental values at 1% level.
- More predictions are in progress.