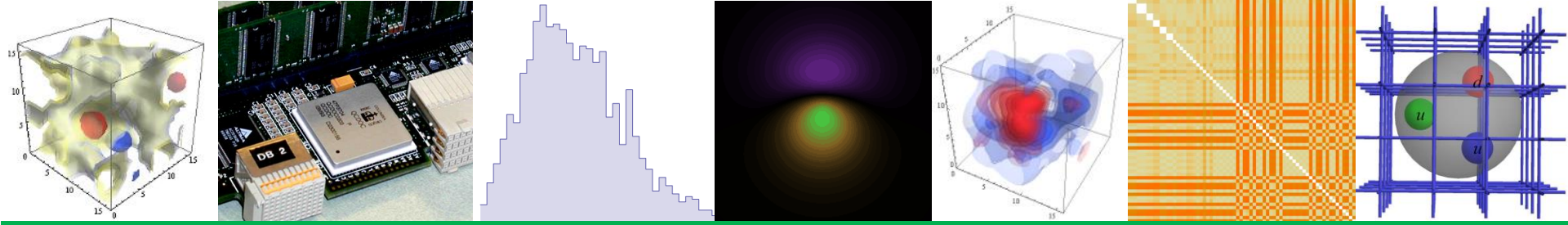


Recent Progress in Lattice Parton Distributions Calculations from MSULat

HUEY-WEN LIN

CTEQ

This work of HL is supported by the NSF under grant PHY 2209424 & 1653405, DOE under DE-SC0024053 and the Research Corporation for Science Advancement through the Cottrell Scholar Award

@LinQCD

NSF

U.S. DEPARTMENT OF ENERGY

RESEARCH CORPORATION for SCIENCE ADVANCEMENT

Outline

§ Lattice QCD and Parton Distribution Functions

§ Selected *x*-Dependent Parton Distributions

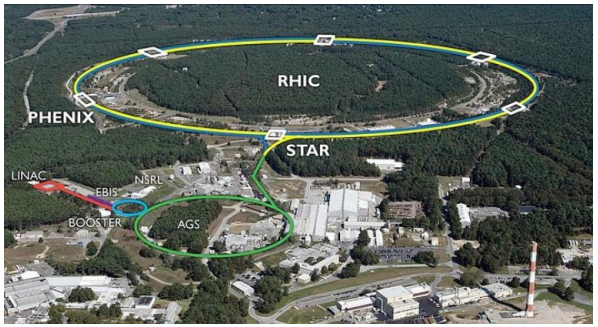
- Flavor non-singlet PDFs
- Gluon PDFs
- Generalized Parton Distributions (GPDs)

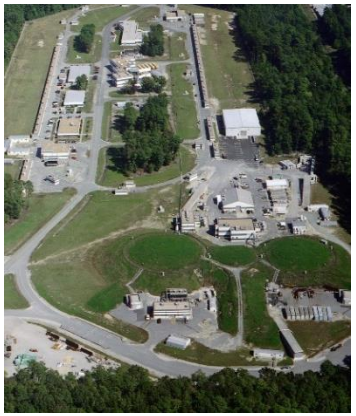
§ Impact of Lattice-QCD PDFs on Global Fits

Parton Distribution Functions

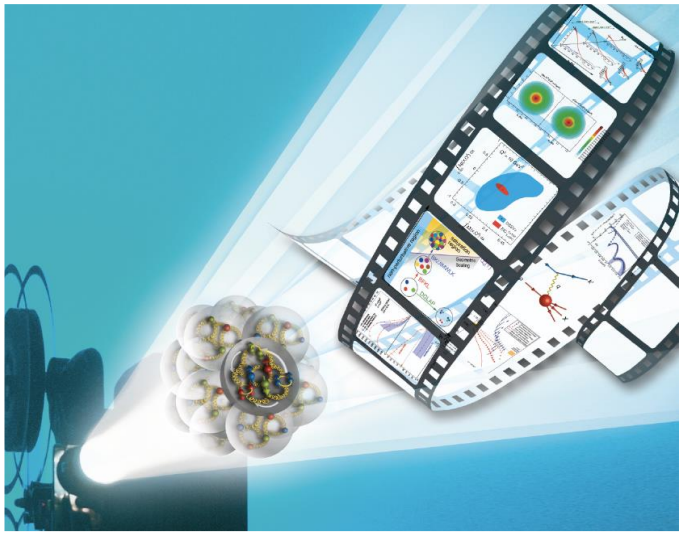
§ PDFs are universal quark/gluon distributions of nucleon

- Many ongoing/planned experiments
  (BNL, JLab, J-PARC, COMPASS, GSI, EIC, EIcC, LHeC, …)





Electron Ion Collider:
The Next QCD Frontier



Imaging of the proton

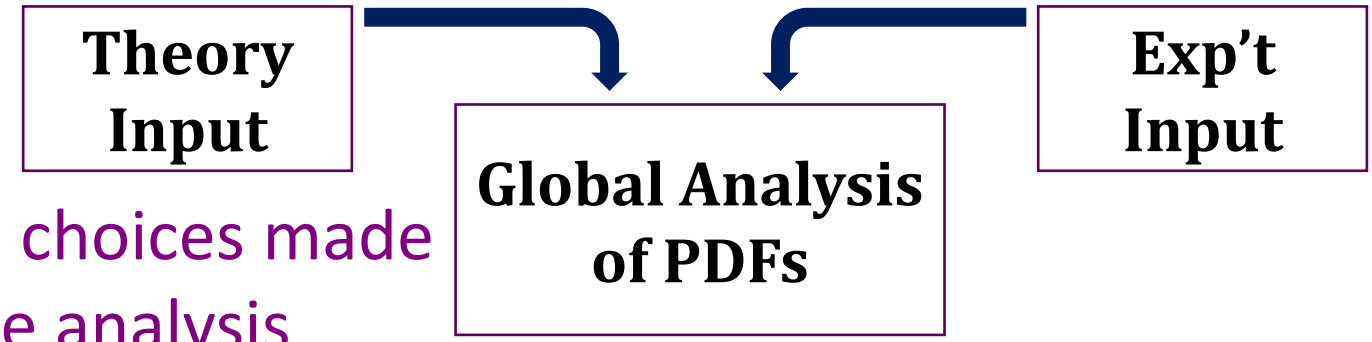
How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?

EIC White Paper, 1212.1701; The Present and Future of QCD (2303.02579)

Global Analysis

§ Experiments cover diverse kinematics of parton variables

- Global analysis takes advantage of all data sets



§ Some choices made for the analysis

- Choice of data sets and kinematic cuts
- Strong coupling constant $\alpha_s(M_Z)$
- How to parametrize the distribution

$$xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2} P(x)$$

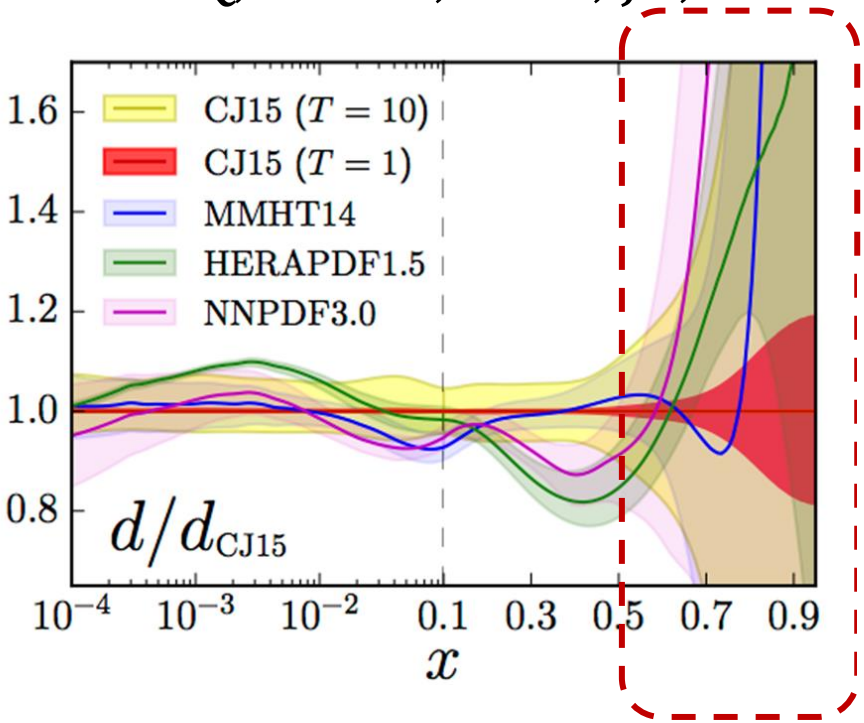
- Assumptions imposed

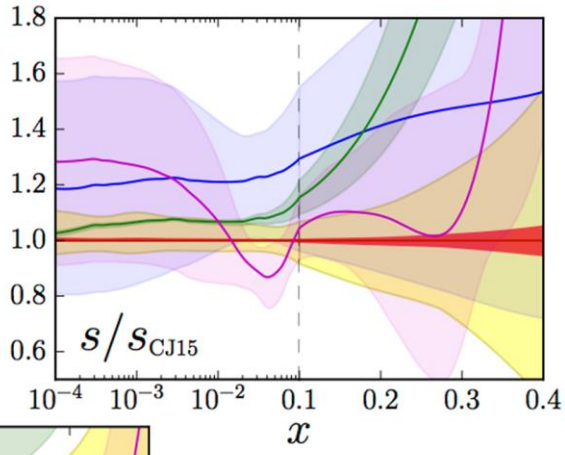
  SU(3) flavor symmetry, charge symmetry, strange and sea distributions

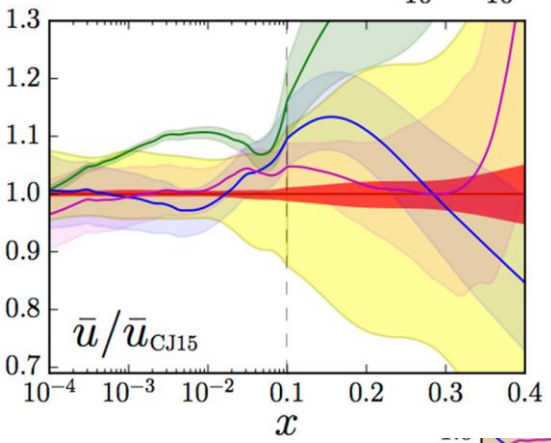
$$s = \bar{s} = \kappa(\bar{u} + \bar{d})$$

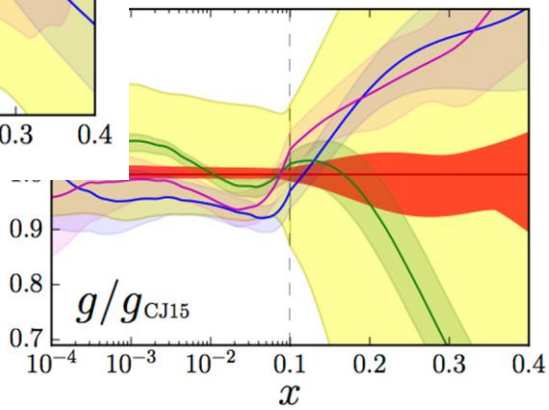
Global Analysis

- Discrepancies appear when data is scarce
- Many groups have tackled the analysis
  - CTEQ, MSTW, ABM, JR, NNPDF, etc.





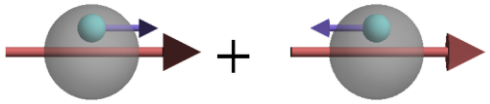




CTEQ-JLAB
https://www.jlab.org/theory/cj/

PDFs on the Lattice

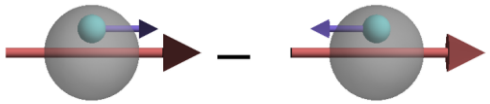
§ Traditional lattice calculations rely on operator product expansion, only provide moments



spin-averaged/unpolarized

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

most well known



spin-dependent longitudinally polarized

$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$

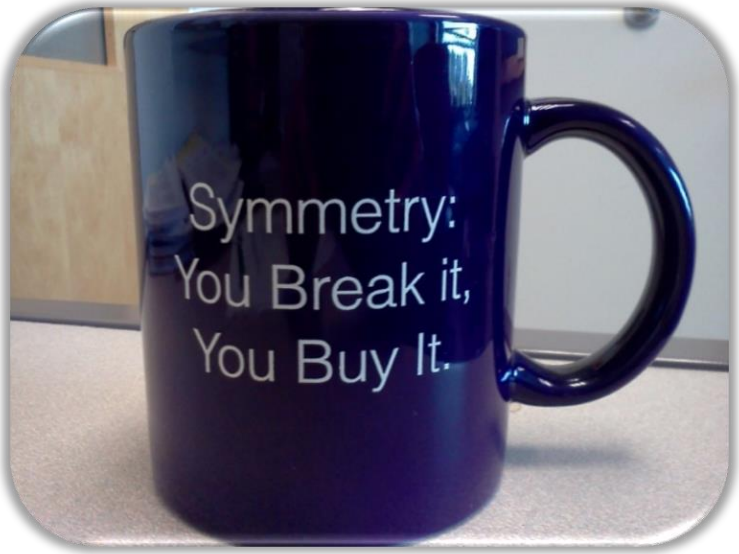
spin-dependent transversely polarized

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$

very poorly known

§ True distribution can only be recovered with all moments

Lattice Structure Limitation



§ Lattice calculations rely on operator product expansion, only provide moments

$$\langle x^{n-1} \rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

§ Limited to the lowest few moments

- For higher moments, all ops mix with lower-dimension ops
- Novel proposals to overcome this problem

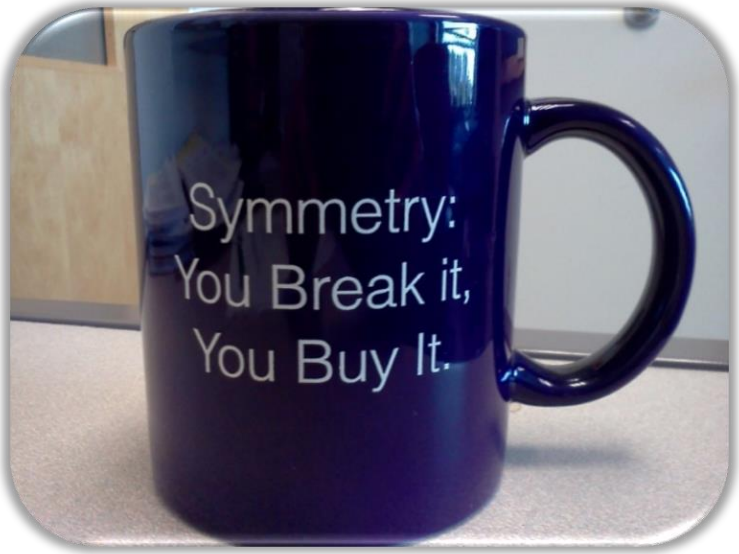
W. Detmold and C. Lin, Phys. Rev. D73 (2006) 014501

THE HOPE COLLABORATION

Z. Davoudi and M. J. Savage, Phys. Rev. D86 (2012) 054505

A. Shindler, arXiv:2311.18704

Lattice Structure Limitation



§ Lattice calculations rely on operator product expansion, only provide moments

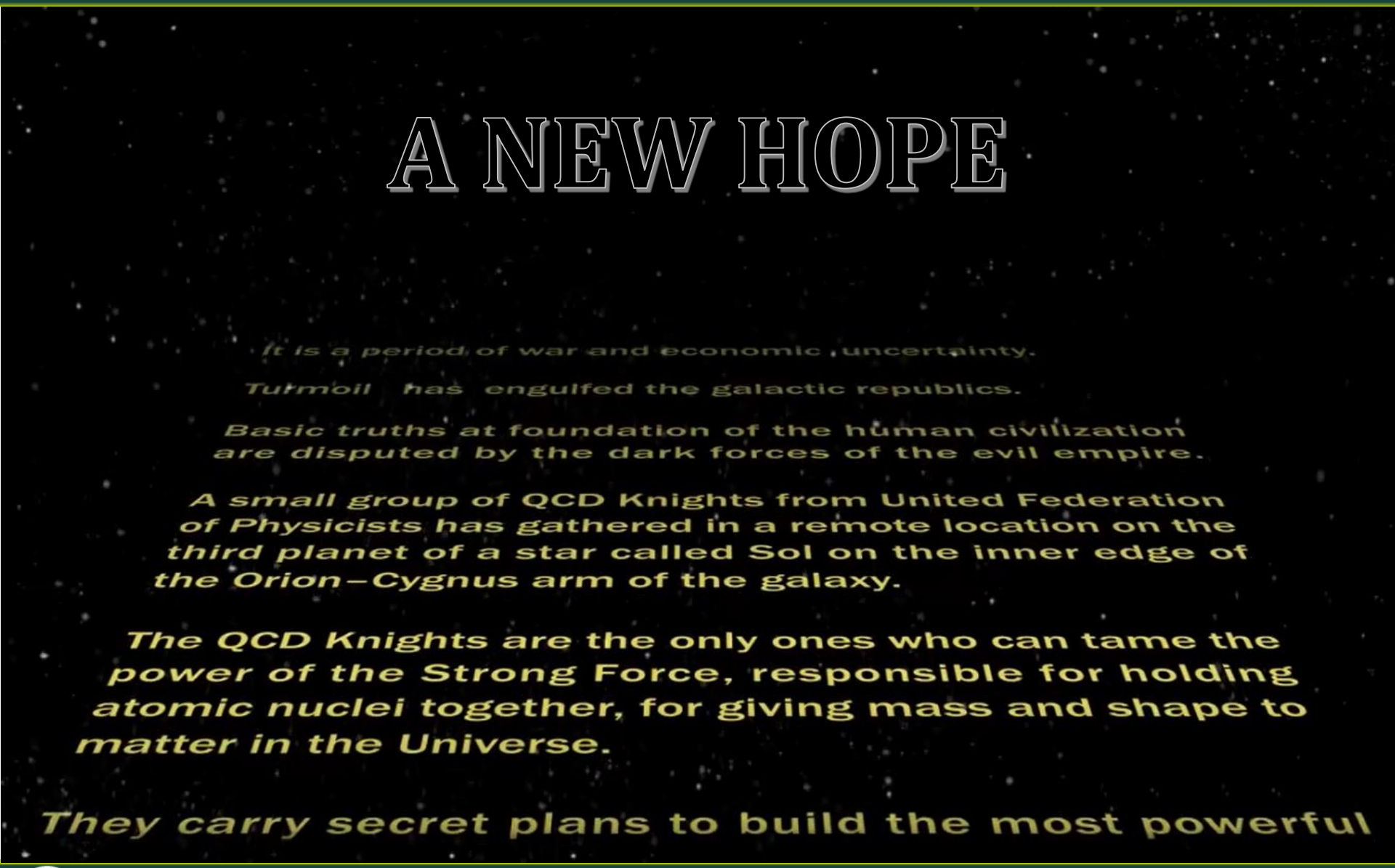
$$\langle x^{n-1} \rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

§ Longstanding obstacle!

- Holy grail of structure calculations

§ Applies to many structure quantities:

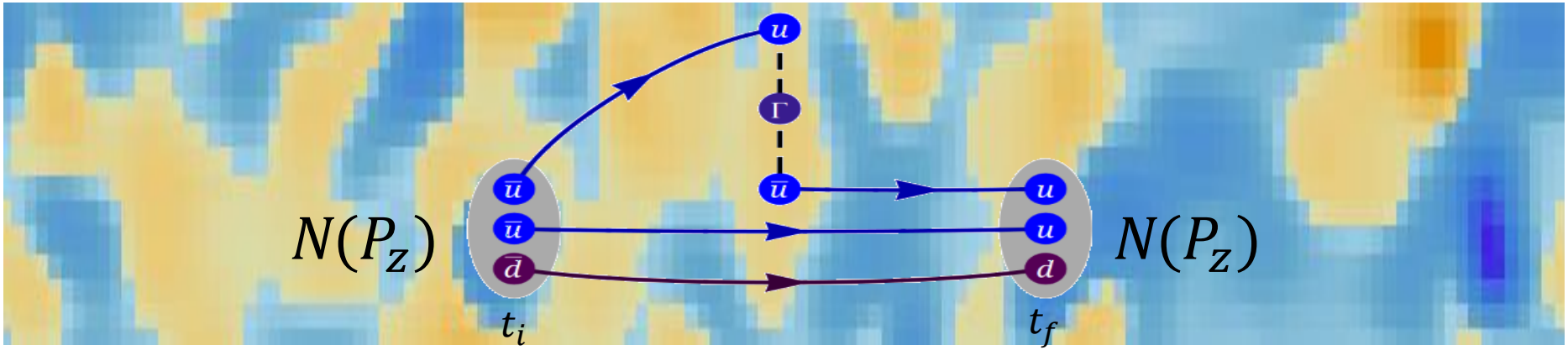
- Parton distribution functions (PDF)
- Generalized parton distributions (GPD)
- Transverse-momentum distributions (TMD)
- ⋮


A NEW HOPE
It is a period of war and economic uncertainty.
Turmoil has engulfed the galactic republics.
Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.
A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.
The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.
They carry secret plans to build the most powerful

Lattice Parton Method

§ Large-momentum effective theory (LaMET)/quasi-PDF

(X. Ji, 2013; See 2004.03543 for review)



§ Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\Gamma \exp\left(-ig\int_0^z dz' A_z(z')\right)\psi(0) \right| P \right\rangle$$

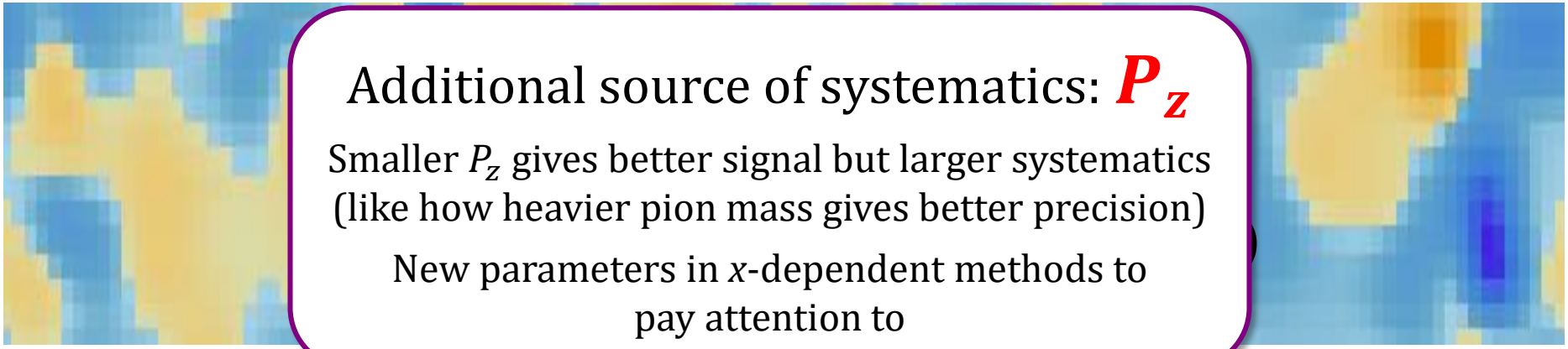
§ Recover true distribution (take $P_z \to \infty$ limit)

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y},\frac{\mu}{P_z}\right) \boldsymbol{q(y,\mu)} + \mathcal{O}\left(\frac{M_N^2}{\boldsymbol{P_z^2}}, \frac{\Lambda_{\rm QCD}^2}{(x\boldsymbol{P_z})^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)\boldsymbol{P_z})^2}\right)$$

X. Xiong e.a., 1310.7471; J.-W. Chen e.a., 1603.06664

Lattice Parton Method

§ Large-momentum effective theory (LaMET)/quasi-PDF
(X. Ji, 2013; See 2004.03543 for review)



Additional source of systematics: $\boldsymbol{P_z}$

Smaller P_z gives better signal but larger systematics (like how heavier pion mass gives better precision)

New parameters in x-dependent methods to pay attention to

§ Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\Gamma \exp\left(-ig\int_0^z dz' A_z(z')\right)\psi(0) \right| P \right\rangle$$

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X. Xiong e.a., 1310.7471; J.-W. Chen e.a., 1603.06664

Direct x-Dependent Structure

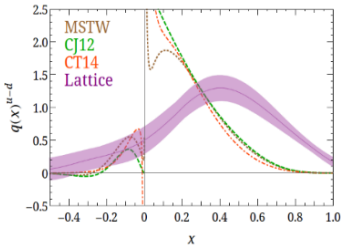
§ Longstanding obstacle to lattice calculations!

- **Quasi-PDF**/large-momentum effective theory (LaMET) (X. Ji, 2013; See 2004.03543 for review)
- **Pseudo-PDF** method: differs in FT (A. Radyushkin, 2017)
- Lattice cross-section method (**LCS**) (Y Ma and J. Qiu, 2014, 2017)
- Compton amplitude method (A.J. Chambers et al., 1703.01153)
- Hadronic tensor currents (Liu et al., hep-ph/9806491, … 1603.07352)
- Euclidean correlation functions (RQCD, 1709.04325)

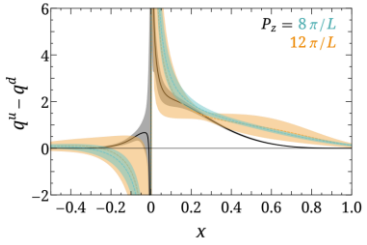
Lattice Parton Calculations

§ Physics-quantity milestones

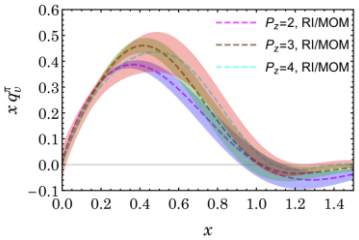
First unpol. lattice PDF



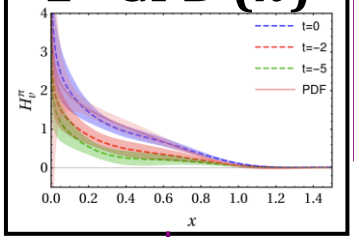
First PDFs at $M_\pi^{\rm phys}$



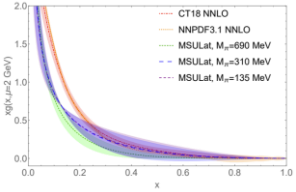
Pion v-PDF



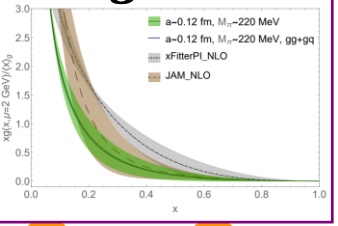
1st GPD (π)



N g-PDF

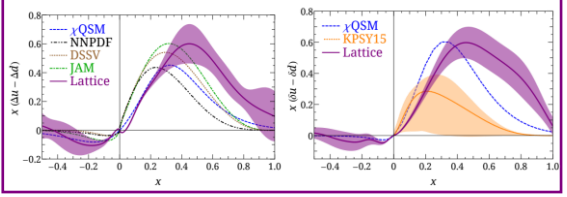


π g-PDF

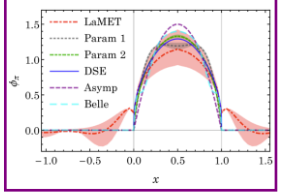


2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023

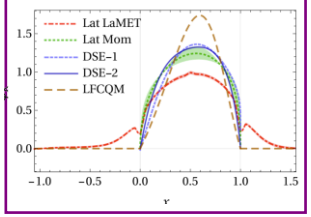
Pol. PDFs and mass corrections



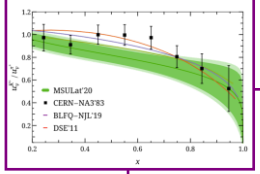
Pion DA



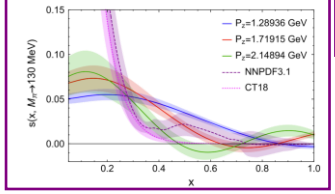
Kaon DA



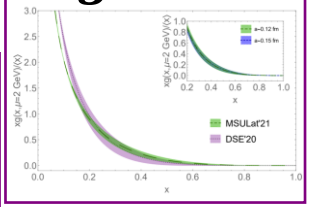
K PDF



s,c PDF



Kaon g-PDF

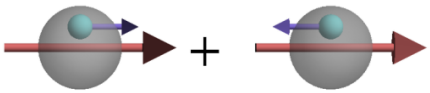


HL, Few Body Syst. 64 (2023) 3, 58

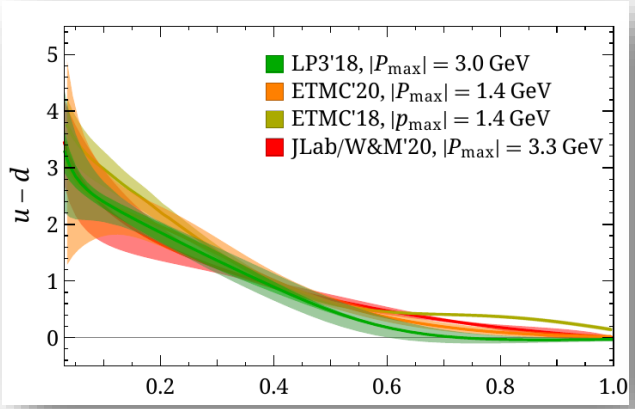
Lattice Example Results

§ Summary of PDF results at physical pion mass

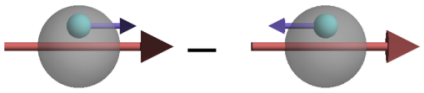
unpolarized



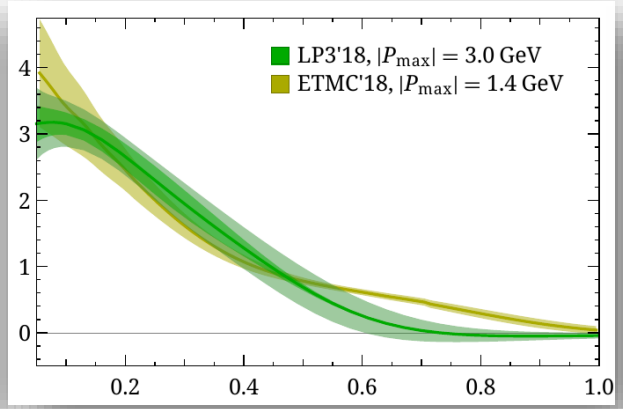
$u(x) - d(x)$



longitudinally polarized

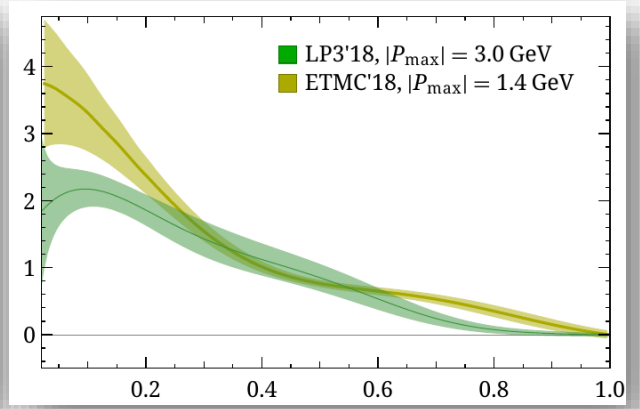


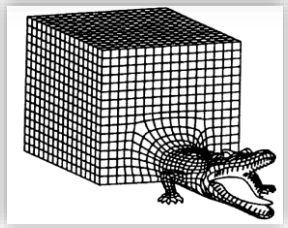
$\Delta u(x) - \Delta d(x)$



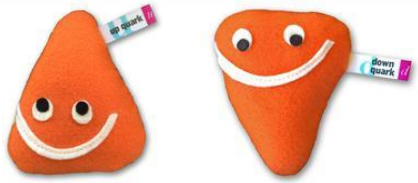
transversely polarized

$\delta u(x) - \delta d(x)$





Finite volume,
Discretization,
…



2006.08636 (PDFLattice2019)

Lattice Example Results

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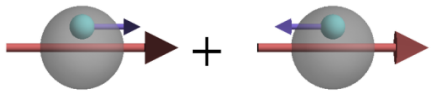
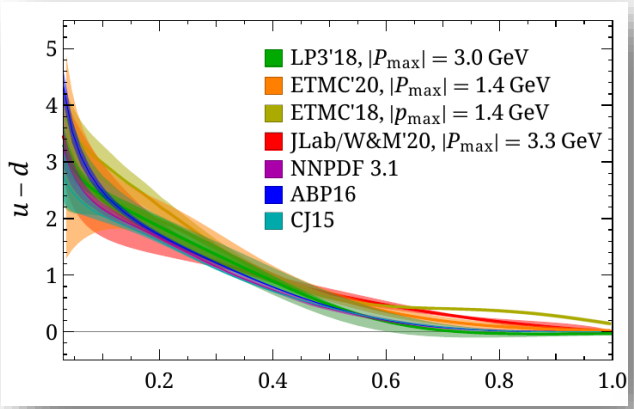
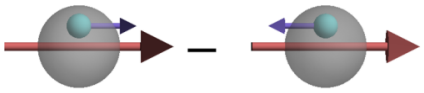
unpolarized



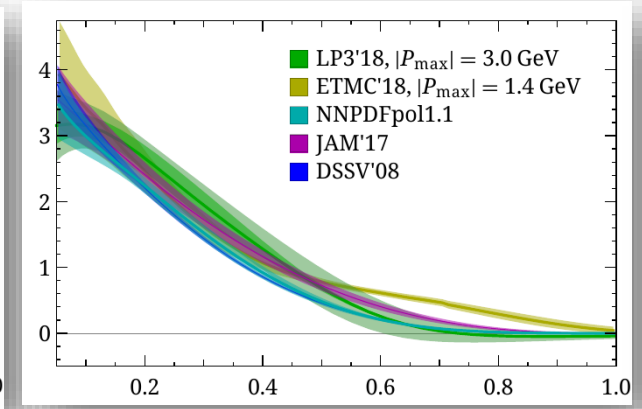
$u(x) - d(x)$



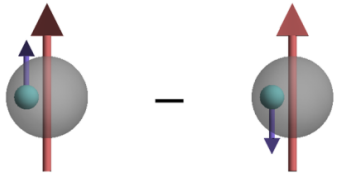
longitudinally polarized



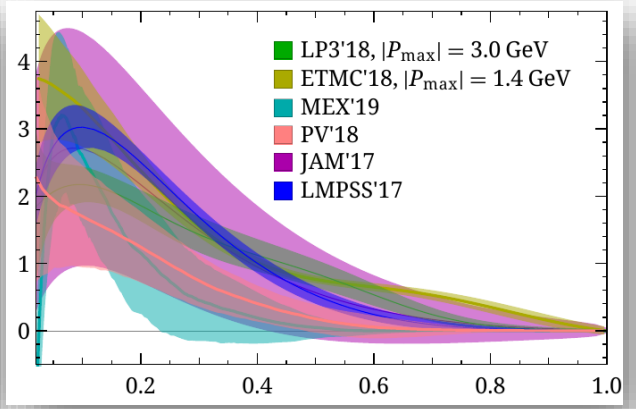
$\Delta u(x) - \Delta d(x)$

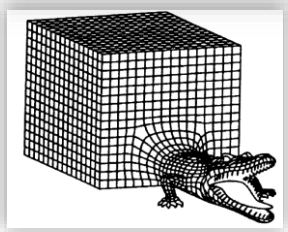


transversely polarized



$\delta u(x) - \delta d(x)$





Finite volume,
Discretization,
…

2006.08636 (PDFLattice2019)

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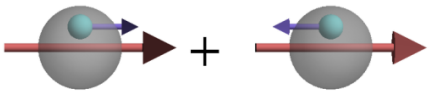
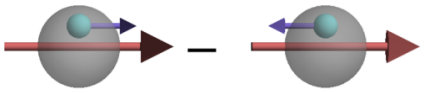
unpolarized



$u(x) - d(x)$

longitudinally polarized



$\Delta u(x) - \Delta d(x)$

transversely polarized

$\delta u(x) - \delta d(x)$

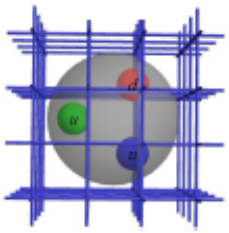
§ Complementary lattice inputs for best PDFs

Theory Input

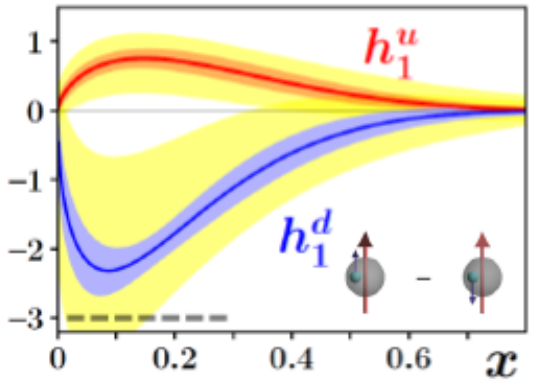
Exp't Input

Global Analysis of PDFs

+



Work has been made in this direction



Phys. Rev. Lett. 120, 152502 (2018)

Isovector PDFs Update

§ Nucleon isovector PDF calculated directly at **physical pion mass**

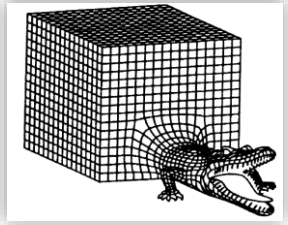
- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

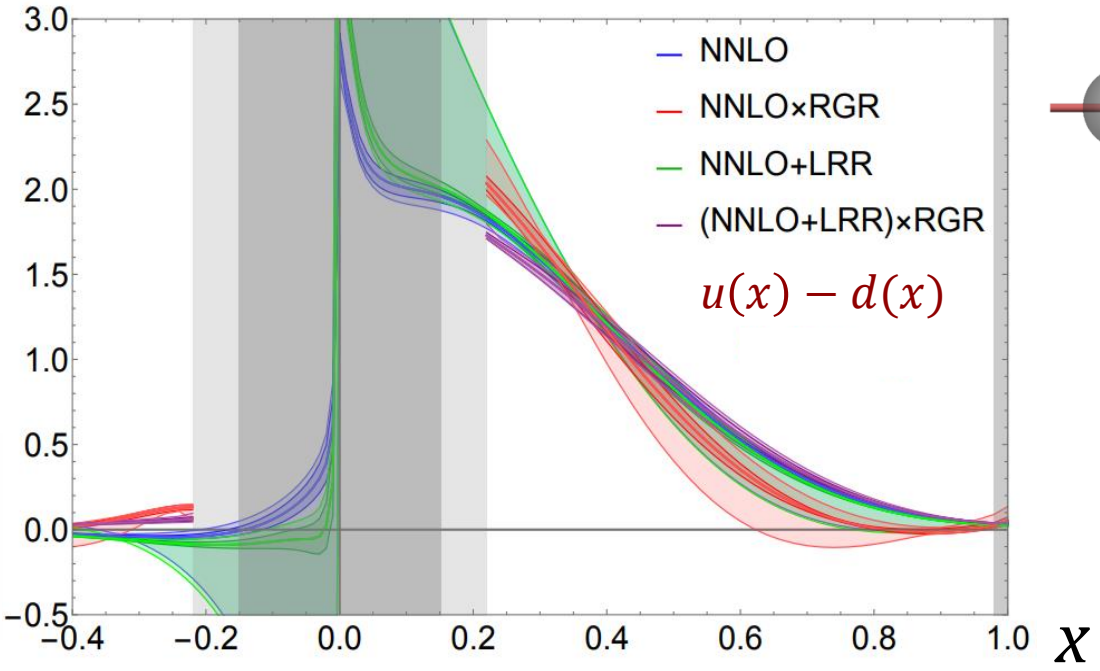
R. Zhang, et. al.
PLB 844, 138081 (2023)

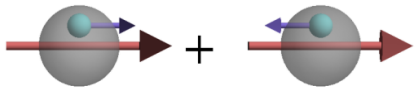
- N_f= 2+1+1 clover/HISQ, $a \approx 0.09$ fm, $P_z \approx 2$ GeV

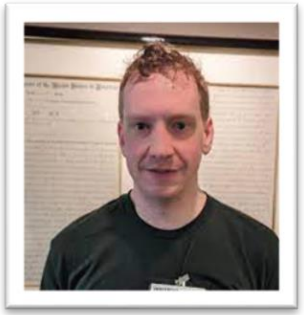
J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

Wanted PDFs, GPDs, etc…









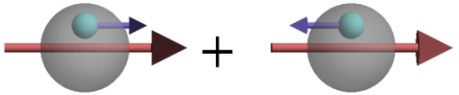
P: Jack Holligan

Continuum PDF

§ Nucleon PDFs using quasi-PDFs in the continuum limit

- Lattice details: clover/2+1+1 HISQ (MSULat)
  - $a \approx \{0.06, 0.09, 0.12\}$ fm
  - $M_\pi \in \{135, 220, 310\}$-MeV pion
  - $M_\pi L \in \{3.3, 5.5\}$
  - $P_z \approx 2$ GeV

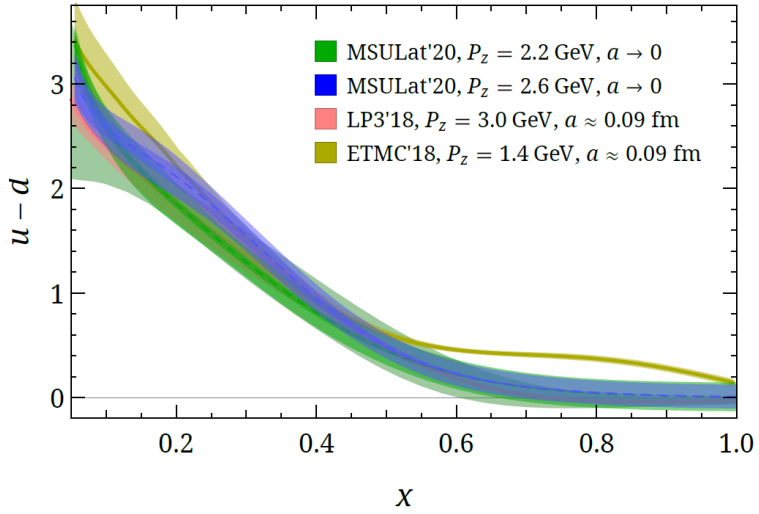
2011.14971, HL et al. (MSULat)

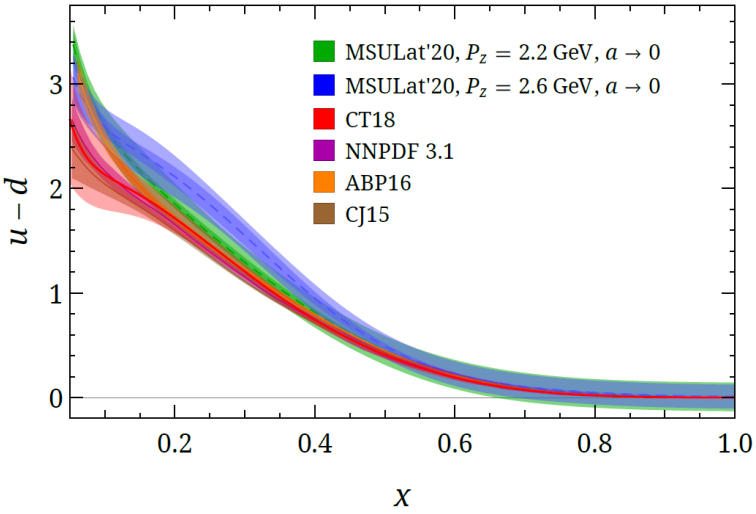


$u(x) - d(x)$

- Naïve extrapolation to physical-continuum limit

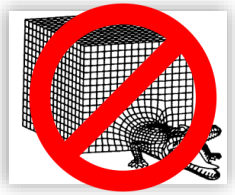
Wanted PDFs, GPDs, etc…





Continuum PDF

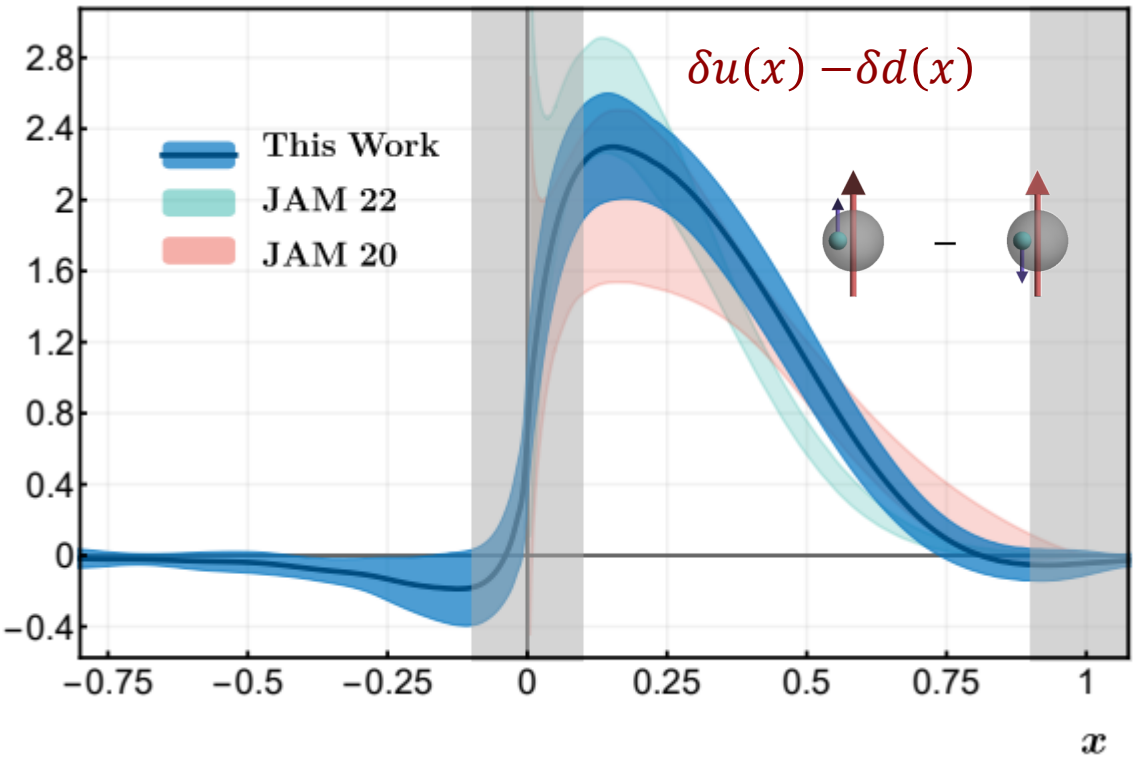
§ Nucleon PDFs using quasi-PDFs in the continuum limit



- Lattice details: clover/2+1 clover (LPC)
  - a ≈ {0.49, 0.64, 0.85, 0.98} fm,
  - $M_\pi \in [222, 354]$-MeV pion,
  - $M_\pi L \in [3.9, 8.1]$
  - $P_z \in [1.8, 2.8]$

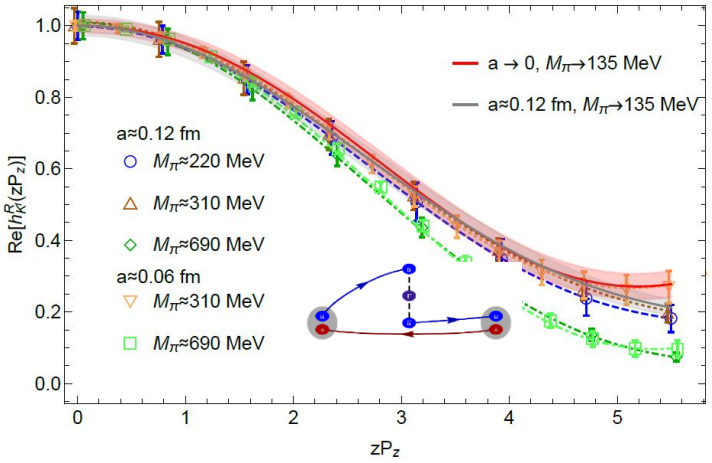
F. Yao et al (LPC), 2208.08008

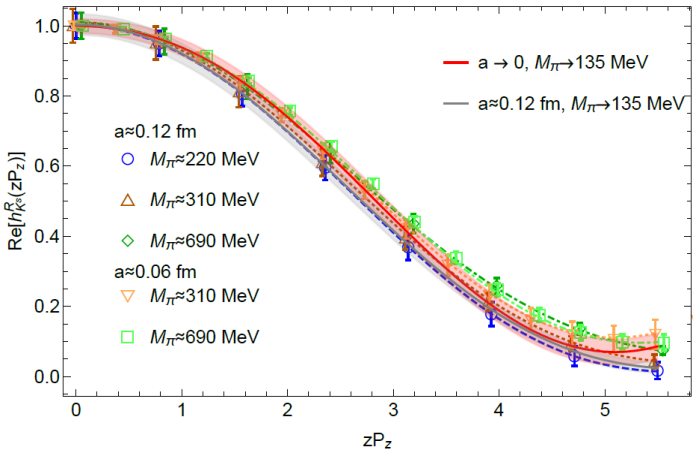
Wanted
PDFs, GPDs,
etc…



Meson Valence-quark PDFs

§ Pion/Kaon PDFs using quasi-PDF in the continuum limit

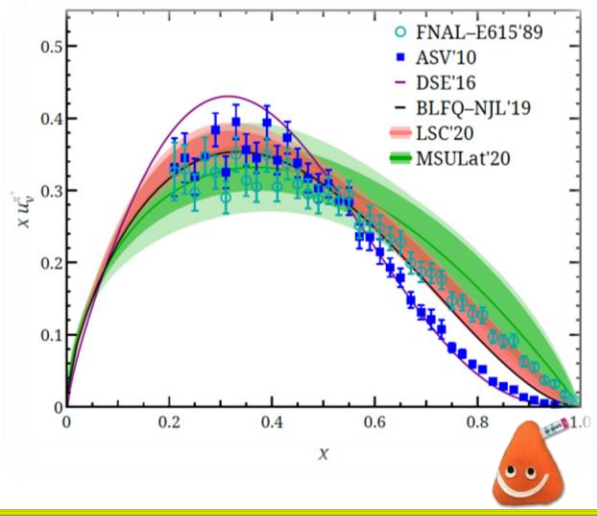


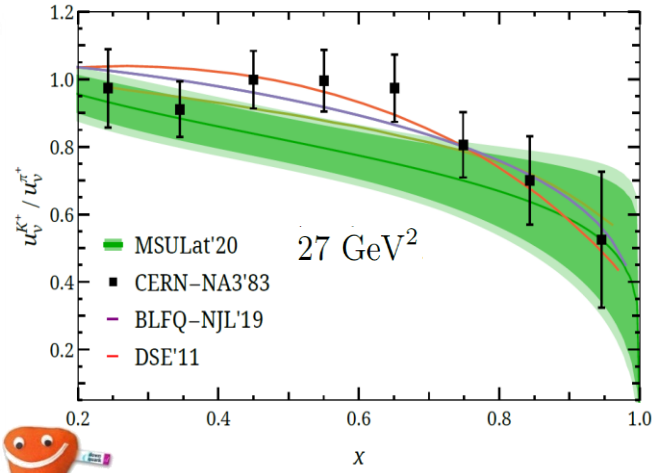


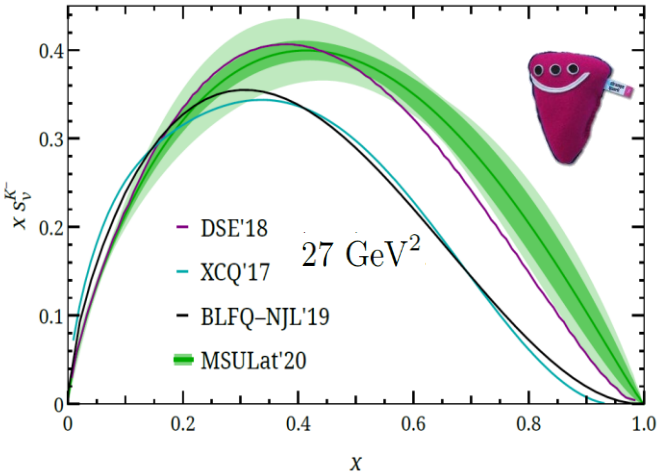
Quantities that can be calculated on the lattice

MSULat, 2003.14128

Wanted PDFs, GPDs, etc…







Valence-quark PDFs Update

§ Pion PDFs calculated directly at physical pion mass

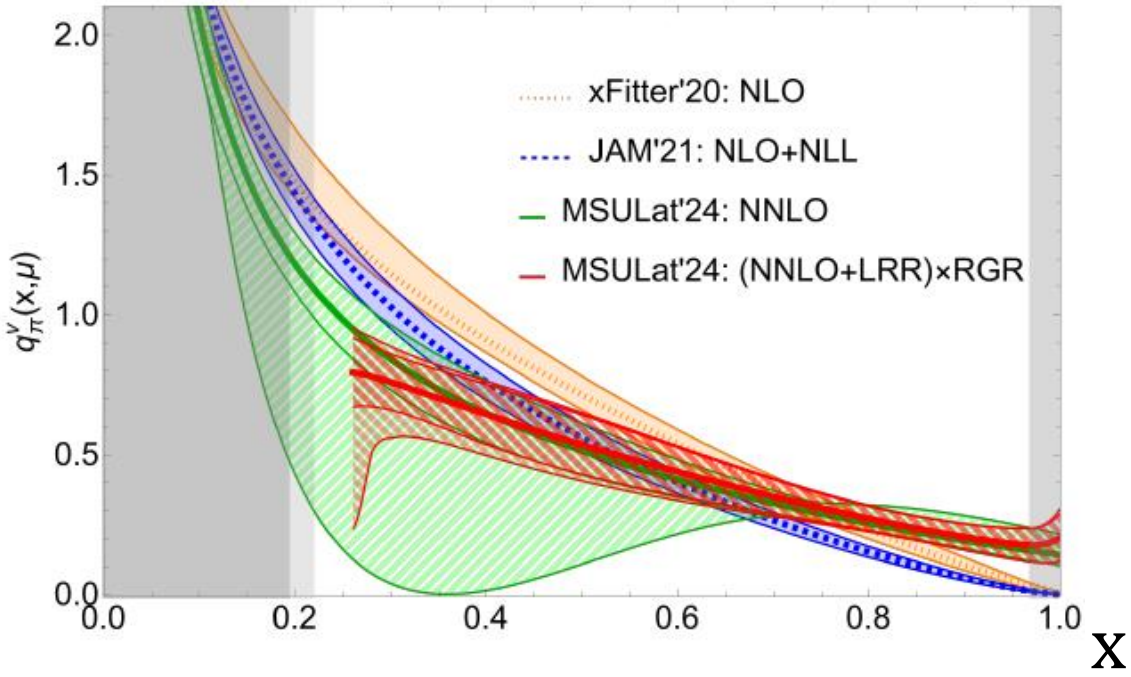
- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

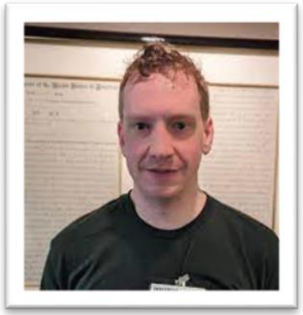
R. Zhang, et. al.
PLB 844, 138081 (2023)

- N_f=2+1+1 clover/HISQ, a~0.09 fm

J. Holligan, HL (MSULat), 10.1088/1361-6471/ad3162

Wanted PDFs, GPDs, etc…





P: Jack Holligan

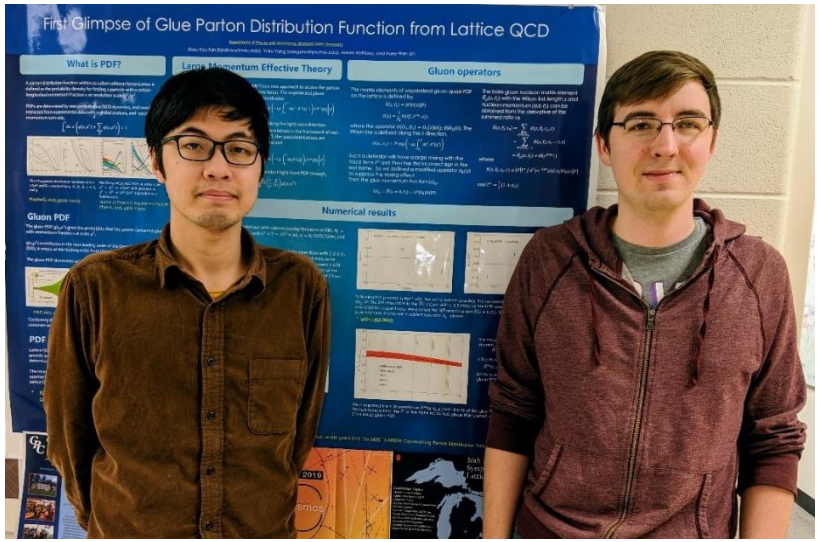
Gluon PDFs

Nucleon Gluon PDF (2018)

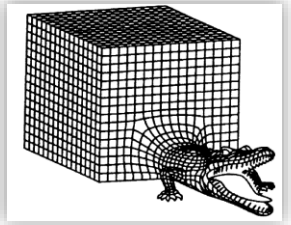
§ Pioneering first glimpse into gluon PDF using LaMET

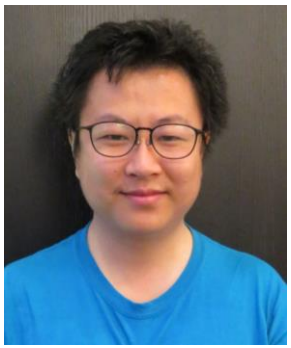
- Lattice details: overlap/2+1DWF, 0.16fm, 340-MeV sea pion mass
- Promising results using coordinate-space comparison, but signal does not go far in z
- Hard numerical problem to be solved

Fan et al, Phys.Rev.Lett. 121, 242001 (2018)

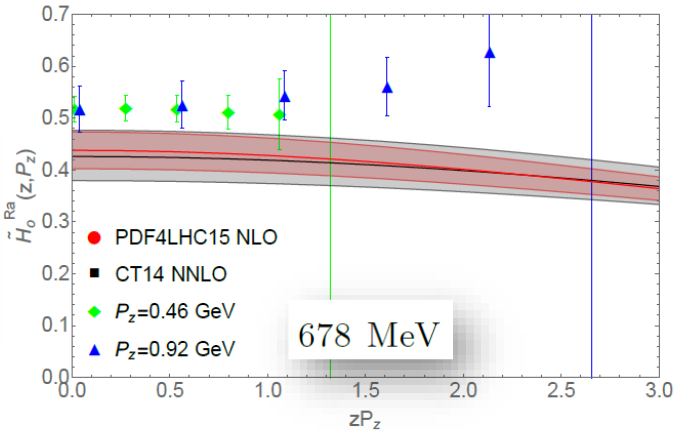


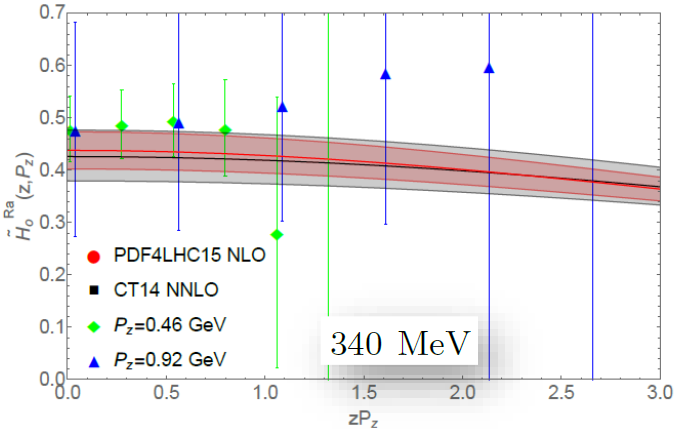
G: Zhouyou Fan G: Adam Antony





P: Yi-Bo Yang





iCER@MSU is crucial for earlier code development and completion of this work

Nucleon Gluon PDF (2020)

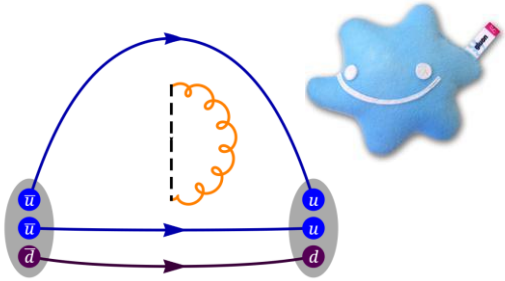
§ Gluon PDF using pseudo-PDF

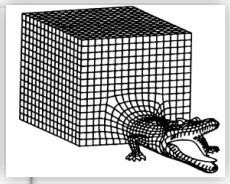
- Lattice details: clover/2+1+1 HISQ 0.12 fm,

  310-MeV sea pion

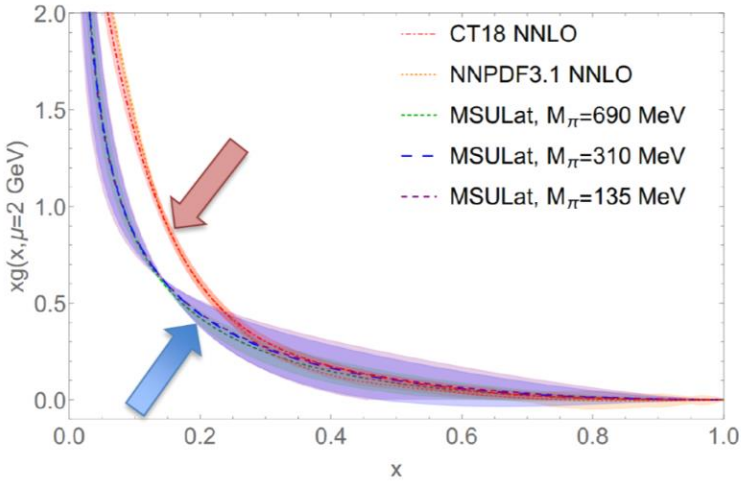
- Study strange/light-quark

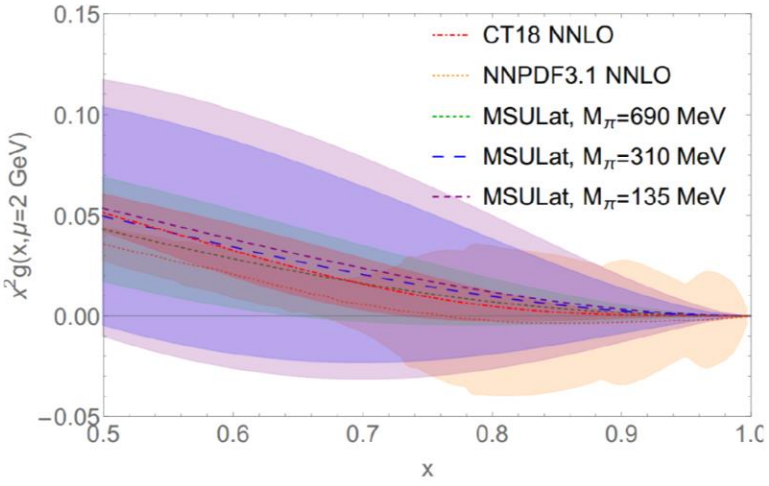
Z. Fan. et al (MSULat), 2007.16113

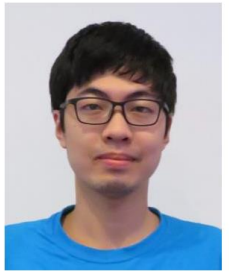




The comparison of the reconstructed unpolarized gluon PDF from the function form with CT18 NNLO and NNPDF3.1 NNLO gluon unpolarized PDF at $\mu = 2\ GeV$ in the $\overline{MS}$ scheme.







G: Zhouyou Fan

Slide by Zhouyou Fan@DNP2020

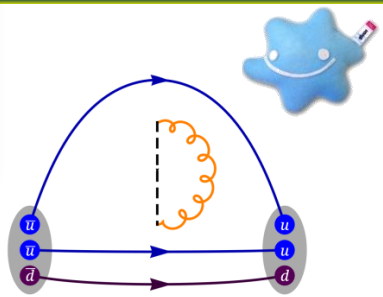
Gluon PDF in Nucleon

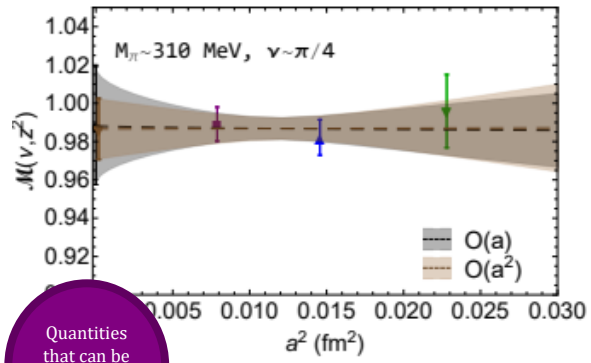
§ Continuum Gluon PDF w/ pseudo-PDF

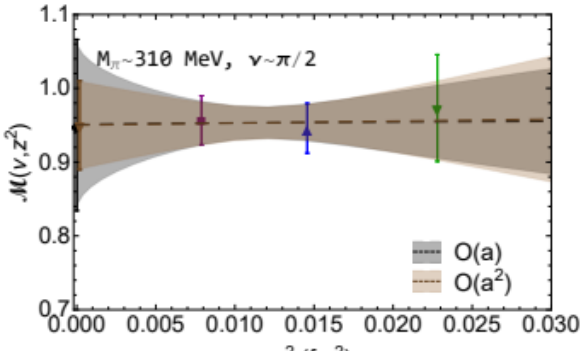
- 2+1+1 HISQ {0.09, 0.12, 0.15} fm

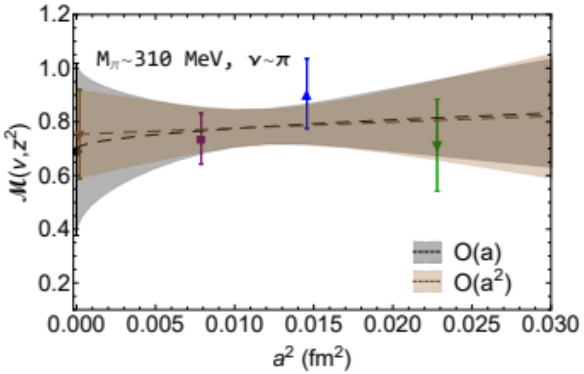
  [220, 310, 700]-MeV pion, 10^5–10^6 statistics

Z. Fan et al (MSULat), 2210.09985

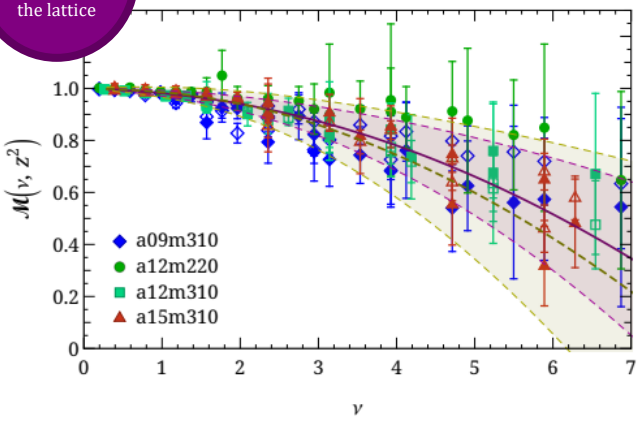


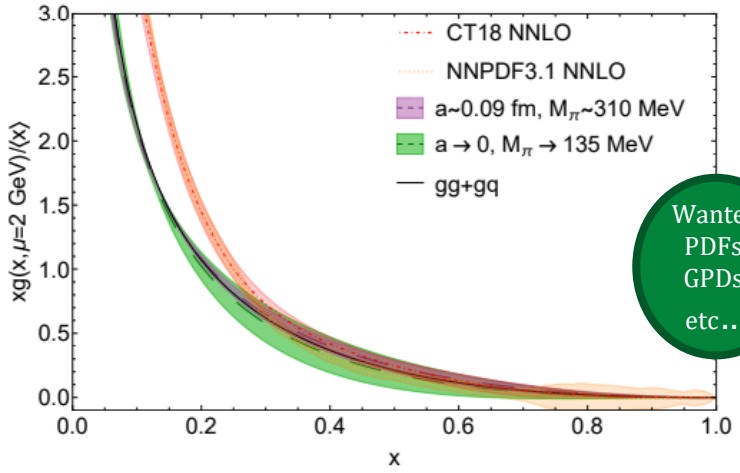






Quantities that can be calculated on the lattice



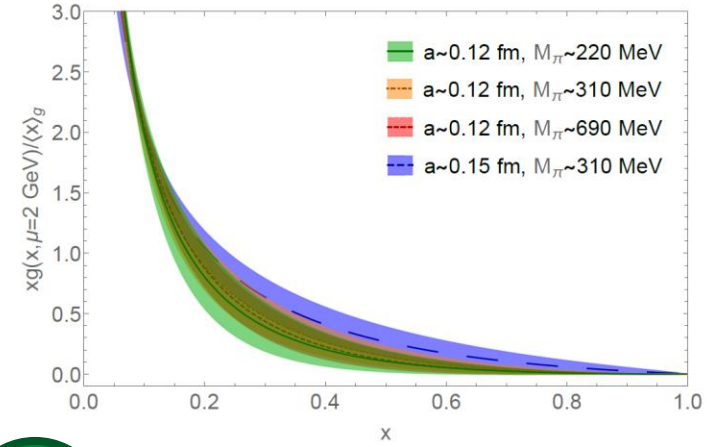


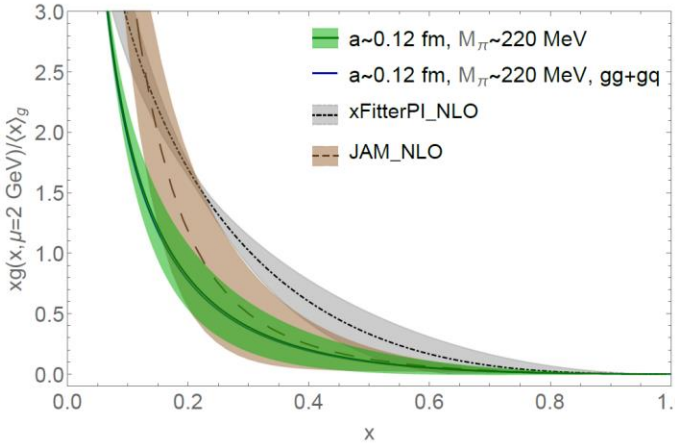
Wanted PDFs, GPDs, etc…

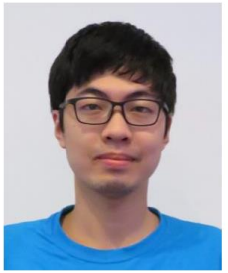
G: Bill Good

Meson Gluon PDFs

§ First pion and kaon gluon PDFs $g(x)/\langle x\rangle$ using pseudo-PDF



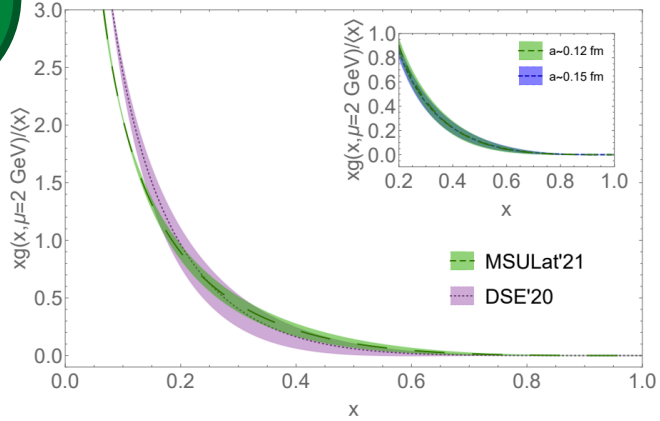




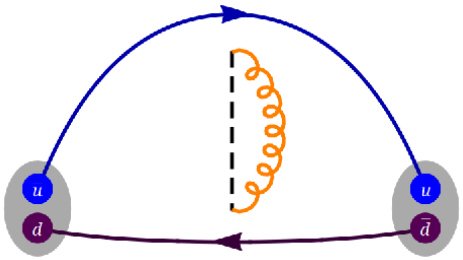
G: Zhouyou Fan

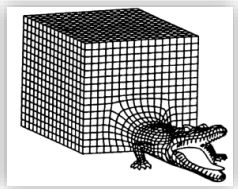
2104.06372, Fan et al. (MSULat); 2112.03124, Salas-Chavira et al. (MSULat)

Wanted PDFs, GPDs, etc…



G: Alejandro Salas-Chavira





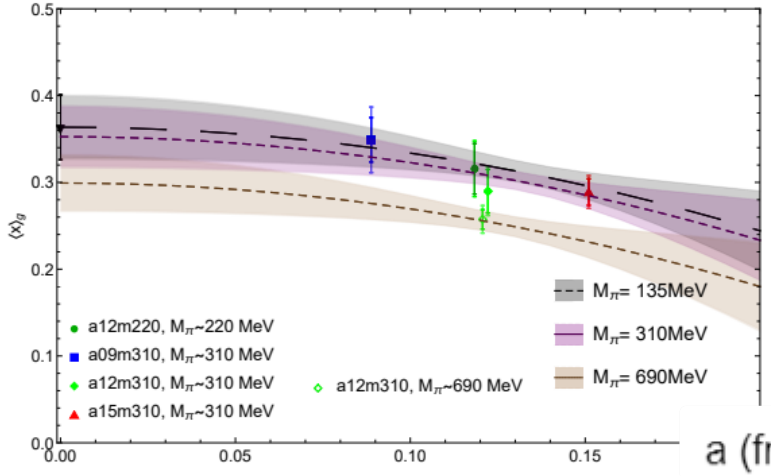
finite-volume, discretization, heavy quark mass, …

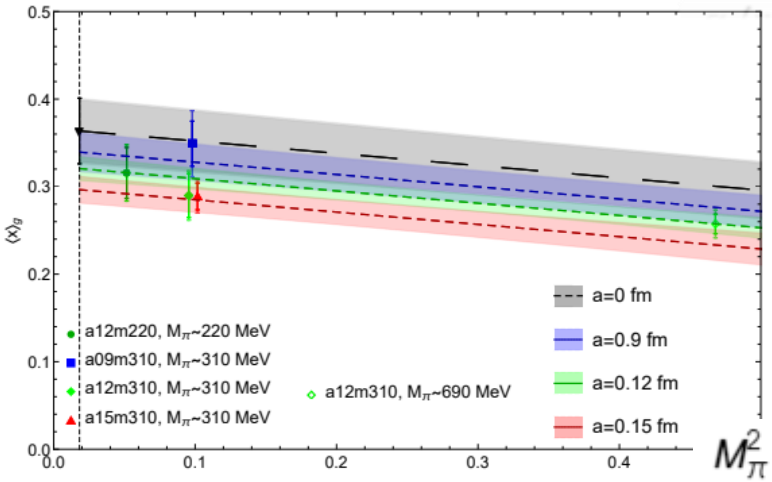
§ What does lattice QCD say about $g(x)$?

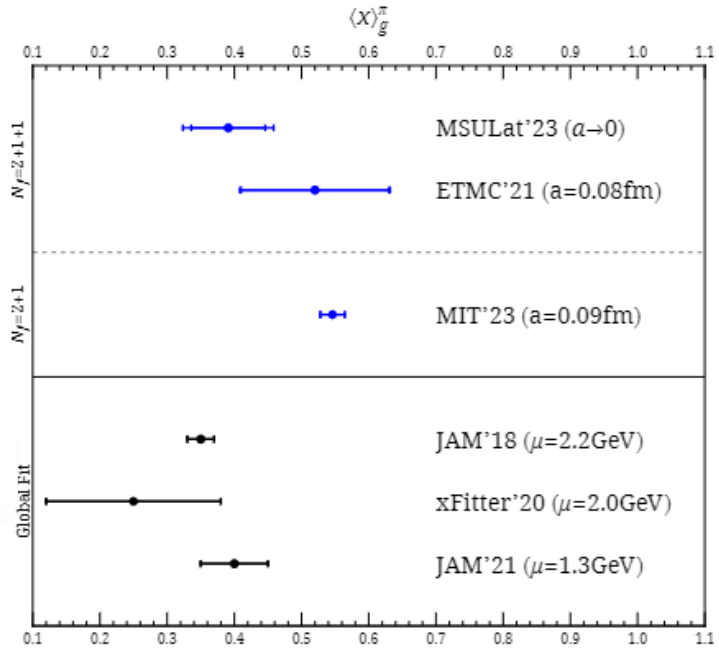
Pion Gluon PDF Update

§ Study discretization systematic in $\langle x \rangle_{\{\pi,g\}}$

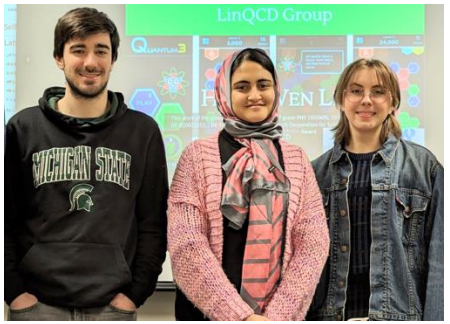
- Lattice details: clover/HI{0.15, 0.12, 0.09} fm







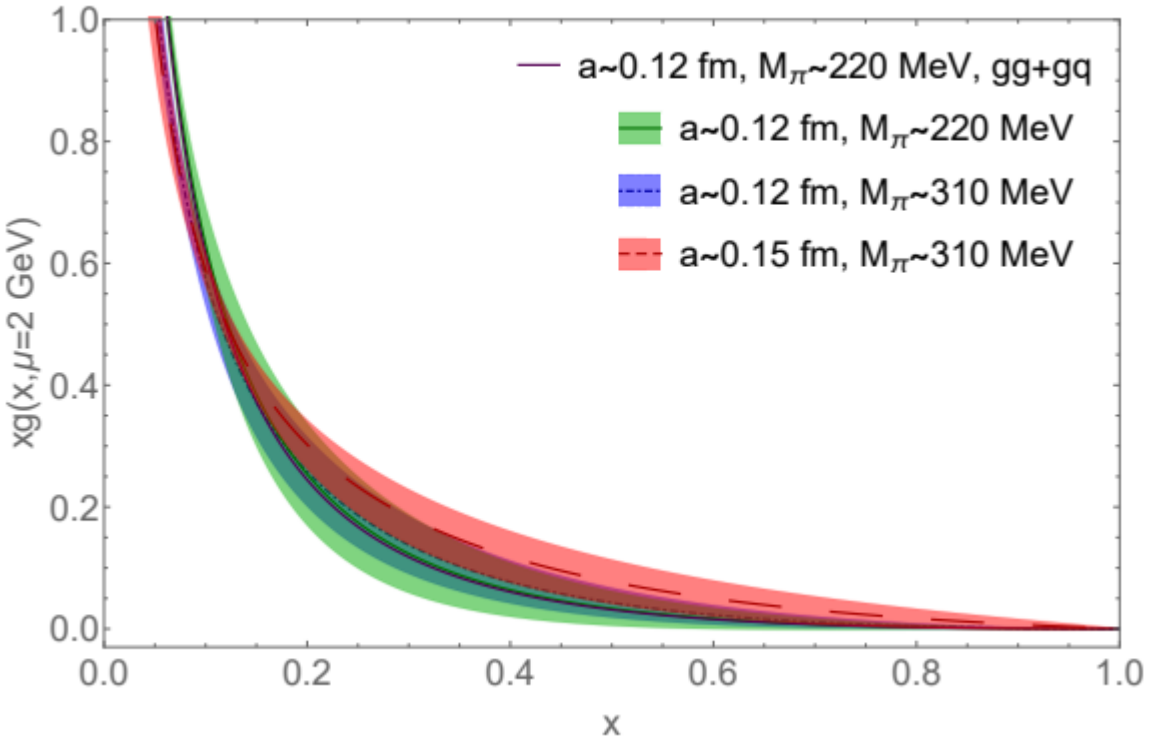
2310.12034, Good et al. (MSULat)



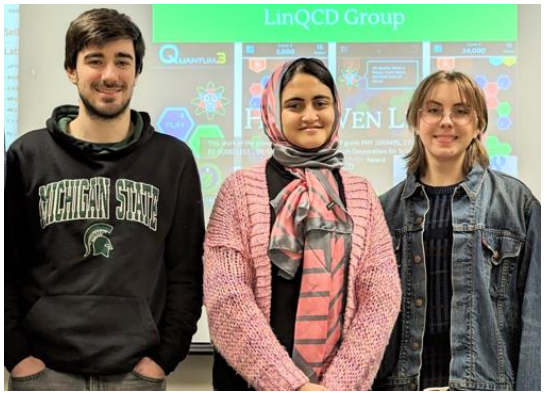
Pion Gluon PDF Update

§ Back to Pion gluon PDF $g(x)$

- Update previous calculated $g(x)/\langle x \rangle$ in 2021



2310.12034, Good et al. (MSULat)



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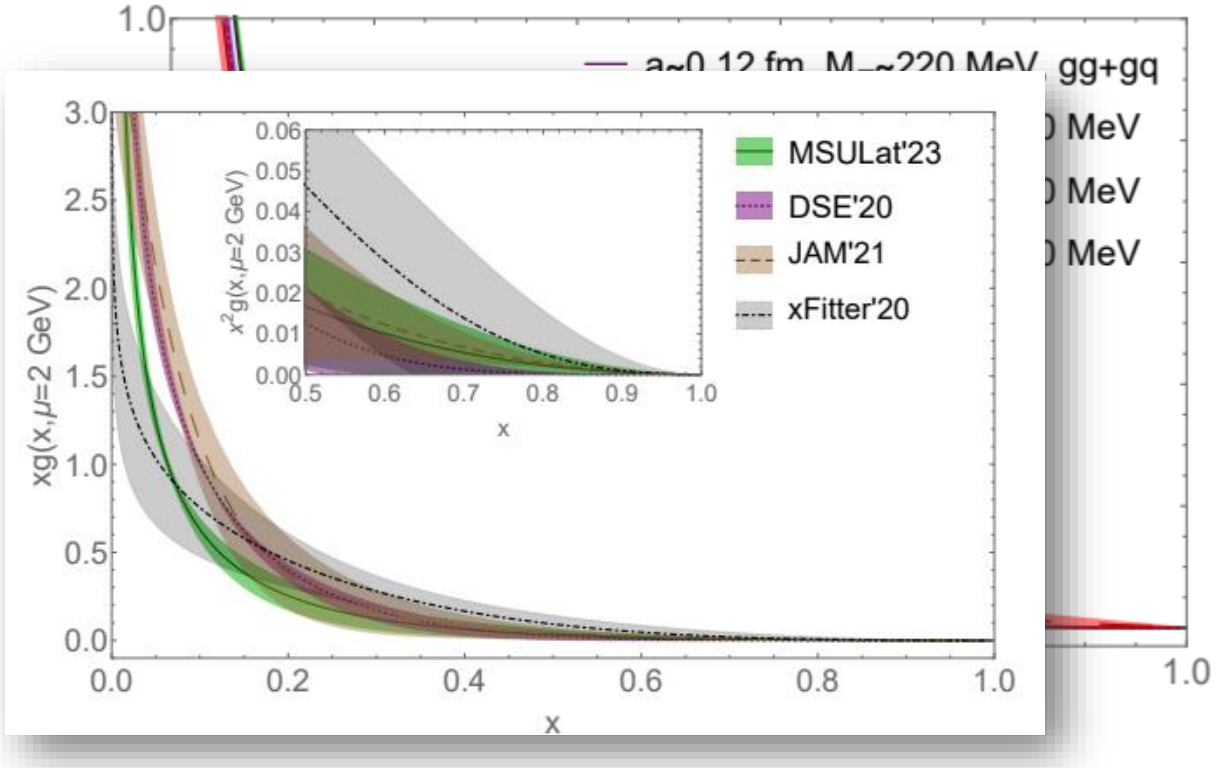
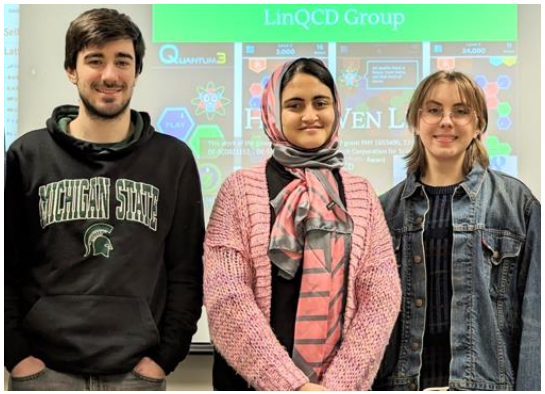
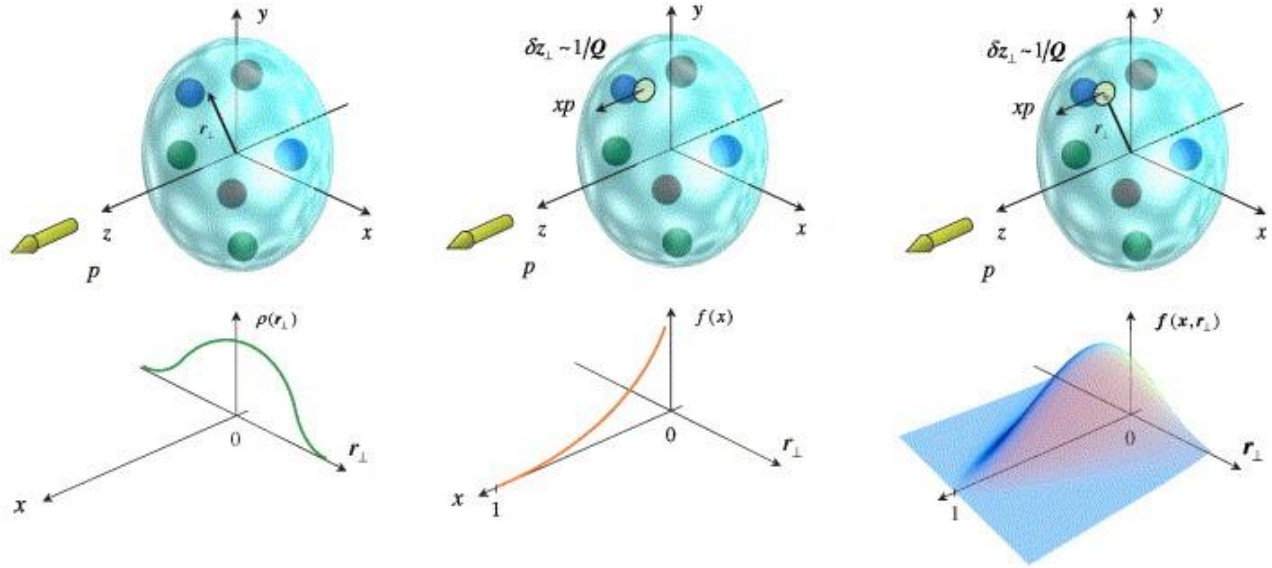


2310.12034, Good et al. (MSULat)



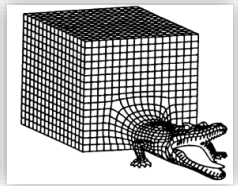
Bjorken-x Dependent GPDs



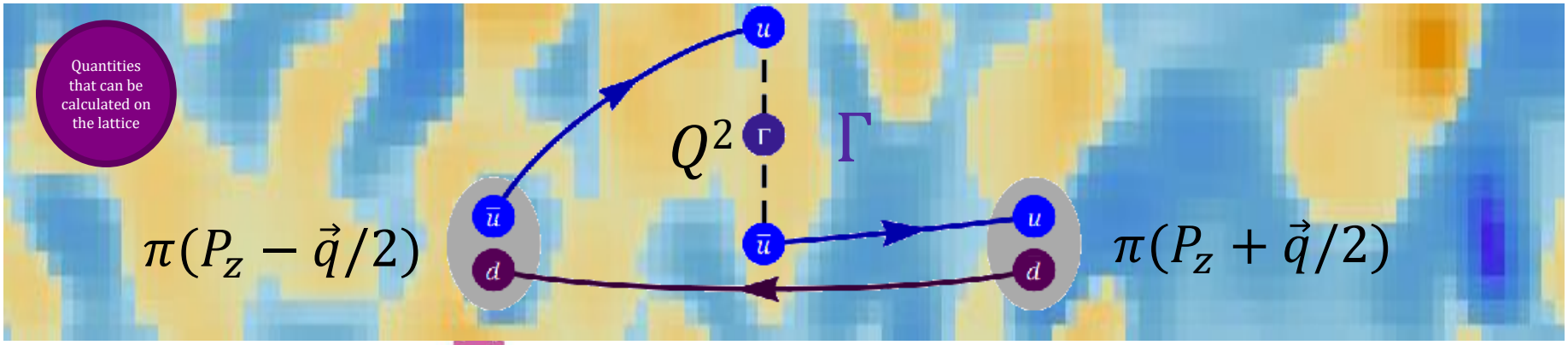
Picture from A. Belitskya and A Radyushkin, Physics Report, 416 (2015)

Generalized Parton Distributions

Single-ensemble result



finite-volume,
discretization,
heavy quark mass,
…



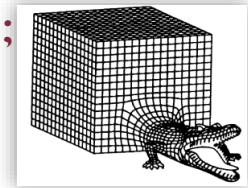
First Lattice GPDs

§ First glimpse into pion GPD using Quasi-PDF/LaMET

- Lattice details: clover/HISQ, **0.12fm**, **310-MeV** pion mass

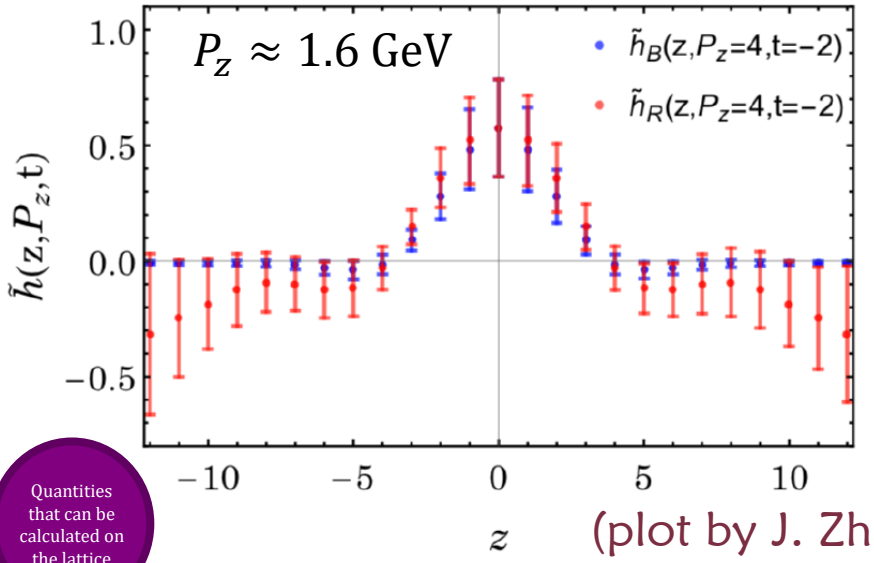
$P_z \approx 1.3, 1.6$ GeV

MILC, Phys. Rev. D, 82 (2010), 074501;
Phys. Rev. D, 87 (2013), 0545056

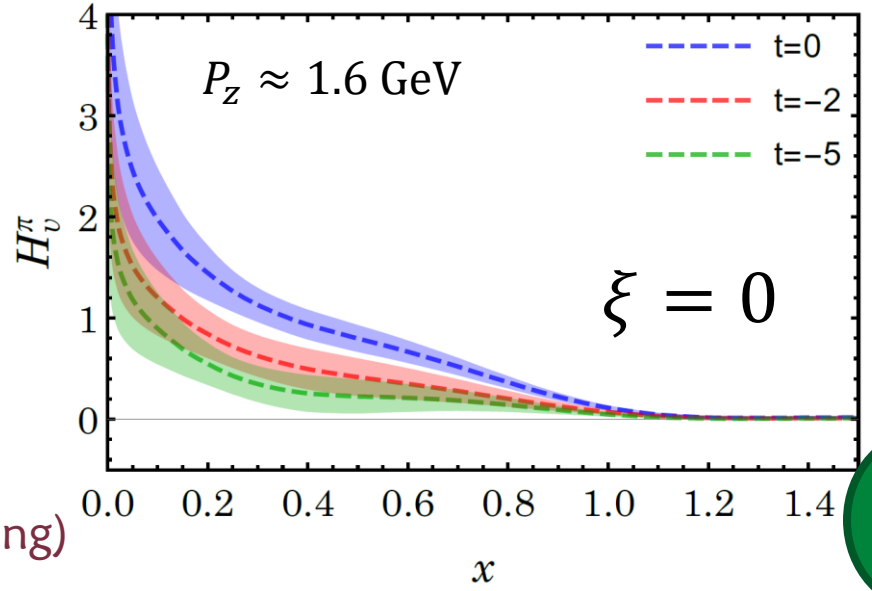


J. Chen, HL, J. Zhang, 1904.1237;

$$H_q^\pi(x,\xi,t,\mu) = \int \frac{d\eta^-}{4\pi} e^{-ix\eta^- P^+} \left\langle \pi(P+\Delta/2) \left| \bar{q}\left(\frac{\eta^-}{2}\right) \gamma^+ \Gamma \left(\frac{\eta^-}{2}, -\frac{\eta^-}{2}\right) q\left(-\frac{\eta^-}{2}\right) \right| \pi(P-\Delta/2) \right\rangle$$



(plot by J. Zhang)



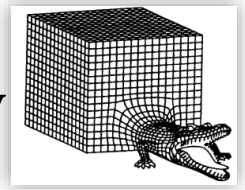
Quantities that can be calculated on the lattice

Wanted PDFs, GPDs, etc…

Valence-Quark Pion GPD

§ Pion GPD (H^π) using quasi-PDFs at physical pion mass

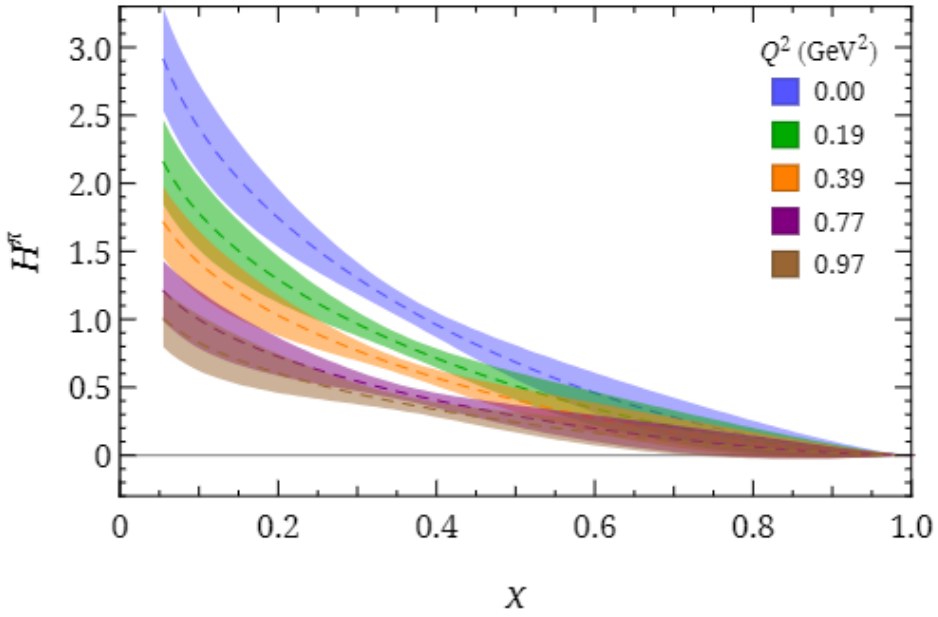
- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV

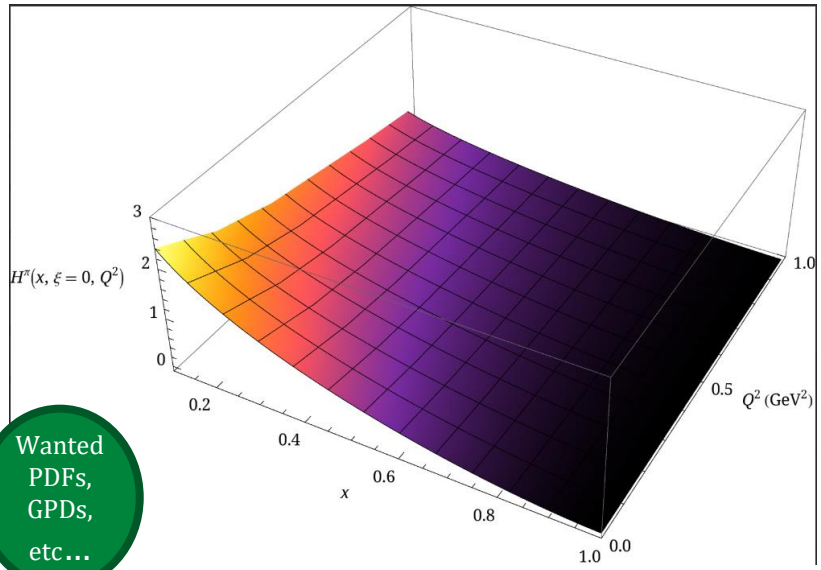


finite-volume, discretization,

- $\xi = 0$ valence-quark Pion GPD results

HL (MSULat), Phys. Lett. B 846 (2023) 138181





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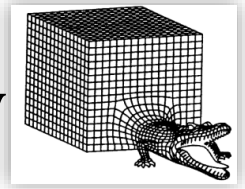
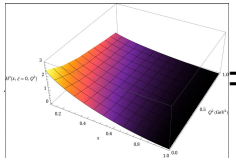
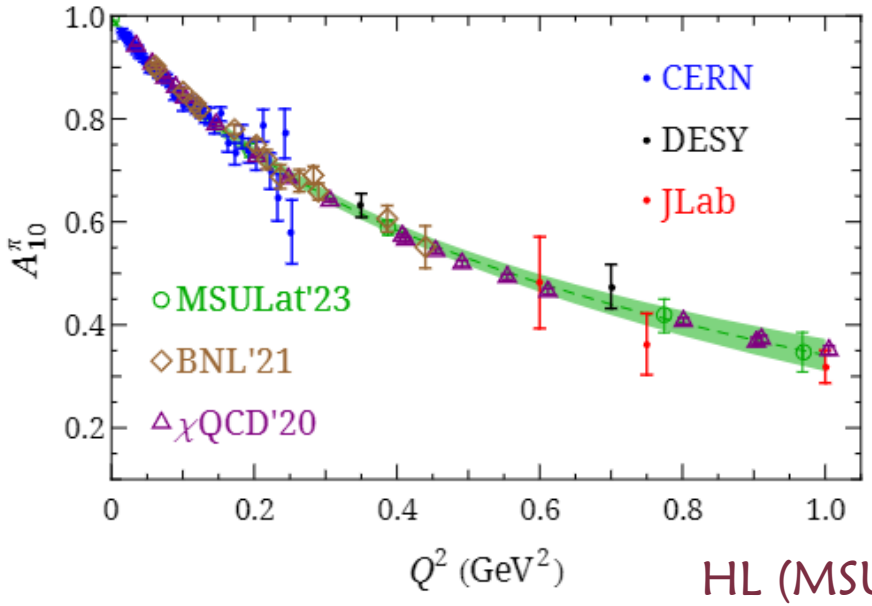


finite-volume, discretization,

- $\xi = 0$ valence-quark Pion GPD results

$$\int_{-1}^{+1} dx\, x^{n-1} \; \text{  } \; = A_{ni}^{\pi}(t)$$

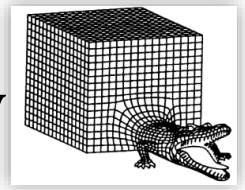


HL (MSULat), Phys. Lett. B 846 (2023) 138181

Pion Tomography

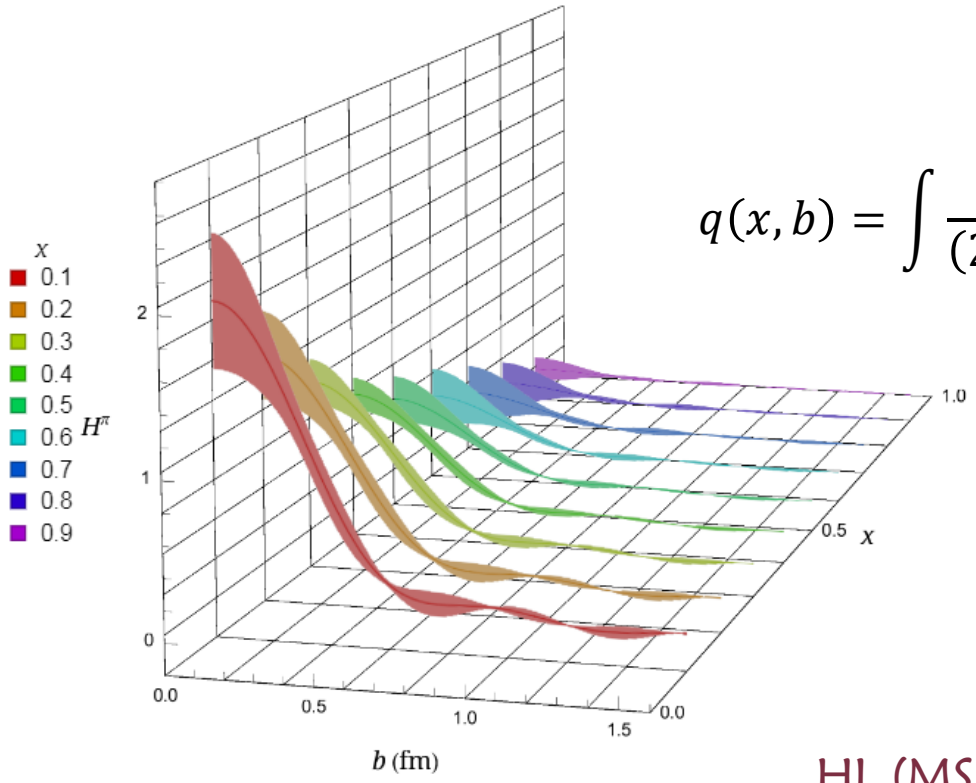
§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV

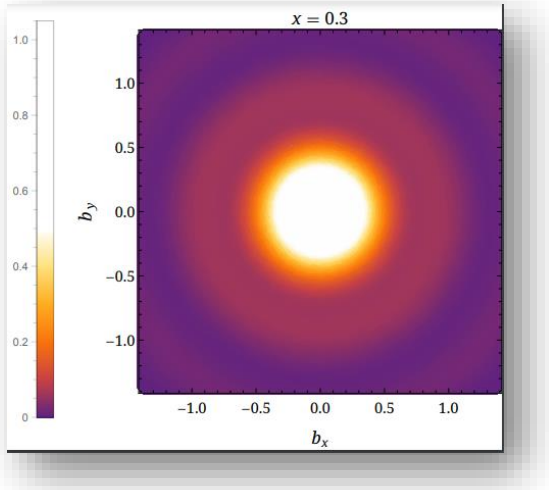


finite-volume, discretization,

- $\xi = 0$ valence-quark Pion GPD results



$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x, \xi = 0, t = -\vec{q}^{\,2}) e^{i\vec{q}\cdot\vec{b}}$$



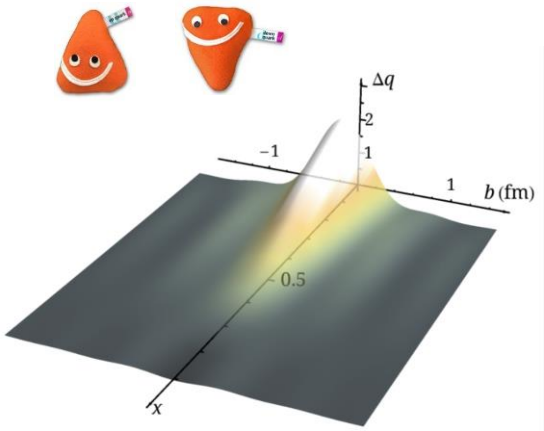
HL (MSULat), Phys. Lett. B 846 (2023) 138181

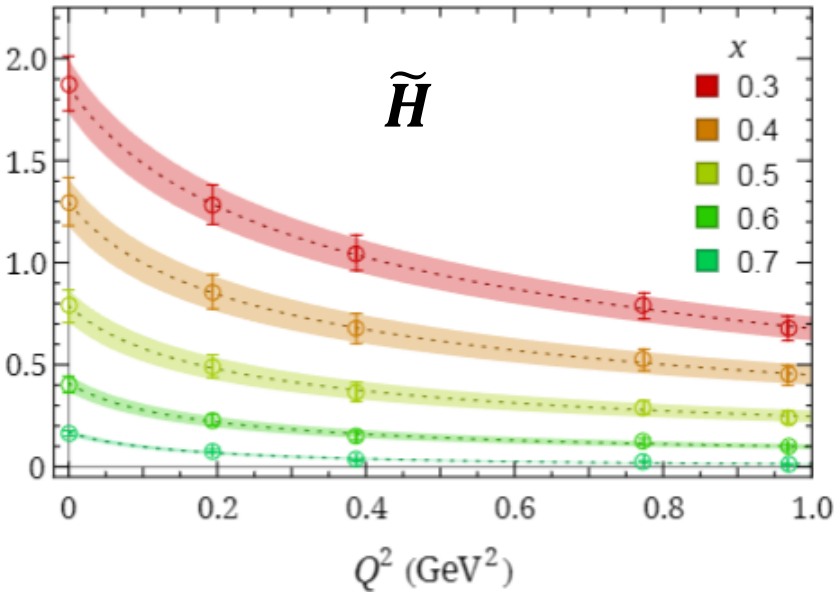
Nucleon Polarized GPDs

§ Helicity GPD ($\widetilde{H}$)using quasi-PDFs at **physical pion mass**

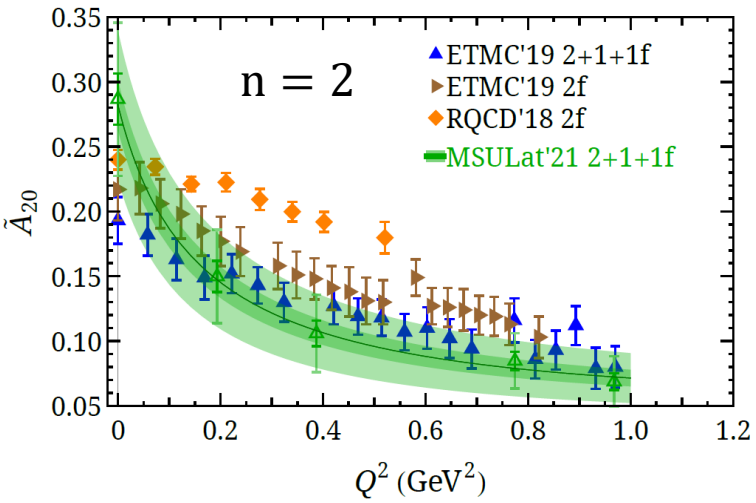
- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon (quasi-)GPD results

HL (MSULat), Phys.Lett.B 824 (2022) 136821





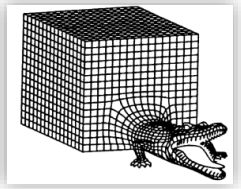
- Take the integral to form moments



Nucleon Tomography

§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV



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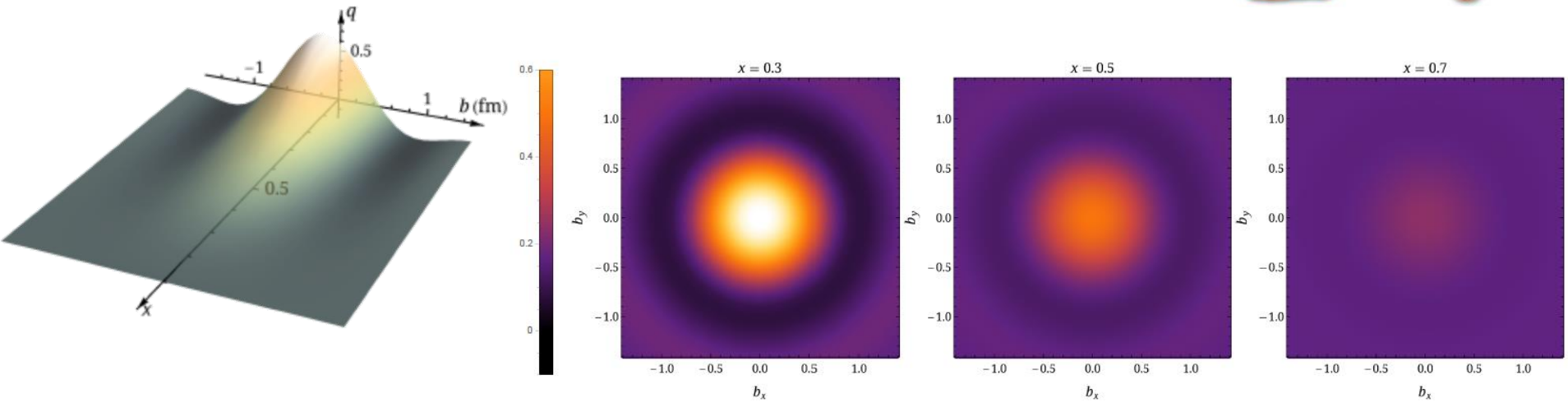


HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Also see work done by ANL/BNL/ETMC, 2209.05373 ,2310.13114

GPD Systematic Update

§ Nucleon isovector GPDs calculated directly at physical pion mass

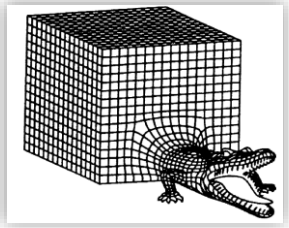
- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

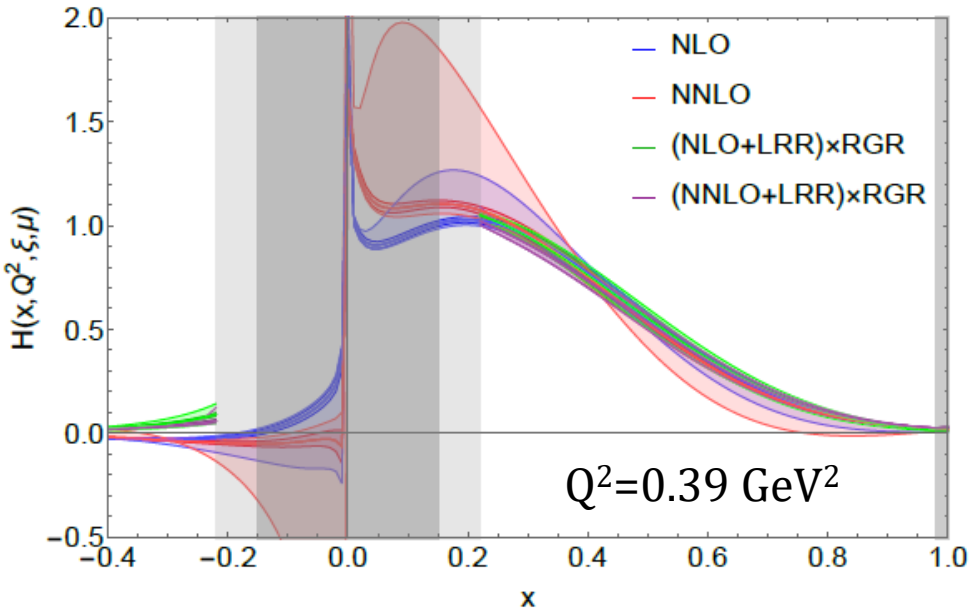
R. Zhang, et. al.
PLB 844, 138081 (2023)

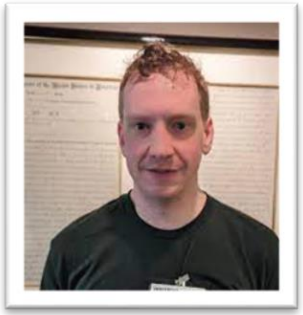
- N_f= 2+1+1 clover/HISQ, $a \approx 0.09$ fm, 135-MeV pion, $P_z \approx 2$ GeV

J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

Wanted PDFs, GPDs, etc…







P: Jack Holligan

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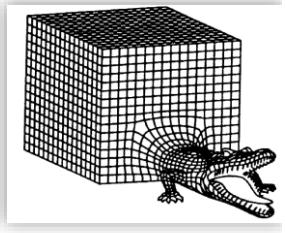

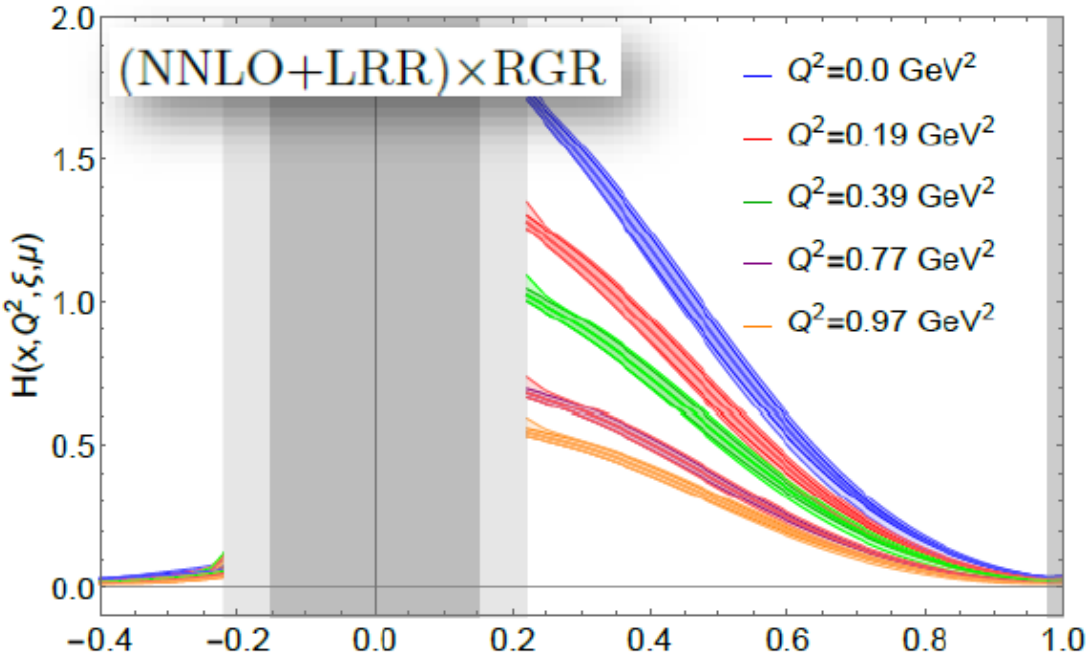

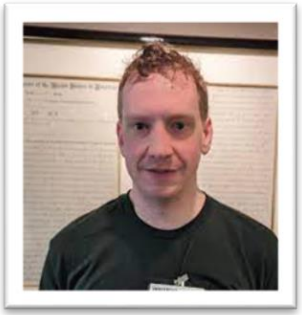

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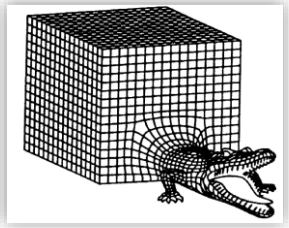
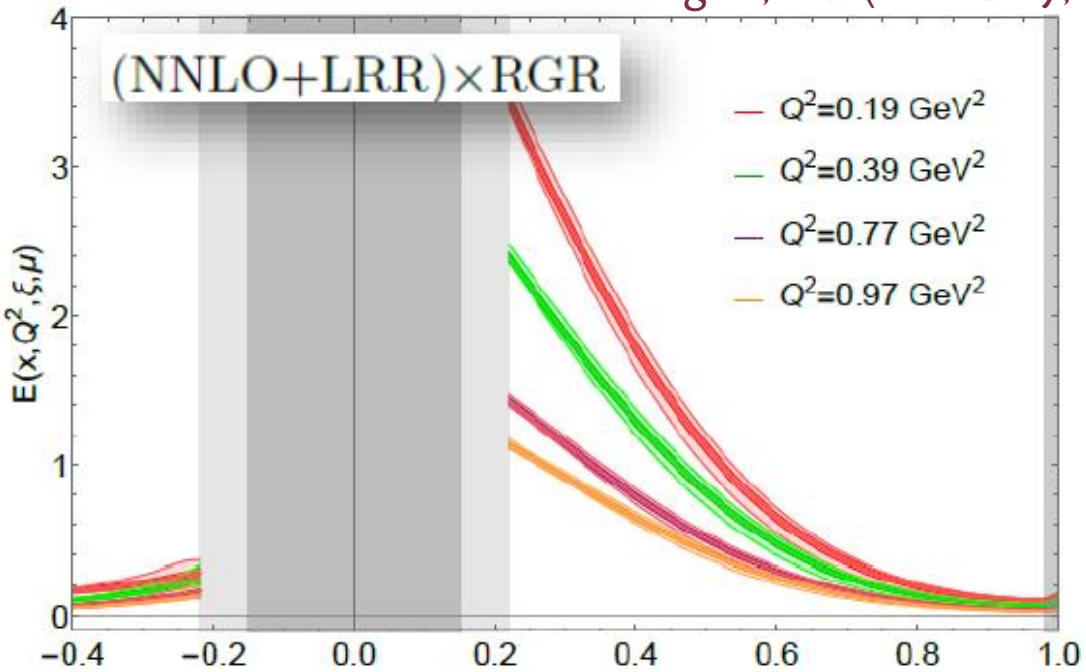
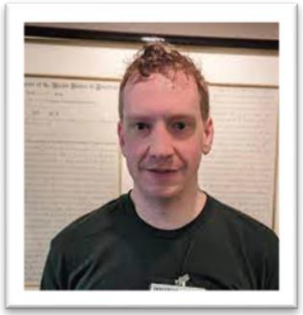






P: Jack Holligan

Outline

§ Lattice QCD and Parton Distribution Functions

§ Selected *x*-Dependent Parton Distributions

- Flavor non-singlet PDFs
- Gluon PDFs
- Generalized Parton Distributions (GPDs)

§ Impact of Lattice-QCD PDFs on Global Fits

CTEQ

First Lattice Strange PDF

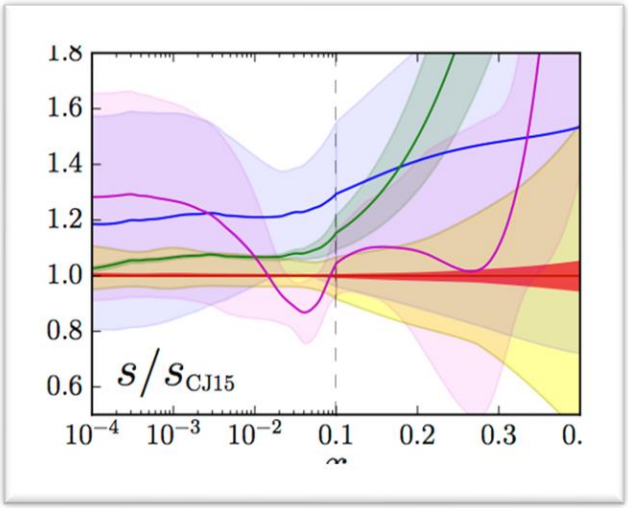
§ Results by MSULat/quasi-PDF method

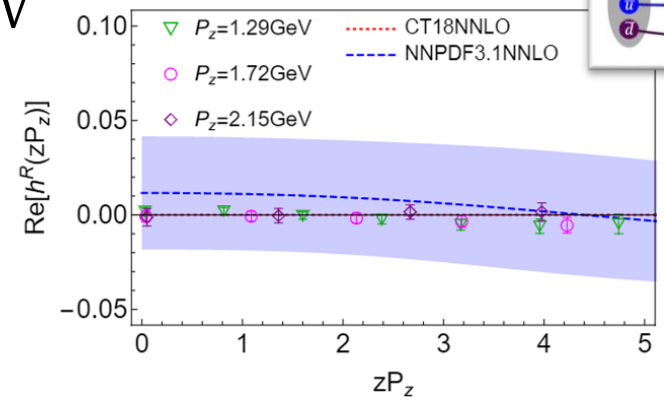
- Clover on 2+1+1 HISQ, 0.12-fm 310-MeV QCD vacuum
- Extrapolated to $M_\pi \approx 140$ MeV

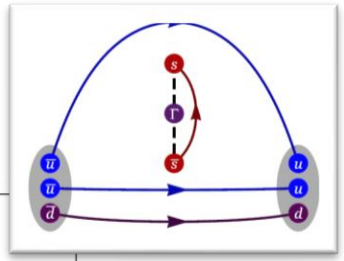
R. Zhang et al (MSULat), 2005.01124

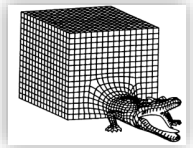
$$\mathrm{Re}[h(z)] \propto \int dx\,(s(x) - \bar{s}(x))\cos(xzP_z)$$

G: Rui Zhang









Quantities that can be calculated on the lattice

Lattice Strangeness Asymmetry Impact

§ Results by MSULat/quasi-PDF method

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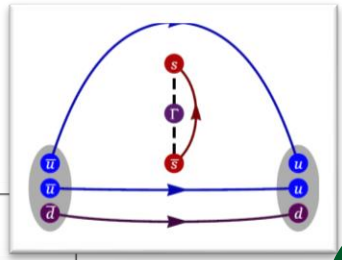
- Extrapolated to $M_\pi \approx 140$ MeV



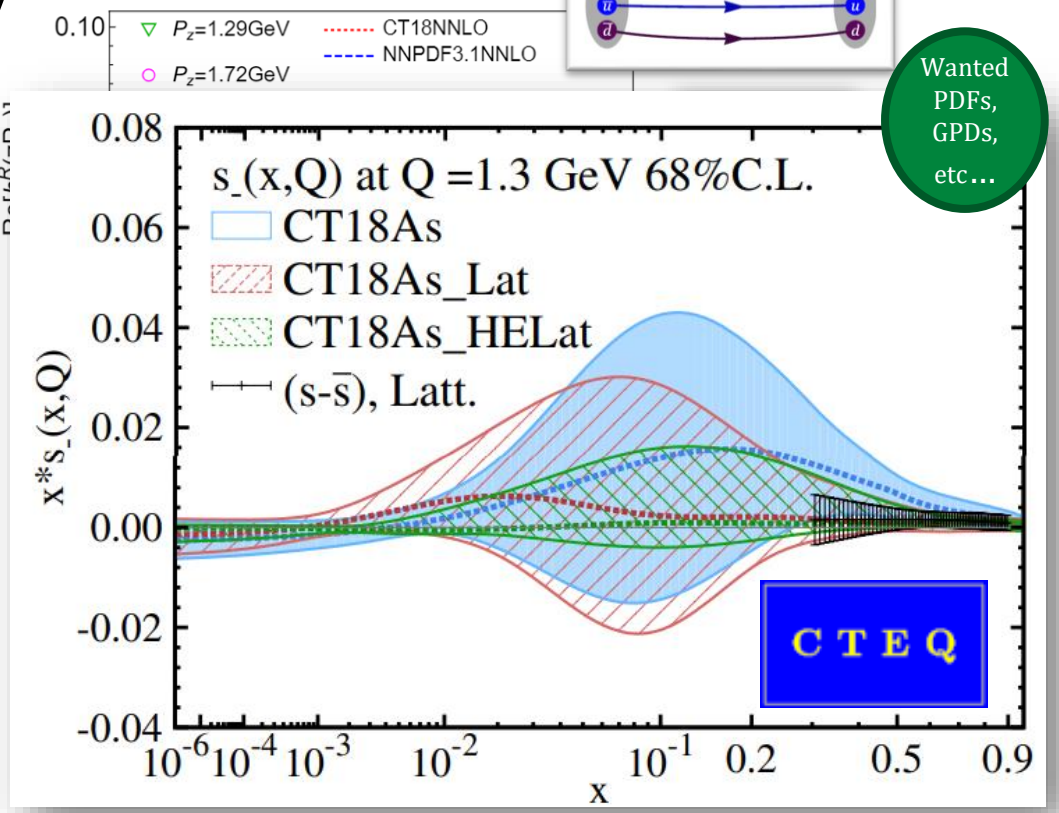
R. Zhang et al (MSULat), 2005.01124

$$\mathrm{Re}[h(z)] \propto \int dx\,(s(x)-\bar{s}(x))\cos(xzP_z)$$

§ From quasi-PDF to PDF

$$\tilde{f}_q(x,P_z) = \int_{-1}^{1}\frac{dy}{|y|}f_q(y)C_{q/q}(x,y,P_z,\mu) + O\left(\frac{\Lambda^2_{\mathrm{QCD}}}{x^2P_z^2},\frac{\Lambda^2_{\mathrm{QCD}}}{(1-x)^2P_z^2}\right)$$

T. Hou, HL, M. Yan, C. Yuan, 2211.11064

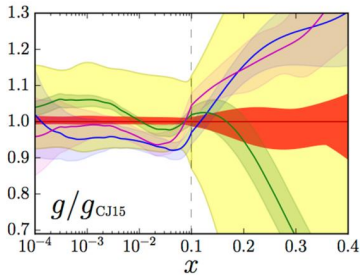


§ The strangeness asymmetry $s(x,Q) - \bar{s}(x,Q)$ at $x > 0.2$ is difficult to measure, but can be predicted in lattice QCD

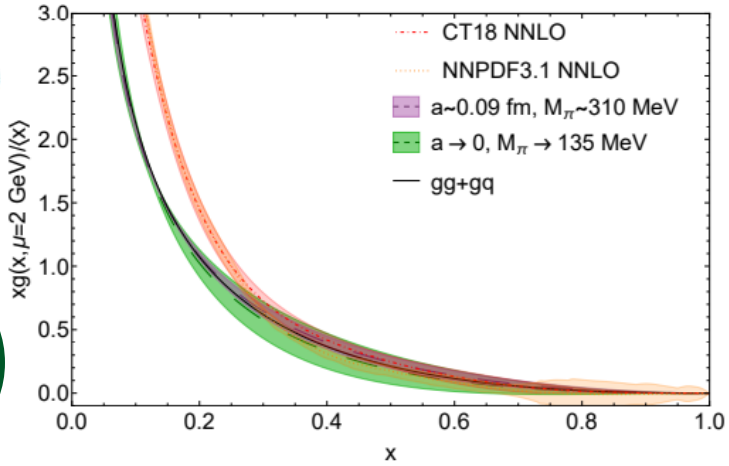
Lattice Gluon PDF Impact

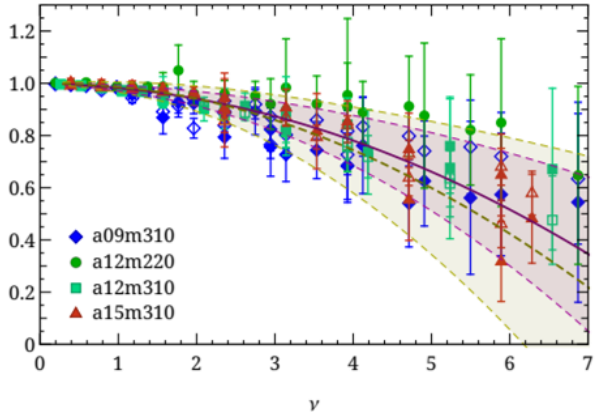
§ First continuum Gluon PDF w/ pseudo-PDF

- 2+1+1 HISQ {0.09, 0.12, 0.15} fm,
  [220,310,700]-MeV pion, 10^5-10^6 statistics



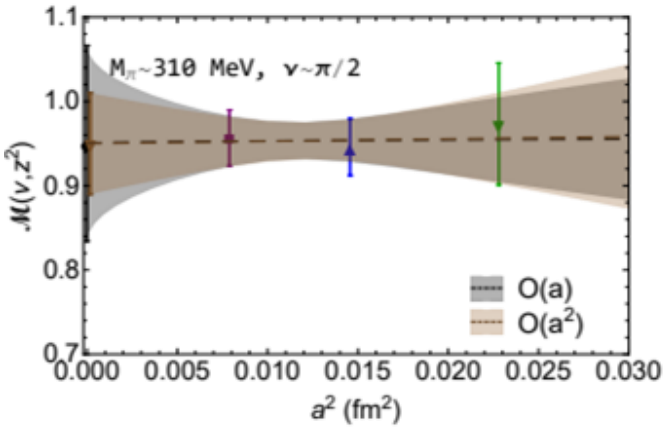
2210.09985, W. Good et al (MSULat)





Quantities that can be calculated on the lattice

Wanted PDFs, GPDs, etc…



§ Can use $a \approx 0.09$ fm results as

- Best estimate of gluon PDF
- Aim for future precision

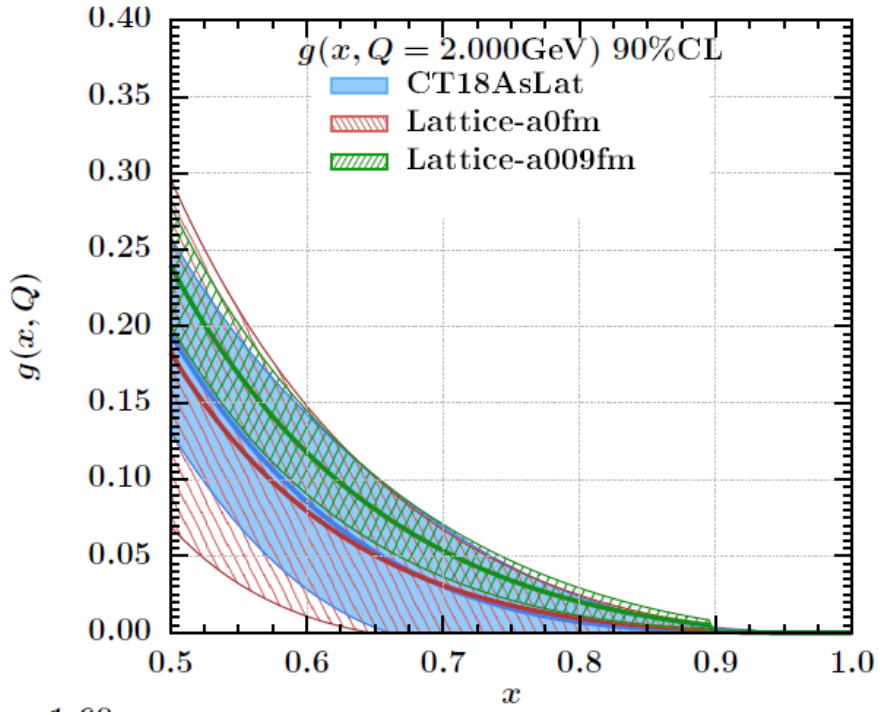
Lattice Gluon PDF Impact

§ Preliminary study with CTEQ-TEA analysis

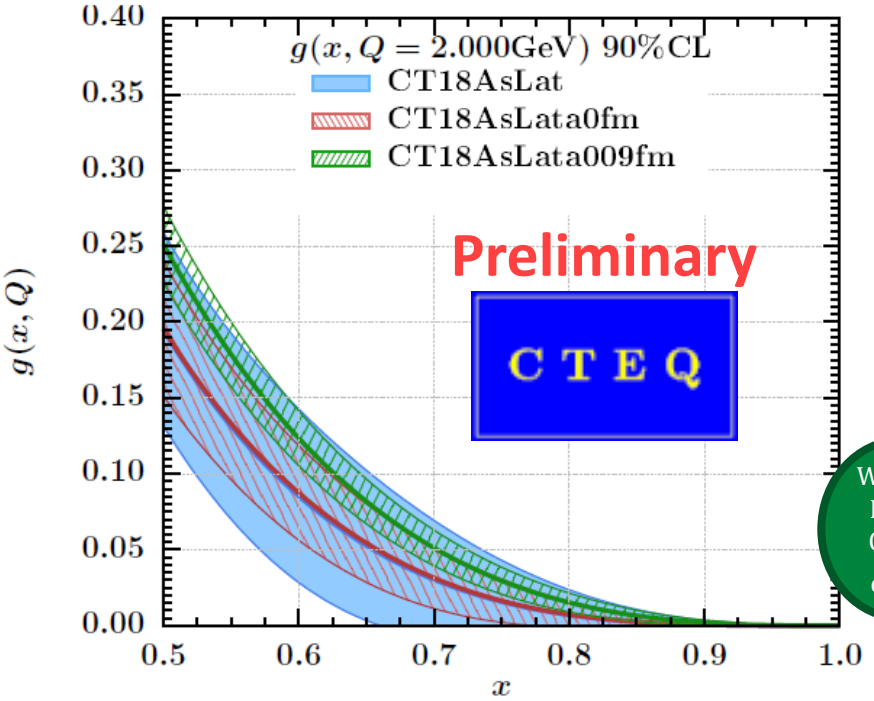
- Take lattice inputs in the region where no strong experimental data constraints, x ∈ [0.4,0.7]
- Using e-pump for re-weighting

Plots by Alim Ablat (Xinjiang U.)

Before



After



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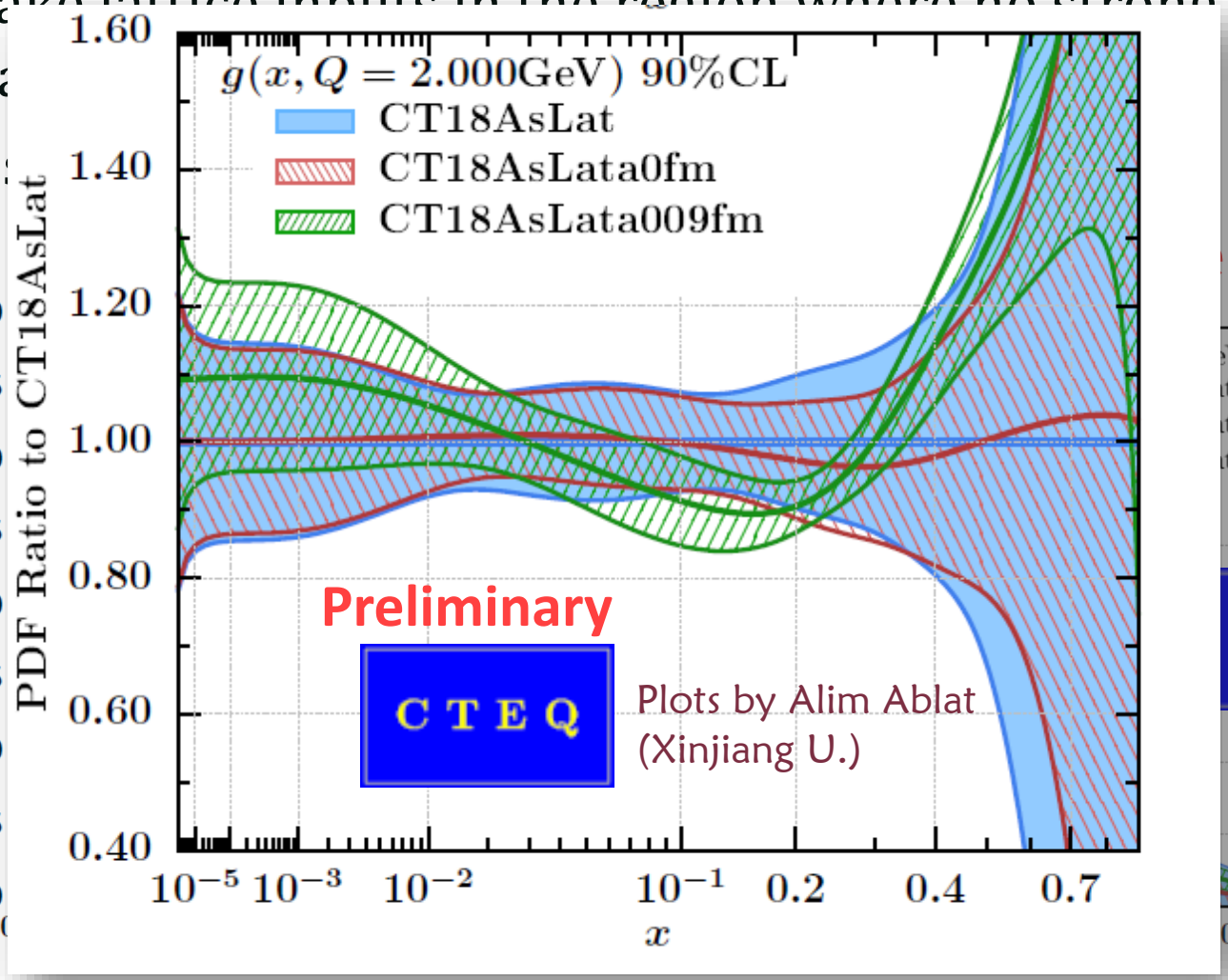
Lattice Gluon PDF Impact

§ Preliminary study with CTEQ-TEA analysis

- Take lattice inputs in the region where no strong experimental da
- Us



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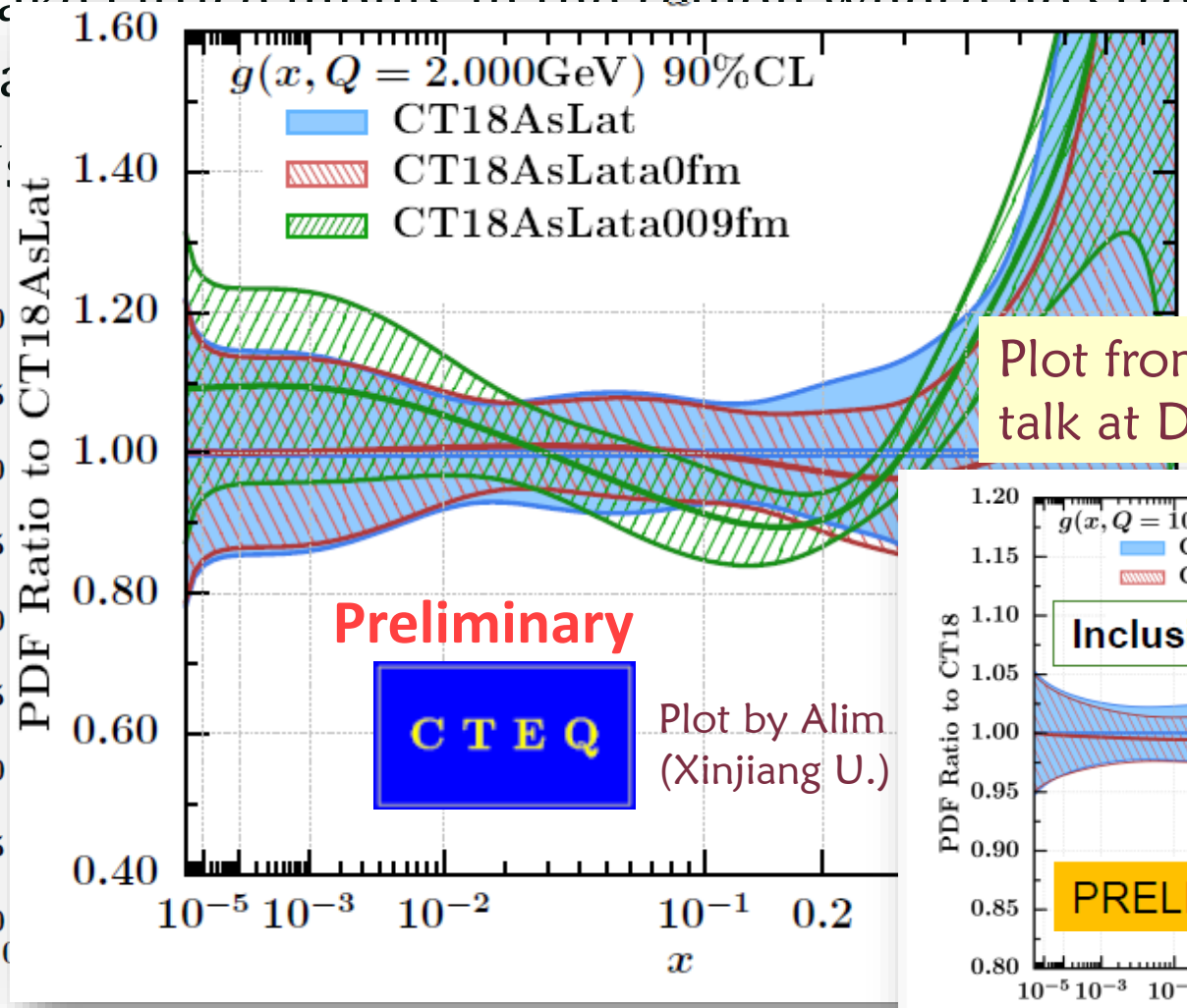
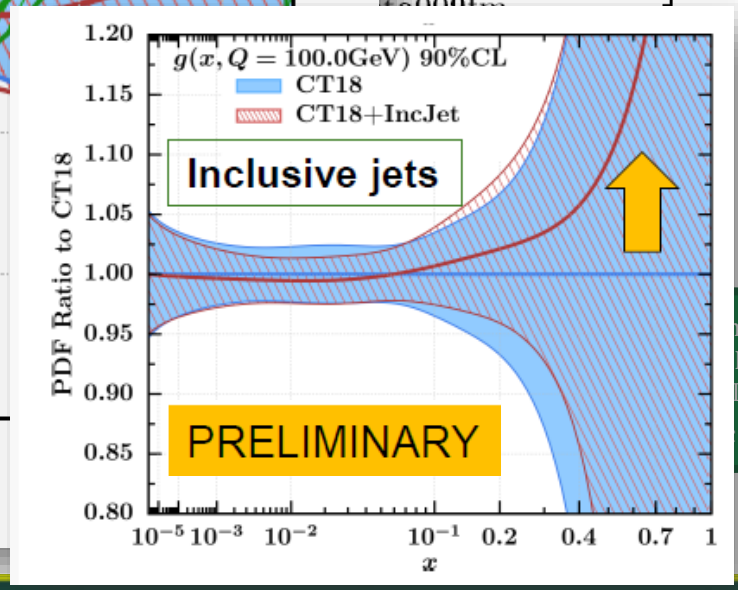


Plot from P. Nadolsky's talk at DIS2024



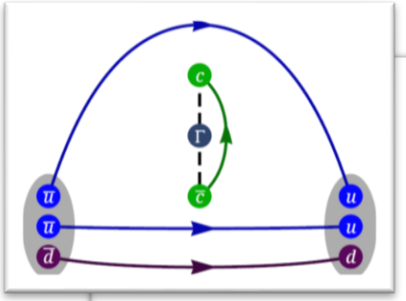
First Lattice Charm PDF

§ Large uncertainties in global PDFs

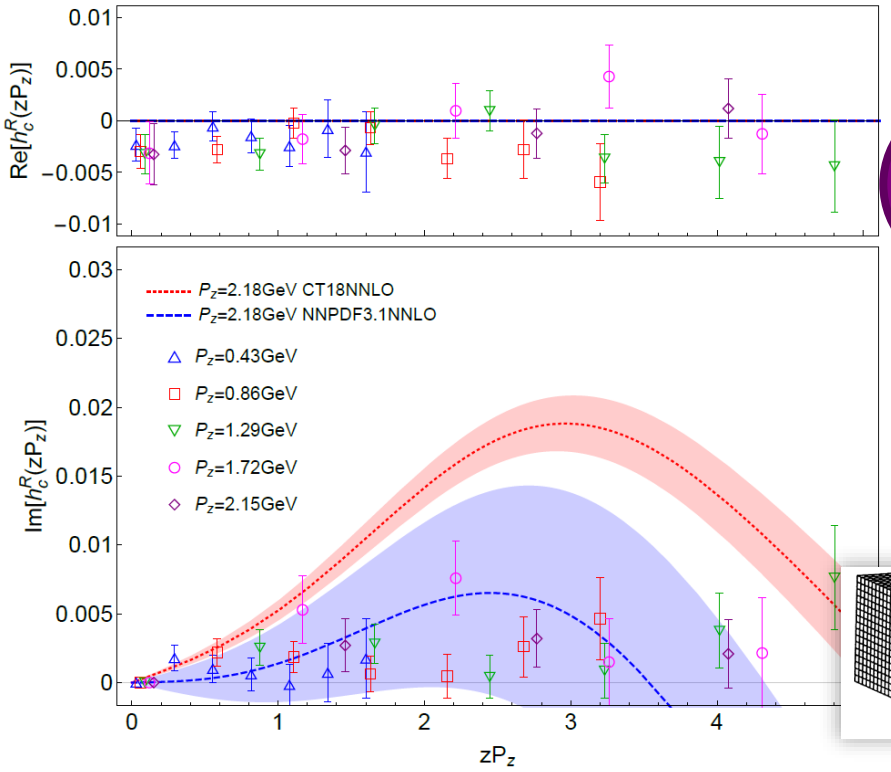
§ Results by MSULat/quasi-PDF method

Clover on 2+1+1 HISQ 0.12-fm 310-MeV QCD vacuum

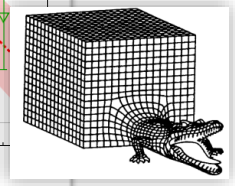
2005.01124, R. Zhang et al (MSULat)



- suggest a symmetric $c - \bar{c}$ distribution
- much smaller than strange PDF



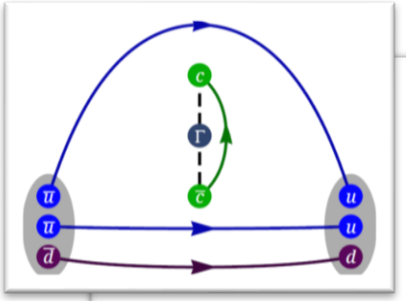
Quantities that can be calculated on the lattice



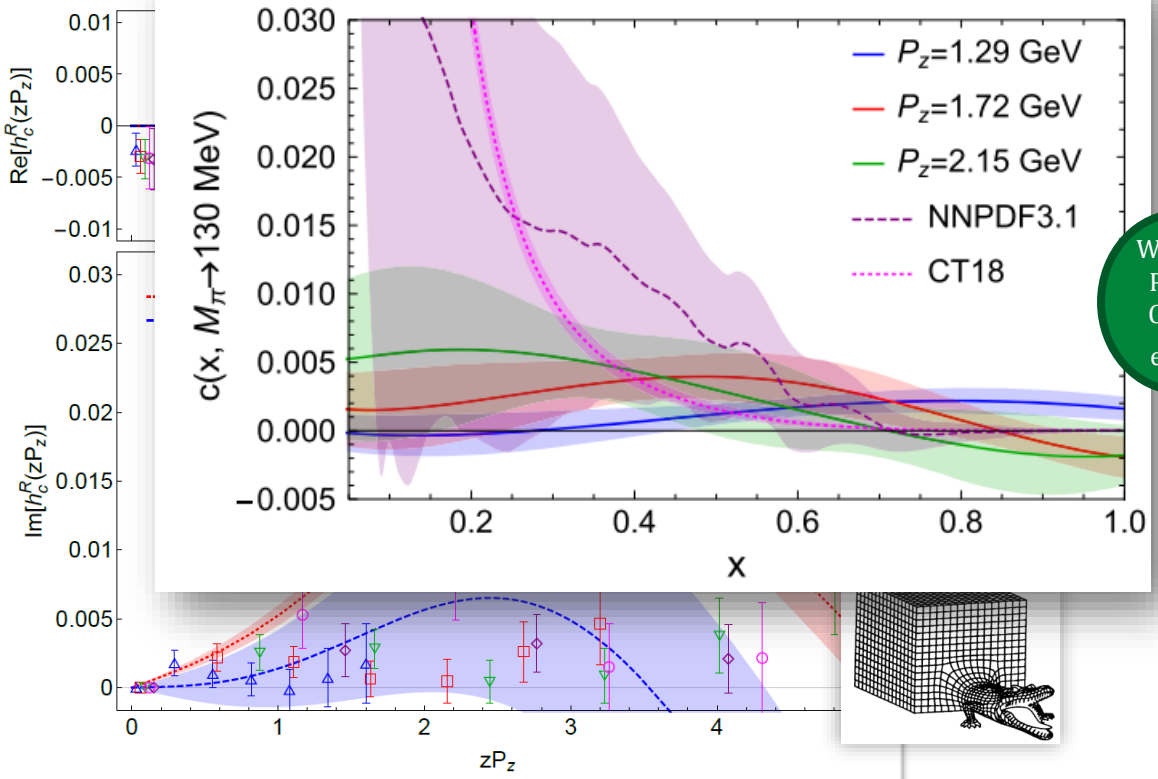
First Lattice Charm PDF

- § Large uncertainties in global PDFs
- § Results by MSULat/quasi-PDF method
  - Clover on 2+1+1 HISQ 0.12-fm 310-MeV QCD vacuum

2005.01124, R. Zhang et al (MSULat)



- suggest a symmetric $c - \bar{c}$ distribution
- much smaller than strange PDF



Wanted PDFs, GPDs, etc…

Lattice Progress & Challenges

§ Beyond the standard twist-2 collinear PDFs

- Generalized parton distributions (GPDs) for the pion and unpolarized/polarized nucleon
- Transverse-momentum– dependent distributions (TMDs)
  - Collins-Soper kernel, soft function and wavefunctions
- Twist-3 PDFs and GPDs

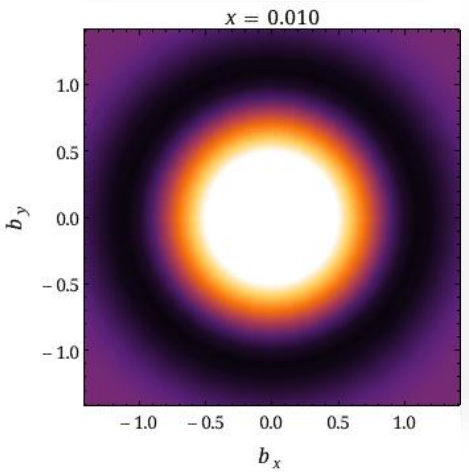
For more details and references, refer to 2202.07193

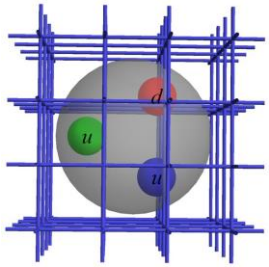
§ Challenges ahead for precision PDFs

- Large momentum is essential
  - With sufficient statistics nucleons may reach 5 GeV
- Methods for signal-to-noise improvement
  - Gluonic observables, new ideas for large momentum
- Access small-x physics; some methods have inverse problem in PDF extraction, more computational resources, etc.

Summary and Outlook

- Exciting era using LQCD to study *x*-dependent PDFs
- Overcoming longstanding limitations
  - Bjorken-*x* dependence of parton distributions now widely studied
  - More study of systematics planned for the near future
- Lattice strange and gluon PDFs can have impacts
  - Treat lattice matrix elements as expt inputs in the futu
- Precision and progress are limited by resources



Theory Input → **Global Analysis of PDFs/GPDs** ← **Exp't Input** + 

EXCLAIM

Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices & USQCD/NSF/DOE for computational resources
This work is partially sponsored by grants NSF PHY 1653405 & 1653405, DOE DE-SC0024053 & RCSA Cottrell Scholar Award

PDFLattice24 Workshop

§ November 18–20, 2024
@ Jefferson Lab
Newport News, VA, USA

- Joint community workshop between global-fit and lattice-QCD practitioners
- Theme: uncertainty quantification on nonperturbative correlator functions in phenomenology and lattice calculations

INT Program

§ Precision QCD with the Electron-Ion Collider

- Week 1: (May 12–16)
  - Precision theory for hard scattering at the EIC
  - Factorization and resummation
- Week 2: (May 19–23)
  - Parton distributions and the interplay of EIC and LHC
  - Lattice QCD meets phenomenology
- Week 3: (May 26–30)
  - Small-*x* physics in the EIC era
- Week 4: Workshop (June 2–6)
  - Bridging theory and experiment
  - Organizers: A. Bacchetta, F. Ringer, A. Stasto, W. Cosyn
- Week 5: (June 9–13)
  - Artificial intelligence and enhanced design
- Week 6: (June 16–20)
  - Jets and semi-inclusive reactions
  - Nucleon and nuclear tomography
- Organizers
  - Renee Fatemi
  - Huey-Wen Lin
  - Werner Vogelsang

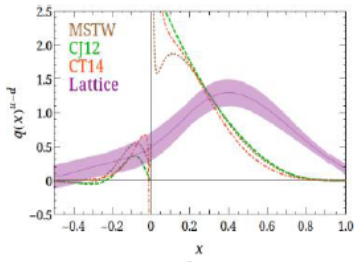
Backup Slides

Lattice Parton Calculations

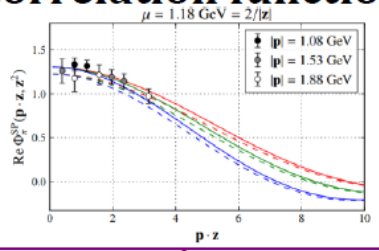
§ Rapid developments!

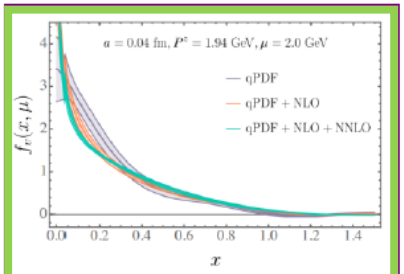
HL, Few Body Syst. 64 (2023) 3, 58

LaMET/quasi-PDF lattice calculation



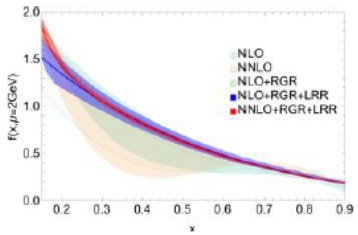
Euclidean correlation functions





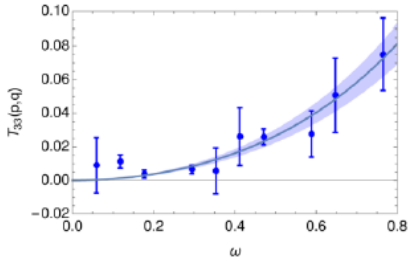
1st NNLO PDF

1stPDF w/ LRR+RGR

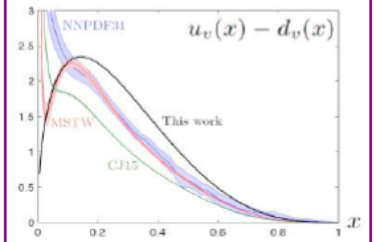


2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023 2024

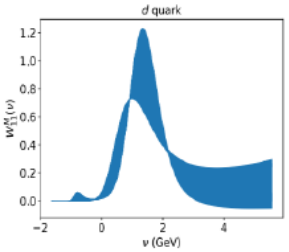
Compton amplitude



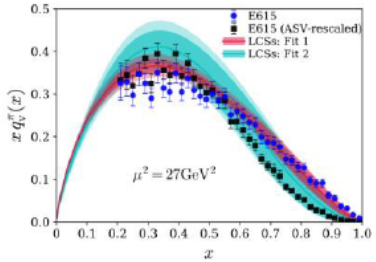
pseudo-PDFs



Hadronic tensor



LCS



CSSM/QCDSF

Isovector PDFs Update

§ Nucleon isovector PDF calculated directly at **physical pion mass**

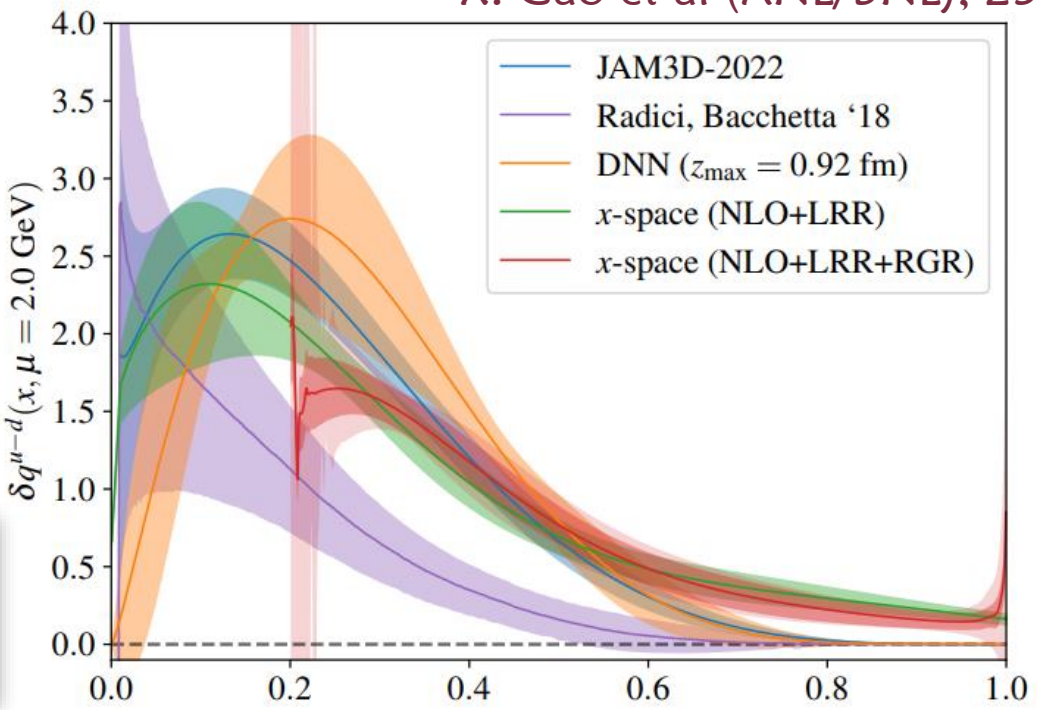
- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

R. Zhang, et. al.
PLB 844, 138081 (2023)

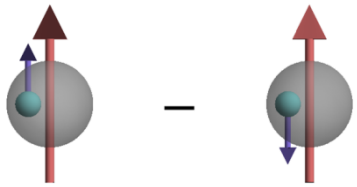
- N_f= 2+1 clover/HISQ, $a \approx 0.076$ fm, $P_z \approx 1.5$ GeV

X. Gao et al (ANL/BNL), 2310.19047 [hep-lat]

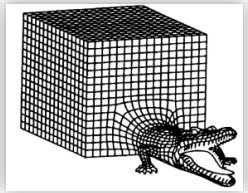
Wanted PDFs, GPDs, etc…



Transversity

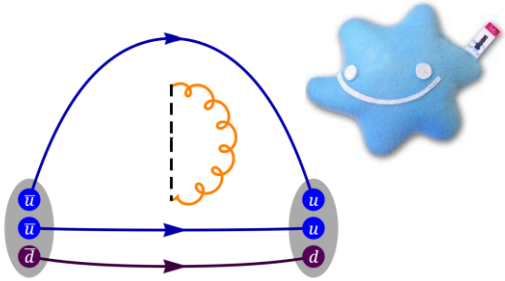


$\delta u(x) - \delta d(x)$

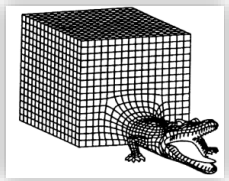


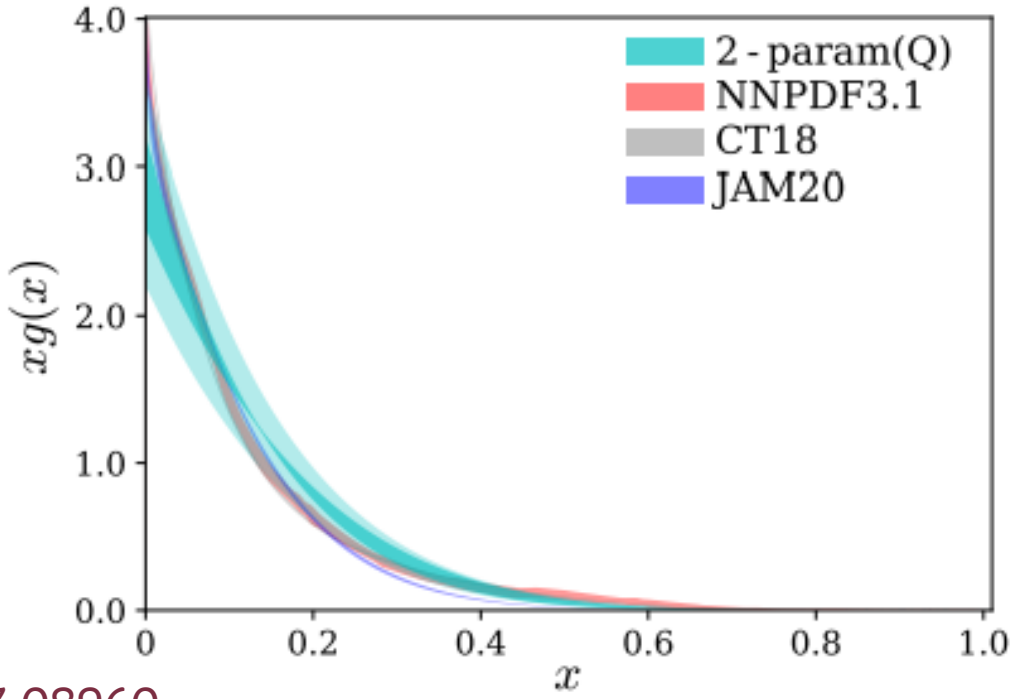
Nucleon Gluon PDF (2021)

§ Gluon PDF using pseudo-PDF



- Lattice details: 2+1 clover, 0.09 fm, 360-MeV sea pion T. Khan et al. (HadStruc), 2107.08960
- Use many nucleon Interpolating operators to improve signal with larger boosted momentum state

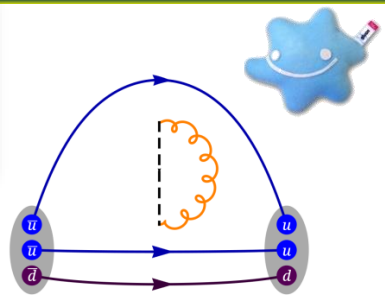




T. Khan et al. (HadStruc), 2107.08960

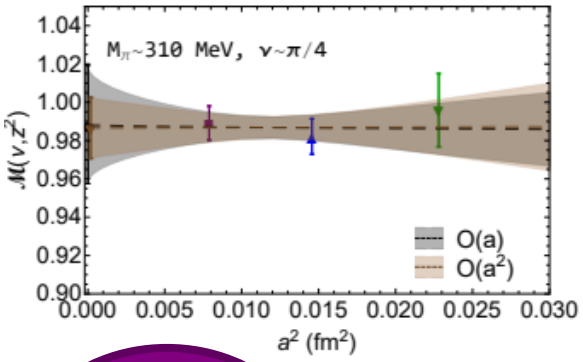
Nucleon Gluon PDF (2022)

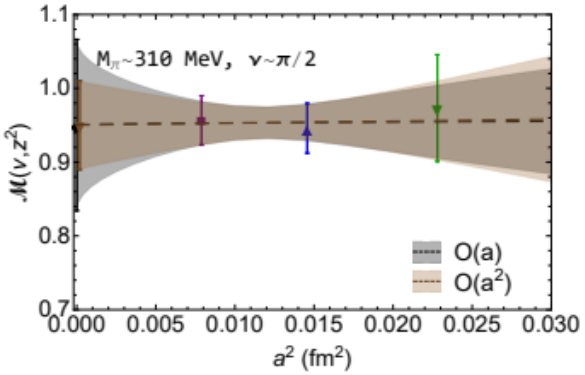
§ Continuum Gluon PDF w/ pseudo-PDF

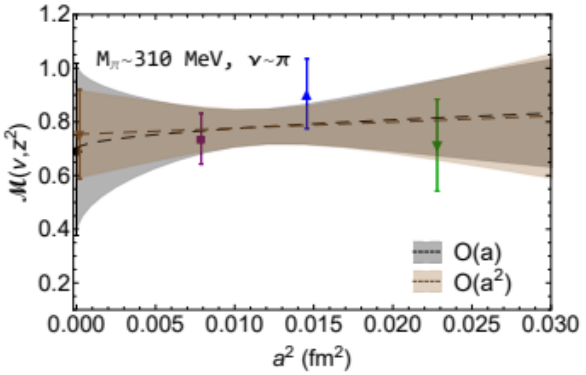


- 2+1+1 HISQ {0.09, 0.12, 0.15} fm,
  [220,310,700]-MeV pion, 10^5-10^6 statistics

  Z. Fan, W. Good, HL (MSULat), 2210.09985







Quantities that can be calculated on the lattice

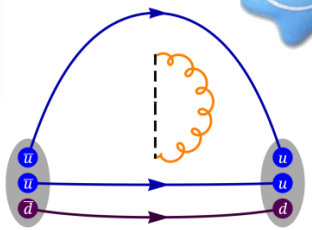
G: Bill Good

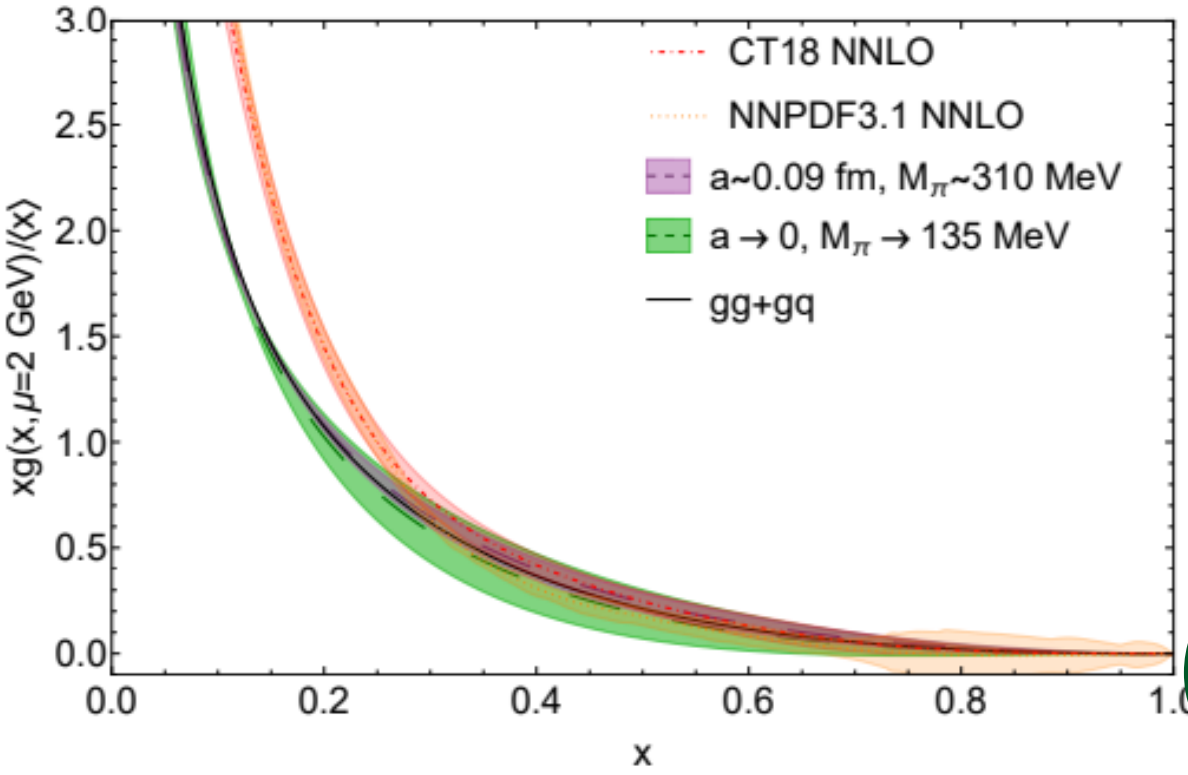
Nucleon Gluon PDF (2022)

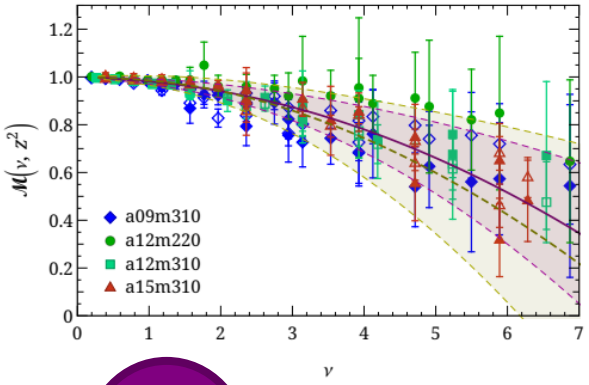
§ Continuum Gluon PDF w/ pseudo-PDF

- 2+1+1 HISQ {0.09, 0.12, 0.15} fm, [220,310,700]-MeV pion, 10^5-10^6 statistics

Z. Fan, W. Good, HL (MSULat), 2210.09985







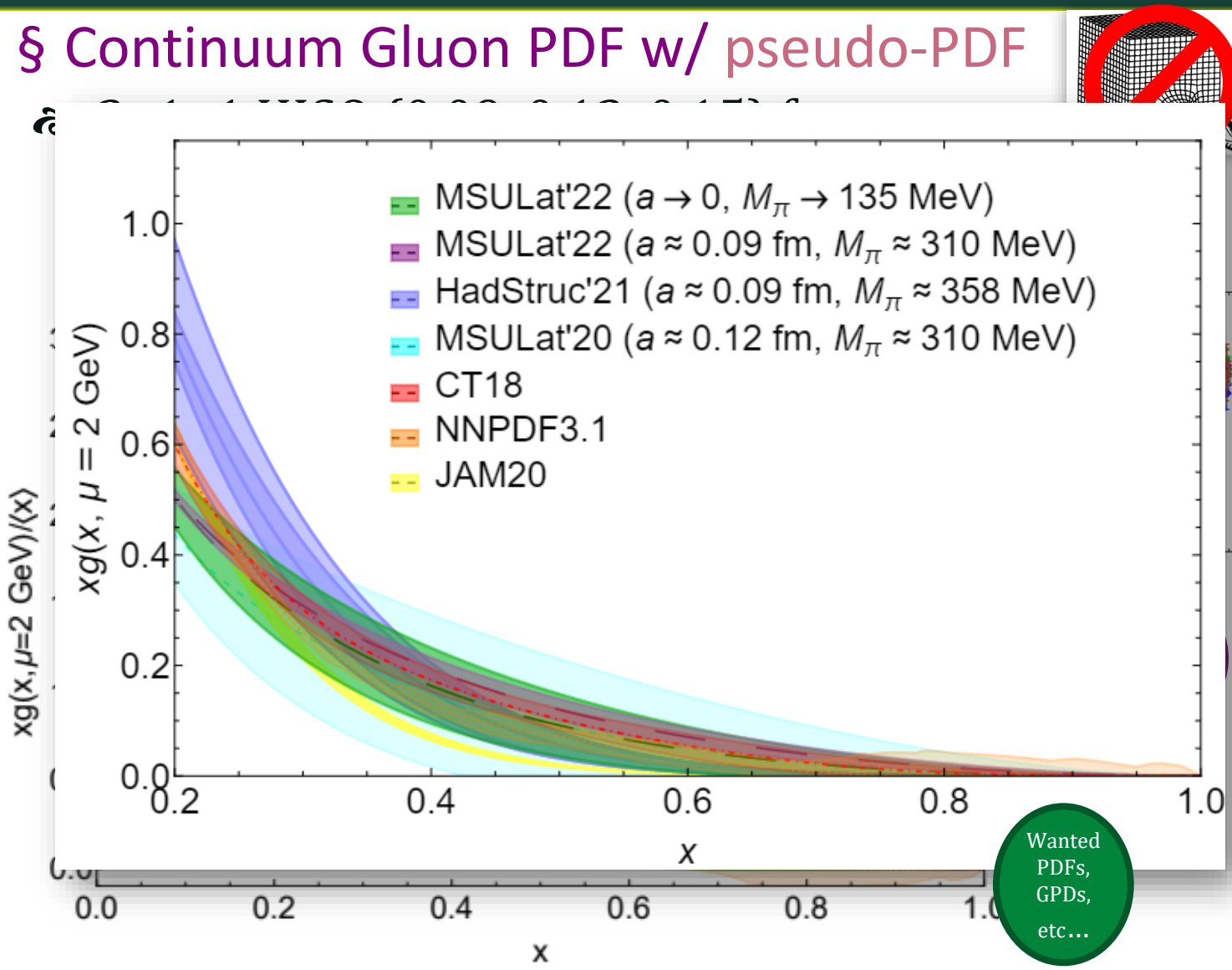
Quantities that can be calculated on the lattice

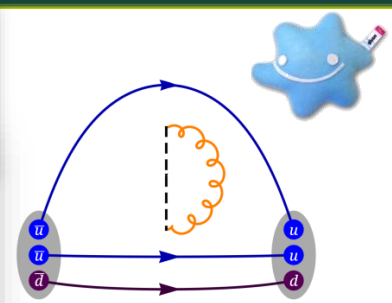
Wanted PDFs, GPDs, etc…

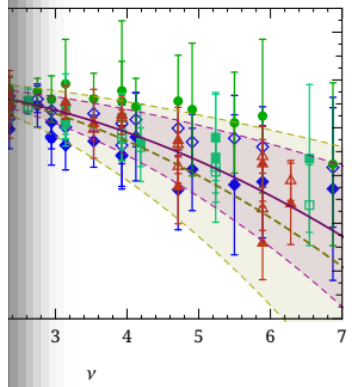
G: Bill Good

Nucleon Gluon PDF (2022)

§ Continuum Gluon PDF w/ pseudo-PDF





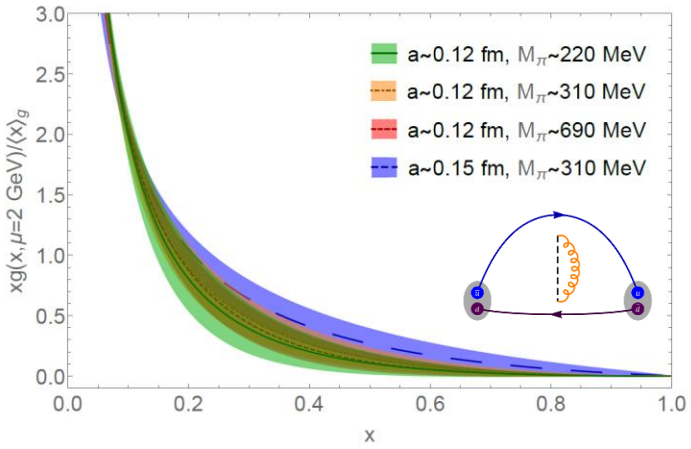


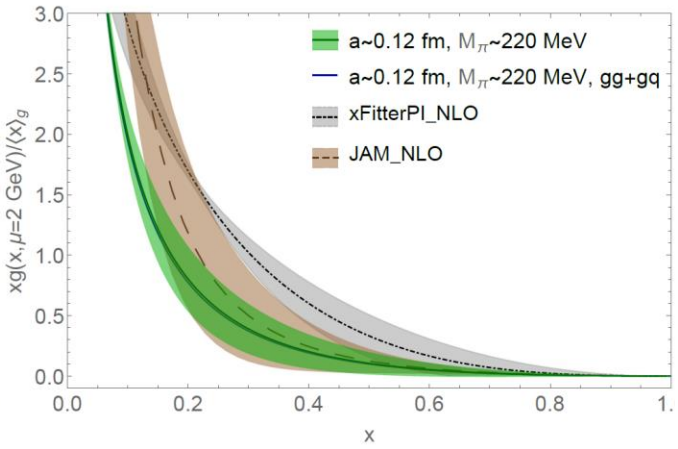
Wanted PDFs, GPDs, etc…

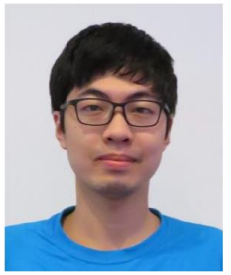
G: Bill Good

Meson Gluon PDFs

§ First pion and kaon gluon PDFs using pseudo-PDF



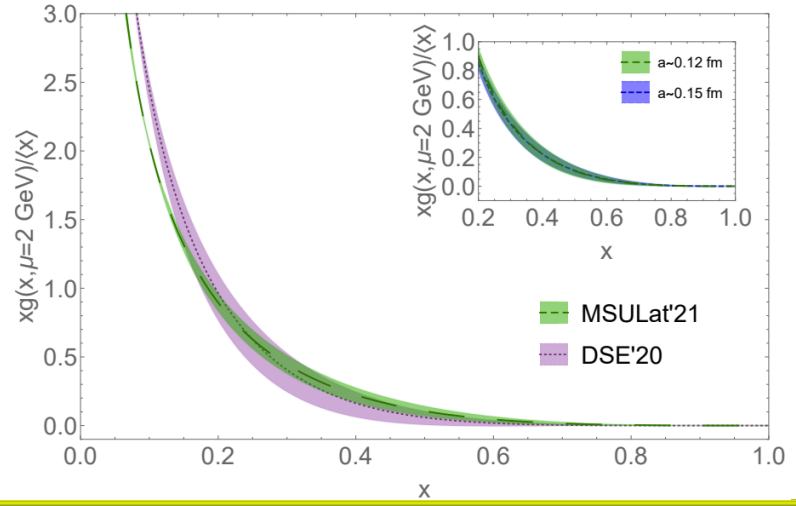




G: Zhouyou Fan

2104.06372, Fan et al (MSULat)

Wanted PDFs, GPDs, etc…



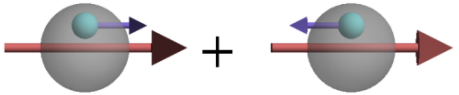
G: Alejandro Salas-Chavira

2112.03124 , Salas-Chavira et al (MSULat)

Continuum PDF

§ Nucleon PDFs using quasi-PDFs in the continuum limit

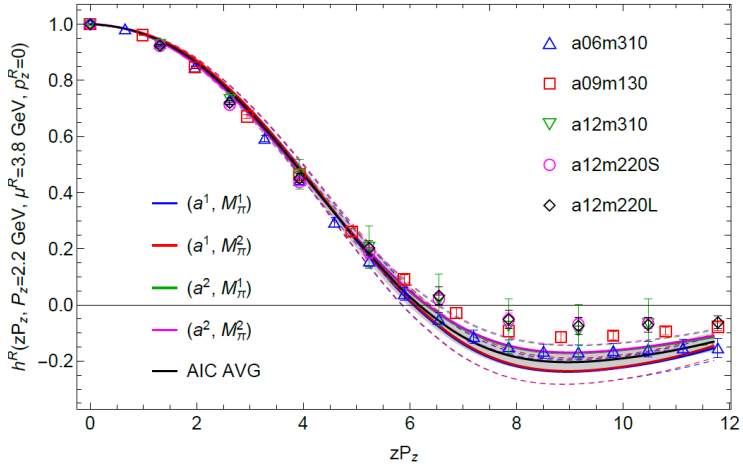
- Lattice details: clover/2+1+1 HISQ (MSULat)
  - $a \approx \{0.06, 0.09, 0.12\}$ fm
  - $M_\pi \in \{\mathbf{135}, 220, 310\}$-MeV pion
  - $M_\pi L \in \{3.3, 5.5\}$
  - $P_z \approx 2$ GeV

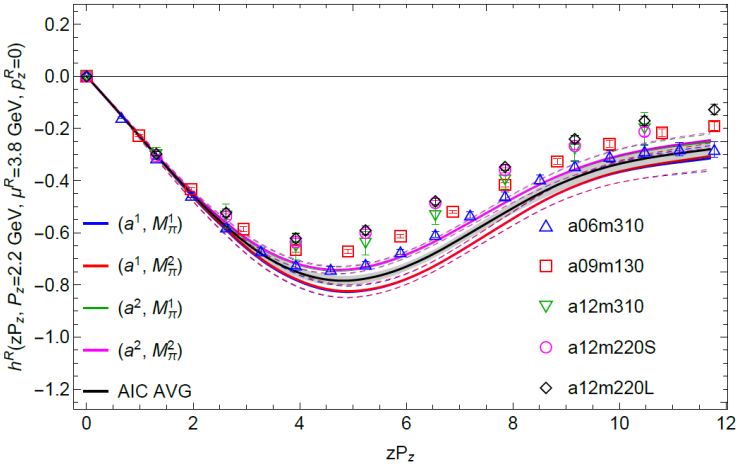


2011.14971, HL et al. (MSULat)

- Naïve extrapolation to physical-continuum limit

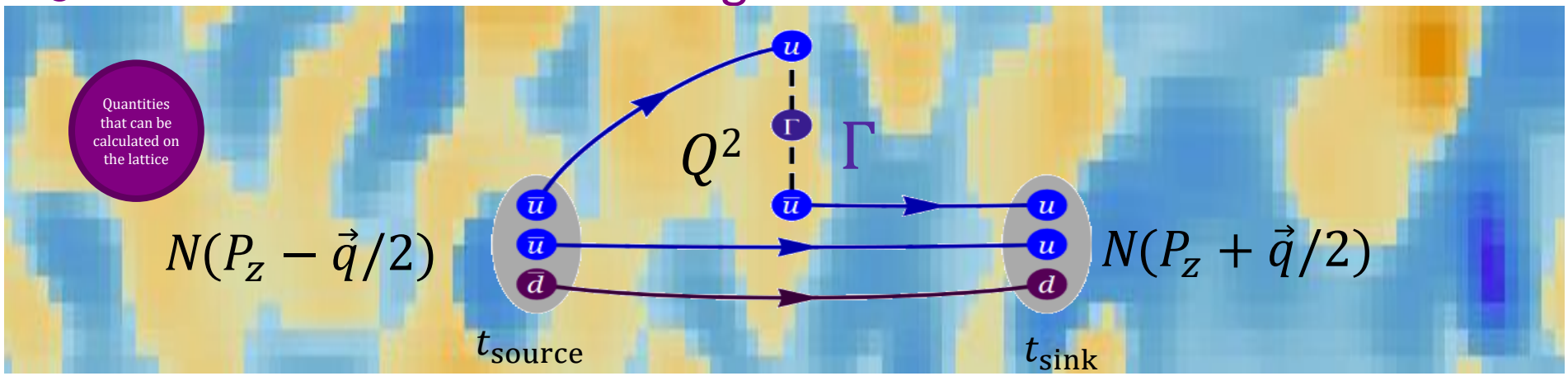
Quantities that can be calculated on the lattice





Generalized Parton Distributions

- Nucleon/pion GPDs using quasi-PDFs at **physical pion mass** calculated at Breit Frame
- One calculates the following matrix elements on the lattice



Nucleon unpolarized GPDs: H and E

$$\tilde{F}\left(x,\tilde{\xi},t,\bar{P}_z\right) = \frac{\bar{P}_z}{\bar{P}_0}\int\frac{dz}{4\pi}e^{ixz\bar{P}z}\langle P'|\tilde{O}_{\gamma_0}(z)|P\rangle = \frac{\bar{u}(P')}{2\bar{P}^0}\left(\boldsymbol{H}\left(x,\tilde{\xi},t,\bar{P}_z\right)\gamma^0 + \boldsymbol{E}\left(x,\tilde{\xi},t,\bar{P}_z\right)\frac{i\sigma^{0\mu}\Delta_\mu}{2M}\right)u(P'')$$

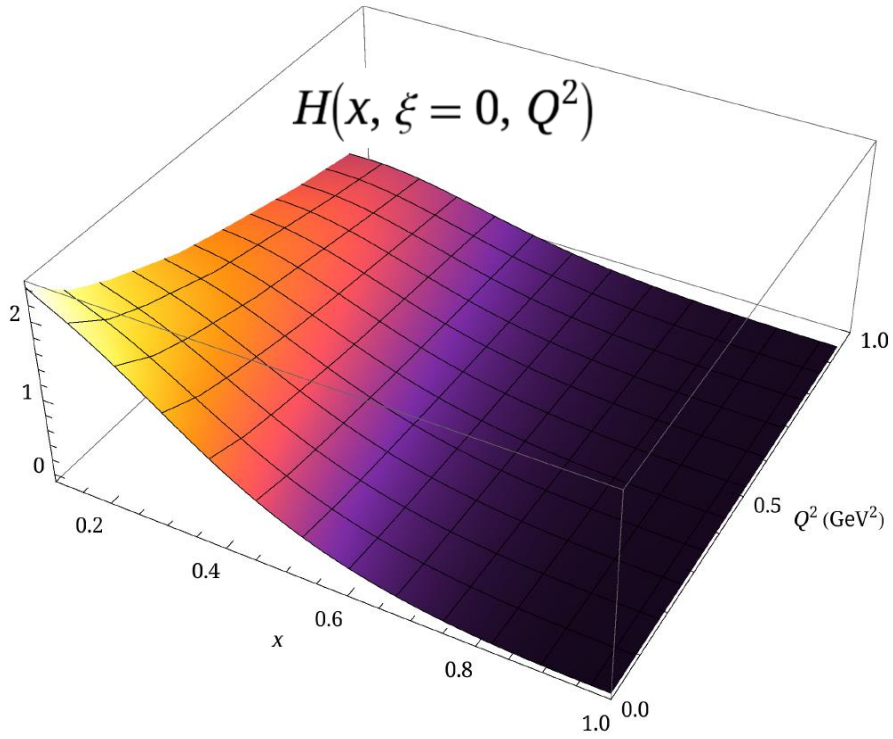
$$p^\mu = \frac{p''^\mu + p'^\mu}{2}, \qquad \Delta^\mu = p''^\mu - p'^\mu, \qquad t = \Delta^2, \qquad \xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$$

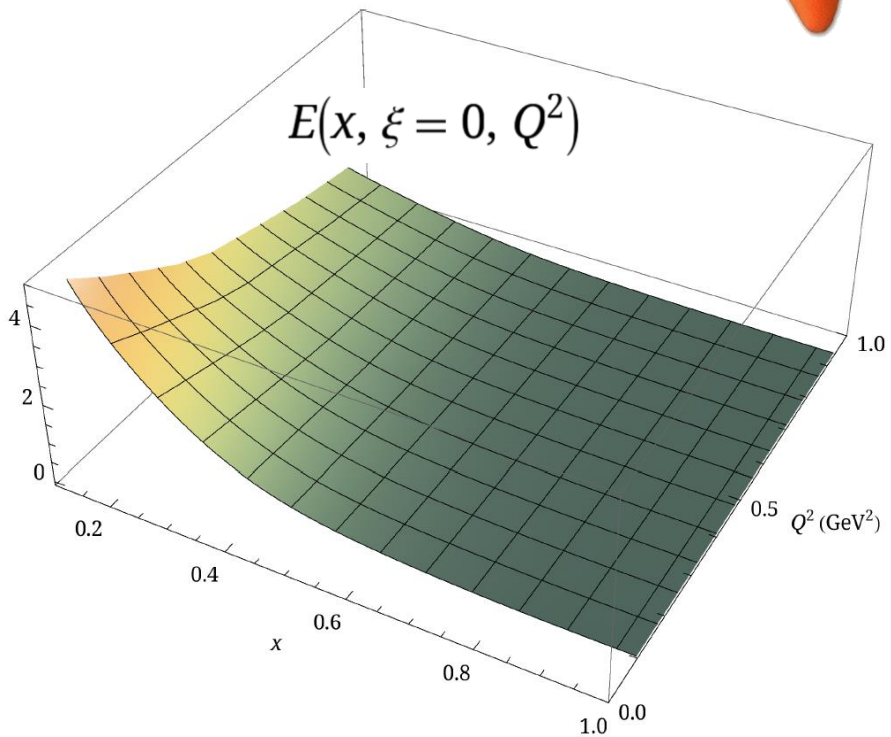
HL, Phys. Rev. Lett. 127 (2021) 18, 182001

Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at **physical pion mass**

- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results





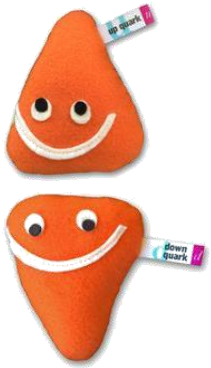
HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Isovector Nucleon GPDs

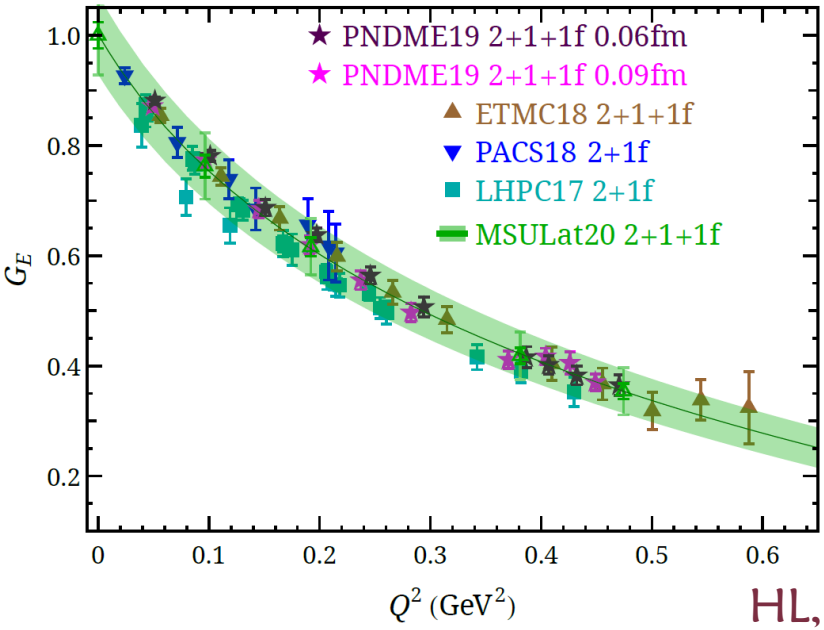
§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ (MSULat)
  0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results

$$\int_{-1}^{+1} dx\, x^{n-1} \; = \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i A_{ni}^q(t) + (-2\xi)^n C_{n0}^q(t)\Big|_{n\ \text{even}}$$



$n = 1$



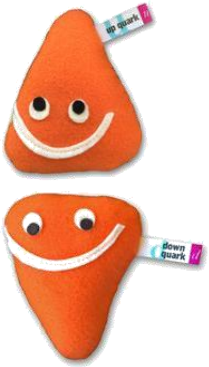
HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Isovector Nucleon GPDs

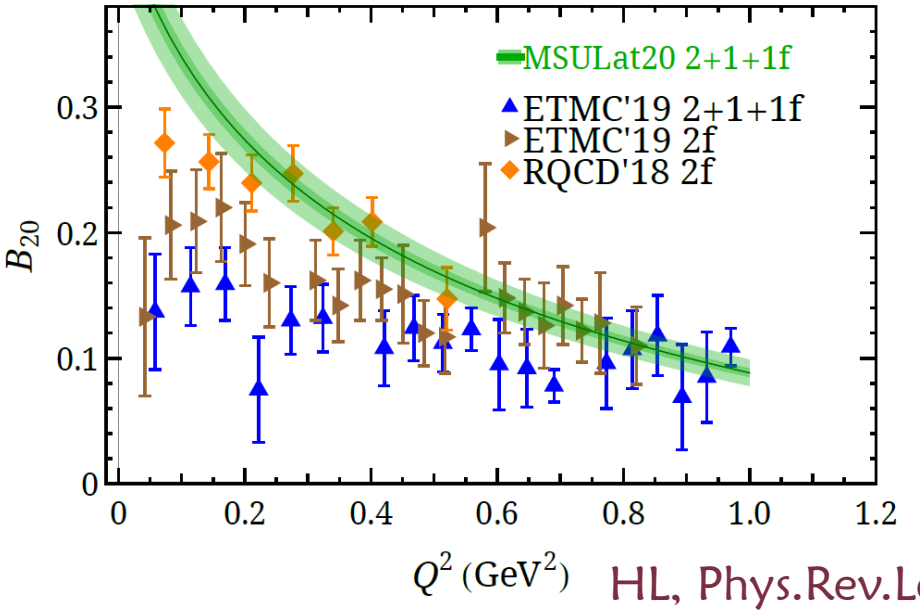
§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ (MSULat)
  0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results

$$\int_{-1}^{+1} dx\, x^{n-1} \; = \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i B^q_{ni}(t) - (-2\xi)^n C^q_{n0}(t)\Big|_{n\ \text{even}}$$



$n = 2$

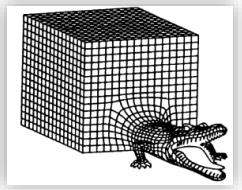


HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Nucleon Tomography

§ Nucleon GPD using quasi-PDFs at physical pion mass

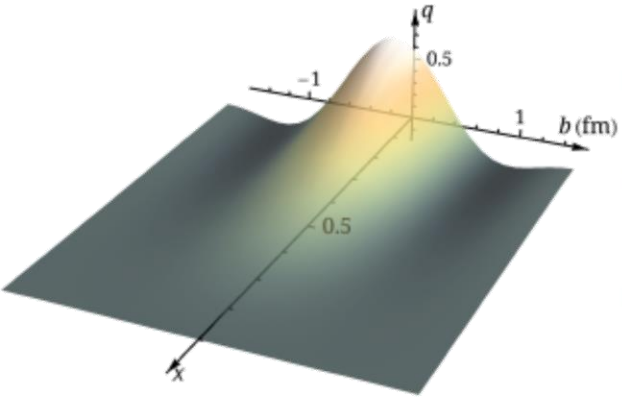
- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results

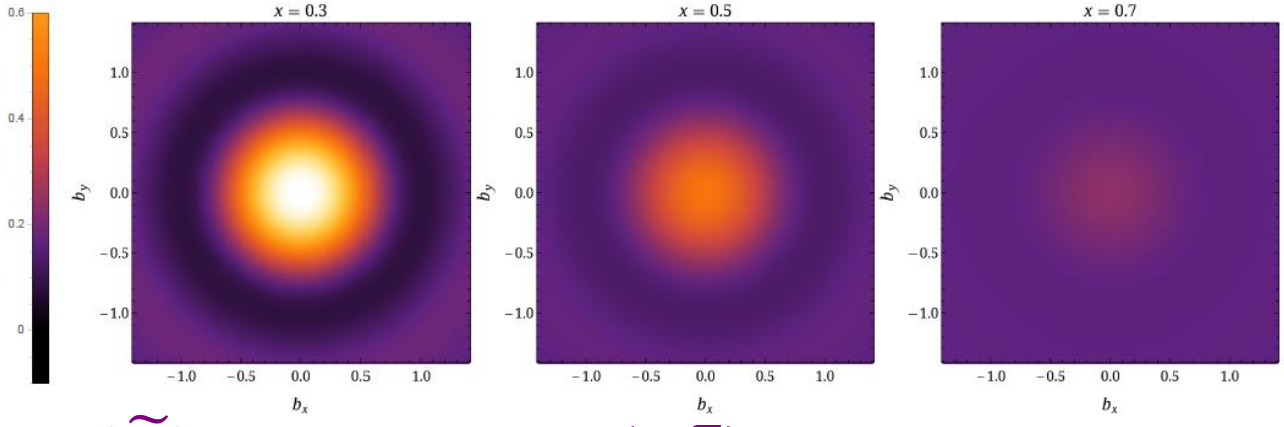


finite-volume, discretization,

$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x, \xi = 0, t = -\vec{q}^2) e^{i\vec{q}\cdot\vec{b}}$$

HL, Phys. Rev. Lett. 127 (2021) 18, 182001





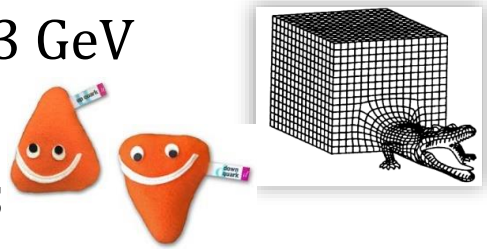
§ Nucleon helicity GPD ($\widetilde{H}$) and pion GPD (H^π) using quasi-PDFs at **physical pion mass**

HL (MSULat), Phys.Lett.B 824 (2022) 136821;
Phys. Lett. B 846 (2023) 138181

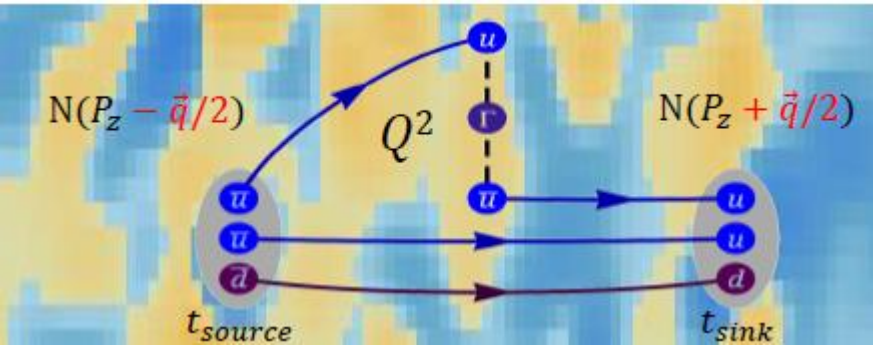
Asymmetric-Frame GPD

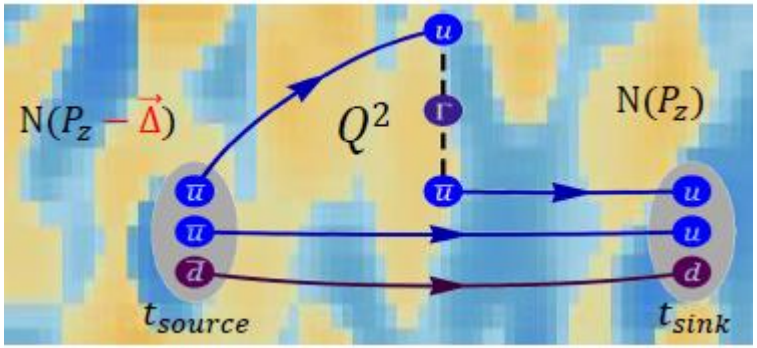
§ New calculations by ANL/BNL/ETMC using asymmetric frame

- 2+1+1 twisted-Wilson, 0.09 fm, $P_z \approx 1.3$ GeV
  260-MeV pion, one source-sink used
- $\xi = 0$ isovector nucleon GPD results



finite-volume, discretization, *heavy quark, excited-state, …*





$$\tilde{F}(x,\tilde{\xi},t,\bar{P}_z) = \frac{\bar{u}(P')}{2\bar{P}^0}\left(H(x,\tilde{\xi},t,\bar{P}_z)\gamma^0 + E(x,\tilde{\xi},t,\bar{P}_z)\frac{i\sigma^{0\mu}\Delta_\mu}{2M}\right)u(P'')$$

$$F^\mu(z,P,\Delta) = \bar{u}(p_f,\lambda')\left[\frac{P^\mu}{m}A_1 + mz^\mu A_2 + \frac{\Delta^\mu}{m}A_3 + im\sigma^{\mu z}A_4 + \frac{i\sigma^{\mu\Delta}}{m}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{m}A_6 + mz^\mu i\sigma^{z\Delta}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{m}A_8\right]u(p_i,\lambda)$$

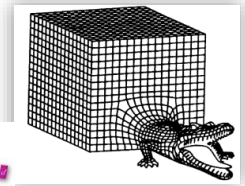
$$H(z\cdot P^{s/a}, z\cdot\Delta^{s/a}, (\Delta^{s/a})^2) = A_1 + \frac{\Delta^{s/a}\cdot z}{P^{s/a}\cdot z}A_3$$

ANL/BNL/ETMC, 2209.05373 ,2310.13114

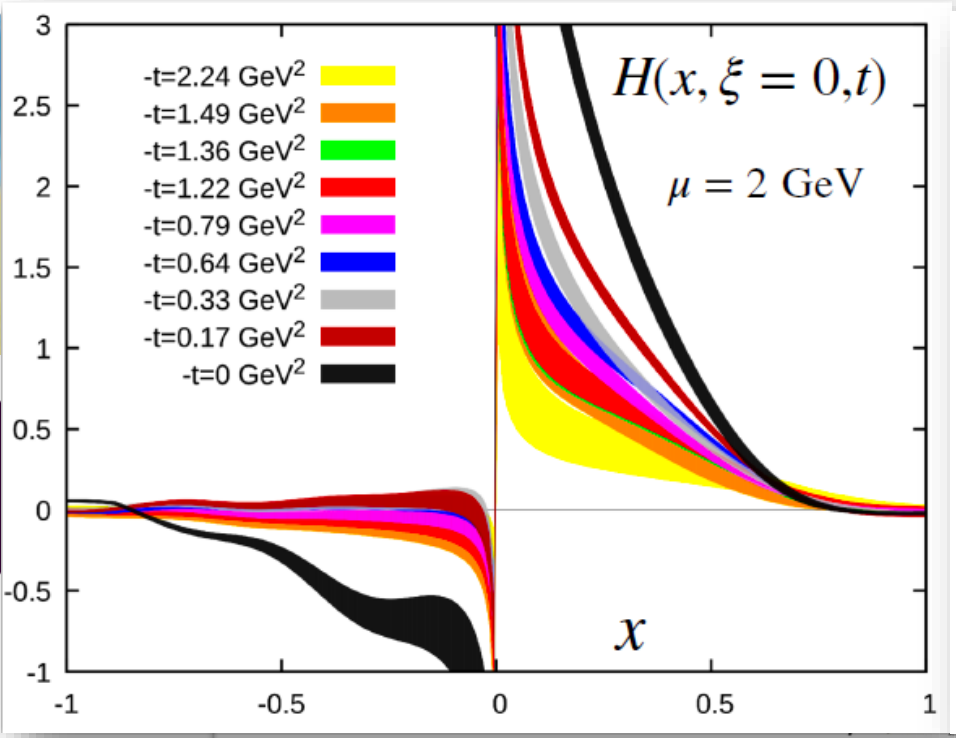
Asymmetric-Frame GPD

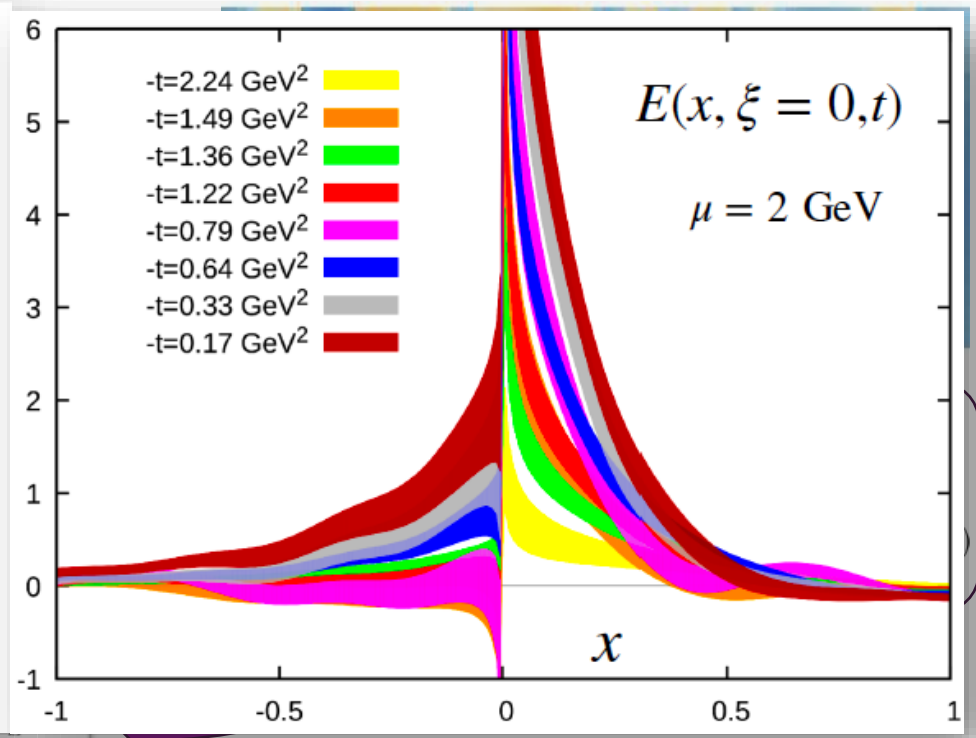
§ New calculations by ANL/BNL/ETMC using asymmetric frame

- 2+1+1 twisted-Wilson, 0.09 fm, $P_z \approx 1.3$ GeV
  260-MeV pion, one source-sink used
- $\xi = 0$ isovector nucleon GPD results



finite-volume, discretization, *heavy quark, excited-state, …*





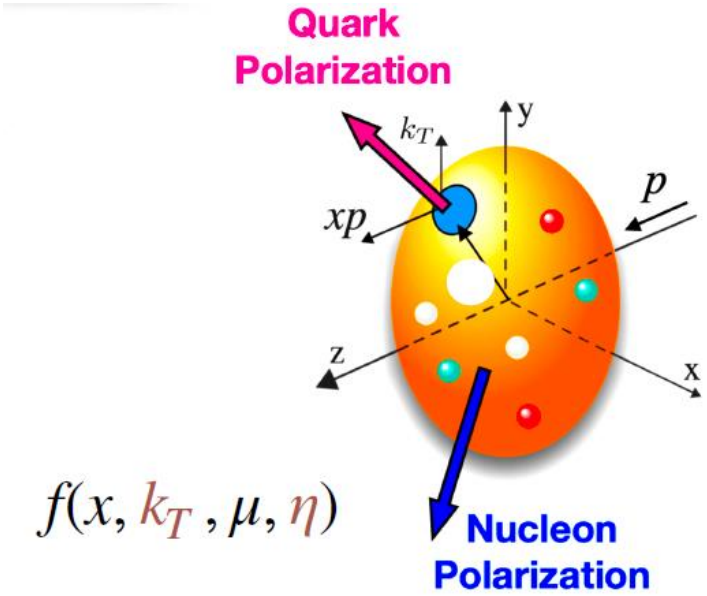
See Shohini Bhattacharya' talks@ Lattice 2024

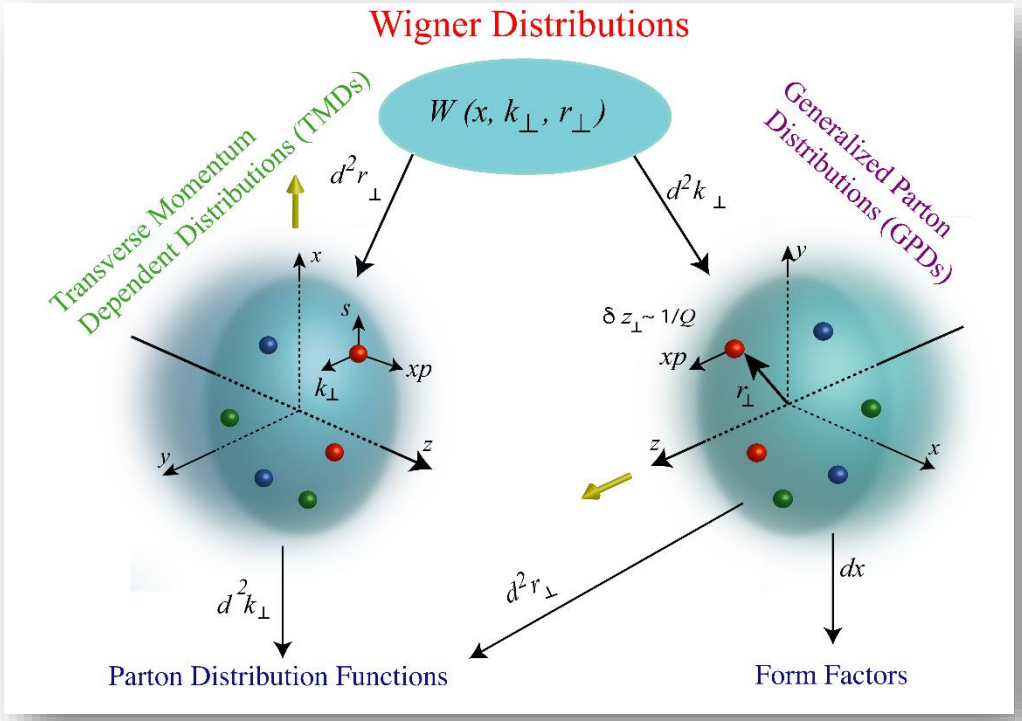
ANL/BNL/ETMC, 2209.05373 ,2310.13114

Transverse Momentum Distributions

§ TMDs inform on confined motion of partons in hadron

- Complementary to structure provided by PDFs and GPDs





Picture from INT Program INT–17–3

§ Mini-review by David Lin @ Mon. 2:30PM

Transverse Momentum Distributions

§ TMDs inform on confined motion of partons in hadron

- Complementary to structure provided by PDFs and GPDs

Relating quasi-TMDPDF to TMDPDF

M.A. Ebert, S.T. Schindler, I.W. Stewart, Y. Zhao, JHEP 04 (2022) 178

$$\tilde{f}^{\mathrm{TMD}}(x,\vec{b}_T,\mu,P^z) = \underbrace{C^{\mathrm{TMD}}(\mu,xP^z)}_{\text{pertub. theo.}}\, g_S(b_T,\mu)\exp\left[\frac{1}{2}K(b_T,\mu)\log\frac{(2xP^z)^2}{\zeta}\right] \times f^{\mathrm{TMD}}(x,\vec{b}_T,\mu,\zeta) + \mathcal{O}\left(\frac{q_T^2}{P_z^2},\frac{\Lambda_{\mathrm{QCD}}^2}{P_z^2}\right)$$

★ To obtain f^{TMD}, one computes $\tilde{f}^{\mathrm{TMD}}$ with lattice QCD

★ Also need non-perturbative calculation of

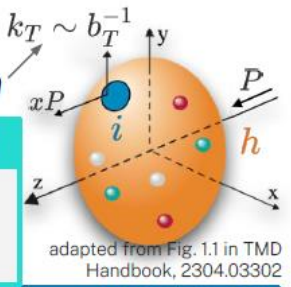
The Collins-Soper kernel, $K(b_T,\mu)$

The soft function, $g_S(b_T,\mu) \sim \sqrt{S_I(b_T,\mu)}$

Slide from D. Lin

Transverse Momentum Distributions

Collins-Soper (CS) kernel from lattice QCD + LaMET complements global analyses in nonperturbative region of *small transverse momentum*



adapted from Fig. 1.1 in TMD Handbook, 2304.03302

CS kernel governs RG evolution of **Transverse-Momentum-dependent Distributions (TMDs)**

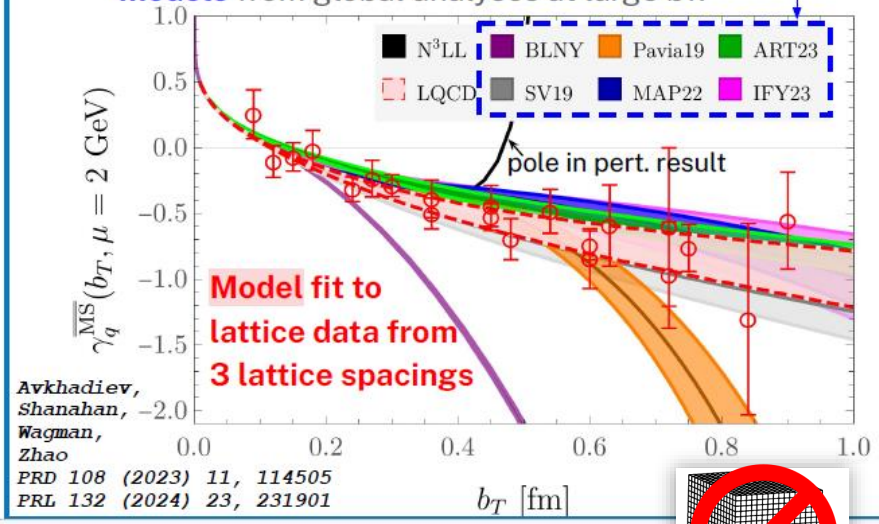
$$\gamma_i(b_T,\mu) = 1/\ln[\boldsymbol{P_1}/\boldsymbol{P_2}] \ln\left[\tilde{f}_{i/h}(x,b_T,\mu,\boldsymbol{P_1}) \Big/ \tilde{f}_{i/h}(x,b_T,\mu,\boldsymbol{P_2})\right] + \text{power corrections}$$

$i \in \{\text{quark}, \text{gluon}\}$

(space-like) **quasi-TMDs** computable in lattice QCD ⟷ LaMET ⟷ light-like TMDs

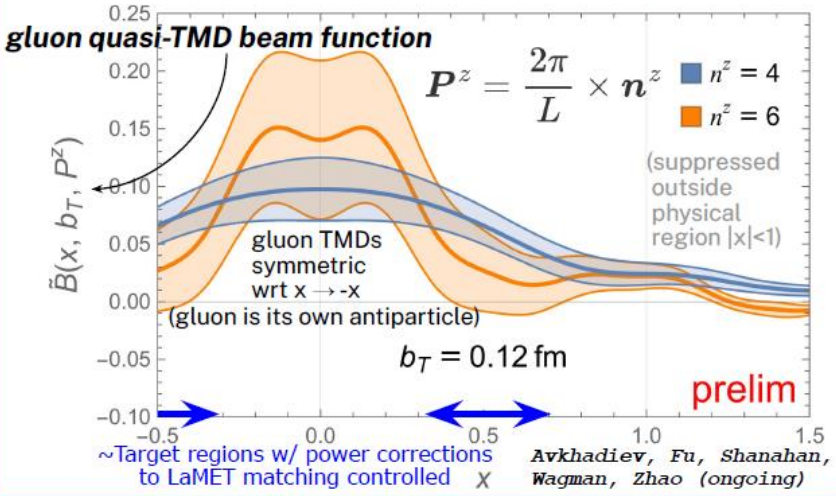
Quark CS kernel — completed

- Systematic control over quark mass, operator mixing, and discretization effects.
- **Sufficiently precise to discriminate between pheno models** from global analyses at large bT.



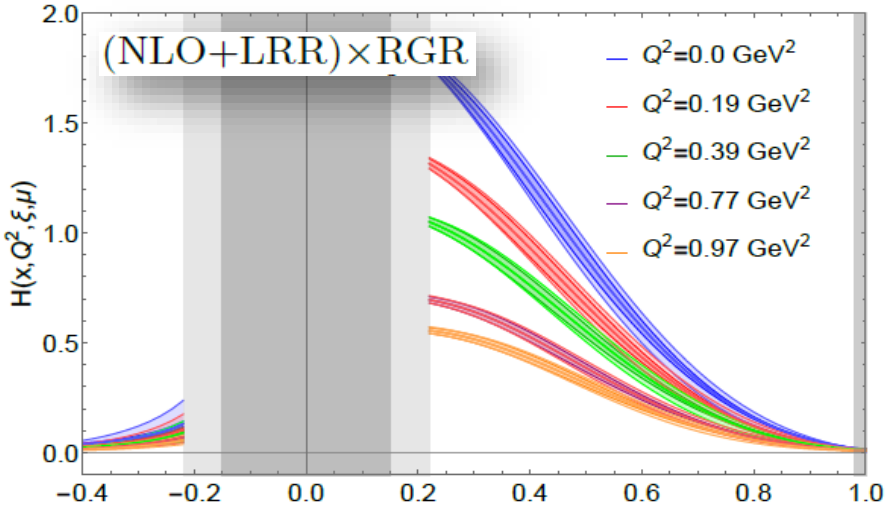
Gluon CS kernel — ongoing

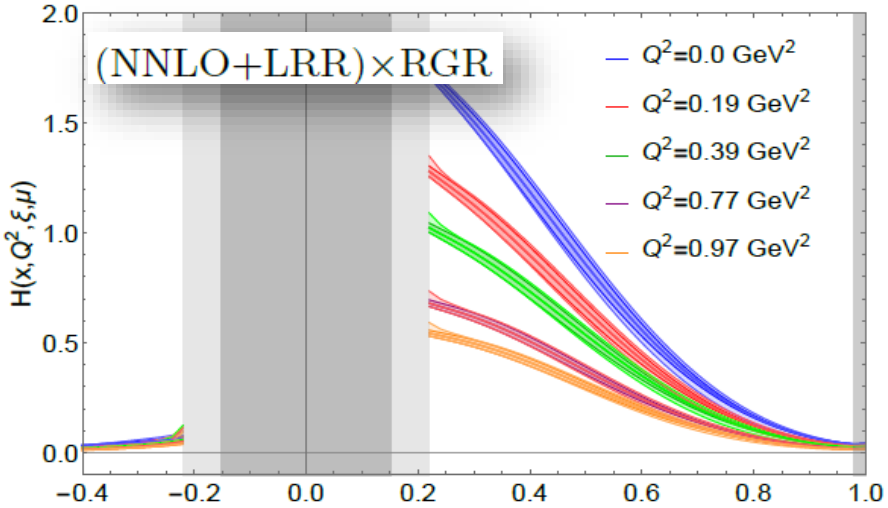
- **Aiming for ~30x more stats** than in the quark project for comparable precision.
- **No global analysis results yet** (expected w/ EIC data) — lattice QCD + LaMET will provide a prediction.

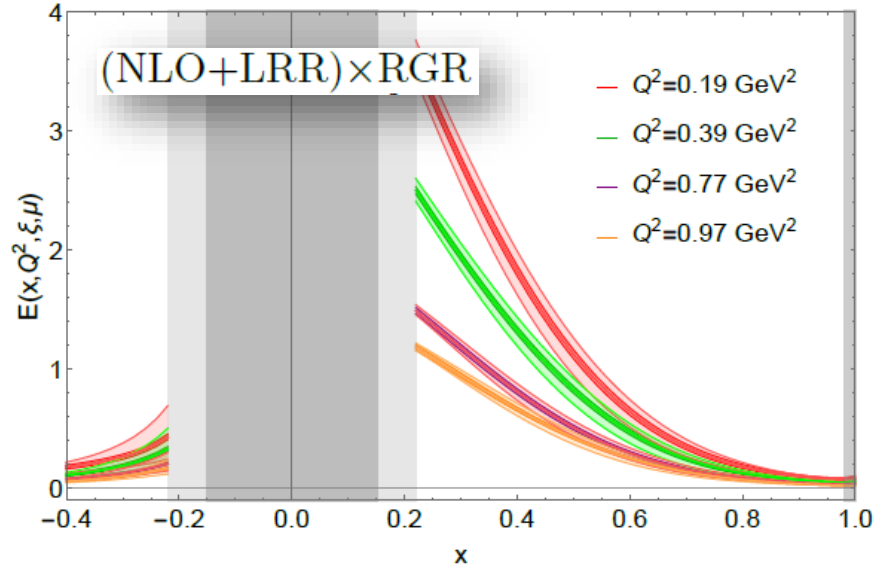


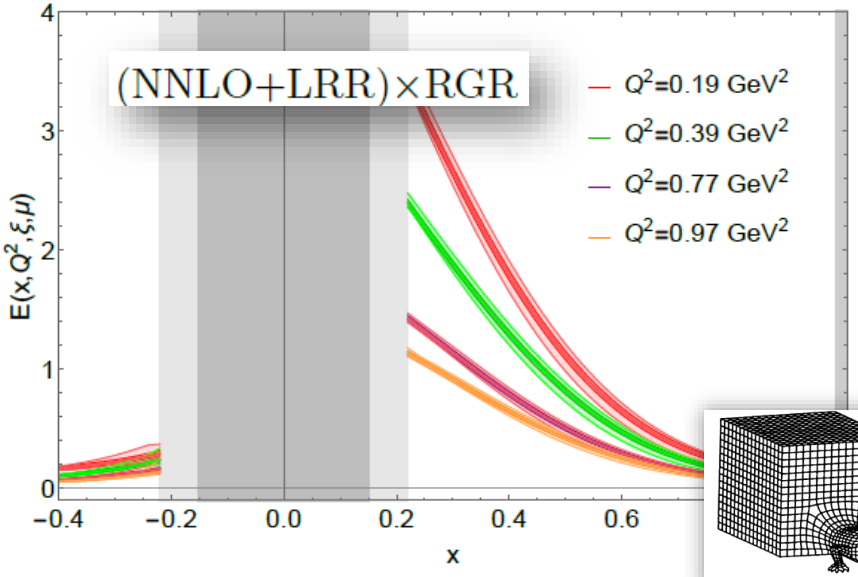
Slide by G: Artur Avkhadiev (MIT)

LRR & RGR ξ=0 GPDs







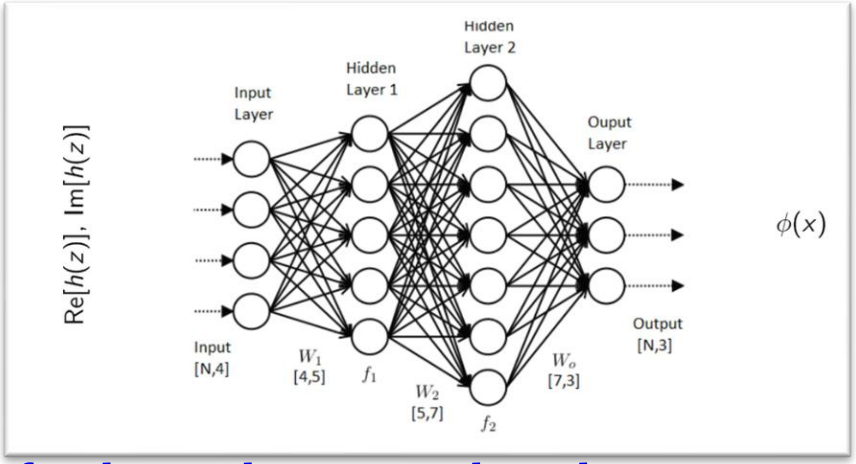


J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

MSULat Pion/Kaon Structure

§ Meson distribution amplitude

- Pion Distribution Amplitude from Lattice,
  Phys. Rev. D 95 (2017) 9, 094514
- Kaon Distribution Amplitude from Lattice QCD and the Flavor SU(3) Symmetry, Nucl. Phys. B 939 (2019) 429-446
- Pion and kaon distribution amplitudes in the continuum limit,
  Phys. Rev. D 102 (2020) 9, 094519



- Precision control in lattice calculation of x-dependent pion distribution amplitude, Nucl. Phys. B 993 (2023) 116282

§ Miscellaneous

- Machine-learning prediction for quasiparton distribution function matrix elements, Phys. Rev. D 101 (2020) 3, 034516

MSULat Pion/Kaon Structure

§ Pion/kaon PDFs

- First direct lattice-QCD calculation of the *x*-dependence of the pion parton distribution function, Phys. Rev. D 100 (2019) 3, 034505
- Valence-Quark Distribution of the Kaon and Pion from Lattice QCD, Phys. Rev. D 103 (2021) 1, 014516
- Gluon parton distribution of the pion from lattice QCD, Phys. Lett. B 823 (2021) 136778
- First glimpse into the kaon gluon parton distribution using lattice QCD, Phys. Rev. D 106 (2022) 9, 094510
- The Gluon Moment and Parton Distribution Function of the Pion from N_f=2+1+1 Lattice QCD, 2310.12034 [hep-lat]
- Pion valence quark distribution at physical pion mass of N_f=2+1+1 LQCD

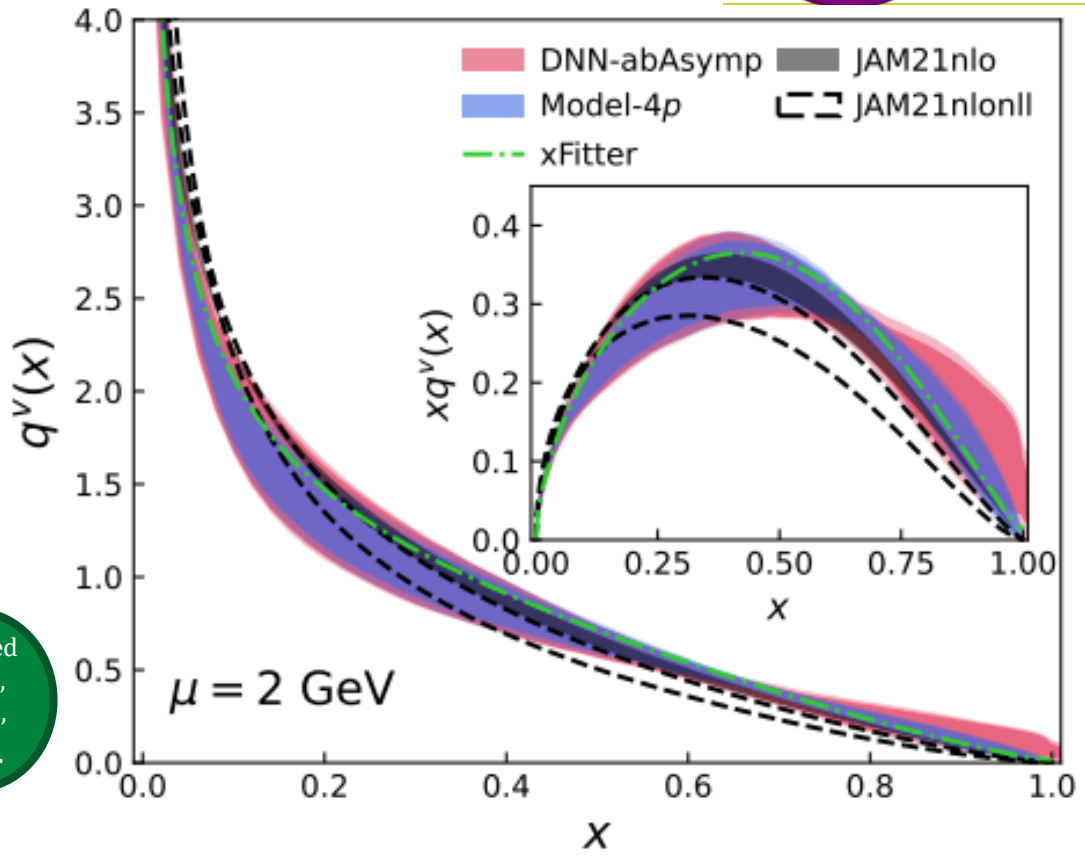
§ Pion GPD

- Pion generalized parton distribution from lattice QCD, Nucl. Phys. B 952 (2020) 114940
- Pion valence-quark generalized parton distribution at physical pion mass , Phys. Lett. B 846 (2023) 138181

Valence-quark PDFs Update

§ Pion PDFs calculated directly at physical pion mass

- with NNLO matching



Wanted PDFs, GPDs, etc…

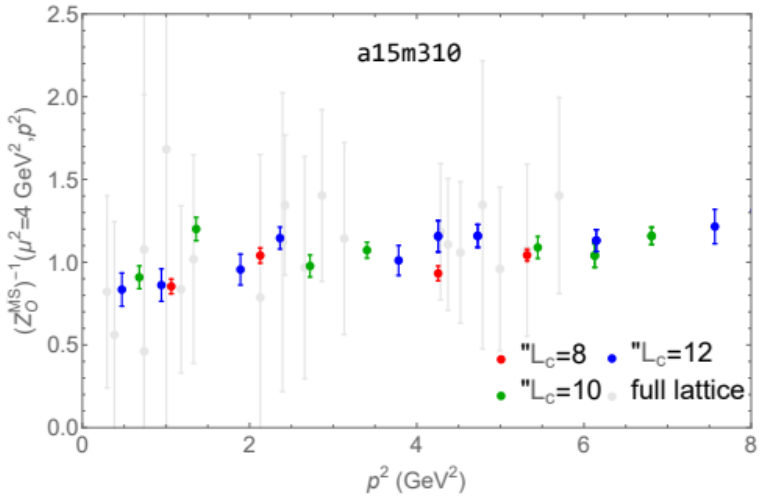
- N_f=2+1 clover/HISQ
  a~0.076 fm

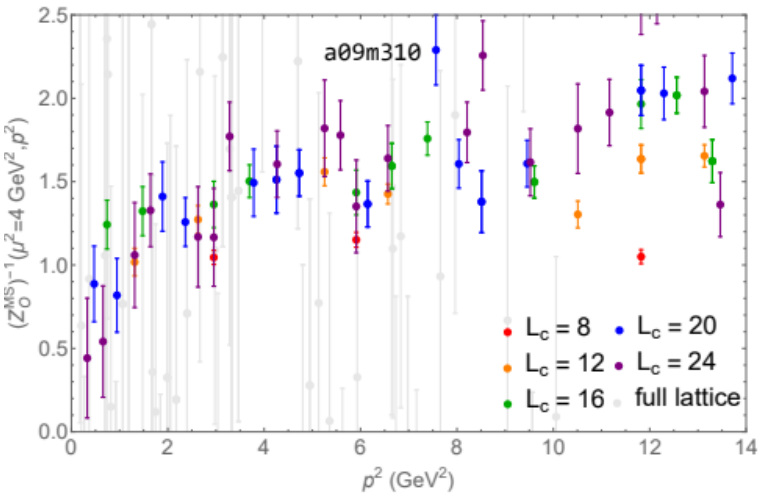
ANL/BNL, Phys. Rev. D 106, 114510 (2022)

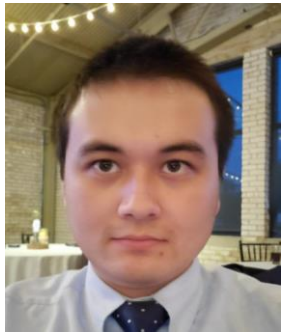
Pion Gluon PDF Update

§ Nonperturbatively renormalized $\langle x \rangle_{\{\pi,g\}}$ at the finer lattice spacing at lighter pion mass is nontrivial

- Using cluster-decomposition error reduction (CDER) to enhance the signal-to-noise ratio 1805.00531, Y. Yang et al. (χQCD)
- Lattice details: clover/HISQ, a ~{0,15, 0.12, 0.09} fm 2208.00980, Fan et al. (MSULat)

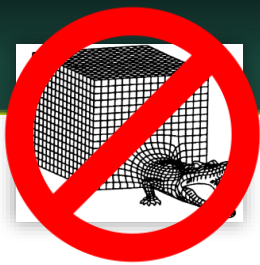




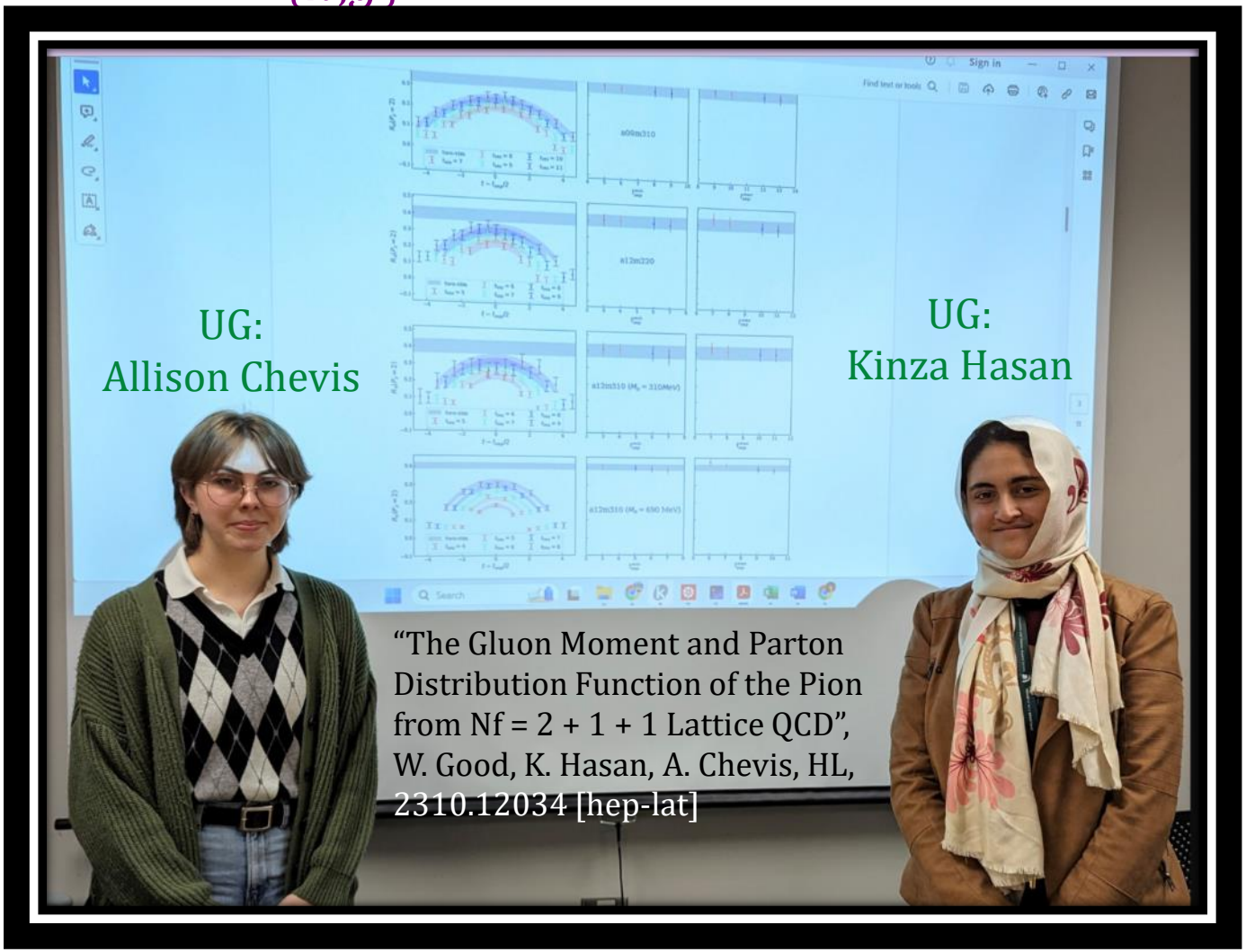


G: Matthew Zeilbeck

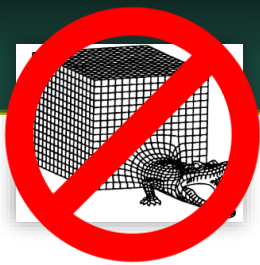
Pion Gluon PDF Update



§ Study Bare $\langle x \rangle_{\{\pi,g\}}$



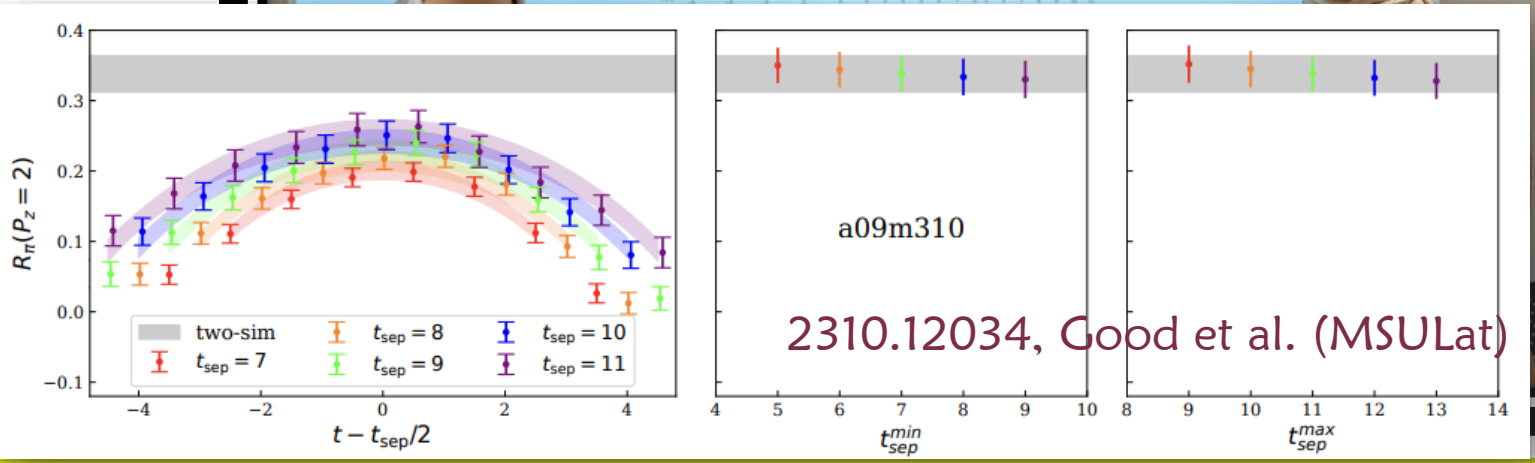
Pion Gluon PDF Update



§ Study Bare $\langle x \rangle_{\{\pi,g\}}$

UG:
Allison Chevis

UG:
Kinza Hasan



2310.12034, Good et al. (MSULat)

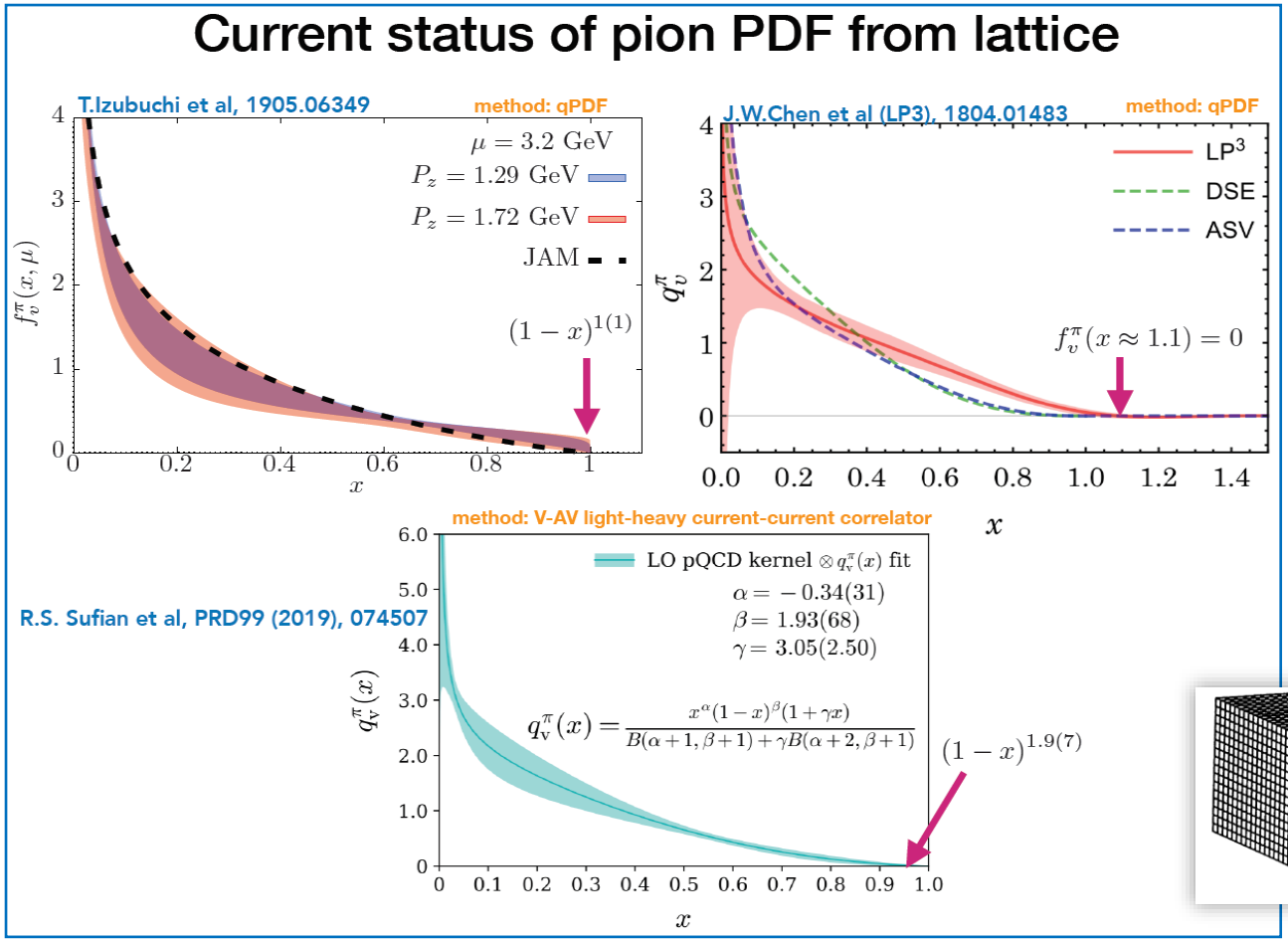
Pion Valence-Quark PDF

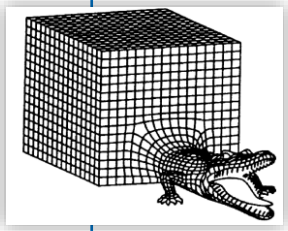
§ Status as of Summer 2019

Slide by Nikhil Karthik @ Lattice 2019

$M_\pi \approx 310$ MeV

$M_\pi \approx 426$ MeV





§ Single-ensemble calculation

- Non-physical pion mass, single lattice spacing, single volume

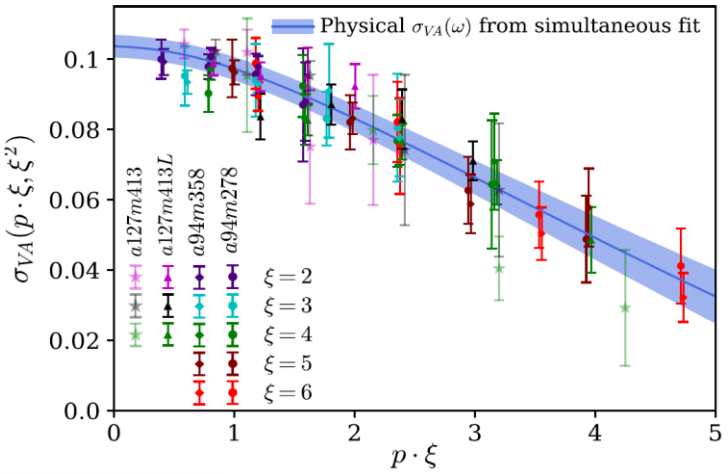
Pion Valence-Quark PDF

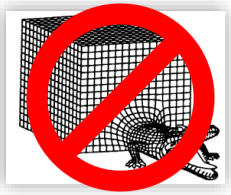
§ Results from JLab-W&M/ **LCS** method

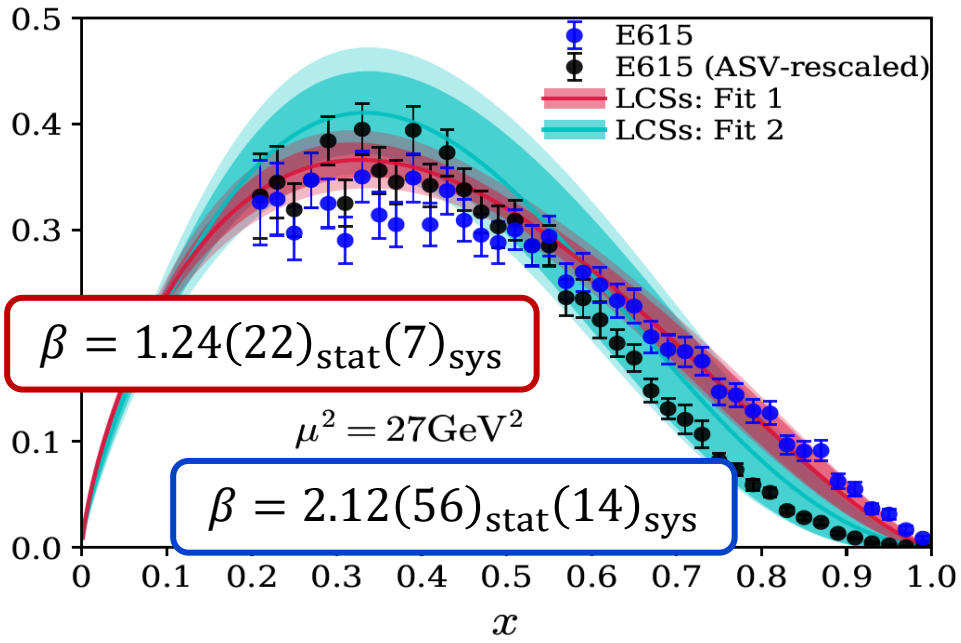
- $M_\pi = \mathbf{278}, 358, 413$ MeV with $a = 0.094, 0.127$ fm
- Extrapolated to physical limit (shown as blue band)
- Renormalized $Z_{V,A}$ in RI/MOM, matched to $\overline{\text{MS}}$, run to 27 GeV^2

R. S. Sufian, et al, 2001.04960

$$q_v^\pi(x) = \frac{x^\alpha (1-x)^\beta (1+\gamma x)}{B(\alpha+1,\beta+1)+\gamma B(\alpha+2,\beta+1)}$$





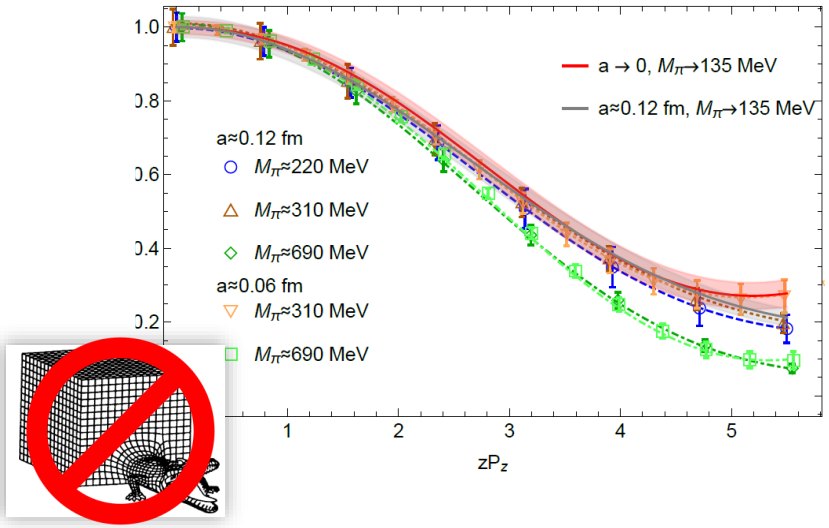


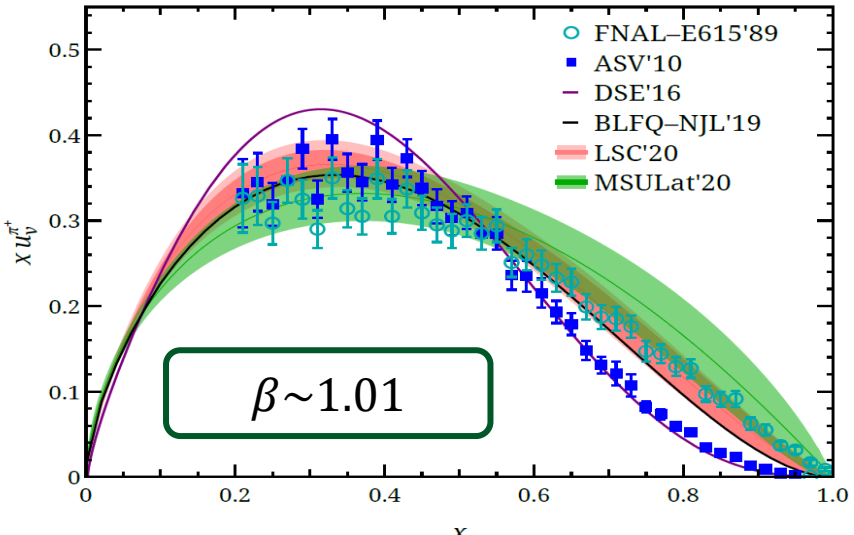
Pion Valence-Quark PDF

§ Results from MSULat/quasi-PDF method

- $M_\pi = \mathbf{220}, 310, 790$ MeV with $a = 0.06, 0.12$ fm
- Extrapolated to physical limit (shown as pink/green band)
- Renormalized in RI/MOM, matched to $\overline{\text{MS}}$, run to 27 GeV^2

H. Lin et al. (MSULat), 2003.14128





J. S. Conway et al., PRD39, 92 (1989).
M. Aicher et al, PRL105, 252003 (2010), 1009.2481.
C. Chen et al, PRD93, 074021 (2016), 1602.01502.

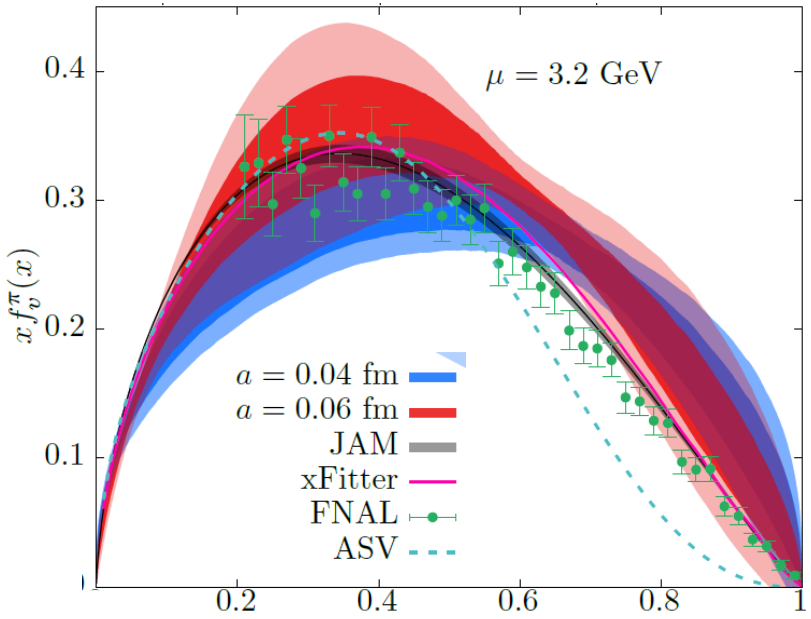
J. Lan, et al, PRL122, 172001 (2019), 1901.11430;
PRD101, 034024 (2020), 1907.01509.
R. S. Sufian, et al, 2001.04960

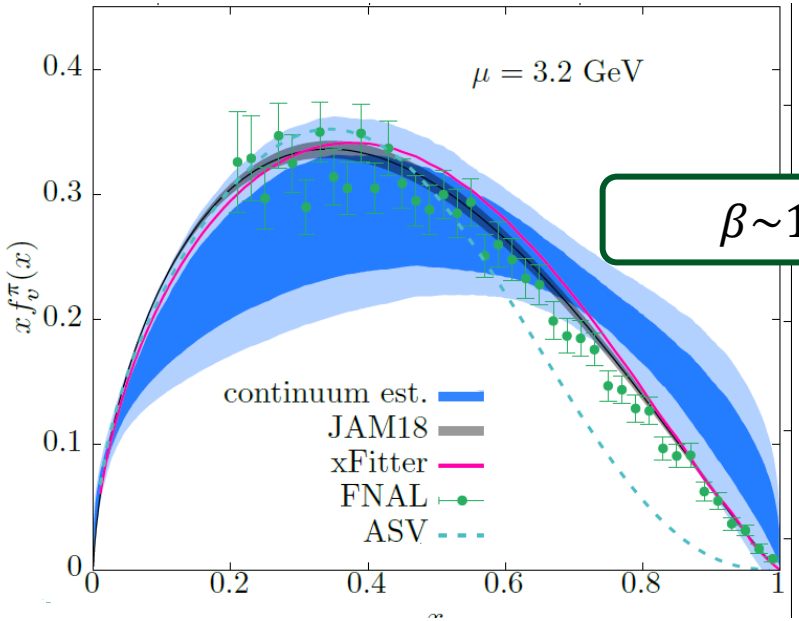
Pion Valence-Quark PDF

§ Results from BNL/quasi-PDF method

- $M_\pi = \mathbf{300}$ MeV with $a = 0.04, 0.06$ fm
- Extrapolated to continuum limit
- Renormalized in RI/MOM, matched to $\overline{\text{MS}}$ at 10 GeV^2

X. Gao et al. 2007.06590



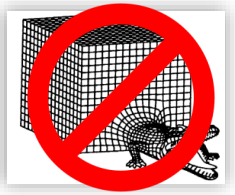


$\beta \sim 1$

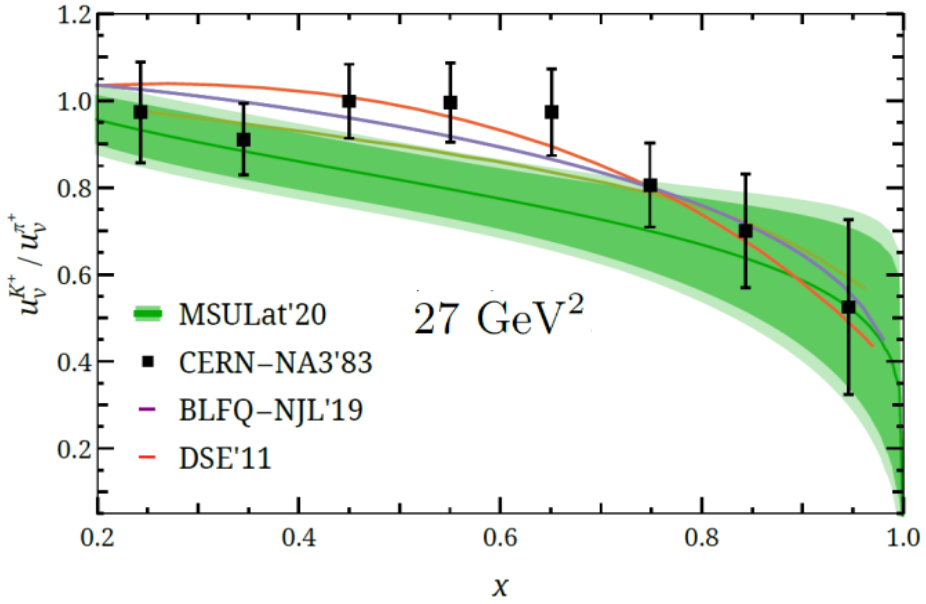
Kaon Valence-Quark PDFs

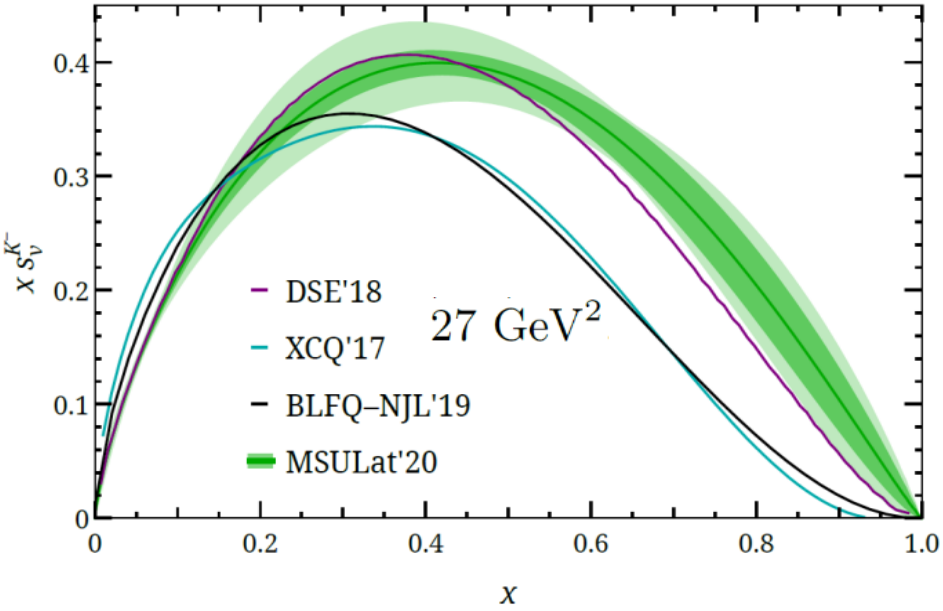
§ Pion/**kaon** PDFs using quasi-PDF in the continuum limit

- Lattice details: clover/2+1+1 HISQ (MSULat)
  $a \approx \{0.06, 0.12\}$ fm,
  $M_\pi \in \{220, 310, 690\}$-MeV pion
  $P_z \approx \{1.3, 1.7\}$ GeV



2003.14128 HL et al (MSULat)

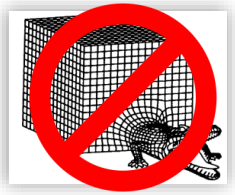




Kaon Valence-Quark PDFs

§ Pion/**kaon** PDFs using quasi-PDF in the continuum limit

- Lattice details: clover/2+1+1 HISQ (MSULat)
  $a \approx \{0.06, 0.12\}$ fm,
  $M_\pi \in \{220, 310, 690\}$-MeV pion
  $P_z \approx \{1.3, 1.7\}$ GeV



§ First LQCD calculation $\langle x^n \rangle$ of $u_v^{K^+}$ and $s_v^{K^-}$ 2003.14128 HL et al (MSULat)

n	$\langle x^n \rangle (u_v^{K^+})$	$\langle x^n \rangle (s_v^{K^-})$
1	$0.192(8)_{\text{stat}}(6)_{\text{syst}}$	$0.261(8)_{\text{stat}}(8)_{\text{syst}}$
2	$0.080(7)_{\text{stat}}(6)_{\text{syst}}$	$0.120(7)_{\text{stat}}(9)_{\text{syst}}$
3	$0.041(6)_{\text{stat}}(4)_{\text{syst}}$	$0.069(6)_{\text{stat}}(8)_{\text{syst}}$

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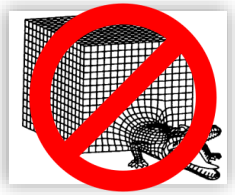

§ First LQCD calculation $\langle x^n \rangle$ of $u_v^{K^+}$ and $s_v^{K^-}$

§ Later ETMC **260**-MeV results on $\langle x^n \rangle$ of $u_v^{K^+}$ and $s_v^{K^-}$

2003.14128 HL et al (MSULat)

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2010.0349, 2104.02247

n	$\langle x^n \rangle (u_v^{K^+})$	$\langle x^n \rangle (s_v^{K^-})$
1	$0.246(2)_{\text{stat}}(2)_{\text{syst}}$	$0.317(2)_{\text{stat}}(1)_{\text{syst}}$
2	$0.093(5)_{\text{stat}}(3)_{\text{syst}}$	$0.134(5)_{\text{stat}}(2)_{\text{syst}}$
3	$0.035(6)_{\text{stat}}(3)_{\text{syst}}$	$0.075(5)_{\text{stat}}(1)_{\text{syst}}$

First Pion Gluon PDF

§ Pion GLUON PDFs using pseudo-PDF

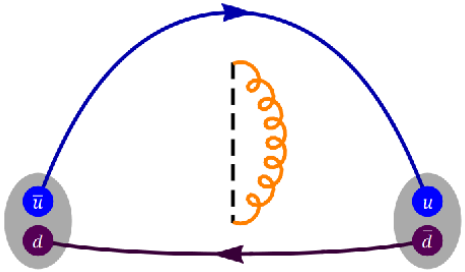
- Lattice details: clover/2+1+1 HISQ (MSULat)

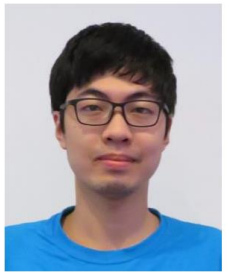
  $a \approx \{0.12, 0.15\}$ fm,

  $M_\pi \in \{220, 310, 690\}$-MeV pion

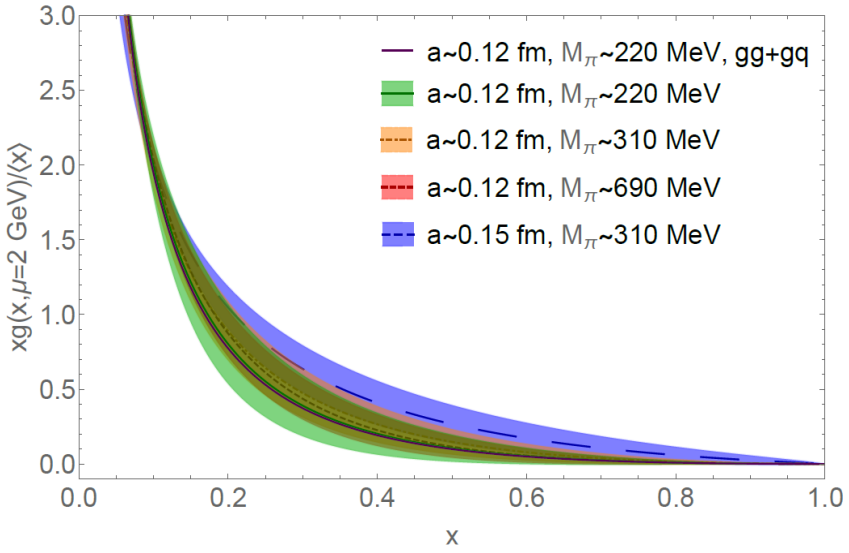
  $P_{z,\max} \approx 2.3$ GeV

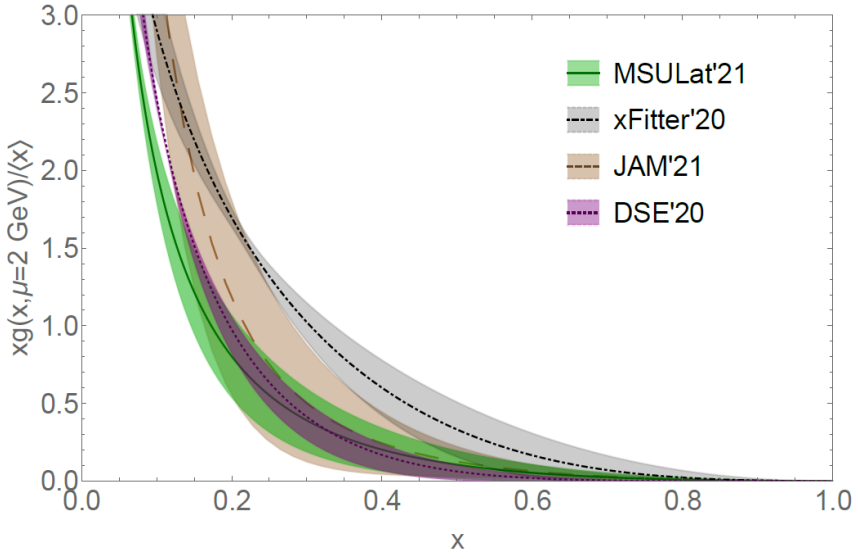
2104.06372, Fan, HL(MSULat)





Zhouyou Fan (MSU)



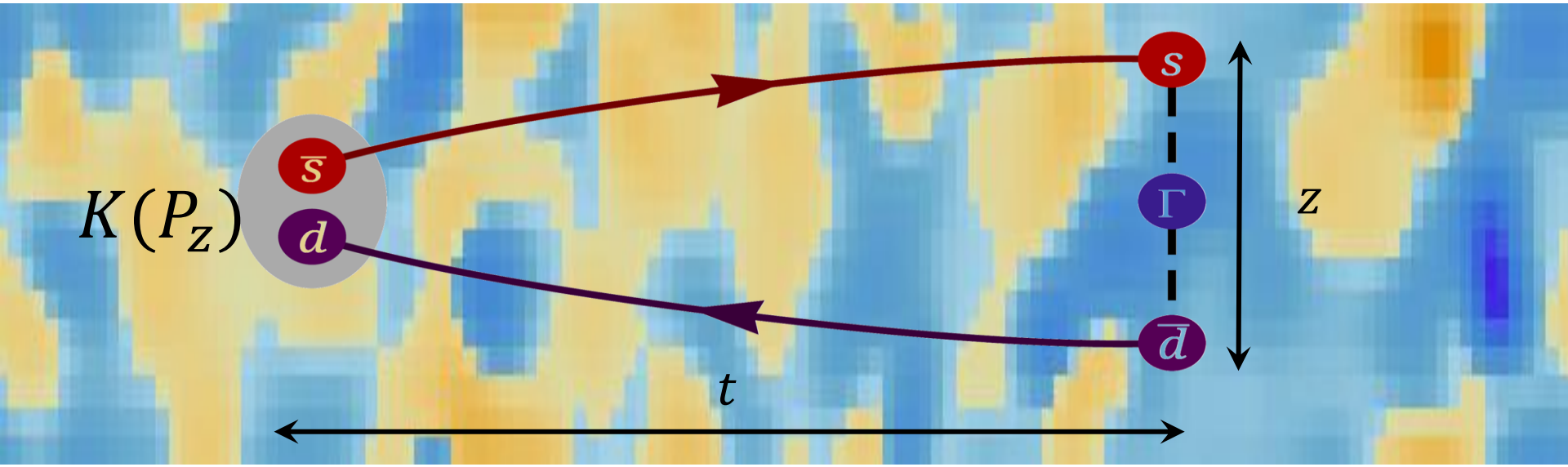


Pion and Kaon DA

§ The first continuum-limit study of *x*-dependent meson DA on the lattice

- $M_\pi \in \{\mathbf{310}, 690\ (\eta_s)\}$ MeV
- $a \in \{0.06, 0.09, 0.12\}$ fm
- $M_\pi^{\min} L = 4.5$

$$C_M^{\mathrm{DA}}(z,P,t) = \left\langle 0 \left| \int d^3y\, e^{i\,\vec{P}\cdot\vec{y}} \bar{\psi}_1(\vec{y},t)\gamma_z\gamma_5 U(\vec{y},\vec{y}+z\,\hat{z})\psi_2(\vec{y}+z\,\hat{z},t)\bar{\psi}_2(0,0)\gamma_5\psi_1(0,0) \right| 0 \right\rangle$$



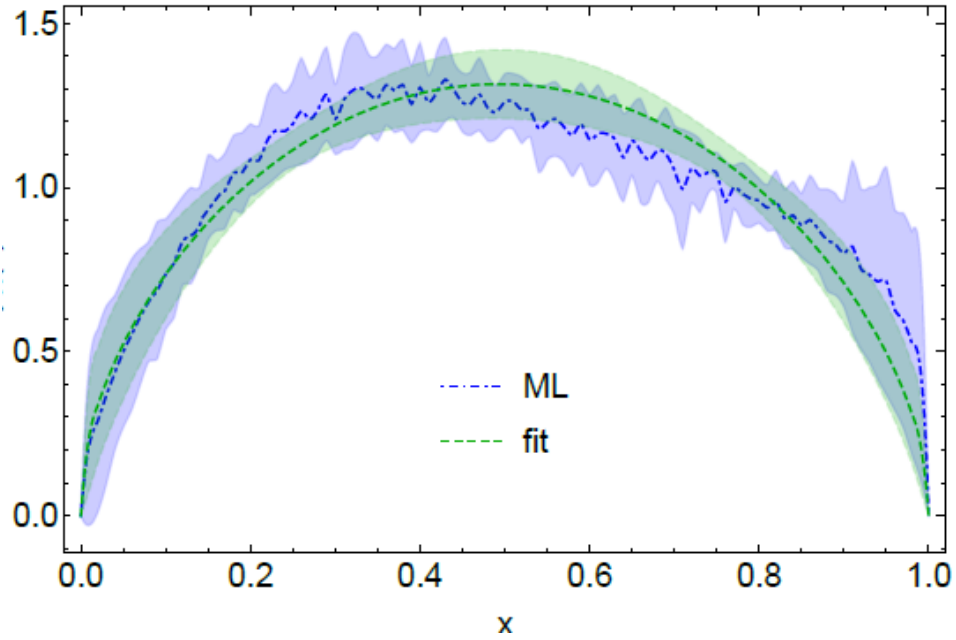
Pion and Kaon DA

§ Extract the DA distribution from the physical-continuum matrix elements

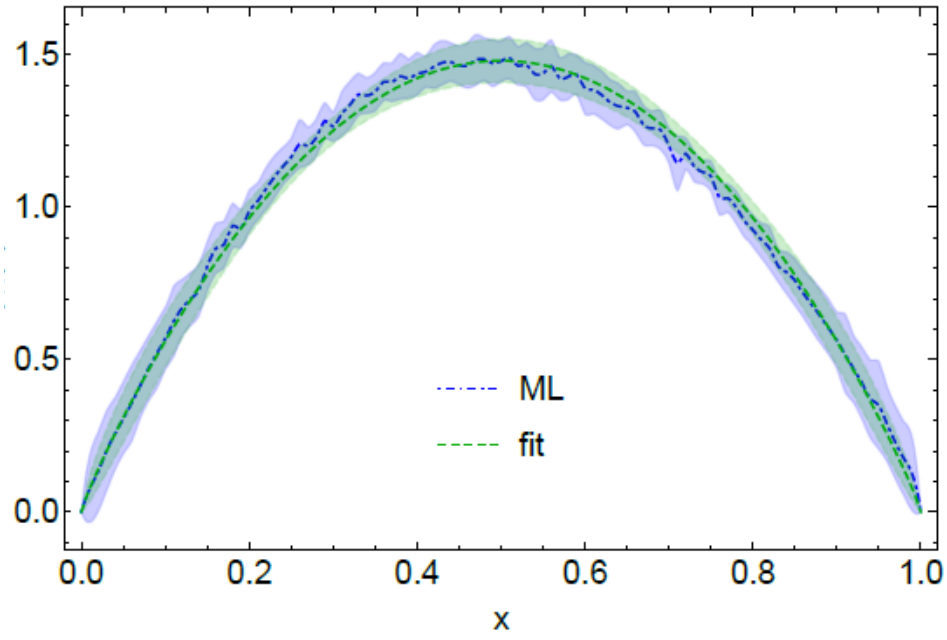
R. Zhang et al. (MSULat), 2005.13955

$$h(z,\mu^R,p_z^R,P_z)=\int_{-\infty}^{\infty}dx\int_0^1 dy\, C\left(x,y,\left(\frac{\mu^R}{p_z^R}\right)^2,\frac{P_z}{\mu^R},\frac{P_z}{p_z^R}\right)f_{m,n}(y)e^{i(1-x)zP_z}$$

Pion



Kaon



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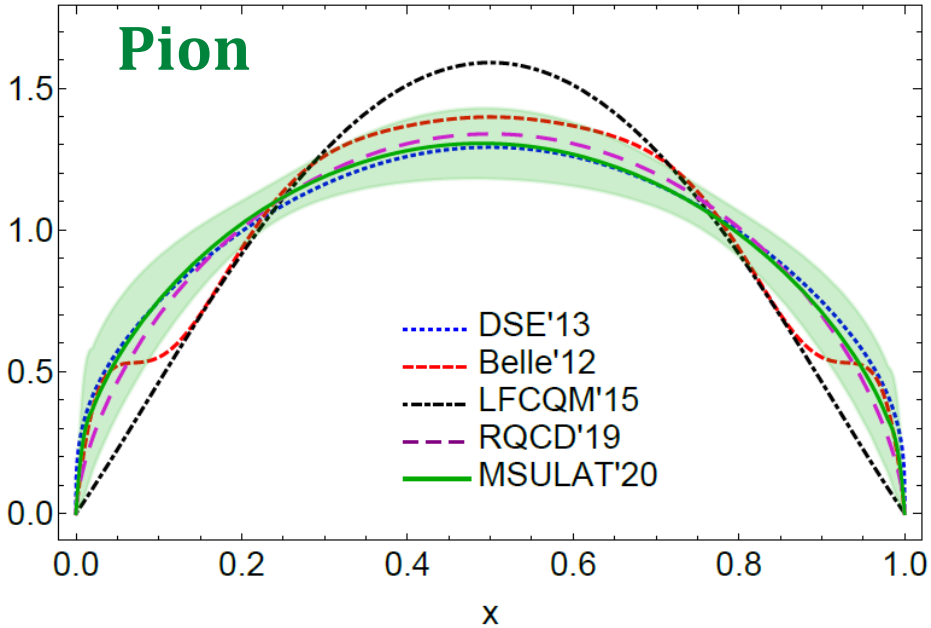
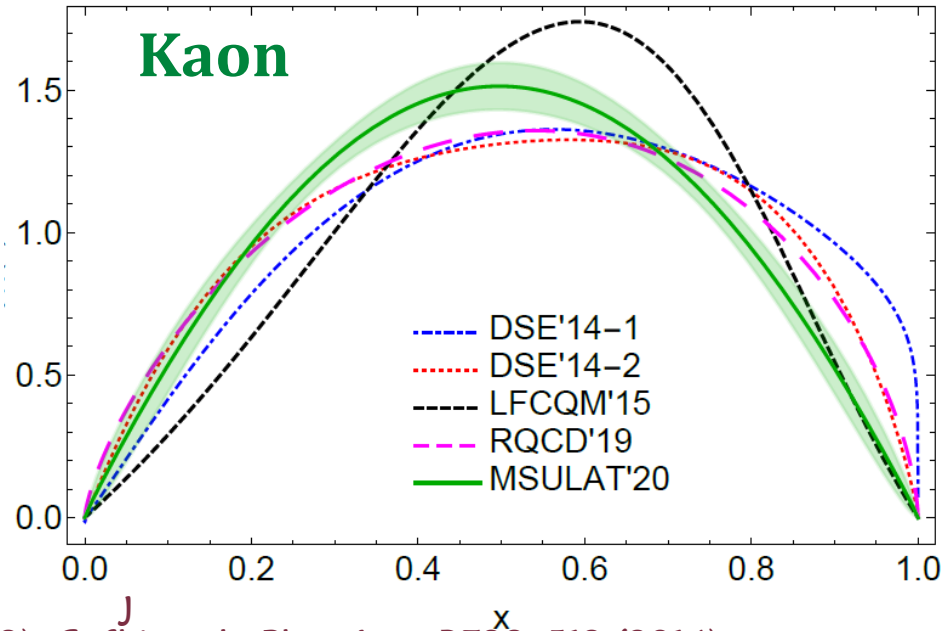


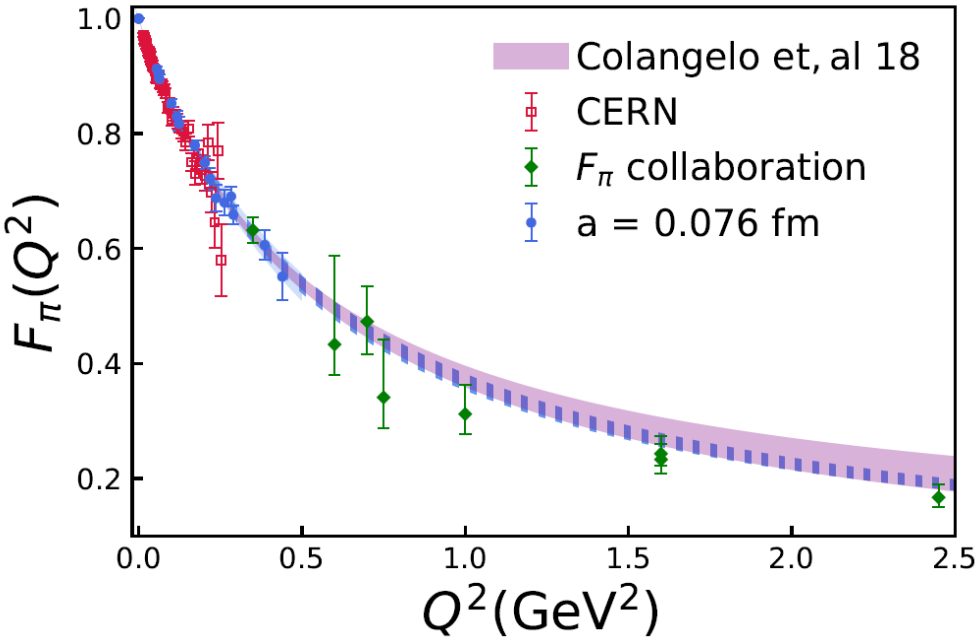


DES'13: L. Chang et al., Phys. Rev. Lett. 110, 132001 (2013); C. Shi et al., Phys. Lett. B738, 512 (2014)
Belle'12: S. Agaev et al., Phys. Rev. D86, 077504 (2012);
LFCQM'15:J. P. B. C. de Melo et al., AIP Conf. Proc. 1735, 080012 (2016);
RQCD'19: G. S. Bali et al.,J HEP 08, 065 (2019); DSE'14:

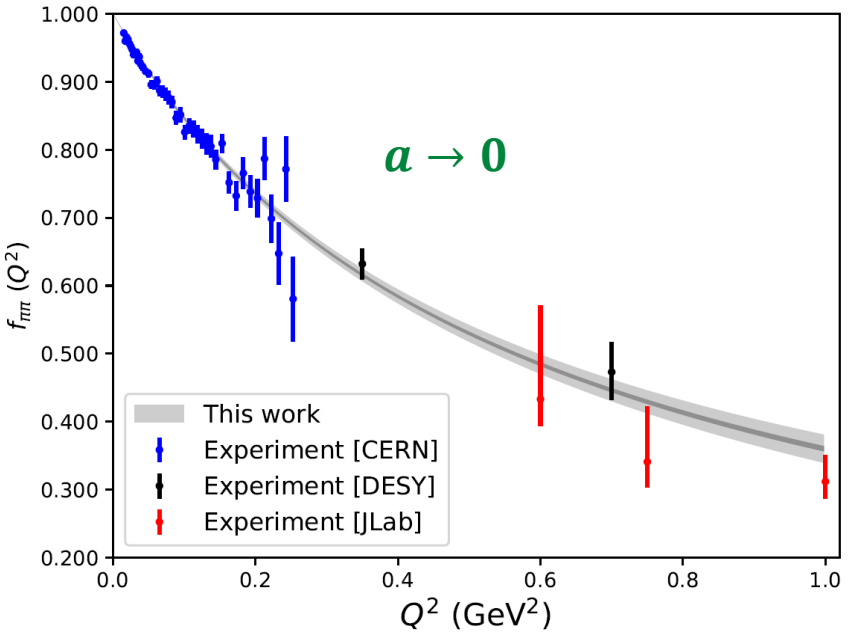
Pion Form Factors

§ Two new lattice pion form factors calcs at physical pion

- χQCD: 2+1f, overlap/DWF,
  $a \approx [0.08,0.2]$ fm, $M_\pi \in [139,340]$-MeV
- BNL: 2+1+1,clover/HISQ,
  $a \approx [0.04,0.08]$ fm, $M_\pi \in \{135, 300\}$-MeV



X. Gao et al (BNL), 2102.06047



G. Wang et al (XQCD), 2006.05431

Caveats

§ Systematics in our earlier quasi-PDF calculation

- Renormalization: non-perturbative RI/MOM renormalization
  - State of the art: hybrid-ratio renormalization

    X. Ji et. al. NPB 964, 115311 (2021)
- Next-leading order (NLO) matching only
  - State of the art: NNLO matching kernel available

    X. Gao, PRL 128, 142003 (2022)
- Did not teat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

    R. Zhang, et. al. PLB 844, 138081 (2023)
- For the rest of this presentation, we will focus on the uncertainties from the above (rather than typical lattice-calculation precision or systematics)

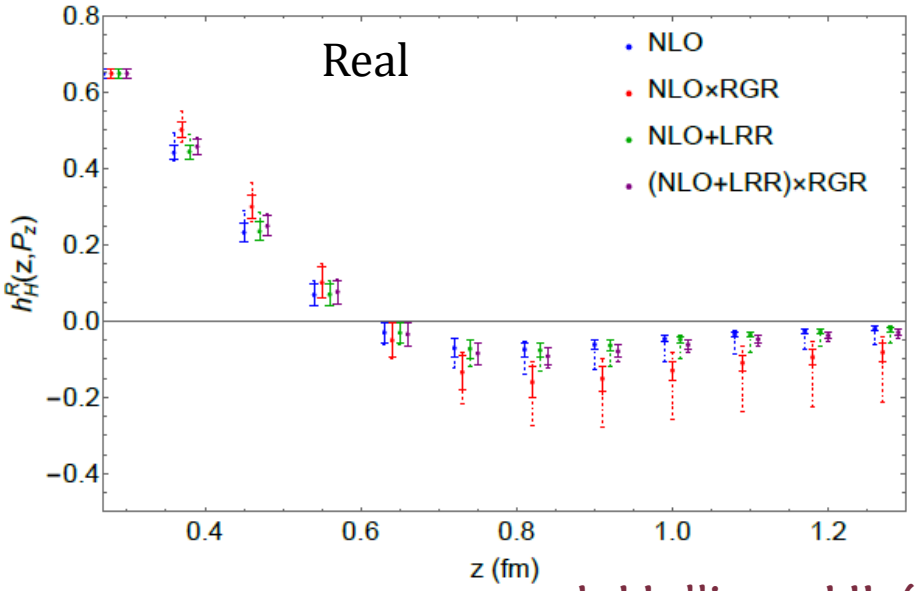
Forward-Limit Case: PDF

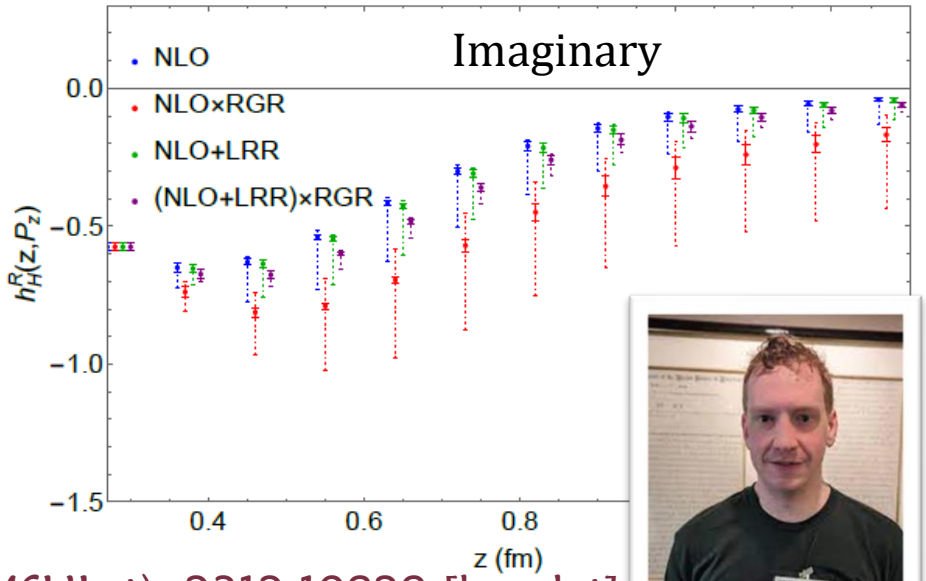
§ NLO hybrid-ratio renormalized matrix elements

$$h^R(z,P_z) = \begin{cases} N\dfrac{h^B(z,P_z)}{h^B(z,P_z=0)} & \text{for } z < z_s \\ Ne^{(\delta m+m_0)(z-z_s)}\dfrac{h^B(z,P_z)}{h^B(z,P_z=0)} & \text{for } z \geq z_s \end{cases}$$

Remove the linear divergence & renormalon ambiguity at large distances

- Vary the scale within $[0.75, 1.5]$: $\approx 15\%$ variation $\alpha_s(\mu = 2.0\ \text{GeV})$
- Systematic errors shown below:



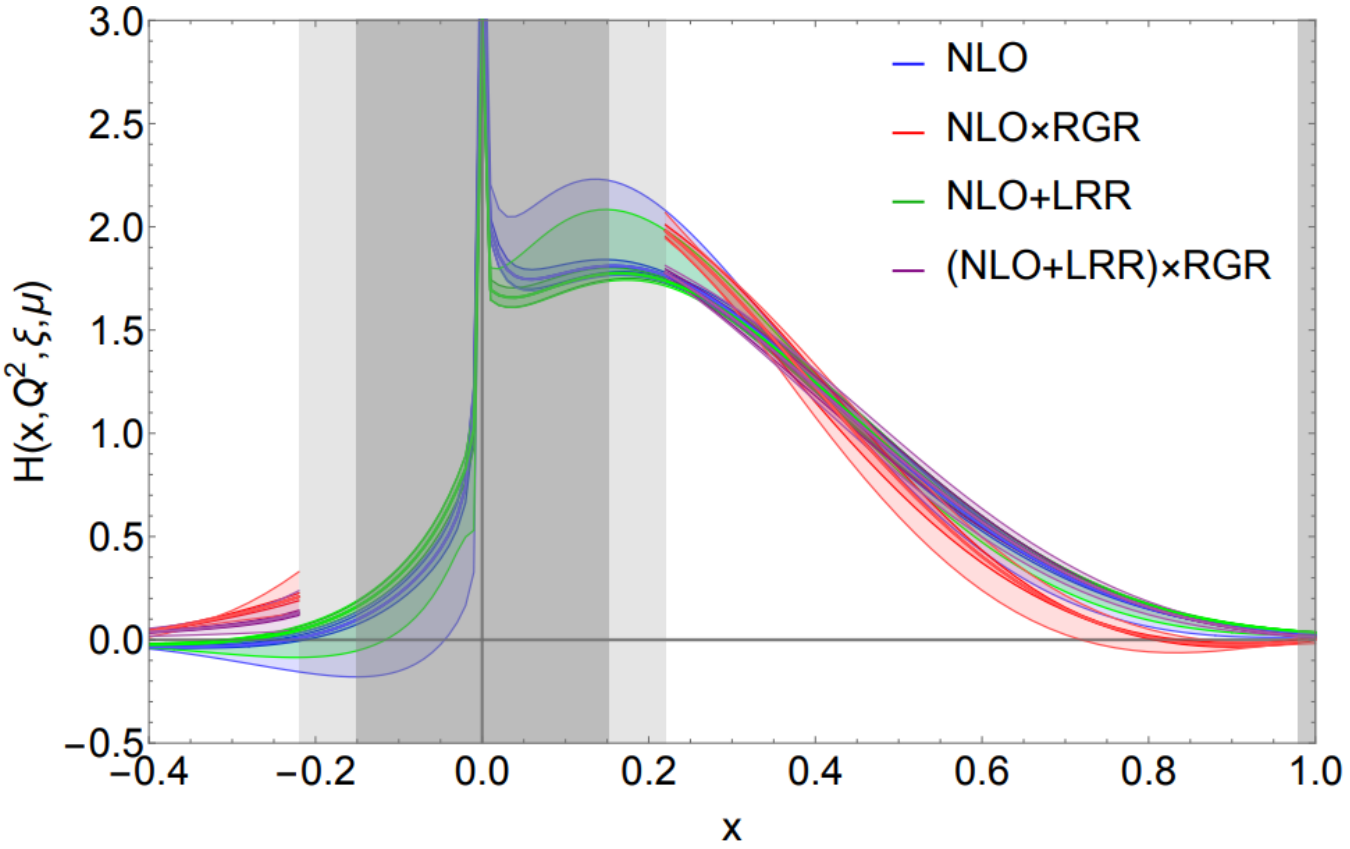


J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

Forward Limit Case: PDF

§ NLO isovector nucleon $H(\xi = 0, Q^2 = 0, x)$

- RGR process: DGLAP equation breaks down for $|x| \lesssim 0.2$ with $\mu = c' \times 2xP_z$



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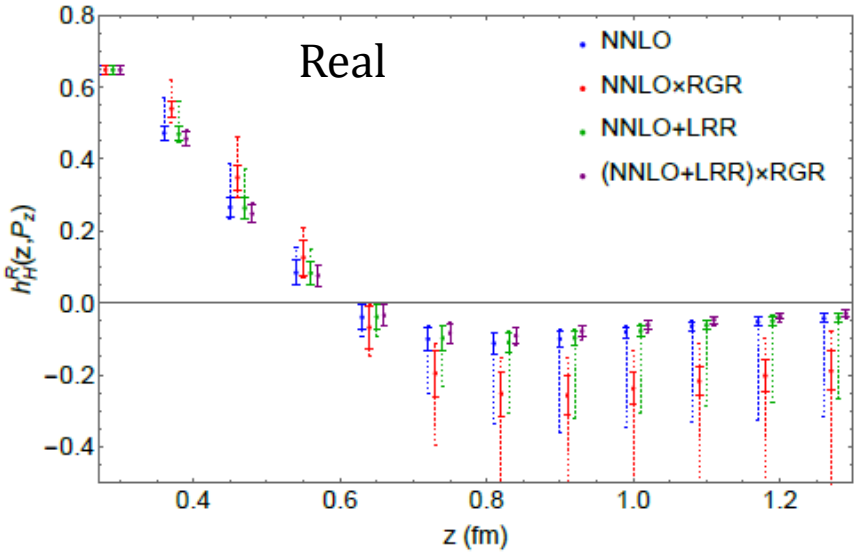

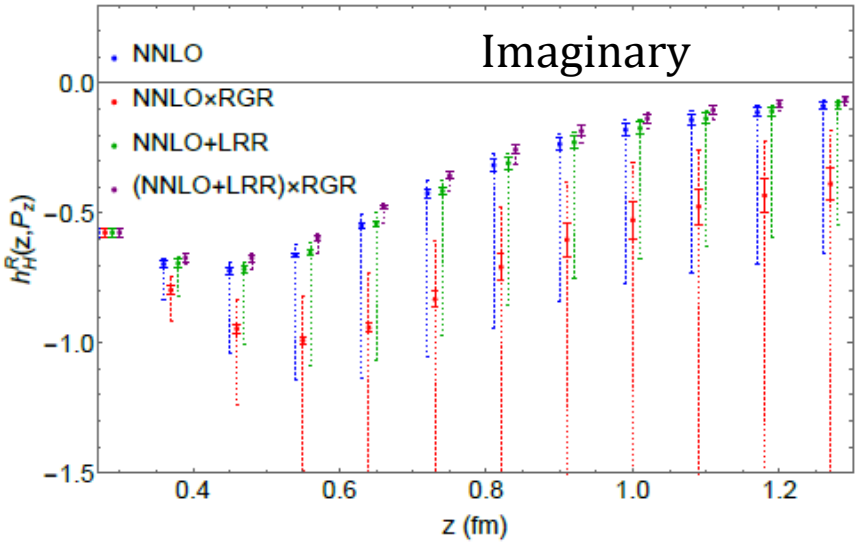

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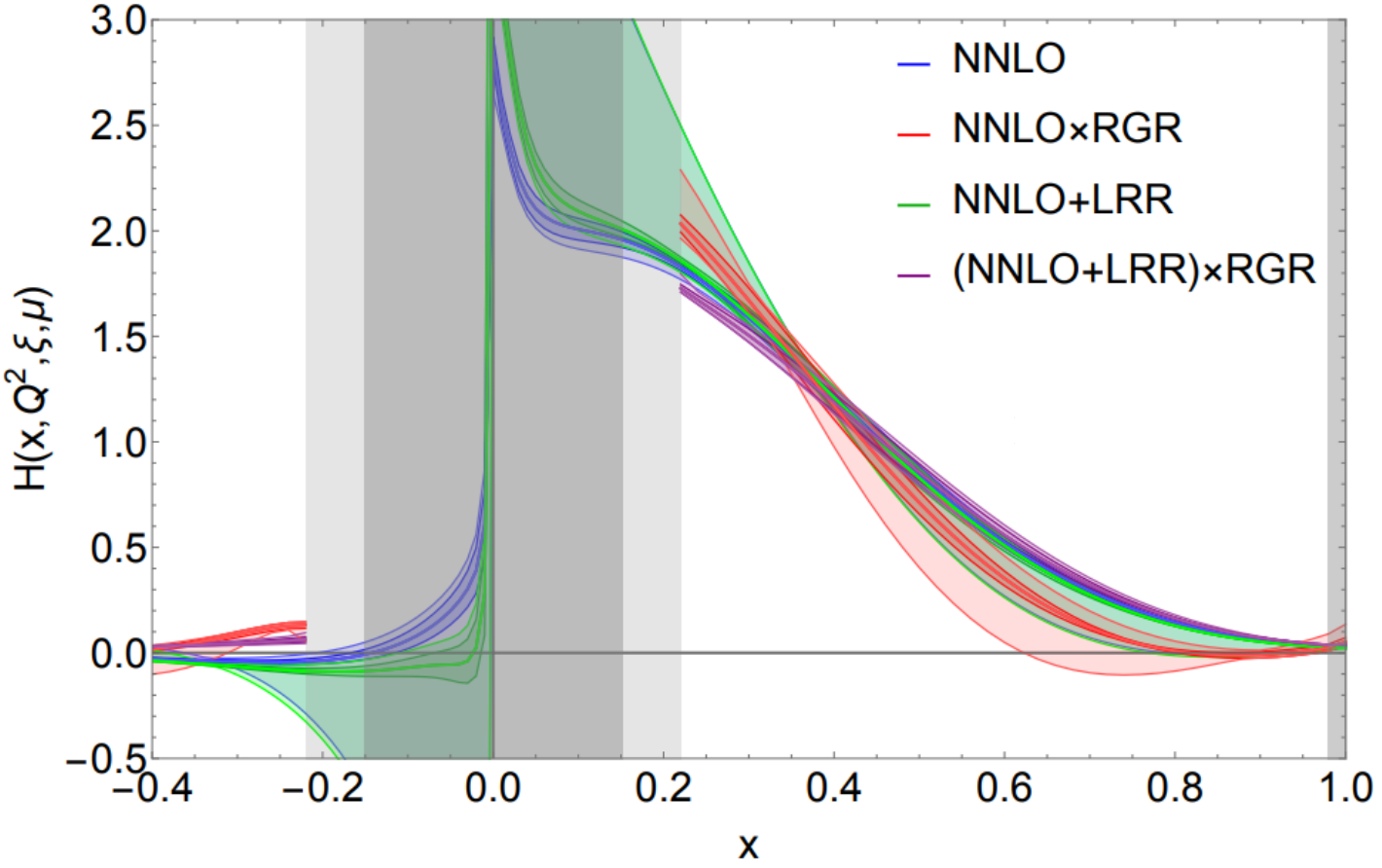
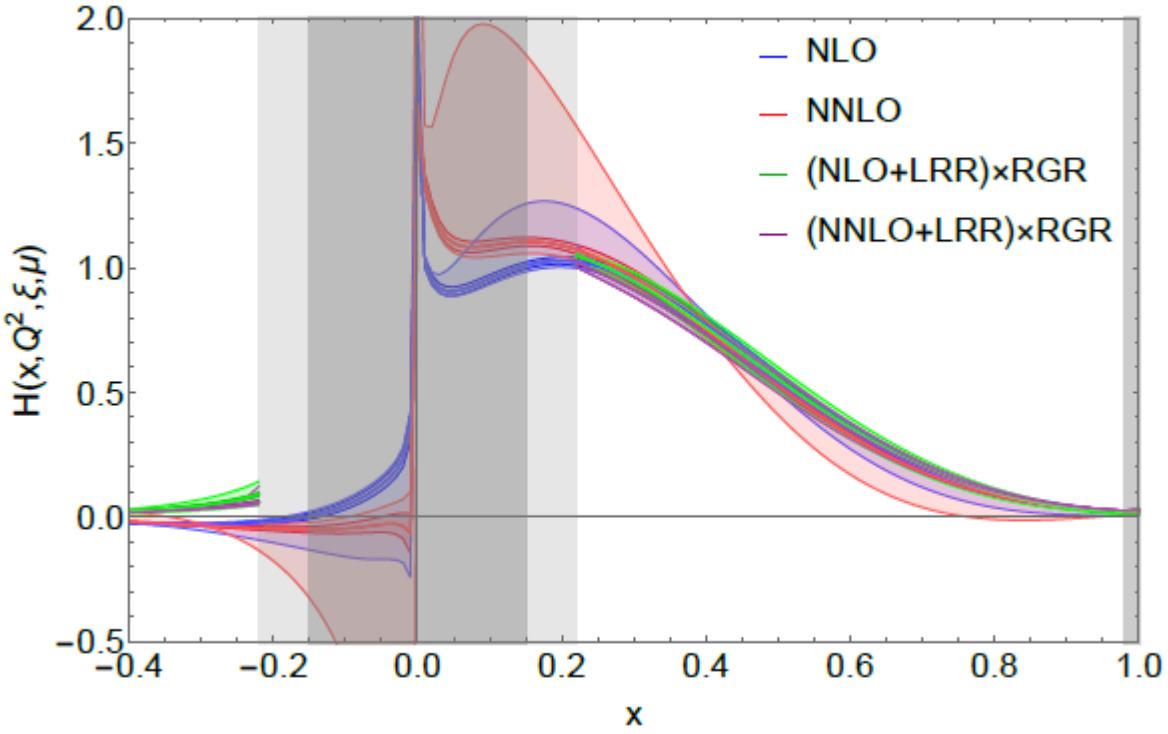


J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

$\xi=0$, $Q^2=0.39$ GeV^2 GPDs

§ Repeat the procedure for nonzero transfer momentum



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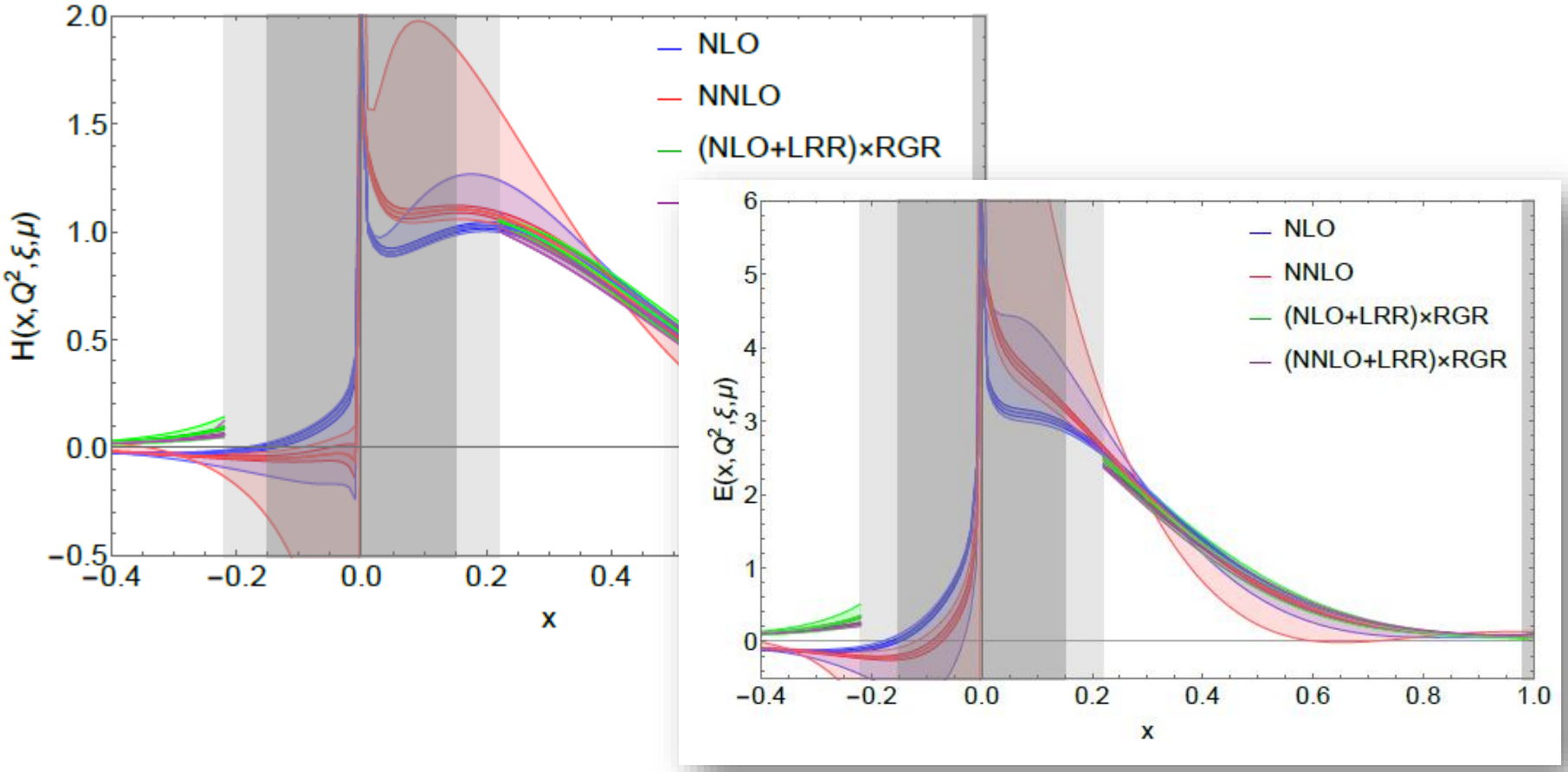
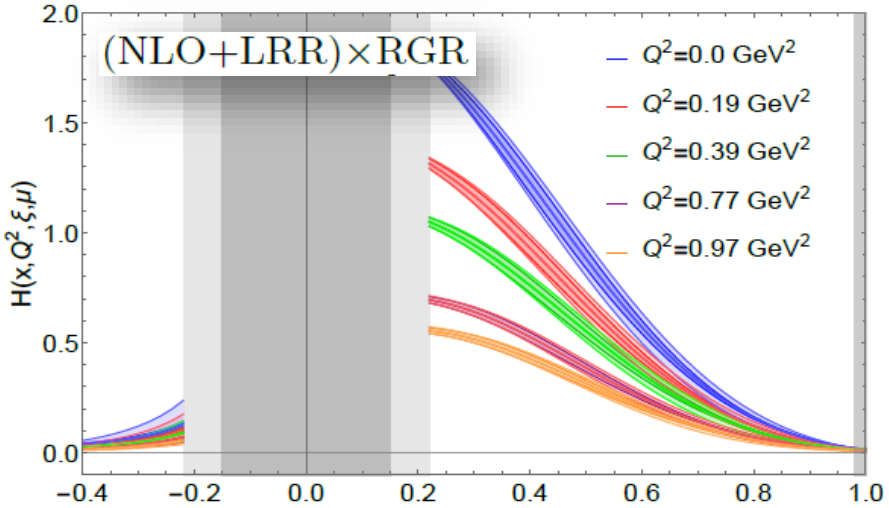
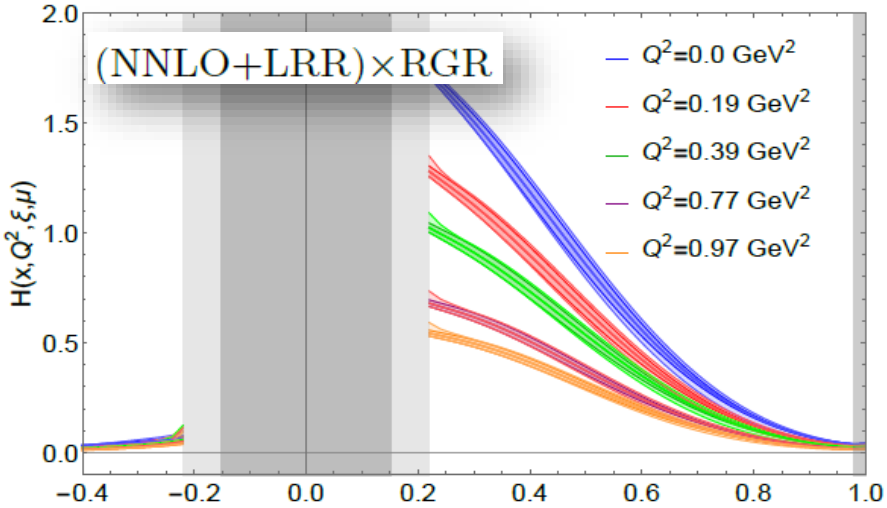
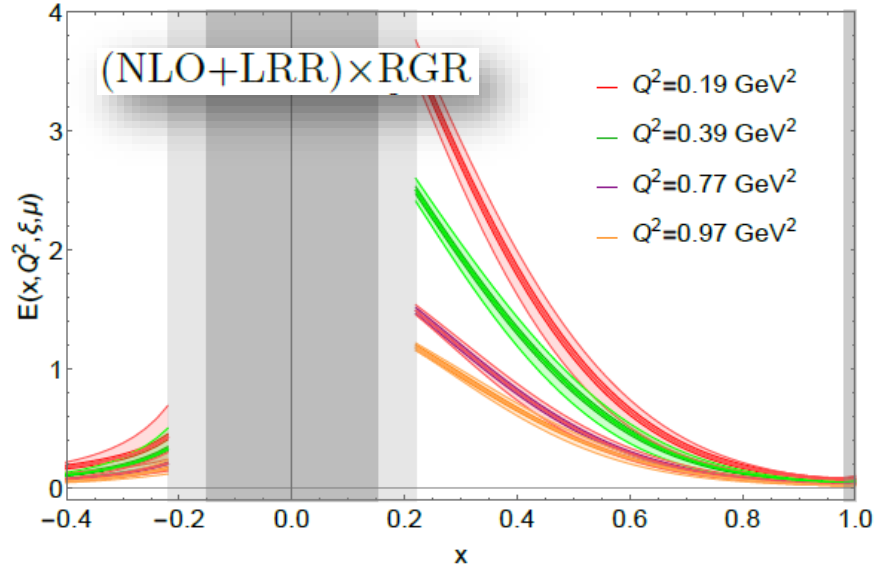


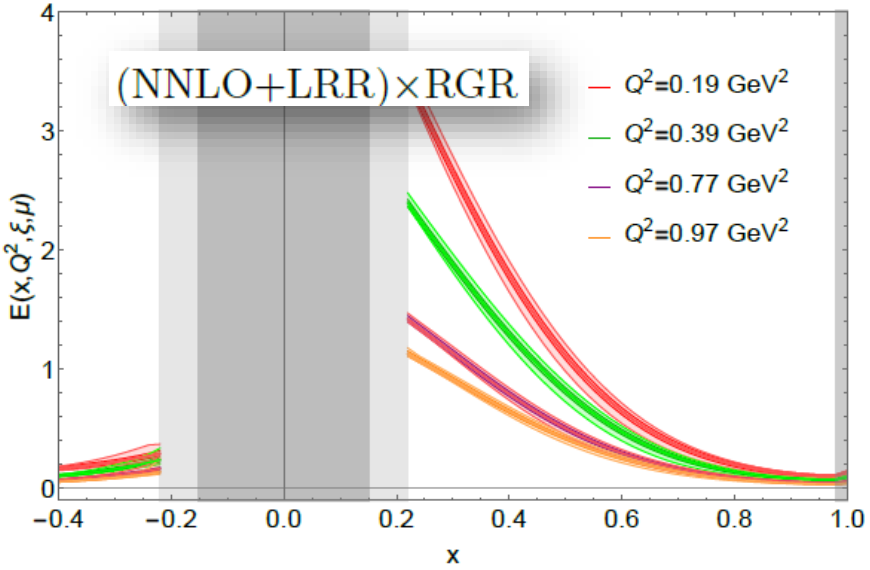
J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

ξ=0 GPDs









J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

$\xi \neq 0$ GPDs

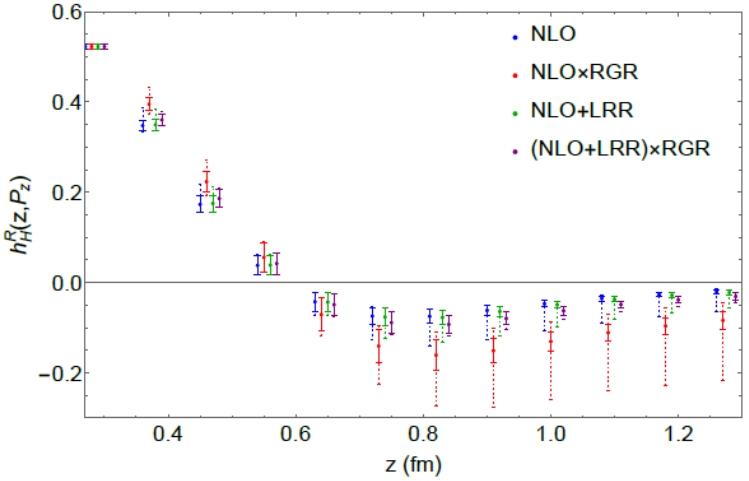
§ Only the NLO matching kernel is available

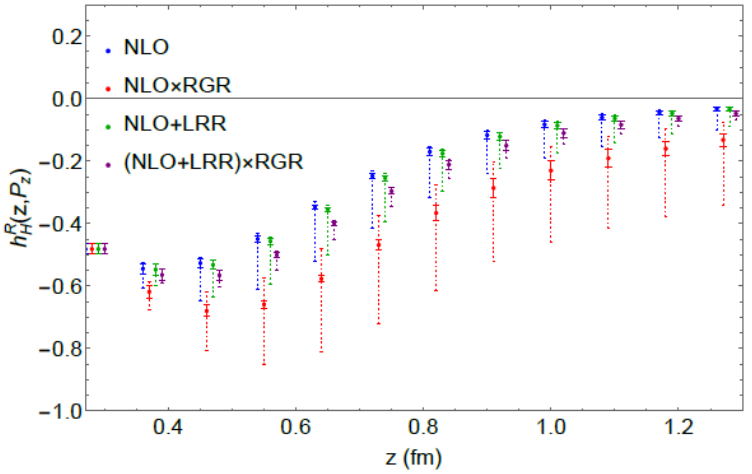
$$
\begin{aligned}
&\mathcal{K}(x, y, \mu, \xi, P_z) \\
&= \delta(x-y) \\
&+ \frac{\alpha_s C_F}{4\pi}\Bigg[\left(\frac{|\xi+x|}{2\xi(\xi+y)} + \frac{|\xi+x|}{(\xi+y)(y-x)}\right)\left(\ln\left(\frac{4y^2(\xi+x)^2P_z^2}{\mu^2}\right)-1\right) \\
&+ \left(\frac{|\xi-x|}{2\xi(\xi-y)} + \frac{|\xi-x|}{(\xi-y)(x-y)}\right)\left(\ln\left(\frac{4y^2(\xi-x)^2P_z^2}{\mu^2}\right)-1\right) \\
&+ \left(\left(\frac{\xi+x}{\xi+y} + \frac{\xi-x}{\xi-y}\right)\frac{1}{|x-y|} - \frac{|x-y|}{\xi^2-y^2}\right)\left(\ln\left(\frac{4y^2(x-y)^2P_z^2}{\mu^2}\right)-1\right)\Bigg]
\end{aligned}
$$

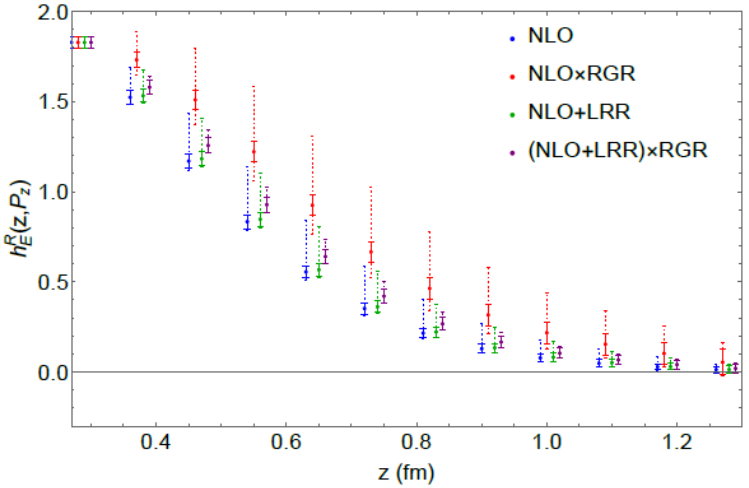
F. Yoa et al, JHEP 11(2023) 021

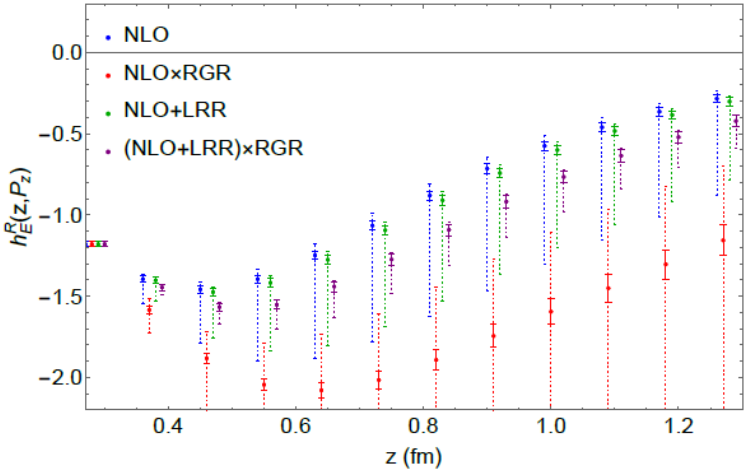
$\xi \neq 0$ GPDs

§ NLO $\xi = 0.1$, $Q^2 = 0.23\ \mathrm{GeV}^2$









J. Holligan, HL (MSULat), in preparation

$\xi \neq 0$ GPDs

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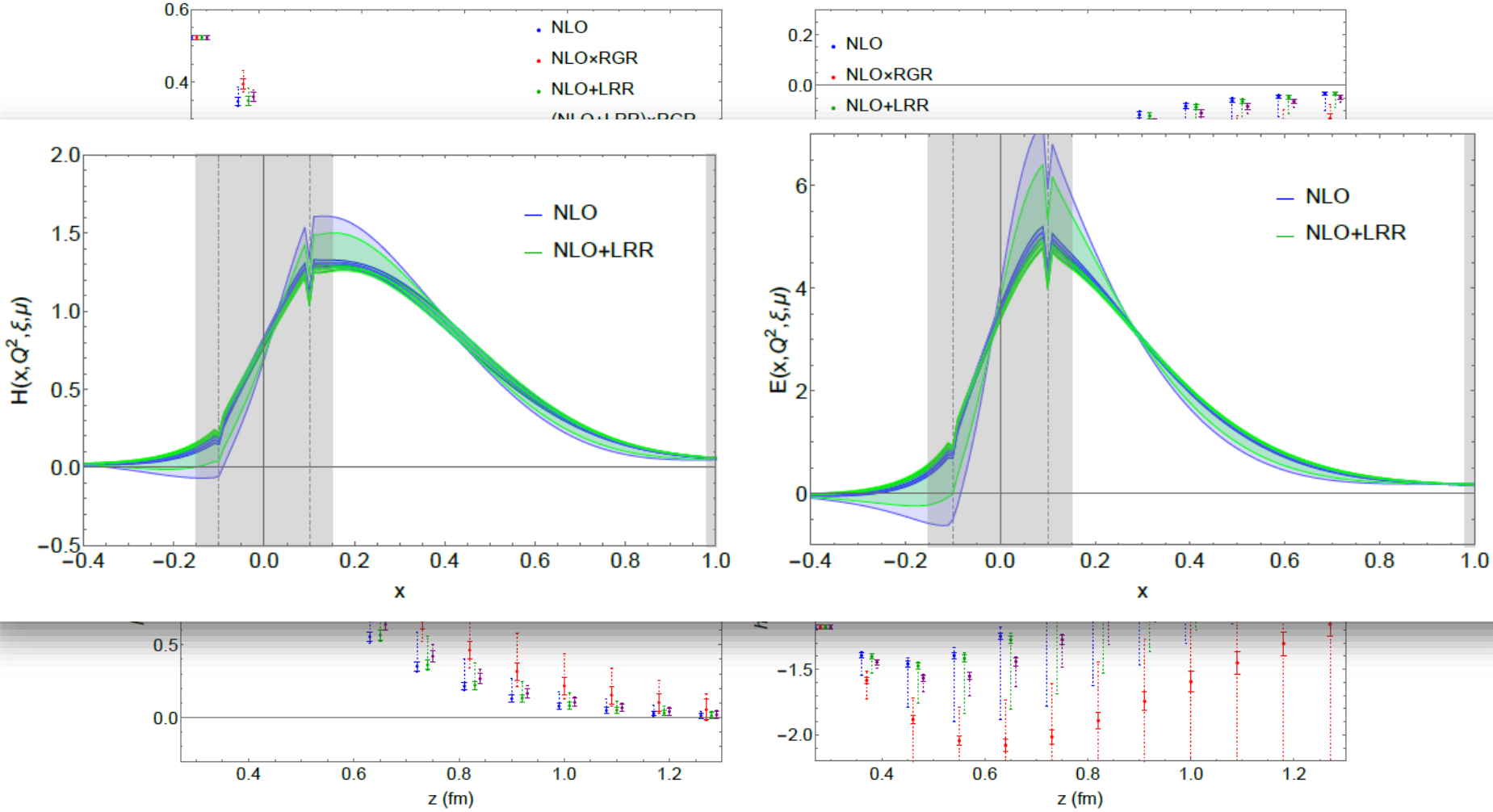
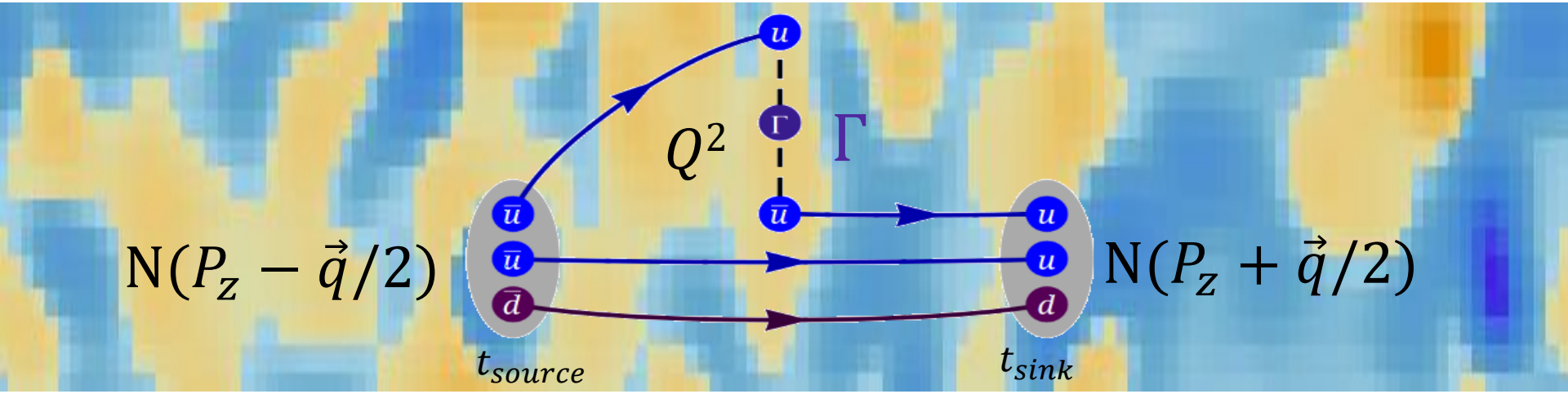


J. Holligan, HL (MSULat),2312.10829 [hep-lat]

Generalized Parton Distributions

§ On the lattice, one needs to calculate the following (nucleon example)

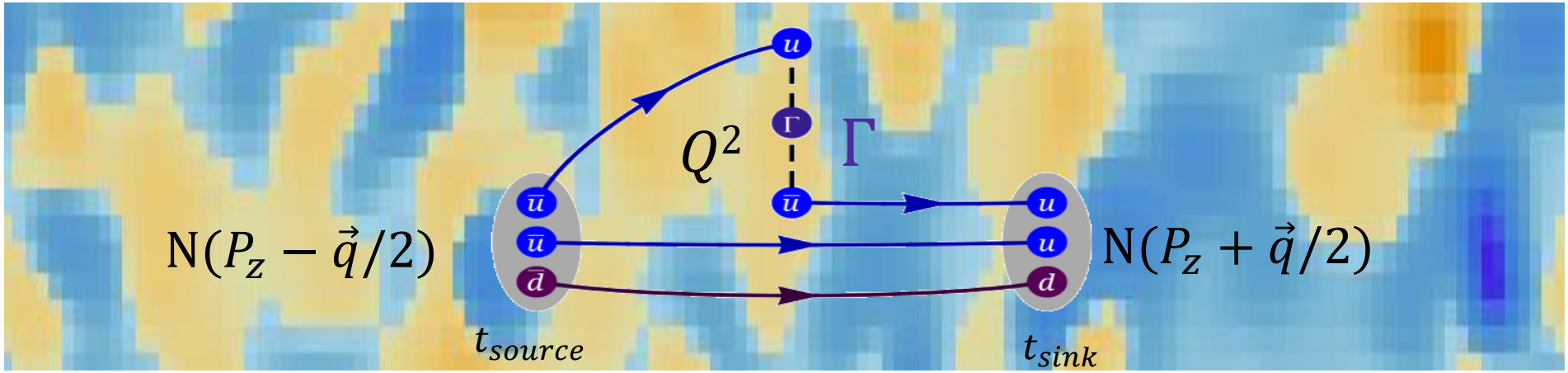


$$\tilde{F}\left(x,\tilde{\xi},t,\bar{P}_z\right) = \frac{\bar{P}_z}{\bar{P}_0}\int\frac{dz}{4\pi}e^{ixz\bar{P}z}\left\langle P'\middle|\tilde{O}_{\gamma_0}(z)\middle|P\right\rangle = \frac{\bar{u}(P')}{2\bar{P}^0}\left(H\left(x,\tilde{\xi},t,\bar{P}_z\right)\gamma^0 + E\left(x,\tilde{\xi},t,\bar{P}_z\right)\frac{i\sigma^{0\mu}\Delta_\mu}{2M}\right)u(P'')$$

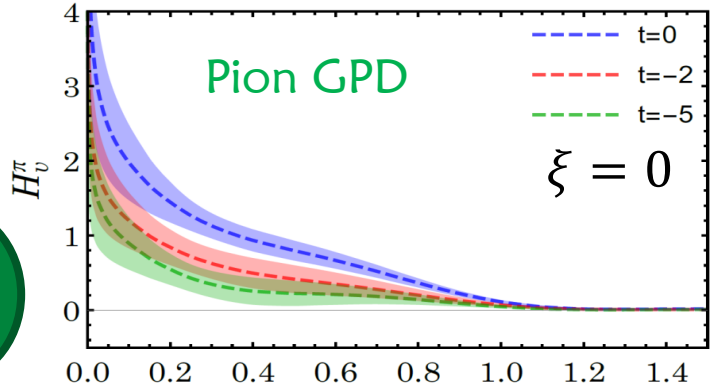
$$p^\mu = \frac{p''^\mu + p'^\mu}{2}, \qquad \Delta^\mu = p''^\mu - p'^\mu, \qquad t = \Delta^2, \qquad \xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$$

Generalized Parton Distributions

§ On the lattice, one needs to calculate the following

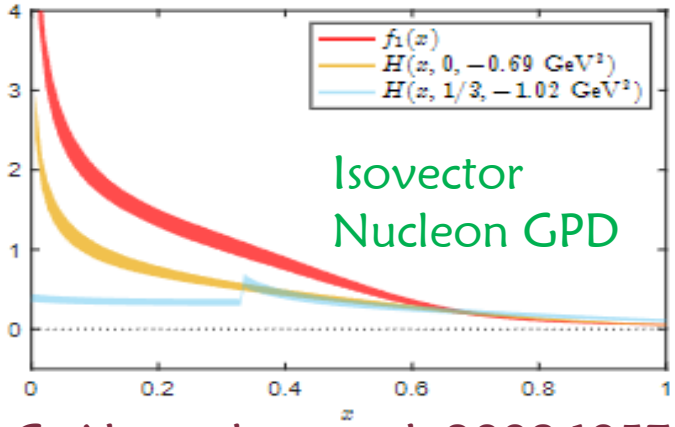


§ Heavy pion-mass results



Wanted PDFs, GPDs, etc…

J. Chen, HL, J. Zhang, 1904.12376



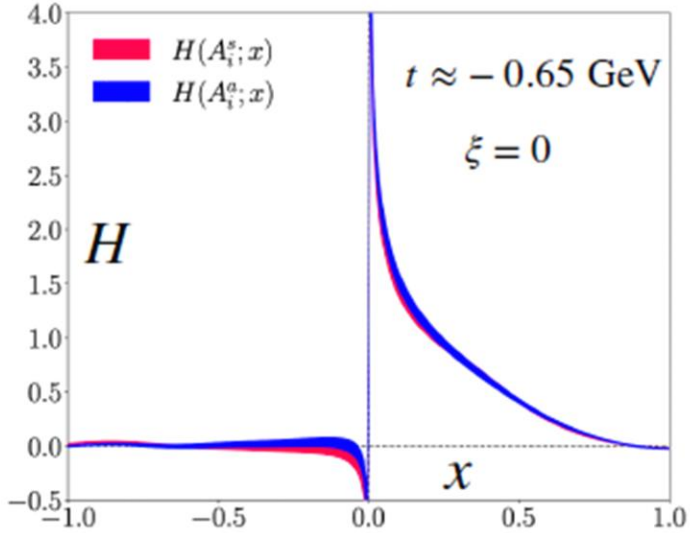
C. Alexandrou et al, 2008.10573

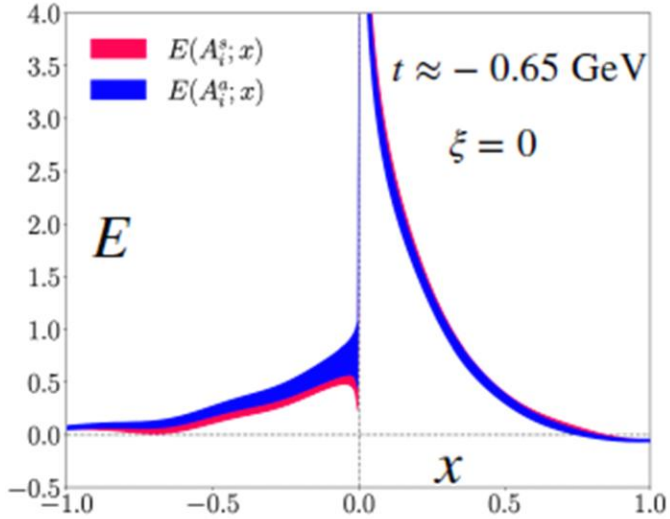
Ge

§ Or

See T

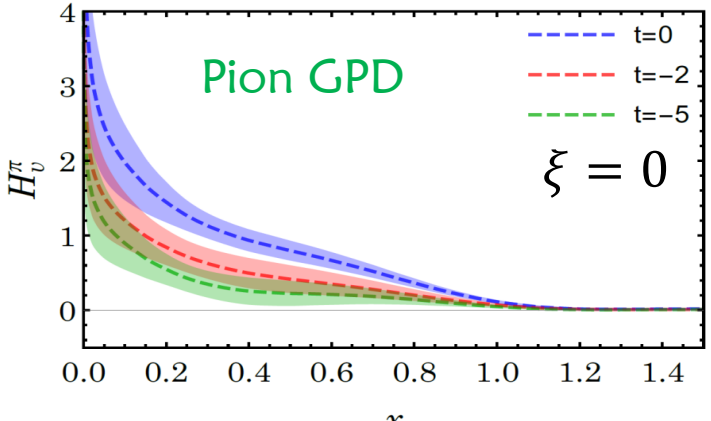
Shohini Bhattacharya *et al.*, arXiv:2209.05373 [hep-lat]



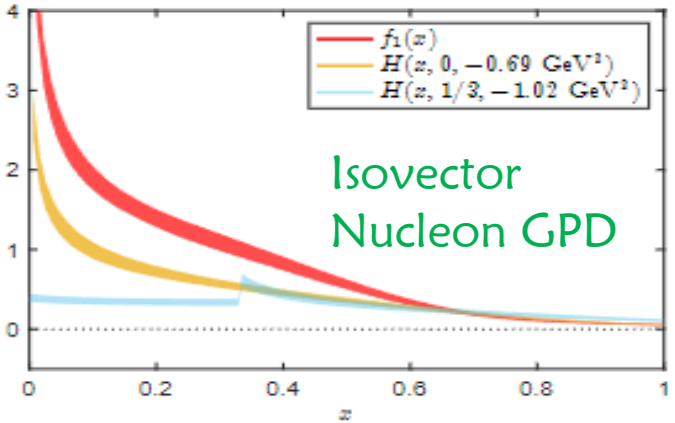


260-MeV results for asymmetric setup for GPD calculations to save computing time

§ Heavy pion-mass results



J. Chen, HL, J. Zhang, 1904.12376



C. Alexandrou et al, 2008.10573

2008.10573

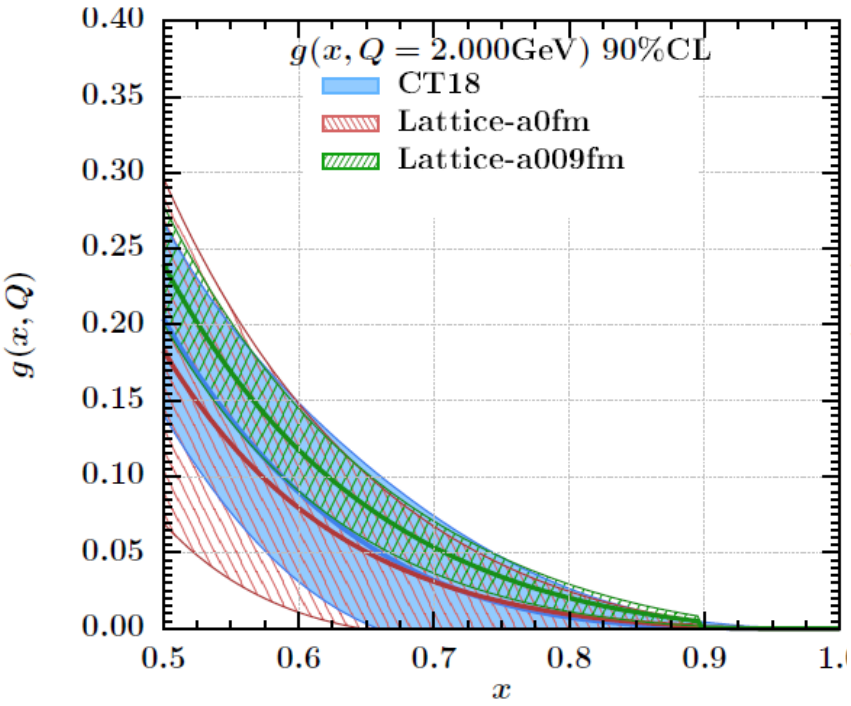
Lattice Gluon PDF Impact

§ Preliminary study with CTEQ-TEA analysis

- Take lattice inputs in the region where no strong experimental data constraints, x ∈ [0.4,0.7]
- Using e-pump for re-weighting

Plots by Alim Ablat (Xinjiang U.)

Before



After