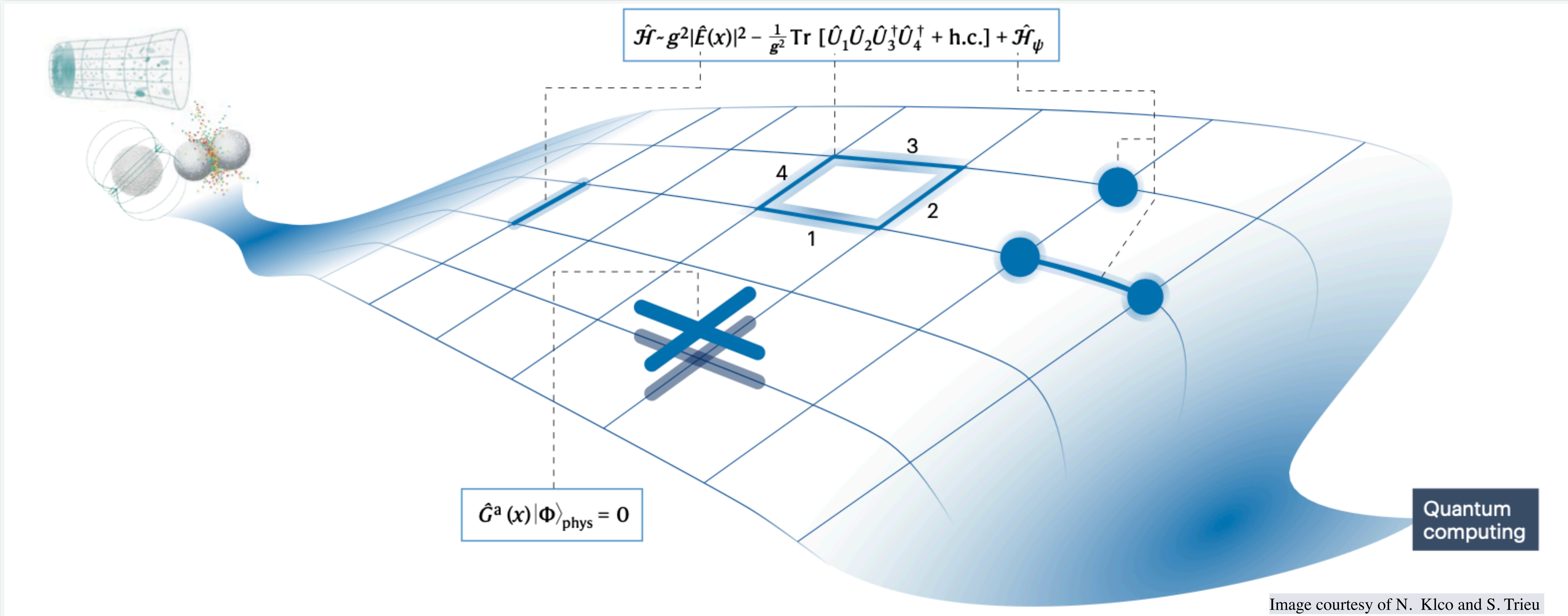


Image courtesy of N. Klco and S. Trieu

Quantum Simulations for Lattice QCD

第四届中国格点量子色动力学研讨会

中国科学技术大学
University of Science and Technology of China

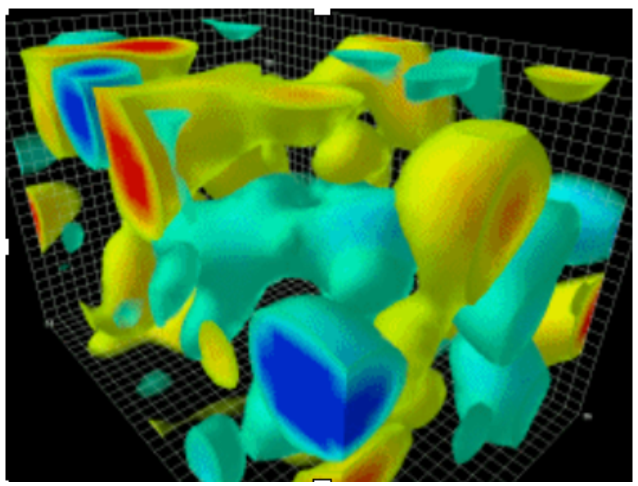
Ying-Ying Li (李英英)

Fermilab: Marcela Carena, Henry Lamm
Judah Unmuth-Yockey
UChicago: Wanqiang Liu
Peking U: Jing Shu, Bin Xu
USTC: Yi-Lin Wang

Simulating the Theory - From First Principles

QFT - path integral in the background of quantum fields

Euclidean Spacetime



field configurations $\mathcal{C}$ on lattice with N_V lattice sites

$$W(\mathcal{C}) \sim \exp(-S(\mathcal{C}))$$

$$\langle O \rangle = \frac{\sum_{\mathcal{C}} O(\mathcal{C}) W(\mathcal{C})}{\sum_{\mathcal{C}} W(\mathcal{C})}$$

From 1970s

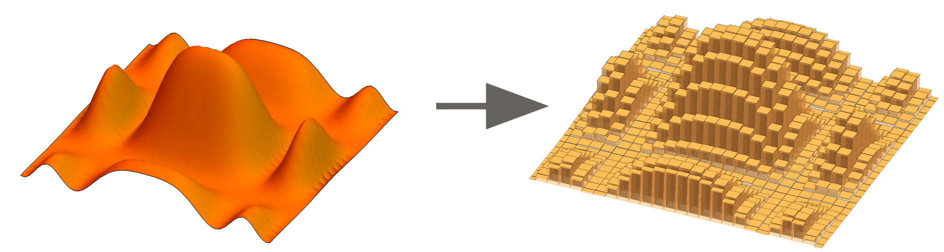
complex $S(\mathcal{C})$ →

Sign problem

CLASSICAL HARD

Real Time

$$\int \mathcal{D}\phi e^{iS} = \langle x | e^{-iHt} | y \rangle$$

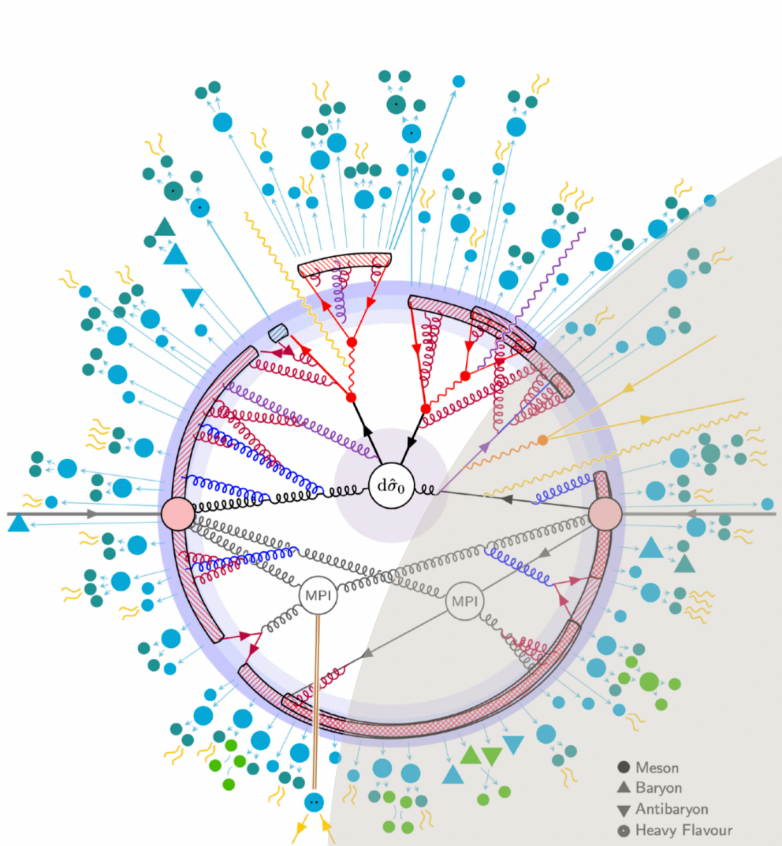


$$\dim H \propto |G|^{N_V}$$

\# of classical bits: exponential function of N_V (*CLASSICAL HARD*)

$$N_q \propto N_V \log |G|$$

\# of quantum bits: power-law function of N_V (*QUANTUM EASY*)



From PYTHIA 8.3

QUANTUM EASY

High Energy Physics

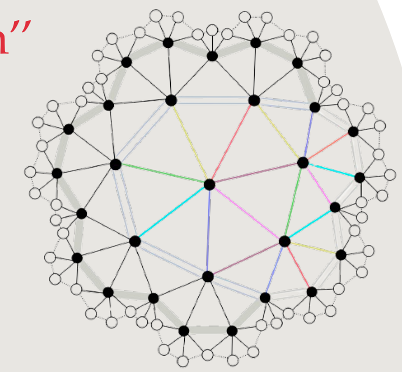
real-time dynamics
finite density
quantum interference
out-of equilibrium
``strongly interacting many-body system''

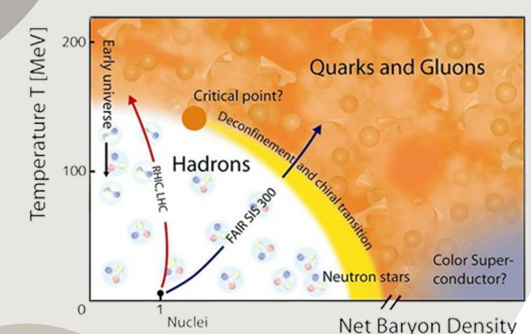
QUANTUM HARD

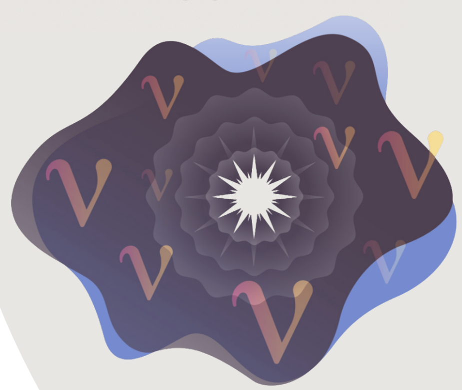
e.g. traveling salesmen problem

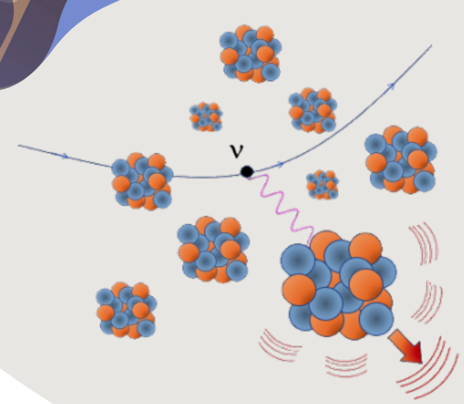
CLASSICAL EASY

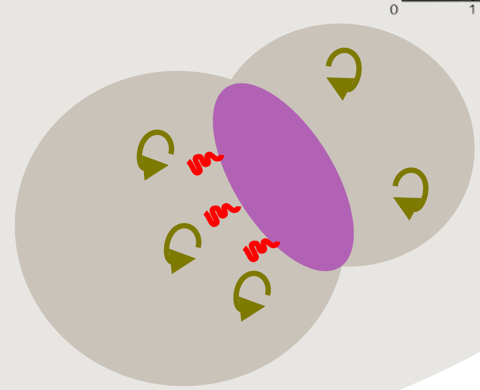
polynomial time











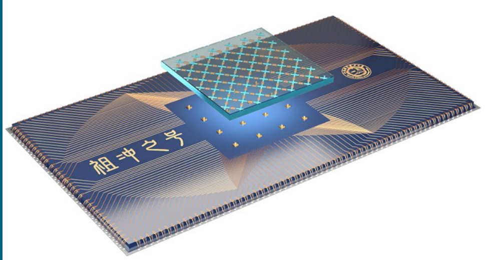
Problems in HEP that are beyond classical easy but are ``QUANTUM EASY''

Quantum Computing

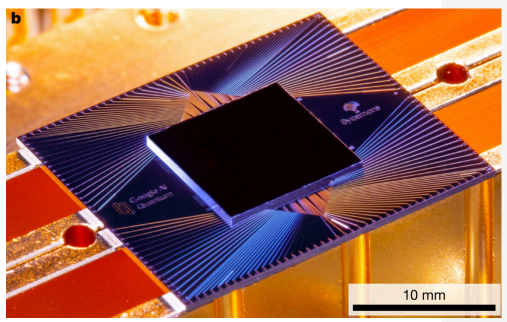
Now - Noisy Intermediate Scale Quantum (NISQ) era

more than 50 well controlled qubits, not error-corrected yet

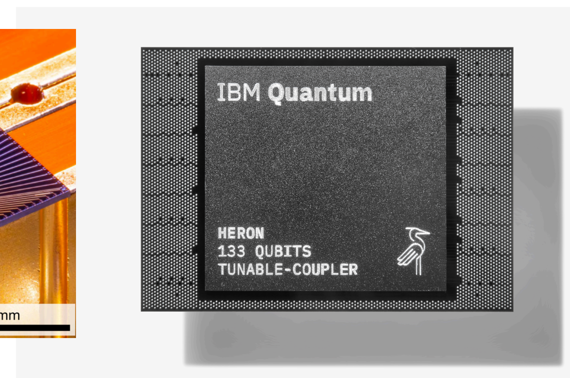
superconducting processor



176 qubits



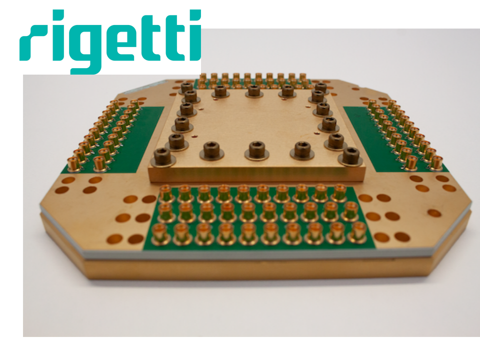
54 qubits



1121 qubits

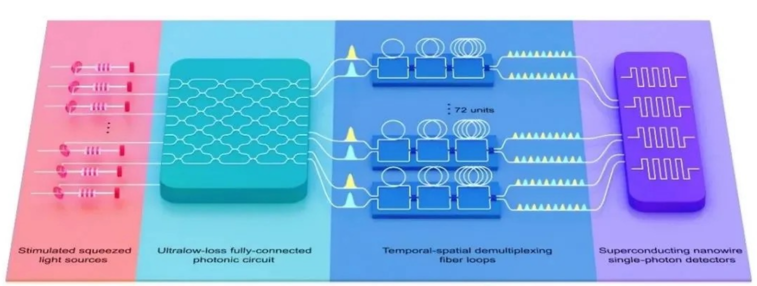
access to 133 qubits

multi-chip quantum processor



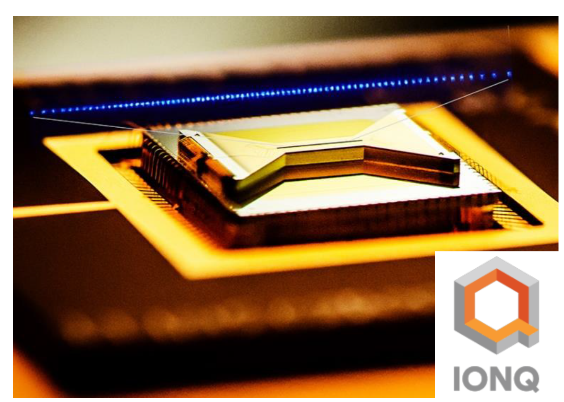
80 qubits

photon qubits



Jiuzhang - 255 qubits

trapped ion qubits

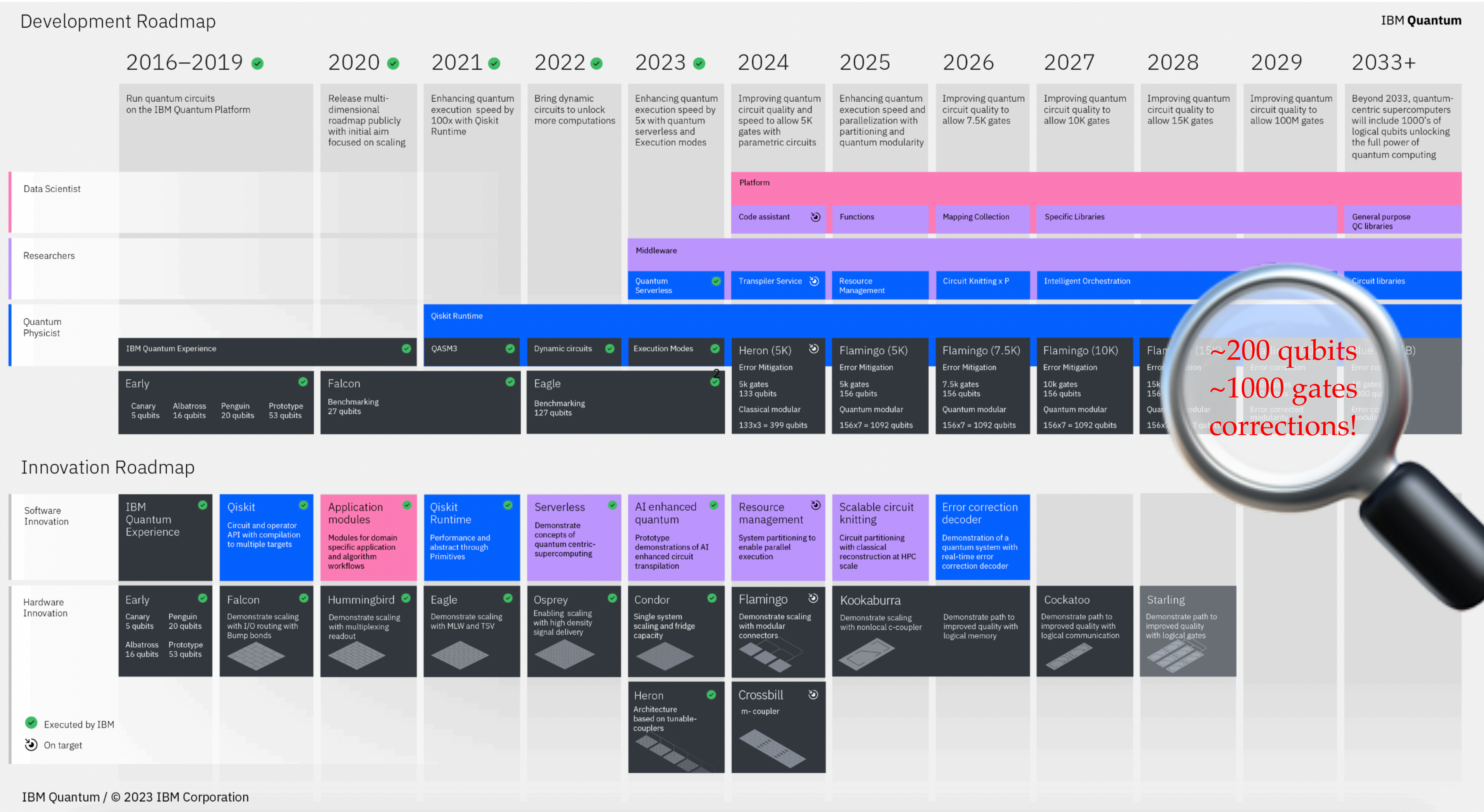


22 qubits

48 logical qubits

Quantum Computing

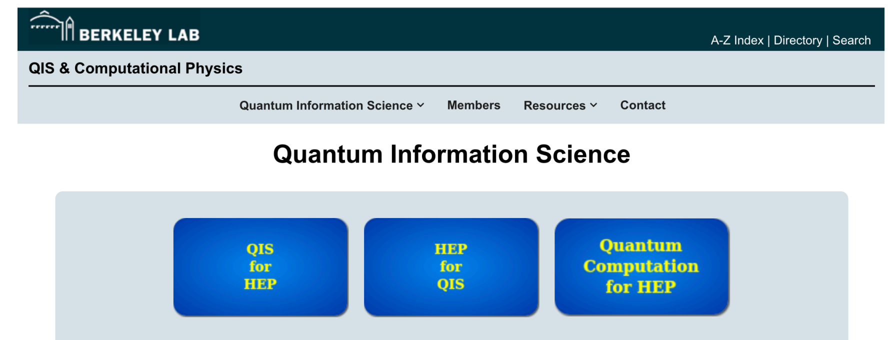
Next decades



世界趋势

- 2023年美国P5 report: “The particle physics community needs to invest now in order to train and retain the next generation of quantum scientists.”

- Quantum simulation and information processing: applications to QCD



DESY. QUANTUM | Center for Quantum Technology and Applications

“offers the fascinating opportunity to solve problems which are extremely hard or even impossible to address on conventional computers”

IHEP seeks quantum opportunities to accelerate fundamental science

China’s Institute of High Energy Physics (IHEP) in Beijing is pioneering innovative approaches in quantum computing and quantum machine learning to open up new research pathways within its particle physics programme, as **Hideki Okawa**, **Weidong Li** and **Jun Cao** explain

世界趋势

[*PRX Quantum* 4 (2023) 2, 027001]

Quantum Simulation for High Energy Physics

Christian W. Bauer,[1,a] Zohreh Davoudi,[2,b] A. Baha Balantekin,[3] Tanmoy Bhattacharya,[4] Marcela Carena,[5,6,7,8] Wibe A. de Jong,[1] Patrick Draper,[9] Aida El-Khadra,[9] Nate Gemelke,[10] Masanori Hanada,[11] Dmitri Kharzeev,[12,13] Henry Lamm,[5] Ying-Ying Li,[5] Junyu Liu,[14,15] Mikhail Lukin,[16] Yannick Meurice,[17] Christopher Monroe,[18,19,20,21] Benjamin Nachman,[1] Guido Pagano,[22] John Preskill,[23] Enrico Rinaldi,[24,25,26] Alessandro Roggero,[27,28] David I. Santiago,[29,30] Martin J. Savage,[31] Irfan Siddiqi,[29,30,32] George Siopsis,[33] David Van Zanten,[5] Nathan Wiebe,[34,35] Yukari Yamauchi,[2] Kübra Yeter-Aydeniz,[36] and Silvia Zorzetti[5]

- Collider Phenomenology
- Matter in and out of Equilibrium
- Neutrino (Astro)physics
- Early Universe and Cosmology
- Quantum Gravity

International Conference on Quantum Technology for High-Energy Physics (QT4HEP), CERN, 2022
Quantum Computing Methods For High Energy Physics, Munich, 2023
Quantum Computing and Machine Learning Workshop, 青岛, 2023
人工智能、量子信息、量子计算在粒子物理、核物理和宇宙学等学科前沿的应用，吉林，2024
Quantum Technologies and Computation for High Energy Physics, Munich, 2024
2nd Edition of QT4HEP, CERN, January 2025

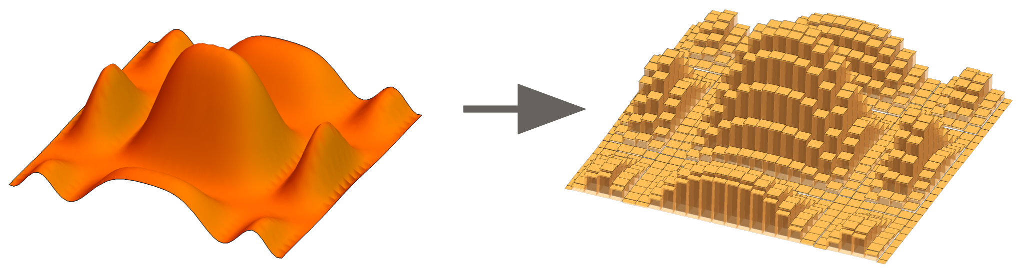
Quark and lepton flavour physics: LT3	Algorithms and artificial intelligence: LT2	Hadronic and nuclear spectrum and interactions: LT1	Theoretical developm... TR4	Structure of hadrons and nuclei: Flex2	QCD at non-zero temperature: TR5	Vacuum structure and confineme... TR6	Quantum computing and quantum information: TR7
Jonathan Flynn	*Scott Lawrence*	*Christopher Thomas*	*Richard Brower*	*Anthony Grebe*	*Anders Tranberg*	*Biagio Lucini*	*Zohreh Davoudi*
11:15 - 13:15	11:15 - 12:55	11:15 - 13:15	11:15 - 13:15	11:15 - 13:15	11:15 - 13:15	11:15 - 13:15	11:15 - 13:15

Lattice 2024 @Liverpool

Quantum Computing for HEP

[Jordan, Lee, Preskill, 2011]

$$\int \mathcal{D}\phi e^{iS} = \langle x| e^{-iHt} |y\rangle$$

Discretization  *infinities in space*

Digitization $|q\rangle^N \to |G\rangle$ *infinities in field variables*

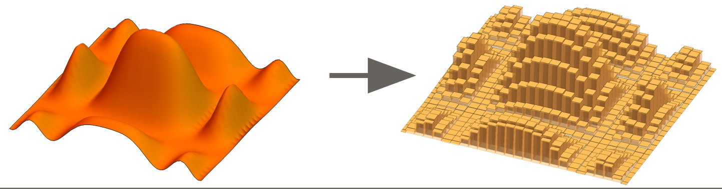
Initialization $\mathcal{U}\,|G\rangle^L \to |\psi_0\rangle$ *ground/thermal/bound state prep*

Propagation $\mathcal{U}\,|\psi_0\rangle \to |\psi(t)\rangle$ *efficiency of time evolutions*

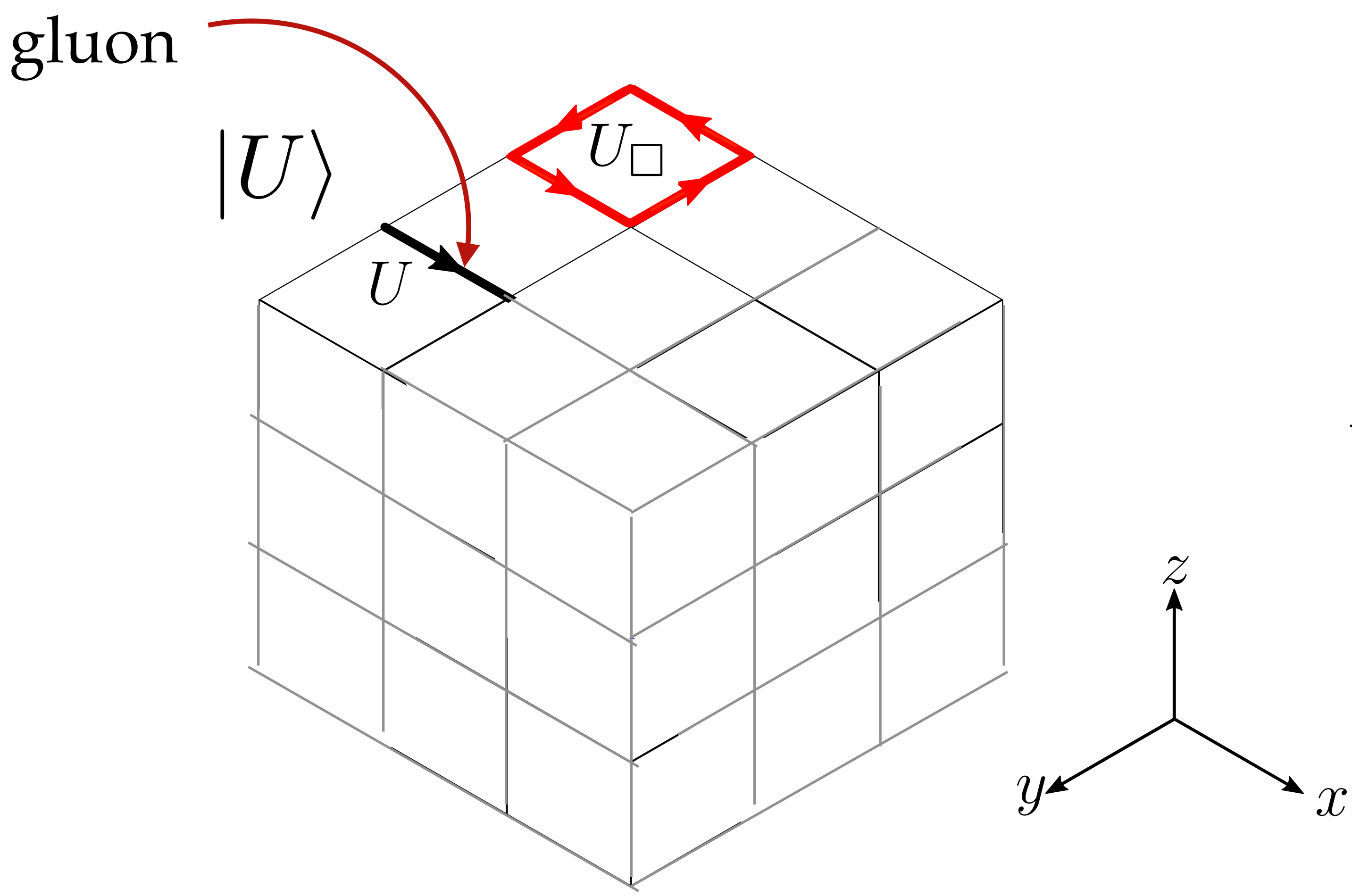
Evaluation $\langle\mathcal{O}\rangle$ *parton distribution function,*

Error mitigation/corrections *gauge symmetry for error corrections*

Discretization



infinities in QFT



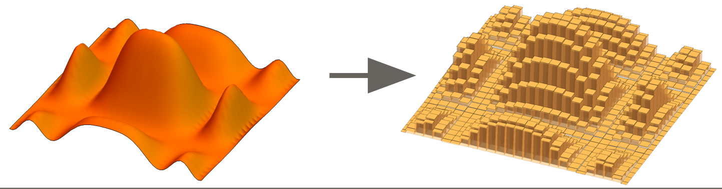
gauge invariant Hamiltonian

KS Hamiltonian [Phys. Rev. D 11, 395 (1975)]

$$H_{KS} = \sum \left(K_L + U_\square \right)$$

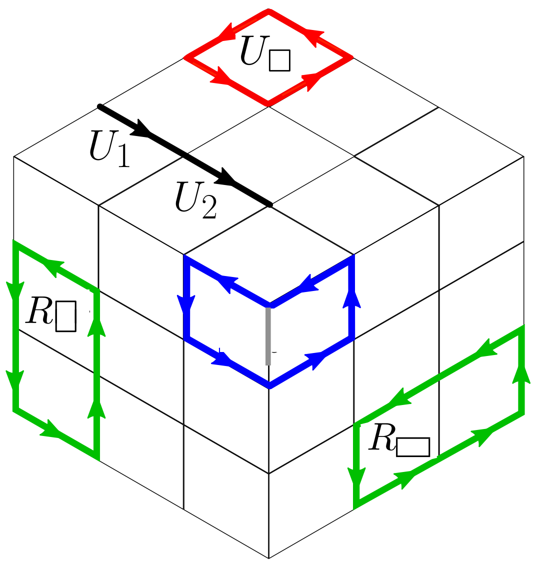
$$H_{KS} \xrightarrow{a \to 0} H = \int dx (E^2 + B^2)$$

Discretization

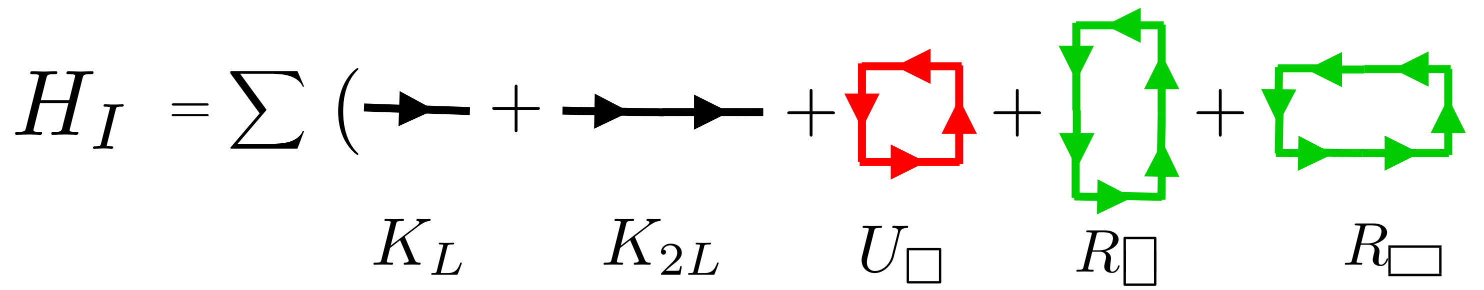


infinities in QFT

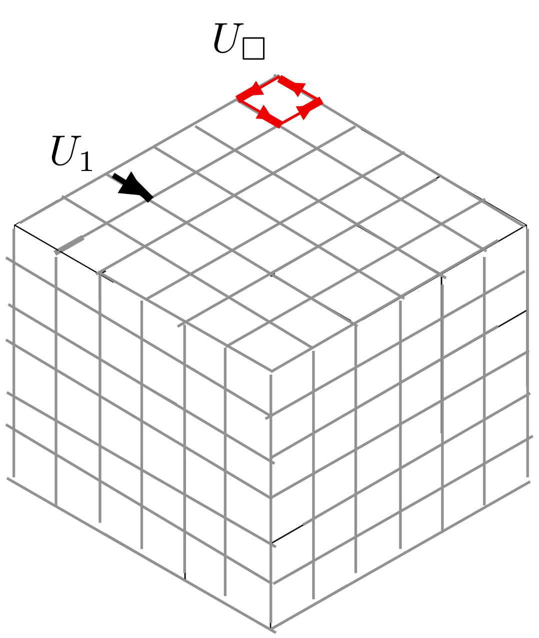
$$K_{2L} = E_i(\mathbf{x})U_i(\mathbf{x})E_i(\mathbf{x}+a\mathbf{i})U_i^{\dagger}(\mathbf{x})$$



Improved Hamiltonian



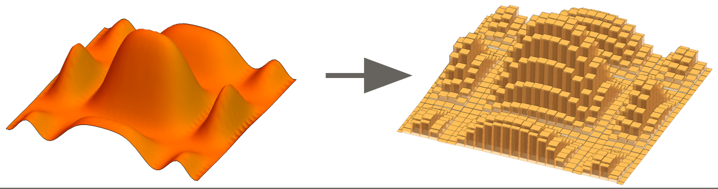
$$|\langle H_{KS}(a) - H\rangle| \sim |\langle H_I(2a) - H\rangle|?$$



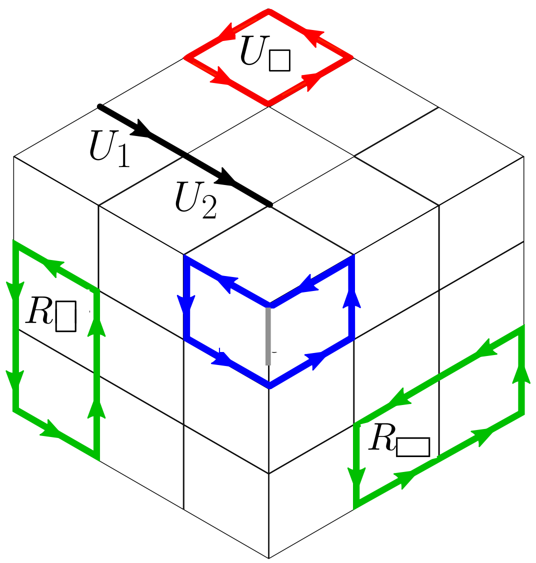
$$N_q \sim \left(\frac{L}{a}\right)^d$$

The number of qubits, also trotter steps can be reduced

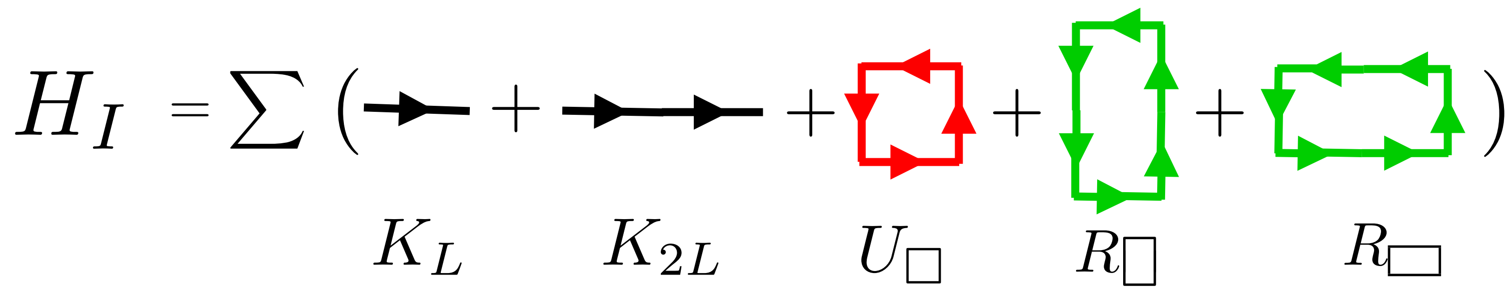
[Carena, Lamm, **YYL**, Liu, PRL. **129**, 051601]

Discretization  infinities in QFT

$$K_{2L} = E_i(\mathbf{x})U_i(\mathbf{x})E_i(\mathbf{x}+a\mathbf{i})U_i^\dagger(\mathbf{x})$$

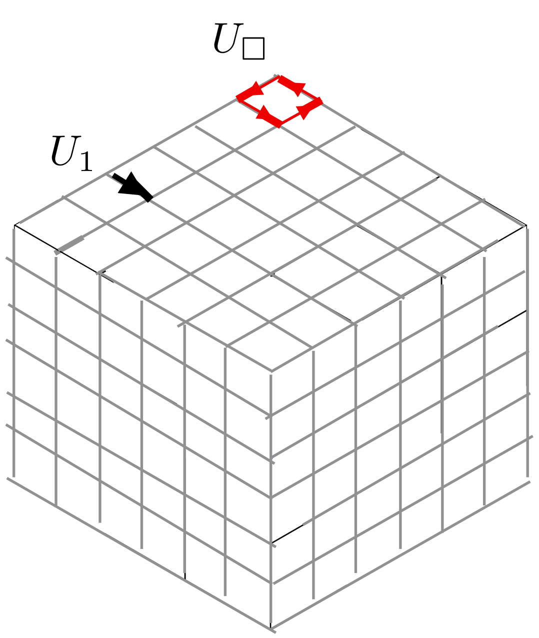


Improved Hamiltonian



quantum circuits

$$\langle U_1', U_2'| \mathcal{U}_{K_{2L}} |U_1, U_2\rangle = \delta_{U_1'U_2', U_1U_2} \langle U_1'| e^{i\theta K_{L_1}} |U_1\rangle$$

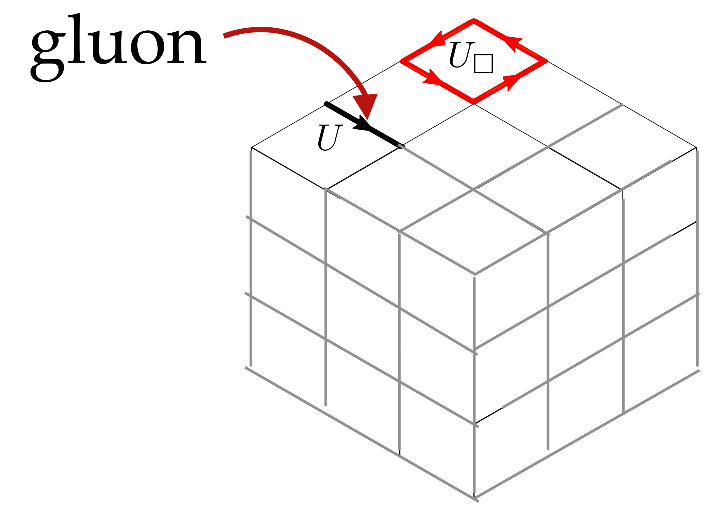


It conserves the product of group elements!
Thus can be reduced to the matrix elements
of the one-link kinetic term

[Carena, Lamm, **YYL**, Liu, PRL. **129**, 051601]

Digitization

infinities in field variables



Formulations & bases: examples

Kogut-Susskind Formulation

— Irrep / angular momentum basis: *Byrnes, Yamamoto, Zohar, Burrell, et al*

— Group-element basis: *Carena, Lamm,* **YYL**, *Liu, Zohar, et al.*

— Mixed basis: *Bauer, D'Andrea, Freytsis, Grabowska*

— Large N truncations: *Bauer, Ciavarella*

— Continuous-Variable quantum computing: *Abel, Spannowsky, Williams, et al.*

***Gauge magnets*/Quantum link models:** *Wiese, Chandrasekhara, et al.*

Casimir variables: *Klco, Savage, Stryker, Ciavarella*

Schwinger boson formulations: *Mathur, Anishetty, Raychowdhury, et al*

Loop-string-hadron formulation: *Raychowdhury, Stryker, Davoudi, Kadam, et al.*

Qubit models: *Chandrasekhara, Singh, et al.*

Dual/rotor formulations: *Kaplan, Stryker, Haase, et al.*

q-deformed Kogut-Susskind: *Zache, Zoller et al*

Dual/rotor formulations: *Kaplan, Stryker, Haase, Dellantonio, et al.*

Theory developments and algorithms are still in very early stages, vibrant research directions

Digitization $|q\rangle^N \to |G\rangle$

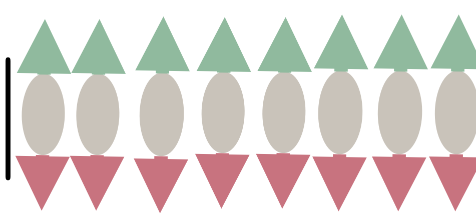
infinities in QFT

discrete subgroup

$$|U\rangle = \left\{ \begin{pmatrix} \alpha & -\overline{\beta} \\ \beta & \overline{\alpha} \end{pmatrix} : \alpha, \beta \in \mathbb{C}, |\alpha|^2 + |\beta|^2 = 1 \right\} \longrightarrow |U\rangle = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a, b, c, d \in \mathbf{F}_3, ad - bc \equiv 1 \pmod 3 \right\}$$

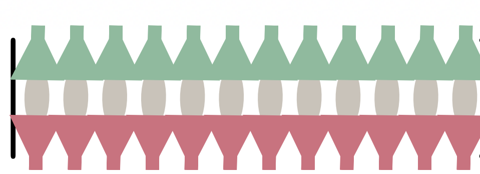
3-sphere

block product encoding: BT

$|U\rangle = |$  $\rangle$

block product encoding: BI

$$|U\rangle = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a, b, c, d \in \mathbf{F}_5, ad - bc \equiv 1 \bmod(5) \right\}$$

$|U\rangle = |$  $\rangle$

qudit?

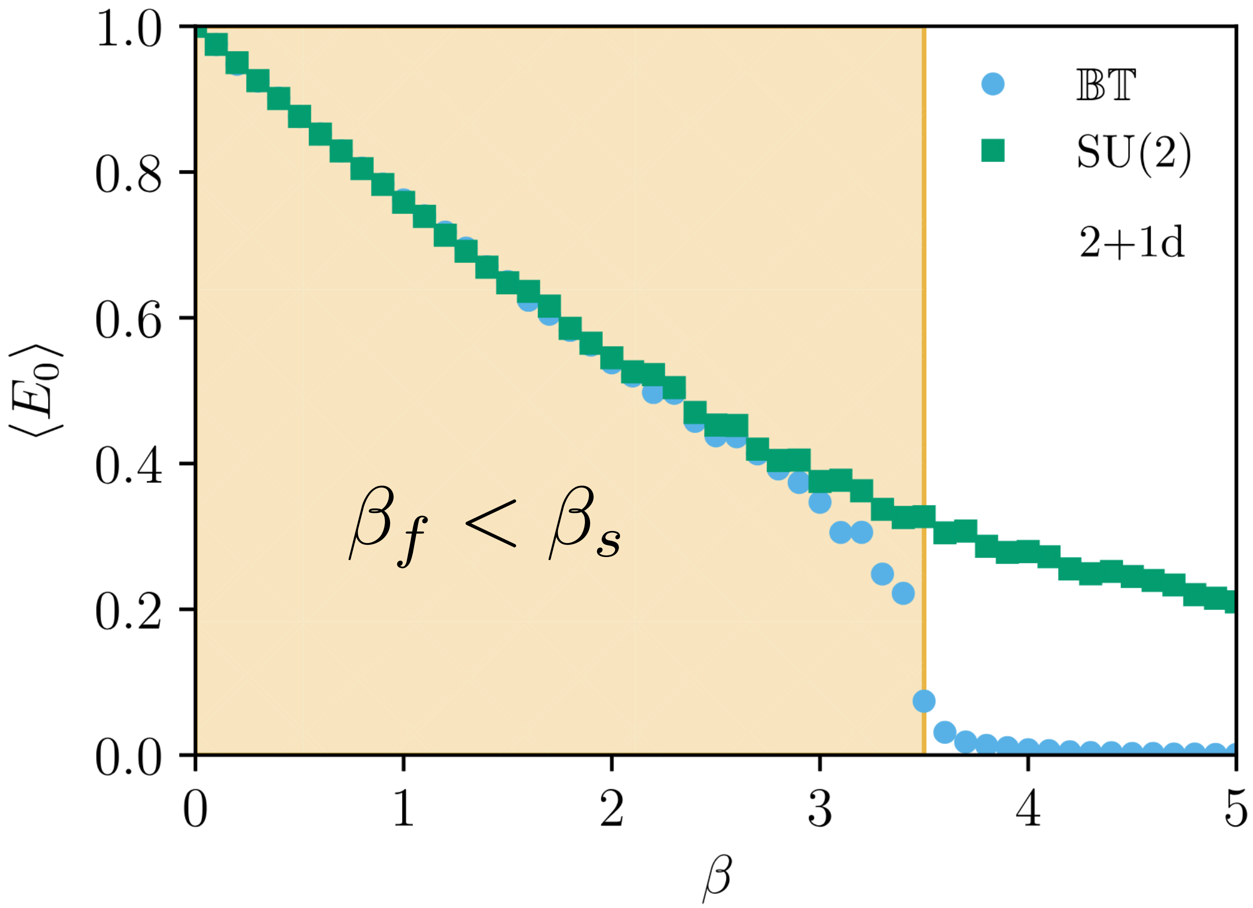
[Lamm,**YYL**, Shu, Wang, Bin, arXiv:2405.12890]

- quantum algorithms developed for its time evolutions

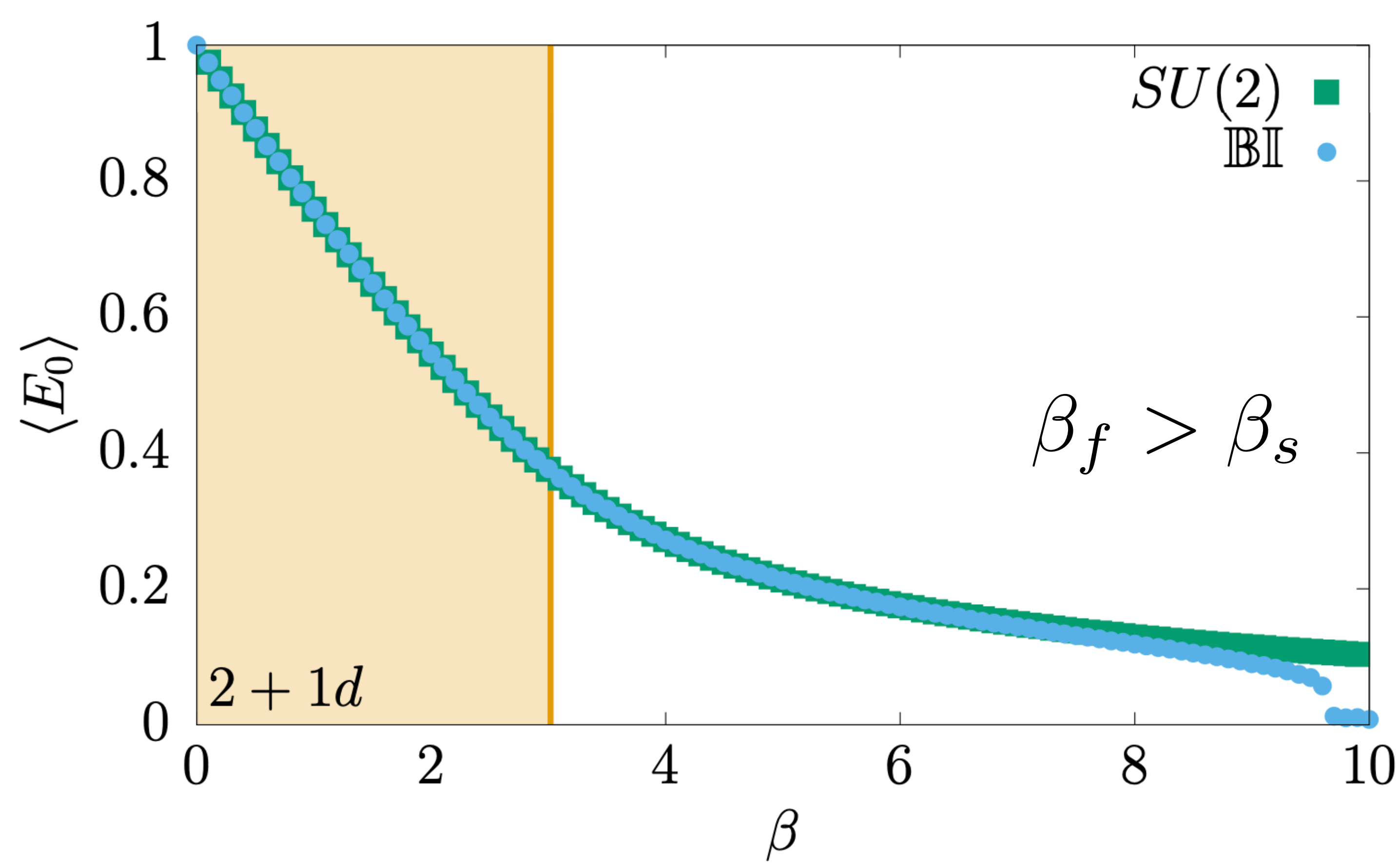
Digitization $|q\rangle^N \to |G\rangle$

infinities in QFT

[Gustafson, Lamm, Lovelace, Mush, PRD **106**, 114501]



[Lamm,**YYL**, Shu, Wang, Bin, arXiv:2405.12890]

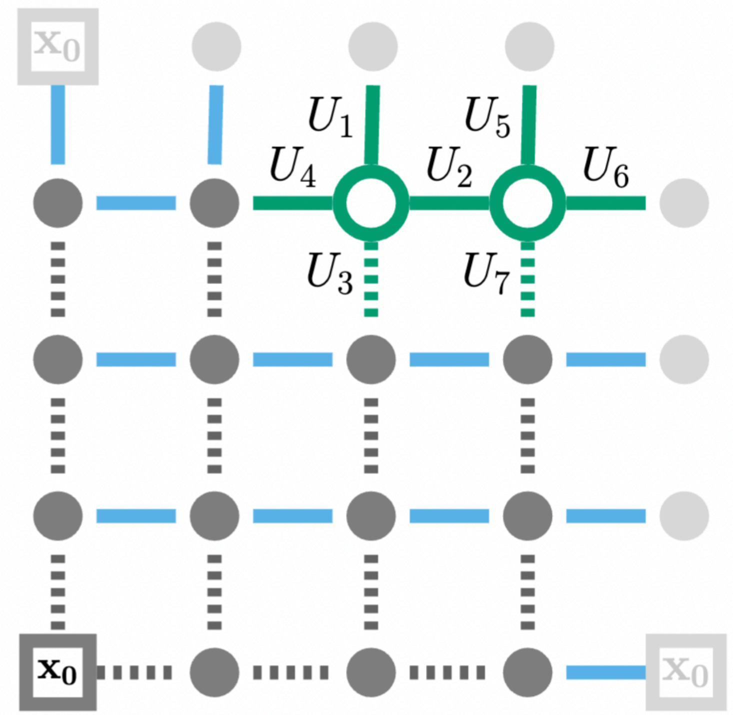


systematic ways to quantify/ improve the errors?

In the Scaling Regime: significantly reduces the errors in simulating SU(2) physics

Digitization $|q\rangle^N \to |G\rangle$

infinities in QFT



Should we keep the redundancies or not?

$$\mathcal{H}_{\text{full}} = \text{span}(\{|U\rangle, U \in G\})^{\otimes N_L}$$

$$\hat{\Theta}_\Omega(x)\,|U_1U_2U_3U_4\rangle = |U_1'U_2'U_3'U_4'\rangle$$

gauge redundant

gauge invariant

$$\hat{\Theta}_\Omega(x)\,|\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle$$

$$\hat{G}^a(x)\,|\psi_{\text{phys}}\rangle = 0 \quad \text{(Gauss's law)}$$

$$\mathcal{H}_{\text{inv}} = \text{span}(\{|U\rangle, U \in G\})^{\otimes N_L - N_V + 1}$$

Digitization $\quad |q\rangle^N \to |G\rangle$

infinities in QFT

Hamiltonian complexity?

$$\mathcal{H}_{\text{full}} = \text{span}(\{|U\rangle, U \in G\})^{\otimes N_L}$$

$$H_{KS} = \sum \left(K_L + U_\Box \right)$$

K_L $\quad$ $U_\Box$

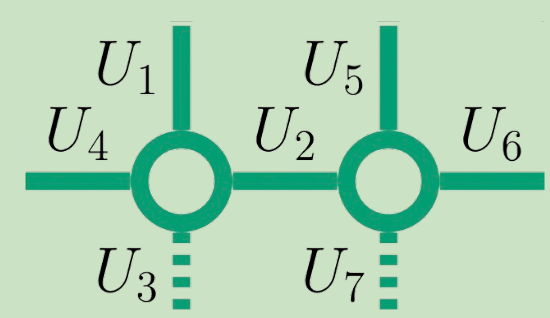
$$\mathcal{H}_{\text{inv}} = \text{span}(\{|U\rangle, U \in G\})^{\otimes N_L - N_V + 1}$$

$$\hat{\Theta}_\Omega(x)\,|\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle$$

$$H_{KS} = \sum \left(K_L + U_\Box \right)$$

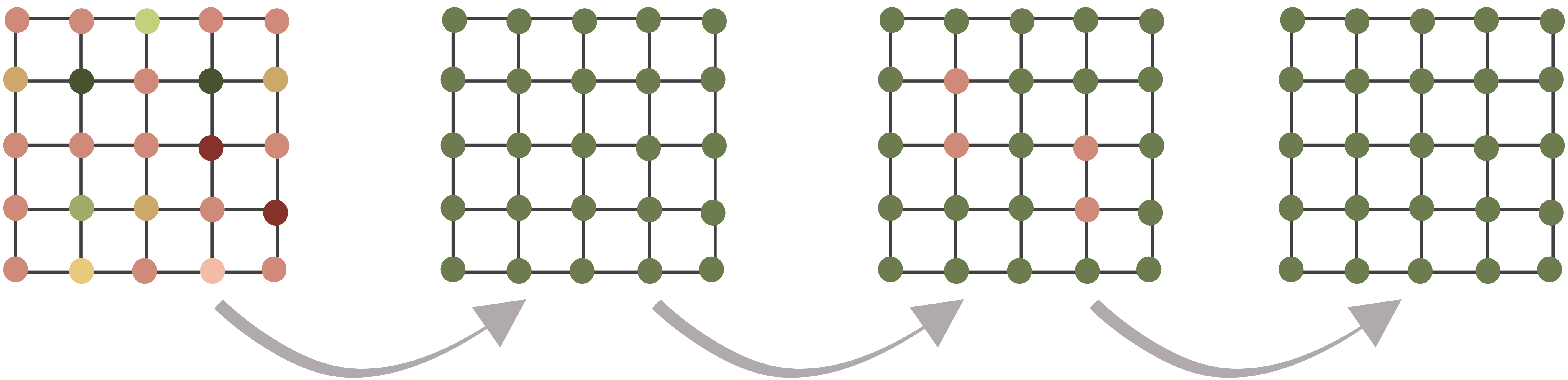
K_L $\quad$ $U_\Box$

kinetic terms for U_3, U_7 depend on other links



Digitization $|q\rangle^N \to |G\rangle$ infinities in QFT

If we keep the redundancies…

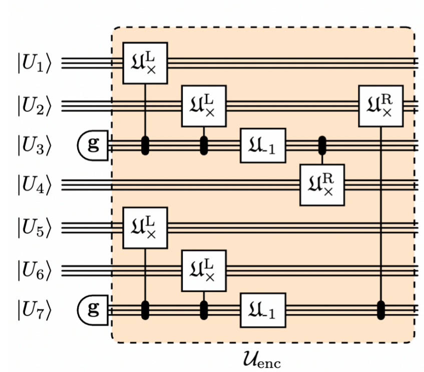


preparing gauge invariant state

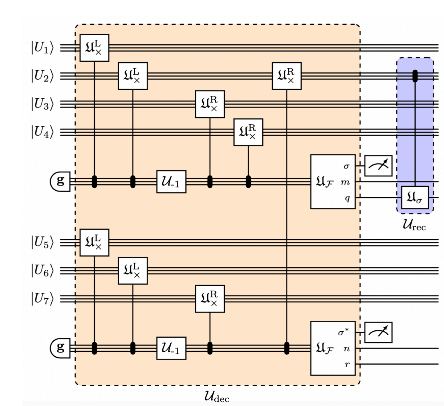
time evolution with quantum noise

Measure Gauss's Law and recover gauge invariance

KL condition



$$\mathcal{U}(t) = e^{-iH_{KS}t}$$

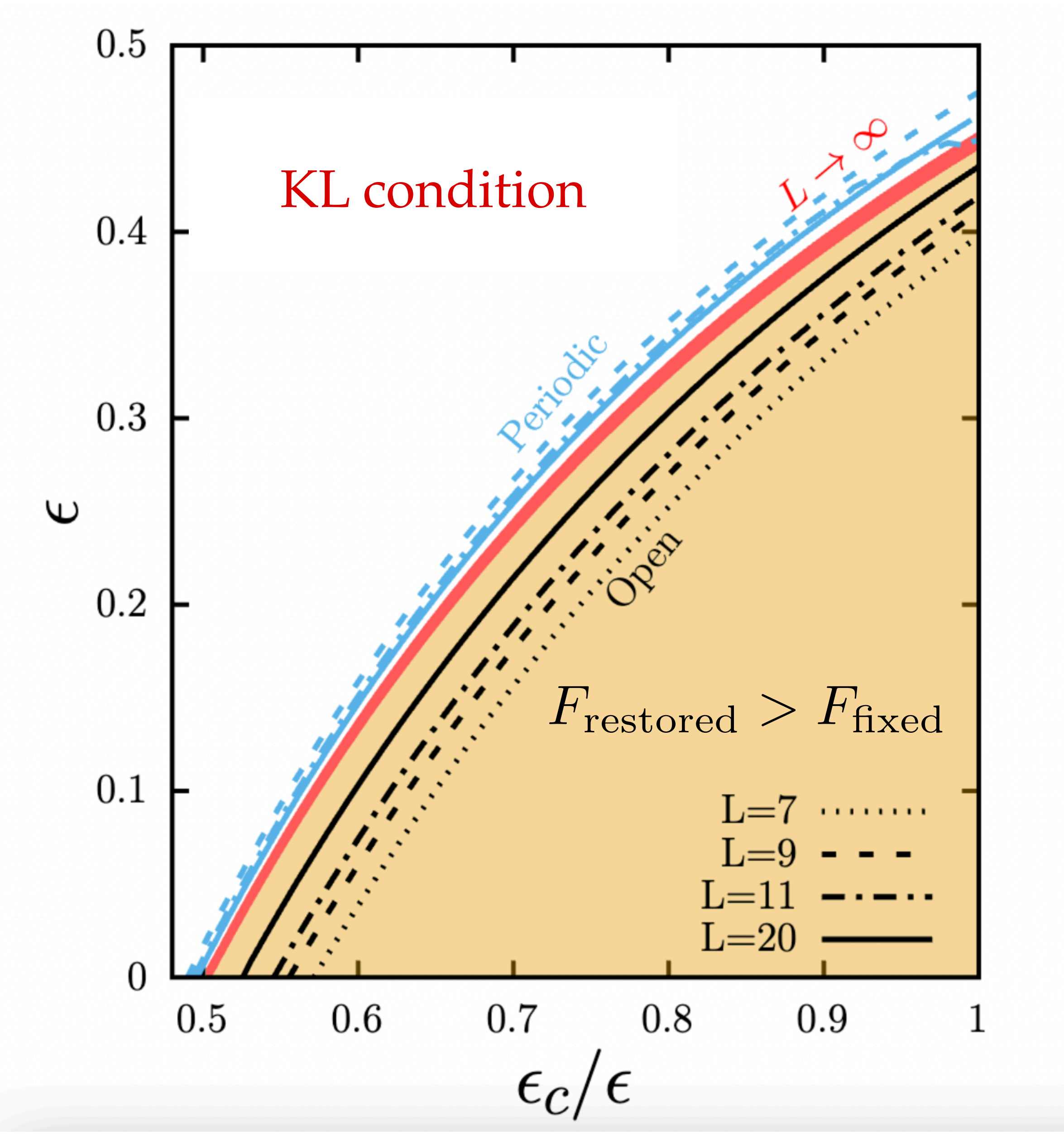


[Carena, Lamm, **YYL**, Liu, arXiv:2402.16780]

- quantum algorithms to correct errors for general gauge theories

Digitization $|q\rangle^N \to |G\rangle$

2d lattice



If we keep the redundancies…

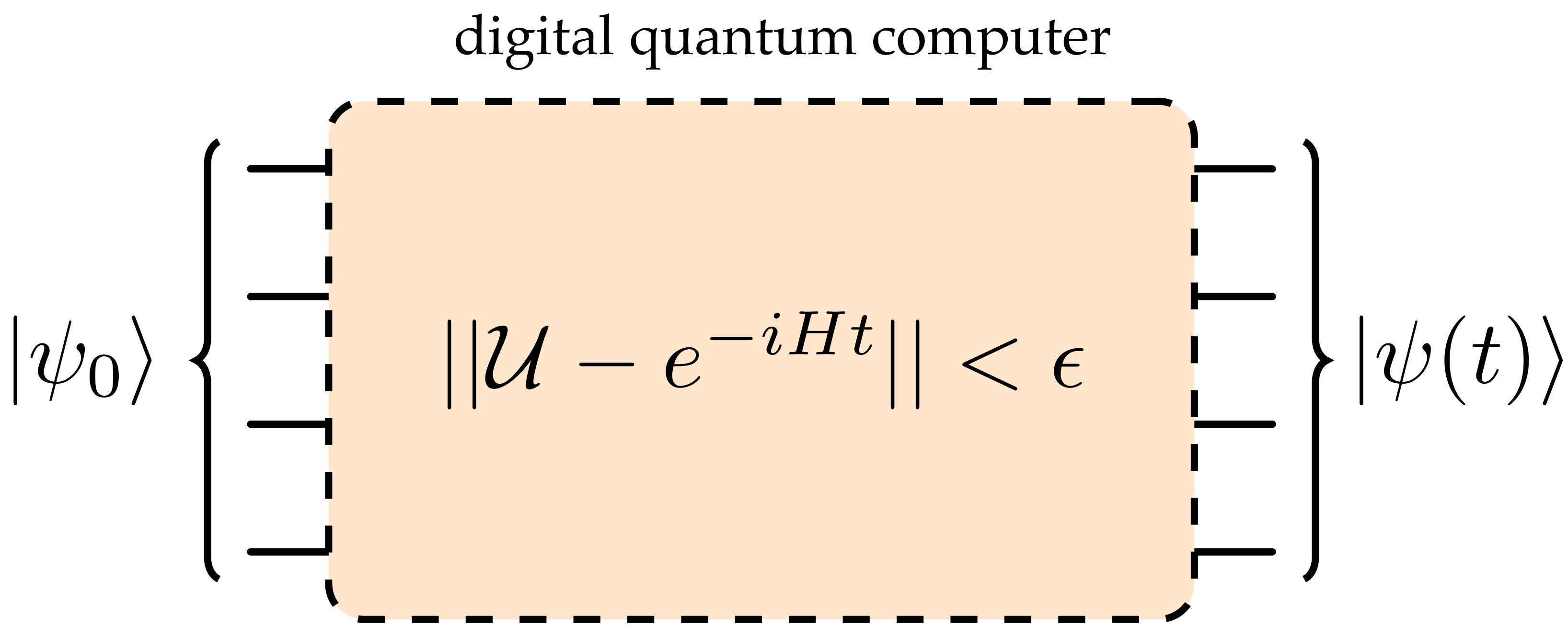
error corrections could lead to HIGHER fidelity

- error thresholds derived as guidelines to keep gauge redundancies

[Carena, Lamm, **YYL**, Liu, arXiv:2402.16780]

All these can be applied to QCD calculations

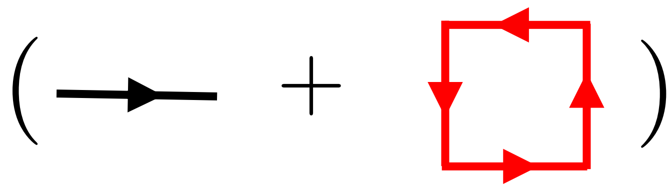
Propagation



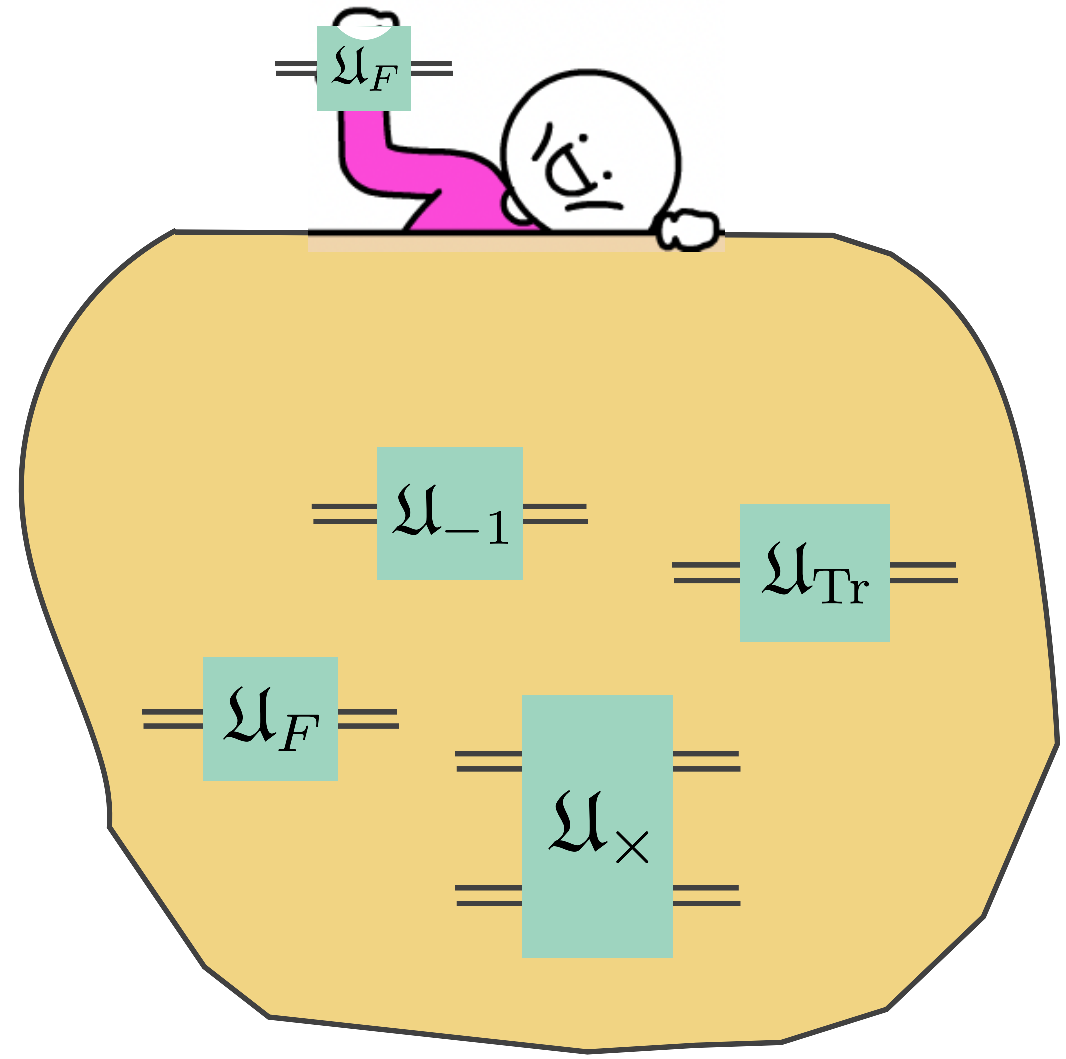
time-evolution while keep the gauge redundancies

Propagation $\mathcal{U}\,|\psi_0\rangle \to |\psi(t)\rangle$

$G-$register : $|U\rangle$ $=\!=$

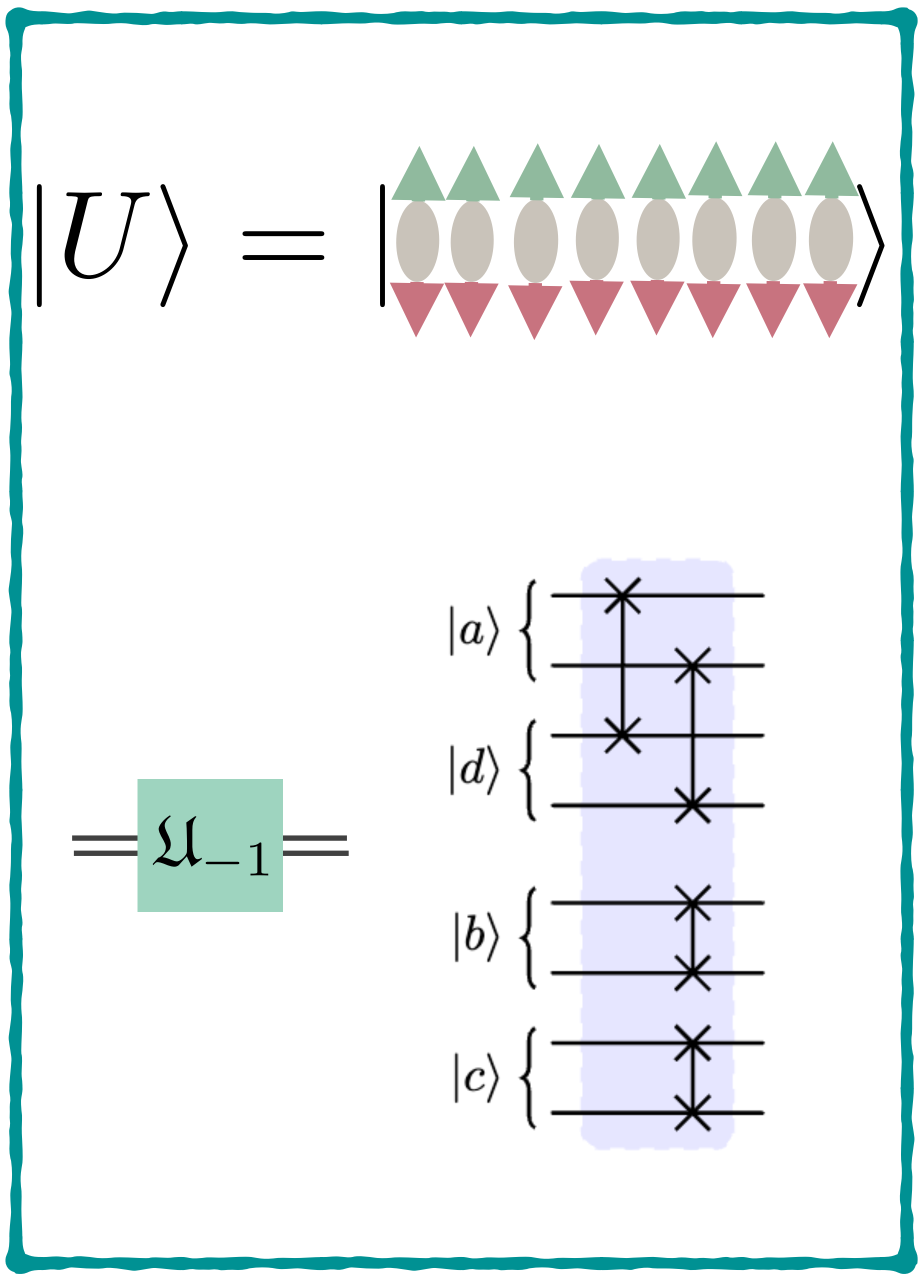
$H_{KS} = \sum$ 

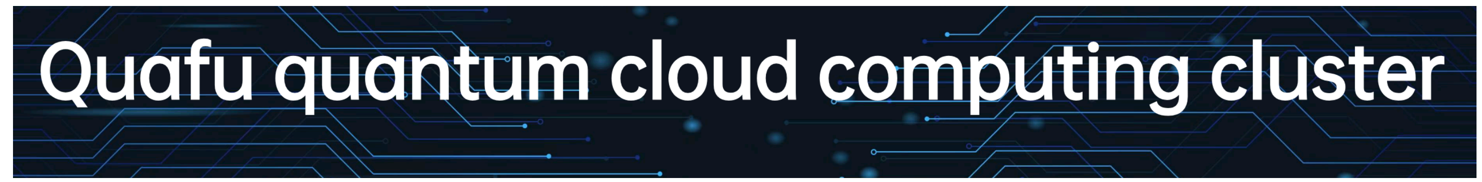
K_L $\quad$ $U_\square$

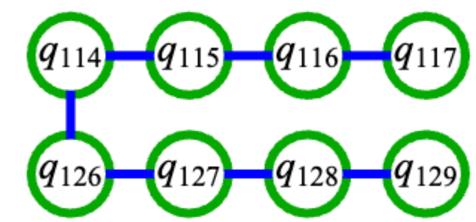


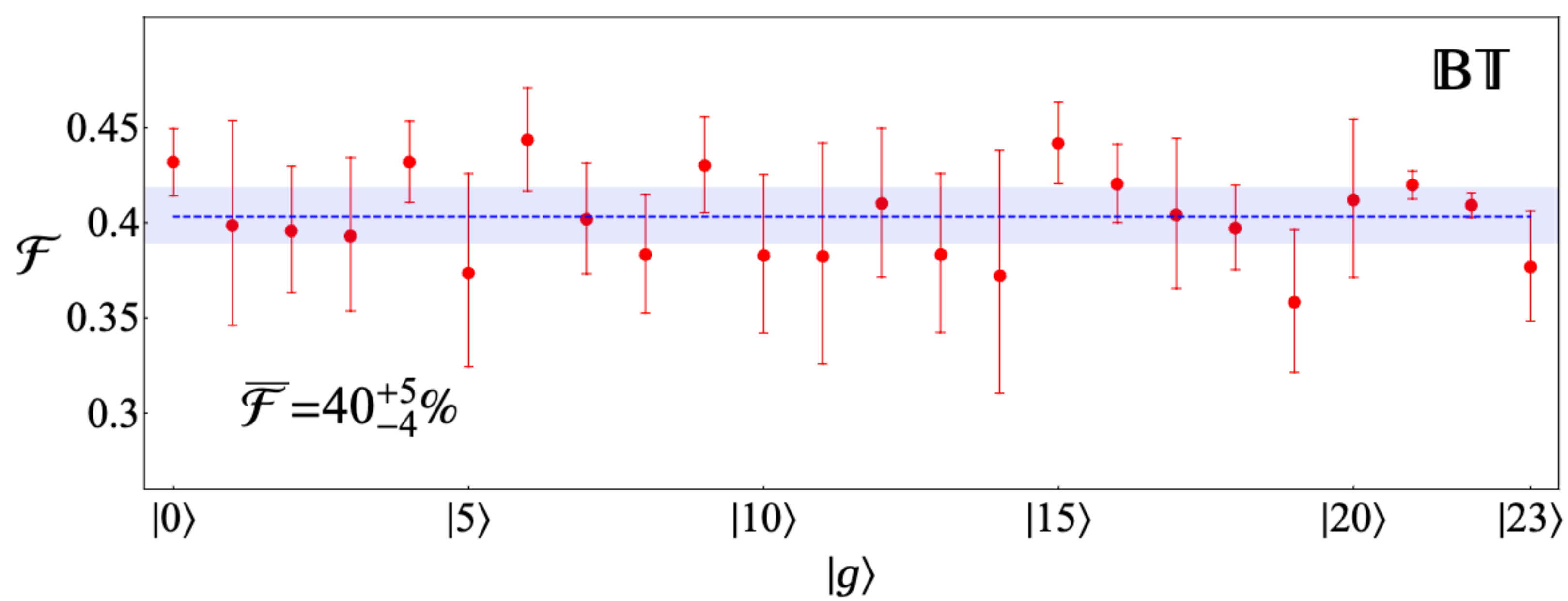
$$\mathcal{U}(t) = e^{-iH_{KS}t}$$
$$\approx \left[e^{-i\delta t K_L} e^{-i\delta t U_\square}\right]^{t/\delta t}$$

Propagation $\mathcal{U}\,|\psi_0\rangle \rightarrow |\psi(t)\rangle$



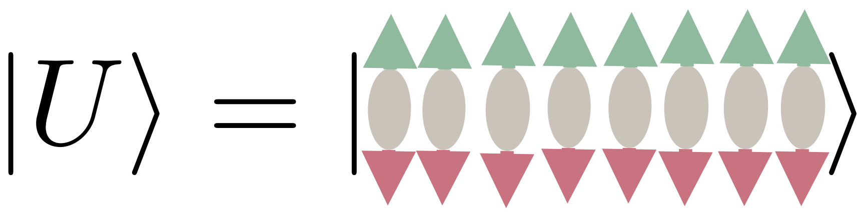


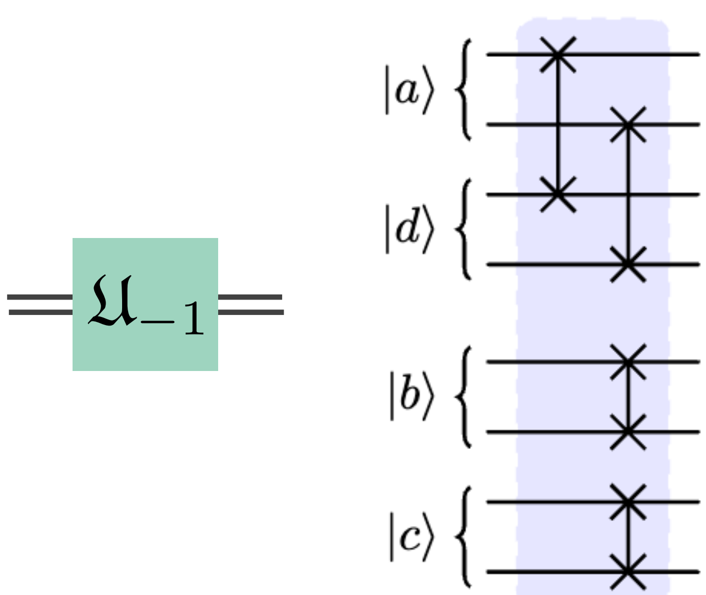




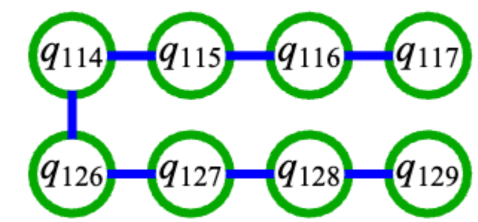
[Lamm, **YYL**, Shu, Wang, Bin, arXiv:2405.12890]

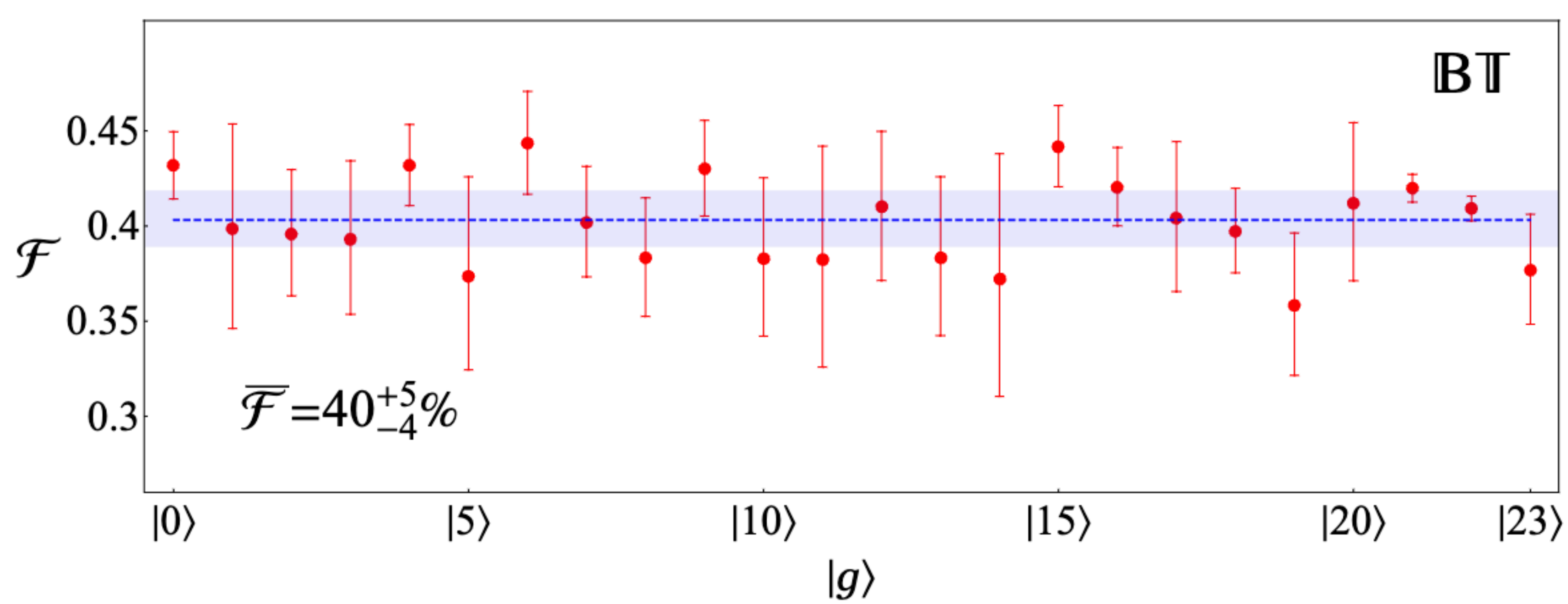
Propagation $\mathcal{U}\,|\psi_0\rangle \rightarrow |\psi(t)\rangle$





Quafu quantum cloud computing cluster

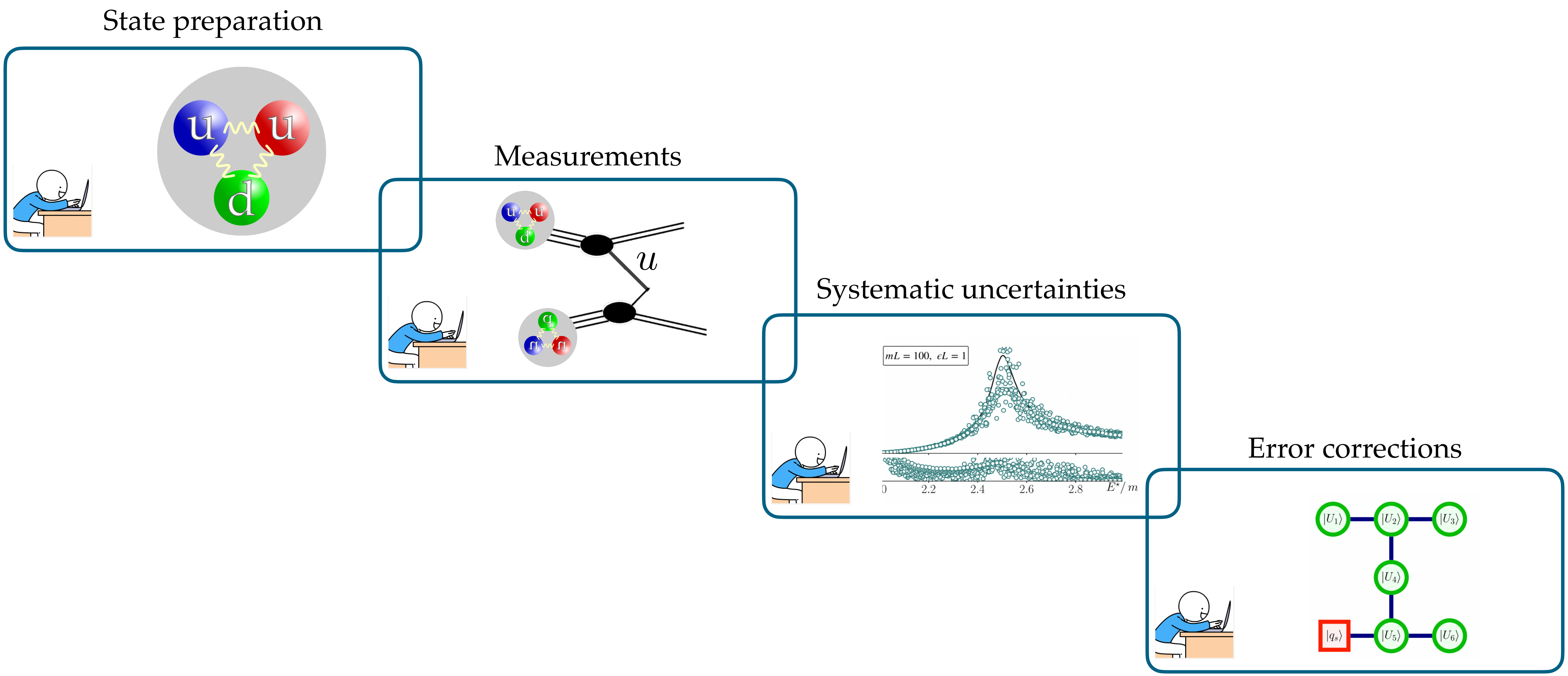




optimization?

[Lamm, **YYL**, Shu, Wang, Bin, arXiv:2405.12890]

To reach the observables — How to do…



and reach the continuum limit

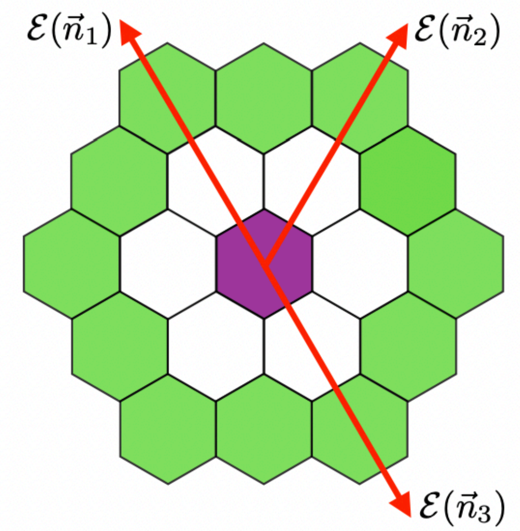
Now - Noisy Intermediate Scale Quantum (NISQ) era

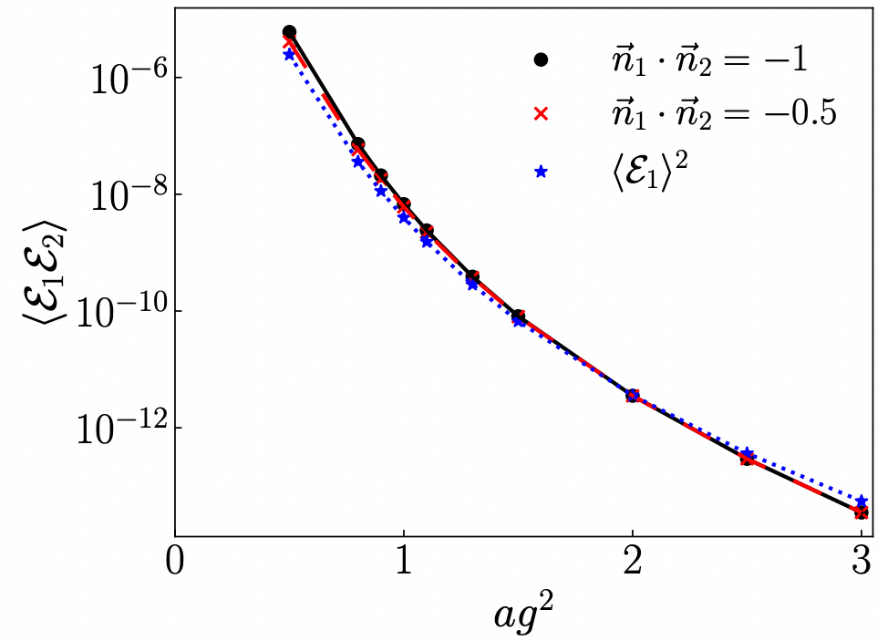
With $\mathcal{O}(100)$ well controlled qubits, not error-corrected yet

Physics Benchmarks

Physics Benchmarks for Quantum Computing

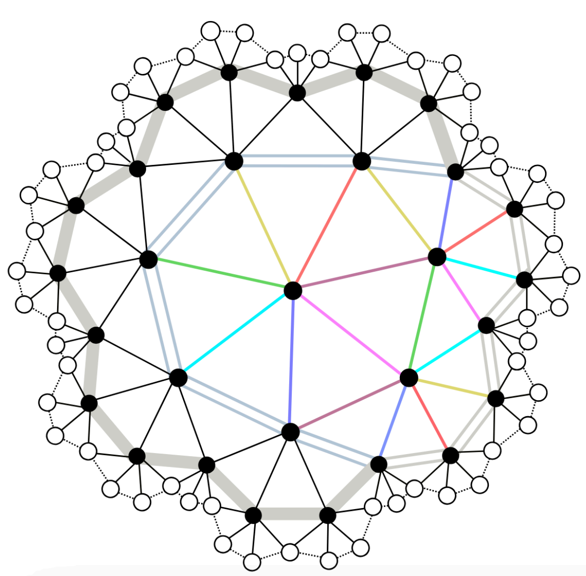
fragmentation dynamics

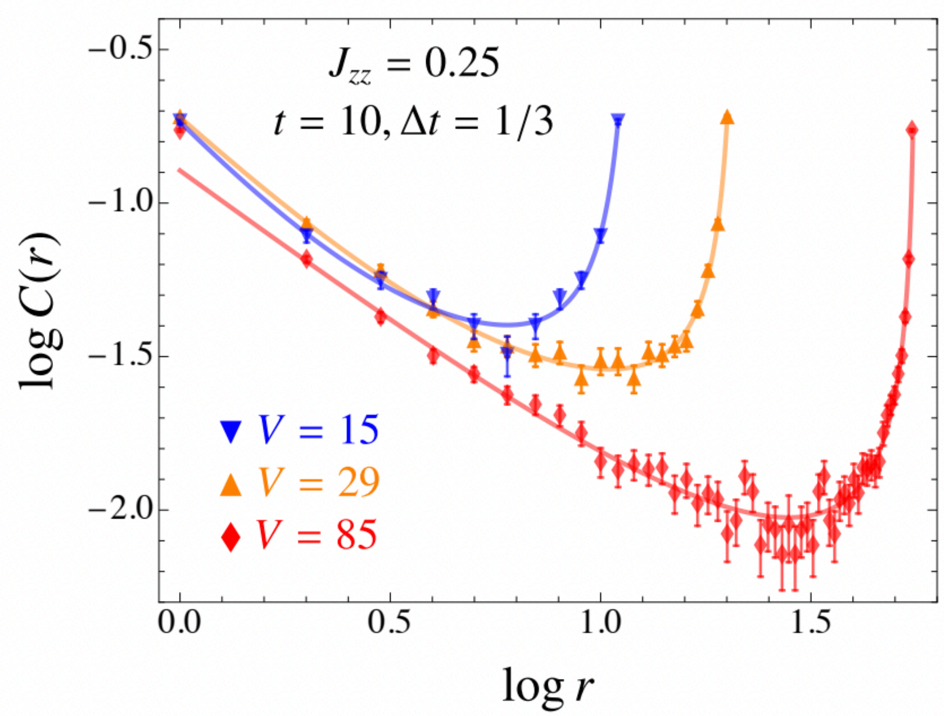




[Lee, Turro, Yao, arXiv:2409.13830]

Holography





[**YYL,** Sajid, Unmuth-Yockey, *Phys.Rev.D* 110 (2024) 3, 034507]

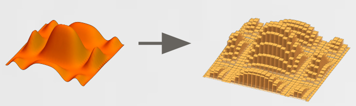
Lager lattices and with higher truncations?

entanglement entropy?

"Quantum potential for first-principle calculations!"

(2030s) narrow down the framework with

- systematic studies of errors from digitization —— phase diagrams with spontaneous symmetry breaking, and for improved H
- hardware co-design —— qudits for blocking encodings and the redundancies
- improving algorithms —— efficient quantum circuits based on qudits
- benchmark studies —— HEP case calculations for experiments
- …



$|q\rangle^N \to |G\rangle$

↓

$\mathcal{U}\,|G\rangle^L \to |\psi_0\rangle$

↓

$\mathcal{U}\,|\psi_0\rangle \to |\psi(t)\rangle$

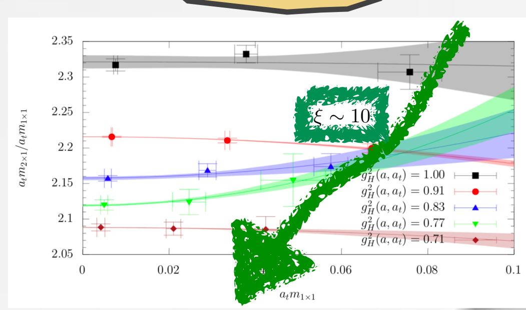
↓

$\langle \mathcal{O} \rangle$

S. P. Jordan,
K. S. M. Lee,
J. Preskill

2011-

various methods



2020 -

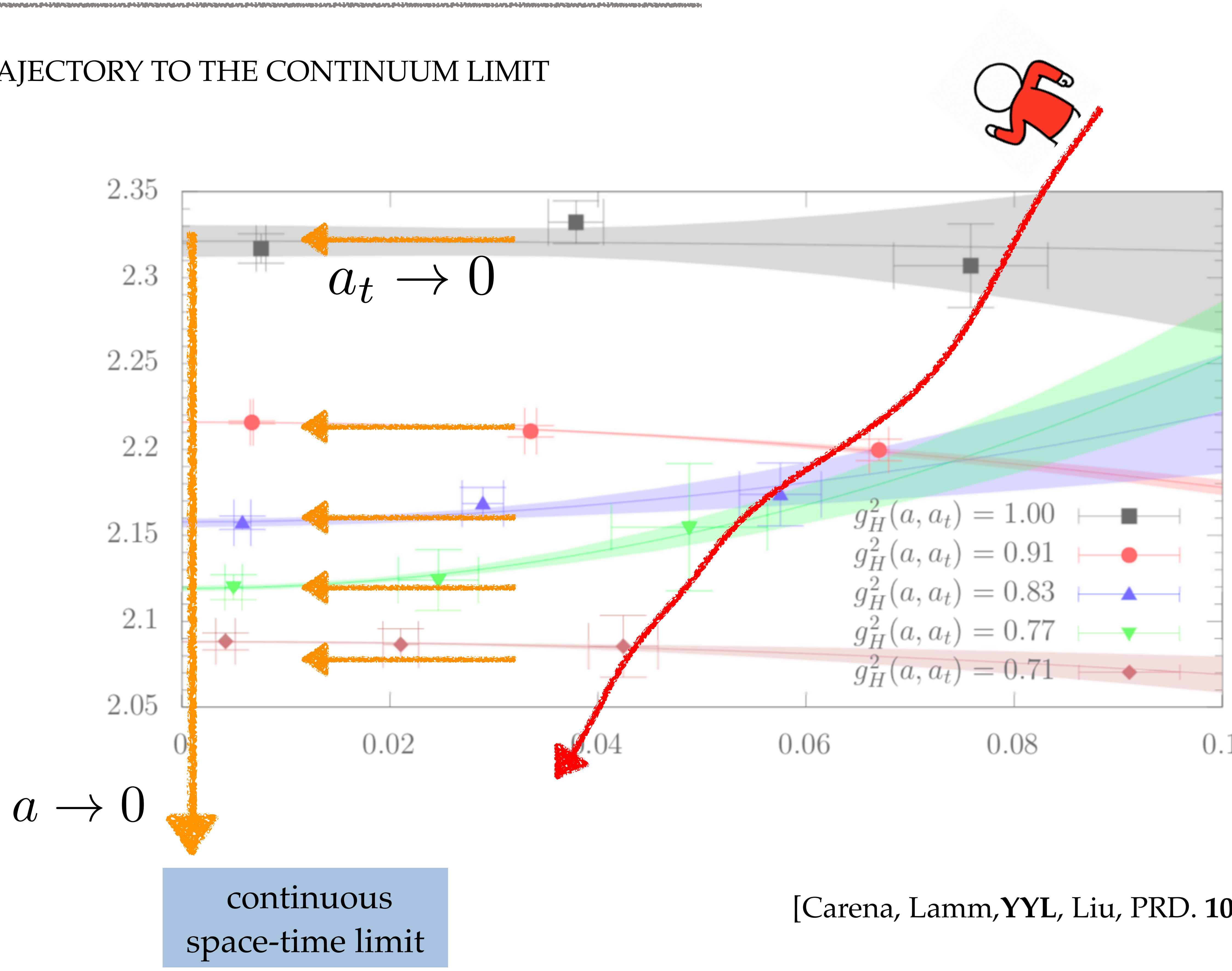
2030s -

Thank you

BACK UP

To reach observables in the continuum limit

TRAJECTORY TO THE CONTINUUM LIMIT



[Carena, Lamm,**YYL**, Liu, PRD. **104**, 094519]