Total Gluon Helicity from Lattice without Effective Theory Matching

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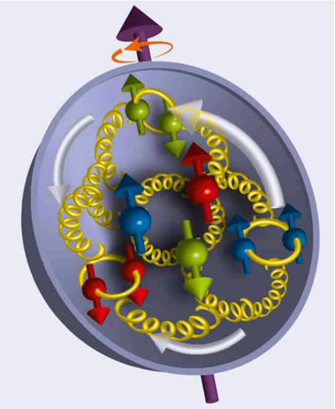
Spin structure of the proton

- Proton is a composite particle with spin 1/2



Jaffe-Manohar sum rule **Jaffe and Manohar, NPB 90'**

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

- Complete decomposition into quark and gluon spin & orbital AM
- Gauge-dependent, but with clear partonic interpretation

Ji sum rule **Ji, PRL 97'**

$$\frac{1}{2} = J_q + J_g = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

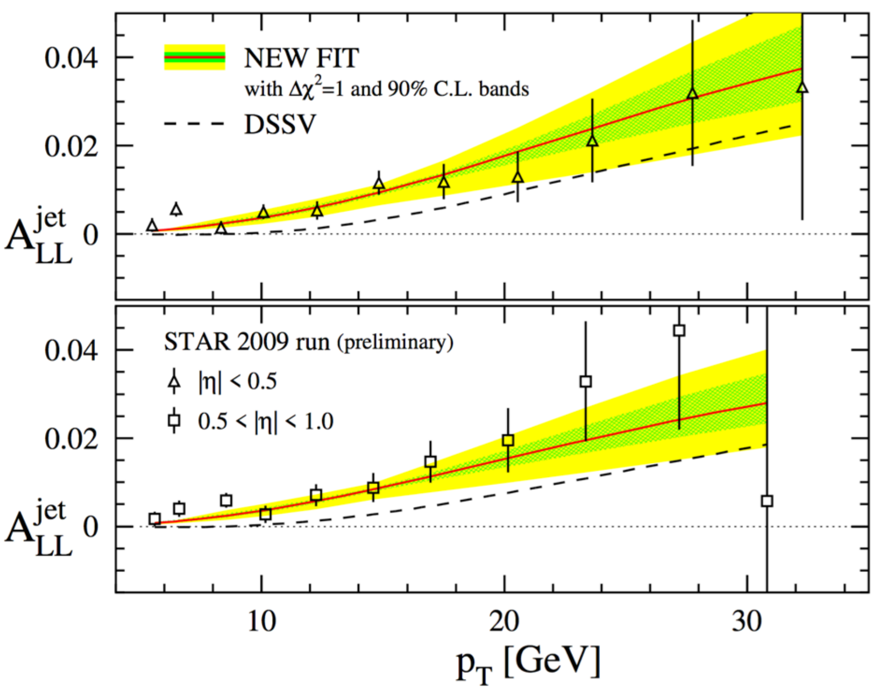
- Frame- and gauge-independent
- Quark and gluon contributions related to the moments of GPDs

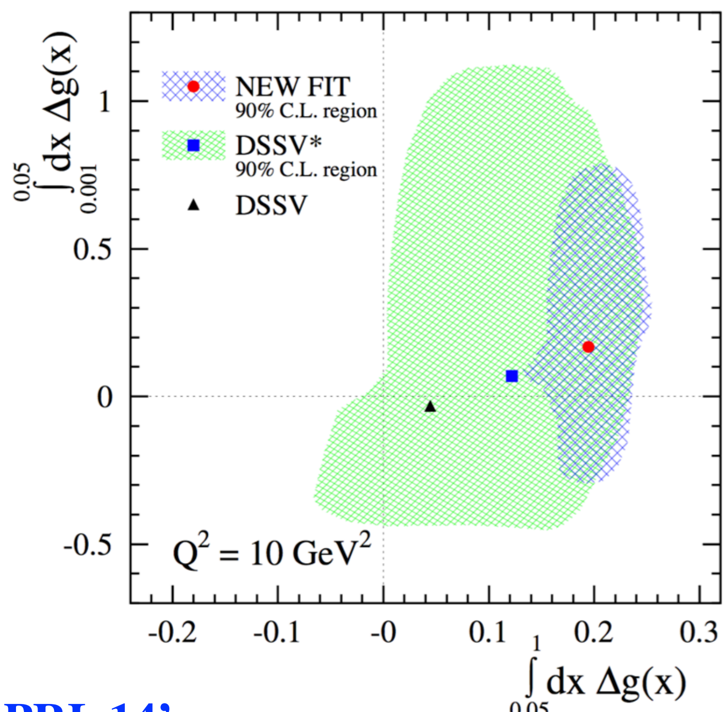
Total gluon helicity from experiments

- The total gluon helicity ΔG can be measured by probing the spin-dependent gluon helicity distribution in polarized high-energy scattering experiments

$$\Delta g(x) = \frac{i}{2xP^+}\int \frac{d\xi^-}{2\pi} e^{-ix\xi^- P^+} \langle PS \,|\, F_a^{+\mu}(\xi^-)\mathcal{L}_{ab}(\xi^-,0)\tilde{F}^+_{b,\mu}(0)\,|PS\rangle$$

$$\Delta G = \int dx \Delta g(x)$$





de Florian et al, PRL 14'

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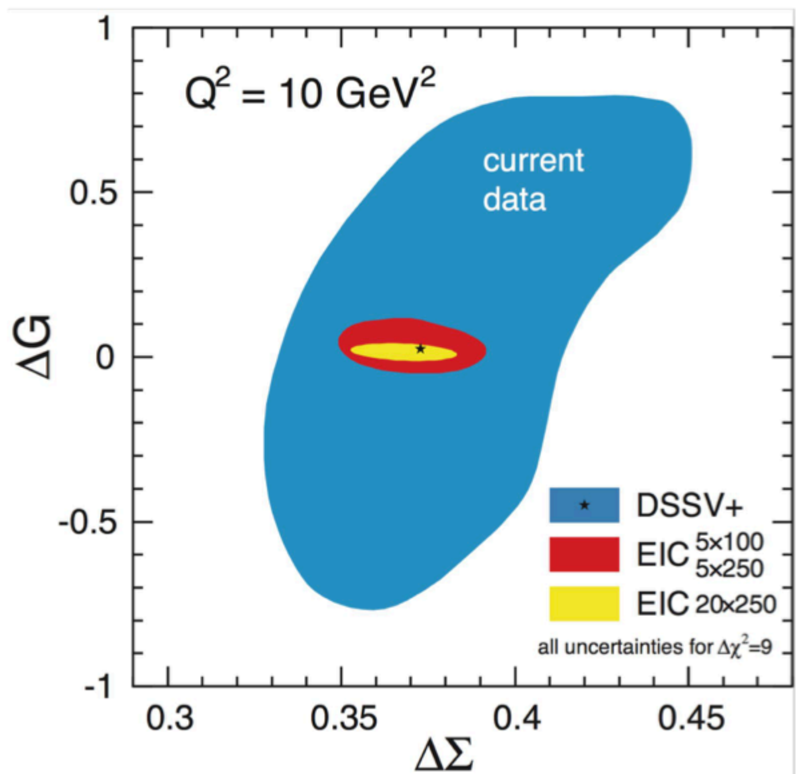

Accardi et al, EPJA 16’

Total gluon helicity from theory

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$$\Delta G = \int dx \Delta g(x)$$

- $\Delta g(x)$ is defined in term of complicated nonlocal lightcone correlations

- Unlike the moments of quark distributions, ΔG is still nonlocal and cannot be readily calculated on a Euclidean lattice

- It reduces to $\vec{E}_a \times \vec{A}_a$ in the lightcone gauge, but the lightcone gauge is difficult to realize on lattice

Total gluon helicity from theory

- We have shown that ΔG can be obtained by boosting the matrix element of the static operator $\vec{E}_a \times \vec{A}_{\perp,a}$ to the infinite momentum frame
  Ji, JHZ, Zhao, PRL 13'
  - $\vec{A}_\perp$ is the transverse part of the gauge field
  - It takes a nonlocal form in general, but reduces to $\vec{A}$ in the Coulomb gauge

- ΔG can be calculated by studying the matrix element

$$\Delta\tilde{G} = < PS|\,\vec{E} \times \vec{A}\,|PS >_{C.G.}$$

in a large momentum nucleon state (subject to a factorization/matching)

$$\begin{tikzcd}[column sep=huge]
\text{total gluon helicity} \arrow[d, "A^+ = 0"] & (\vec{E}_a \times \vec{A}_{\perp,a})^z \arrow[l, swap, "\text{Boost to IMF}"] \arrow[d, "\nabla \cdot A = 0"] \\
(P_z \to \infty)\quad (\vec{E}_a \times \vec{A}_a)^z & (\vec{E}_a \times \vec{A}_a)^z \quad (P_z \text{ is finite}) \arrow[l, swap, "\text{Matching}"]
\end{tikzcd}$$

Total gluon helicity from lattice

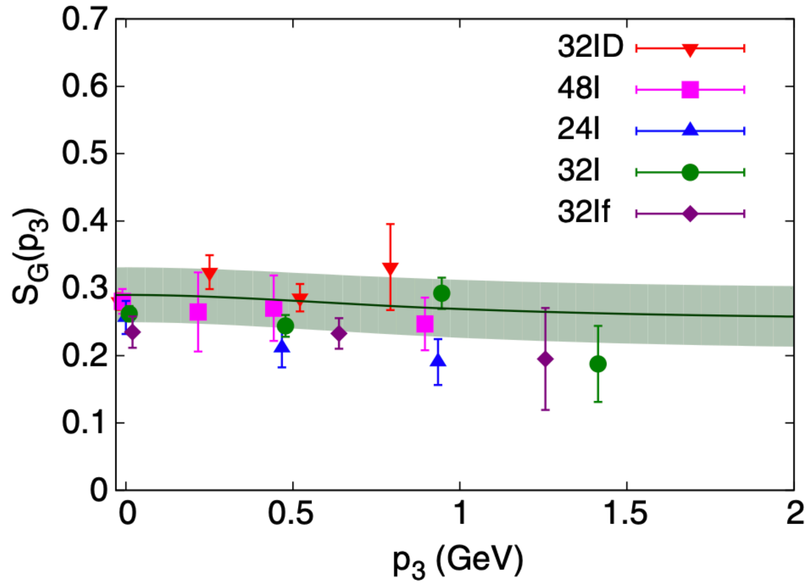
- Factorization for $\Delta\tilde{G} = \langle PS|\vec{E}\times\vec{A}|PS\rangle_{C.G.}$ Ji, JHZ, Zhao, PRL 13'

$$\Delta\tilde{G} = C_{gg}\Delta G + C_{gq}\Delta\Sigma + h.t.,$$
$$C_{gg} = 1 + a_s C_A \frac{7}{3}\ln\frac{P_z^2}{\mu^2} + \text{fin.}, \qquad C_{gq} = a_s C_F \frac{4}{3}\ln\frac{P_z^2}{\mu^2} + \text{fin.}.$$

- Lattice calculation Yang et al, PRL 17'

Potential improvements:

- Nonperturbative renormalization
- Perturbative matching rather than an empirical fit
- Resummations
- Control of power corrections



What if ΔG is extracted from the gluon helicity distribution?

Total gluon helicity from lattice

- Factorization for $\Delta\tilde{G} = \langle PS|\vec{E}\times\vec{A}|PS\rangle_{C.G.}$ **Ji, JHZ, Zhao, PRL 13'**

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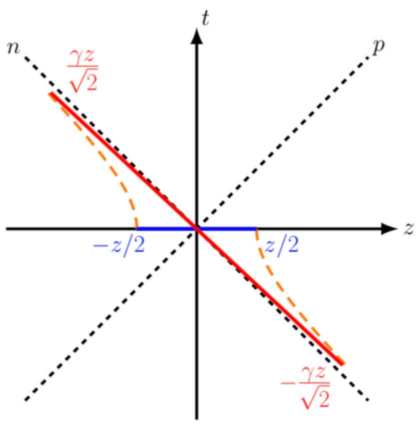
- Factorization based on the gluon helicity distribution

Yao, JHZ et al, JHEP 23'

$$\Delta\tilde{g}(x) = \int\frac{dy}{|y|}C_{gg}\left(\frac{x}{y},\frac{\mu}{yP_z}\right)\Delta g(y) + \int\frac{dy}{|y|}C_{gq}\left(\frac{x}{y},\frac{\mu}{yP_z}\right)\Delta q(y) + h.t.,$$
$$C_{gg}\supset\delta(1-\frac{x}{y})+4a_sC_A\theta(x)\theta(y-x)\left\{\frac{(2x^2-3xy+2y^2)}{(x-y)y}\left(\ln\frac{\mu^2}{4y^2P_z^2}-\ln\frac{x(y-x)}{y^2}\right)+fin.\right\}.$$



$$\tilde{h}(z,P_z,1/a) = \langle PS|F^{3\mu}(z)\mathcal{L}(z,0)\tilde{F}^0_\mu(0)|PS\rangle,$$
$$\Delta\tilde{g}(x,P_z,1/a) = \frac{i}{2xP_z}\int\frac{dz}{2\pi}e^{ixzP_z}\tilde{h}(z,P_z,1/a),$$

Ji, PRL 13' & SCPMA 14', Ji, JHZ et al, RMP 21'

Total gluon helicity from lattice

- Factorization for $\Delta\tilde{G} = \langle PS|\vec{E}\times\vec{A}|PS\rangle_{C.G.}$ **Ji, JHZ, Zhao, PRL 13'**

$$\Delta\tilde{G} = C_{gg}\Delta G + C_{gq}\Delta\Sigma + h.t.,$$
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$$\Delta\tilde{g}(x) = \int\frac{dy}{|y|}C_{gg}\left(\frac{x}{y},\frac{\mu}{yP_z}\right)\Delta g(y) + \int\frac{dy}{|y|}C_{gq}\left(\frac{x}{y},\frac{\mu}{yP_z}\right)\Delta q(y) + h.t.,$$
$$C_{gg} \supset \delta\left(1-\frac{x}{y}\right)+4a_s C_A\theta(x)\theta(y-x)\left\{\frac{(2x^2-3xy+2y^2)}{(x-y)y}\left(\ln\frac{\mu^2}{4y^2P_z^2}-\ln\frac{x(y-x)}{y^2}\right)+fin.\right\}.$$

- **Inconsistency:**
  - The intrinsic momentum scale in the matching shall be the parton momentum yP_z, not the proton momentum P_z
  - Alternatively, $\int dx\Delta\tilde{g}(x) \neq C_{gg}\Delta G + C_{gq}\Delta\Sigma + h.t.$ is, in general, a convolution rather than a multiplication

Total gluon helicity without EFT matching

- This inconsistency can be resolved for certain choices of gluon operators
  Pang, Yao, JHZ, JHEP 24'

$$\tilde{h}(z, P_z, \tfrac{1}{a}) \xrightarrow{\text{hybrid scheme}} \tilde{h}_R^{\text{hyb.}}(z, P_z) \xrightarrow{Z_T} \tilde{h}_R^{\overline{\text{MS}}}(z, P_z) \xrightarrow{\mathcal{F}, 1_{\text{th}}\ \text{moment}} \Delta G$$

$(\langle PS|F^{3\mu}\mathcal{L}(z,0)\tilde{F}^0_\mu(0)|PS\rangle)$

Matching coefficients between $\tilde{h}_R^{\overline{\text{MS}}}(z, P_z)$ and $h_R^{\overline{\text{MS}}}(z, P_z)$:

$$C_{gg}(\alpha, z, \mu) = \delta(\alpha) + 2a_s C_A\Big\{\Big(4\alpha\bar{\alpha} + 2\Big[\tfrac{\bar{\alpha}^2}{\alpha}\Big]_+\Big)(\mathrm{L_z} - 1) + 6\alpha\bar{\alpha} - 4\Big[\tfrac{\ln(\alpha)}{\alpha}\Big]_+ + (-3\mathrm{L_z} + 2)\delta(\alpha)\Big\},$$

$$C_{gq}(\alpha, z, \mu) = \tfrac{-2ia_s C_F}{z}\Big\{-2\alpha(\mathrm{L_z} + 1) - 4\bar{\alpha} + \mathrm{L_z}\delta(\alpha)\Big\},$$

Matching between $\Delta\tilde{G}$ and ΔG is trivial:

$$\Delta\tilde{G}(P_z, \mu) = \tfrac{1}{2P_z}\int_0^\infty dz\,\tilde{h}(z, P_z, \mu)$$
$$= \int d\lambda \int_0^1 \tfrac{d\alpha}{\bar{\alpha}}\Big[C_{gg}\big(\alpha, \tfrac{\lambda}{\bar{\alpha}P_z}, \mu\big)h_g(\lambda, \mu) + C_{gq}\big(\alpha, \tfrac{\lambda}{\bar{\alpha}P_z}, \mu\big)h_q(\lambda, \mu)\Big] + h.t. = \Delta G + h.t.,$$

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- Relation to the matrix element of the topological current

$$\int_0^\infty dz \langle PS | m^{3\mu;0\mu} | PS \rangle = \langle PS | A^i B^i | PS \rangle |_{A^z=0} = \langle PS | K^0 | PS \rangle |_{A^z=0}$$

$$m^{3\mu;0\mu} = F^{3\mu}(z) \mathscr{L}(z,0) \tilde{F}^0_\mu(0)$$

Trivial matching to $\langle PS | K^+ | PS \rangle |_{A^+=0} = 4S^+ \Delta G$

Hatta et al, PRD 14'

- This is similar to fixing an axial gauge on the lattice when calculating the matrix element of the topological current

- Similar conclusion also exists for other suitably chosen operators

Total gluon helicity without EFT matching

- From local operator matrix element in a fixed gauge **Pang, Yao, JHZ, JHEP 24'**

$$\langle P'S | K^{0/z} | PS \rangle_{\text{C.G.}} \xrightarrow{\text{IMF}} \langle P'S | K^{0/z} | PS \rangle_{A^+=0}$$

- But we shall start from the non-forward matrix element and take a special forward limit
- The forward limit suffers from some subtlety that can be best elucidated by examining its non-forward matrix element. In Coulomb gauge

$$\langle P'S | K^\mu | PS \rangle_{\nabla\cdot A=0,\text{finite}} = S^\mu a_1 + \text{h.t.},$$
$$\langle P'S | K^\mu | PS \rangle_{\nabla\cdot A=0,\text{pole}} = \frac{S^\mu}{S\cdot q} b_1 + \text{h.t.}$$

- b_1 can lead to contributions that contaminate ΔG, and can be safely ignored by taking the forward limit along the direction $q^+ \gg \{q_\perp, q^-\}$

Total gluon helicity without EFT matching

- From local operator matrix element in a fixed gauge **Pang, Yao, JHZ, JHEP 24'**

$$\langle P'S|K^{0/z}|PS\rangle_{\text{C.G.}} \xrightarrow{\text{IMF}} \langle P'S|K^{0/z}|PS\rangle_{A^+=0}$$

- But we shall start from the non-forward matrix element and take a special forward limit

The bare result $\lim_{q\to 0}\langle P'S|K^{0/z}|PS\rangle_{\text{C.G.}}$ needs to be renormalized, we adopt RI/MOM scheme:

$$\begin{pmatrix} \Delta G_{\mathrm{R}}^{\mathrm{RI}} \\ \Delta\Sigma_{\mathrm{R}}^{\mathrm{RI}} \end{pmatrix} = \begin{pmatrix} Z_{11}^{\mathrm{RI}} & Z_{12}^{\mathrm{RI}} \\ Z_{21}^{\mathrm{RI}} & Z_{22}^{\mathrm{RI}} \end{pmatrix} \begin{pmatrix} \Delta G_B \\ \Delta\Sigma_B \end{pmatrix},$$

The perturbative gauge-invariance of $\langle PS|K^{0/z}|PS\rangle$ has been used.

The conversion factor from RI scheme to $\overline{\mathrm{MS}}$ scheme is derived at one-loop:

$$R_{11}^{\overline{MS},\mathrm{RI}}(\mu_R^2,\mu^2) = 1 + a_s\left[\beta_0 \ln\left(\frac{\mu_R^2}{\mu^2}\right) - \frac{367}{36}C_A + \frac{10}{9}n_f\right],$$

$$R_{12}^{\overline{\mathrm{MS}},\mathrm{RI}}(\mu_R^2,\mu^2) = a_s C_F\left[3\ln\left(\frac{\mu_R^2}{\mu^2}\right) - 6\right],$$

$$R_{21}^{\overline{\mathrm{MS}},\mathrm{RI}}(\mu_R^2,\mu^2) = -2a_s C_F,$$

$$R_{22}^{\overline{MS},\mathrm{RI}}(\mu_R^2,\mu^2) = 1.$$

Summary and outlook

- Understanding the spin structure of the proton is an important goal of the EicC/EIC program

- The total gluon helicity ΔG can be accessed on the lattice, two different types of approaches
  - From local operator matrix element in an appropriately fixed gauge
  - From the gluon helicity distribution $\Delta g(x)$

- Inconsistency in the factorization relations is resolved by utilizing suitable gluon operators that do not require an EFT matching
  - In particular, no Fourier transform is needed in the second approach

- Many systematic improvements need to be done