Determining heavy meson LCDAs from lattice QCD

Based on 2403.17492 and 2410.xxxx

In collaboration with LPC members and C.D. Lü, J. Xu, S. Zhao, et al.

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Outline

- **Motivation**
- **Theoretical framework for the two-step factorization method**
- **Lattice verification**
  - Lattice setup
  - Bare matrix elements and hybrid-ratio renormalization
  - Matching from quasi DAs to QCD LCDAs
  - Determination of HQET LCDA
- **Phenomenological discussions**
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  - Determination of the inverse and inverse-logarithmic moments
  - Impact on $B \to V$ form factors
- **Summary and prospect**

Motivation

Main tasks of heavy flavor physics:

- Precisely testing the standard model
- Indirect search for new physics
- Study on CP violation

- $B \to \pi\pi$: ***Beneke, Buchalla, Neubert, Sachrajda, 1999; 1422 citations***
- $B \to \pi K$: ***Beneke, Buchalla, Neubert, Sachrajda, 2001; 1177 citations***

- $B \to \pi\ell\nu$: ***Becher, Hill, 2005; 215 citations***
  Khodjamirian, Mannel, Offen, Wang, 2011; 192 citations
- $B \to K^{(*)}\ell\ell$: ***Khodjamirian, Mannel, Pivavorov, Wang, 2010; 486 citations***
- $B \to D\ell\nu$: ***HPQCD Collaboration, 2015; 387 citations***

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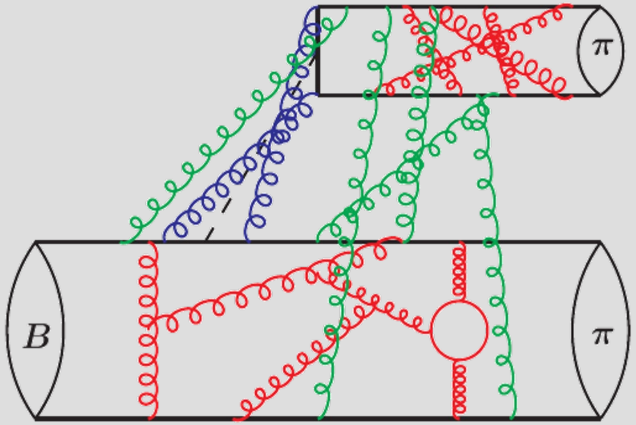
A multi-scale problem: factorization

$$\langle \pi(p')\,\pi(q)\,|Q_i|\,\bar{B}(p)\rangle = f^{B\to\pi}(q^2)\int_0^1 dx T_i^{\mathrm{I}}(x)\phi_\pi(x) + \int_0^1 d\xi dx dy T_i^{\mathrm{II}}(\xi,x,y)\phi_B(\xi)\phi_\pi(x)\phi_\pi(y)$$



- Perturbative： matching, resummation, evolution
- Nonperturbative： Lattice QCD, sum rules, SU(3) symmetry, Quark model

Towards the precision frontier?

- The uncertainty of $B \to \pi,\ K^*$ form factors from LCSRs:

[Gao, Lu, Shen, Wang, Wei, 2020; Cui, Huang, Shen, Wang, 2023]

$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}{}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

$$f^+_{B\to\pi}(0) = 0.122 \times \Big[1 \pm 0.07\Big|_{S_0^\pi} \pm 0.11\Big|_{\Lambda_q} \pm 0.02\Big|_{\lambda_E^2/\lambda_H^2}{}^{+0.05}_{-0.06}\Big|_{M^2} \pm 0.05\Big|_{2\lambda_E^2+\lambda_H^2}$$
$$\qquad {}^{+0.06}_{-0.10}\Big|_{\mu_h} \pm 0.04\Big|_{\mu}{}^{+1.36}_{-0.56}\Big|_{\lambda_B}{}^{+0.25}_{-0.43}\Big|_{\sigma_1,\sigma_2}\Big].$$

λ_B and σ_1: the first inverse and inverse-log moments,

$\varphi_B^\pm$: uncertainties from different parameterizations of the B meson LCDA.

Without reliable B LCDA, it is impossible to discuss precision calculation!

What do we know about heavy meson LCDAs?

- Equation of motion: [Kawamura, Kodaira, Qiao, Tanaka, 2001]

- Evolution behavior: [Lange, Neubert, 2003; Bell, Feldmann, 2008]

$$\frac{d}{d\ln\mu}\varphi_B^+(\omega,\mu) = -\frac{\alpha_s C_F}{4\pi}\int_0^\infty d\omega' \gamma_+^{(1)}(\omega,\omega',\mu)\,\varphi_B^+(\omega',\mu) + \mathcal{O}\left(\alpha_s^2\right)$$

$$\gamma_+^{(1)}(\omega,\omega',\mu) = \left(\Gamma_{\text{cusp}}^{(1)}\ \ln\frac{\mu}{\omega} - 2\right)\delta(\omega-\omega') - \Gamma_{\text{cusp}}^{(1)}\,\omega\left[\frac{\theta(\omega'-\omega)}{\omega'(\omega'-\omega)} + \frac{\theta(\omega-\omega')}{\omega(\omega-\omega')}\right]_+$$

  - Solution of evolution equations; [Bell, Feldmann, Wang and Yip, 2013; Braun, Manashov, 2014]
  - RG equations of $\phi_B^+(\omega,\mu)$ at two-loops; [Braun, Ji, Manashov, 2019; Liu, Neubert, 2020]
  - RG equations of the higher-twist B-meson distribution amplitudes; [Braun, Ji, Manashov, 2017]
  - NNLO QCD correction to relevant hadronic B-meson decays. [Bell, Beneke, Huber, Li, 2020]

- Perturbative constraint at large ω: [Lee, Neubert, 2005]

$$\varphi^+(\omega,\mu) = \frac{C_F\alpha_s}{\pi\omega}\left[\left(\frac{1}{2} - \ln\frac{\omega}{\mu}\right) + \frac{4\bar{\Lambda}}{3\omega}\left(2 - \ln\frac{\omega}{\mu}\right)\right]$$

But for the fully distribution……

- Models for heavy meson LCDAs

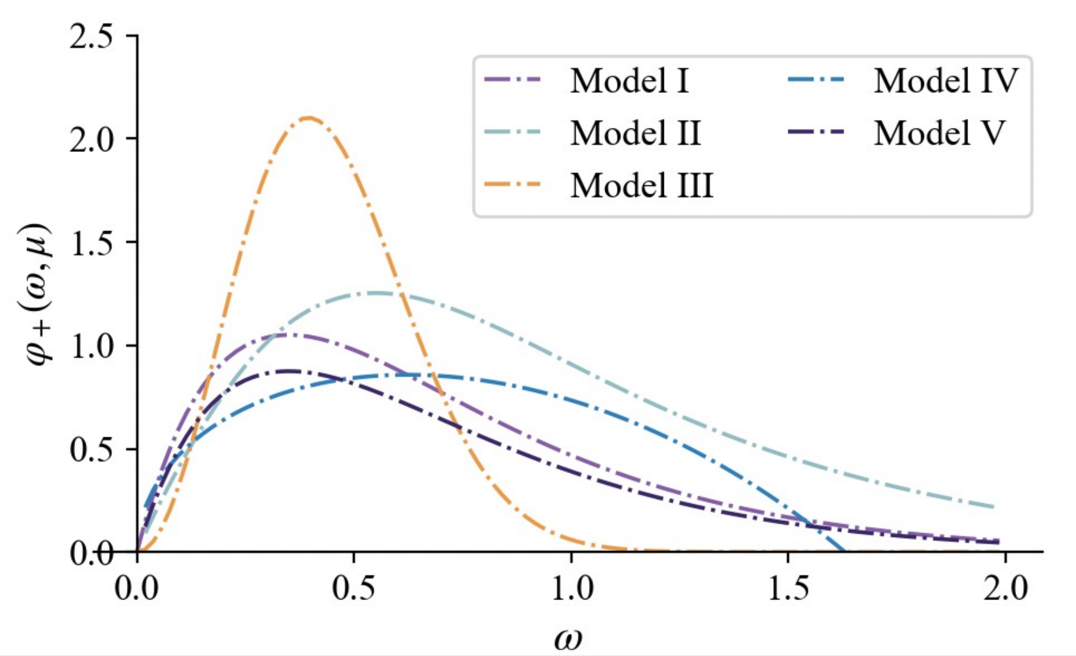
$$\varphi_{\rm I}^{+}(\omega,\mu_0)=\frac{\omega}{\omega_0^2}e^{-\omega/\omega_0},$$

$$\varphi_{\rm II}^{+}(\omega,\mu_0)=\frac{4}{\pi\omega_0}\frac{k}{k^2+1}\left[\frac{1}{k^2+1}-\frac{2\left(\sigma_B^{(1)}-1\right)}{\pi^2}\ln k\right],$$

$$\varphi_{\rm III}^{+}(\omega,\mu_0)=\frac{2\omega^2}{\omega_0\omega_1^2}e^{-(\omega/\omega_1)^2},$$

$$\varphi_{\rm IV}^{+}(\omega,\mu_0)=\frac{\omega}{\omega_0\omega_2}\frac{\omega_2-\omega}{\sqrt{\omega\left(2\omega_2-\omega\right)}}\theta\left(\omega_2-\omega\right),$$

$$\varphi_{\rm V}^{+}(\omega,\mu_0)=\frac{\Gamma(\beta)}{\Gamma(\alpha)}\frac{\omega}{\omega_0^2}e^{-\omega/\omega_0}U(\beta-\alpha,3-\alpha,\omega/\omega_0).$$

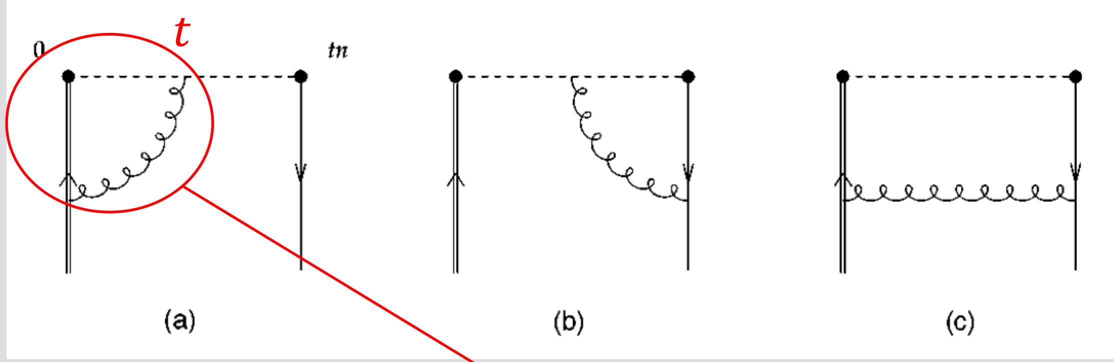


This leads to the largest systematic error in $B\to V$ form factors: [Gao, Lu, Shen, Wang, Wei, 2020]

$$\mathcal{V}_{B\to K^*}(0)=0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}{}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

Difficulties in first principle determinations

$$\langle H(p_H)|\, \bar{h}_v(0)\hbar_+\gamma_5\,[0,tn_+]\, q_s(tn_+)\,|0\rangle = -i\tilde{f}_H m_H n_+\cdot v\int_0^\infty d\omega e^{i\omega t n_+\cdot v}\varphi_+(\omega;\mu)$$



Diverge at $t\to 0$!

$$O_+^{\rm ren}(t,\mu) = O_+^{\rm bare}(t) + \frac{\alpha_s C_F}{4\pi}\left\{\left(\frac{4}{\hat\epsilon^2}+\frac{4}{\hat\epsilon}\ln(it\mu)\right)O_+^{\rm bare}(t) - \frac{4}{\hat\epsilon}\int_0^1 du\frac{u}{1-u}\left[O_+^{\rm bare}(ut)-O_+^{\rm bare}(t)\right]\right.$$

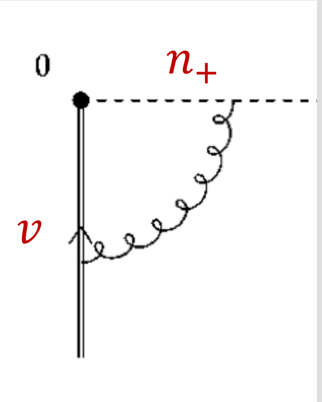
[Braun, Ivanov, Korchemsky, 2004]

Cusp divergence: No local limit!

- Non-negative moments are not related to OPE, and actually they diverge
- Cannot obtain φ_B from lattice QCD through their moments.

How to solve this problem?

Cusp divergence:



$$\cosh\theta = \frac{n_+ \cdot v}{\sqrt{n_+^2}\sqrt{v^2}}.$$

[Korchemskaya, Korchemsky, 1992]

Light cone $n_+^2 = 0 \Rightarrow$ divergence!

✓ Off light-cone Wilson line $n_+^2 \neq 0$, still heavy quark field h_v

[Wang^2, Xu, Zhao, 2020; Xu, Zhang, 2022; Hu, Wang, Xu, Zhao, 2024]

🤔 Difficult to realize on lattice QCD.

✓ No h_v : QCD heavy quark. $\Rightarrow$ This work

[Han, Wang, Zhang, et.al, 2403.17492; Han, Wang, Zhang, Zhang, 2408.13486; Deng, Wang, Wei, Zeng, 2409.00632]

A two-step factorization method

- Start from Quasi DA, calculable from LQCD 3 characteristic scales: $P^z, m_H, \Lambda_{\rm QCD}$

$(P^z, m_H, \Lambda_{\rm QCD})$ **Quasi DA** → **LaMET** → $(m_H, \Lambda_{\rm QCD})$ **LCDA in QCD** → **bHQET** → $(\Lambda_{\rm QCD})$ **LCDA in HQET**

LaMET: $\mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(xP^z, \bar{x}P^z)^2}\right)$

bHQET: $\mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_H}\right)$

- A multi-scale processes:

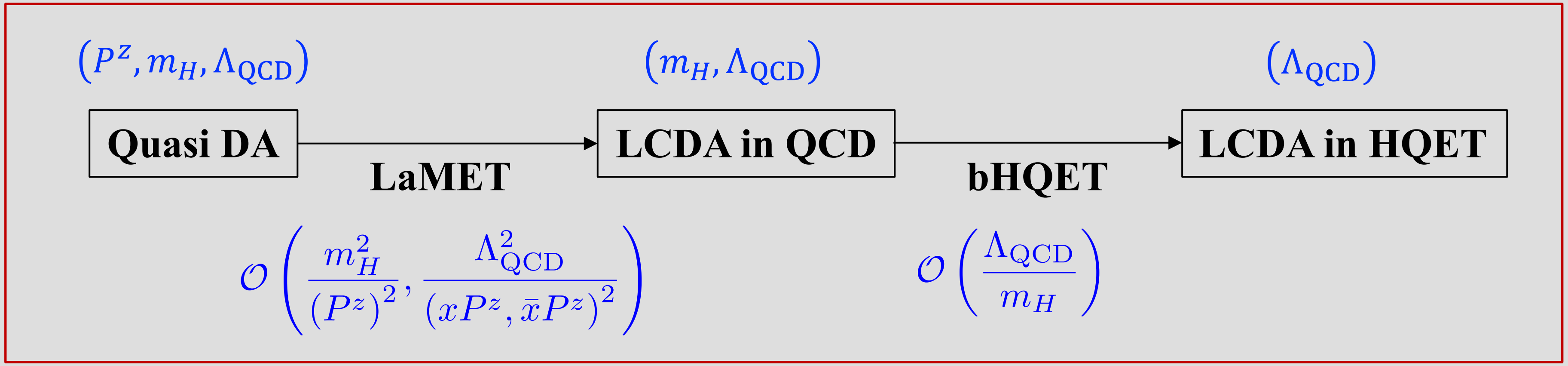
  1. LaMET requires $\Lambda_{\rm QCD}, m_H \ll P^z$ and finally integrate out P^z;
  2. bHQET requires $\Lambda_{\rm QCD} \ll m_H$ and integrate out m_H;

⇒ **Hierarchy $\boldsymbol{\Lambda_{\rm QCD} \ll m_H \ll P^z}$.**

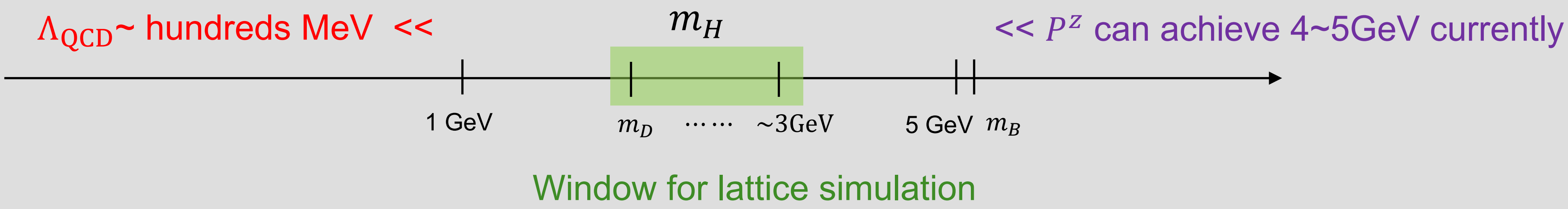
A two-step factorization method

- Start from Quasi DA, calculable from LQCD

3 characteristic scales: $P^z, m_H, \Lambda_{\rm QCD}$



⇒ **Hierarchy $\boldsymbol{\Lambda_{\rm QCD} \ll m_H \ll P^z}$:** Still a big challenge for lattice simulation



Lattice QCD verification

- Finest CLQCD ensemble currently:

  $n_s^3 \times n_t = 48^3 \times 144,\ a \simeq 0.052\text{fm};$

- $m_\pi \simeq 317\text{MeV},\ m_D \simeq 1.92\text{GeV};$

- Coulomb gauge fixed plane source:

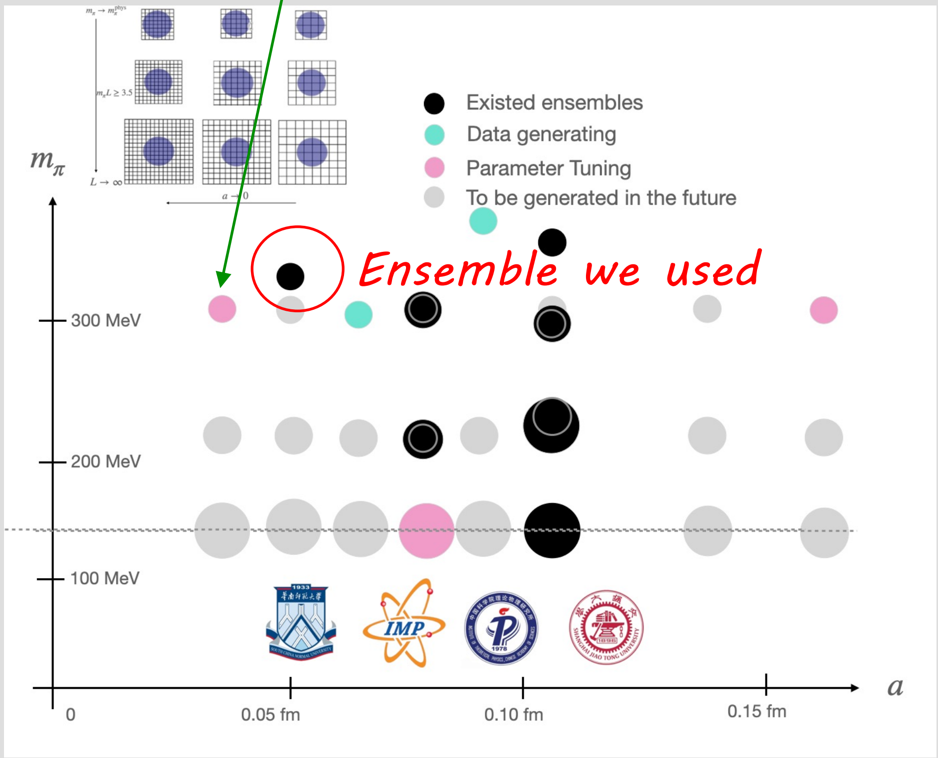
  Summing over all x & y sites to improve signal

  Fixed position along z axis to project arbitrary P^z

- 549 configurations × 16measurements;

- $P^z = \{2.99, 3.49, 3.98\}$GeV up to about 4GeV.

We are looking forward to this finer one!



[Hu, et·al·, 2004]

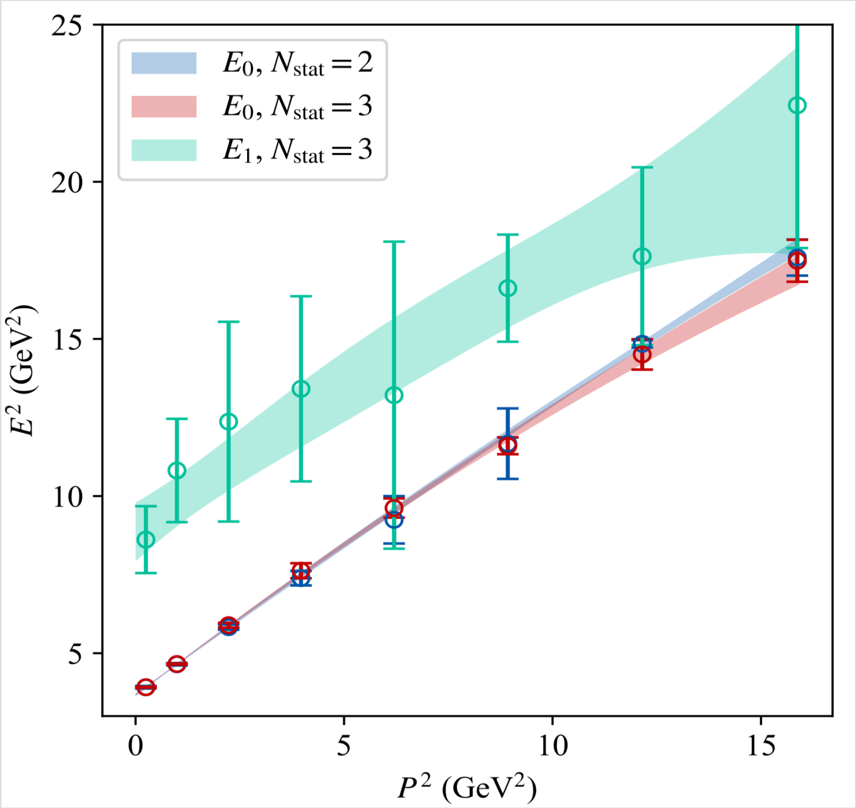
Lattice QCD verification

➢ Dispersion relation:

$$E(P^z) = \sqrt{m^2 + c_1\,(P^z)^2 + c_2\,(P^z)^4\,a^2}$$

$N_{\text{stat}} = 2$	$m = 1.917(5)\text{GeV}$	$c_1 = 0.977(23)$	$c_2 = -0.072(25)$
$N_{\text{stat}} = 3$	$m = 1.915(6)\text{GeV}$	$c_1 = 1.007(30)$	$c_2 = -0.124(45)$
	$E_1 = 2.98(16)\text{GeV}$	$c_1 = 1.00(43)$	$c_2 = -0.21(49)$

Consistent with the relativistic dispersion relation up to possible discretization error.

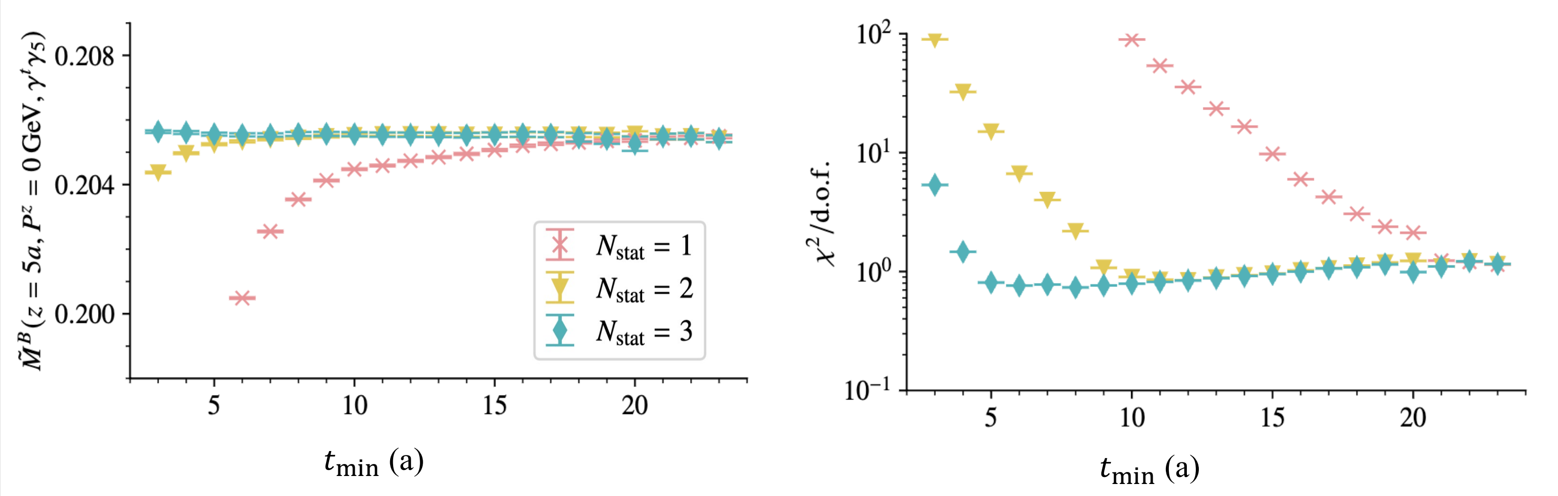


Extraction of bare matrix elements

- Result of zero-momentum matrix element from N-stat fit:

$$C_2(z, P^z, t; \Gamma) = \sum_{x^3} e^{iP^z x^3} \left\langle G_q(x^3+z, t, 0)\Gamma W_c(x^3+z, x^3)\gamma_5 G_Q^\dagger(x^3, t, 0)\right\rangle$$
$$= \sum_n \frac{1}{2E_n} e^{-E_n t} \langle n(P^z)|\bar{Q}(0)\gamma_5 q(0)|0\rangle \langle 0|\bar{q}(z)\Gamma W_c(z,0)Q(0)|n(P^z)\rangle .$$

$$\frac{C_2(z, P^z, t; \Gamma)}{C_2(0, P^z, t; \Gamma)} = \tilde{M}^B(z, P^z; \Gamma) e^{-E_0 t}\left[\sum_{n=0}^{N_{\rm stat}-1} A_n e^{-(E_n - E_0)t}\right]$$

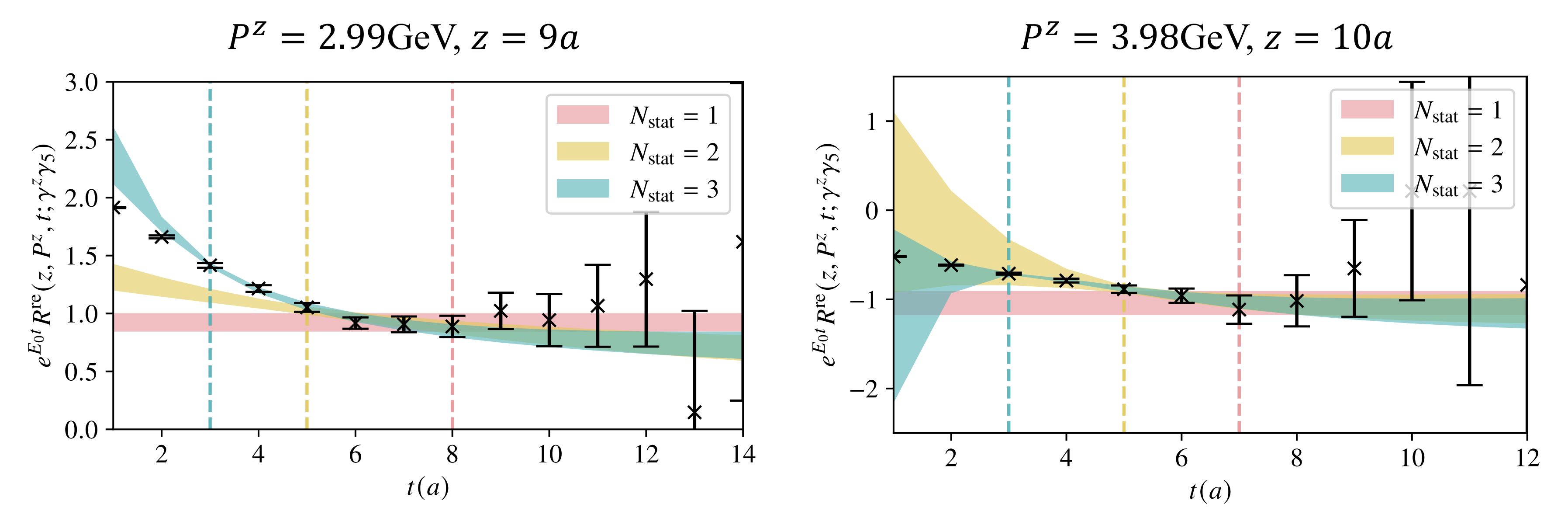


- $N_{\rm stat}$ fit from $t \in [t_{\min}, 31a]$;
- All the fit results are consistent at proper fit ranges;
- Different fit strategy valid at different ranges.

Extraction of bare matrix elements

➢ Fit result for the 2pts at large P^z and z:

$$e^{E_0 t}\frac{C_2(z,P^z,t;\gamma^z\gamma_5)}{C_2(0,P^z,t;\gamma^z\gamma_5)} = \tilde{M}^B(z,P^z;\Gamma)\left[1 + A_1 e^{-\Delta E_1 t} + A_2 e^{-\Delta E_2 t} + \cdots\right]$$



- We take a conservative approach involves first performing the ground state fit over a sufficiently large range of t, after excluding excited-state contaminations.

Renormalization in the hybrid-ratio scheme

➤ Hybrid-ratio scheme on a single lattice spacing: [Ji, et·al·, NPB964, 2021; Holligan, Lin, PRD110, 2024; Baker, et·al, JHEP07, 2024]

$$\tilde{M}^R(z,P^z) = \begin{cases} \frac{\tilde{M}^B(z,P^z;\gamma^z\gamma_5)}{\tilde{M}^B(z,P^z=0;\gamma^t\gamma_5)} & |z| < z_s \\ e^{(\delta m+m_0)(z-z_s)}\frac{\tilde{M}^B(z,P^z;\gamma^z\gamma_5)}{\tilde{M}^B(z_s,P^z=0;\gamma^t\gamma_5)} & |z| \geq z_s \end{cases}.$$

- **Main spirit:** UV divergence comes from the correction of quark operator, linear divergence from the self-energy correction of Wilson line. Both of them are insensitive to the external momentum.

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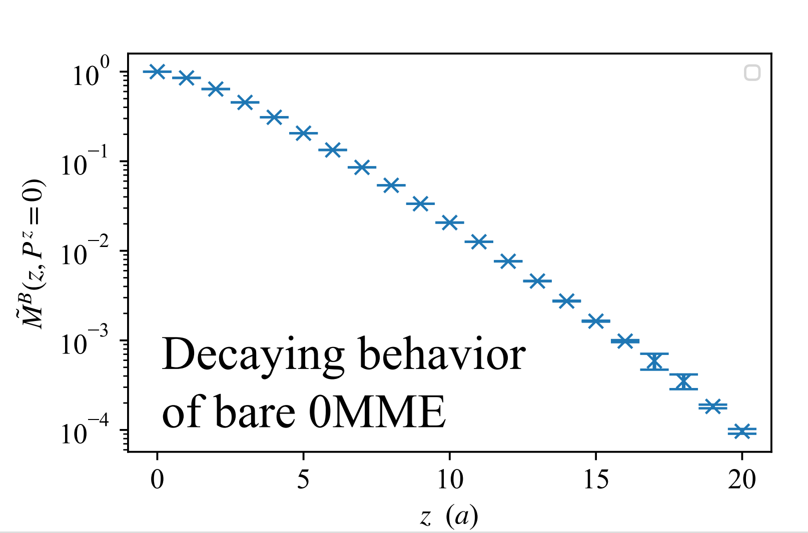
- **Main spirit:** UV divergence comes from the correction of quark operator, linear divergence from the self-energy correction of Wilson line. Both of them are insensitive to the external momentum.
- **At short distance region:** Ratio scheme, subtract the UV and linear divergences by dividing zero-momentum matrix element (0MME).
- **At long distance region:** Ratio will introduce additional nonperturbative effect. We use the self renormalization to parameterize the divergences from fitting and then subtract them.
  - δm: linear divergence
  - m_0: Regularization scheme-specific renormalon ambiguity

Renormalization in the hybrid-ratio scheme

➢ δm governs the exponential decay behavior of long-range correlations, can be extracted from fitting the 0MME at large z.



Renormalization in the hybrid-ratio scheme

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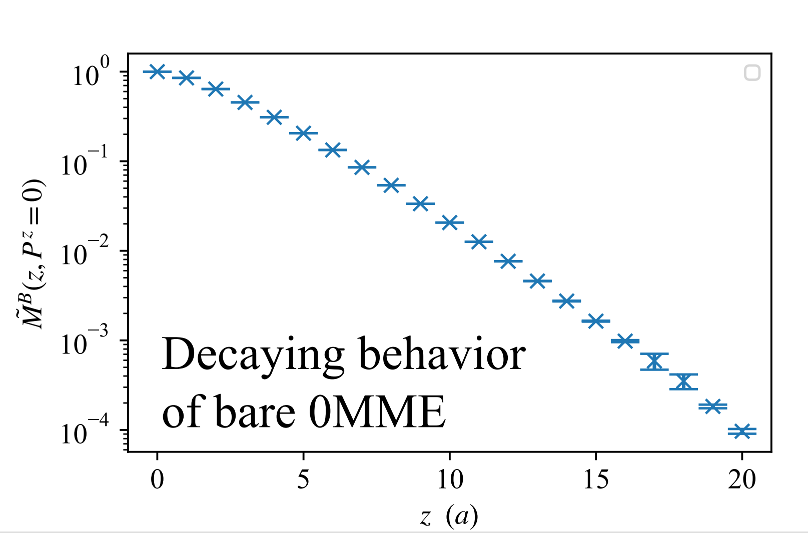
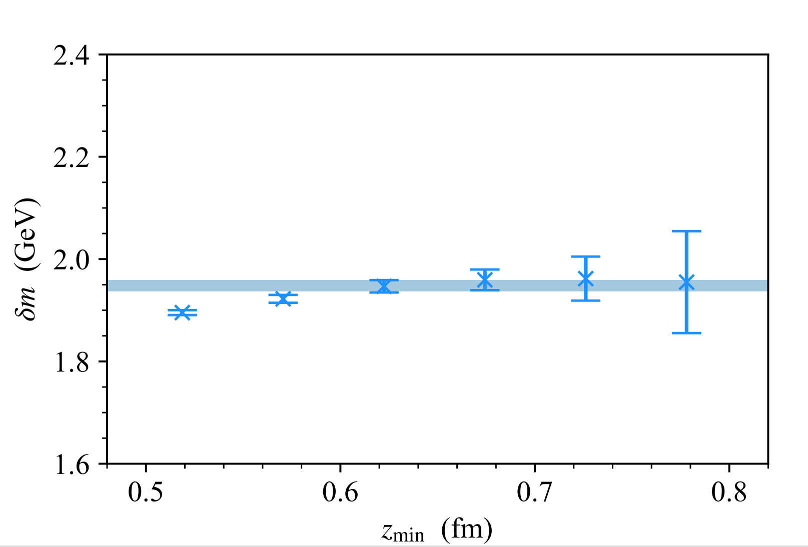
$$\widetilde{M}^B(z, P^z = 0) \sim Ae^{-\delta m z} \text{ at large z}$$

- Extraction δm from fitting 0MME in range $z \in [z_{\min}, z_{\min} + 4a]$;
- The results of δm converge after $z_{\min} \simeq 0.62\text{fm}$.





Renormalization in the hybrid-ratio scheme

- m_0 collects the renormalon ambiguity and other effects after removing the linear divergence, can be extracted from matching the renormalized 0MME to its perturbative results.

$$\tilde{M}^B(z,0;a)e^{(\delta m(a)+m_0)z} = C_0\left(z,\mu_0\right)e^{-\mathcal{I}(\mu_0)}e^{\mathcal{I}^{\rm lat}\left(a^{-1}\right)}.$$

Renormalized lattice results   Perturbative $\overline{\rm MS}$ results

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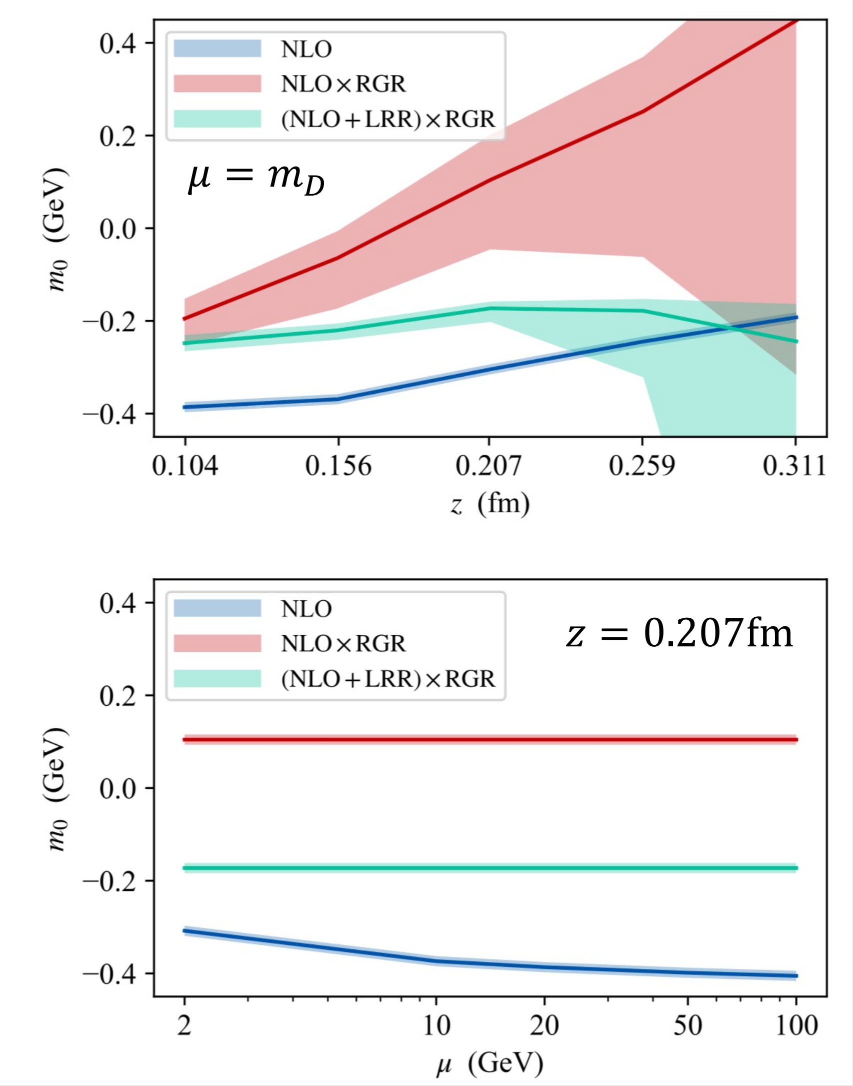
Renormalized lattice results   Perturbative $\overline{\mathrm{MS}}$ results

- At a single lattice spacing:

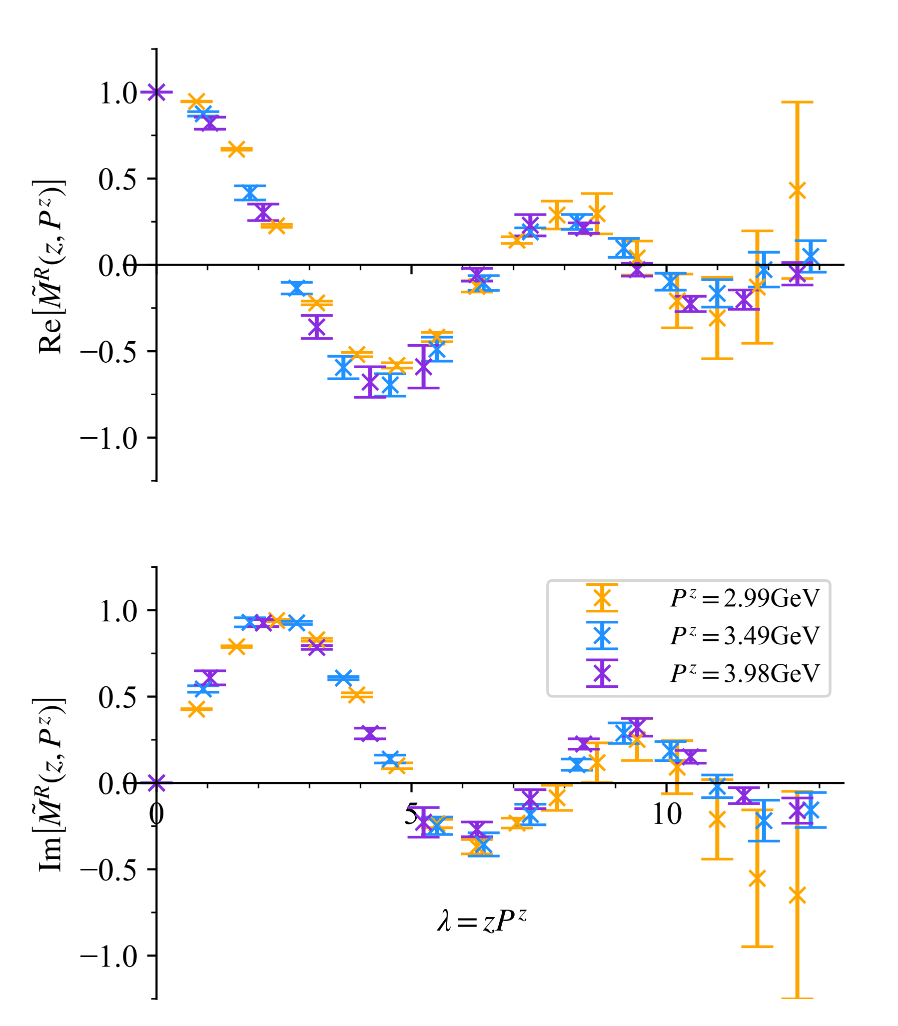
$$(m_0+\delta m)z - I_0 = \ln\left[\frac{C_0^{\mathrm{NLO}(+\mathrm{LRR})(\times\mathrm{RGR})}(z,\mu)}{\tilde{M}^B(z,0)}\right]$$

- We consider the RGR and LRR improvement for C_0.



Extrapolating the long-range correlations

- Renormalized quasi DA matrix elements:
  - Truncated at finite $\lambda = zP^z$
  - Signal-to-noise ratio exponentially increasing



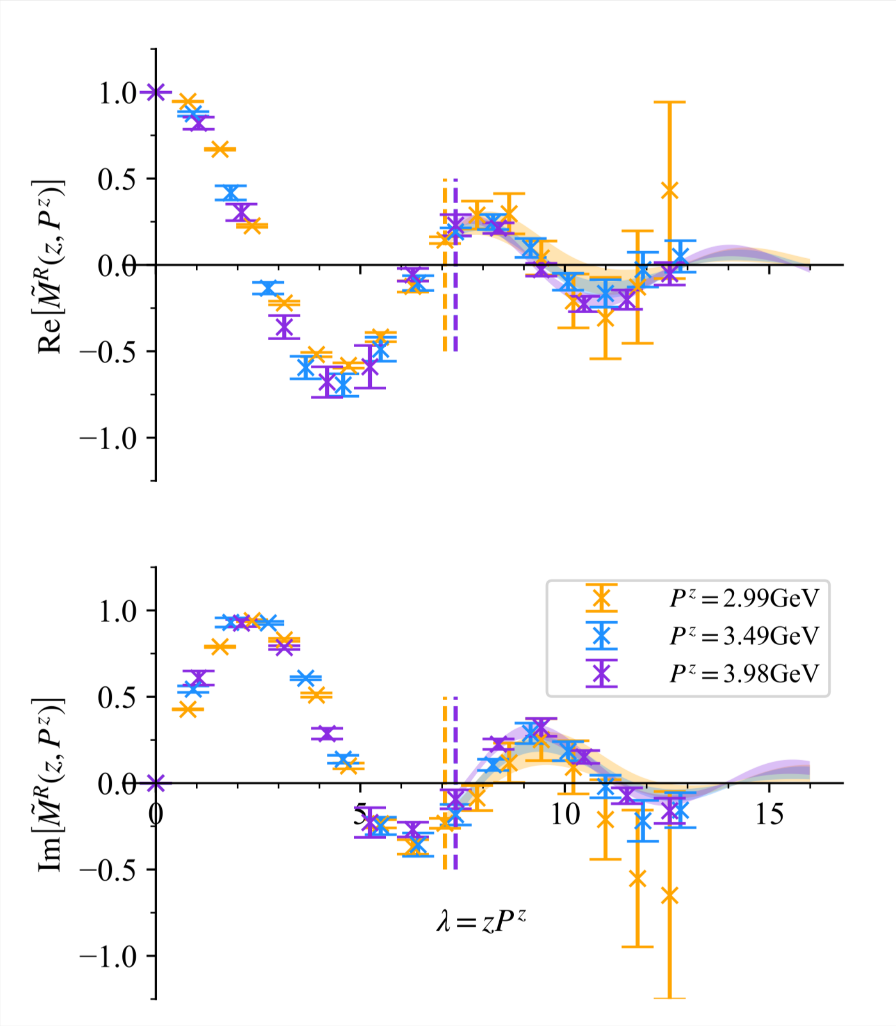
Extrapolating the long-range correlations

- Renormalized quasi DA matrix elements:
  - Truncated at finite $\lambda = zP^z$
  - Signal-to-noise ratio exponentially increasing

- To construct the fully distributions:

$$\tilde{M}^R(\lambda) = \left[\frac{c_1}{(-i\lambda_1)^{d_1}} + e^{i\lambda}\frac{c_2}{(i\lambda_2)^{d_2}}\right] e^{-\lambda/\lambda_0}$$

  - algebraic behavior motivated by the Regge behavior of the light-cone distributions at endpoint regions;
  - governed by the decaying $\propto e^{-\delta m z}$ at long-tail region.



Matching I: from quasi DAs to LCDAs in QCD

- D meson quasi DA $\tilde{\phi}(x, P^z)$, include the scales $\Lambda_{\rm QCD} \ll m_D \ll P^z$

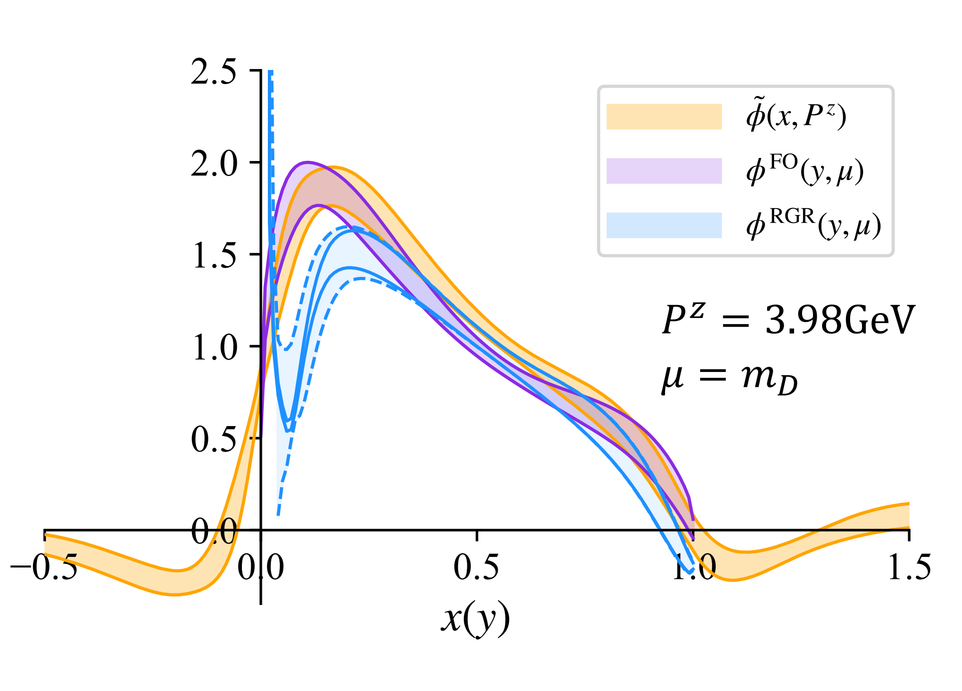
$$\tilde{\phi}(x, P^z) = \int \frac{dz}{2\pi} e^{-ixP^z z} \tilde{M}(z, P^z)$$

- Matching formula in LaMET:

$$\tilde{\phi}(x, P^z) = \int_0^1 C\left(x, y, \frac{\mu}{P^z}\right) \phi(y, \mu) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(xP^z, \bar{x}P^z)^2}\right)$$

Liu, Wang, Xu, **QAZ**, Zhao, 2019;
Han, Hua, Ji, Lu, Wang, Xu, **QAZ**, Zhao, 2024

- FO: matching from fixed-order perturbation theory;
- RGR: resuming the large logs in C by using the ERBL evolution equation.



Systematic error from RGR: scale variation of $\mu_0 = 2yP^z$ with factor 0.8-1.2.

Matching I: from quasi DAs to LCDAs in QCD

- The power correction within the LaMET matching:

$$\tilde{\phi}(x, P^z) = \int_0^1 C\left(x, y, \frac{\mu}{P^z}\right) \phi(y, \mu) + \mathcal{O}\left(\frac{m_H^2}{P^{z2}}, \frac{\Lambda_{\rm QCD}^2}{(xP^z, \bar{x}P^z)^2}\right)$$

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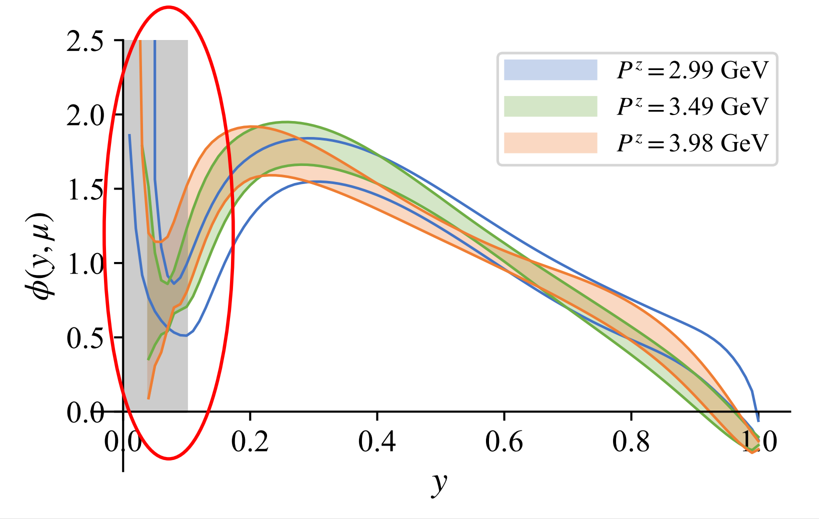

- Power correction $\Lambda_{\rm QCD}^2/(xP^z)^2$:

  Significant at end-point region

  Can be improved by considering the LRR, …

  [Su, Holligan, Ji, Yao, Zhang, Zhang, 2023]

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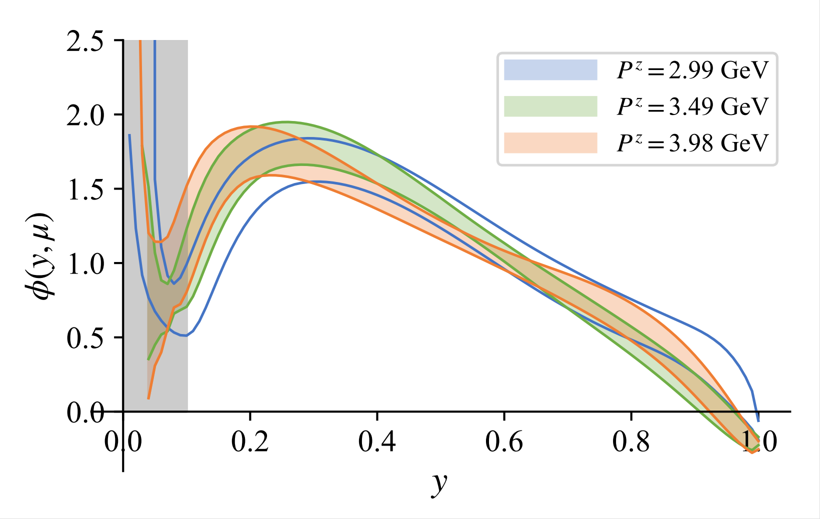
➢ The power correction within the LaMET matching:

$$\tilde{\phi}(x, P^z) = \int_0^1 C\left(x, y, \frac{\mu}{P^z}\right) \phi(y, \mu) + \mathcal{O}\left(\frac{m_H^2}{P^{z2}}, \frac{\Lambda_{\rm QCD}^2}{(xP^z, \bar{x}P^z)^2}\right)$$



- Power correction $\Lambda_{\rm QCD}^2/(xP^z)^2$:

  Significant at end-point region

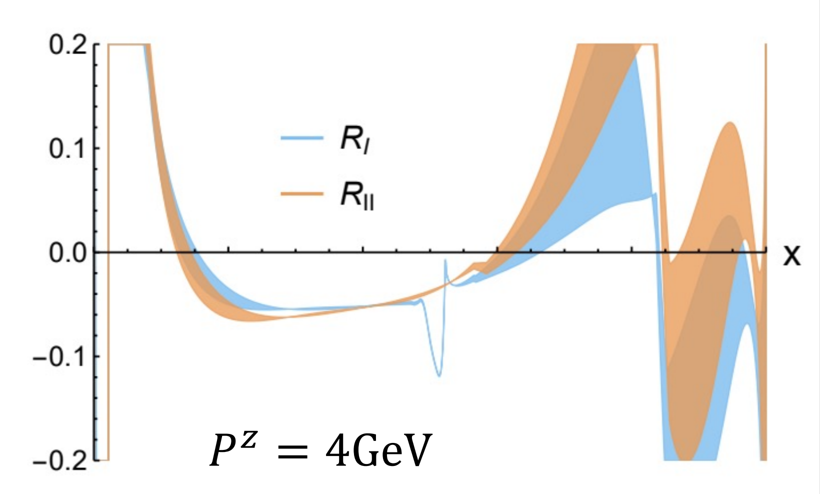
  Can be improved by considering the LRR, …

  [Su, Holligan, Ji, Yao, Zhang, Zhang, 2023]

- Mass correction $m_H^2/(P^z)^2$:

  Smaller than 20% in most region, and smaller than 10% in the region $y \in [0.1, 0.5]$

  [Han, Wang, Zhang, Zhang, 2024]



Matching I: from quasi DAs to LCDAs in QCD

- The regions of QCD LCDA $\phi(y,\mu;m_H)$:
  - The shape of curves dominated by m_H and μ.

    At very large scale $\mu \gg m_H$, asymptotic form.

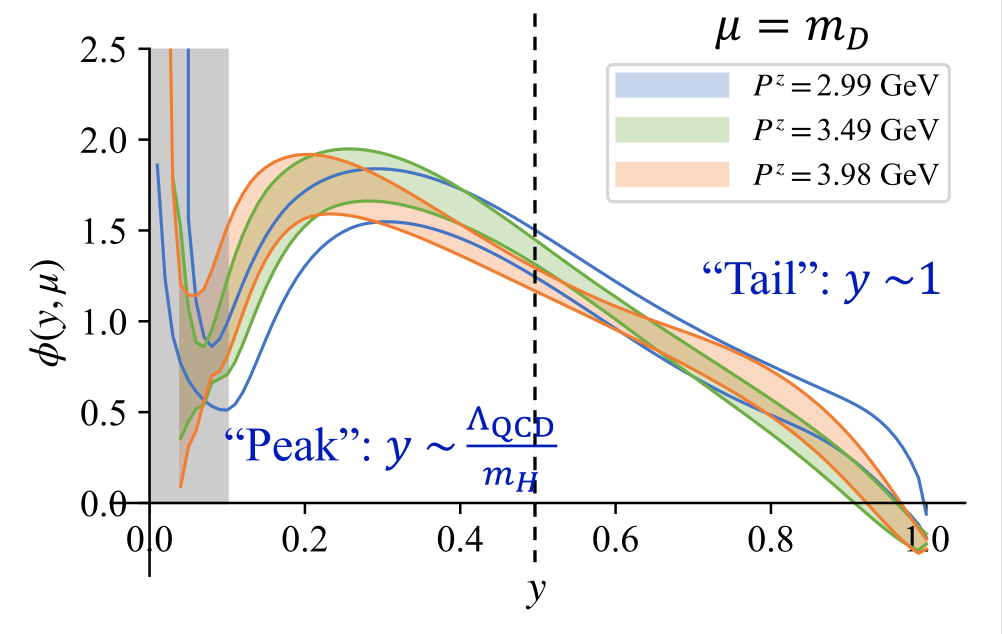
  - For the scale $\mu \lesssim m_Q$:

    Light quark carries small momentum fraction $y \sim \Lambda_{\rm QCD}/m_H$

    $\Rightarrow$ peak region, related to the HQET LCDA;

    [Ishaq, Jia, Xiong, Yang, 2020; Beneke, Finauri, Vos, Wei, 2023]

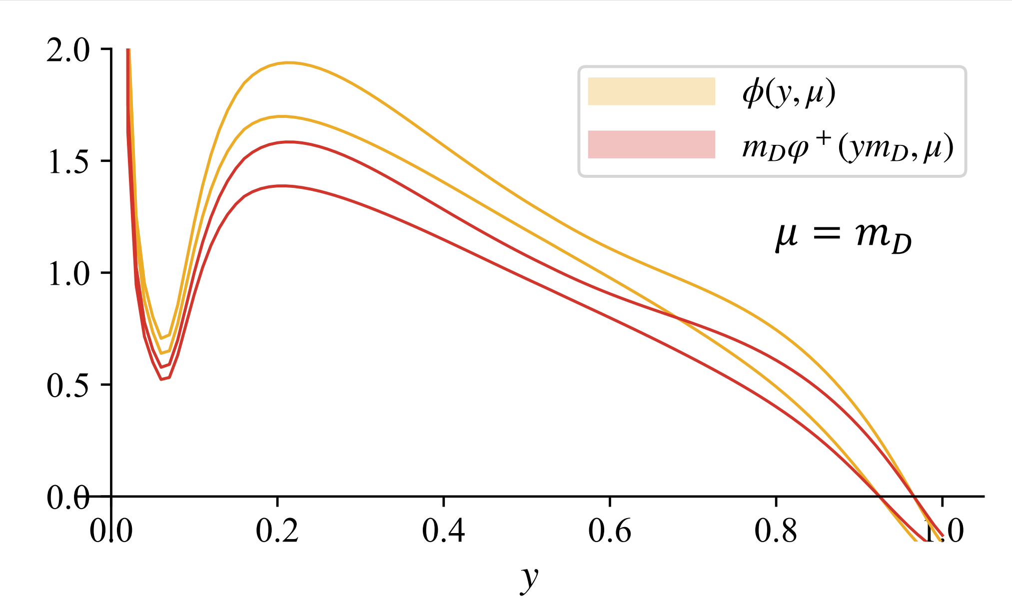
    $y \sim O(1) \Rightarrow$ tail region, contain only hard-collinear physics, suppressed in LCDA.



Matching II: connecting LCDAs in QCD and HQET

- In the peak region, HQET LCDA φ^+ connected with QCD LCDA ϕ through a multiplicative factorization: [Beneke, Finauri, Vos, Wei, 2023]

$$\varphi^+_{\rm peak}(\omega,\mu) = \frac{f_H}{\tilde{f}_H}\frac{1}{\mathcal{J}_{\rm peak}}\phi(y,\mu;m_H) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_H}\right)$$



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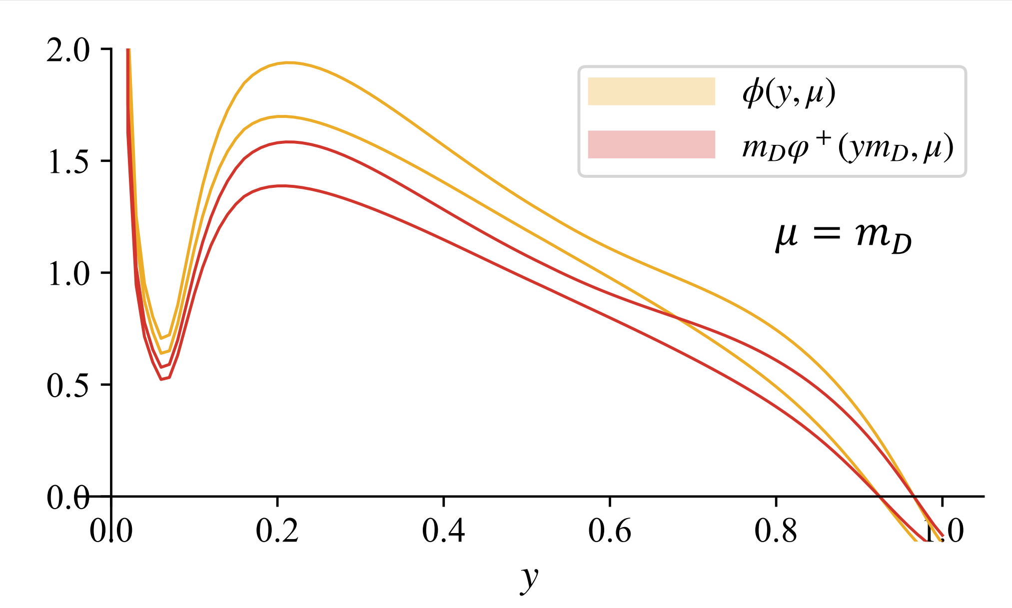
- HQET LCDA is independent on heavy quark mass, $m_Q = m_b$ or m_c are same at leading power.
- For simulating the D meson, power correction $\Lambda_{\rm QCD}/m_H$ still large:

A possible solution proposed in [Deng, Wang, Wei, Zeng, 2024]

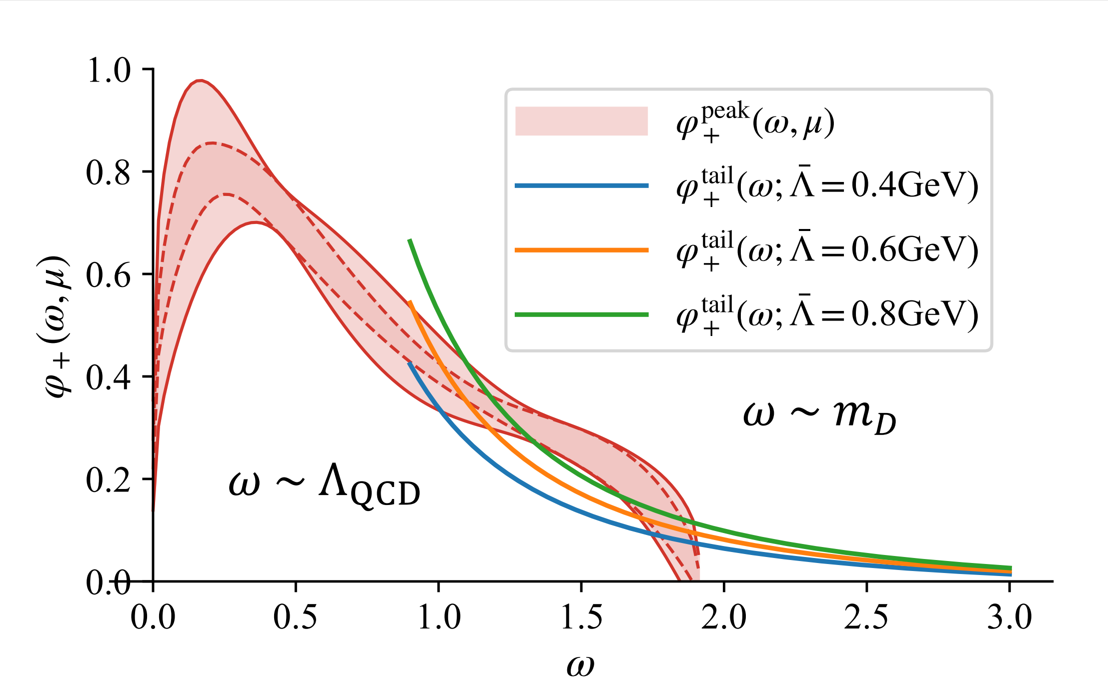


Matching II: connecting LCDAs in QCD and HQET

➢ The tail region of HQET LCDA is perturbative: [Lee, Neubert, 2005]

$$\varphi^{+}_{\rm tail}(\omega,\mu) = \frac{\alpha_s C_F}{\pi\omega}\left[\left(\frac{1}{2} - \ln\frac{\omega}{\mu}\right) + \frac{4\bar{\Lambda}}{3\omega}\left(2 - \ln\frac{\omega}{\mu}\right)\right]$$

where $\bar{\Lambda} \equiv m_H - m_Q^{\rm pole}$ reflect the power correction, and usually be chosen as hundreds of MeV.



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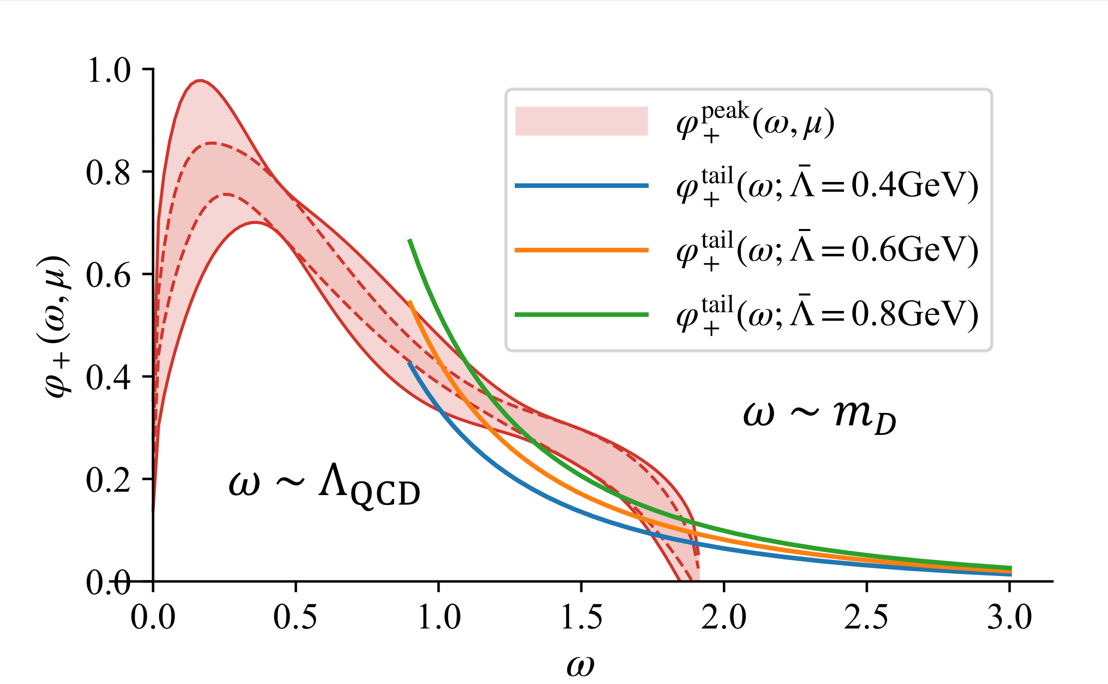


- Merging the peak and tail region:

$$\varphi^{+}(\omega,\mu)=\varphi^{+}_{\text{peak}}(\omega,\mu)\theta(\omega_b-\omega)+\varphi^{+}_{\text{tail}}(\omega,\mu)\theta(\omega-\omega_b)$$

We joint the peak and tail region, and use the Savitzky-Golay filter to smooth the data within a vicinity of $\delta=0.05\text{GeV}$ around the intersection position ω_b.

Final result of leading twist HQET LCDA

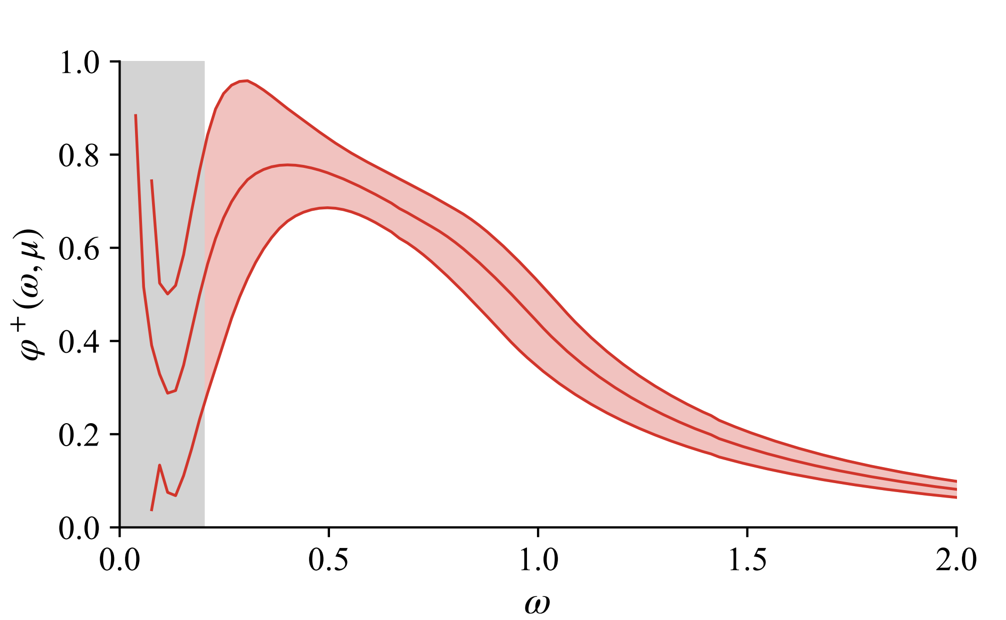
- Finally, we obtain the final result of HQET LCDA.

  - Just a verification of the two-step factorization method, the numerical result is still preliminary.
  - Considered the systematic errors in lattice analysis:

    From extrapolation, scale uncertainty in matching, large momentum limit, ……
  - Some key systematic errors are still absent:

    Only one lattice spacing,

    Power corrections within two matchings are still significant, ……



Although the current result is preliminary, it still warrants some phenomenological discussions...

Phenomenological Discussions I: comparison with models

- Models for heavy meson LCDAs

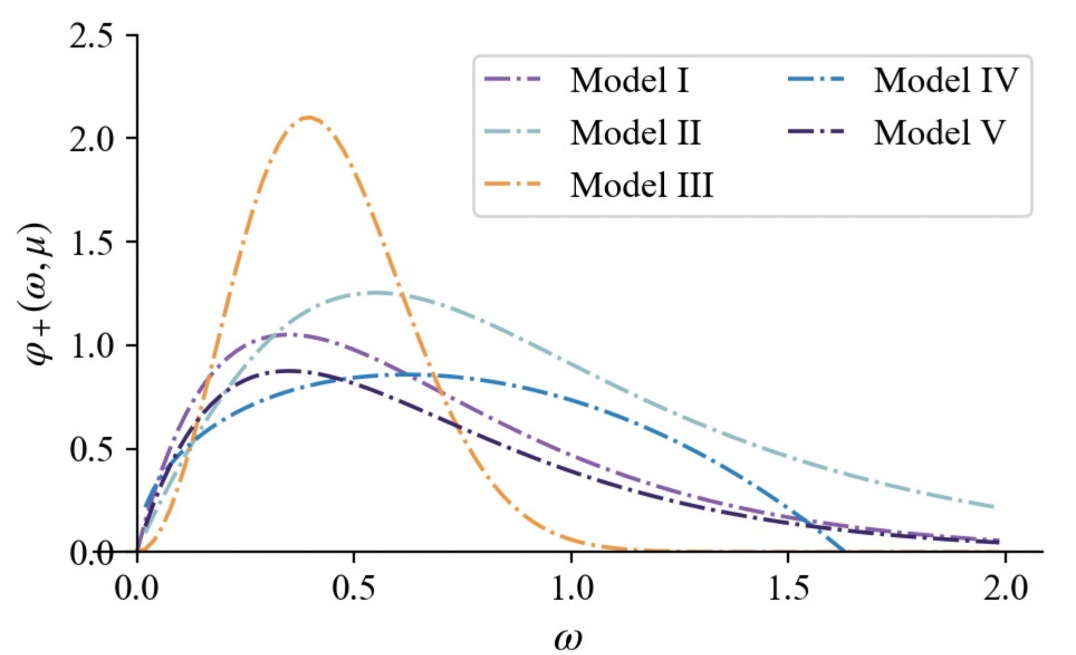
$$\varphi_{\mathrm{I}}^{+}(\omega,\mu_0)=\frac{\omega}{\omega_0^2}e^{-\omega/\omega_0},$$

$$\varphi_{\mathrm{II}}^{+}(\omega,\mu_0)=\frac{4}{\pi\omega_0}\frac{k}{k^2+1}\left[\frac{1}{k^2+1}-\frac{2\left(\sigma_B^{(1)}-1\right)}{\pi^2}\ln k\right],$$

$$\varphi_{\mathrm{III}}^{+}(\omega,\mu_0)=\frac{2\omega^2}{\omega_0\omega_1^2}e^{-(\omega/\omega_1)^2},$$

$$\varphi_{\mathrm{IV}}^{+}(\omega,\mu_0)=\frac{\omega}{\omega_0\omega_2}\frac{\omega_2-\omega}{\sqrt{\omega\left(2\omega_2-\omega\right)}}\theta\left(\omega_2-\omega\right),$$

$$\varphi_{\mathrm{V}}^{+}(\omega,\mu_0)=\frac{\Gamma(\beta)}{\Gamma(\alpha)}\frac{\omega}{\omega_0^2}e^{-\omega/\omega_0}U(\beta-\alpha,3-\alpha,\omega/\omega_0).$$



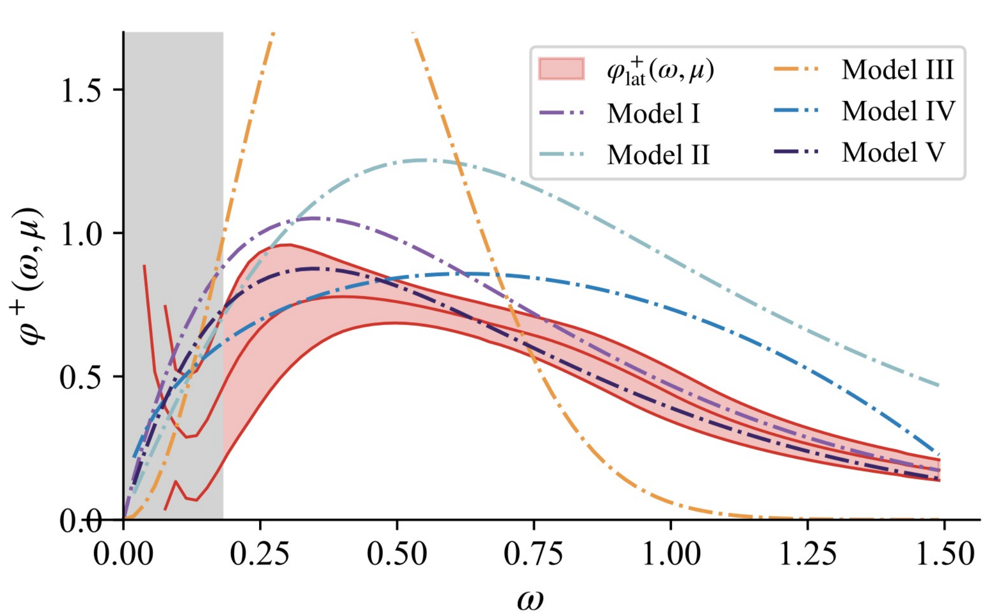
This leads to the largest systematic error:

[Gao, Lu, Shen, Wang, Wei, 2020]

$$\mathcal{V}_{B\to K^*}(0)=0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}{}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

Pheno discussions I: Comparison with models

- Our results are consistent with the model estimates. Especially agree with model V, which constrained by the RG evolution of HQET LCDA.
- We fitting our data based on the models, to show the constrain for the parameters. (Usually $\omega_0 = \lambda_B$).



Models	I	II	III	IV	V
This work	$\omega_0 = 478 \pm 48$MeV	$\omega_0 = 680 \pm 119$MeV	———	$\omega_0 = 396 \pm 29$MeV	$\omega_0 = 482 \pm 44$MeV
		$\sigma_B^{(1)} = 2.53 \pm 1.51$GeV			
References	$\omega_0 = 354^{+38}_{-30}$MeV	$\omega_0 = 368^{+42}_{-32}$MeV	$\omega_0 = 389^{+35}_{-28}$MeV	$\omega_0 = 303^{+35}_{-26}$MeV	$\omega_0 = 350$MeV
		$\sigma_B^{(1)} = 1.4 \pm 0.4$GeV			

Pheno discussions II: Inverse and inverse-logarithmic moments

- Significant uncertainties from λ_B and σ_1: [Gao, Lu, Shen, Wang, Wei, 2020]

$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}{}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

- Definition of Inverse and inverse-logarithmic moments:

$$\lambda_B^{-1}(\mu) = \int_0^\infty \frac{d\omega}{\omega}\varphi^+(\omega,\mu),$$

$$\sigma_B^{(n)}(\mu) = \lambda_B(\mu)\int_0^\infty \frac{d\omega}{\omega}\ln\left(\frac{\mu}{\omega}\right)^{(n)}\varphi^+(\omega,\mu).$$

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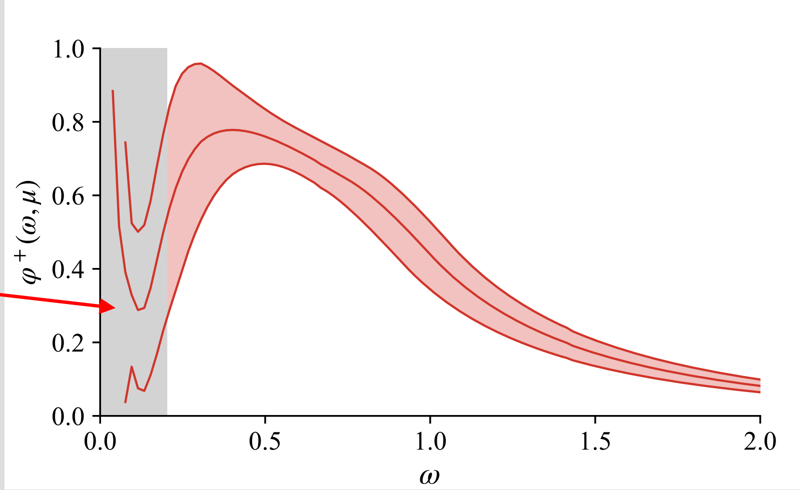
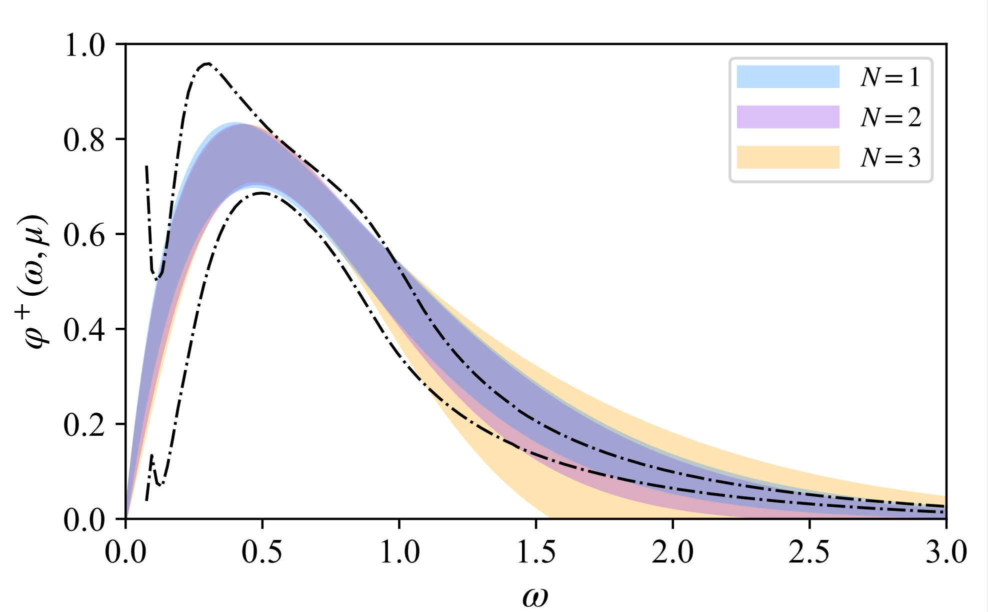
The power corrections at small ω makes the integral non-computable.



Pheno discussions II: Inverse and inverse-logarithmic moments

- A model-independent parametrization form:

$$\varphi^+(\omega,\mu) = \sum_{n=1}^{N} c_n \frac{\omega^n}{\omega_0^{n+1}} e^{-\omega/\omega_0}$$
$$= \frac{c_1\omega}{\omega_0^2}\left[1 + c_2'\frac{\omega}{\omega_0} + c_3'\left(\frac{\omega}{\omega_0}\right)^2 + \cdots\right] e^{-\omega/\omega_0},$$



Fit results of the N-th order:

$$\begin{aligned} N=1:&\ \ \omega_0 = 0.433(55),\ c_1 = 0.899(87); \\ N=2:&\ \ \omega_0 = 0.381(95),\ c_1 = 0.68(37), \\ &\ \ c_2' = 0.16(32); \\ N=3:&\ \ \omega_0 = 0.35(16),\ c_1 = 0.62(44), \\ &\ \ c_2' = 0.10(37),\ c_3' = 0.05(19). \end{aligned}$$

Pheno discussions II: Inverse and inverse-logarithmic moments

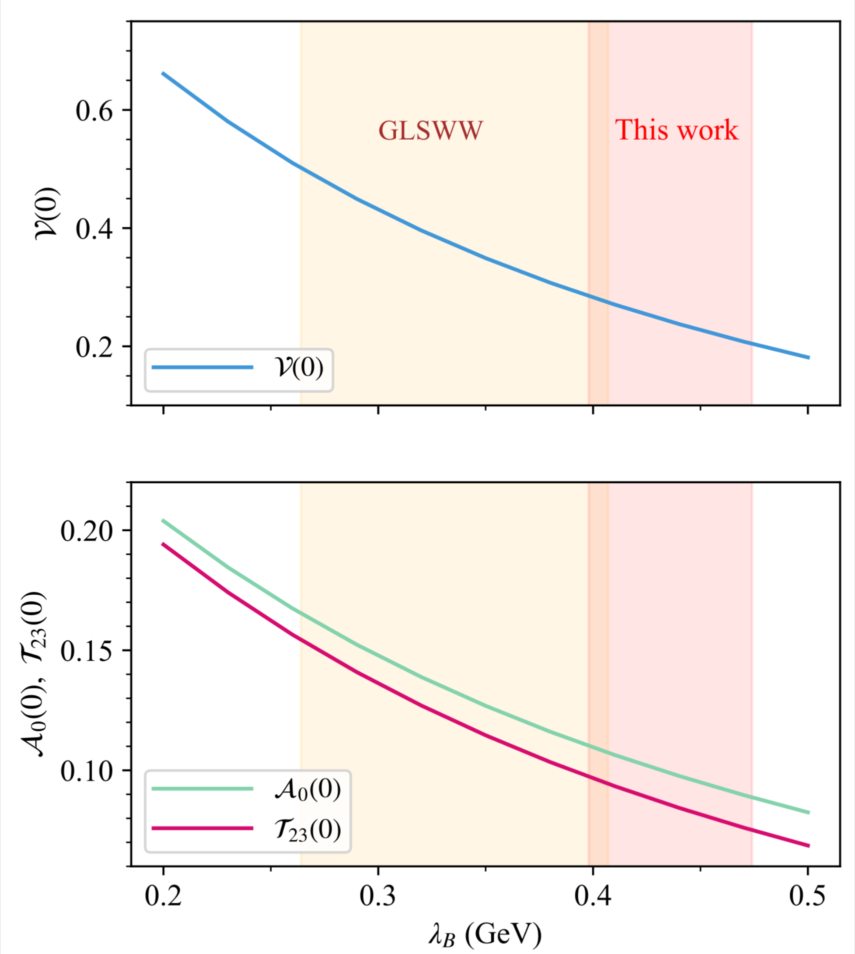
- Numerical results of λ_B and $\sigma_1^{(1)}$ at $\mu = 1\text{GeV}$:

		λ_B (GeV)	$\sigma_B^{(1)}$
Our results	*N=1*	0.436(38)	1.55(10)
	N=2	0.455(59)	1.52(10)
	N=3	0.470(84)	1.51(9)
Experiment	*Belle 2018*	> 0.24	
Other theoretical approach	*Khodjamirian, Mandal, Mannel, 2020*	0.383(153)	
	Lee, Neubert, 2005	0.48(11)	1.6(2)
	Braun, Ivanov, Korchemsky, 2004	0.46(11)	1.4(4)
	Grozin, Neubert, 1997	0.35(15)	
	Mandal, Nandi, Ray, 2024	0.338(68)	

Pheno discussions III: Impact on $B \to V$ form factors

➢ An accurate λ_B will significantly improve the prediction for the $B \to K^*$ form factors: [Gao, Lu, Shen, Wang, Wei, 2020]

	GLSWW		our result
λ_B :	0.343^{+64}_{-79}GeV	$\to$	$0.482(42)$GeV
Error of $\mathcal{V}(0)$:	0.23	$\to$	0.08



We greatly thank Yuming Wang's code for this form factor calculation.

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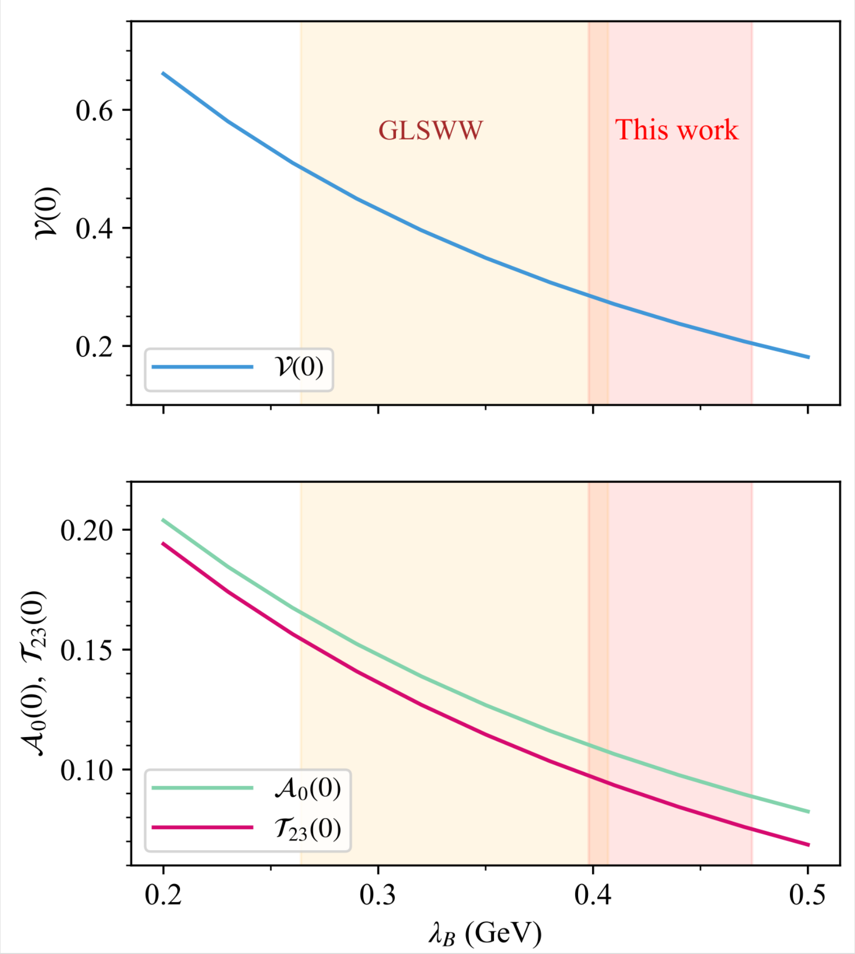
$$\text{Error of } \mathcal{V}(0): \quad 0.23 \quad \to \quad 0.08$$

GLSWW — our result

- We are looking forward to a more precise analysis of the form factors and accordingly physical observables.

$$\mathcal{V}_{B\to K^*}(0) = 0.359\,{}^{+0.141}_{-0.085}\Big|_{\lambda_B}\,{}^{+0.019}_{-0.019}\Big|_{\sigma_1}\,{}^{+0.001}_{-0.062}\Big|_{\mu}\,{}^{+0.010}_{-0.004}\Big|_{M^2}\,{}^{+0.016}_{-0.017}\Big|_{s_0}\,{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$



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Summary and outlook

- ✓ We present a first lattice-implementable method to extract the heavy meson LCDA, and implement it on a CLQCD ensemble.
- ✓ Although the results are preliminary, they can be continually improved.
- ✓ The phenomenological implications demonstrate that our results will significantly advance the theoretical studies towards the frontier of high precision.

More importantly, improving the reliability of our results for the next stage:

- How to properly control the power corrections within two step factorization?
- More systematic lattice QCD calculations: more a, larger P^z, …

Thanks for your attention!