Lattice Parton Collaboration

The Baryon LCDA from Lattice QCD

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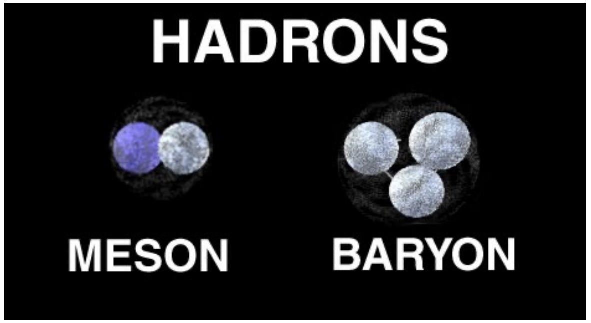
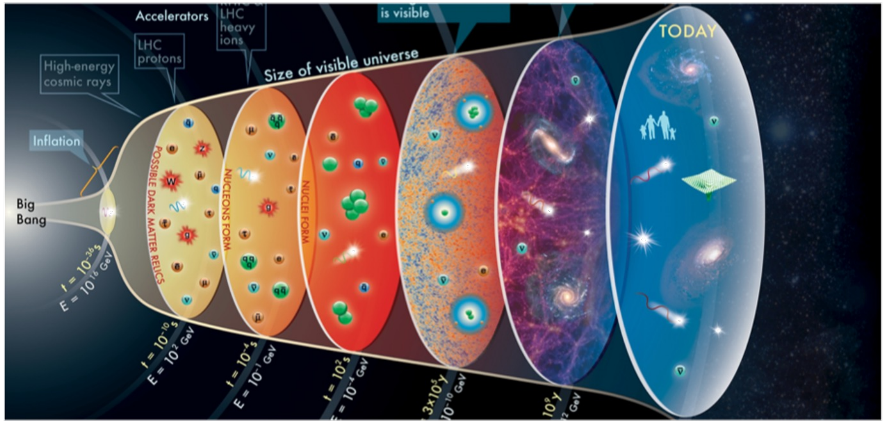
第四届中国格点量子色动力学研讨会

CONTENTS

Motivation

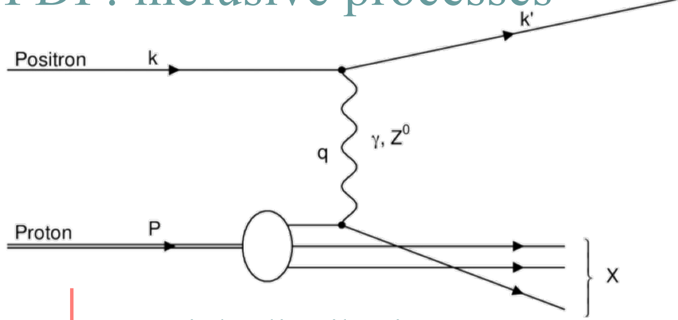
- The visible matter of the Universe is mainly made of baryons.
- Baryons play an important role in the evolution of the Universe, such as baryogenesis and big-bang nucleosythesis.



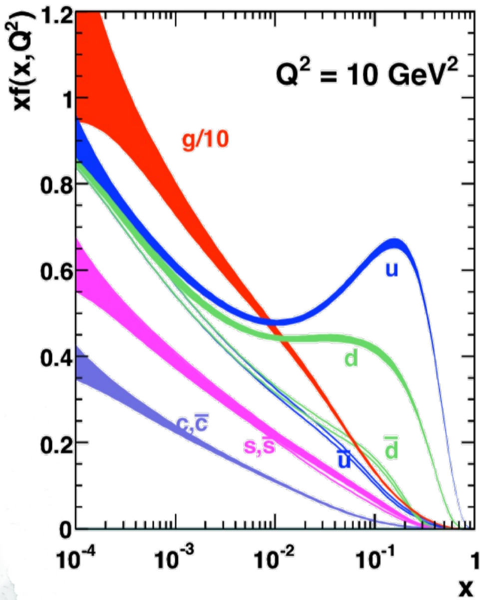


Motivation

PDF: inclusive processes

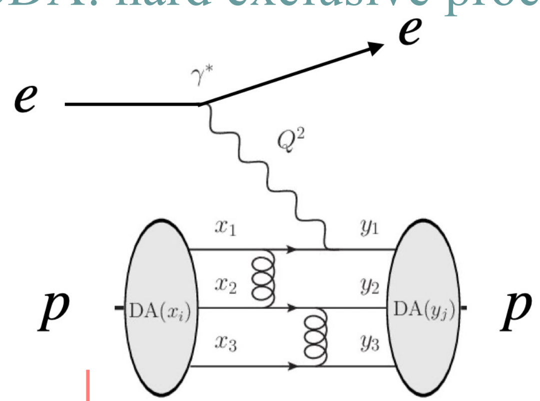


1-particle distributions

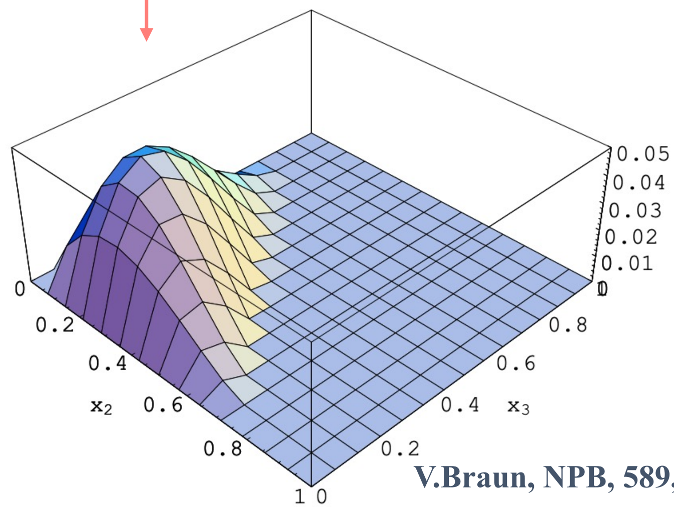


Complementary

LCDA: hard exclusive processes



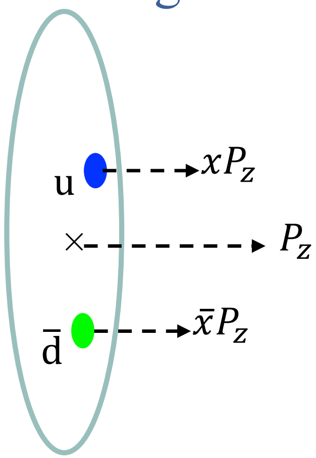
3-particle distributions



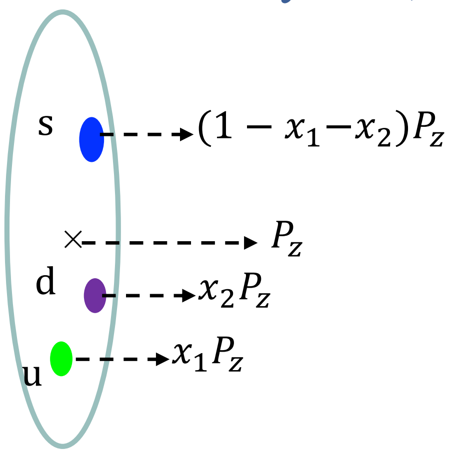
V.Braun, NPB, 589,381(2000)

Motivation

Light meson $\pi/K\ldots$



Baryon Λ, proton…



- **CKM matrix**
- **CP violation**
- New physics ···

- Sakharov conditions for Baryogenesis:
  1) Baryon number violation
  2) C and CP violation
  3) Out of thermal equilibrium

- CPV well established in K, B and D mesons
- But CPV never established in any baryon
- Indications of CPV $\Lambda_b^0 \to p\pi^-\pi^+\pi^-$, (LHCb) NP13, 391 (2017)

Motivation

➢ Light meson LCDAs have been extensively pursued: (1970s - now)

- **Asymptotic LCDAs**
  Chernyak, Zhitnitsky, 1977; Lepage, Brodsky, 1979; Efremov, Radyushkin, 1980
- **Dyson-Schwinger Equation**
  Chang, Cloet, Cobos-Martinez, Roberts, Schmidt, 2013; Gao, Chang, Liu, Roberts, Schmidt, 2014; Roberts, Richards, Chang, 2021
- **Sum rules**
  Chernyak, Zhitnitsky, 1982; Braun, Filyanov, 1989; Ball, Braun, Koike, Tanaka, 1998; Ball, Braun, 1998; Khodjamirian, Mannel, Melcher, 2004; Ball,, Lenz, 2007
- **Inverse Method**
  Li, 2022
- **Models**
  Arriola, Broniowski, 2002, 2006; Zhong, Zhu, Fu, Wu, Huang, 2021;
- **Global Fits**
  Stefanis, 2020; Cheng, Khodjamirian, Rusov, 2020; Hua, Li, Lu, Wang, Xing, 2021
- **Lattice with current-current correlation**
  Bali, Braun, Gläßle, Göckeler, Gruber, 2017, 2018;
- **Lattice with OPE**
  Martinelli, Sachrajda, 1987; Braun, Bruns, et al., 2016; RQCD collaboration, 2019, 2020
- **Lattice with LaMET**
  Zhang, Chen, Ji, Jin, Lin, 2017; LP3 Collaboration, 2019; Zhang, Honkala, Lin, Chen, 2020; Lin, Chen, Fan, Zhang^2, 2021; LPC Collaboration, 2021, 2022
- **Quantum Computing**
  QuNu Collaboration, 2023, 2024

Motivation

➢ Light Baryon LCDAs: (1980s - now)

- **Asymptotic LCDAs**
  Chernyak, Zhitnitsky, 1983

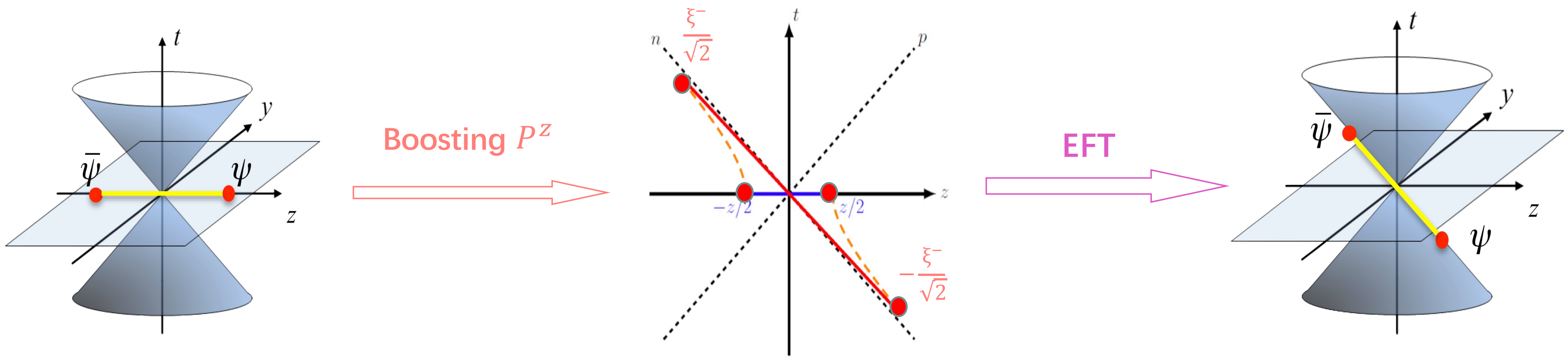
- **Sum rules**
  King, Sachrajda, 1987; Chernyak, Ogloblin, Zhitnitsky 1989;
  Stefanis, Bergmann, 1993

- **Lattice with OPE**
  QCDSF collaboration, 2008, 2009;
  RQCD collaboration, 2016, 2019

- **What's next ? …**

Quasi Distribution

Large Momentum Effective Theory (LaMET)



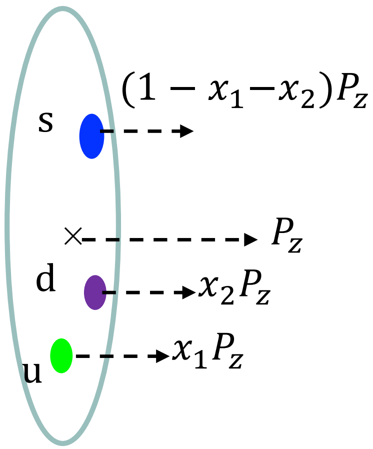
- Effective field theory:
  - Instead of taking $P^z \to \infty$ calcuation, one can perform an expansion for large but finite P^z:

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C(x, y, P^z, \mu)\, q(y, \mu) + \mathcal{O}\left(\frac{\Lambda^2, M^2}{(P^z)^2}\right)$$

Quasi-DA   Matching kernel   LCDA   High power correction

X.Ji PRL110 262002 (2013)

Quasi Distribution



- Definition of light cone baryon LCDA:

$$\int \frac{d\xi_1^-}{2\pi}\frac{d\xi_2^-}{2\pi} e^{ix_1p^+\xi_1^-} e^{ix_2p^+\xi_2^-} \epsilon^{ijk} \left\langle 0 \left| W^{ii'}(\infty,\xi_1^-)\psi_\alpha^{i'}(\xi_1^-)\Gamma_{\alpha\beta}W^{jj'}(\infty,\xi_2^-)\,\psi_\beta^{j'}(\xi_2^-)\,\psi_\gamma^{j}(\infty,0)\right| M(P)\right\rangle$$
$$= if_M(p_1\cdot n)(p_2\cdot n)\phi_M(x_1,x_2).$$

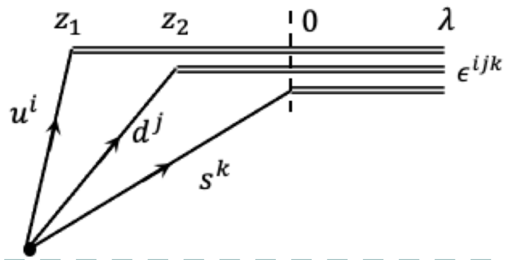
- Corresponding quasi-DA on Euclidean lattice:

$$f_M(p_1,p_2)\tilde{\Phi}^0(x_1,x_2) = \int \frac{p_{z1}dz_1}{2\pi}\frac{p_{z2}dz_2}{2\pi} e^{-i(x_1p_{z1}z_1+x_2p_{z2}z_2)} \left\langle 0\left|\tilde{O}\left(z_1,z_2,\tilde{\Gamma}\right)\right|P^z\right\rangle$$

$$\tilde{O}^\Lambda\left(z_1,z_2,\tilde{\Gamma}\right) = \varepsilon^{ijk}U^i(z_1)\,\tilde{\Gamma}D^j(z_2)\,S^k(0)$$
$$= \varepsilon^{ijk}W^{ii'}(\lambda,z_1)\,u_\alpha^{i'}(z_1)\,\Gamma_{\alpha\beta}W^{jj'}(\lambda,z_2)\,d_\beta^{j'}(z_2)\,s_\gamma^k(0)$$

$$\epsilon^{ijk}U^{ii'}U^{jj'}U^{kk'} = det(U)\epsilon^{i'j'k'}$$

$$det(U) = 1$$



Lattice Setup

- Nonlocal 2pt related to baryon quasi DA:

$$C_2^{\Lambda}(z_1, z_2, t; \tilde{\Gamma}, T) \;=\; \mathrm{tr}\{T * \mathrm{tr}_c[\mathrm{tr}_s(\mathrm{qC13}[W(\lambda, z_1)G^u(z_1)C\gamma_5, \tilde{\Gamma}W(\lambda, z_2)G^d(z_2)]) * G^s(0)]\}$$

$\tilde{\Gamma} = C\gamma_5\gamma^t/\gamma^z, T = (1+\gamma^4)/2$

- Lattice setup:

CLQCD ensemble, 2 + 1 flavor stout smeared clover fermions and Symanzik gauge action

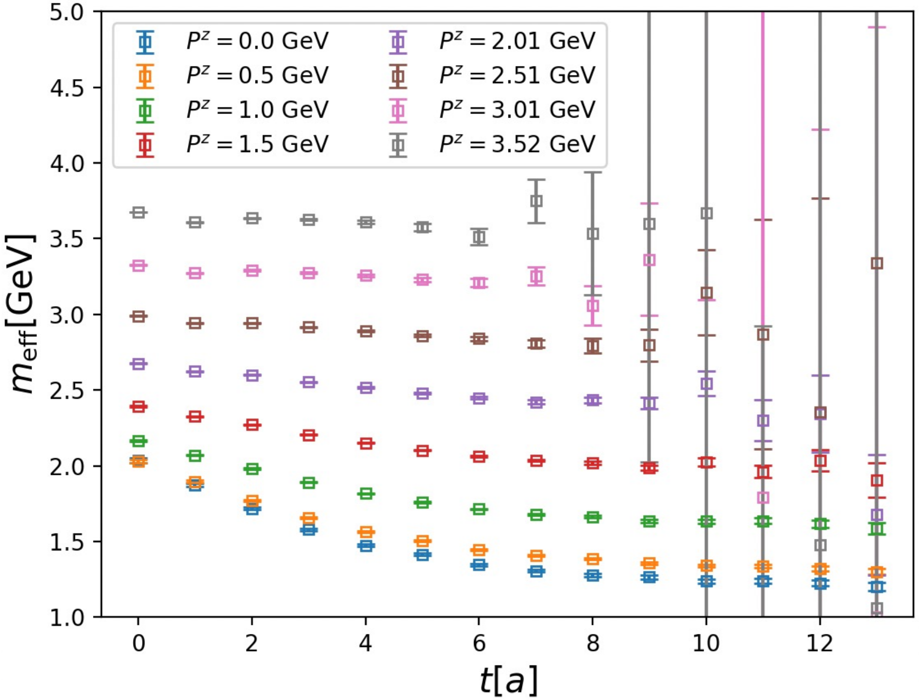
Ensemble	Volume	Lattice spacing	π mass	η_s mass	conf
F32P30	$32^3\times96$	0.077	290MeV	640MeV	777(*32)
H48P32	$48^3\times144$	0.055	300MeV	650MeV	except

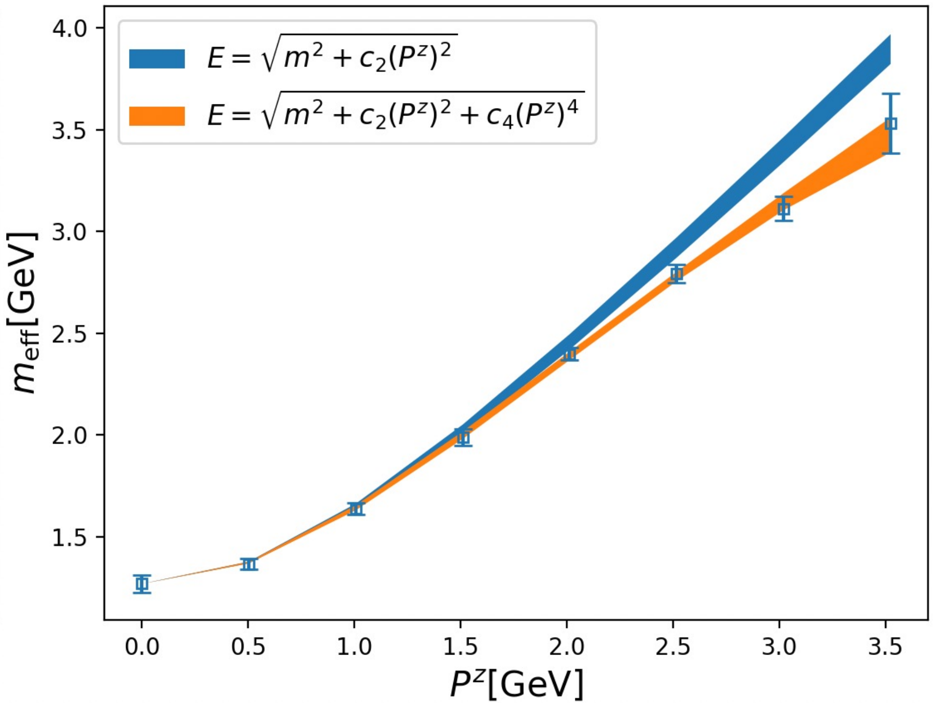
3 momentum: 2.01, 2.51, 3.02 GeV

Lattice Setup

- Dispersion relation:

c2=1.093 (46), c4=-0.0202 (68)





Quasi Distribution

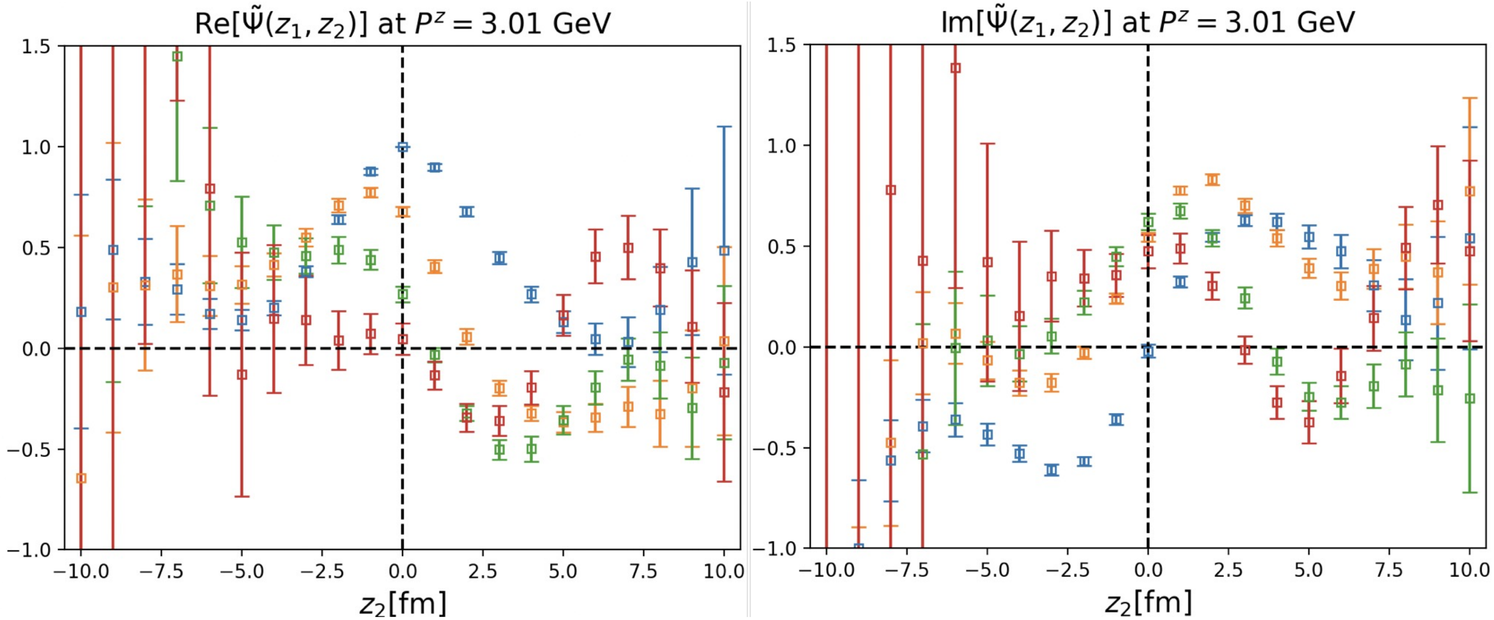
- Baryon (Λ) non-local 2pt correlation function:

$$C_2^{\Lambda}\left(z_1, z_2, t; \tilde{\Gamma}, T\right) = \mathrm{tr}\left\{T * \mathrm{tr}_c\left[\mathrm{tr}_s\left(\mathrm{qC13}\left[G^u\left(z_1\right) C\gamma_5, \tilde{\Gamma} W\left(z_1, z_2\right) G^d\left(z_2\right)\right]\right) * G^s(0)\right]\right\}$$

$$\tilde{\Gamma} = C\gamma_5\gamma^t/\gamma^z$$

$$T = (1+\gamma^4)/2$$

- Renormalized(Preliminary in ratio scheme) quasi-DA (fixed z_1=0.24fm)



Quasi Distribution

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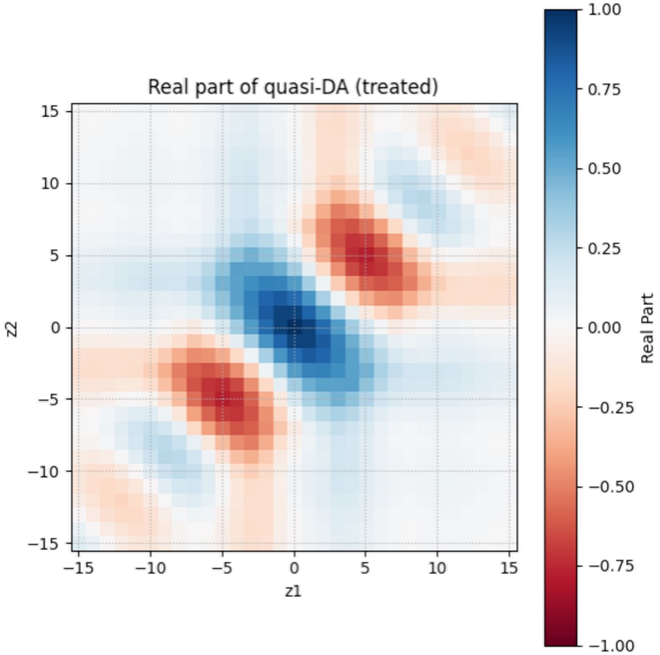
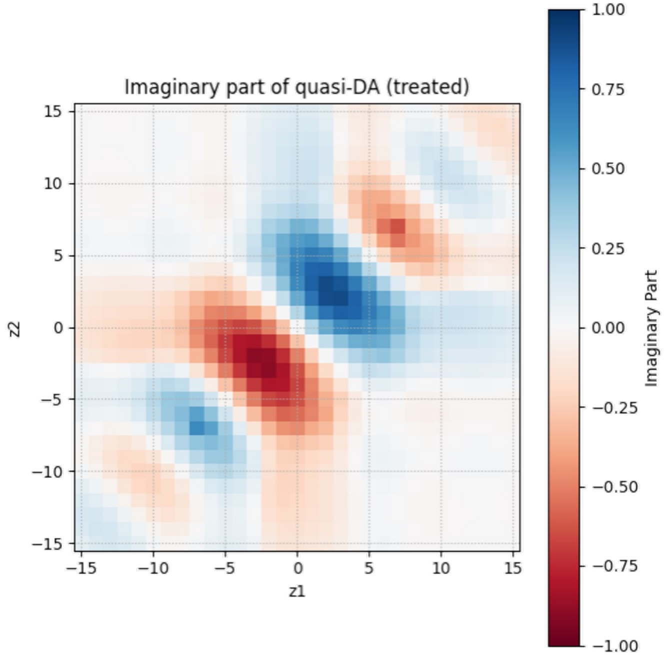
$$\tilde{\Gamma} = C\gamma_5\gamma^t/\gamma^z$$

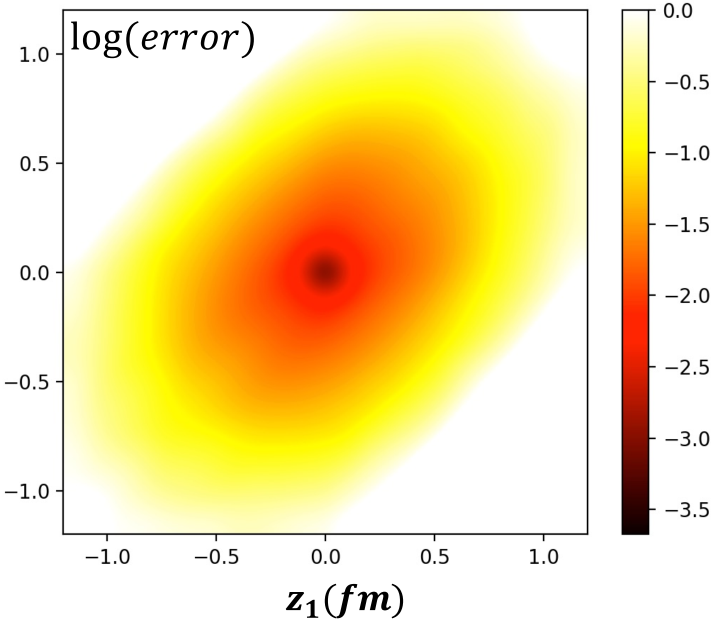
$$T = (1+\gamma^4)/2$$

- Renormalized quasi-DA of Λ :

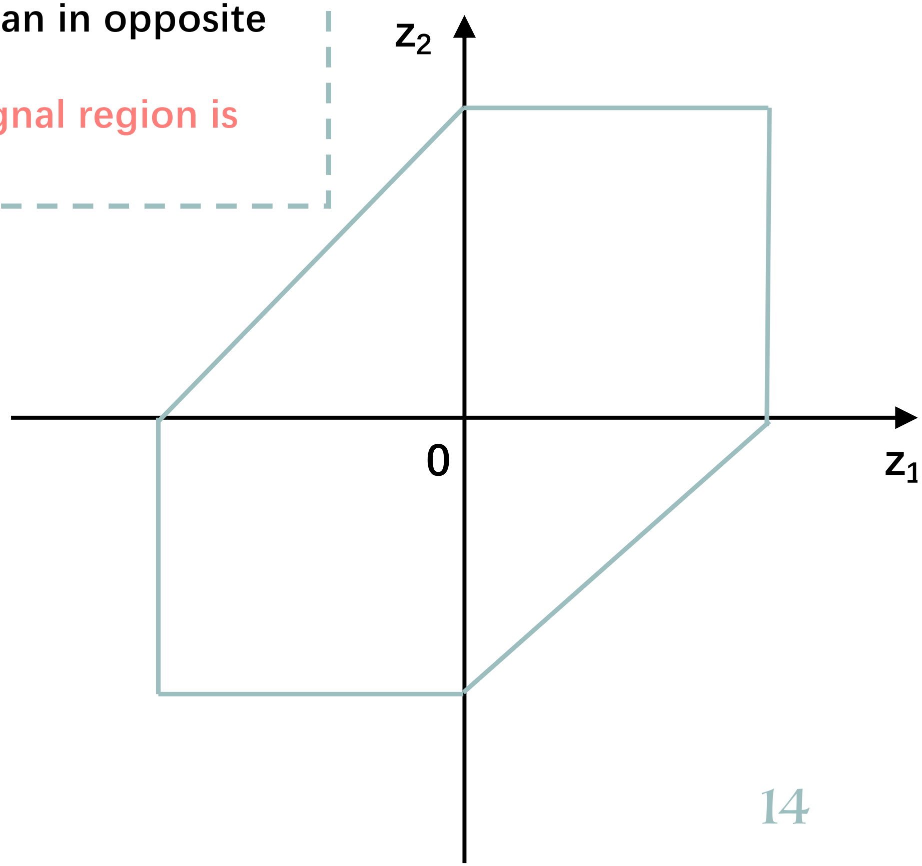


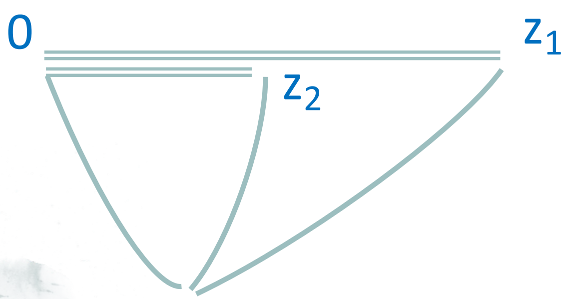


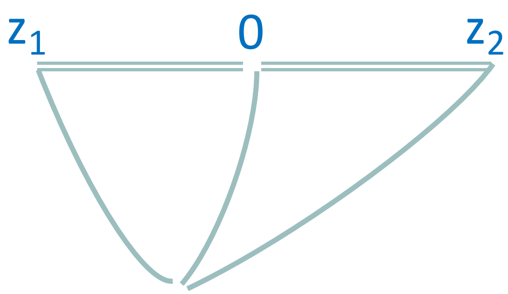
Quasi Distribution



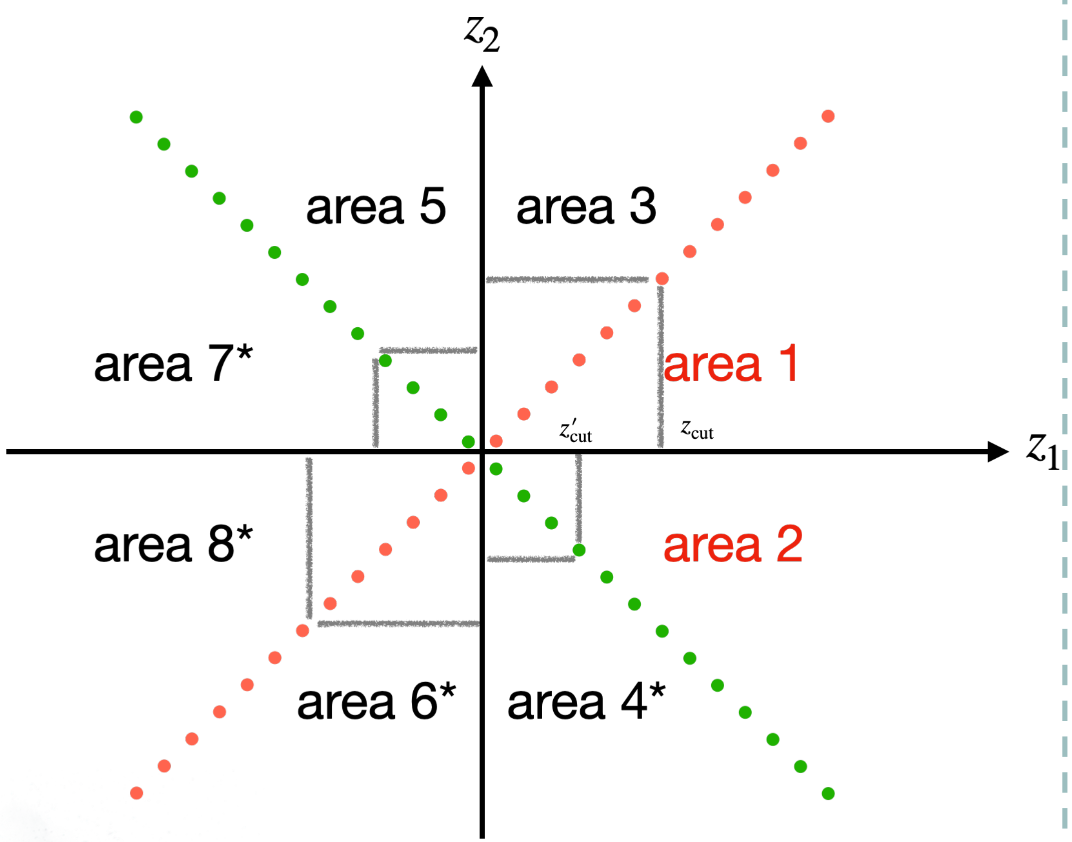
- The Wilson line length(non-local separation) is smaller with z1z2 in same direction than in opposite
- Thus the good signal region is rhombic







Quasi Distribution



➢ Two symmetries for Quasi-DA:

- iso-spin symmetry for "u, d" quarks:

$$\tilde{\phi}(z_1, z_2) = \tilde{\phi}(z_2, z_1)$$

- The constrain by real $\tilde{\phi}(x_1, x_2)$:

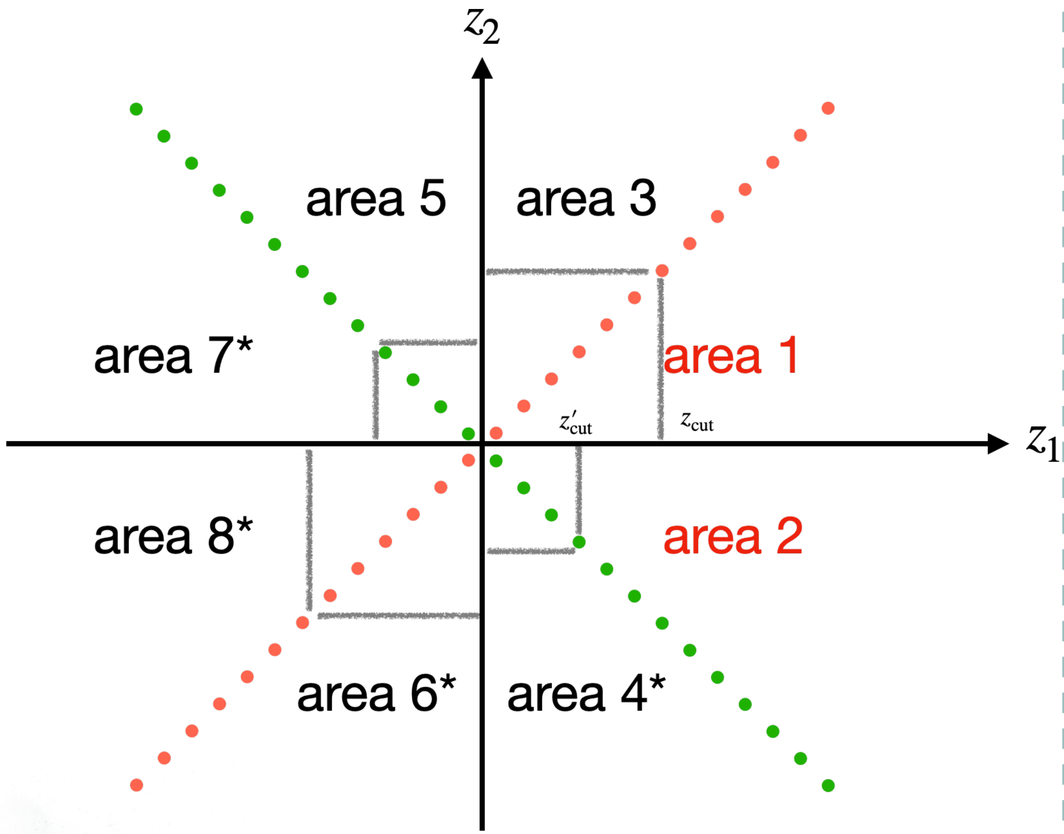
$$\tilde{\phi}(z_1, z_2) = \tilde{\phi}^*(-z_1, -z_2)$$

- ☐ Thus for these areas: $1 = 3 = 6^* = 8^*$, $2 = 4^* = 5 = 7^*$

  only area 1,2 are independent

- ☐ $\tilde{\phi}(z_1, -z_1) = 0$

Quasi Distribution



➤ Renormalization scheme:

- Perturbative 0 momentum quasi-DA:

$$\hat{M}_p(z_1, z_2, 0, 0, \mu) = 1 + \frac{\alpha_s C_F}{2\pi}$$

$$\left[\frac{1}{8}\ln\left(\frac{z_1^2\mu^2 e^{2\gamma_E}}{4}\right) + \frac{1}{8}\ln\left(\frac{z_2^2\mu^2 e^{2\gamma_E}}{4}\right) + \frac{1}{4}\ln\left(\frac{(z_1 - z_2)^2\mu^2 e^{2\gamma_E}}{4}\right) + 4\right]$$

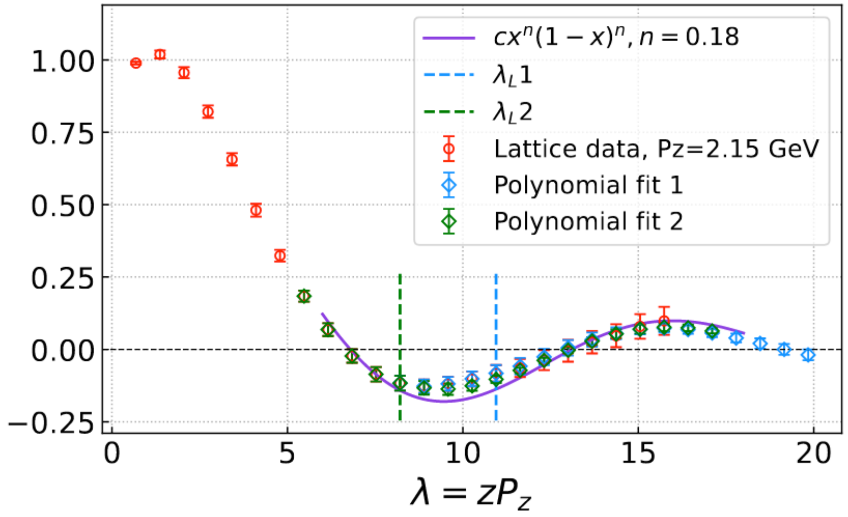
1 pole at $z_1 = z_2$

- Hybrid scheme need a match with the perturbative quasi-DA

  At least $a < 0.06fm$ to apply hybrid scheme

- For simplification: Ratio scheme $\frac{\tilde{\psi}(z_1, z_2, P^z, a)}{\tilde{\psi}(z_1, z_2, P^z = 0, a)}$

Quasi Distribution



(LPC)PRL129 132001(2022)

➢ Extrapolation & Fourier transformation(1D):

- For meson LCDA :
  - Asymptotic in momentum space:

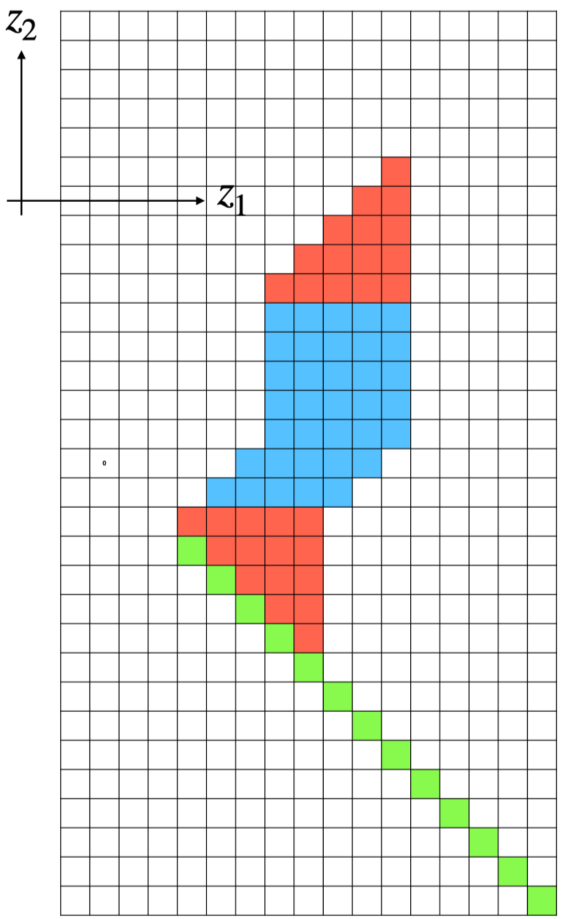
$$F(x) = cx^{d_1}(1-x)^{d_2}$$

↓ Analytic FT then simplify

  - Extrapolation form in coordinate space

$$H_m^{\mathrm{R}}(z, P_z) = \left[\frac{c_1}{(i\lambda)^a} + e^{-i\lambda}\frac{c_2}{(-i\lambda)^b}\right]e^{-\lambda/\lambda_0},$$

Quasi Distribution



- **Extrapolation & Fourier transformation(2D):**
  - Asymptotic in momentum space:

$$F(x_1, x_2, d_1, d_2) = Cx_1^{d_1} x_2^{d_1} (1 - x_1 - x_2)^{d_2}$$

↓ FT

$$\tilde{\phi}(z_1, z_2; d_1, d_2) = \int_0^1 dx_1 \int_0^1 dx_2 \, e^{-ix_1 z_1 P^z} e^{-ix_2 z_2 P^z} C x_1^{d_1} x_2^{d_1} (1 - x_1 - x_2)^{d_2}$$

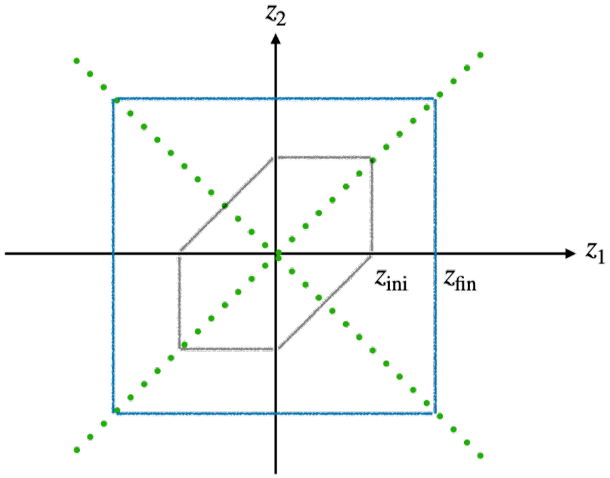
- Numerically FT as the fit from [computing source cost]
- Analytica FT and simplified as the fit form [complicated form for baryon]

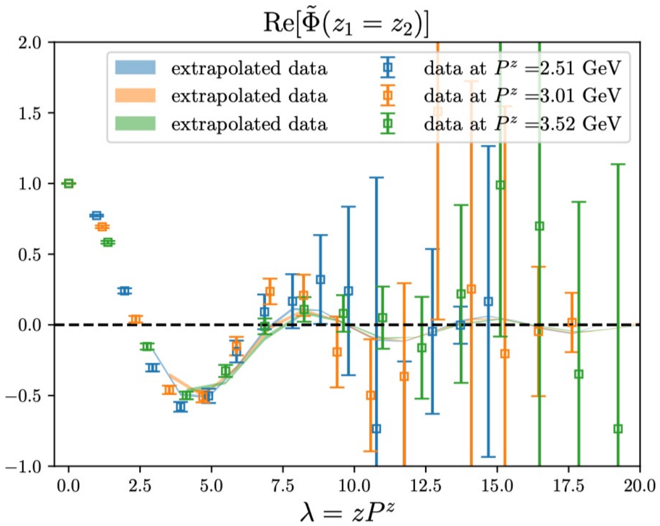
Area2[$z_1 \gg 0, z_2 \gg 0$]:

$$\frac{\Psi(\lambda_1, \lambda_2)}{e^{-\frac{|\lambda_1|}{\lambda_0} - \frac{|\lambda_2|}{\lambda_0} - \frac{|\lambda_1 - \lambda_2|}{\lambda_0}}} = c_1 \left[\frac{1}{(\lambda_1 - \lambda_2)^{d_1} \lambda_1^{d_1}} + \frac{1}{(\lambda_1 - \lambda_2)^{d_1} (-\lambda_2)^{d1}} \right] + c_2 \frac{\cos\left[\frac{1}{2}(d_1\pi + d_2\pi - 2\lambda_1)\right]}{(\lambda_1 - \lambda_2)^{d_1} \lambda_1^{d_2}} + c_2 \frac{\cos\left[\frac{1}{2}(d_1\pi + d_2\pi + 2\lambda_2)\right]}{(\lambda_1 - \lambda_2)^{d_1} (-\lambda_2)^{d_2}}$$

$$-i \left(c_2 \frac{\sin\left[\frac{1}{2}(d_1\pi + d_2\pi - 2\lambda_1)\right]}{(\lambda_1 - \lambda_2)^{d_1} \lambda_1^{d_2}} - c_2 \frac{\sin\left[\frac{1}{2}(d_1\pi + d_2\pi + 2\lambda_2)\right]}{(\lambda_1 - \lambda_2)^{d_1} (-\lambda_2)^{d_2}} \right)$$

Quasi Distribution





➢ Extrapolation & Fourier transformation(2D):

- Asymptotic in momentum space:

$$F(x_1, x_2, d_1, d_2) = C x_1^{d_1} x_2^{d_1} (1 - x_1 - x_2)^{d_2}$$

↓ FT

$$\tilde{\phi}(z_1, z_2; d_1, d_2) = \int_0^1 dx_1 \int_0^1 dx_2\, e^{-ix_1 z_1 P^z} e^{-ix_2 z_2 P^z} C x_1^{d_1} x_2^{d_1} (1 - x_1 - x_2)^{d_2}$$

  - Numerically FT as the fit from
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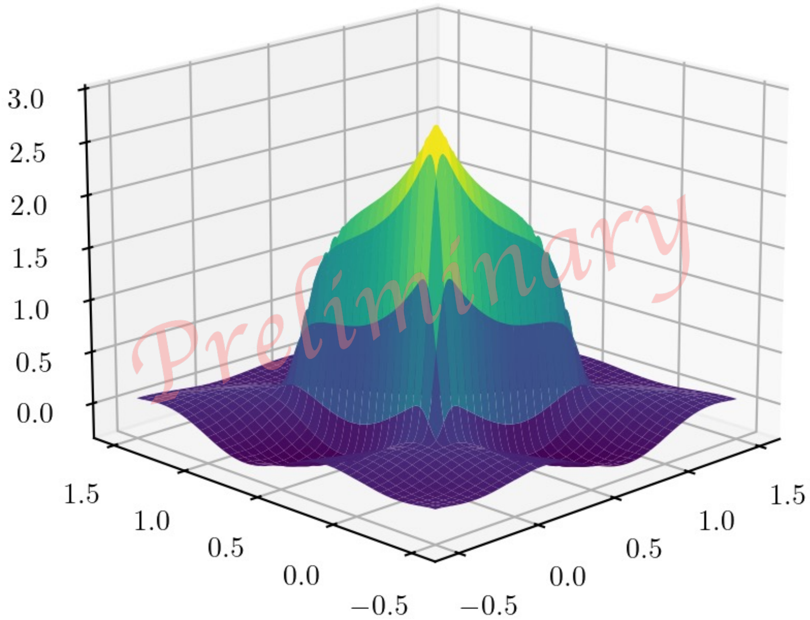
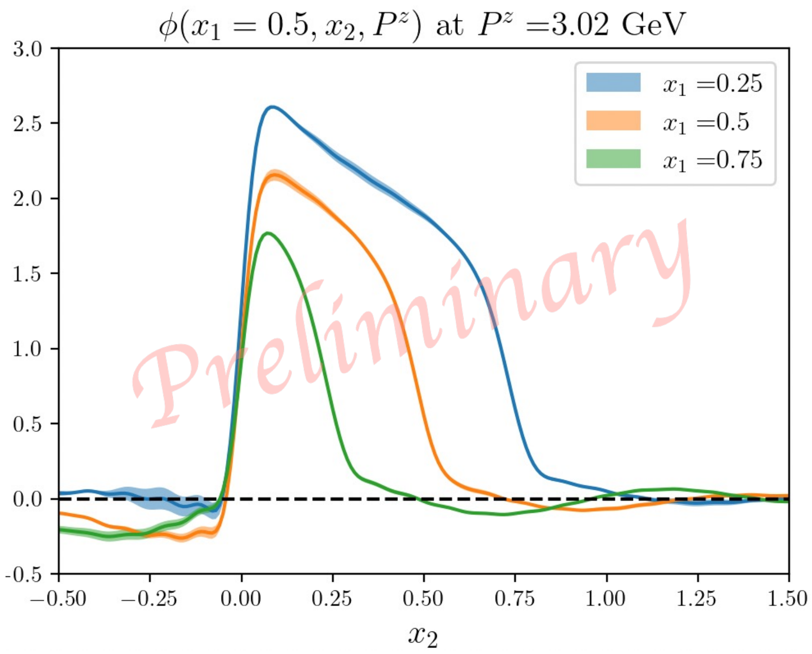
$$\frac{\Psi(\lambda_1,\lambda_2)}{e^{-\frac{|\lambda_1|}{\lambda_0}-\frac{|\lambda_2|}{\lambda_0}-\frac{|\lambda_1-\lambda_2|}{\lambda_0}}} = c_1\left[\frac{1}{(\lambda_1-\lambda_2)^{d_1}\lambda_1^{d_1}} + \frac{1}{(\lambda_1-\lambda_2)^{d_1}(-\lambda_2)^{d1}}\right] + c_2\frac{\cos\left[\frac{1}{2}(d_1\pi+d_2\pi-2\lambda_1)\right]}{(\lambda_1-\lambda_2)^{d_1}\lambda_1^{d_2}} + c_2\frac{\cos\left[\frac{1}{2}(d_1\pi+d_2\pi+2\lambda_2)\right]}{(\lambda_1-\lambda_2)^{d_1}(-\lambda_2)^{d_2}}$$
$$-i\left(c_2\frac{\sin\left[\frac{1}{2}(d_1\pi+d_2\pi-2\lambda_1)\right]}{(\lambda_1-\lambda_2)^{d_1}\lambda_1^{d_2}} - c_2\frac{\sin\left[\frac{1}{2}(d_1\pi+d_2\pi+2\lambda_2)\right]}{(\lambda_1-\lambda_2)^{d_1}(-\lambda_2)^{d_2}}\right)$$

2-Dimensional Matching

- Quasi Distribution in momentum space after FT:





2-Dimensional Matching

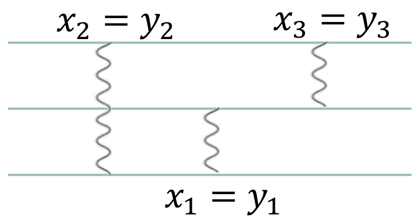
- LaMET factorization for Baryon LCDA:

$$\tilde{\phi}(x_1,x_2)=\int_0^1 dy_1\int_0^{1-y_1} dy_2 C(x_1,x_2,y_1,y_2)\phi(y_1,y_2)+\mathcal{O}\left(\frac{1}{(x_1P^z)^2},\frac{1}{(x_2P^z)^2},\frac{1}{[(1-x_1-x_2)P^z]^2}\right)$$

  - Matching kernel:

Double plus function

$$\begin{aligned}C\left(x_1,x_2,y_1,y_2,\mu\right)=&\ \delta\left(x_1-y_1\right)\delta\left(x_2-y_2\right)\\&+\frac{\alpha_s C_F}{2\pi}\left[\left(\frac{1}{4}C_2\left(x_1,x_2,y_1,y_2\right)-\frac{7}{8}\frac{-1}{|x1-y1|}\right)\delta\left(x_2-y_2\right)\right.\\&+\left(\frac{1}{4}C_2\left(x_2,x_1,y_2,y_1\right)-\frac{7}{8}\frac{-1}{|x2-y2|}\right)\delta\left(x_1-y_1\right)\\&\left.+\left(\frac{1}{4}C_3\left(x_1,x_2,y_1,y_2\right)+\frac{1}{4}C_3\left(x_2,x_1,y_2,y_1\right)-\frac{3}{4}\frac{-2}{|x_1-y_1-x_2+y_2|}\right)\delta\left(x_1+x_2-y_1-y_2\right)\right]_{\oplus},\end{aligned}$$



C.Han et.al. JHEP 12 044 (2023), JHEP 07 019 (2024)

$$\left[g\left(x_1,x_2,y_1,y_2\right)\right]_{\oplus}=g\left(x_1,x_2,y_1,y_2\right)-\delta\left(x_1-y_1\right)\delta\left(x_2-y_2\right)\int dz_1dz_2g\left(z_1,z_2,y_1,y_2\right)$$

2-Dimensional Matching

➢ LaMET factorization for Baryon LCDA:

$$\tilde{\phi}(x_1, x_2) = \int_0^1 dy_1 \int_0^{1-y_1} dy_2 C(x_1, x_2, y_1, y_2) \phi(y_1, y_2) + \mathcal{O}\left(\frac{1}{(x_1 P^z)^2}, \frac{1}{(x_2 P^z)^2}, \frac{1}{[(1-x_1-x_2)P^z]^2}\right)$$

- Inverse matching:

$C(x_1, x_2, y_1, y_2) \rightarrow$ 4 Dimensional tensor $\rightarrow$ Reduce to 2D matrix $\rightarrow$ inverse

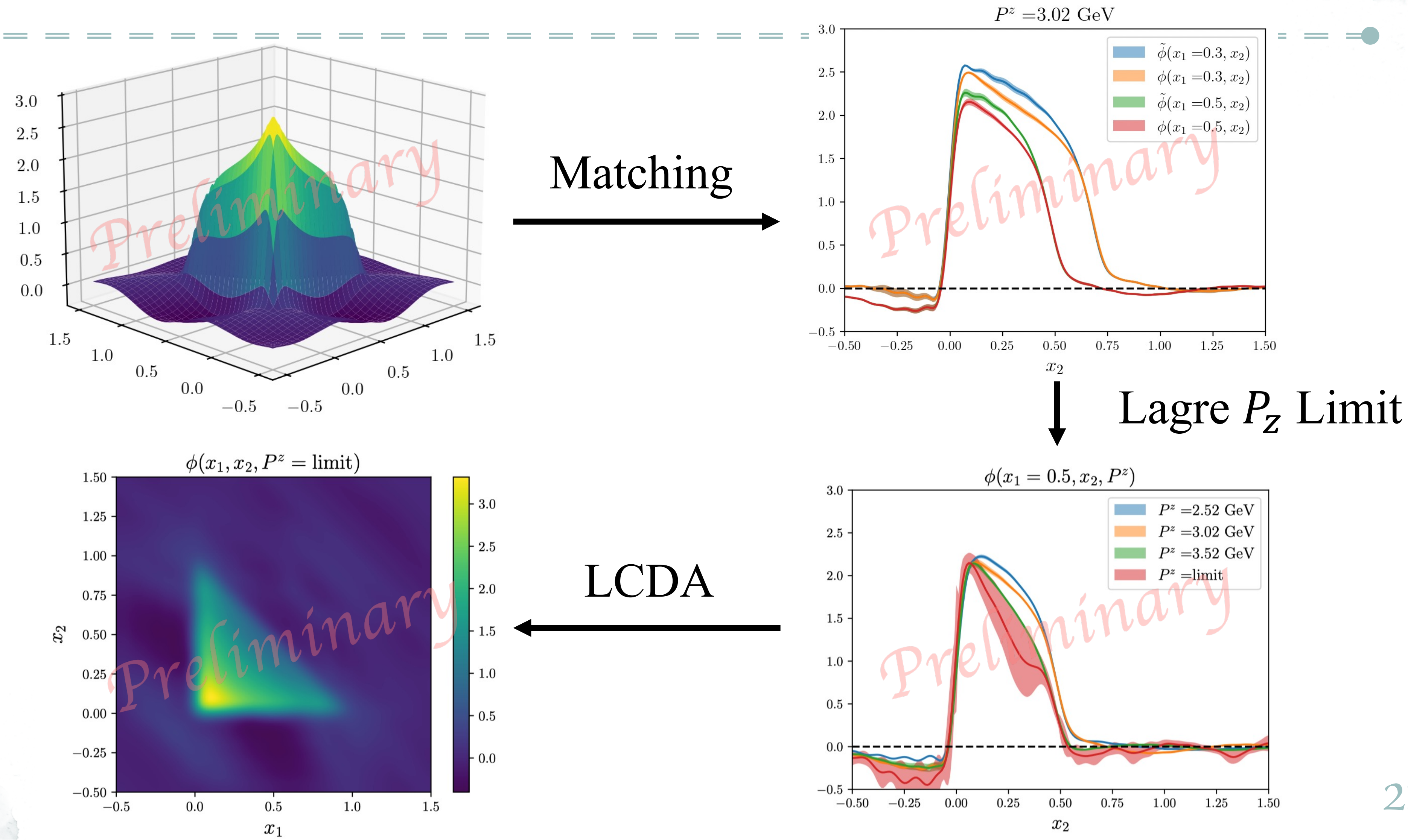
- Iterative matching:

$$\tilde{\phi}(x_1, x_2) = \phi(x_1, x_2) + \frac{\alpha_s C_F}{2\pi} \int_0^1 dy_1 \int_0^{1-y_1} dy_2 c^{(1)}(x_1, x_2, y_1, y_2) \phi(y_1, y_2) + \mathcal{O}(\alpha_s^2)$$

The difference between $\tilde{\phi}(x_1, x_2)$ and $\phi(x_1, x_2)$ introduces error at higher order

$$\phi(x_1, x_2) = \tilde{\phi}(x_1, x_2) - \frac{\alpha_s C_F}{2\pi} \int_0^1 dy_1 \int_0^{1-y_1} dy_2 c^{(1)}(x_1, x_2, y_1, y_2) \tilde{\phi}(y_1, y_2) + \mathcal{O}(\alpha_s^2)$$

Numerical results



Summary and outlook

- ➢ We made the first attempt to implement the numerical computation of baryon LCDA in the LaMET framework.
- ➢ The 3-particle distribution cased 3D structure complexity in several parts:
  - Hybrid renormalization (Match with Perturbative Quasi)
  - Extrapolation and Fourier transformation
  - Matching implementation

- ☐ Calculation with smaller lattice spacing (at least < 0.6 fm)
- ☐ Calculation for all leading twist structure Proton and Lambda LCDA (Based on PyQUDA)

Thanks For Your Attention！