Pion 介子形状因子的微扰计算

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Overview

Pion form factor

Pion form factor

"In elementary particle physics and mathematical physics, in particular in effective field theory, a **form factor** is a function that encapsulates the properties of a certain interaction without including all of the underlying physics, but instead, providing the momentum dependence of a suitable matrix elements. Its further measured experimentally in confirmation or specification of a theory."

Momenta Redistribution

$\Downarrow$ QCD is believed to confine

hadron structures $\otimes$ hard scattering

$\Downarrow$

factorisation theorem, EFT; CKM, g-2, B anomalies

Pion form factor

PION is the lightest and one of the most simplest hadrons,
hence the ideal probe to extremely rich QCD dynamics, including confine and chiral symmetry breaking.

实验室/组	数据类型	能量区间 $q^2(\mathrm{GeV}^2)$	数据点个数	主要参考文献
PRad	微分散射截面 $d\sigma/d\Omega$	[-0.06, -0.0002]	71	Nature 575(7781),147-150(2019)
MAMI	微分散射截面 $d\sigma/d\Omega$	[-0.98, -0.004]	1422	Phys.Rev. Lett.105,242001(2010) Phys.Rev. C.90,015200(2014)
JLab	形状因子比值 $\mu_p G_E^p/G_M^p$	[-8.49, -1.18]	16	Phys.Rev. Lett.104,242301(2010) Phys.Rev. C.85,045203(2012)
多组实验平均值	电形状因子G_E^n磁形状因子G_M^n	[-1.47, -0.14] [-10.0, -0.07]	25 23	Euro.Phys.J. A 48,151(2012)
BABAR	形状因子比值 $\|G_E^p/G_M^p\|$	[3.52, 9.00]	6	Phys.Rev.D. 87, 092005(2013)
BESIII	微分散射截面 $d\sigma/d\Omega$	[3.52, 3.80]	10	Nature Phys 17(2021)11, 1200-1204
多组实验平均值	有效形状因子G_{eff}^p 有效形状因子G_{eff}^n	[3.52, 20.3] [3.53, 9.49]	153 32	Phys.Rev. Lett.96,261803(2005) Phys. Lett. B759,634-640(2016) EPJ Web Conf. 212,0700(2019)

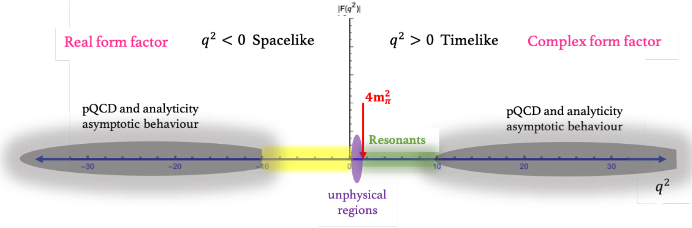
$\Rightarrow$ not so much for F_π

Rich measurements of F_N in different energy regions

Pion form factor

Measurements of F_π in different energy regions

- Spacelike data is available in the narrow region $q^2 \in [-2.5, -0.25]$ GeV^2
  Jefferson Lab 2006,2008, $\cdots$, NA7 1996, CLEO 2005
- Timelike data is dominated by the resonant states, have not extend to large momentum transfers (perturbative QCD available)



Whole region of momentum transfers for electromagnetic form factor

- Mismatch between the QCD based calculation and the available data
- could be restored by employing the dispersion relation
- pQCD prediction at large $|q^2|$ is indispensable

perturbative QCD approach

i Dispersion relations

ii Pion LCDAs

iii Three-scale factorization

Dispersion relations

- QCD calculations are valid in the intermediate/large $|q^2|$

  N^2LO calculation in collinear factorization $\sim$ NLO [Chen^2, Feng, Jia 2312.17228]

  spacelike data is available in the narrow region $q^2 \in [-2.5, -0.25]$ GeV^2
- the mismatch destroys the direct extracting programme from $F_\pi(q^2 < 0)$
- timelike data $F_\pi(q^2 > 0)$ provides another opportunity

  △ $e^+e^- \to \pi^+\pi^-(\gamma)$, $4m_\pi^2 \leqslant q^2 \lesssim 9$ GeV^2 [BABAR 2012]

  △ $\tau \to \pi\pi\nu_\tau$, $4m_\pi^2 \leqslant q^2 \leqslant 3.125$ GeV^2 [Belle 2008]

  △ $e^+e^-(\gamma) \to \pi^+\pi^-$, $0.6 \leqslant Q^2 \leqslant 0.9\,\mathrm{GeV}^2$ with ISR [BESIII 2016]
- The standard dispersion relation and The modulus representation

$$F_\pi(q^2 < s_0) = \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im}F_\pi(s)}{s - q^2 - i\epsilon}$$

⇓ [**SC**, Khodjamirian, Rosov 2007.05550]

$$F_\pi(q^2 < s_0) = \exp\left[\frac{q^2\sqrt{s_0 - q^2}}{2\pi}\int_{s_0}^{\infty} \frac{ds\,\ln|F_\pi(s)|^2}{s\sqrt{s - s_0}\,(s - q^2)}\right]$$

$$|\mathcal{F}_\pi(s)|^2 = \Theta(s_{\max} - s)\,|\mathcal{F}^{\mathrm{data}}_{\pi,\mathrm{Inter.}}(s)|^2 + \Theta(s - s_{\max})\,|\mathcal{F}^{\mathrm{pQCD}}_\pi(s)|^2$$

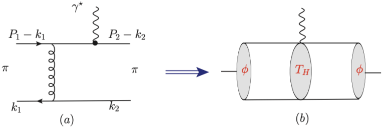
Factorization formular

(spacelike) electromagnetic form factor

$$\langle \pi^-(p_2) \big| J_\mu^{\rm em} \big| \pi^-(p_1) \rangle = e_q \left(p_1 + p_2\right)_\mu F_\pi(Q^2)$$

- the interaction distance of $J_\mu^{\rm em}$ is decided by the external reason Q^2
- at large Q^2, $\bar{z}_i \bar{p}_i \sim 1$, the expansion is performed in twist

- Separate the hard partonic physics out of the hadronic physics (soft, nonperturbative objects) in exclusive processes **Factorization**



$$\frac{d\sigma}{d\Omega} = \int_u^1 \frac{d\zeta}{\zeta} \mathcal{H}^{(t)}(\zeta) f^{(t)}\left(\frac{u}{\zeta}\right)$$

微扰量子色动力学应用到遍举过程中的几个问题	曹俊	1998	黄涛	博士

- The universal nonperturbative objects can be studied by QCD-based analytical (QCDSRs, χPT, instanton) and numerical approaches (LQCD)
- also by performing global fit, CETQ, CT, MMHT, NNPDF, ABM, JAM, et.al.

Pion LCDAs

In the exclusive processes with large momentum transfers, pion is described by the **light-cone distribution amplitudes (LCDAs)**

- Wave function of bound state in terms of Fock states

$$|\pi\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle + \psi_{q\bar{q}gg}|q\bar{q}gg\rangle + \cdots$$
$$\psi_\pi^n(u_i, k_{\perp i}, \lambda_i) = \langle n, u_i, k_{\perp i}, \lambda_i|\pi\rangle$$

- large Q^2, $k_\perp$ is neglected/integrated $\psi_\pi^n(u_i, \mu, \lambda_i)$
- separate out the spin to obtain the LCDAs $\phi_\pi^n(u_i, \mu, Q)$
- LCDAs are dimensionless functions of u_i and renormalization scale μ

△ describe the probability amplitudes to find the π in a state with minimal number of constitutes and have small transversal separation of order $1/\mu$

△ Expansion in power of large momentum transfer is governed by contributions from small transversal separations x^2 between constituents

- The definition of LCDAs is the application of conformal symmetry in massless QCD [Braun, Korchemsky, Müller 2003]

Pion LCDAs

Conformal symmetry in massless QCD

- the underlying idea of *conformal expansion of LCDAs* is similar to *partial-wave expansion of wave function in quantum mechanism*
- *invariance of massless QCD* *under conformal trans.* *VS* rotation symmetry
- longitudinal $\otimes$ transversal dofs *VS* angular $\otimes$ radial dofs
- the transversal-momentum dependence (scale dependence of the relevant operators) is governed by the RGE
- the longitudinal-momentum dependence (orthogonal polynomials) is described in terms of irreducible representations of the corresponding symmetry group
  collinear subgroup of conformal group $SL(2,R) \cong SU(1,1) \cong SO(2,1)$

- **LCDAs are defined at different twists**

$$\langle 0|\bar{u}(x)\gamma_\mu\gamma_5 d(-x)|\pi^-(P)\rangle = f_\pi \int_0^1 du\, e^{i\zeta P\cdot x}\left[iP_\mu\left(\phi(u)+\frac{x^2}{4}g_1(u,\mu)\right)+\left(x_\mu-\frac{x^2 P_\mu}{2P\cdot x}\right)g_2\right]$$

$$\langle 0|\bar{u}(x)i\gamma_5 d(-x)|\pi^-(P)\rangle = f_\pi m_0^\pi \int_0^1 du\, e^{i\zeta P\cdot x}\phi^p(u,\mu)$$

$$\langle 0|\bar{u}(x)i\sigma_{\mu\nu}\gamma_5 d(-x)|\pi^-(P)\rangle = -\frac{if_\pi m_0^\pi}{3}\left(P_\mu x_\nu - P_\nu x_\mu\right)\int_0^1 du\, e^{i\zeta P\cdot x}\phi^\sigma(u,\mu)$$

△ collinear twist: dimension - spin projection on the plus-direction △ geometric twist: dimension - spin

Pion LCDAs

LCDAs are defined at different twists

- quark fields are decomposed into "good" and "bad" components based on light-cone quantization formalism [Kogut and Soper 1970, Jaffe and Ji 1992]

$$\psi = \psi_+ + \psi_-, \quad \psi_+ = \frac{1}{2}\gamma_*\gamma_\cdot\psi, \quad \psi_- = \frac{1}{2}\gamma_\cdot\gamma_*\psi$$
$$\gamma_* = \gamma_\mu z^\mu, \; \gamma_\cdot = \gamma_\mu p^\mu/(p\cdot z)$$

- a bad component introduce one unit of twist
- composite operators of type $\bar{u}d$ contains twist 2 ($\bar{u}_+ d_+$), twist 3 ($\bar{u}_+ d_-, \bar{u}_- d_+$) and twist 4 ($\bar{u}_- d_-$)

- $\phi(x)$ and $\phi^{p,t}(u)$ are the twist 2 and twist 3 LCDAs

$$\phi(u,\mu) = 6u(1-u)\sum_{n=0} a_n^\pi(\mu) C_n^{3/2}(u)$$
$$\phi^\sigma(u) = 6u(1-u)\left[1 + 5\eta_{3\pi} C_2^{3/2}(u)\right]$$
$$\phi^p(u,\mu) = \left[1 + 30\eta_{3\pi} C_2^{1/2}(u) - 3\eta_{3\pi}\omega_{3\pi} C_4^{1/2}(u)\right]$$

Pion LCDAs

- **Three sources of high twist LCDAs**

  † "bad" components in WFs in particular of those with "wrong" spin projection

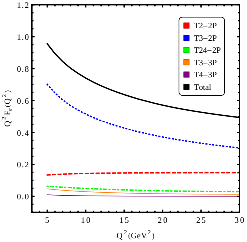
  † transversal motion of $q(\bar{q})$ in the leading twist components

  given by the integrals with additional factors of $k_\perp^2$

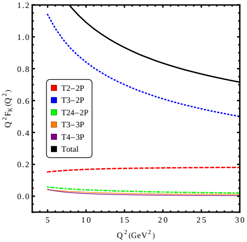
  † higher Fock states with additional g and $q\bar{q}$ pairs

- higher twist contributions to exclusive QCD processes are commonly power suppressed $\mathcal{O}(1/Q)$

- but twist 3 contribution are dominate in the π, K evolved processes due to chiral enhancement $\mathcal{O}(m_0/(x_i Q))$



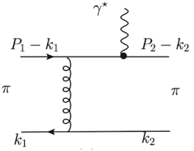
[**SC**, 1905.05059]



Three scale factorization

- end-point singularities appear in exclusive QCD processes

  † $m_{1,2}^2 \ll Q^2$, light-cone coordinate $p_2 = (\frac{Q}{\sqrt{2}}, 0, 0_{\mathrm{T}})$, $p_3 = (0, \frac{Q}{\sqrt{2}}, 0_{\mathrm{T}})$,
  (anti-)valence quarks: $k_2 = x_2 p_2$, $\bar{k}_2 = \bar{x}_2 p_2$



$\phi \propto u(1-u)$, $m_0^\pi \phi^{P,\sigma} \propto m_0^\pi$

$$\propto \sum_t \int du_1 du_2 \kappa_t(u_i) \frac{\alpha_s(\mu)\phi_1^t(u_1)\phi_2^t(u_2)}{u_1 u_2 Q^2 u_2 Q^2}$$

- pick up k_T in the internal propagators

$$\mathcal{M} \propto \sum_{t=2,3,4} \int du_1 du_2 dk_{1T} dk_{2T} \kappa_t(u_i) \frac{\alpha_s(\mu)\phi_1^t(u_1)\phi_2^t(u_2)}{u_1 u_2 Q^2 - (k_{1T} - k_{2T})^2}$$

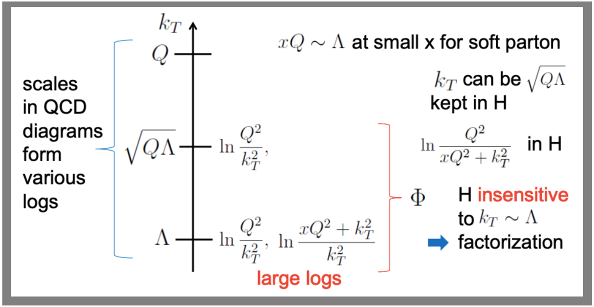
- end-point singularity at leading and subleading powers

$$\mathcal{H} \propto \frac{\alpha_s(\mu)}{u_1 u_2 Q^2 - k_T^2} \sim \frac{\alpha_s(\mu)}{u_1 u_2 Q^2} - \frac{\alpha_s(\mu) k_T^2}{(u_1 u_2 Q^2)^2} + \cdots$$

- the power suppressed TMD terms becomes important at the end-points

Three scale factorization

- introduce k_T to regularize the end-point singularity [Huang 1991]
- scales of transversal momentum and the large logarithms [borrowed from H.N Li]



- multiple scales, hence large single logarithms in $\mathcal{H}/\Phi$ by QCD correction
- double logs in the soft-collinear regions $\alpha_s(\mu)\,\ln^2(k_T^2/m_B^2)$

Three scale factorization

- in order to repair the perturbative expansion, do **resummation**
- k_T resummation for $\mathcal{H}$ to obtain $S(x_i, b_i, Q)$ [Botts 1989, Li 92]

  † decreases the inverse power of the q^2 in the divergence amplitude

  † exhibits high suppression for large transversal distances (small k_T) interactions

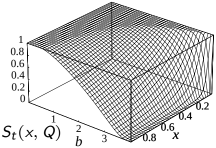
- integrating over k_T, large log $\ln^2(x_i)$ when intermediate gluon is on shell
- threshold resummation for Φ to obtain $S_t(x_i, Q)$ [Li 1999]

  † suppresses the small x_i regions

  † repairs the self-consistency between $\alpha_s(t)$ and hard log $\ln(x_1 x_3 Q^2/t^2)$

‡ dynamics with $k_T < \sqrt{Q\Lambda}$ is organized into $S(x, b, Q)$

‡ dynamics in small x is suppressed by $S_t(x, Q)$



$S(x, b, Q)S_t(x, Q)$

$$\mathcal{M} = \sum_t \phi^t(u_1, b_1) \otimes \mathcal{H}_i(t, b) \otimes \phi^t(u_2, b_2)\, Exp\left[-s(p^+, b) - \int_{1/b}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_\phi(\alpha_s(\bar{\mu}))\right]$$

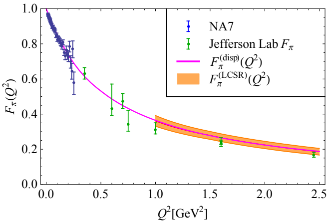
Form factors

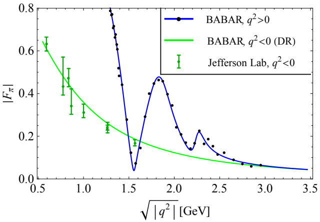
Pion electromagnetic form factor

- the precise pQCD calculation, up to twist 4 at NLO
  [Chai, **SC**, Hua 2209.13312]

- the most interesting parameters in leading twist LCDAs is moments
- up-to-date LCSRs calculation of spacelike form factor
  LCSRs calculation does not involve odd twist LCDAs under the chiral symmetry limit
- fit to the modular dispersion relation result with timelike data

- $a_2 = 0.275 \pm 0.055, \ a_4 = 0.185 \pm 0.065$ [**SC**, Khodjamirian, Rosov 2007.05550]
  △ Pion deviates from the purely asymptotic one △ a_2^π is not enough
  △ $0.258^{+0.079}_{-0.052}$ [LPC 2201.09173[hep-lat]], $0.249^{+0.005}_{-0.006}$ [Li 2205.06746]





Pion electromagnetic form factor

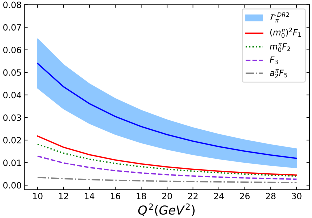
- twist three LCDAs are accompanied by a chiral mass

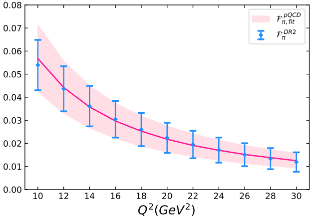
$$\chi^2 = \sum_{i=1}^{11} \frac{\left[\mathcal{F}_\pi^{\rm DR2}(Q_i^2) - \mathcal{F}_\pi^{\rm pQCD}(Q_i^2)\right]^2}{\left[\delta\mathcal{F}_\pi^{\rm DR2}(Q_i^2)\right]^2}$$

- the most precise pQCD calculation, fit to the modular dispersion relation
- $m_0^\pi = 1.37^{+0.29}_{-0.32}$ GeV [2209.13312]

△ smaller than the χPT result $m_0^\pi(1\,\mathrm{GeV}) = 1.89$ GeV [Leutwyler 1996]

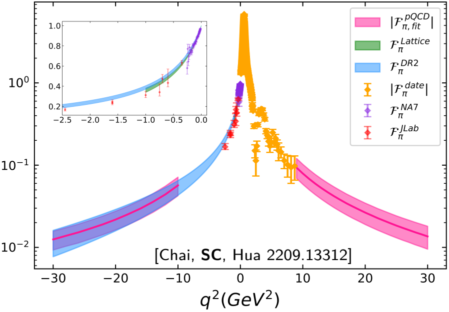
△ also smaller than the value obtained with $\overline{\rm MS}$ current quark masses $m_0^\pi = m_\pi^2/(m_u + m_d)$





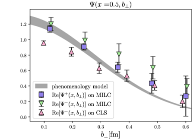
Pion electromagnetic form factor

- the precise pQCD calculation
- modular dispersion relation with e^+e^- annihilation data
- a comprehensive description of $F_\pi(q^2)$ in the whole kinematics

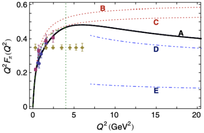


- a slight derivation in the small region

- intrinsic transverse momentum ?[LPC 2302.09961]



- dynamical chiral symmetry breaking ?[Chang et.al. 1307.0026]



Pion electromagnetic form factor

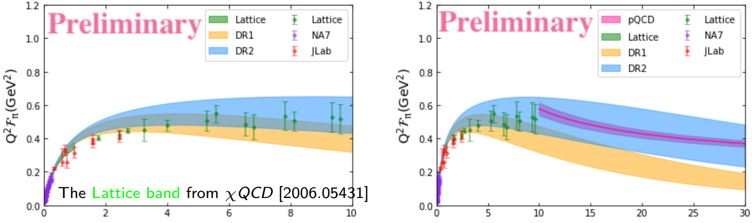
- consider contribution from the iTMD dynamic at $k_T \sim \lambda_{\rm QCD}$

$$\frac{f_\pi m_0^{\mathcal{P}}}{2\sqrt{6}}\phi^p(u,\mu) = \int \frac{d^2\vec{k}_T}{16\pi^3}\phi^p_{2p}(u,\vec{k}_T) + \int \frac{d^2\vec{k}_{T1}}{16\pi^3}\frac{d^2\vec{k}_{T2}}{4\pi^2}\phi^p_{3p}(u,\vec{k}_{T1},\vec{k}_{T2}).$$

$$\psi^p_{2p}(u,\vec{k}_T) = \frac{f_\pi m_0^{\mathcal{P}}}{2\sqrt{6}}\phi^p_{2p}(u,\mu)\Sigma(u,\vec{k}_T),$$

$$\psi^p_{3p}(u,\vec{k}_{1T},\vec{k}_{2T}) = \frac{f_\pi m_0^{\mathcal{P}}}{2\sqrt{6}}\eta_{3\pi}\phi^p_{3p}(u,\mu)\Sigma'(\alpha_i,\vec{k}_{1T},\vec{k}_{2T}).$$

- comparison with the impressive LQCD calculation [H.T Ding et.al, 2404.04412]

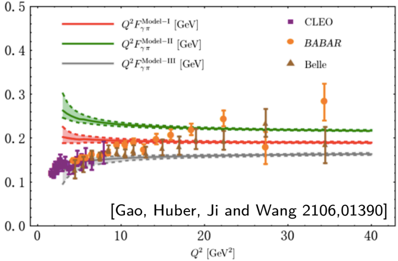


[Chai, **SC** to be appear]

- the slight derivation is still there despite its sensitive to iTMD in the small q^2
- form factor of K meson is also studied

Pion transition form factor

- $F_{\pi\gamma\gamma^*}$ is the theoretically most clean observable $\propto a_n^\pi$
- Two-loop calculation of $F_{\pi\gamma\gamma^*}$ in hard-collinear factorization theorem
  $N^2LO \sim NLO$

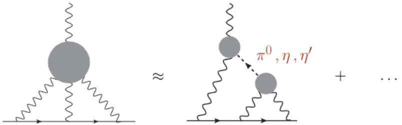


[Gao, Huber, Ji and Wang 2106,01390]

Model-I [Brodsky, Teramond 0707.3859, RQCD 1903.08038]

Model-II [**SC**, Khodjamirian, Rosov 2007.05550]

Model-III [Mikhailov, Pimikov, Stefanis 1604.06391]



[Gérardin, Meyer, Nyffeler 1607.08174]

Hadronic light-by-light scattering (HLbL) contribution to $a_\mu^{HLbL;\pi^0}$

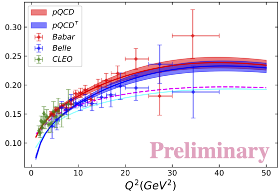
- NLO pQCD calculation with the iTMD contribution

† improve the pQCD power in the intermediate momentum transfers

† modification in the small and intermediate regions is significant

- $\eta^{(\prime)}$, η_q and η_s TFF are is also studied

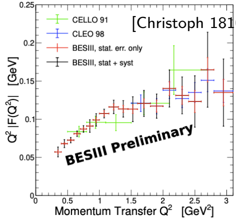
[Chai, **SC** to be appear]



Conclusion

- in the light-cone dominated processes, hadron structure is well studied in terms of **LCDAs**
- a comprehensive studies of $F_\pi(q^2)$ with pQCD calculation and modular dispersion relation
- help to reveal inner structure of pion (moments, iTMD)
- settle down the "fat pion" issue in $F_{\pi\gamma\gamma^*}$

Belle II,BESIII,JLab and future e^+e^- colliders



[Christoph 1810.00654[hep-ex]]

Thank you for your patience.

Dispersion relations

- Introducing an auxiliary function $g_\pi(q^2) \equiv \frac{\ln F_\pi(q^2)}{q^2\sqrt{s_0-q^2}}$ [Geshkenbein 1998]
- Cauchy theorem and Schwartz reflection principle

$$g_\pi(q^2) = \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im}\, g_\pi(s)}{s-q^2-i\epsilon}$$

- At $s > s_0$ on the real axis, the imaginary part of g_π reads as

$$\mathrm{Im}\, g_\pi(s) = \mathrm{Im}\left[\frac{\ln(|F_\pi(s)|e^{i\delta_\pi(s)})}{-is\sqrt{s-s_0}}\right] = \frac{\ln|F_\pi(s)|}{s\sqrt{s-s_0}},$$

- Substituting $g_\pi(q^2)$ and $\mathrm{Im}\, g_\pi(s)$ into the dispersion relation, for $q^2 < s_0$

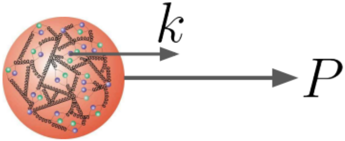
$$\frac{\ln F_\pi(q^2)}{q^2\sqrt{s_0-q^2}} = \frac{1}{2\pi}\int_{s_0}^{\infty}\frac{ds\,\ln|F_\pi(s)|^2}{s\sqrt{s-s_0}\,(s-q^2)}$$

- The modulus representation [**SC**, Khodjamirian, Rosov 2007.05550]

$$F_\pi(q^2<s_0) = \exp\left[\frac{q^2\sqrt{s_0-q^2}}{2\pi}\int_{s_0}^{\infty}\frac{ds\,\ln|F_\pi(s)|^2}{s\sqrt{s-s_0}\,(s-q^2)}\right]$$

Pion PDF,TMD,GPD

Definitions of pion distribution



One dimension PDF

△ $f_i(\zeta) = \int \frac{dz^-}{4\pi} e^{-i\zeta P^+ z^-} \langle \pi | \bar{\psi}_i(0, z^-, 0_T)\gamma^+ \psi_i(0) | \pi \rangle$

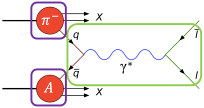
△ $\zeta = \frac{k^+}{P^+}$, the parton momentum fraction

△ $f_i(\zeta) \sim \sum_\alpha \int dk_T^2 \langle \pi | b^\dagger_{k,\alpha} b_{k,\alpha} (\zeta P^+, k_T, \alpha) | \pi \rangle$

number operator

△ Transversal momentum distributions (TMD) $f(\zeta, k_T)$

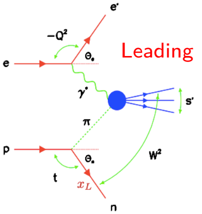
△ Generalized parton distributions (GPD) $f(\zeta, b_T)$



Drell-Yan

$$\sigma \propto \sum_{i,j} f_i^\pi(x_\pi, \mu) \otimes f_j^A(x_A, \mu) \otimes C_{i,j}(x_\pi, x_A, Q/\mu)$$

Extracted from **fixed target πA data**



Leading neutrons

Deeply virtuality meson production

△ TDIS at 12GeV JLab, leading proton observable, fixed target instead of collider (HERA);

△ EIC, EIcC, great integrated luminosity to reduce the systematics uncertainties;

△ COMPASS++/AMBER give π-induced DY data.