The ω-meson from lattice QCD

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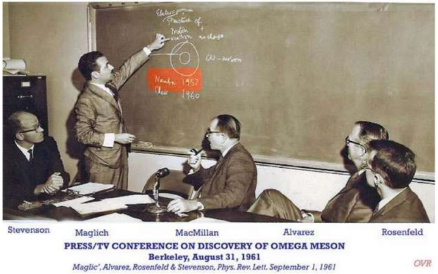
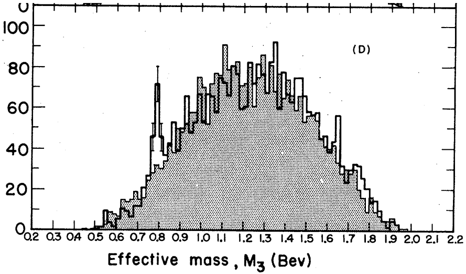
ω: the first neutral vector meson (1961)[1]



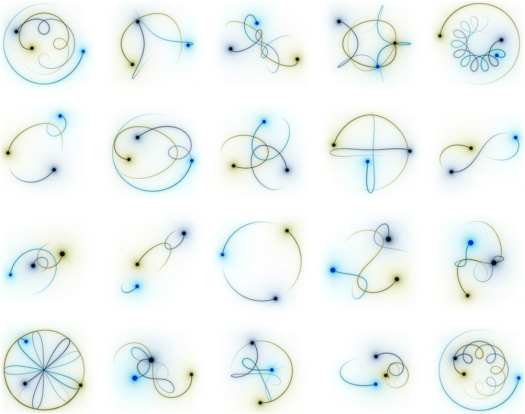


Phenomenologically[2],

- ω is the lightest hadron decaying into three particles: $\omega \to 3\pi$
- ω dominates the isoscalar response within the VMD picture of the photon-nucleon interactions
- ω generates the observed repulsion at $< 1\,\mathrm{fm}$ in the one-boson-exchange picture of the $N - N$ interaction
- ω mixes with the ρ and leads to marked effects in the pion vector form factor
- $\omega - \rho$ mass splitting is phenomenologically interesting, for instance muon $g - 2$ and dark matter

[1] Maglic *et al.* (1961).
[2] Sakurai (1960); Erkelenz (1974); Brown and Jackson (1976); Barkov *et al.*, 1985; Connell *et al.* (1997); Bazavov *et al.* (2021).

Three-body problem

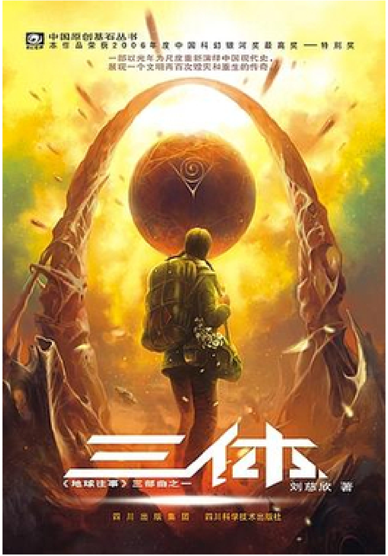


Gravitational three-body problem

- Goal: space-time trajectories – **unsolvable**
- Birth of mathematical chaos

Quantum mechanical three-body problem

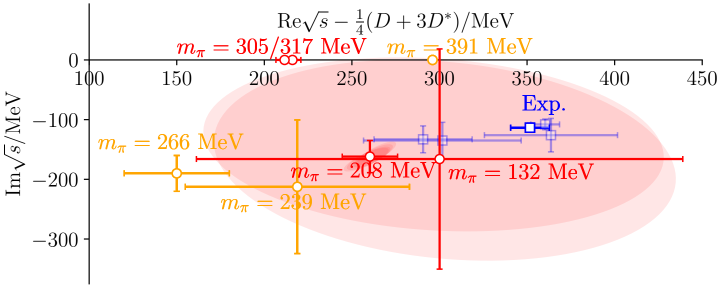
- Goal: rigorous scattering theory – **solvable**
- Spectra from lattice QCD
- Tests of the fundamental theory

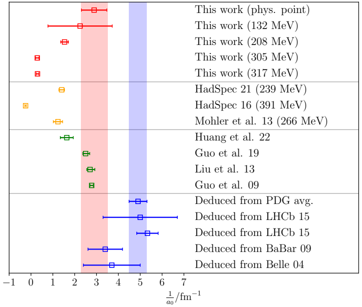


For ω-meson

- Three-body problem with resonances in two-body problem
  - $\pi\pi\pi \to \omega$
  - $\pi\pi \to \rho$
- **Most challenging isospin** in the $\pi\pi\pi$ channel
- e.g. $I = 0$ $\pi\pi \to \sigma$
- Spectroscopy in CLQCD is promising

$D\pi$ scattering at the physical point[2]





- Two-pole structure is to be found in the coupled $D\pi$–$D\eta$–$D_s\bar{K}$ scattering
- $1/a_0$ at the physical point: not consistent with the PDG value (there are no real measurements actually)
- An clear trend for the motion of the $D_0^*(2300)$ pole is identified.

bound state $\to$ virtual state $\to$ resonance

[2] Haobo Yan, Chuan Liu, Liuming Liu, Yu Meng, and Hanyang Xing, $D\pi$ scattering, arXiv:2404.13479.

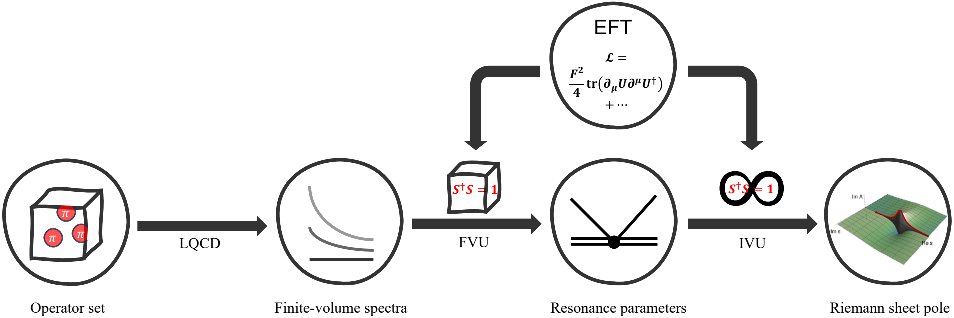
Three-body problem on the lattice – History

Two-body problem: looks good :)

- 2019: $\pi\pi\pi$ at maximal isospin, PRL 122, 062503,
- 2020: $\pi\pi\pi$ at maximal isospin, PRL 124, 032001
- 2020: $\pi\pi\pi$ at maximal isospin, PRD 101, 114507
- 2020: KKK at maximal isospin, $I = \frac{3}{2}$ KKK
- 2021: $\pi\pi\pi$ at maximal isospin, PRL 126, 012001
- 2021: $\pi\pi\pi$ at maximal isospin, EPJC 81, 436
- **2021:** $\pi\pi\pi \to a_1(1260)$**, PRL 127, 222001**
- 2023: $\pi\pi K$ and $KK\pi$ at maximal isospin, JHEP 05, 137

Three-body problem: all repulsive except one study :(

The methodology



- $I = 1$ $\pi\pi$ and $I = 0$ $\pi\pi\pi$ spectra
- Develop the formalism to map finite to infinite volumes
- Establish the pertinent EFT and parametrize the three-body force
- Solve the integral equations and search the poles

Disclaimer

- $\omega \to \pi^+\pi^-$ is forbidden due to G-parity but only 2%
- Mixing with the ϕ or $\omega(1420)$ are ignored since they are too high to play a role in our analysis
- Also, strangeness is not considered
- F32P21 has a very small $m_\pi L \approx 2.6$
- Number of energy levels is limited
- Continuum extrapolations are challenging
- ChPT assumptions are to be tested (*e.g.* the universality assumption $g_{\rho\pi\pi} = g_{\omega\rho\pi}$)
- There are cutoffs in the quantization condition

Lattice setup

Ensemble	Volume	M_π/MeV	N_{confs}
F32P21	$32^3 \times 64$	$206.8(2.1)$	459
F48P21	$48^3 \times 96$	$207.58(76)$	221
F32P30	$32^3 \times 96$	$303.61(71)$	777
F48P30	$48^3 \times 96$	$304.95(49)$	201

- CLQCD ensembles with $N_\mathrm{f} = 2+1$ Clover fermions [CLQCD, 2024]
- Two pion masses with two volumes
- At the same lattice spacing $a = 0.07746(18)\,\mathrm{fm}$

Operator construction

- Construction tool **OpTion**[3] is utilized to generate general N-hadron operators with arbitrary momenta
- P-wave between all $\pi \to$ irrep T_1^-
- Isospin addition $1 \otimes 1 \otimes 1 = (0 \oplus 1 \oplus 2) \otimes 1 = 0 \oplus 1^3 \oplus 2^2 \oplus 3$
- Project to $I = 0$

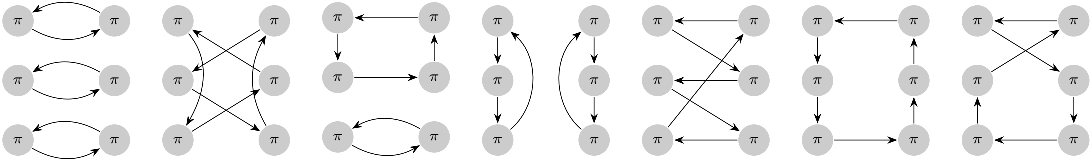
$$\begin{cases} |\omega\rangle & = \frac{1}{\sqrt{2}}(\omega^u + \omega^d), \\ |\rho\pi\rangle & = -\frac{1}{\sqrt{3}}\left[\rho^+\pi^- + \rho^-\pi^+ + \frac{1}{2}[\rho^u\pi^u - \rho^u\pi^d - \rho^d\pi^u + \rho^d\pi^d]\right], \\ |\pi\pi\pi\rangle & = \frac{1}{\sqrt{12}}\left[-\pi^+\pi^u\pi^- + \pi^+\pi^d\pi^- + \pi^u\pi^+\pi^- - \pi^d\pi^+\pi^- + \pi^+\pi^-\pi^u - \pi^+\pi^-\pi^d \right. \\ & \quad \left. -\pi^-\pi^+\pi^u + \pi^-\pi^+\pi^d - \pi^u\pi^-\pi^+ + \pi^d\pi^-\pi^+ + \pi^-\pi^u\pi^+ - \pi^-\pi^d\pi^+\right], \end{cases}$$

- Necessary to have all kinds of operators for low-lying levels

[3] https://github.com/wittscien/OpTion

Contraction topologies

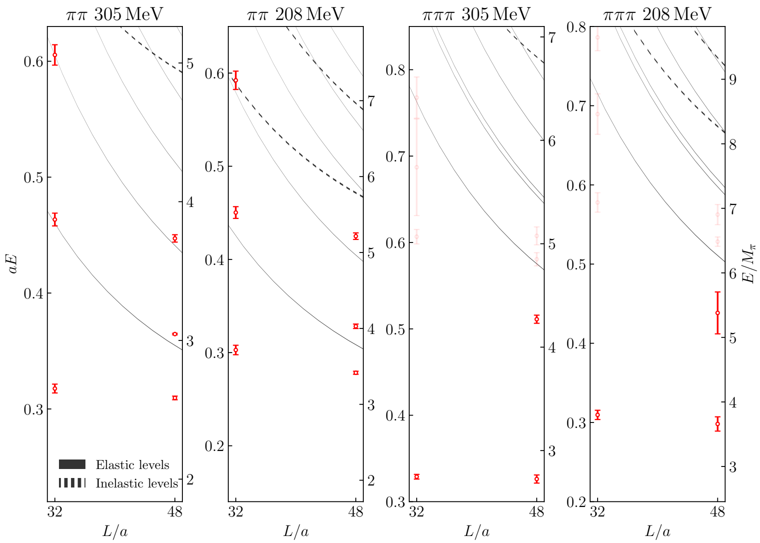
- Insanely many diagrams (202 for only $\pi\pi\pi \to \pi\pi\pi$, only 9 for the two-body problem)
- The topologies for $\pi\pi\pi \to \pi\pi\pi$



- Distillations [Peardon *et al.*, 2009] for the vast number of annihilation diagrams[4]
- Collect all operators with different momentum configurations and do GEVP
- The spectra are stable against more non-local operators / thermal pollution / N_v

[4] Haobo Yan, Chuan Liu, Liuming Liu, Yu Meng, and Hanyang Xing, $D\pi$ scattering, arXiv:2404.13479.

Finite-volume spectra

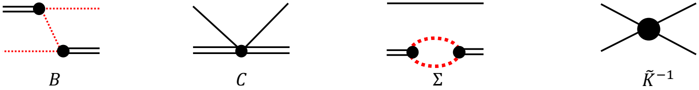


- Strong **attraction** in both the $\pi\pi$ and $\pi\pi\pi$ channels
- In the $\pi\pi\pi$ channel, the ground levels indicate **bound** ω at $M_\pi \approx 305\,\mathrm{MeV}$ and **resonating** ω at $M_\pi \approx 208\,\mathrm{MeV}$
- Restricted to be below the $\omega(1420)$ region

Quantization condition

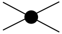
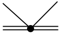
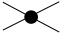
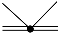
- Using **FVU** (Finite-Volume Unitarity) of all state-of-art formalisms
  - FVU [Mai and Döring, 2017]
  - RFT [Hansen and Sharpe, 2014]
  - NREFT [Hammer, Pang, and Rusetsky, 2017]

$$\begin{cases} \tilde{K}^{-1}(\sigma) - \Sigma^{FV}(\sigma) = 0 \\ \det[(\tilde{K}^{-1}(s) - \Sigma^{FV}(s))E_L - (\tilde{B}(s) + \tilde{C}(s))] = 0 \end{cases}$$



- Two-body quantization condition is equivalent to Lüscher's equation
- Project to T_1^- irrep with the coefficients of the operators from **OpTion**
- Spectator momentum cutoff: $\vec{p}_{\max} = [0, 1, 1]$
- Two-body input cutoff: $\tilde{K}^{-1} \to (1 + e^{-(\sigma - \sigma_0)/M_\pi^2})\tilde{K}^{-1}, \sigma_0 = 2$
- Other $\vec{p}_{\max}$ and σ_0 are tested and have **no relevant effect** on the extracted observables

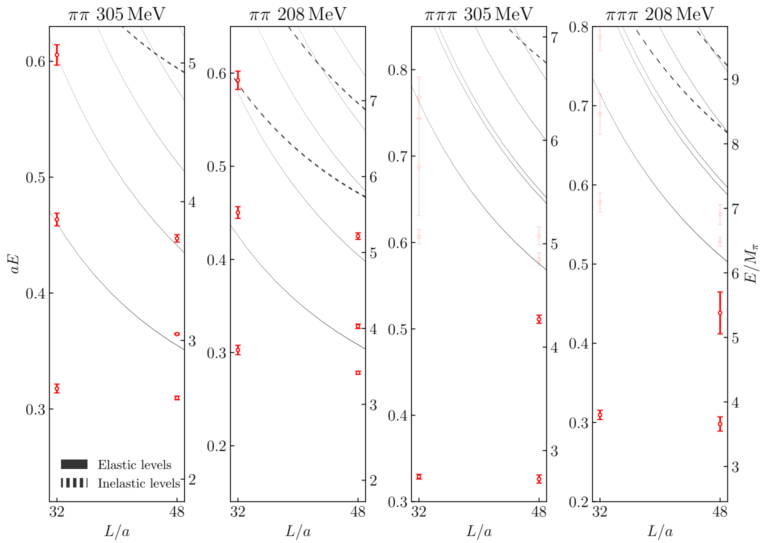
Combined fit for $\pi\pi$ and $\pi\pi\pi$ spectra

1. GEN
   -  $\sim a_0 + a_1\sigma + \cdots$
   -  $\sim \frac{c_0}{s - M_\omega^2} + c_1 + \cdots$
2. EFT[5]
   -  $\sim \frac{\sigma - M_\rho^2}{g^2}$
   -  $\sim \frac{s(M_\rho^2 - \sigma_q + 6g^2 f_\pi^2)(M_\rho^2 - \sigma_p + 6g^2 f_\pi^2)}{g^2 f_\pi^6 (s - M_\omega^2)}$

- Fit each M_π
  - GEN: $[a_0, a_1, c_0, c_1]$
- Fit all M_π's
  - EFT2: $[g, \delta]$: $(M_\rho = \sqrt{2} g f_\pi,\ M_\omega = M_\rho + \delta)$
  - **EFT4** (main): $[g, \delta, M_V, a]$: $(M_\rho = M_V + a\, M_\pi^2,\ M_\omega = M_\rho + \delta)$

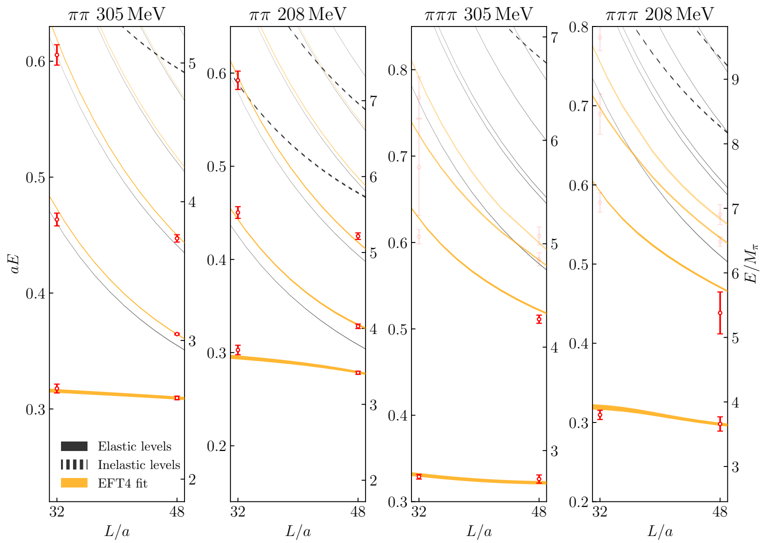
[5] Meißner (1988).

Finite-volume spectra revisited



- $\chi^2_{\mathrm{dof}}(\mathrm{EFT4}) = 2.3$
- Continuous spectra from FVU
- High-lying energies above the cutoff are also well-predicted
- EFT4 could be improved by including further chiral corrections
- Exponential effects from F32P21 are tested
- High-lying energies are tested

Finite-volume spectra revisited



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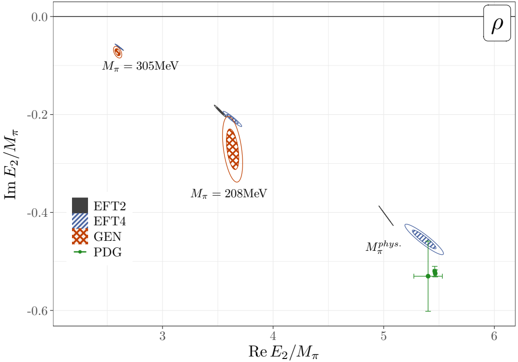
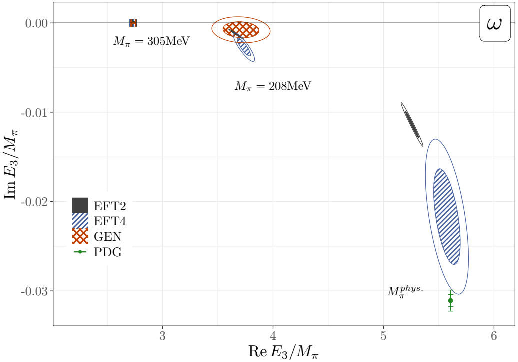
Pole positions





- Solve the integral equation [Mai and Döring, 2017] $T = B + C + \int \frac{d^3 l}{(2\pi)^3} \frac{B+C}{2E_l(\tilde{K}^{-1} - \Sigma^{IV})} T$
- ω is indeed a **bound state** at $M_\pi \approx 305\,\mathrm{MeV}$ and a **resonance** at $M_\pi \approx 208\,\mathrm{MeV}$
- 1σ agreement of $\mathrm{Re}\, M_\omega$ between all three methods
- Extrapolate to the **physical pion mass**, the poles agree astonishingly well with the PDG values

Summary

- First-ever determination of the ω-meson pole from lattice QCD
- Development of the FVU, matching EFT and FVU
- Paved the way to study heavier three-hadron resonances
- The lattice artifacts are to be investigated
- The ρ and ω pole positions at the physical point

$$\sqrt{s_\rho} = (748.9(10.0) - i63.5(1.8))\,\text{MeV}$$
$$\sqrt{s_\omega} = (778.0(11.2) - i3.0(5))\quad\text{MeV}$$

Outlook

For three-body, the following is planned with CLQCD ensembles

- $\pi\pi\pi$ with $I = 1, 2, 3$
- $KK\pi, K\pi\pi$
- Not a challenge: $T_{cc} \to DD\pi$
- Challenge: Roper $\to N\pi, N\pi\pi$

Apart from spectroscopy, there are millions we, you and me, can do...

- **Collaborations are very welcomed**

Thank you!

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ω also thanks you!

$(\geqq \omega \leqq)$