Heavy Flavors at Finite Temperature

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第四届中国格点量子色动力学研讨会

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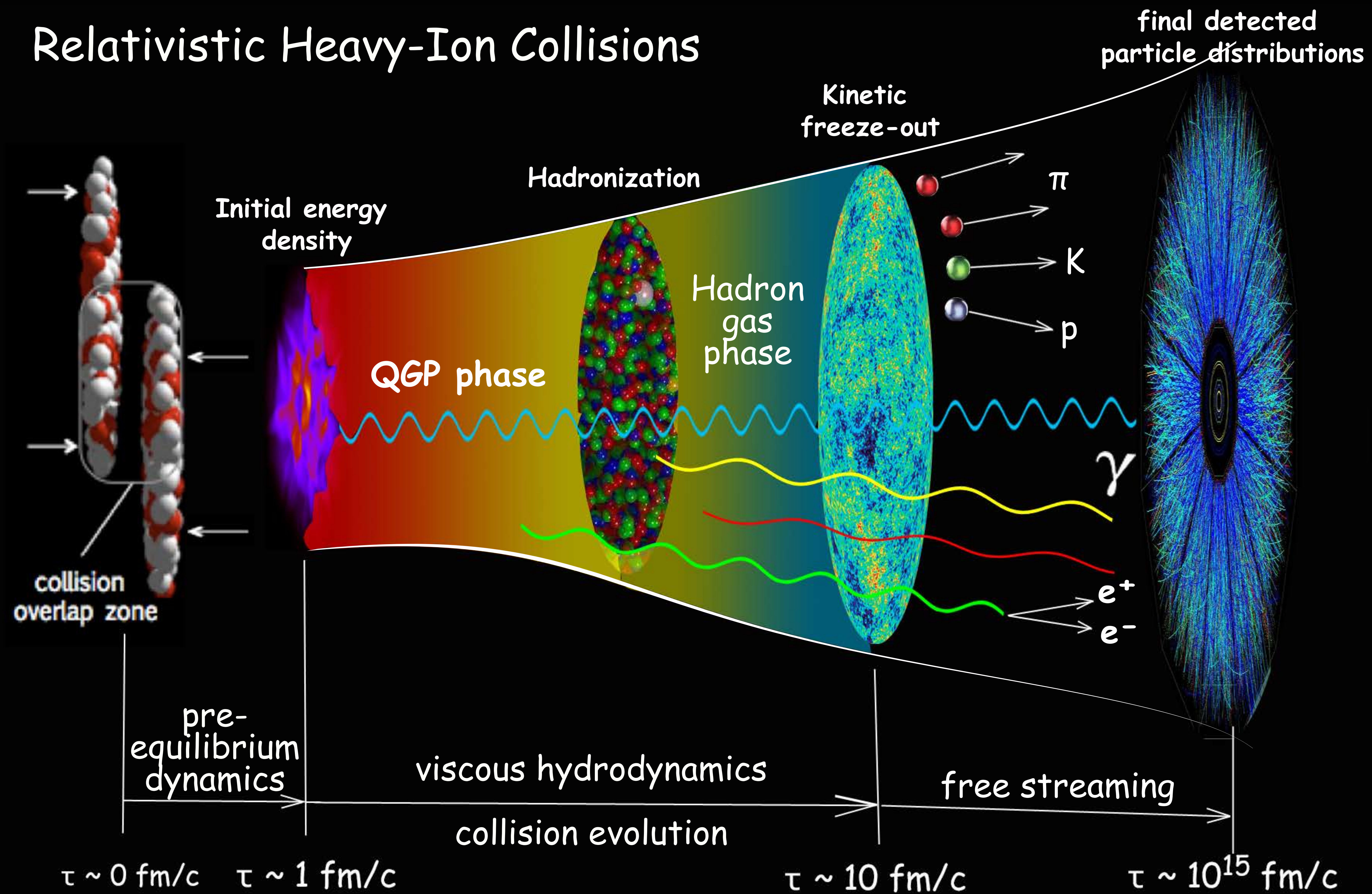
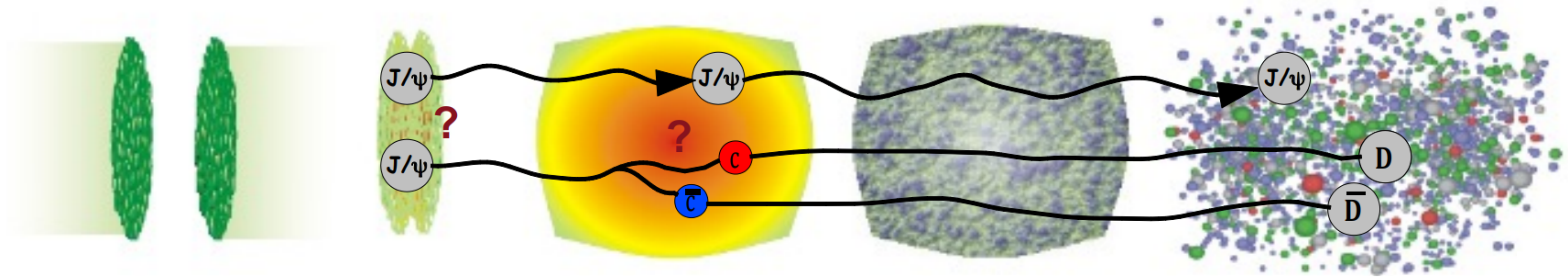
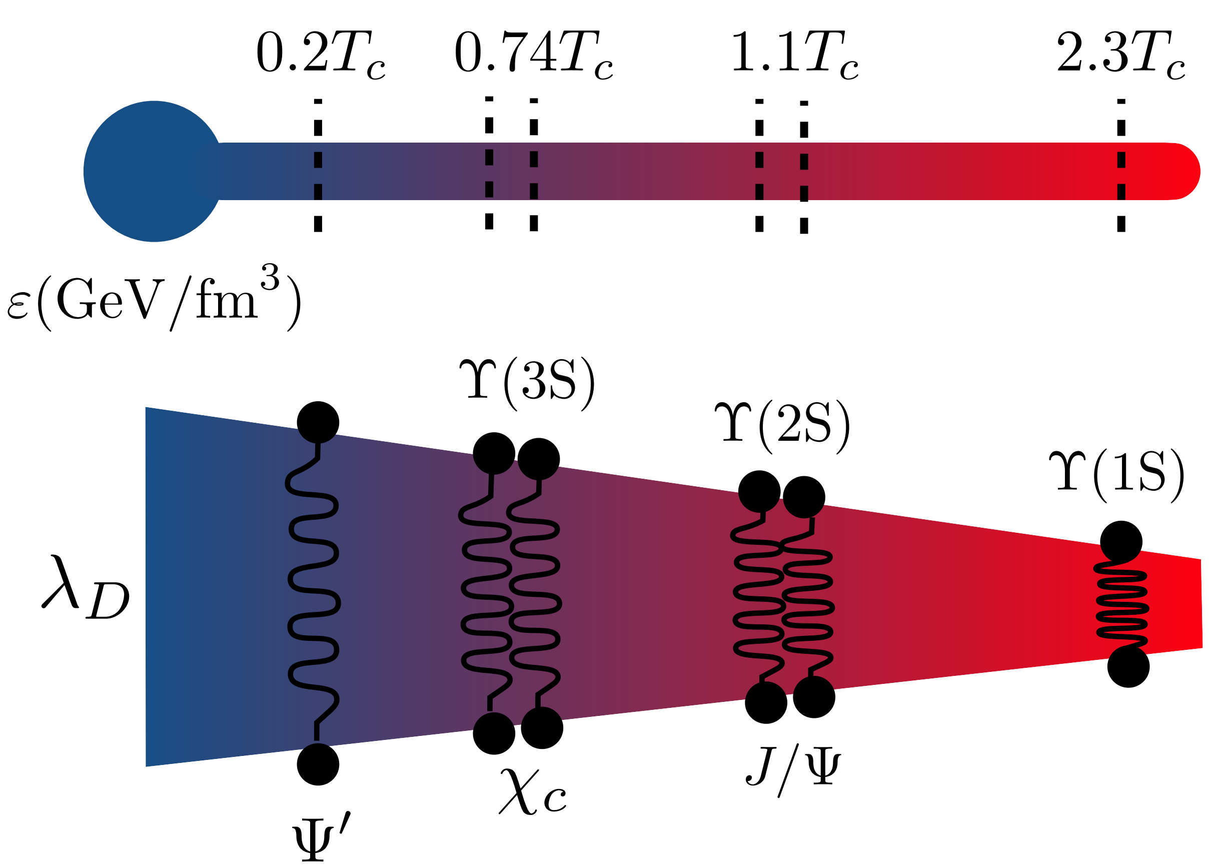
Heavy-ion collisions at LHC and RHIC



Heavy flavor probes



Probe the hot medium with different length scales



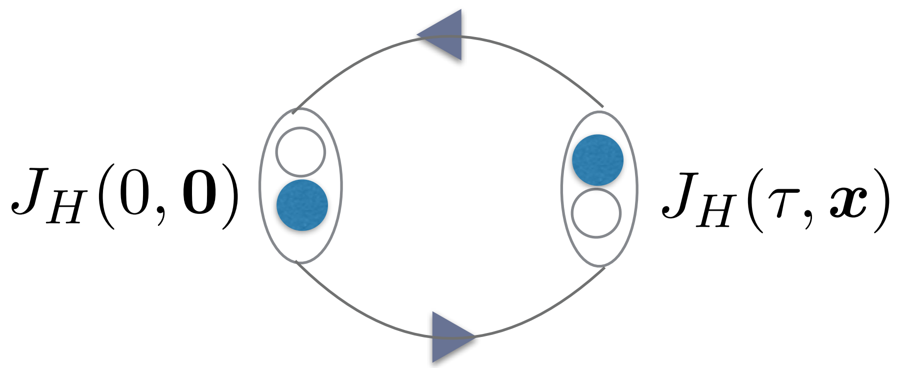
2015 Long Run Plan Nuclear Science

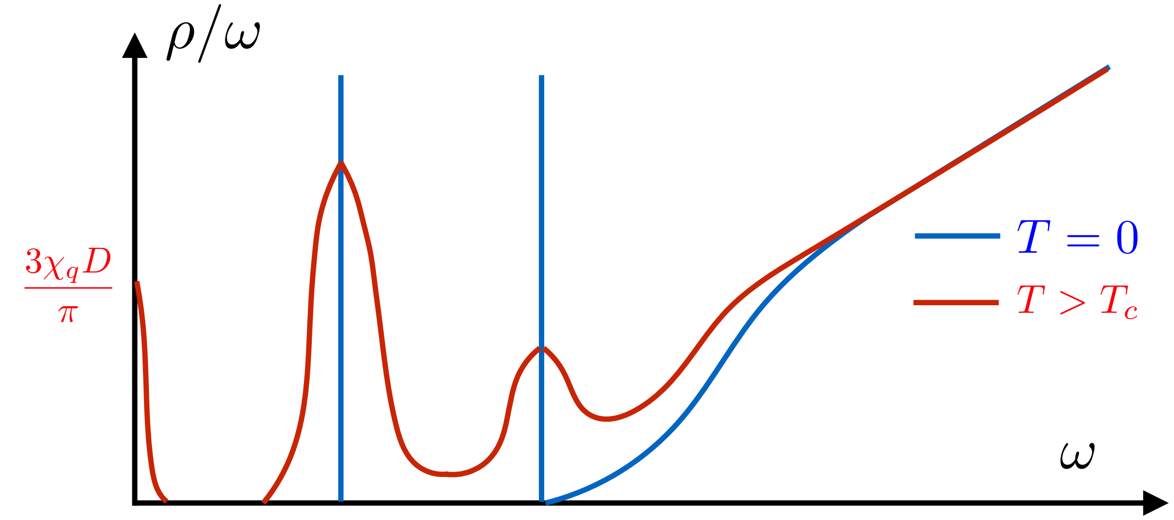
Where Lattice can help …

- In-medium quarkonium properties: masses, widths, melting T
- Complex quark-antiquark potential: $\mathrm{Re}[V]$, $\mathrm{Im}[V]$
- Heavy quark diffusion: D_s

From Imag.-time lattice to real-time physics

An example: meson spectral function





$$G_H(\tau,\vec{p}) = \sum_{x,y,z} exp(-i\,\vec{p}\cdot\vec{x})\langle J_H(0,\vec{0})J_H^{\dagger}(\tau,\vec{x})\rangle = \int \frac{d\omega}{2\pi}\frac{\cosh(\omega(\tau T-1/2)/T)}{\sinh(\omega/2T)}\rho_H(\omega,\vec{p},T)$$

$K(\omega,\tau,T)$

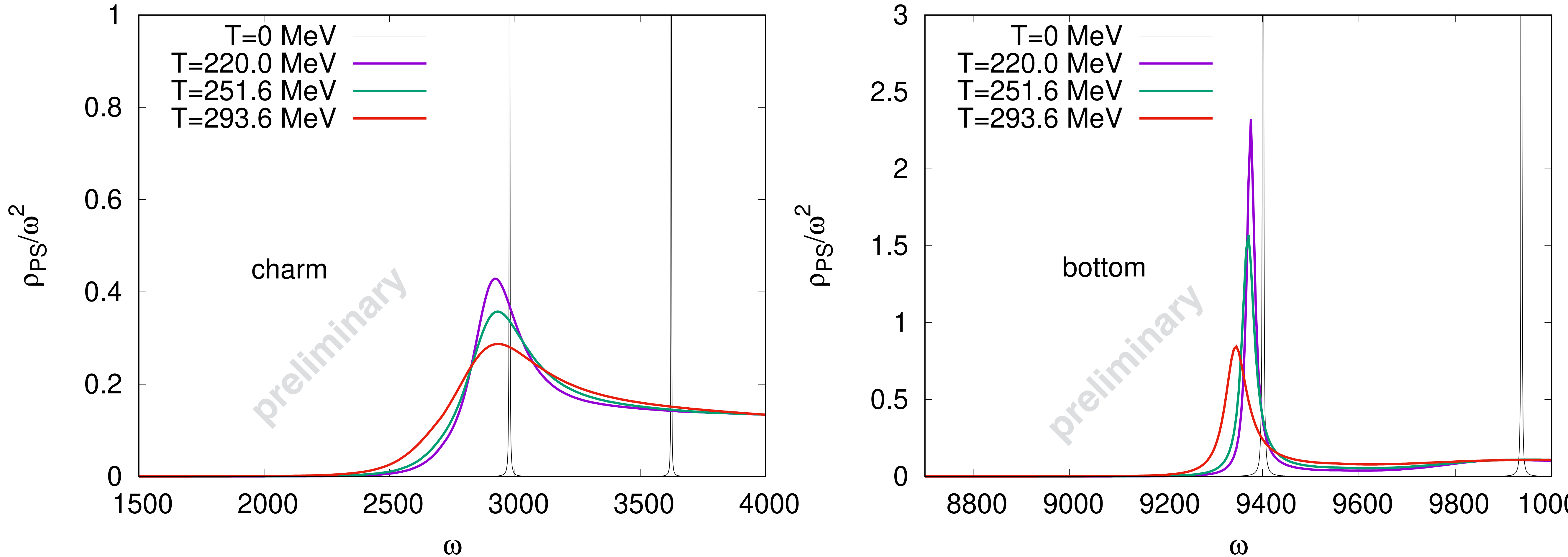
- Meson spectral function tells melting temperature and heavy quark diffusion

Quarkonium spectral function (relativistic HQ)

Need very fine and large lattice for heavy quark $T = 1/(aN_\tau)$

First full QCD calculation with relativistic heavy quarks

[hotQCD, Few Body Syst. 64(2023) 3, 52]
[S. Ali, D. Bala, et al., in prep.]



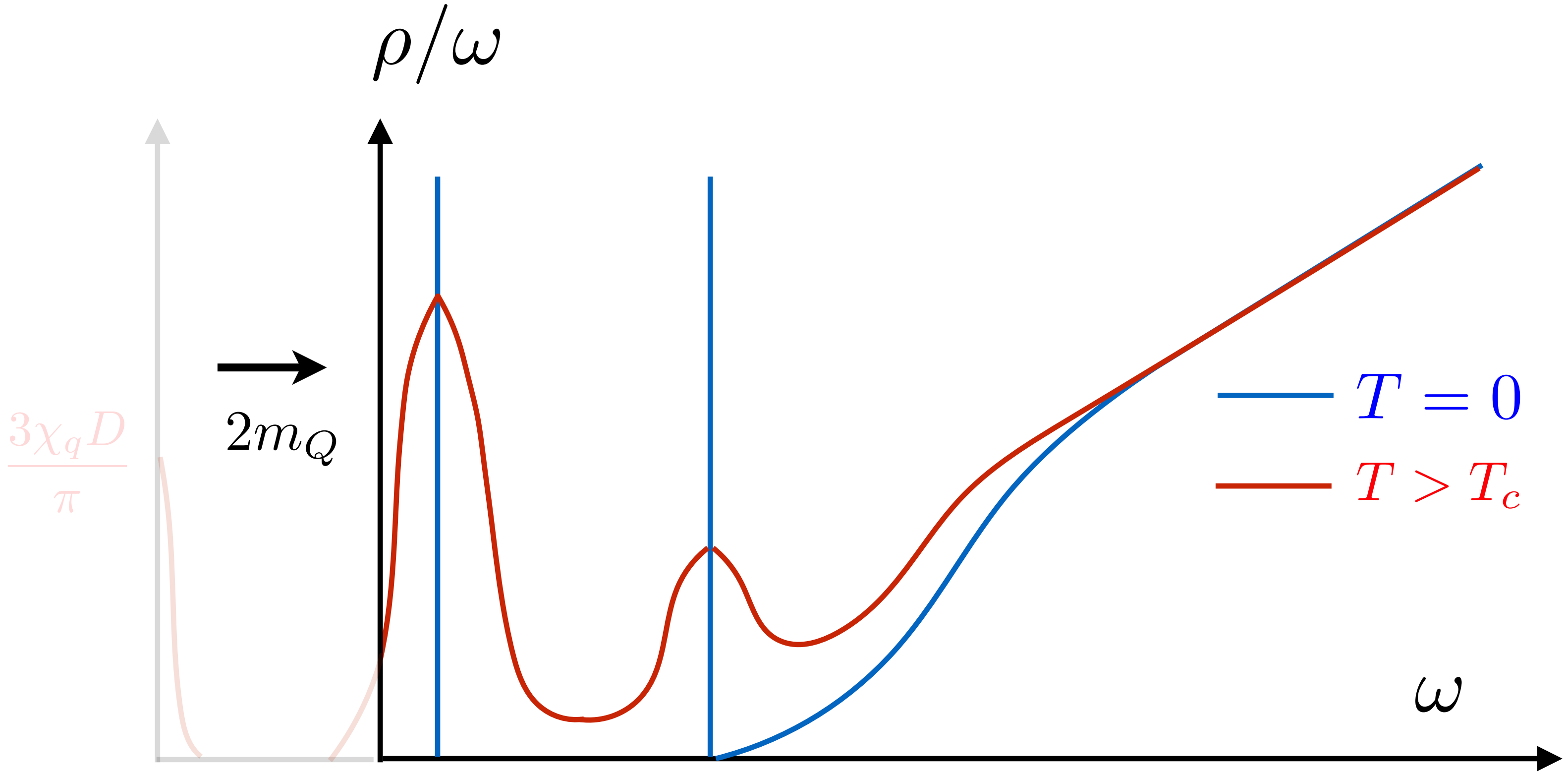
- Thermal broadening for both charm and bottom
- More broadening for charm than bottom
- Determination of dissociation temperature needs more investigation

Non-relativistic heavy quark

NRQCD becomes possible due to scale separation: $m_Q \gg m_Q v \gg m_Q v^2$

~~m_Q : hard scale, quark creation and annihilation~~
$m_Q v$: soft scale, momentum exchange between $Q\bar{Q}$
$m_Q v^2$: ultrasoft scale, binding/melting of $Q\bar{Q}$

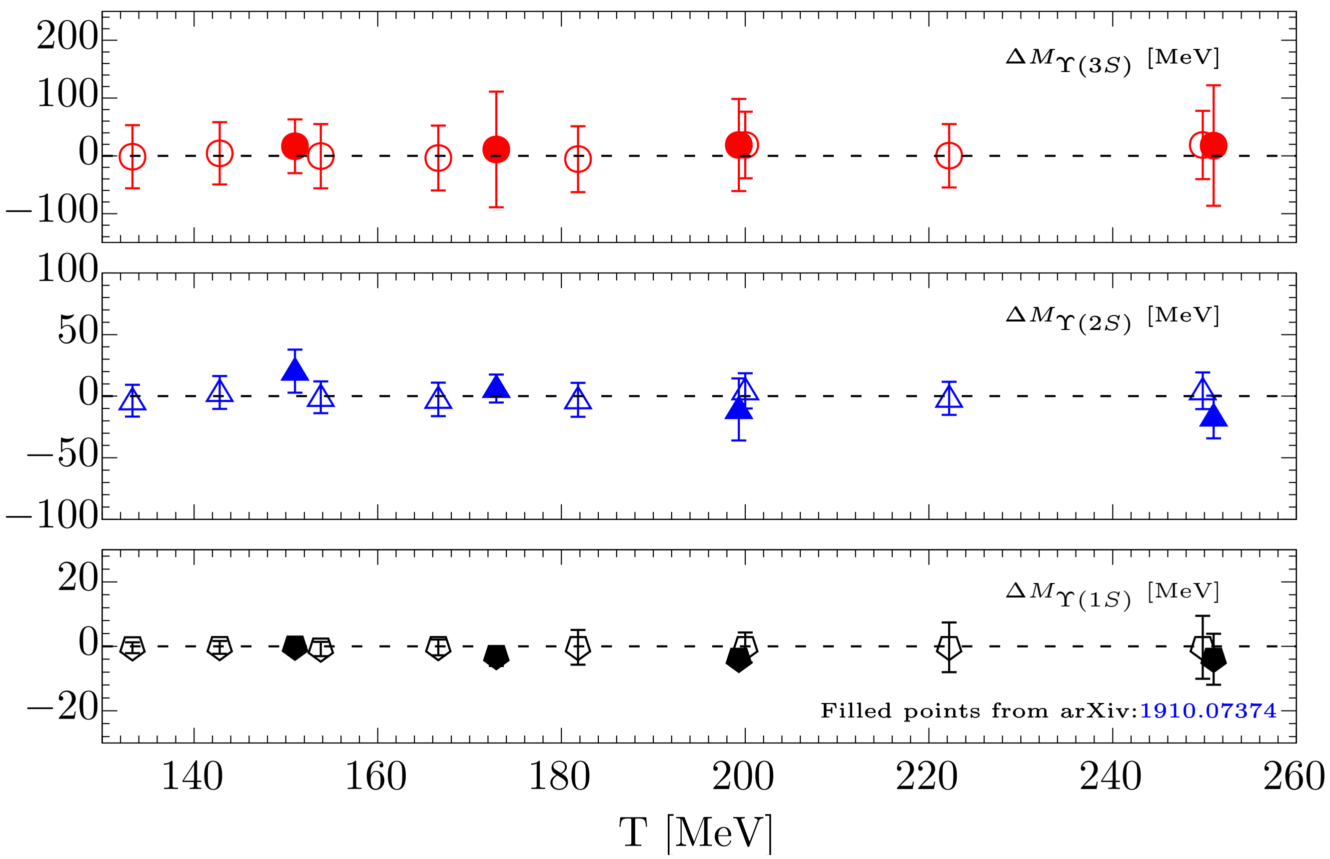
G. Aarts, et al., JHEP 07 (2014) 097
S. Kim, et al., JHEP 11 (2018) 088
R. Larsen, et al., PRD 100 (2019) 7, 074506
R. Larsen, et al., PLB 800 (2020) 135119

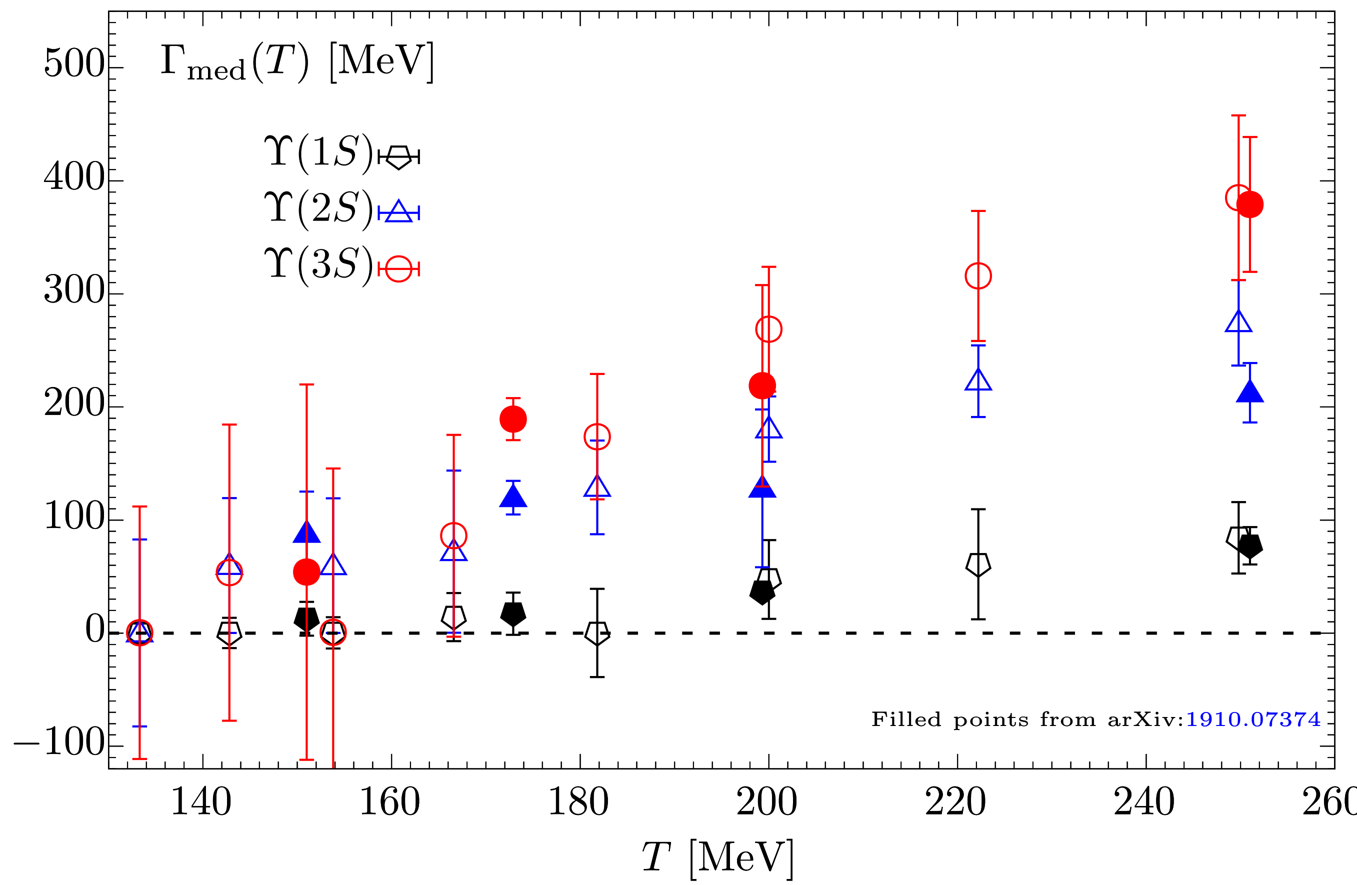


Quarkonium correlators can be computed with better resolution!

Mass shift & thermal width of bottomonium

Model spectra: $\rho_\alpha^{\mathrm{med}}(\omega,T) = A_\alpha^{\mathrm{cut}}(T)\,\delta\left(\omega-\omega_\alpha^{\mathrm{cut}}(T)\right) + A_\alpha(T)\exp\left(-\frac{[\omega - M_\alpha(T)]^2}{2{\Gamma_\alpha}^2(T)}\right)$





Filled points

a [fm]	0.1088	0.0951	0.0825	0.0655	0.0493
T [MeV]	151	173	199	251	341

→

Open points

a [fm]	0.0493	0.0493	0.0493	0.0493	0.0493
T [MeV]	151	173	199	251	341

R. Larsen, et al., PLB 800 (2020) 135119

Wei-Ping Huang's poster

- No mass shift for all states
- Increasing thermal width with temperature for all states
- Thermal broadening follows the hierarchical increasing pattern

Static quark potential

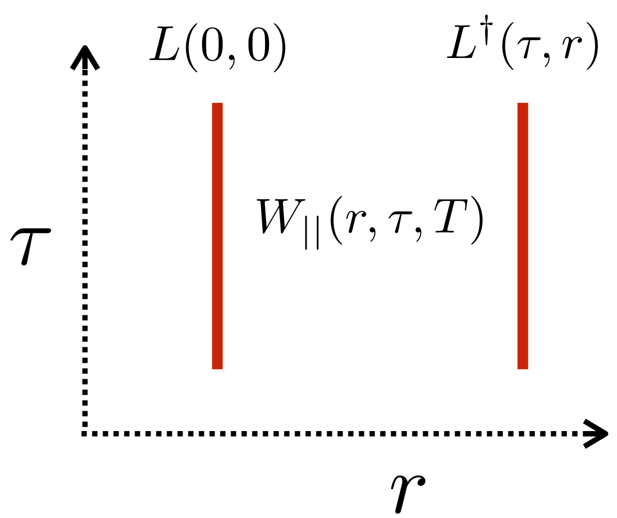
- Hard Thermal Loop resummed perturbation theory M. Laine, JHEP0703,054 (2007)

$$\lim_{t\to\infty} V_{>}^{(2)}(t,r) = -\frac{g^2 C_F}{4\pi}\left[m_\mathrm{D} + \frac{\exp(-m_\mathrm{D} r)}{r}\right] - \frac{i g^2 T C_F}{4\pi}\phi(m_\mathrm{D} r)$$

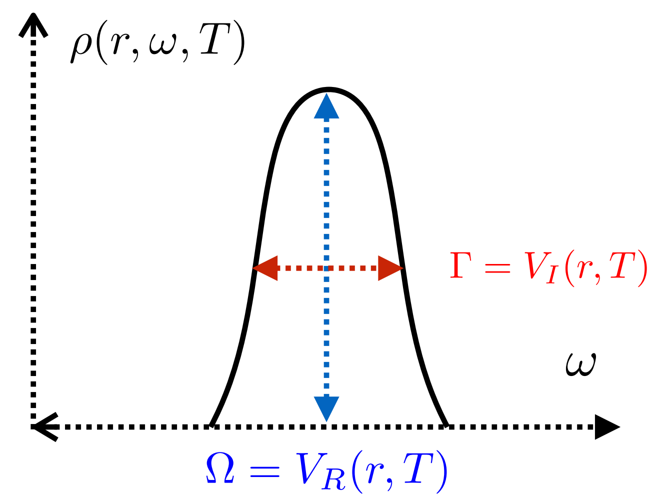
Imaginary part becomes important for physical bottomonium at T>250 MeV!

- Non-perturbative determination matters at around and above Tc

Wilson loop/thermal Wilson line correlators in Coulomb gauge A. Rothkopf et al., PRL. 108 (2012) 162001



$$W_{||}(r,\tau,T) = \int d\omega \rho(r,\omega,T) e^{-\omega\tau}$$



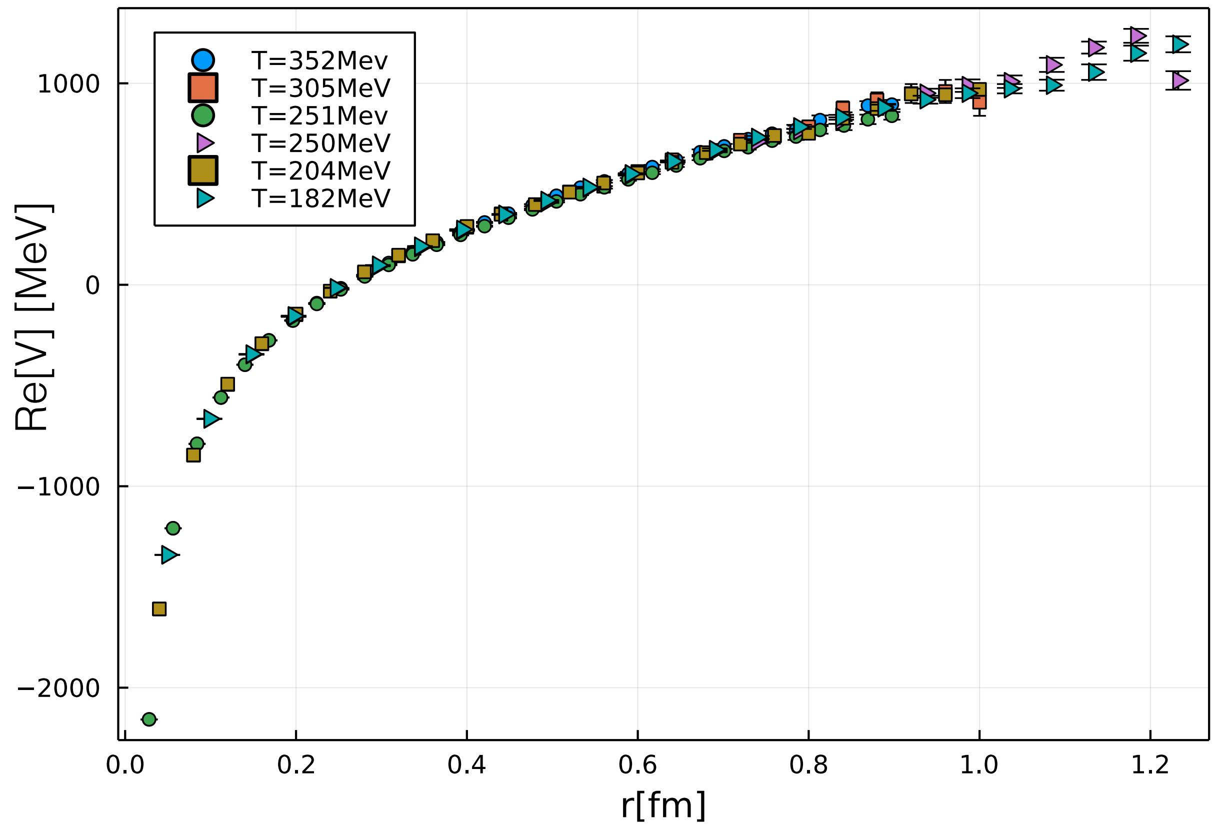
- Subtract continuum contribution from zero temperature correlators
- Model the potential as arguments of spectral function

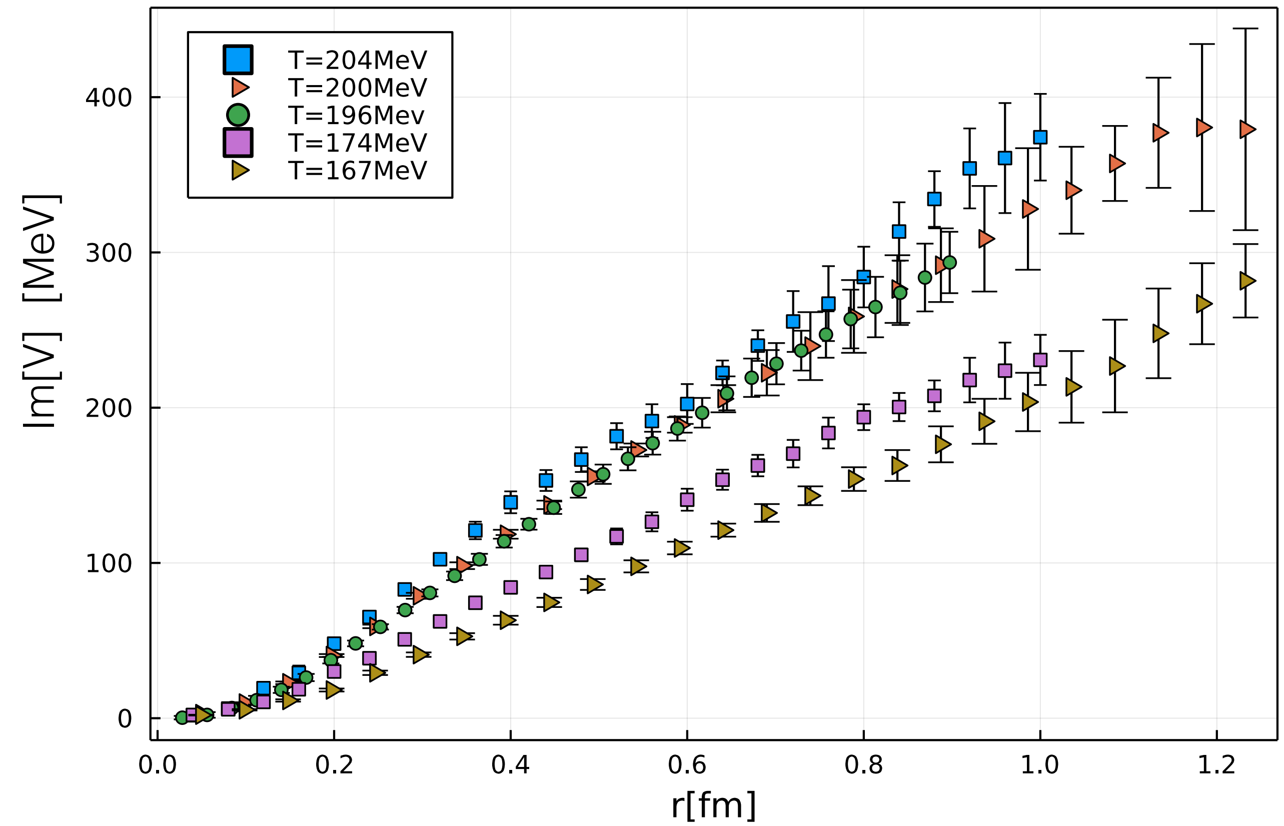
Static quark potential in hot QCD

$$N_f = 2+1, m_\pi = 160\ \mathrm{MeV}, a \to 0$$

Real part: temperature insensitive

Imag. part: increasing with T & r





[HotQCD, PRD 109 (2024) 7, 074504]

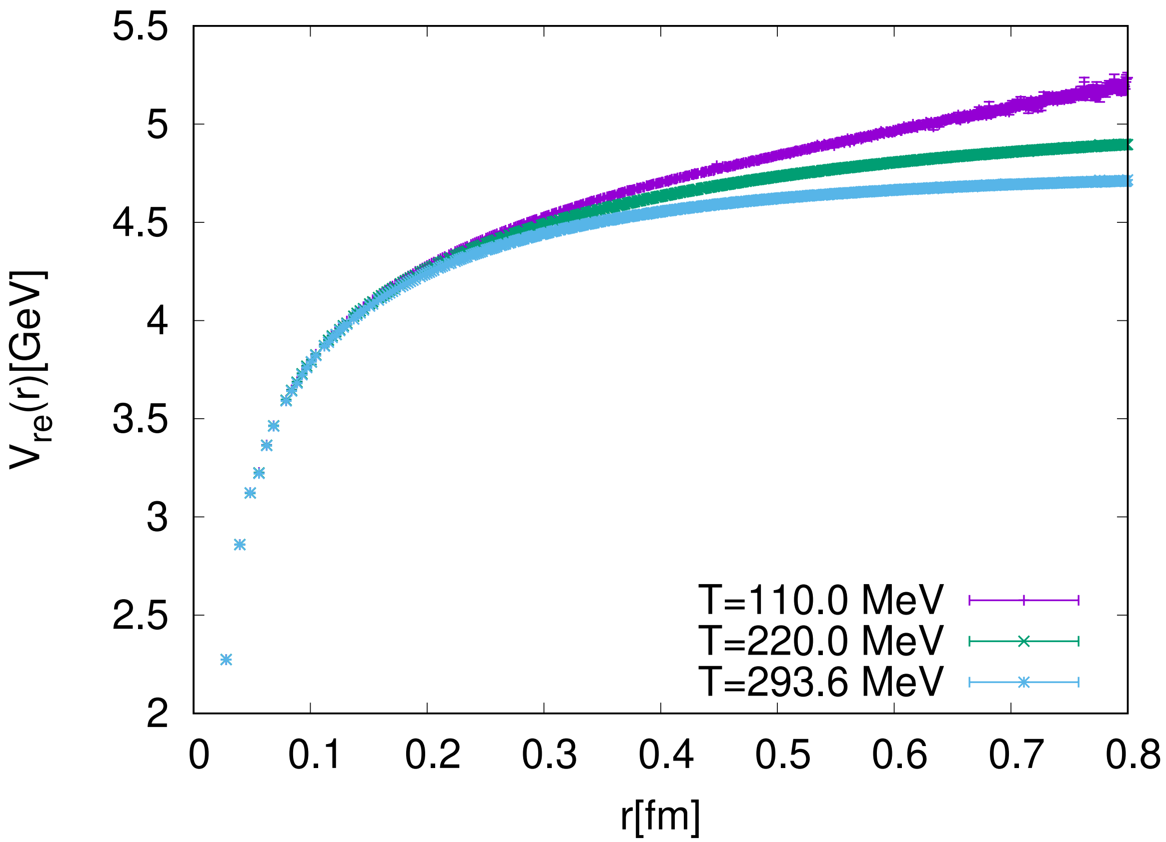
$$\rho(r,\omega) = \frac{1}{\pi}\mathrm{Im}\frac{A(T)}{\omega - \mathrm{Re}V(r,T) - i\Gamma(\omega,r,T)}$$

- No color screening from cut-Lorentzian model SPF (no mass shift for bottomonium)

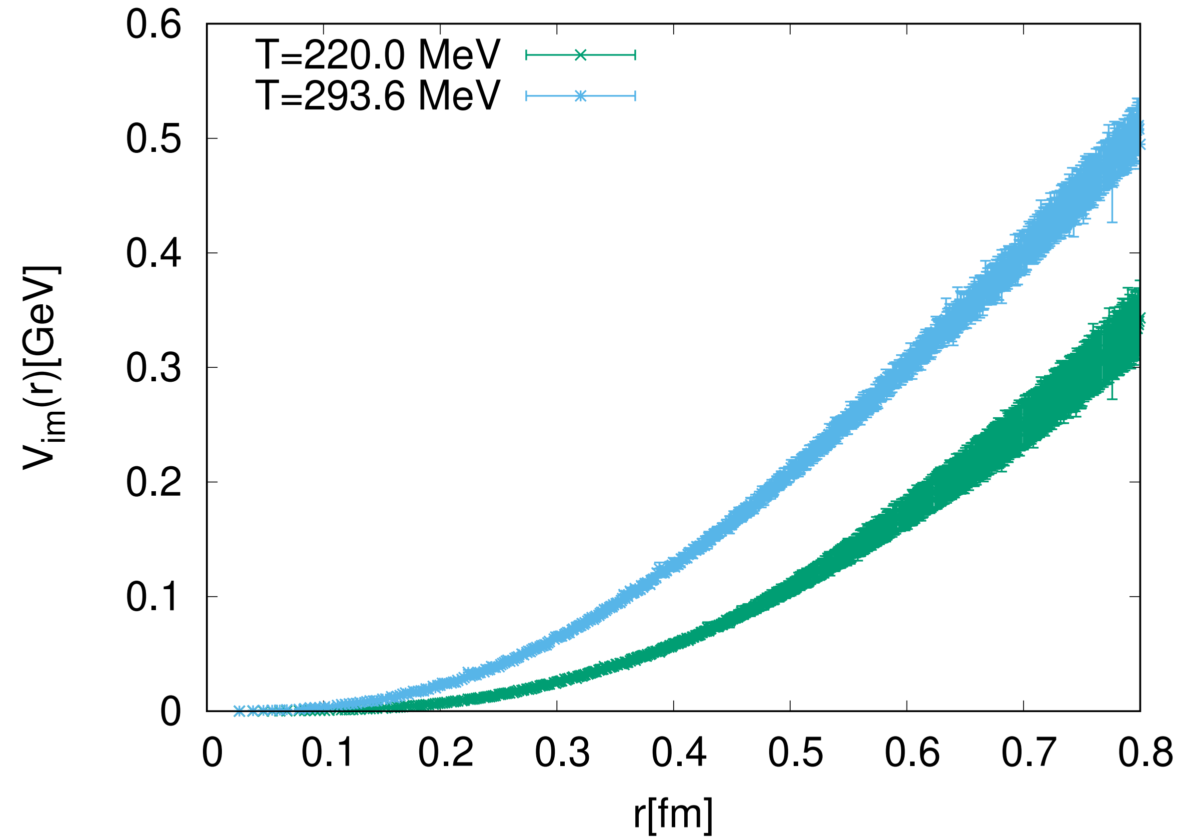
Static quark potential in hot QCD

$$N_f = 2+1, m_\pi = 160\ \mathrm{MeV}, a \to 0$$

Real part: temperature sensitive



Imag. part: increasing with T & r



[Dibyendu Bala, et al, in prep.]

$$\rho(r,\omega) = n_b(\omega)\left(c_{-1}/\omega + \sum_{l=0}^{\infty} c_{2l+1}\omega^{2l+1}\right)$$

- Color screening evident from HTL-inspired model SPF

Heavy quark diffusion via HQEFT

- Langevin equations of heavy quark motion

$$\partial_t p_i = -\eta_D p_i + \xi_i(t) \qquad \langle \xi_i(t)\xi_j(t')\rangle = \kappa \delta_{ij}\delta(t-t')$$

- Mass dependent **momentum** diffusion coefficient

$$\kappa^{(M)} \equiv \frac{M^2\omega^2}{3T\chi_q}\sum_i \frac{2T\rho_V^{ii}(\omega)}{\omega}\Big|^{\eta \ll |\omega| \lesssim \omega_{\rm UV}}$$

- Large quark mass limit in HQ effective field theory

$$\kappa \equiv \frac{\beta}{3}\sum_{i=1}^{3}\lim_{\omega\to 0}\left[\lim_{M\to\infty}\frac{M^2}{\chi_q}\int_{-\infty}^{\infty} {\rm d}t\, e^{i\omega(t-t')}\int {\rm d}^3\vec{x}\,\left\langle \frac{1}{2}\{\mathcal{F}^i(t,\vec{x}),\mathcal{F}^i(0,\vec{0})\}\right\rangle\right]$$

J. Casalderrey-Solana and D. Teaney, PRD 74, 085012

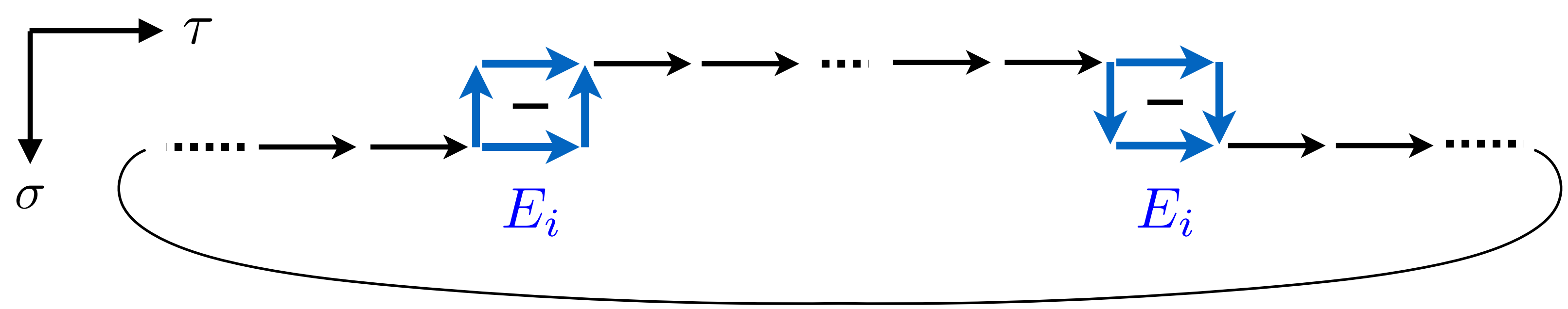
S. Caron-Huot et al., JHEP 0904 (2009) 053

A. Bouttefeux, M. Laine, JHEP 12 (2020) 150

$$\partial_t \mathbf{p} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) = \mathbf{F}$$

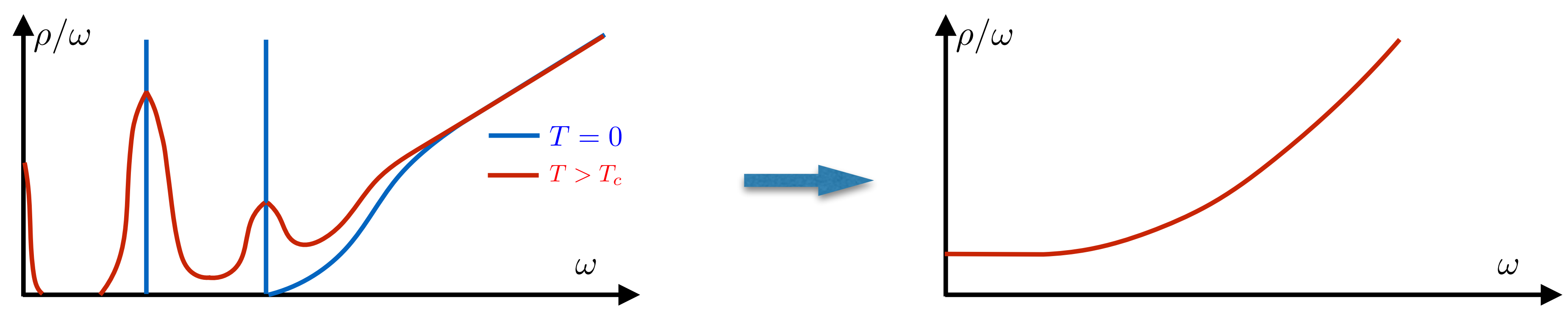
Momentum diffusion on the lattice

$$\kappa = \kappa_E + \frac{2}{3}\langle \mathbf{v^2}\rangle \kappa_{\mathbf{B}} \qquad \langle \mathbf{v}^2 \rangle = \frac{3T}{M} \qquad \frac{1}{2\pi T D} = \frac{\kappa}{4\pi T^3} = \frac{1}{2\pi T^2} \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$



Color-electric field correlation function

$$G(\tau, T) = \int \frac{d\omega}{\pi} K(\omega, \tau, T) \rho(\omega, T)$$

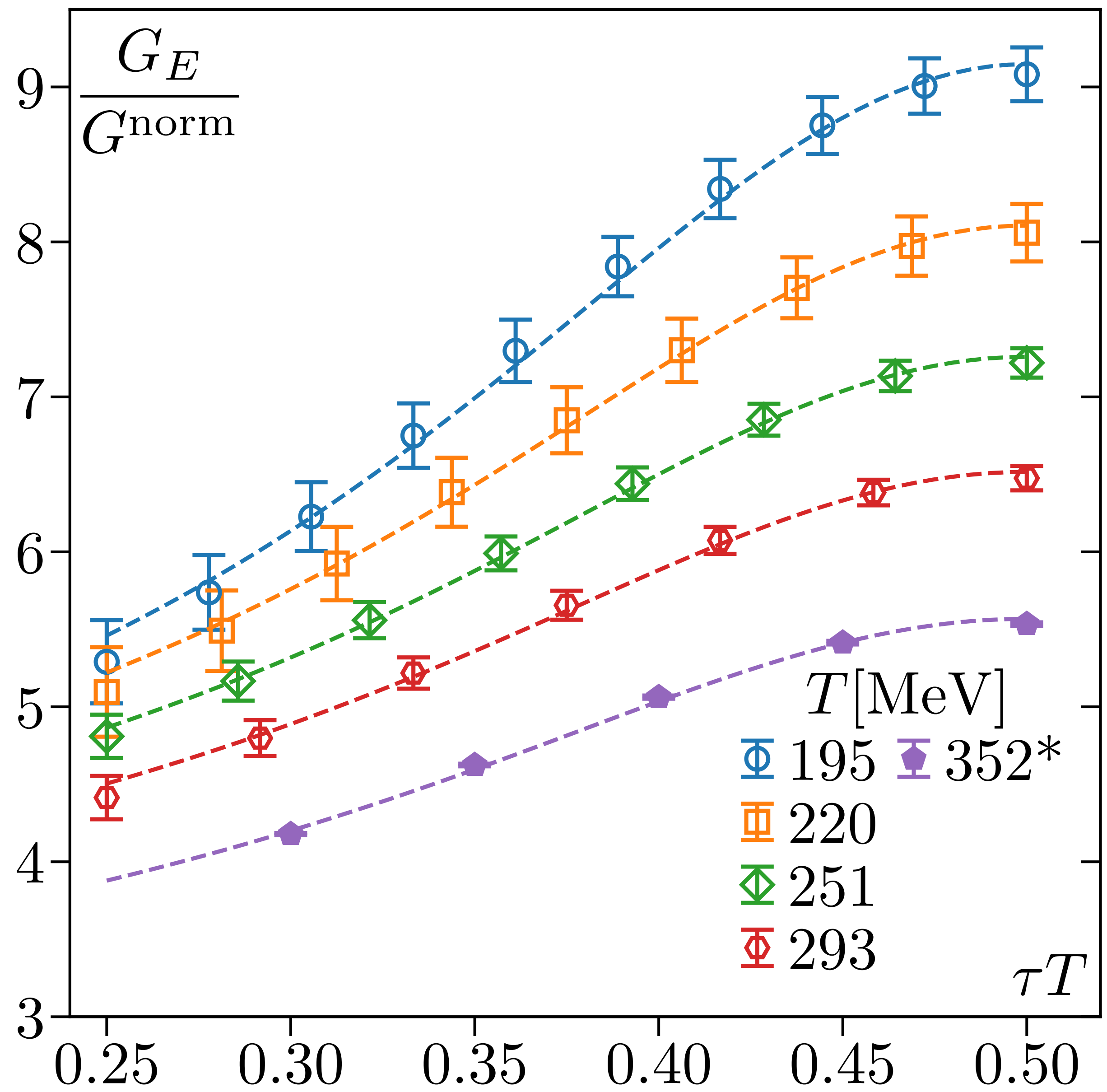


- Cheaper to measure on the lattice
- No peak structures in spectral functions
- Absence of transport peak

First full QCD calculation of EE correlators

First full QCD calculation of κ, only possible via Gradient Flow!

Luscher & Weisz, JHEP1102(2011)051
Narayanan & Neuberger, JHEP0603(2006)064

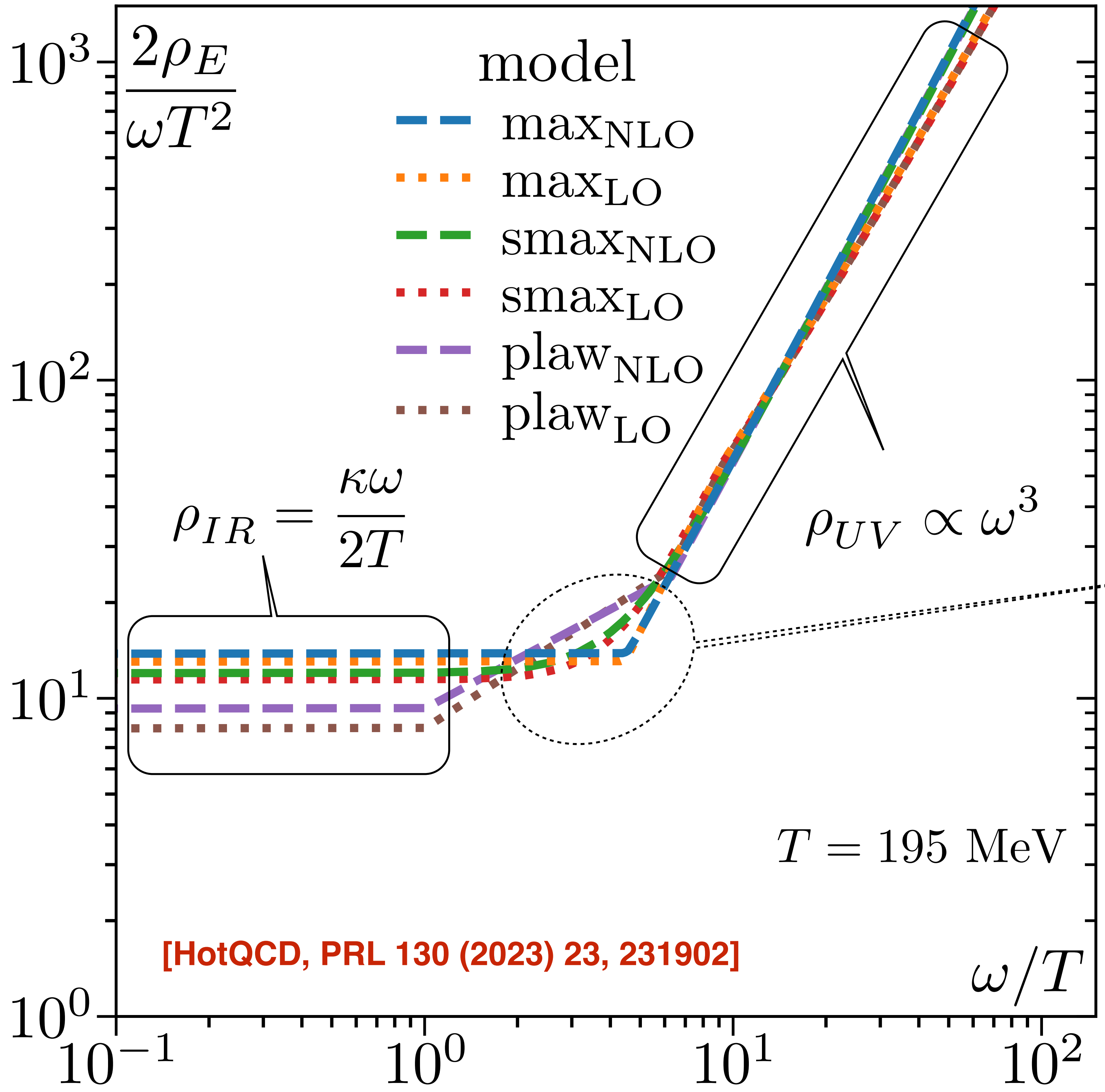


[HotQCD, PRL 130 (2023) 23, 231902]

- 195 MeV <= T <= 352 MeV
- 2+1 flavor in the sea
- pion mass 320 MeV

For application of gradient flow on gluon plasmas's viscosity, see Cheng Zhang's poster

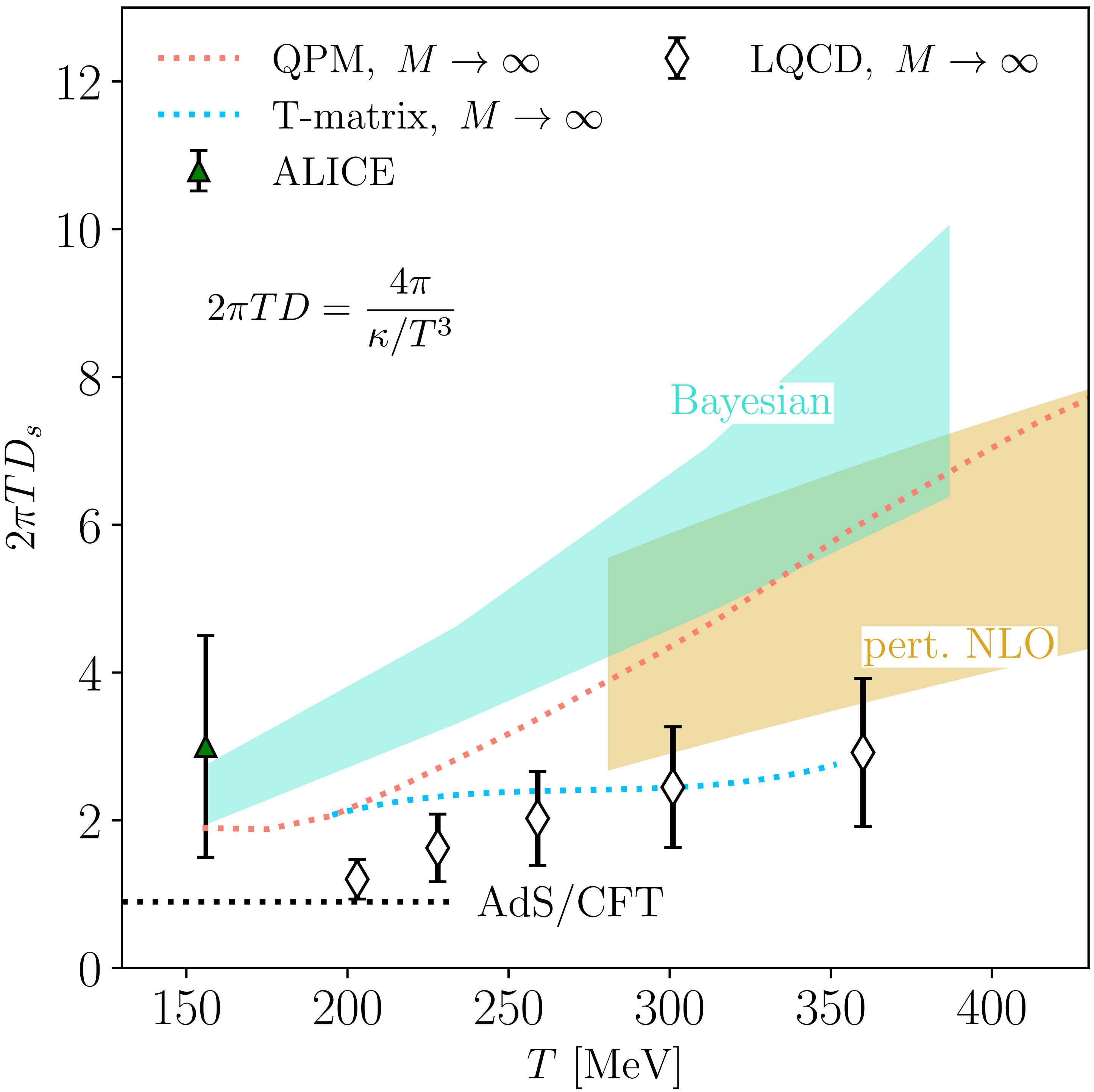
Spectra analysis



$$G(\tau, T) = \int \frac{d\omega}{\pi} K(\omega, \tau, T)\rho(\omega, T)$$

$$\begin{cases} \rho_{\max} \equiv \max(\phi_{IR}, \phi_{UV}) \\ \rho_{\text{smax}} \equiv \sqrt{\phi_{IR}^2 + \phi_{UV}^2} \\ \rho_{\text{plaw}} \equiv \begin{cases} \phi_{IR} & \omega \le \omega_{IR}, \\ a\omega^b \quad \text{for} & \omega_{IR} < \omega < \omega_{UV}, \\ \phi_{UV} & \omega \ge \omega_{UV}, \end{cases} \end{cases}$$

HQ diffusion coefficient at HQ mass limit



[HotQCD, PRL 130 (2023) 23, 231902]

- Agree with AdS/CFT at $\sim T_c$
- Agree with T-matrix estimate at moderate and high temperature
- Agree with NLO perturbative estimate at high temperature
- Mild temperature dependence
- Rapid equilibrium <—> QGP is near perfect fluid

$$\tau_{\text{kin}} = \frac{1}{\eta_D} = \frac{1}{\kappa/T^3}\left(\frac{T_c}{T}\right)^2\left(\frac{M}{1.5\ \text{GeV}}\right)\frac{3\ \text{GeV}}{T_c^2}$$

Finite mass correction

Physical charm & bottom quark not infinitely heavy!

$$M_c : \sim 1.3 \text{ GeV}$$
$$M_b : \sim 4.5 \text{ GeV}$$
$$\kappa_E : \ M_Q \to \infty$$

D. Guazzini, et al., JHEP 10 (2007) 081

$$\langle \mathcal{F}(t')\mathcal{F}(t)\rangle = q^2 \left\{ \langle E_i(t')E_j(t)\rangle + \frac{1}{3}\langle \mathbf{v}^2\rangle \langle \delta_{ij} B_k(t')B_k(t) - B_j(t')B_i(t)\rangle \right\}$$

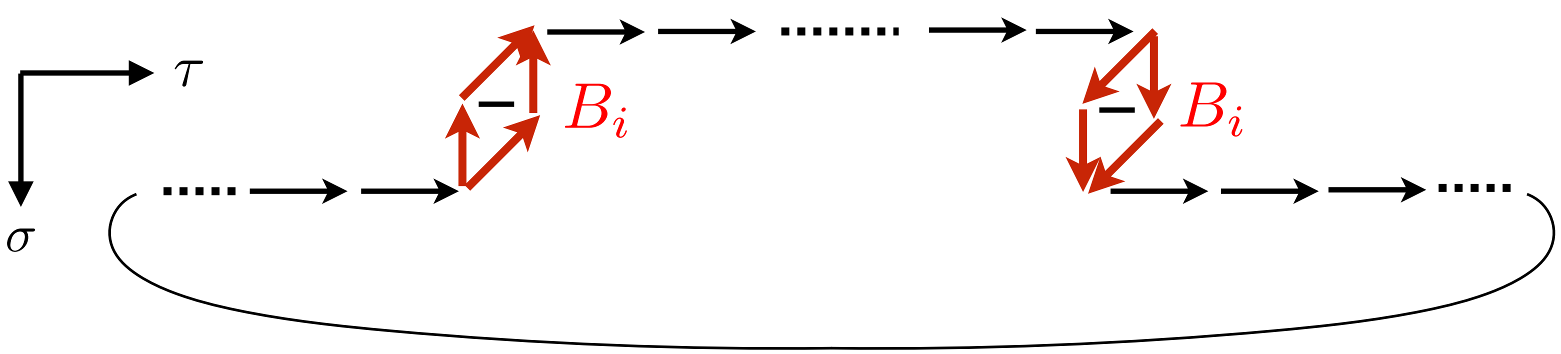
Infinite heavy limit

Finite mass correction

$$\kappa = \kappa_E + \frac{2}{3}\langle \mathbf{v^2}\rangle \kappa_{\mathbf{B}}$$

$$2/3 \cdot \langle \mathbf{v}^2\rangle_{\text{charm}} : \ 18\% \sim 30\%$$
$$2/3 \cdot \langle \mathbf{v}^2\rangle_{\text{bottom}} : \ 7\% \sim 13\%$$

$$D_s = \frac{2T^2}{\kappa} \Longrightarrow D_s = \frac{2T^2}{\kappa} \cdot \frac{\langle p^2\rangle}{3MT}$$



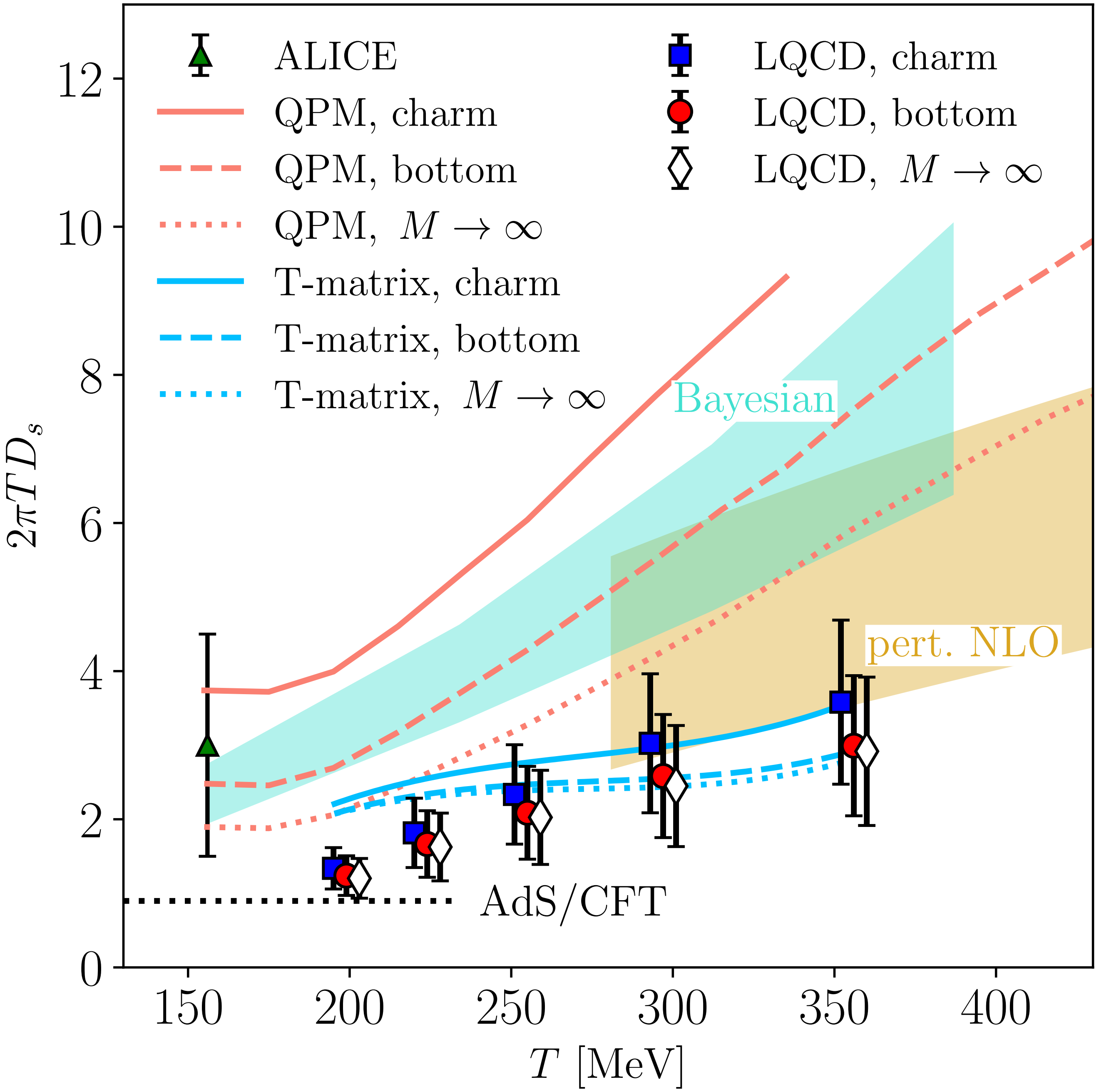
$$G_B^{\text{phys.}} = Z_{\text{match}} G_B^{\text{flow}}(\tau_{\text{F}})$$

Color-magnetic field correlation function

A. Bouttefeux, M. Laine, JHEP 12 (2020) 150

Charm and Bottom quark diffusion

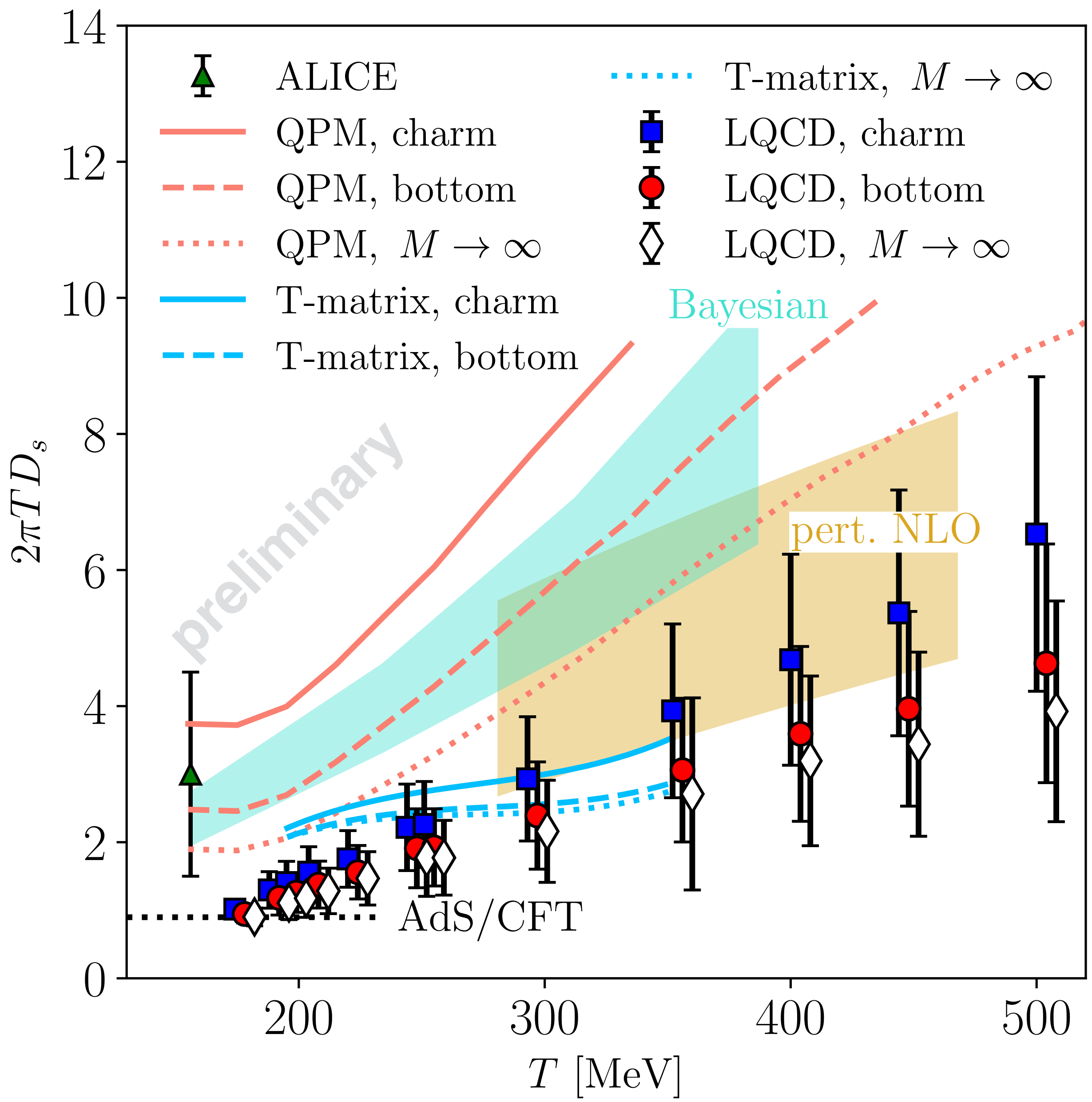
First full QCD determination of charm&bottom quark diffusion!



[HotQCD, PRL 132 (2024) 5, 051902]

- HQ mass dependence of HQ diffusion: mild
- Universal change pattern with quark mass
- Weak quark mass dependence in LQCD & T-matrix
- Weaker than quasi-particle model (QPM) calculations

HQ diffusion at the physical point



[J.D. Golan, Swagato Mukherjee, P. Petreczky, HTS, J.H. Webber, work in progress]

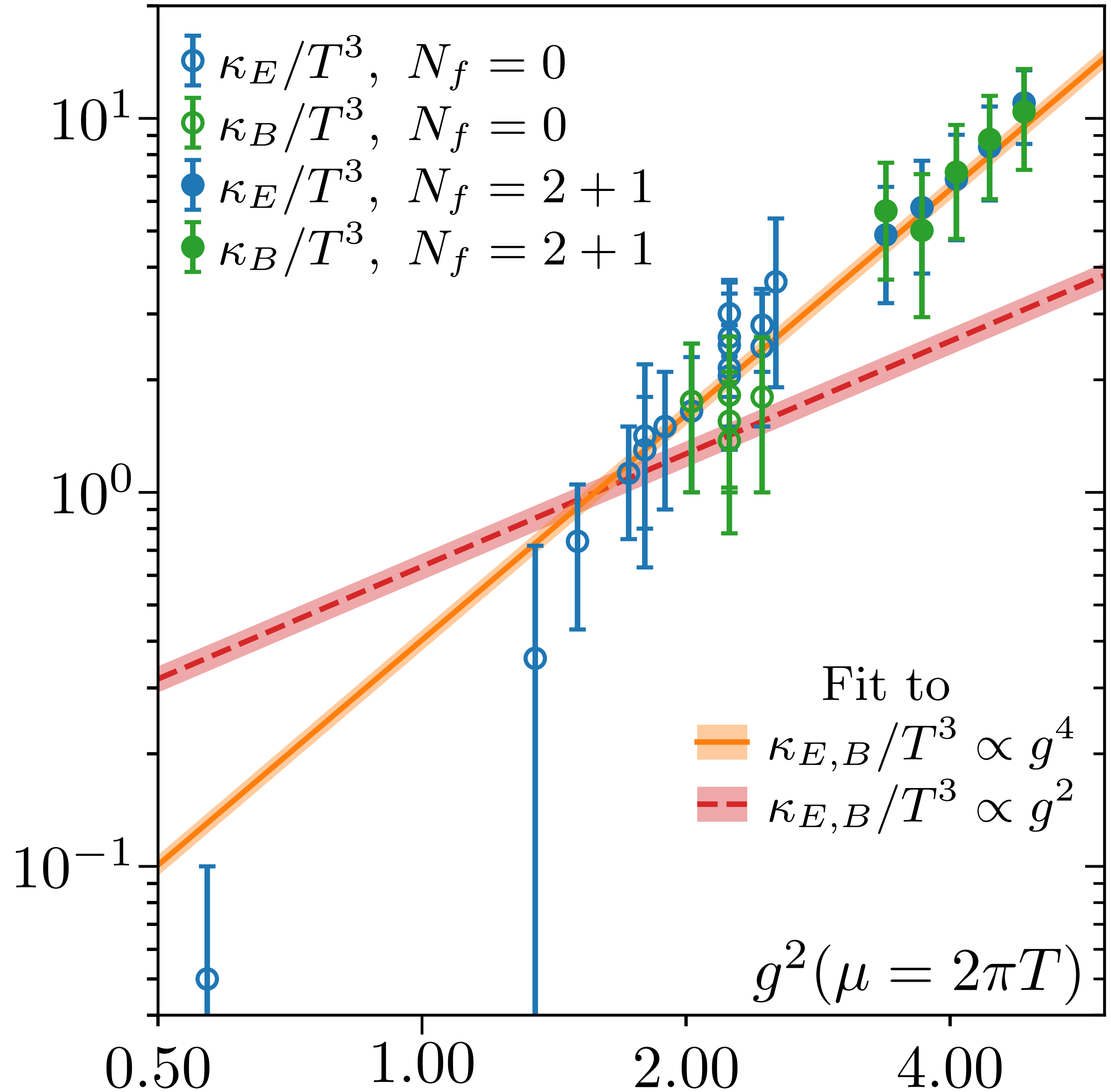
$195 \text{ MeV} \le T \le 352 \text{ MeV}$ $m_\pi \sim 320\text{MeV}$

$174 \text{ MeV} \le T \le 500 \text{ MeV}$ $m_\pi \sim 160\text{MeV}$

Increasing stat. for $T = 153 \text{ MeV}, 164 \text{ MeV}$

- Physical pion mass
- Wide temperature range down to 174 MeV and up to 500 MeV
- Consistent observation as previous studies
- Almost invisible light quark mass dependence

κ_E V.S. κ_B



[L. Altenkort, D. Cruz, O. Kaczmarek, GD. Moore, HTS, PRD 109 (2024) 11, 114505]

Quenched results from:

A. Francis, et al., PRD92, 116003

B. L. Altenkort, et al., PRD103,014511

D. Banerjee, et al., Nucl.Phys.A.2023.122721

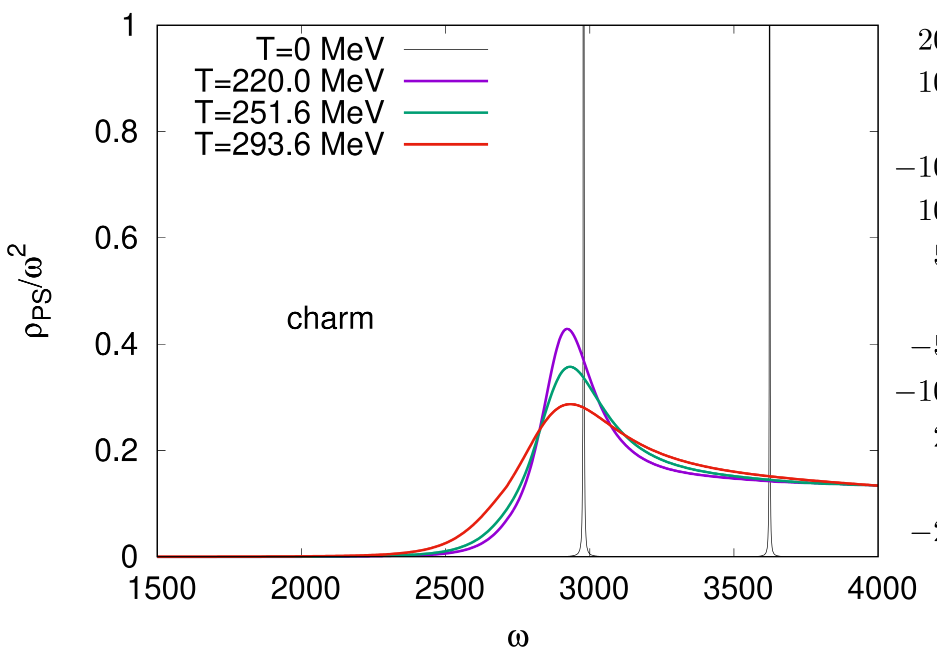
D. Banerjee, et al., JHEP 08 (2022) 128

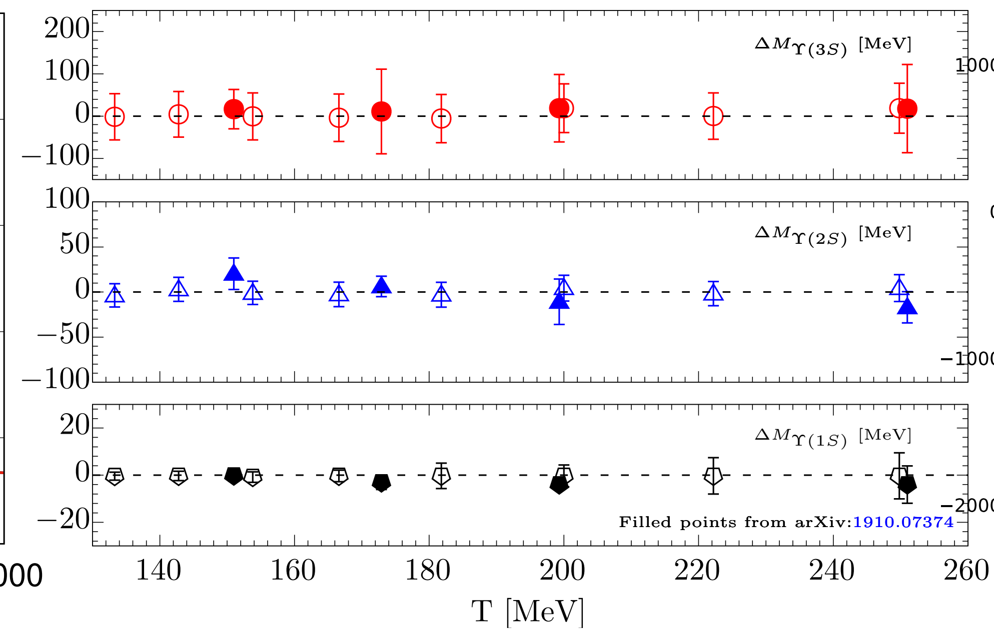
N. Brambilla, et al., PRD107, 054508

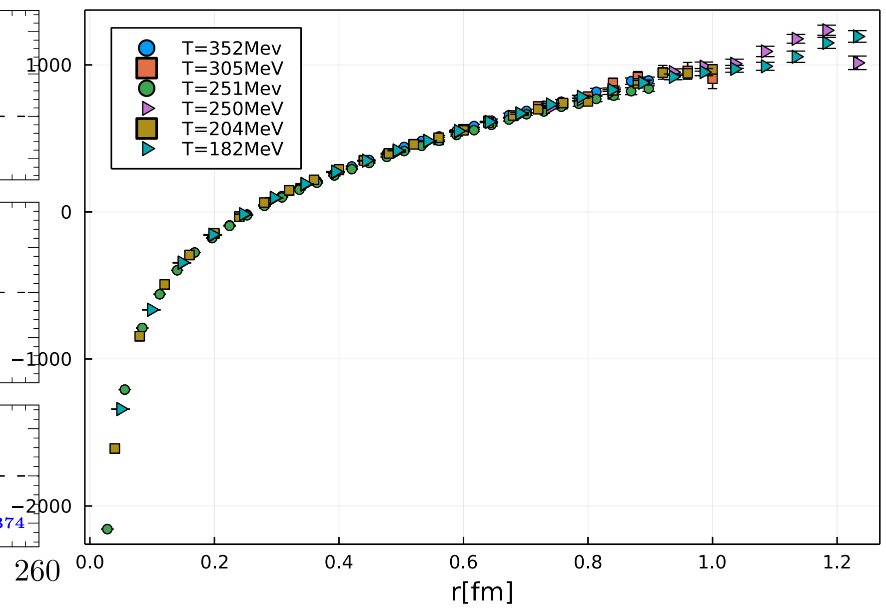
==> S. Caron-Huot and G. D. Moore, PRL. 100, 052301 (2008)

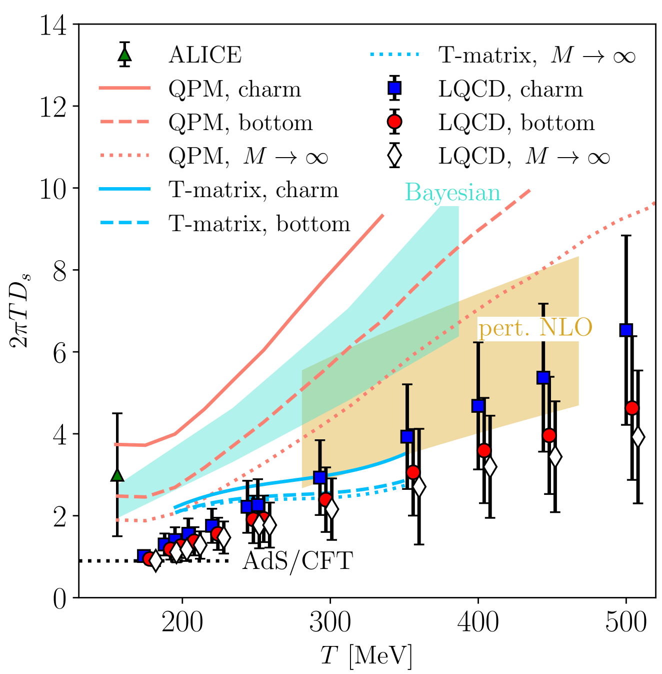
- Similar magnitude for κ_E and κ_B in full QCD & quenched
- Smooth connection between quenched and full QCD in temperature
- Lattice results confirms the form suggested by pert. computations

Major achievements from HF @ finite T





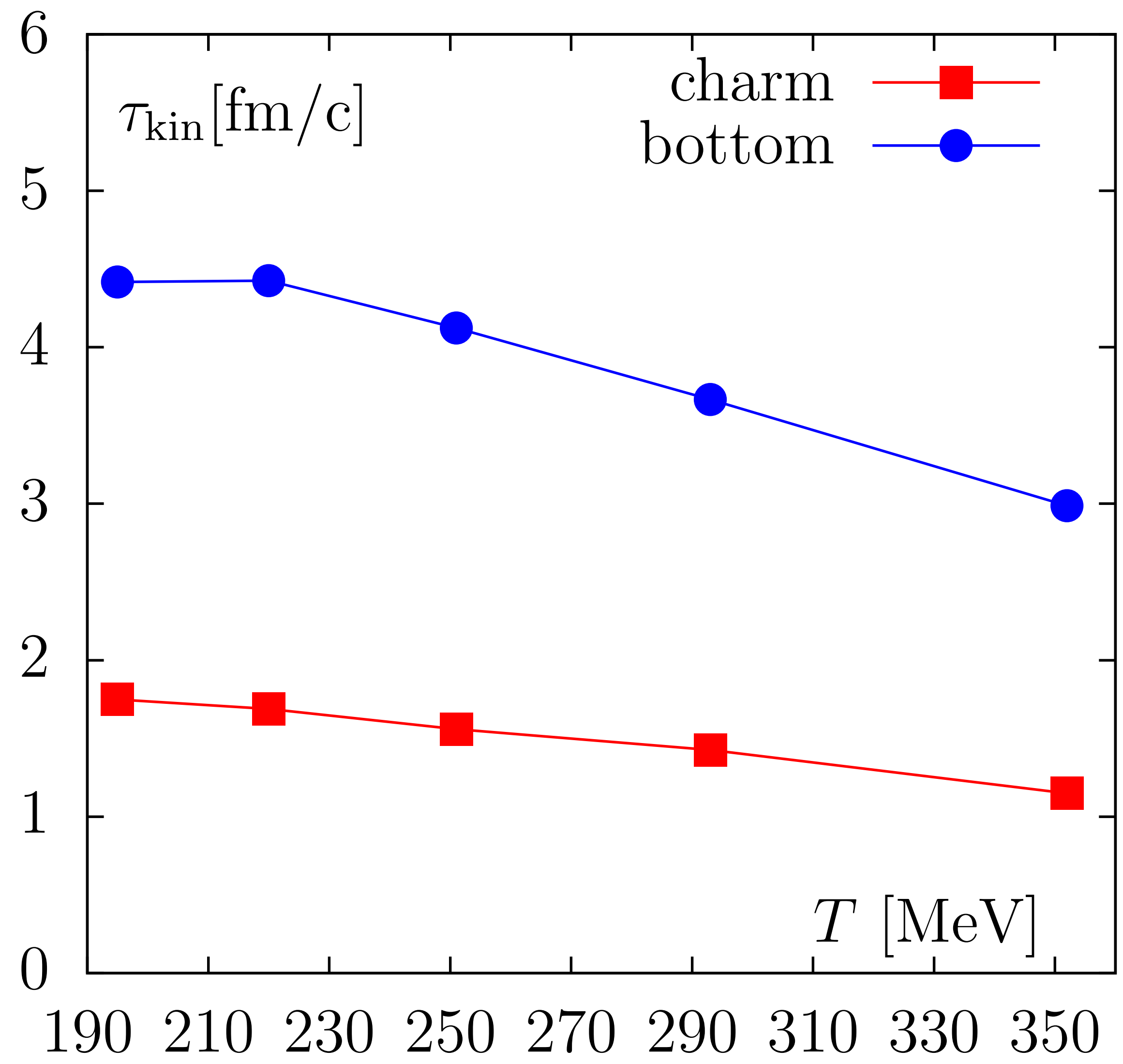




- First full QCD study of quarkonium using relativistic heavy quarks
- No mass shift for bottomonium & increasing thermal width
- Screening (or not?) for $\mathrm{Re}[V]$ & increasing $\mathrm{Im}[V]$ with T & r
- First full QCD calculation of HQ diffusion coefficient at the physical point in $174\ \mathrm{MeV} \leq T \leq 500\ \mathrm{MeV}$

Heavy flavors serve as hard probe to provide accurate and realistic inputs for HIC phenomenology

Backup: equilibration time



$$\tau_{\rm kin} = \frac{1}{\eta_D} = \frac{1}{\kappa/T^3}\left(\frac{T_c}{T}\right)^2\left(\frac{M}{1.5\ {\rm GeV}}\right)\frac{3\ {\rm GeV}}{T_c^2}$$

- Equilibration time of charm quark favors the experimental estimate (~1 fm/c for all)

Backup: identify the heavy quark diffusion

Phenomenological diffusion picture of classical particle

Equilibrium -> Relaxation -> Equilibrium

$$\langle A(\mathbf{x})\rangle_{\rm eq} = 0 \qquad \partial_t \langle A(\mathbf{x},t)\rangle = D\nabla^2 \langle A(\mathbf{x},t)\rangle$$

Solution:

$$\langle A(\mathbf{k},\omega)\rangle = \frac{i}{\omega + iD\mathbf{k}^2}\langle A(\mathbf{k},t=0)\rangle$$

Linear response theory

Perturbation to Hamiltonian:

$$H(t) = H_0 - \int \mathrm{d}\mathbf{x}\ A(\mathbf{x})h(x)e^{\epsilon t}\Theta(-t)$$

Solution:

$$\frac{\partial}{\partial t}\Big(\delta\langle A(\mathbf{k},t=0)\rangle\Big) = -\frac{G_R(\mathbf{k},t)}{\chi_q(\mathbf{k})}\delta\langle A(\mathbf{k},0)\rangle$$

Kubo formula: $$G_R(\mathbf{k},\omega) = \frac{iD\mathbf{k}^2}{\omega + iD\mathbf{k}^2}\chi_q(\mathbf{k}) \sim \rho(\vec{k},\omega)$$

$$A \to J^\mu = \bar{\psi}\gamma^\mu\psi$$

$$D = \frac{1}{3\chi_q}\lim_{\omega\to 0}\sum_{i=1}^{3}\frac{\rho^{ii}(\omega)}{\omega}$$

Backup: full QCD setup

$$N_f = 2+1,\ \text{HISQ},\ m_\pi = 320\ \text{MeV}$$

T [MeV]	β	am_s	am_l	N_σ	N_τ	# conf.
195	7.570	0.01973	0.003946	64	20	5899
	7.777	0.01601	0.003202	64	24	3435
	8.249	0.01011	0.002022	96	36	2256
220	7.704	0.01723	0.003446	64	20	7923
	7.913	0.01400	0.002800	64	24	2715
	8.249	0.01011	0.002022	96	32	912
251	7.857	0.01479	0.002958	64	20	6786
	8.068	0.01204	0.002408	64	24	5325
	8.249	0.01011	0.002022	96	28	1680
293	8.036	0.01241	0.002482	64	20	6534
	8.147	0.01115	0.002230	64	22	9101
	8.249	0.01011	0.002022	96	24	688
352	8.249	0.01011	0.002022	96	20	2488

- Wide temperature range
- Different lattice spacings
- Large lattices towards thermodynamic limit

Backup: anomalous dimension of B-field

- Anomalous dimension in MSbar-scheme

$$Z_B = 1 + \frac{g^2 C_A}{(4\pi)^2}\left[\frac{1}{\epsilon} + 2\ln\left(\frac{\bar{\mu}e^{\gamma_E}}{4\pi T}\right) - 2\right] + \mathcal{O}(g^4)$$

- Gradient flow-scheme —> MSbar-scheme —> physical values

- Scale dependence must go for "WeWant" and $\langle BB\rangle_{\tau_F}$

$$\mathrm{WeWant} = Z_B^2 \langle BB\rangle_{\mathrm{MS}}$$

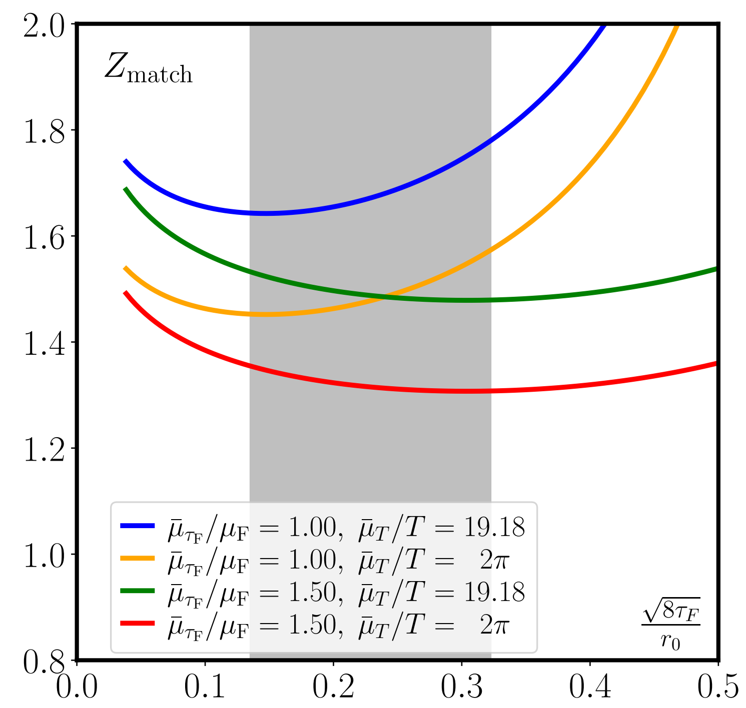
$$\langle BB\rangle_{\tau_F} \equiv Z^{-2}\langle BB\rangle_{\mathrm{MS}}$$

$$\mathrm{WeWant} = Z_B^2 Z^2 \langle BB\rangle_{\tau_F}$$

$$Z^2 = \left(1 - 2\frac{g^2 C_A}{16\pi^2}\ln(\mu^2\tau_F)\right)\left(1 + 2K\frac{g^2 C_A}{16\pi^2}\right) \equiv Z_f^2 Z_K^2$$

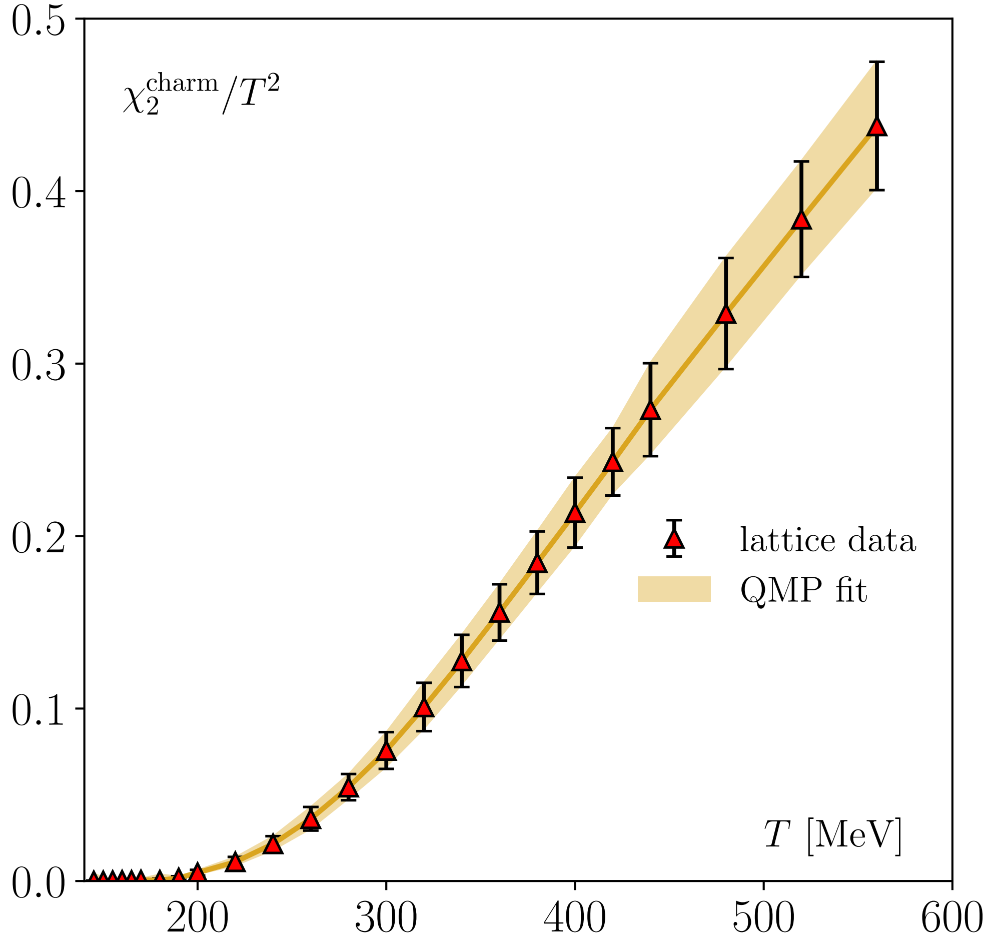
- Determination of the matching factor

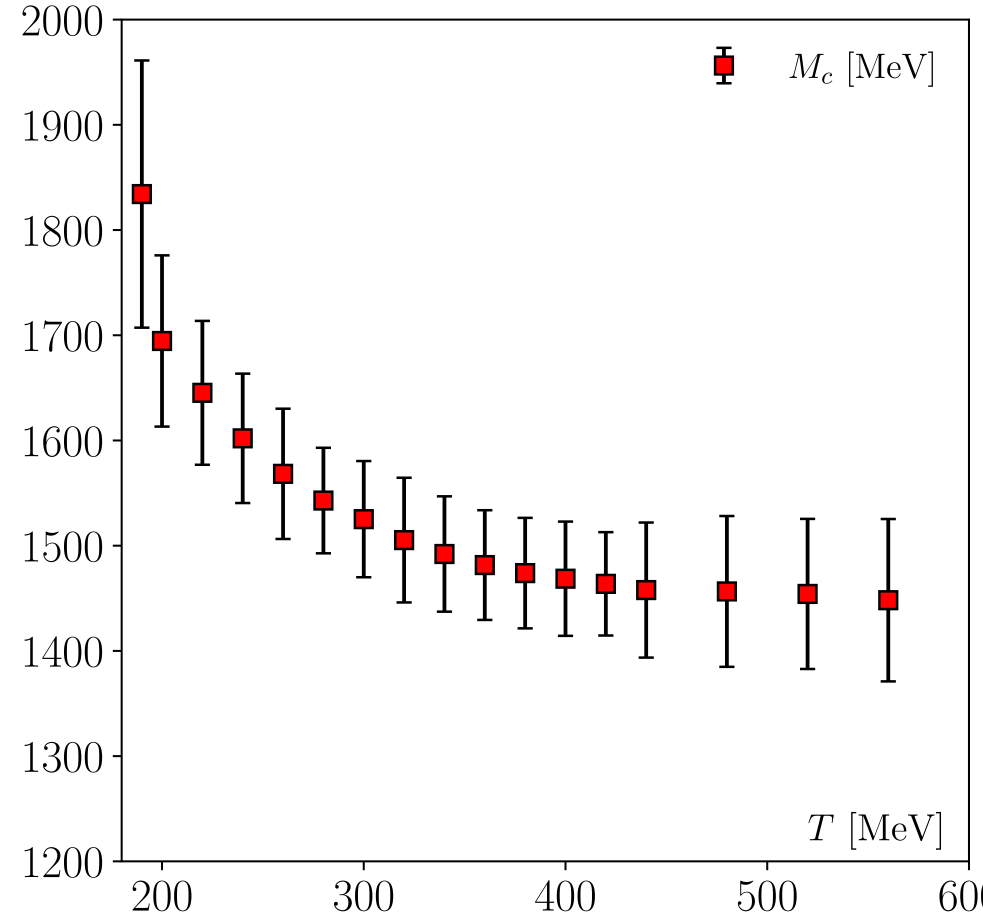
$$\ln Z_{\mathrm{match}} = \int_{\bar{\mu}_T^2}^{\bar{\mu}_{\tau_F}^2} \gamma_0 g^2_{\overline{\mathrm{MS}}}(\bar{\mu})\frac{d\bar{\mu}^2}{\bar{\mu}^2} + \gamma_0 g^2_{\overline{\mathrm{MS}}}(\bar{\mu}_T)\left[\ln\frac{\bar{\mu}_T^2}{(4\pi T)^2} - 2 + 2\gamma_{\mathrm{E}}\right] - \gamma_0 g^2_{\overline{\mathrm{MS}}}(\bar{\mu}_{\tau_{\mathrm{F}}})\left[\ln\frac{\bar{\mu}_{\tau_{\mathrm{F}}}^2}{4\,\mu_{\mathrm{F}}^2} + \gamma_{\mathrm{E}}\right]$$



[PRL 132 (2024) 5, 051902]

Backup: T-dependent charm quark mass





[PRL 132 (2024) 5, 051902]

$$\frac{\chi_2^{\rm charm}}{T^2} = \frac{4N_c}{(2\pi T)^3}\int d^3p\; e^{-E_p/T}$$

$$E_p^2(T) = m^2(T) + p^2$$

$\longrightarrow m(T)$

$$\langle v^2 \rangle = \left(\int d^3p\; \frac{p^2}{E_p^2} e^{-E_p/T}\right) / \left(\int d^3p\; e^{-E_p/T}\right)$$

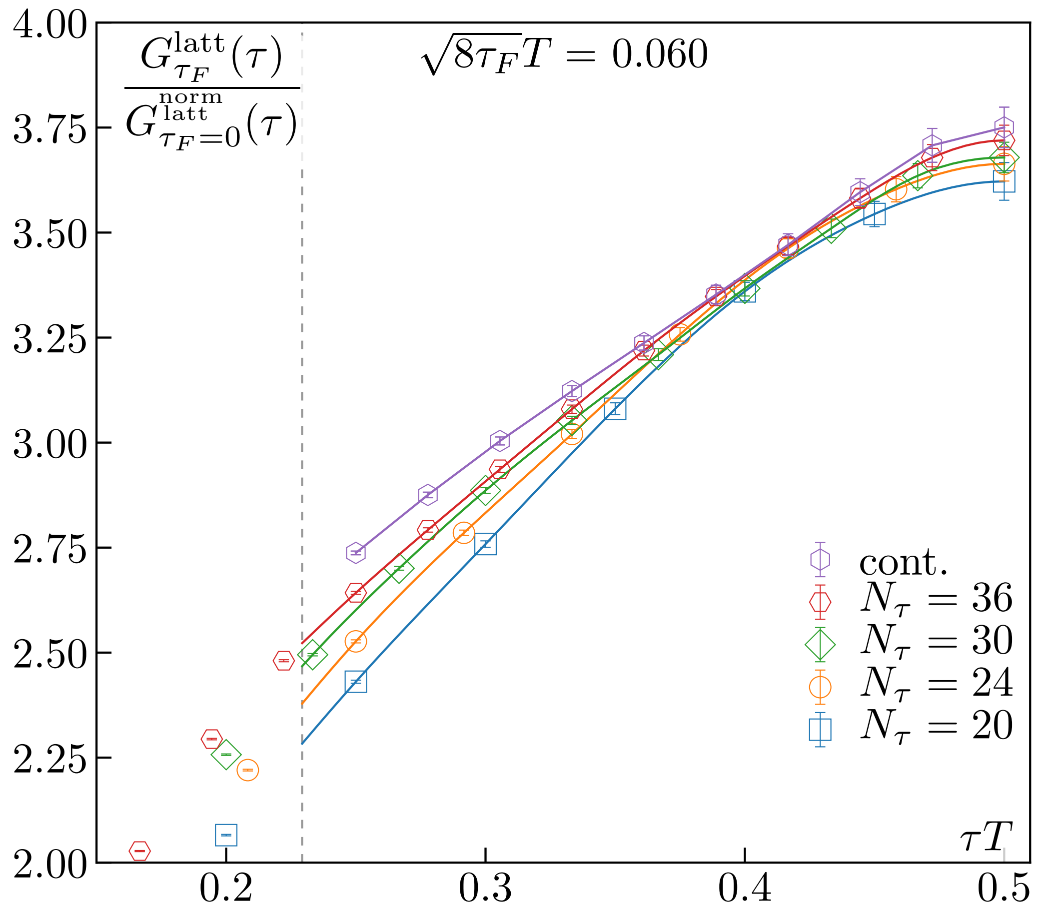
$$\langle p^2 \rangle = \left(\int d^3p\; p^2 e^{-E_p/T}\right) / \left(\int d^3p\; e^{-E_p/T}\right)$$

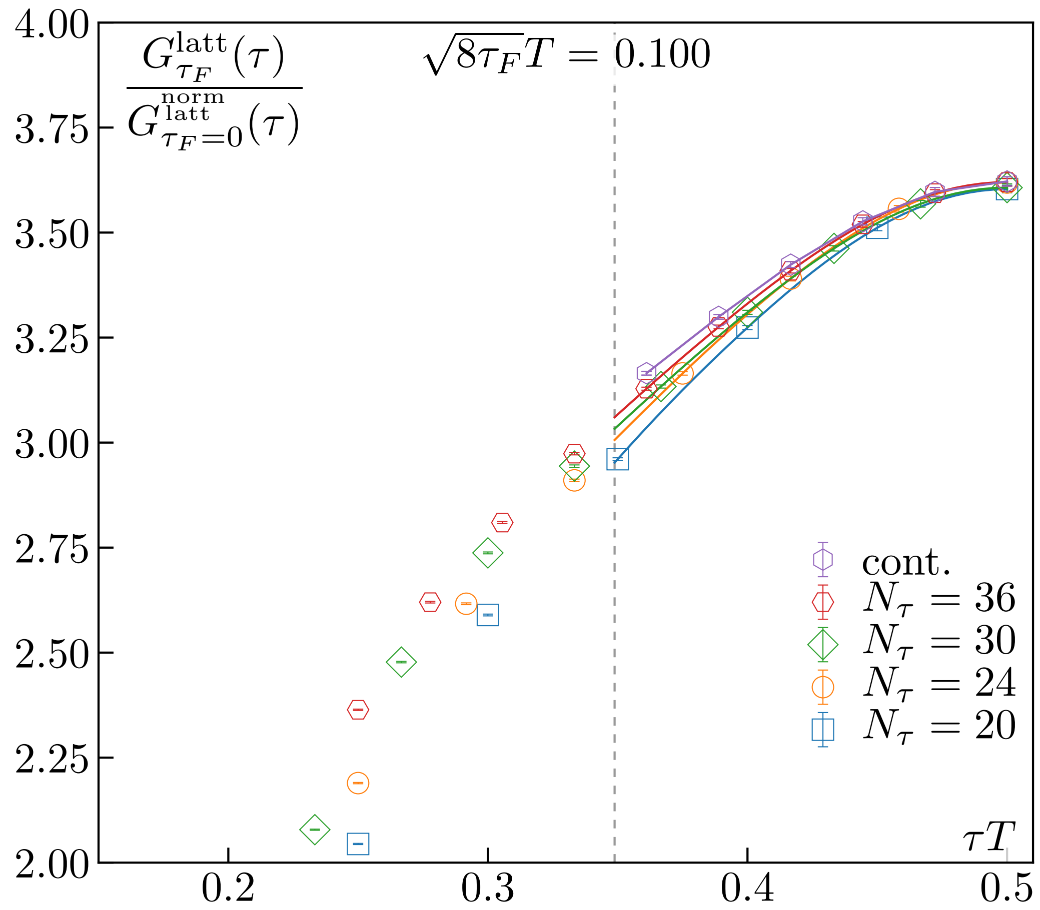
$$\kappa = \kappa_E + \frac{2}{3}\langle \mathbf{v^2} \rangle \kappa_{\mathbf{B}}$$

$$D_s = \frac{2T^2}{\kappa} \cdot \frac{\langle p^2 \rangle}{3MT}$$

Backup: smearing effects of gradient flow

[HTS et al., PRD103(2021) 1, 014511]

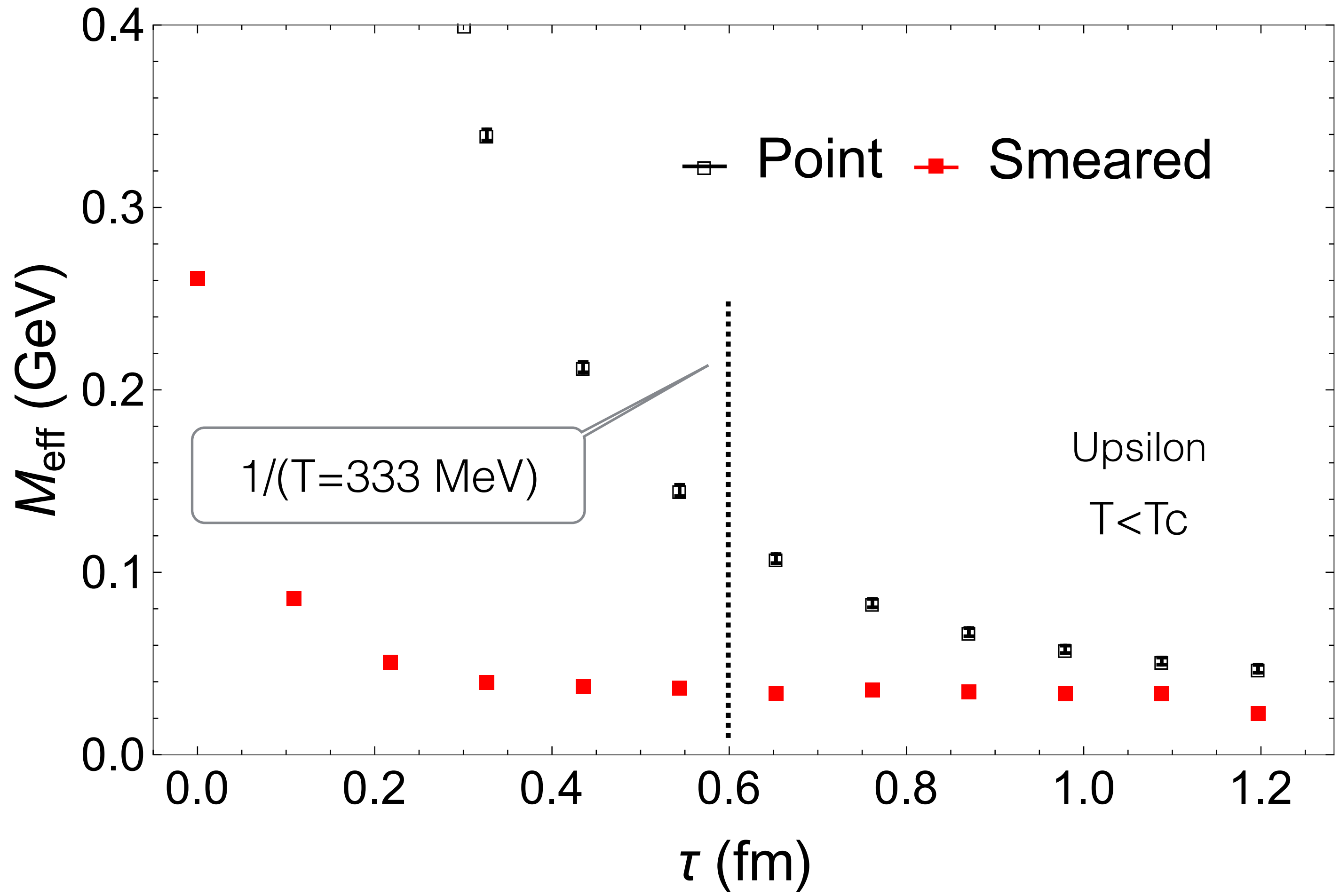




- Gradient flow reduces the noise in correlators
- Gradient flow removes the lattice effects (disordering)
- Need proper flow time range

Extended meson source

Excited states accessible from extended meson operator



R. Larsen, et al., PRD 100 (2019) 7, 074506

Backup: scattering from various models