Investigation of pion-nucleon contributions to nucleon matrix elements

arXiv:2408.03893

Constantia Alexandrou, Giannis Koutsou, Yan Li,
Marcus Petschlies, Ferenc Pittler

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Background

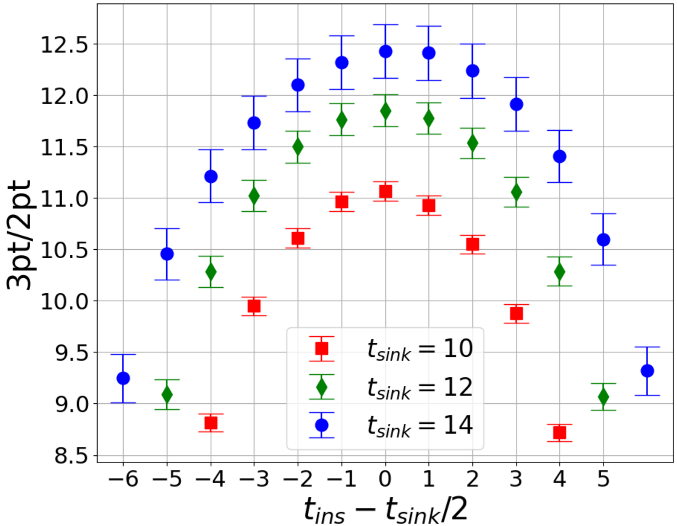
➢ Nucleon structure: nucleon matrix elements

$$R_{\mathcal{O}} := \frac{\langle \mathcal{J}_N(t_{\text{sink}})\mathcal{O}(t_{\text{ins}})\mathcal{J}_N^\dagger(0)\rangle}{\langle \mathcal{J}_N(t_{\text{sink}})\mathcal{J}_N^\dagger(0)\rangle} \xrightarrow{\text{all } t \text{ well-separated}} \langle N|\mathcal{O}|N\rangle$$

➢ Time-dependence indicates contamination from excited states

➢ Lowest excited state is a **Nucleon–Pion** state

$\mathcal{O} = S^{u+d} = \bar{u}u + \bar{d}d \quad \vec{q} = (0, 0, 0)$



Simulation details

Ensembles	Flavors	$N_L^3 \times N_T$	m_π (MeV)	L (fm)	$m_\pi L$	$N_{\rm cfg}$
cA2.09.48	2	$48^3 \times 96$	131	4.50	2.98	1200

- Physical-point twisted-mass ensemble

  An ensemble with $m_\pi = 346$ was also studied (see preprint)

- Interpolating fields used:

  $$J_p; \qquad J_{N\pi}^{1/2} = \sqrt{2/3}\, J_{n\pi^+} - \sqrt{1/3}\, J_{p\pi^0}$$

- Generalized eigenvalue problem (GEVP)
  - Do GEVP on 2pt functions
  - Use the results to improve 3pt functions

2pt functions and GEVP

- 2pt functions:

$$\begin{bmatrix} \langle \mathcal{J}_N \mathcal{J}_N^\dagger \rangle & \langle \mathcal{J}_N \mathcal{J}_{N\pi}^\dagger \rangle \\ \langle \mathcal{J}_{N\pi} \mathcal{J}_N^\dagger \rangle & \langle \mathcal{J}_{N\pi} \mathcal{J}_{N\pi}^\dagger \rangle \end{bmatrix}$$

$$C_{ij}(t) = \langle \mathcal{J}_i(t) \mathcal{J}_j^\dagger(0) \rangle$$

- GEVP returns eigenvalues and eigenvectors:

$$C_{ij}(t) v_j^n = \lambda^n(t, t_0) C_{ij}(t_0) v_j^n$$

$$\lambda^n(t, t_0) = e^{-E_n(t-t_0)}, \quad v_j^n \mathcal{J}_j^\dagger(0) |0\rangle = |n\rangle$$

- We determine the optimal interpolating field:

$$\tilde{\mathcal{J}}_N |0\rangle = (\mathcal{J}_N + v_{N\pi}^N \mathcal{J}_{N\pi}) |0\rangle \propto |N\rangle$$

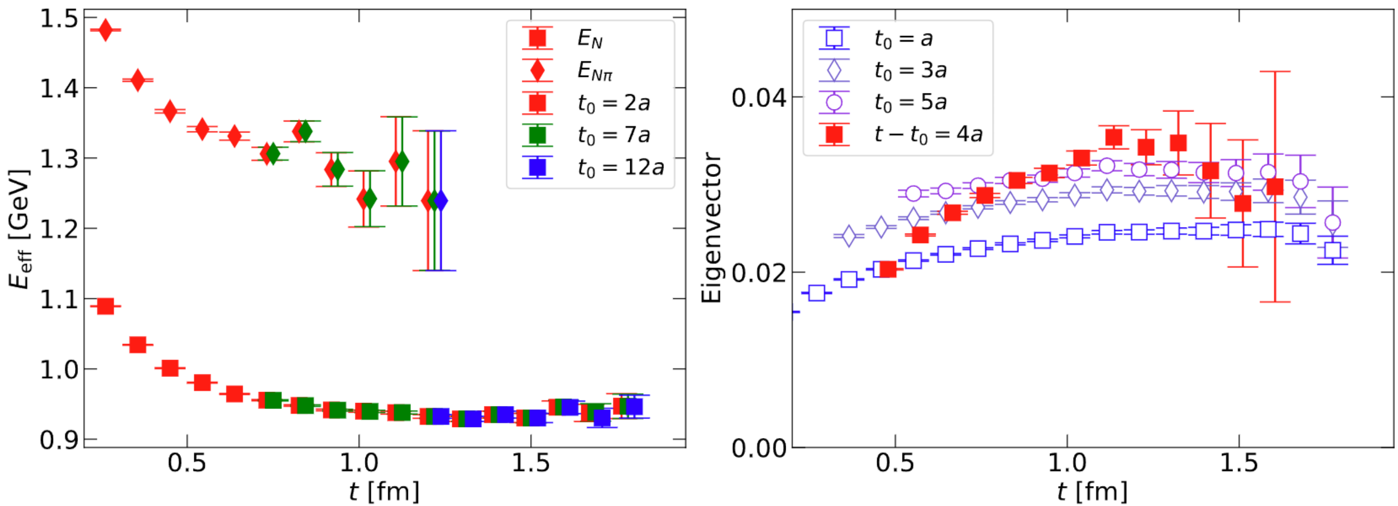
- We can use it to improve matrix elements:

$$\frac{\langle \mathcal{J}_N \mathcal{O} \mathcal{J}_N^\dagger \rangle}{\langle \mathcal{J}_N \mathcal{J}_N^\dagger \rangle} \xrightarrow{\text{GEVP improved}} \frac{\langle \tilde{\mathcal{J}}_N \mathcal{O} \tilde{\mathcal{J}}_N^\dagger \rangle}{\langle \tilde{\mathcal{J}}_N \tilde{\mathcal{J}}_N^\dagger \rangle}$$

t_0 dependence of GEVP

$$C_{ij}(t)v_j^n = \lambda^n(t,t_0)C_{ij}(t_0)v_j^n$$

ALPHA Collaboration JHEP04(2009)094



- E_{eff} are not sensitive to t_0
- Eigenvectors are sensitive to t_0
- With small t_0, it converges to a wrong value
- In this work, we fix $t - t_0$, and do plateau fits to determine eigenvectors

GEVP results

- $\vec{p} = (0,0,0)$:

2 interpolators

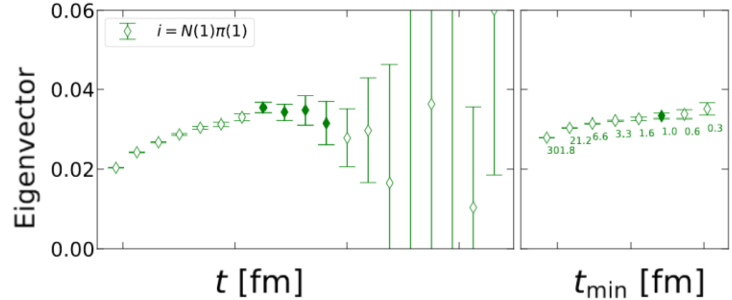
$N(0), N(1)\pi(1)$

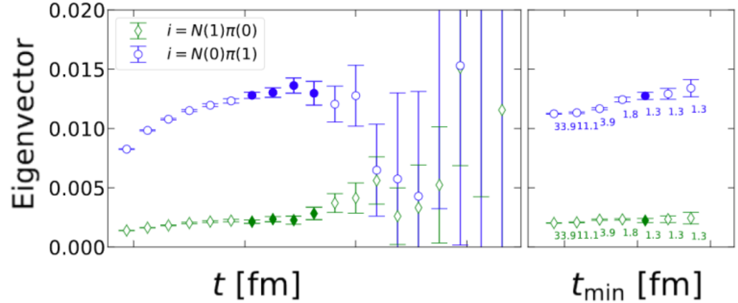
- $\vec{p} = (0,0,1)$:

3 interpolators

$N(1), N(1)\pi(0), N(0)\pi(1)$

Eigenvectors fits





Filled points: best fits

3pt functions and GEVP improvement

$$\frac{\langle \mathcal{J}_N \mathcal{O} \mathcal{J}_N^\dagger \rangle}{\langle \mathcal{J}_N \mathcal{J}_N^\dagger \rangle} \xrightarrow{\text{GEVP improved}} \frac{\langle \tilde{\mathcal{J}}_N \mathcal{O} \tilde{\mathcal{J}}_N^\dagger \rangle}{\langle \tilde{\mathcal{J}}_N \tilde{\mathcal{J}}_N^\dagger \rangle} \rightarrow \begin{bmatrix} \langle \mathcal{J}_N \mathcal{O} \mathcal{J}_N^\dagger \rangle & \langle \mathcal{J}_N \mathcal{O} \mathcal{J}_{N\pi}^\dagger \rangle \\ \langle \mathcal{J}_{N\pi} \mathcal{O} \mathcal{J}_N^\dagger \rangle & \cancel{\langle \mathcal{J}_{N\pi} \mathcal{O} \mathcal{J}_{N\pi}^\dagger \rangle} \end{bmatrix}$$

$$\tilde{\mathcal{J}}_N = \mathcal{J}_N + v_{N\pi}^N \mathcal{J}_{N\pi}$$

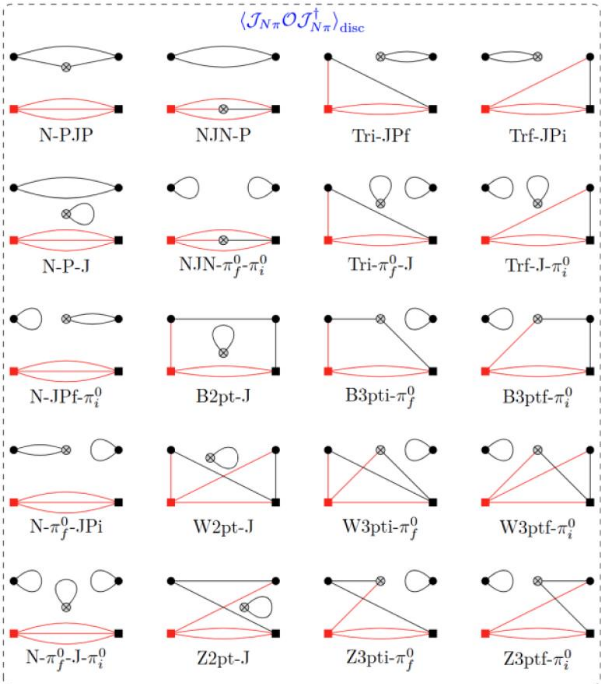
- We compute everything except $\cancel{\langle \mathcal{J}_{N\pi} \mathcal{O} \mathcal{J}_{N\pi}^\dagger \rangle}$.
- RQCD: Last term is subleading by ChPT

Barca, Bali, Collins PRD 107, L051505 (2023)
Bar PRD 99, 054506 (2018) and 100, 054507 (2019)

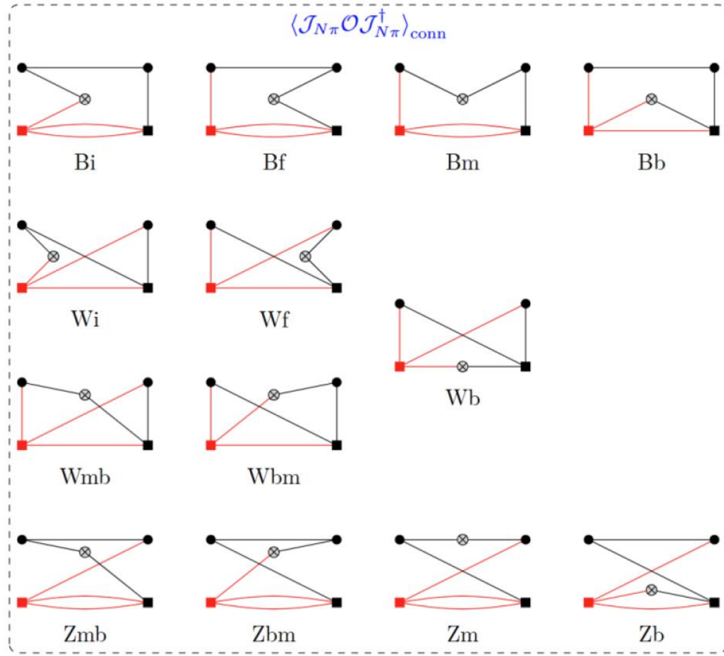
- This work:

We found a new method that doesn't require such term

Topologies



Not done



New method

- 3pt function without GEVP: $I_0 = \langle \mathcal{J}_N \mathcal{O} \mathcal{J}_N^\dagger \rangle$
- Fully GEVP improved 3pt function:

$$I = \langle \tilde{\mathcal{J}}_N \mathcal{O} \tilde{\mathcal{J}}_N^\dagger \rangle = v_{N,N}\, v_{N,N}^* \langle \mathcal{J}_N \mathcal{O} \mathcal{J}_N^\dagger \rangle + v_{N,N}\, v_{N,N\pi}^* \langle \mathcal{J}_N \mathcal{O} \mathcal{J}_{N\pi}^\dagger \rangle$$
$$+ v_{N,N\pi}\, v_{N,N}^* \langle \mathcal{J}_{N\pi} \mathcal{O} \mathcal{J}_N^\dagger \rangle + v_{N,N\pi}\, v_{N,N\pi}^* \langle \mathcal{J}_{N\pi} \mathcal{O} \mathcal{J}_{N\pi}^\dagger \rangle$$

- New method:

$$I_d = d_{N,N}\, v_{N,N}\, v_{N,N}^* \langle \mathcal{J}_N \mathcal{O} \mathcal{J}_N^\dagger \rangle + d_{N,N\pi}\, v_{N,N}\, v_{N,N\pi}^* \langle \mathcal{J}_N \mathcal{O} \mathcal{J}_{N\pi}^\dagger \rangle$$
$$+ d_{N\pi,N}\, v_{N,N\pi}\, v_{N,N}^* \langle \mathcal{J}_{N\pi} \mathcal{O} \mathcal{J}_N^\dagger \rangle + 0\times v_{N,N\pi}\, v_{N,N\pi}^* \langle \mathcal{J}_{N\pi} \mathcal{O} \mathcal{J}_{N\pi}^\dagger \rangle$$

$d_{N,N}$ ↑ $1 - W^*(\vec{p}\,')\, W(\vec{p})$

$d_{N,N\pi}$ ↑ $1 + W^*(\vec{p}\,')$

$d_{N\pi,N}$ ↓ $1 + W(\vec{p})$

$$W(\vec{p}) = \frac{1}{[v]_{0N}(\vec{p})\,[v^{-1}]_{N0}(\vec{p})} - 1$$

New method

- 3pt function without GEVP: $I_0 = \langle \mathcal{J}_N \mathcal{O} \mathcal{J}_N^\dagger \rangle$
- Fully GEVP improved 3pt function:

$$I = \langle \tilde{\mathcal{J}}_N \mathcal{O} \tilde{\mathcal{J}}_N^\dagger \rangle = v_{N,N}\, v_{N,N}^* \langle \mathcal{J}_N \mathcal{O} \mathcal{J}_N^\dagger \rangle + v_{N,N}\, v_{N,N\pi}^* \langle \mathcal{J}_N \mathcal{O} \mathcal{J}_{N\pi}^\dagger \rangle + v_{N,N\pi}\, v_{N,N}^* \langle \mathcal{J}_{N\pi} \mathcal{O} \mathcal{J}_N^\dagger \rangle + v_{N,N\pi}\, v_{N,N\pi}^* \langle \mathcal{J}_{N\pi} \mathcal{O} \mathcal{J}_{N\pi}^\dagger \rangle$$

- New method:

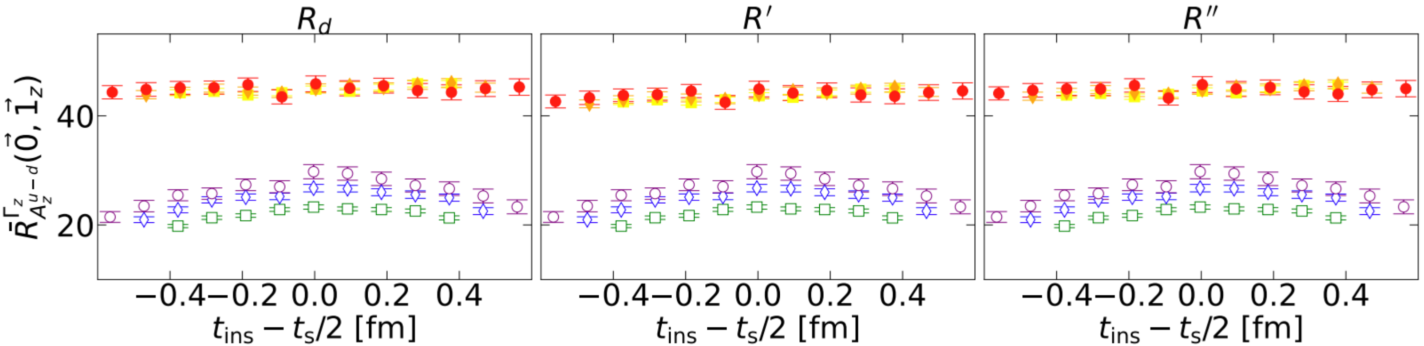
$$I_d = d_{N,N}\, v_{N,N}\, v_{N,N}^* \langle \mathcal{J}_N \mathcal{O} \mathcal{J}_N^\dagger \rangle + d_{N,N\pi}\, v_{N,N}\, v_{N,N\pi}^* \langle \mathcal{J}_N \mathcal{O} \mathcal{J}_{N\pi}^\dagger \rangle + d_{N\pi,N}\, v_{N,N\pi}\, v_{N,N}^* \langle \mathcal{J}_{N\pi} \mathcal{O} \mathcal{J}_N^\dagger \rangle + 0 \times v_{N,N\pi}\, v_{N,N\pi}^* \langle \mathcal{J}_{N\pi} \mathcal{O} \mathcal{J}_{N\pi}^\dagger \rangle$$

- Coefficients d can be determined by GEVP, do not depend on the insertion operator $\mathcal{O}$
- I_d can remove the leading contamination terms

$$R_0 = \langle N|\mathcal{O}|N\rangle + a(e^{-\Delta E\, t_{\rm ins}} + e^{-\Delta E\,(t_{\rm sink}-t_{\rm ins})}) + b\, e^{-\Delta E\, t_{\rm sink}}$$
$$R = \langle N|\mathcal{O}|N\rangle$$
$$R_d = \langle N|\mathcal{O}|N\rangle - b\, e^{-\Delta E\, t_{\rm sink}}$$

New method vs. others

- Before GEVP: Open
- After GEVP: Filled



$$I_d = d_{N,N}\, I_{N,N} + d_{N,N\pi}\, I_{N,N\pi} + d_{N\pi,N}\, I_{N\pi,N}$$

$$I' = I_{N,N} + I_{N,N\pi} + I_{N\pi,N}$$

$$I'' = I_{N,N} + I_{N,N\pi} + I_{N\pi,N} + I^{\text{disc}}_{N\pi,N\pi}$$

Overview of results

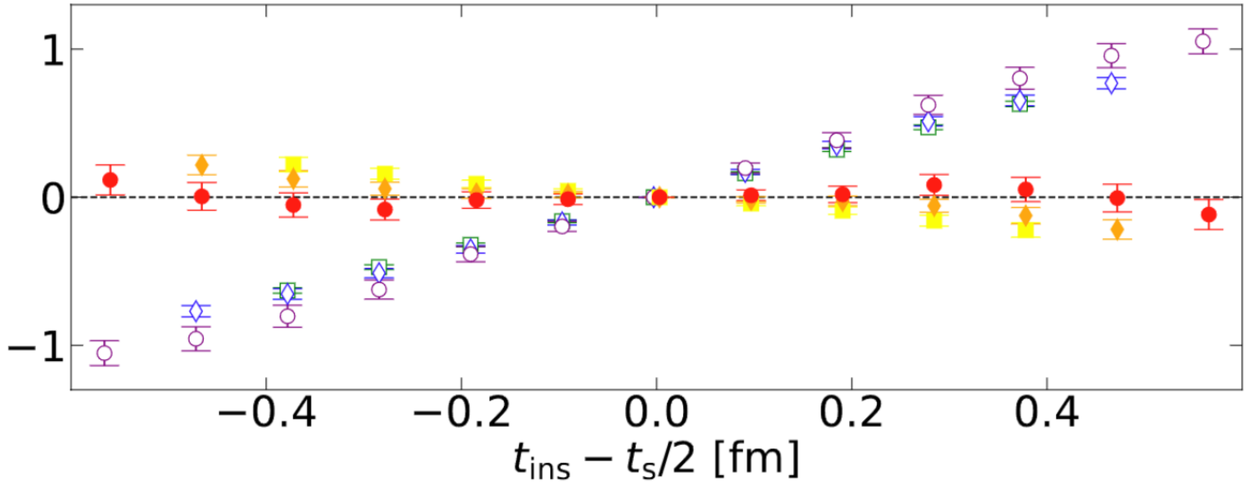
$(\vec{p}_{sink}, \vec{p}_{src}) = (\vec{0},\vec{0}), (\vec{0},\vec{1}), (\vec{1},\vec{1})$

- We have investigated on 52 cases
  {u+d & u−d} × $\{S, V_\mu, P, A_\mu, \sigma_{\mu\nu}\}$ × {various kinematics}
- We include both connected and disconnected contributions for both isoscalar & isovector
  (disc u−d is nonzero for tmlQCD)
- 46 cases
  - No significant changes observed
  - Including the sigma term $\sigma_{\pi N}$
- 6 cases: (u−d) × $\{\boldsymbol{P}, \boldsymbol{A_\mu}\}$
  - Isovector pseudoscalar (2) and axial (4) currents
  - Significant changes observed
  - Insertion operator has same quantum number with the pion

Pseudoscalar at 0 momentum transfer

- Converges to 0 (Parity symmetry)
- Before GEVP: Open
- After GEVP: Filled

Zero



Axial charge g_A^{u-d}

Open: no GEVP Filled: GEVP
Grey: ETMC23, PRD 109 (2024) 3, 034503
Dashed black: experiment

- Four cases for g_A^{u-d}

Ratio (left) and two-state fits (right) =>

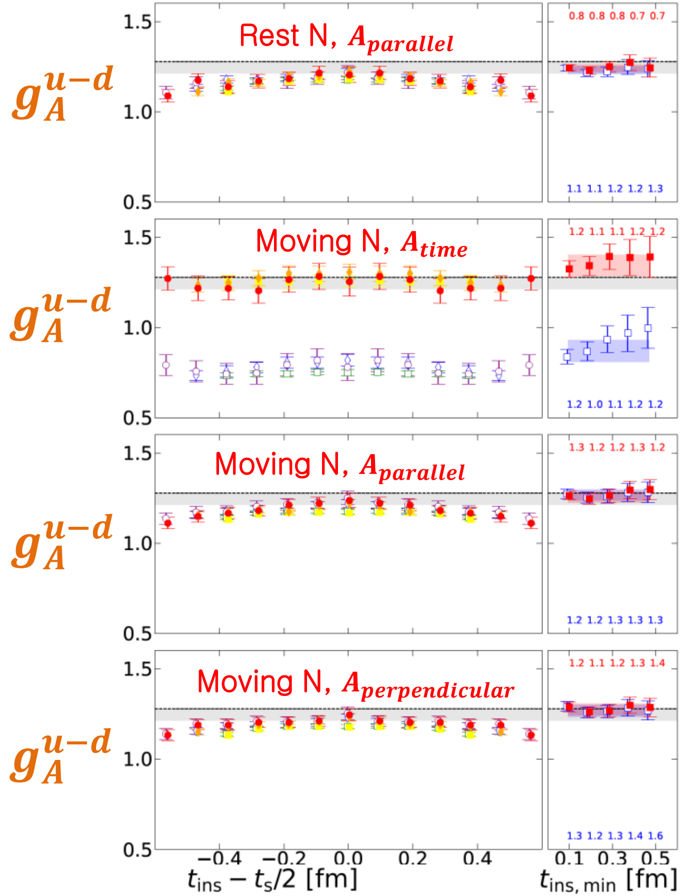
- ETMC23: Band

(Excited state analysis & continuum limit)
PRD 109 (2024) 3, 034503

Small lattice artefact for g_A^{u-d}

- No changes for 3 cases
- GEVP brings 2nd case agreement with the other 3

This work: 1.258(18)
Exp: 1.27641(56)



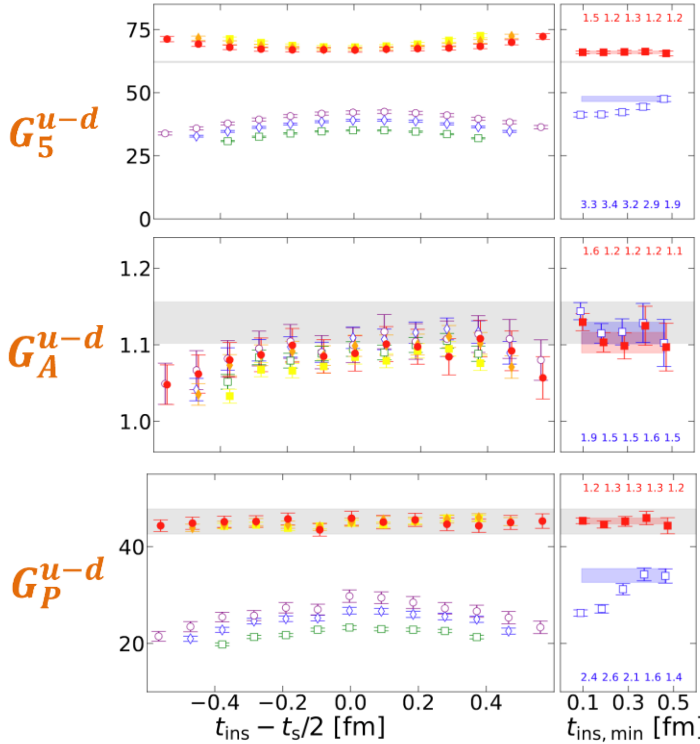
PCAC related quantities @ 1−unit transfer

G_5^{u-d}, G_A^{u-d}, G_P^{u-d}

Open: no GEVP   Filled: GEVP

Grey: ETMC23, PRD 109 (2024) 3, 034503

- Significant improvement observed for G_5^{u-d}, G_P^{u-d}
- Large lattice artefact expected for G_5^{u-d}
- Small lattice artefact for G_P^{u-d} after including the isovector insertion loop (nonzero for tmlQCD)



Conclusions

- Strong t_0 dependence for GEVP eigenvectors
- New method without requiring $\langle \mathcal{J}_{N\pi} \mathcal{O} \mathcal{J}^\dagger_{N\pi} \rangle$
- We investigate on 52 cases = 46 (no) + 6 (yes)
  - 46 includes $\sigma_{\pi N}$
  - 6 are with isovector pseudoscalar & axial currents
- Reduced lattice artefacts with isovector insertion loop

THANKS

Με τη συγχρηματοδότηση της Ευρωπαϊκής Ένωσης

Κυπριακή Δημοκρατία

ΙΔΡΥΜΑ ΕΡΕΥΝΑΣ ΚΑΙ ΚΑΙΝΟΤΟΜΙΑΣ

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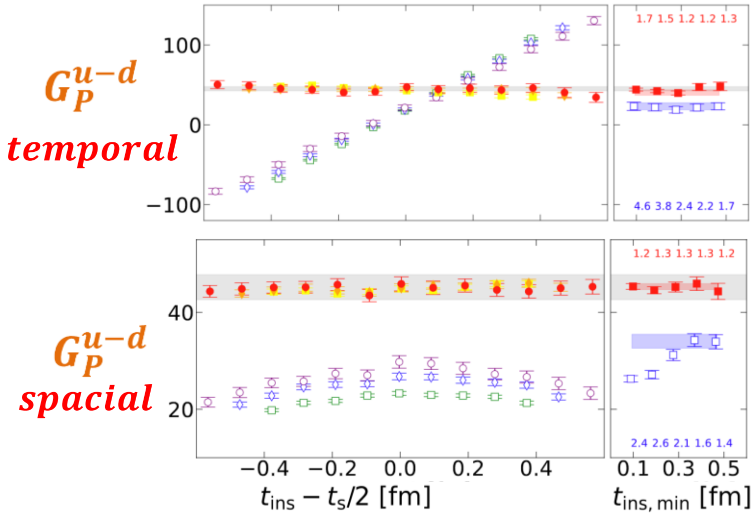
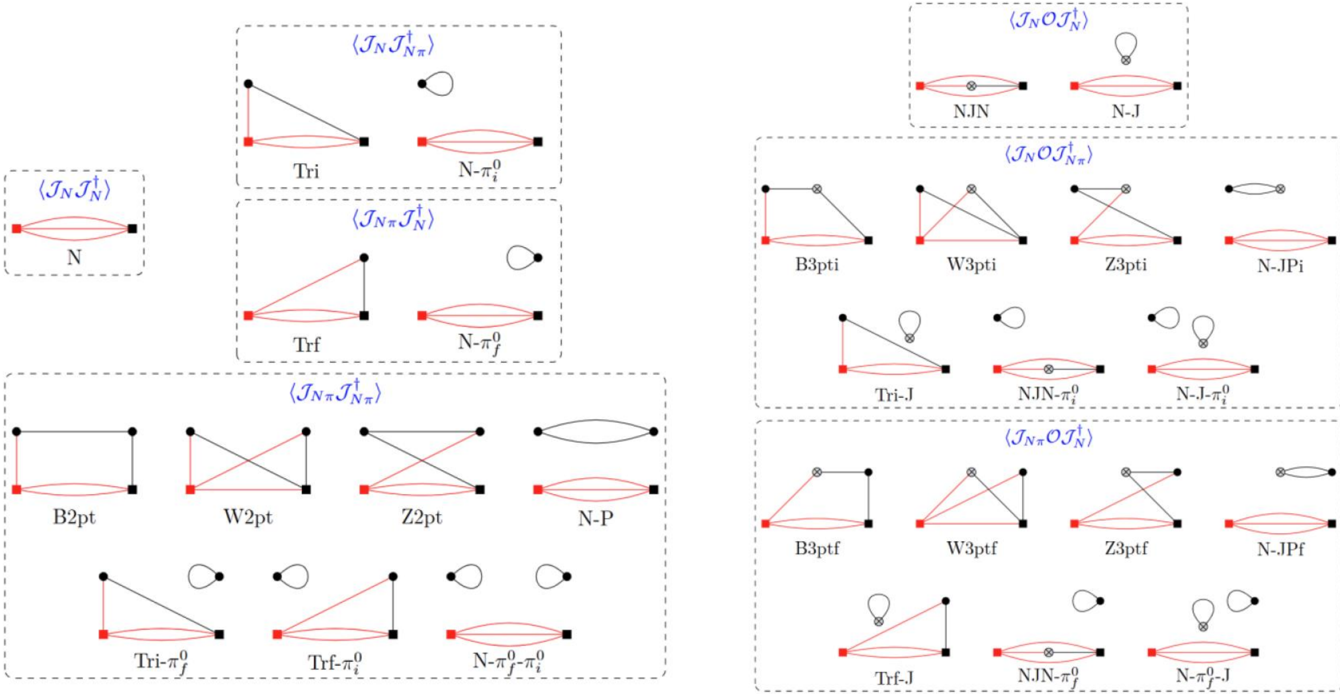
The other case for G_P^{u-d}

Open: no GEVP
Filled: GEVP
Grey: ETMC23, PRD 109 (2024) 3, 034503

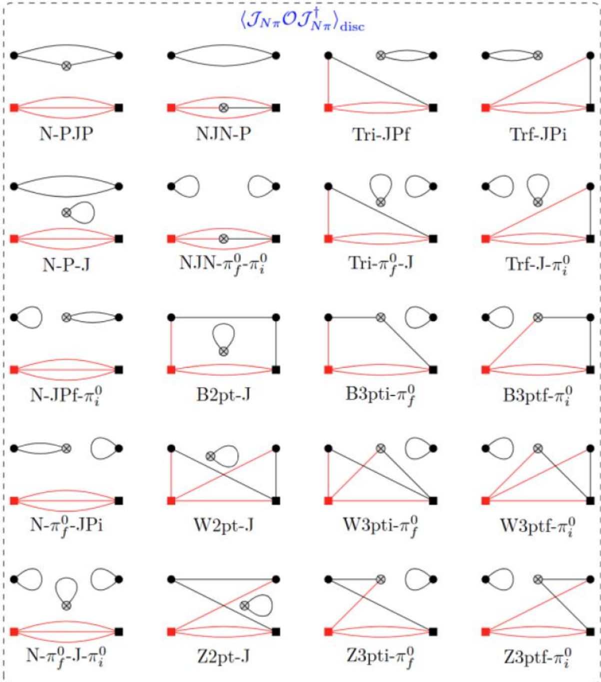


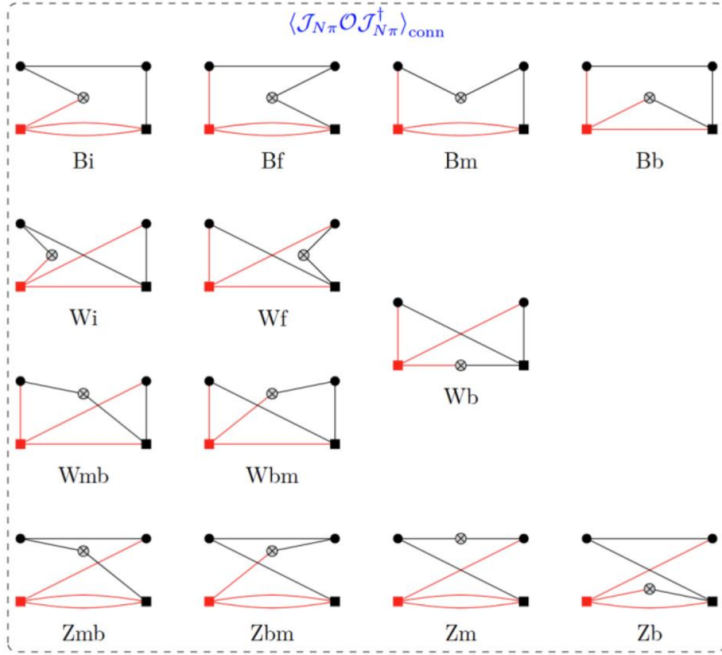
Topologies


$\langle \mathcal{J}_N \mathcal{J}_N^\dagger \rangle$
N
$\langle \mathcal{J}_N \mathcal{J}_{N\pi}^\dagger \rangle$
Tri
N-π_i^0
$\langle \mathcal{J}_{N\pi} \mathcal{J}_N^\dagger \rangle$
Trf
N-π_f^0
$\langle \mathcal{J}_{N\pi} \mathcal{J}_{N\pi}^\dagger \rangle$
B2pt
W2pt
Z2pt
N-P
Tri-π_f^0
Trf-π_i^0
N-π_f^0-π_i^0
$\langle \mathcal{J}_N \mathcal{O} \mathcal{J}_N^\dagger \rangle$
NJN
N-J
$\langle \mathcal{J}_N \mathcal{O} \mathcal{J}_{N\pi}^\dagger \rangle$
B3pti
W3pti
Z3pti
N-JPi
Tri-J
NJN-π_i^0
N-J-π_i^0
$\langle \mathcal{J}_{N\pi} \mathcal{O} \mathcal{J}_N^\dagger \rangle$
B3ptf
W3ptf
Z3ptf
N-JPf
Trf-J
NJN-π_f^0
N-π_f^0-J

Topologies

Not done

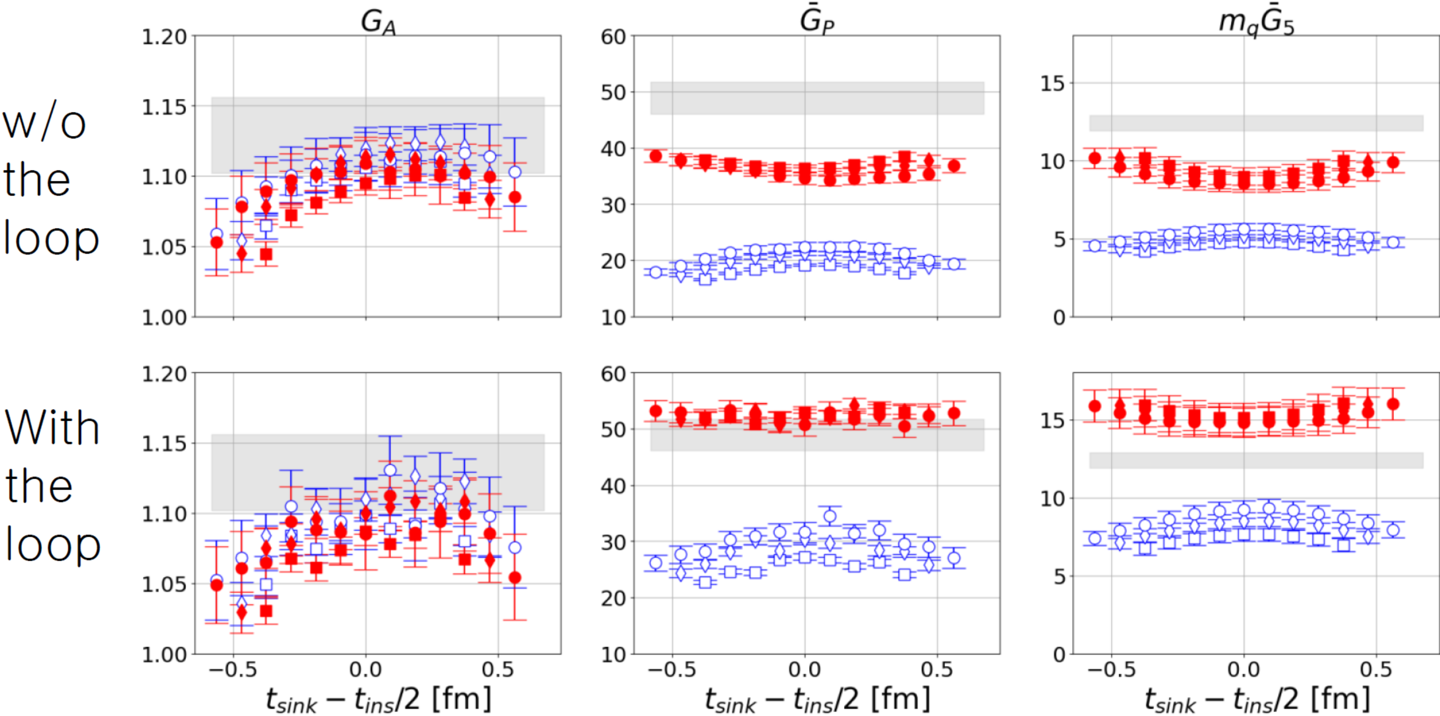




Statistics

Number	Subset	cA211.530.24	cA2.09.48
N_{cfg}	all cases	2467	1228
N_{src}	N	121	272
	NJN	16	8
	NJN (tensor)	4	1
	Tri	96	16
	B, W, Z	9	8
	BWZ3pt (tensor)	3	4
N_{stoc}	π^0	200	100
	J	200	400
	B, W	12	12
N_{oet}	P, JP, Z	1	1

Isovector insertion loop: Form factors



Isovector insertion loop: PCAC ratio

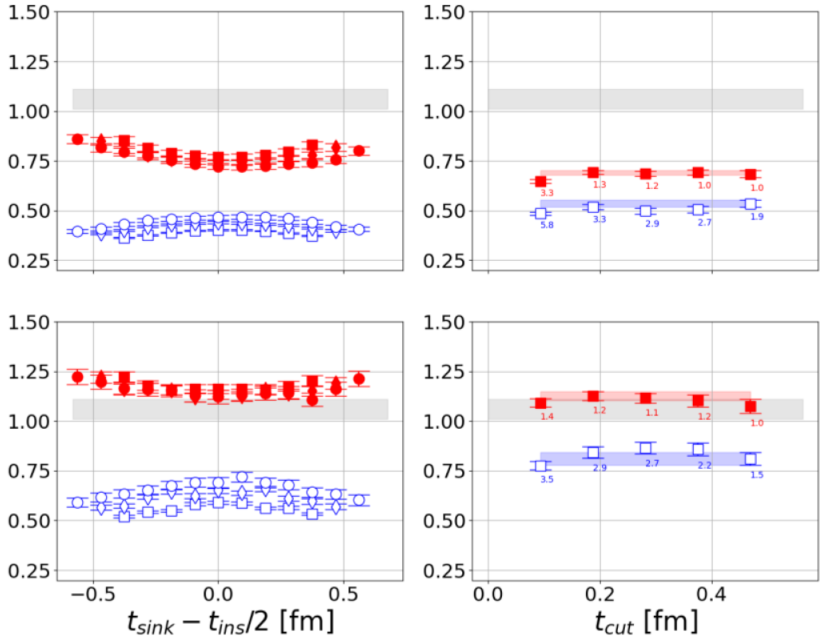
$$r_{\mathrm{PCAC}}(Q^2) = \frac{\frac{m_q}{m_N} G_5(Q^2) + \frac{Q^2}{4m_N^2} G_P(Q^2)}{G_A(Q^2)}$$

Unfilled Blue: no GEVP   Filled Red: GEVP

Grey: arXiv:2309.05774 @ continuum limit

w/o the loop

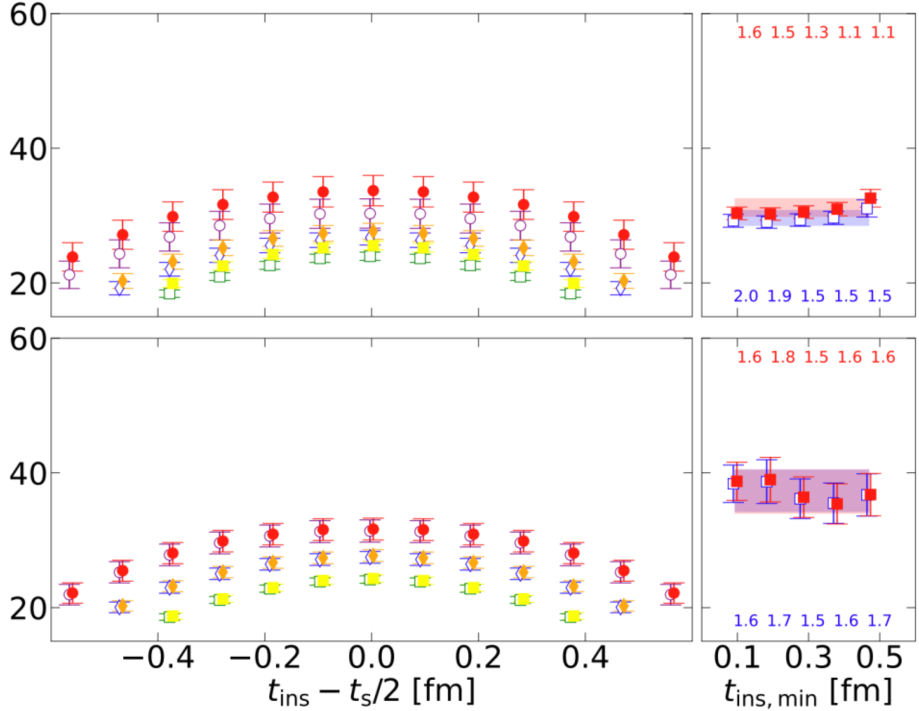
With the loop



Sigma term

Rest

Moving



π^0–loop is necessary for isospin symmetry

$$\langle 0|O_p \begin{bmatrix} \bar{O}_{n\pi^+} = \sqrt{\tfrac{2}{3}}O_{\frac{1}{2}} + \cancel{\sqrt{\tfrac{1}{3}}O_{\frac{3}{2}}} \\ \bar{O}_{p\pi^0} = -\sqrt{\tfrac{1}{3}}O_{\frac{1}{2}} + \cancel{\sqrt{\tfrac{2}{3}}O_{\frac{3}{2}}} \end{bmatrix} |0\rangle \qquad \langle 0|O_p\bar{O}_{n\pi^+}|0\rangle = \langle 0|O_p\bar{O}_{p\pi^0}|0\rangle \times (-\sqrt{2})$$