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Extracting ρ meson from Lattice at the physical mass and continuum limit

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Motivation

- Study the m_π dependence of ρ-meson properties using Hamiltonian Effective Field Theory (HEFT) by the spectra from lattice QCD

$$m_\rho = c_0 + c_1 m_\pi^2 + \xi a^2$$

- Same a, different m_π, √
- Same m_π, different a, ?

CLQCD

M. Werner, et al., Eur.Phys.J.A 56 (2020) 2, 61

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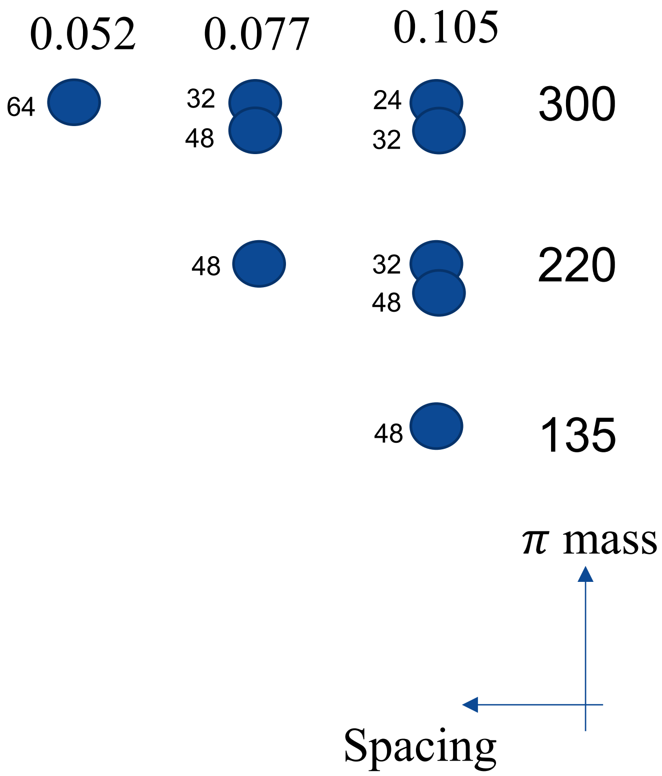
Configurations

Name	Volume	Spacing	β	m_π/MeV	$m_\pi L$
C24P29	$24^3 \times 72$	$0.10530fm$	6.20	292	3.75
C32P29	$32^3 \times 64$			292	5.01
C32P23	$32^3 \times 64$			228	3.91
C48P23	$48^3 \times 96$			225	5.79
C48P14	$48^3 \times 96$			135	3.56
F32P30	$32^3 \times 96$	$0.07746fm$	6.41	303	3.81
F48P30	$48^3 \times 96$			303	5.72
F48P21	$48^3 \times 96$			207	3.91
H48P32	$48^3 \times 144$	$0.05187fm$	6.72	321	4.06



Zhi-Cheng Hu, et al., Phys.Rev.D 109 (2024) 5, 054507

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Spectroscopy on lattice

- Build large basis of operators $\{\mathcal{O}_1, \mathcal{O}_2, \ldots\}$ with desired quantum numbers, construct the matrix of correlation function:

$$C_{ij} = \left\langle 0 \middle| \mathcal{O}_i \mathcal{O}_j^\dagger \middle| 0 \right\rangle = \sum_n Z_i^n Z_j^{n^*} e^{-E_n t}$$

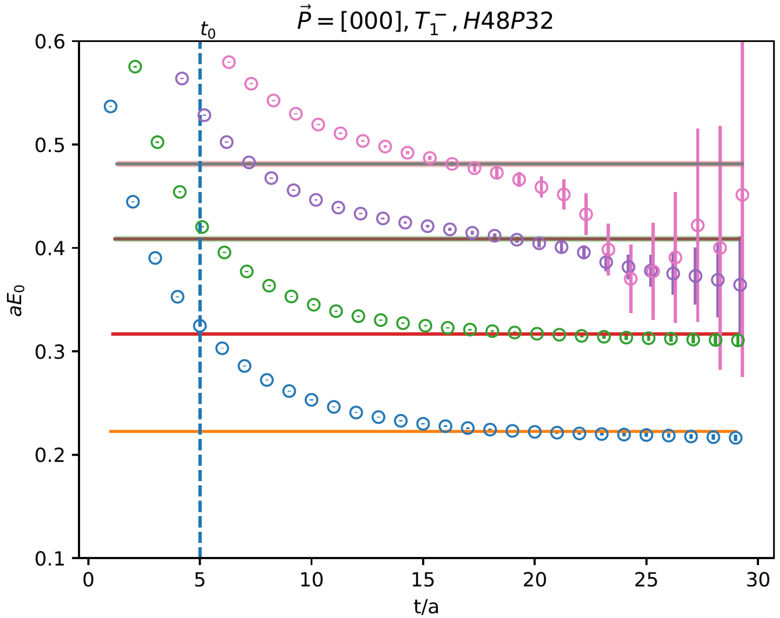
$\rho, \pi\pi, \pi\omega, K\overline{K}, \ldots$

- Solve the generalized eigenvalue problem(GEVP):

$$C_{ij} v_j^n(t) = \lambda_n(t) C_{ij}^0 v_j^n(t)$$

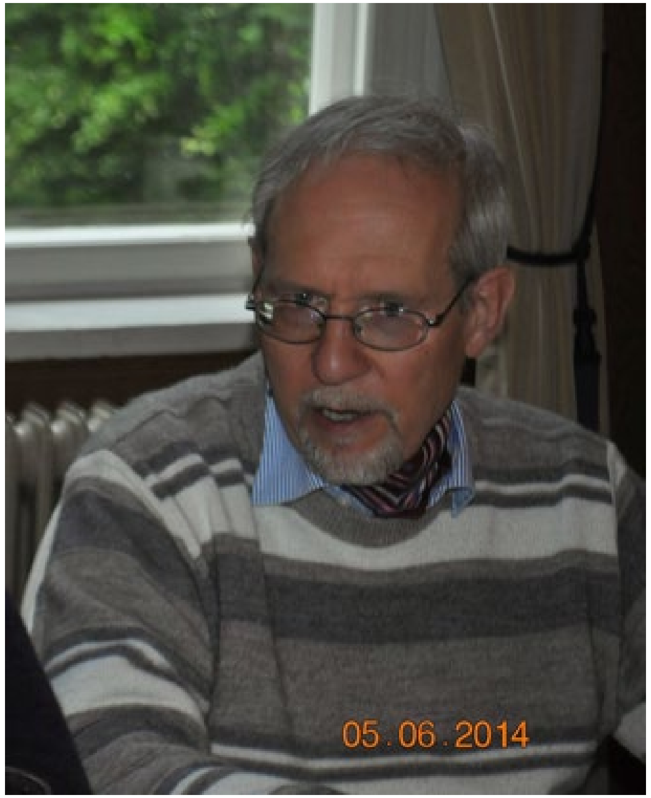
- Eigenvalues:

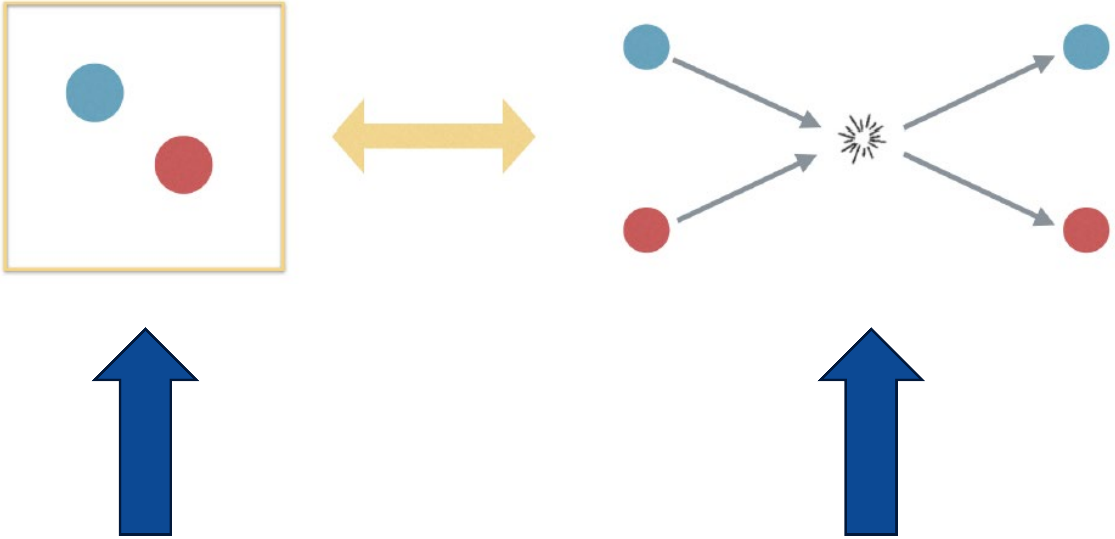
$$\lambda_n(t) \sim e^{-E_n t}\left(1 + e^{-\Delta E t}\right)$$



Scattering on lattice

M. Lüscher, Nucl. Phys. B354, 531(1991)





Finite volume
spectrum

Infinite volume
scattering parameters

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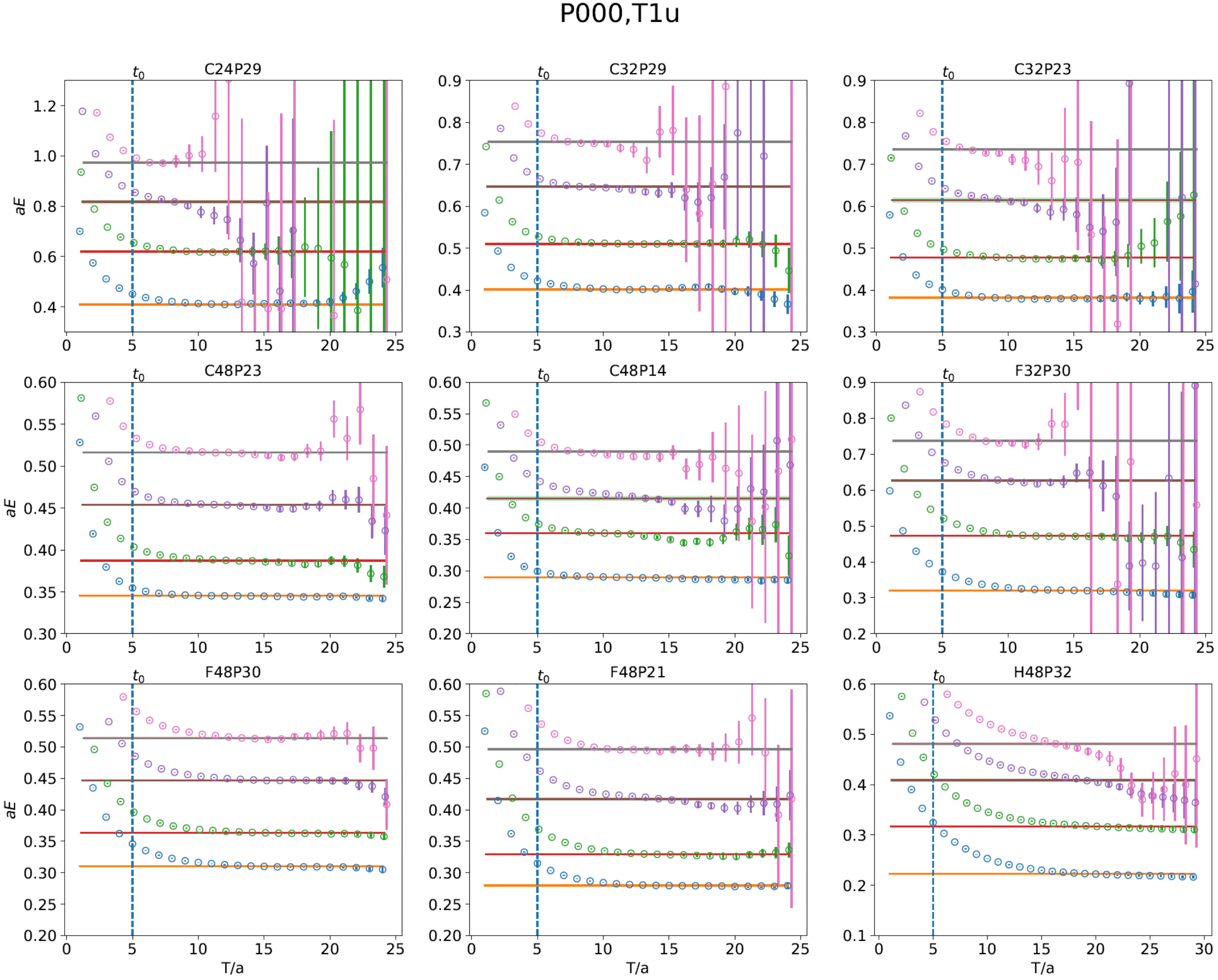
Operators

$[000]T_1^-$	$[001]A_1$	$[001]E_2$	$[011]A_1$
$\rho[000]$	$\rho[001]$	$\rho[001]$	$\rho[011]$
$\pi[001]\pi[00{-1}]$	$\pi[000]\pi[001]$	$\pi[0{-1}0]\pi[011]$	$\pi[000]\pi[011]$
$\pi[011]\pi[0{-1}{-1}]$	$\pi[0{-1}0]\pi[011]$	$\pi[0{-1}{-1}]\pi[111]$	$\pi[{-1}00]\pi[111]$
$\pi[111]\pi[{-1}{-1}{-1}]$	$\pi[{-1}{-1}0]\pi[111]$		$\pi[01{-1}]\pi[002]$
	$\pi[00{-1}]\pi[002]$		

$[011]B_1$	$[011]B_2$	$[111]A_1$	$[002]A_1$
$\rho[011]$	$\rho[011]$	$\rho[111]$	$\rho[002]$
$\pi[010]\pi[001]$	$\pi[{-1}00]\pi[111]$	$\pi[000]\pi[111]$	$\pi[000]\pi[002]$
$\pi[110]\pi[{-1}01]$	$\pi[110]\pi[{-1}01]$	$\pi[100]\pi[011]$	
$\pi[0{-1}1]\pi[002]$		$\pi[200]\pi[{-1}11]$	

$\rho, \pi\pi, \pi\omega, K\bar{K}, \ldots$

Variational analysis: P000/T1u



Totally, we have 9 different configurations, and 8 different spectra for each configuration with different total momentum and irreps, including $|p|^2 = 0$ only T_1^-, $|p|^2 = 1$ with A_1 and E_2, $|p|^2 = 2$ with A_1, B_1 and B_2, and $|p|^2 = 3$ and 4 only with A_1 irreps.

We should mention that since the operator basis we have used does not sample well the inelastic spectrum, we will restrict our analysis to the elastic region below $K\bar{K}$ and $\pi\omega$

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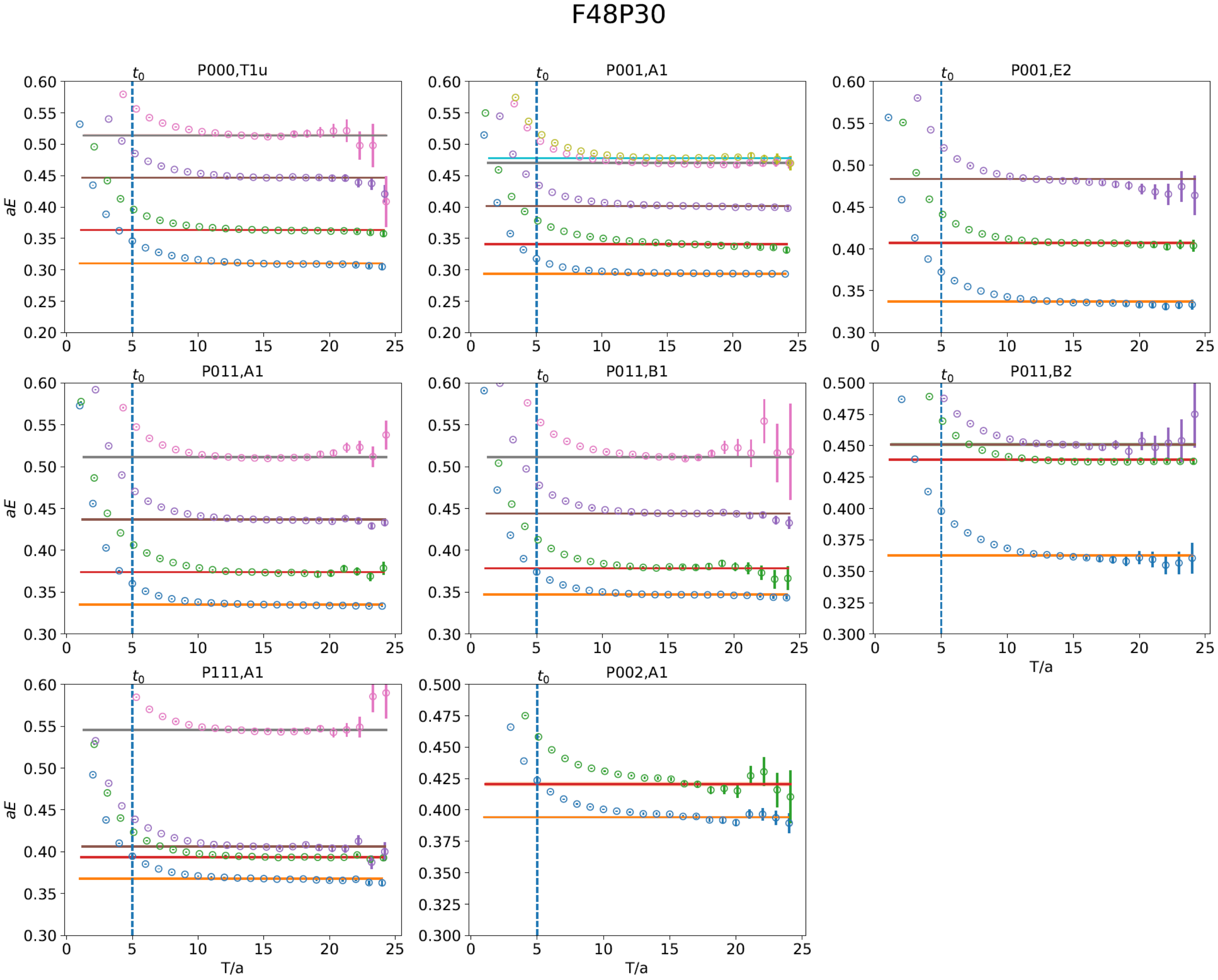
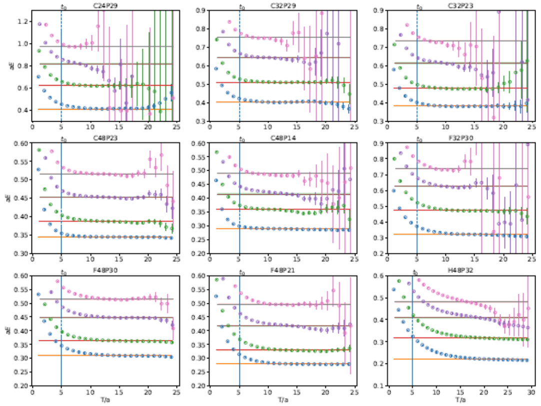
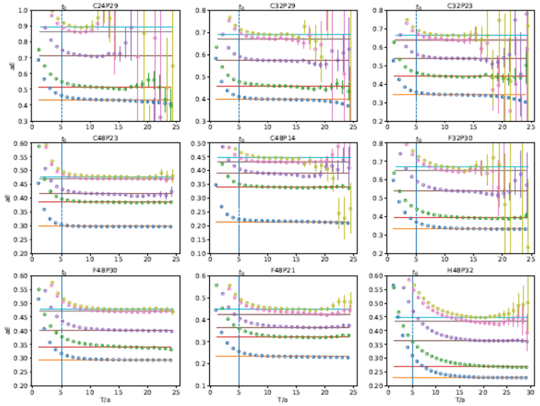
Variational analysis: F48P30



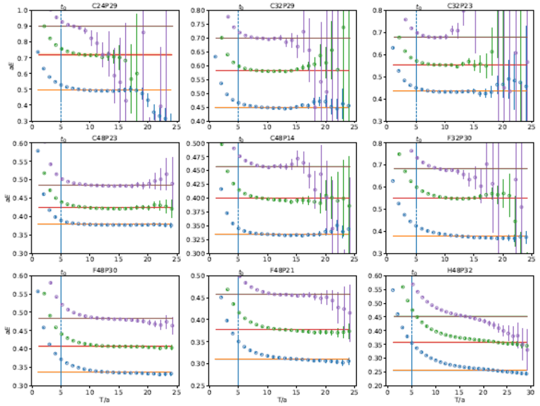
P000,T1u



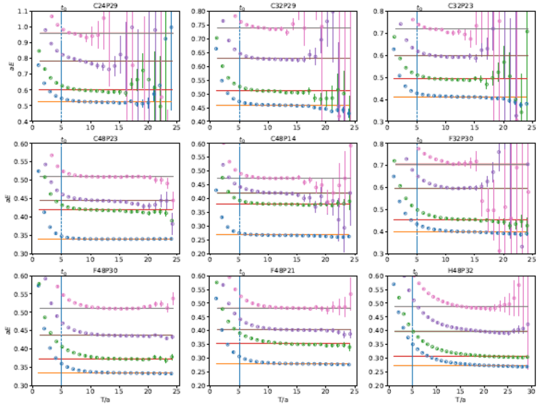
P001,A1



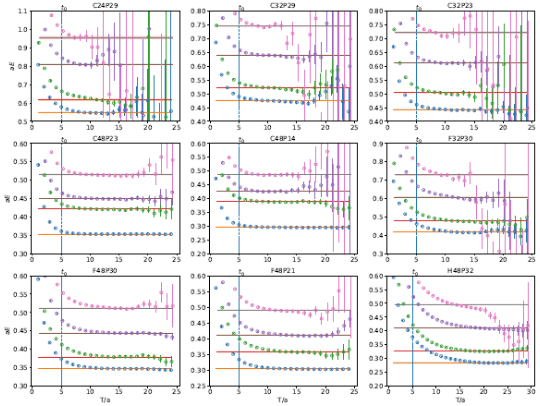
P001,E2



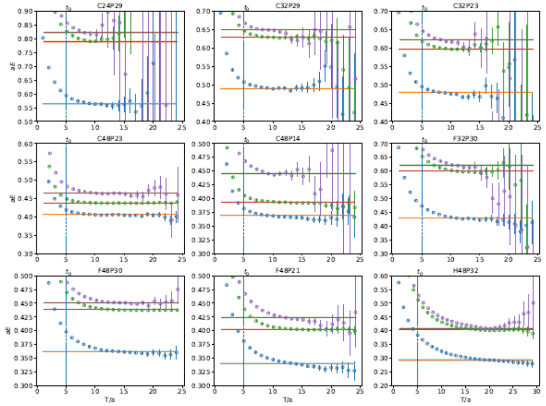
P011,A1



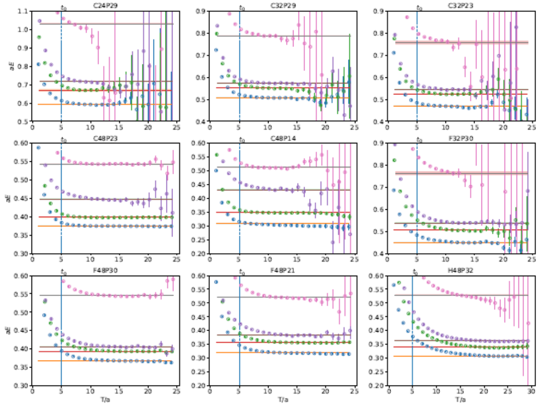
P011,B1



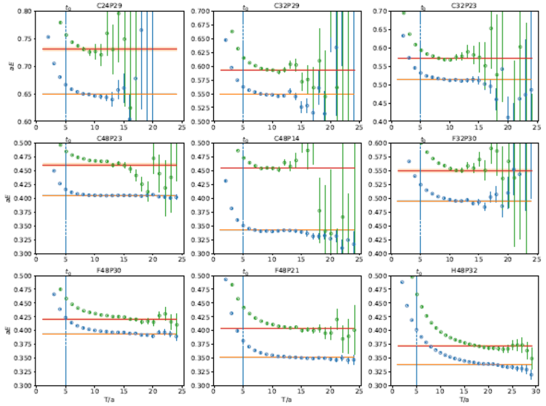
P011,B2



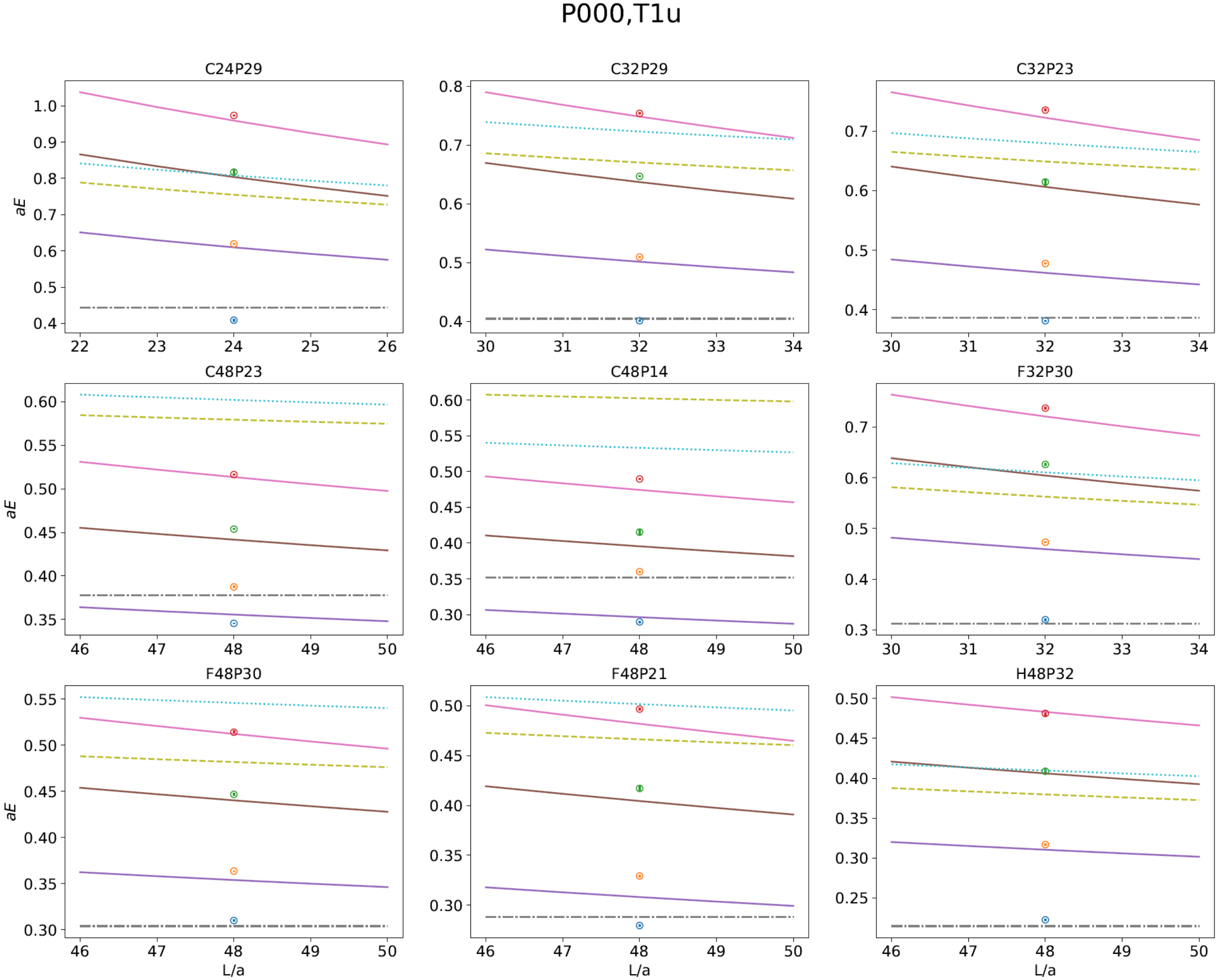
P111,A1



P002,A1

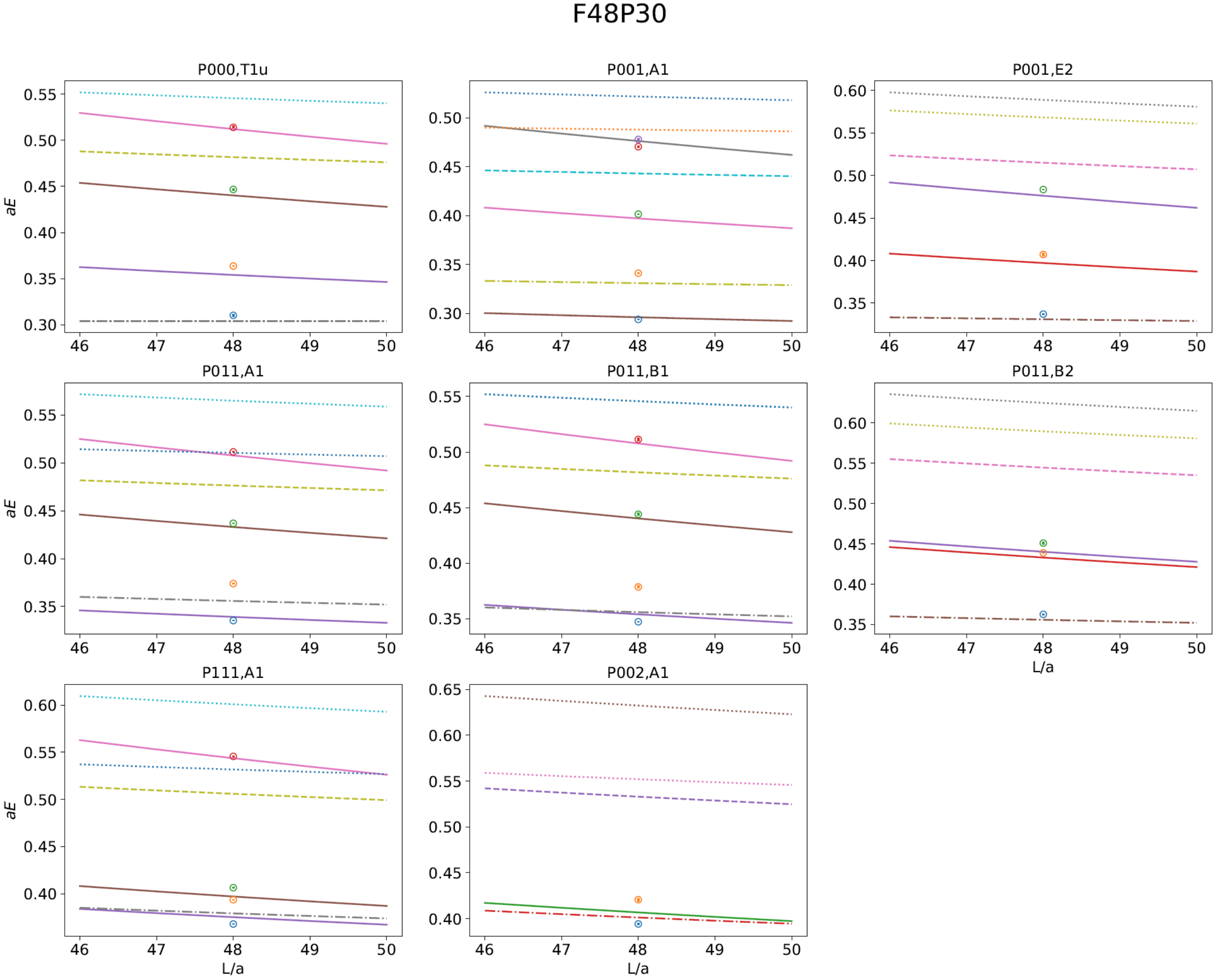


Finite volume spectra: P000/T1u



1. The ρ can decay in some lattice
2. The spectra above ρ lie above the free energy levels of $\pi\pi$, spectra below ρ lie below the free energy levels of $\pi\pi$
3. There is only one energy level between the two free levels of $\pi\pi$
4. Energy levels decrease as the size of the lattice L increases
5. The ρ is more likely to decay in moving systems
6. The energy levels farther from the mass of the ρ being near the free energy levels of $\pi\pi$

Finite volume spectra: F48P30



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Phase shifts from finite-volumn spectra

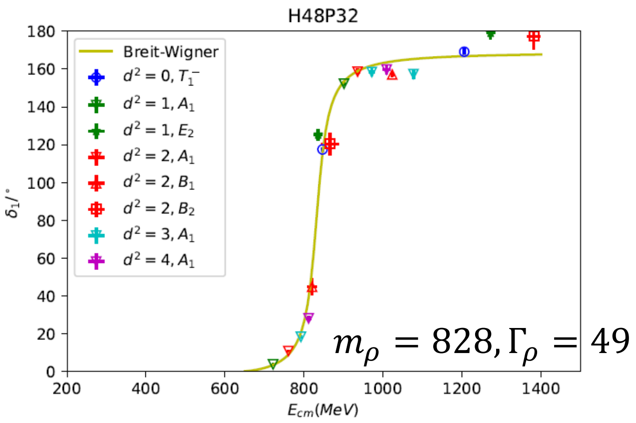
$$\det\left(M_{ln,l'n'}(k) - \delta_{ll'}\delta_{nn'}\cot(\delta_l)\right) = 0,$$

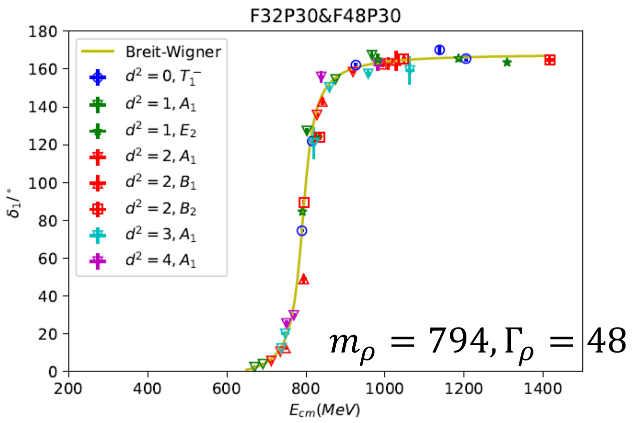
$$\delta_1 = \operatorname{arccot} M_{11,11}^{(\vec{P},\Lambda,\mu)},$$

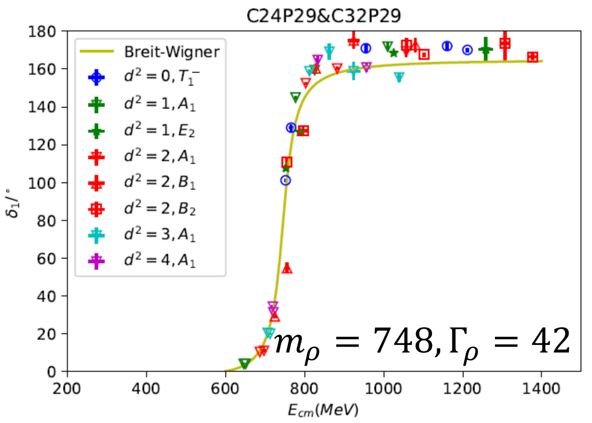
$$w_{j,s} = \frac{\mathcal{Z}_{js}(1,q^2)}{\pi^{3/2}\sqrt{2j+1}\gamma q^{j+1}}, \qquad q = \frac{kL}{2\pi},$$

$\boldsymbol{d}^2$	Λ, μ	$M_{11,11}^{(\vec{P},\Lambda,\mu)}$
0	$T_1^-, 2$	$w_{0,0}$
1	$A_1, 1$	$w_{0,0} + 2w_{2,0}$
1	$E_2, 1$	$w_{0,0} - w_{2,0}$
1	$E_2, 2$	$w_{0,0} - w_{2,0}$
2	$A_1, 1$	$w_{0,0} + \frac{1}{2}w_{2,0} + i\sqrt{6}w_{2,1} - \frac{\sqrt{6}}{2}w_{2,2}$
2	$B_1, 1$	$w_{0,0} + \frac{1}{2}w_{2,0} - i\sqrt{6}w_{2,1} - \frac{\sqrt{6}}{2}w_{2,2}$
2	$B_2, 1$	$w_{0,0} - w_{2,0} + \sqrt{6}w_{2,2}$
3	$A_1, 1$	$w_{0,0} - i2\sqrt{6}w_{2,2}$

Assuming that all $\delta_{l\geq 3}$ are negligible

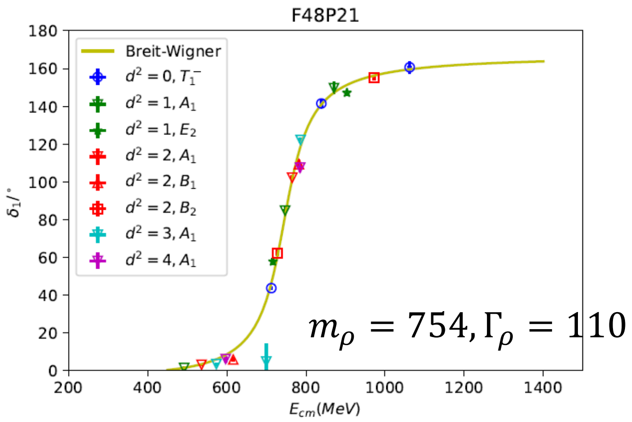


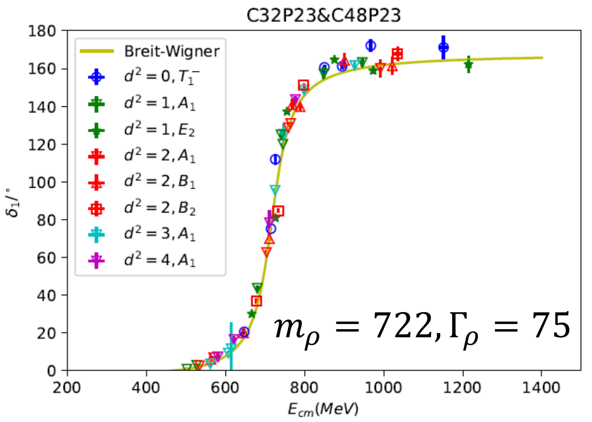




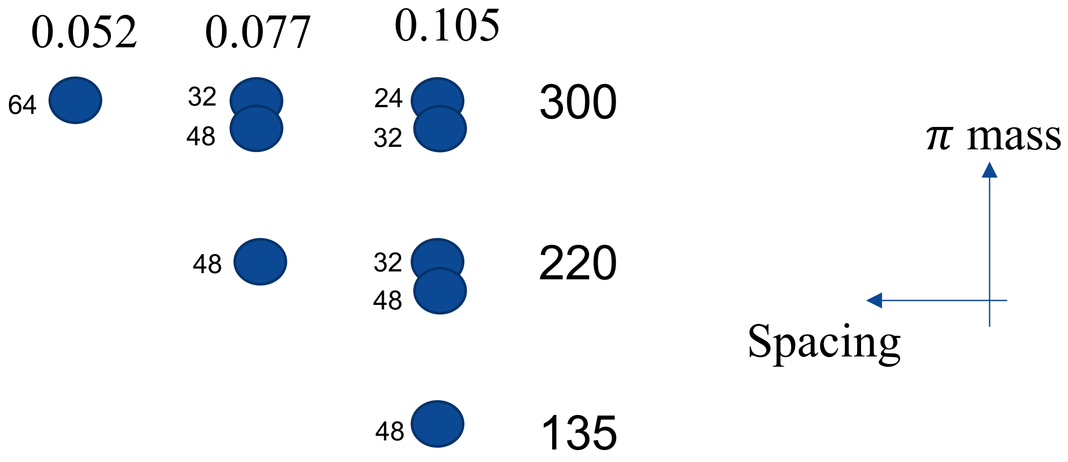
$$a_1 = \frac{-\sqrt{s}\Gamma(s)}{s - m_\rho^2 + \mathrm{i}\sqrt{s}\Gamma(s)} = \mathrm{e}^{\mathrm{i}\delta(s)} \sin\delta(s),$$

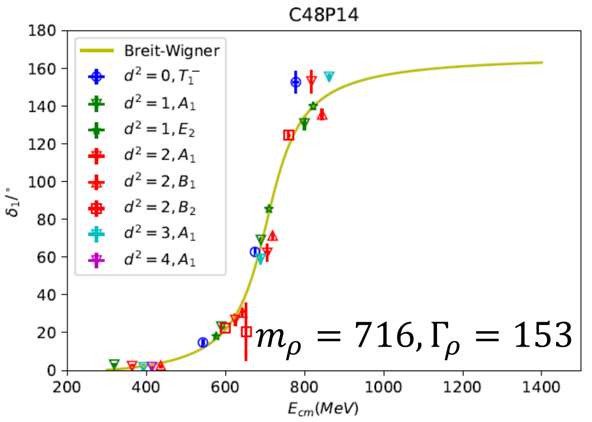
$$\Gamma(s) = \frac{p^{*3}}{s} \frac{g_{\rho\pi\pi}^2}{6\pi}.$$

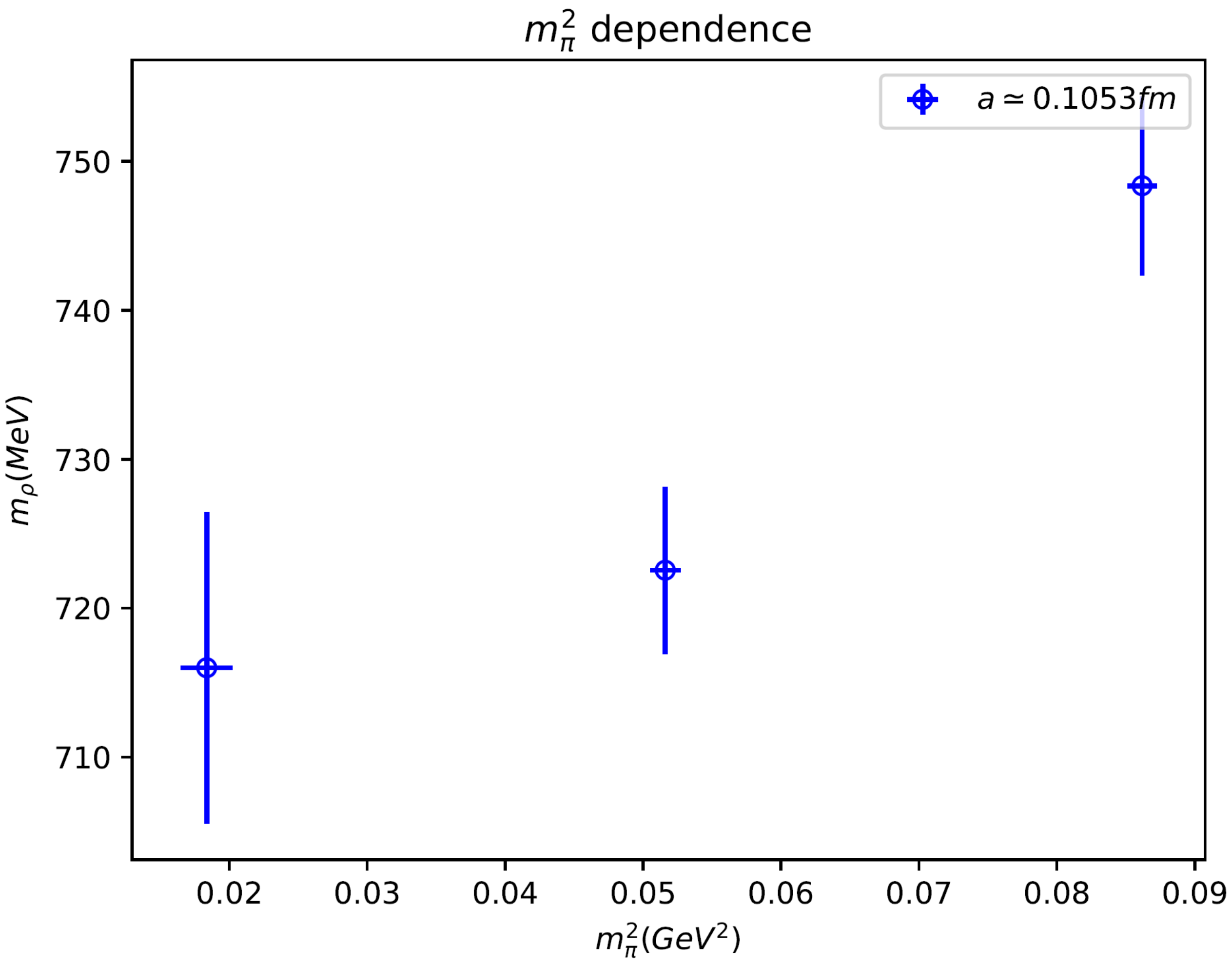


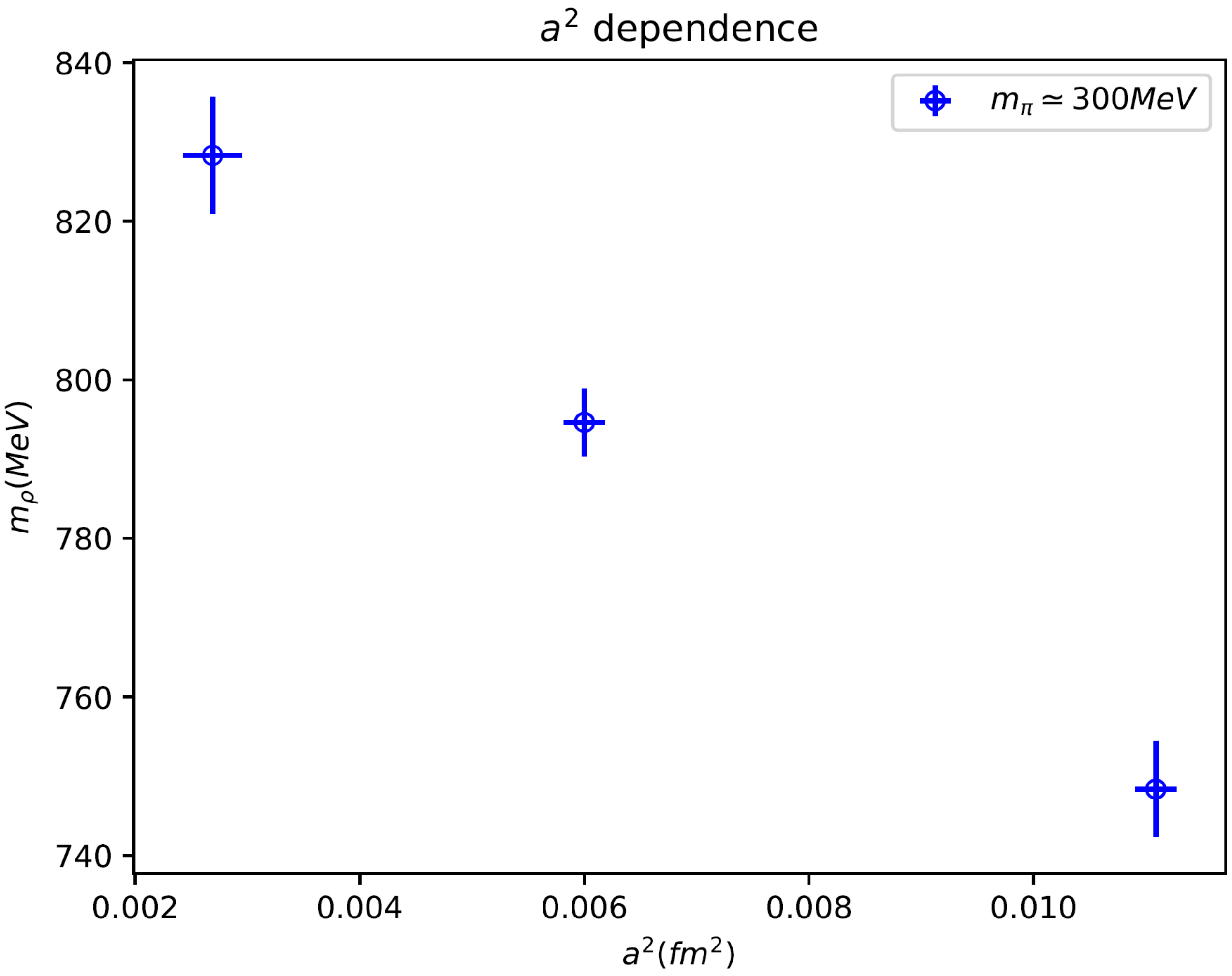


1. The lineshape of the phase shift from large lattice size are even more smooth since $m_\pi L$ is larger
2. There exists the jump of phase shift from 0 degree to 180 degrees









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How important $m_\pi L$

Name	Volume	Spacing	β	m_π/MeV	$m_\pi L$
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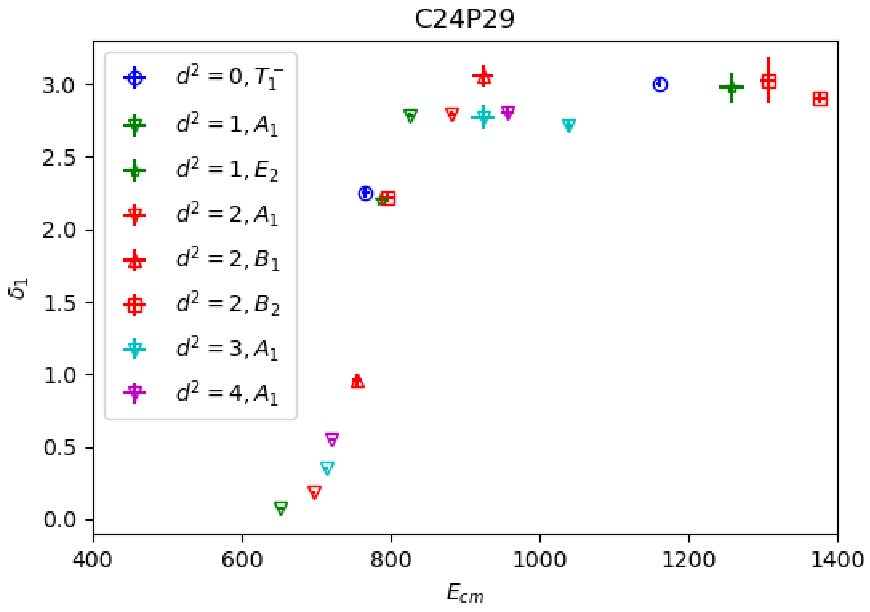

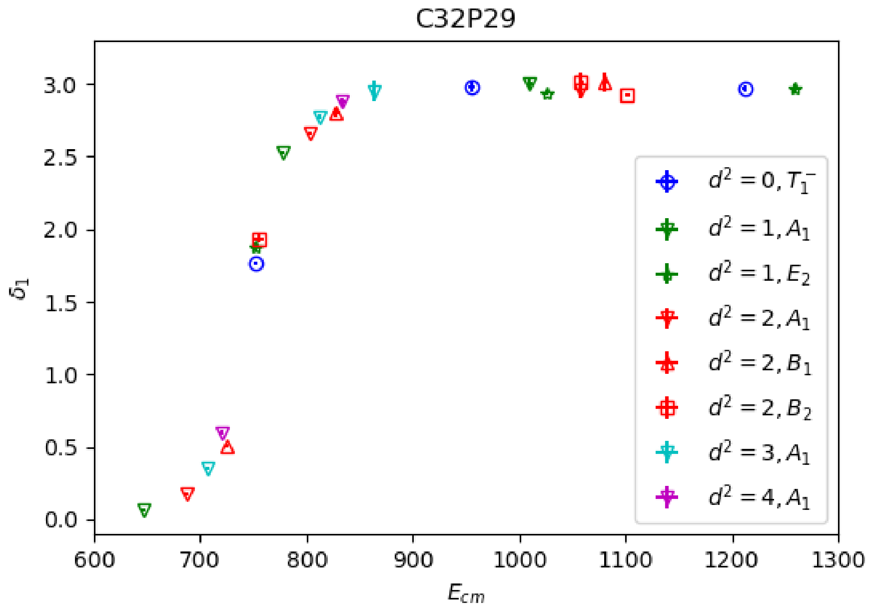

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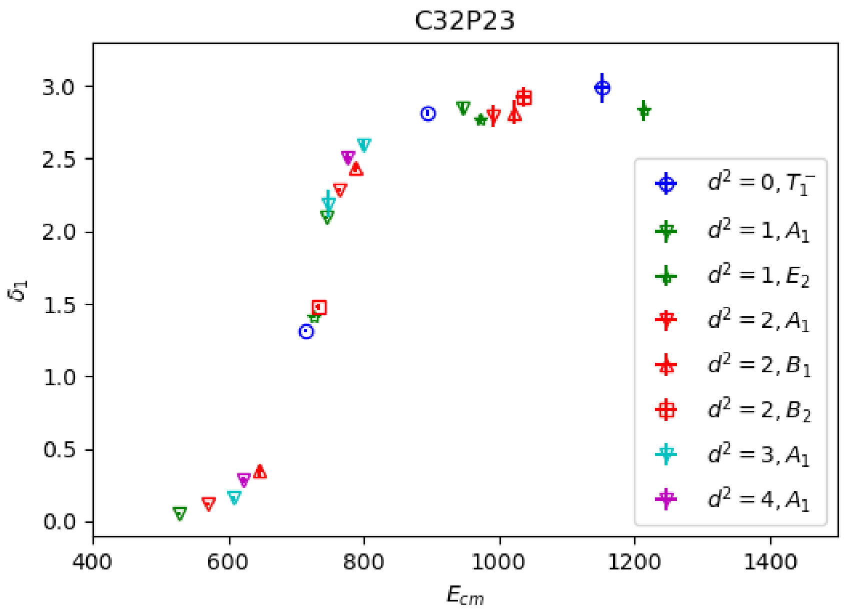

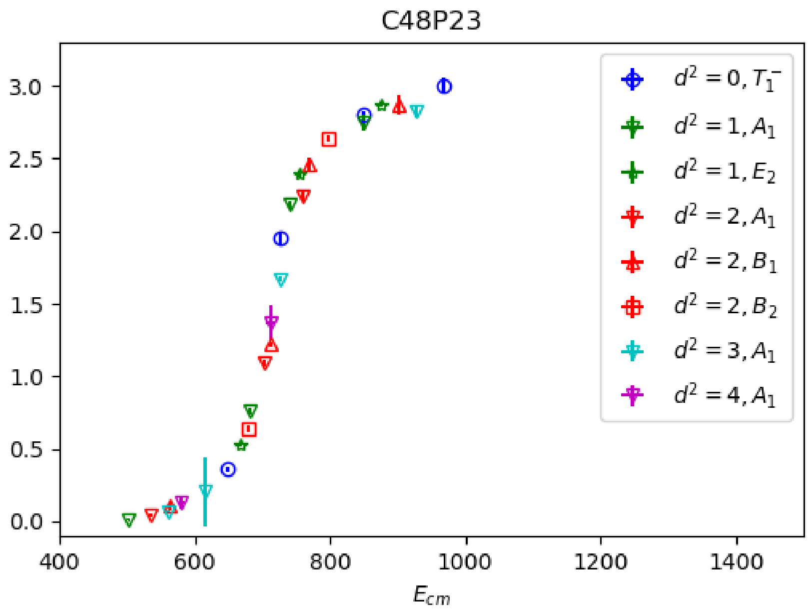

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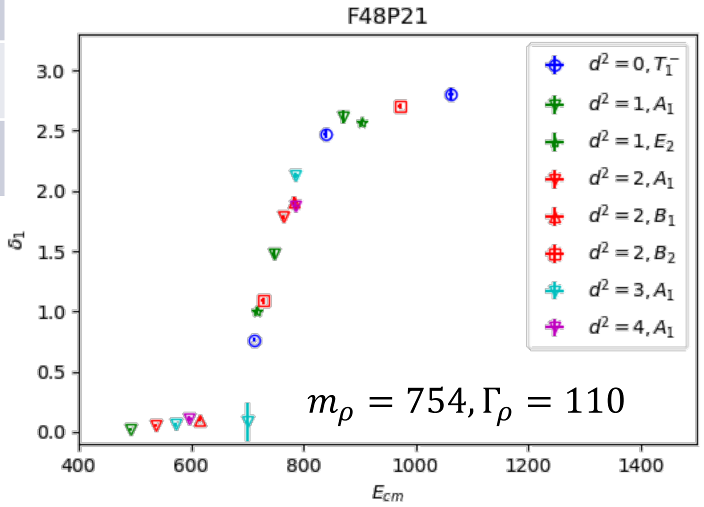

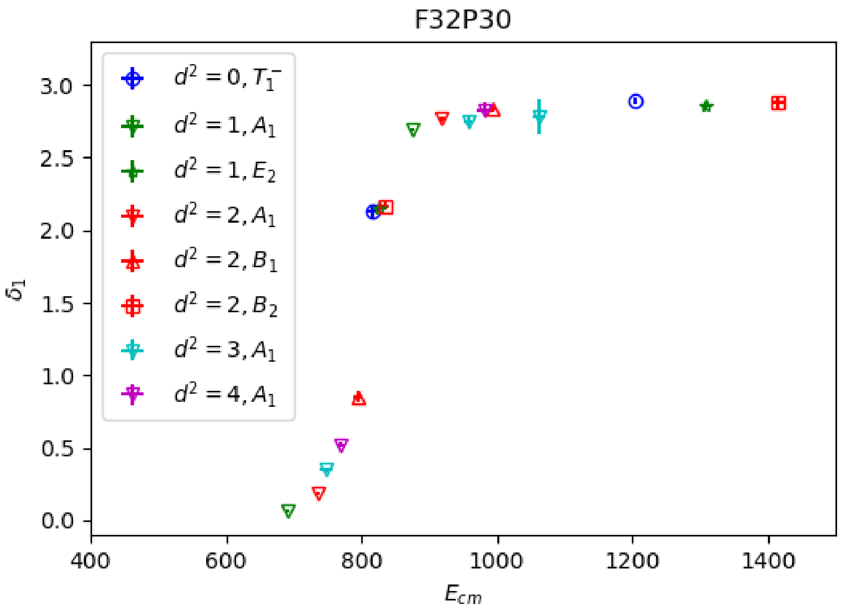

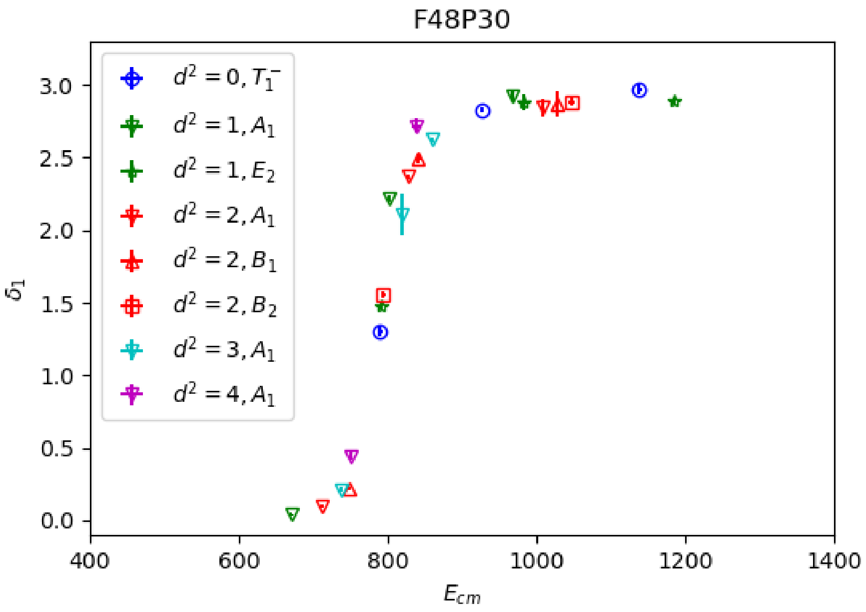

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Chiral extrapolation

$$m_\rho = c_0 + c_1 m_\pi^2 + c_2 a^2$$

$$m_\rho = 775.8(8.5)\mathrm{MeV}, \quad \Gamma_\rho(m_\rho) = 160(10)\mathrm{MeV},$$

$$Z_{\mathrm{pole}} = 766.7(8.6) - i76.5(4.4)\mathrm{MeV}.$$

Here we take $m_{\pi-phy} = m_{\pi^0} \sim 135$ MeV, because here we do not consider isospin breaking and $\pi^\pm$ should suffer large effect from electromagnet interaction.

$\rho(770)$ T-MATRIX POLE $\sqrt{s}$	$(761-765) - i(71-74)$ MeV
$\rho(770)$ **MASS**	
NEUTRAL ONLY, e^+e^-	775.26 ± 0.23 MeV
CHARGED ONLY, τ DECAYS and e^+e^-	775.11 ± 0.34 MeV
MIXED CHARGES, OTHER REACTIONS	763.0 ± 1.2 MeV
CHARGED ONLY, HADROPRODUCED	766.5 ± 1.1 MeV
NEUTRAL ONLY, PHOTOPRODUCED	769.2 ± 0.9 MeV
NEUTRAL ONLY, OTHER REACTIONS	769.0 ± 0.9 MeV (S = 1.4)
$m_{\rho(770)^0} - m_{\rho(770)^\pm}$	-0.7 ± 0.8 MeV (S = 1.5)
$m_{\rho(770)^+} - m_{\rho(770)^-}$	
$\rho(770)$ RANGE PARAMETER	$5.3^{+0.9}_{-0.7}$ GeV^{-1}
$\rho(770)$ **WIDTH**	
NEUTRAL ONLY, e^+e^-	147.4 ± 0.8 MeV (S = 2.0)
CHARGED ONLY, τ DECAYS and e^+e^-	149.1 ± 0.8 MeV
MIXED CHARGES, OTHER REACTIONS	149.5 ± 1.3 MeV
CHARGED ONLY, HADROPRODUCED	150.2 ± 2.4 MeV
NEUTRAL ONLY, PHOTOPRODUCED	$151.5^{+1.9}_{-2.1}$ MeV
NEUTRAL ONLY, OTHER REACTIONS	150.9 ± 1.7 MeV (S = 1.1)
$\Gamma_{\rho(770)^0} - \Gamma_{\rho(770)^\pm}$	0.3 ± 1.3 MeV (S = 1.4)
$\Gamma_{\rho(770)^+} - \Gamma_{\rho(770)^-}$	1.8 ± 2.1

Summary

- We have presented an investigation of the ρ properties using lattice QCD with $N_f = 2 + 1$ Wilson-Clover ensembles including three values of the lattice spacing and a range of pion mass values. Through the analysis of the lattice spectra of various different pion masses and lattice spacings, we obtain the pole position of ρ at physical pion mass and continuum limit simultaneously
- This is a good example demonstrating that QCD can accurately describe the properties of strong interaction-hadrons in the low-energy region
- We will study the role of $K\bar{K}$ channel in future work

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感谢！