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Accurate quark mass determination using the exact chiral fermion

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CONTENT

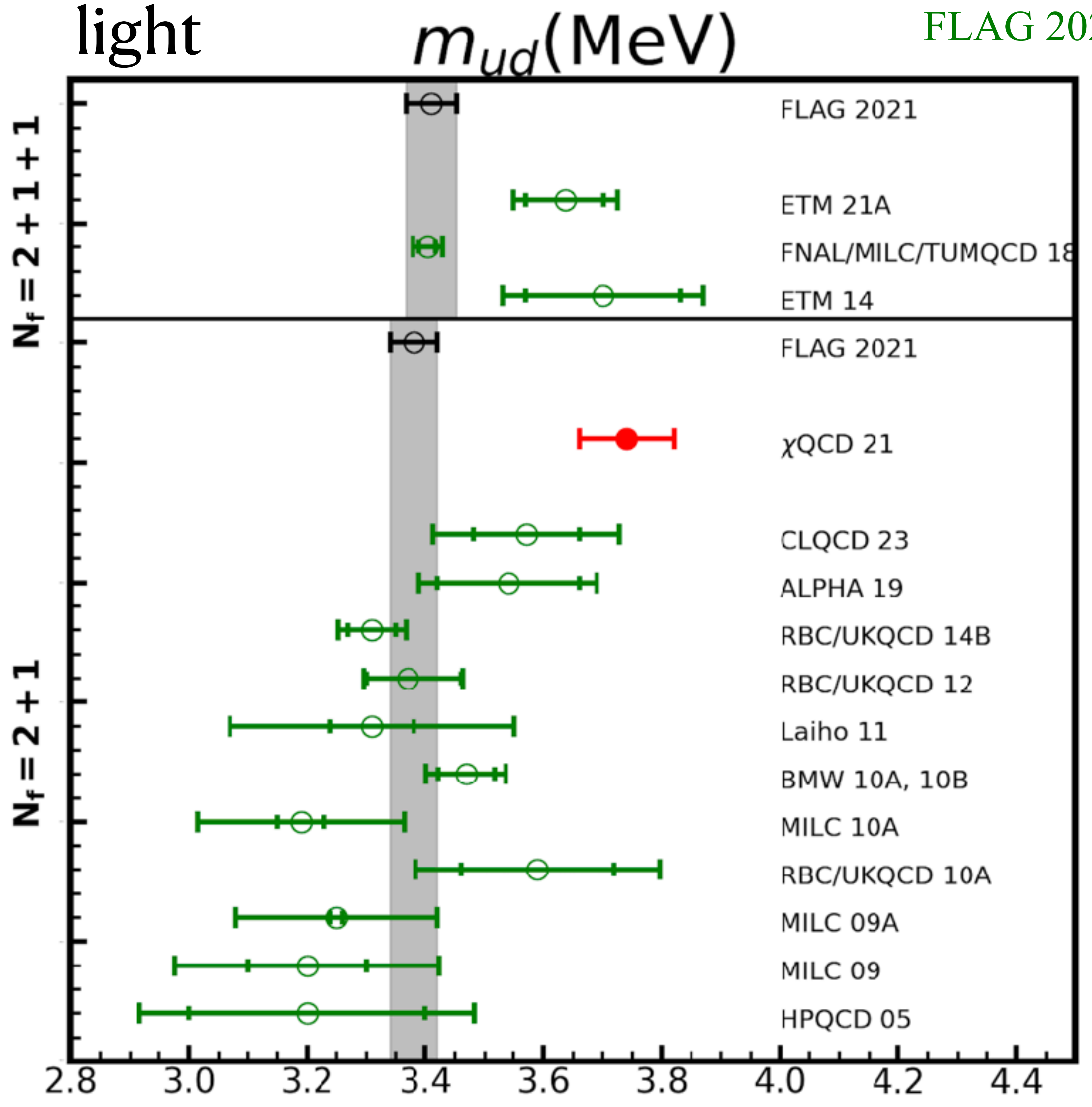
1.1. Discretized fermion scheme

Fermion Type	**Staggered (HISQ)**	**Wilson (Clover)**	**Domain Wall Fermion**	**Overlap**
	Reduce discrete error (→ HISQ, → Clover)			
Collaboration	MILC, …	CLS, CLQCD, …	RBC/UKQCD, …	χQCD, JLQCD, …
Fermion Doubling	4 times degeneracy	✅	$N_5 \to \infty$ $D_{GDW} \to D_{OV}$	✅
Chiral Symmetry	$\gamma_5 \to \eta_5 = (-1)^{\sum_{i=1}^{4} x_i}$ $D_{ST}\eta_5 + \eta_5 D_{ST} = 0$	❌		Ginsparg-Wilson ✅
Computational Complexity	Lv. 1	Lv. 2	Lv. 3	Lv. 4

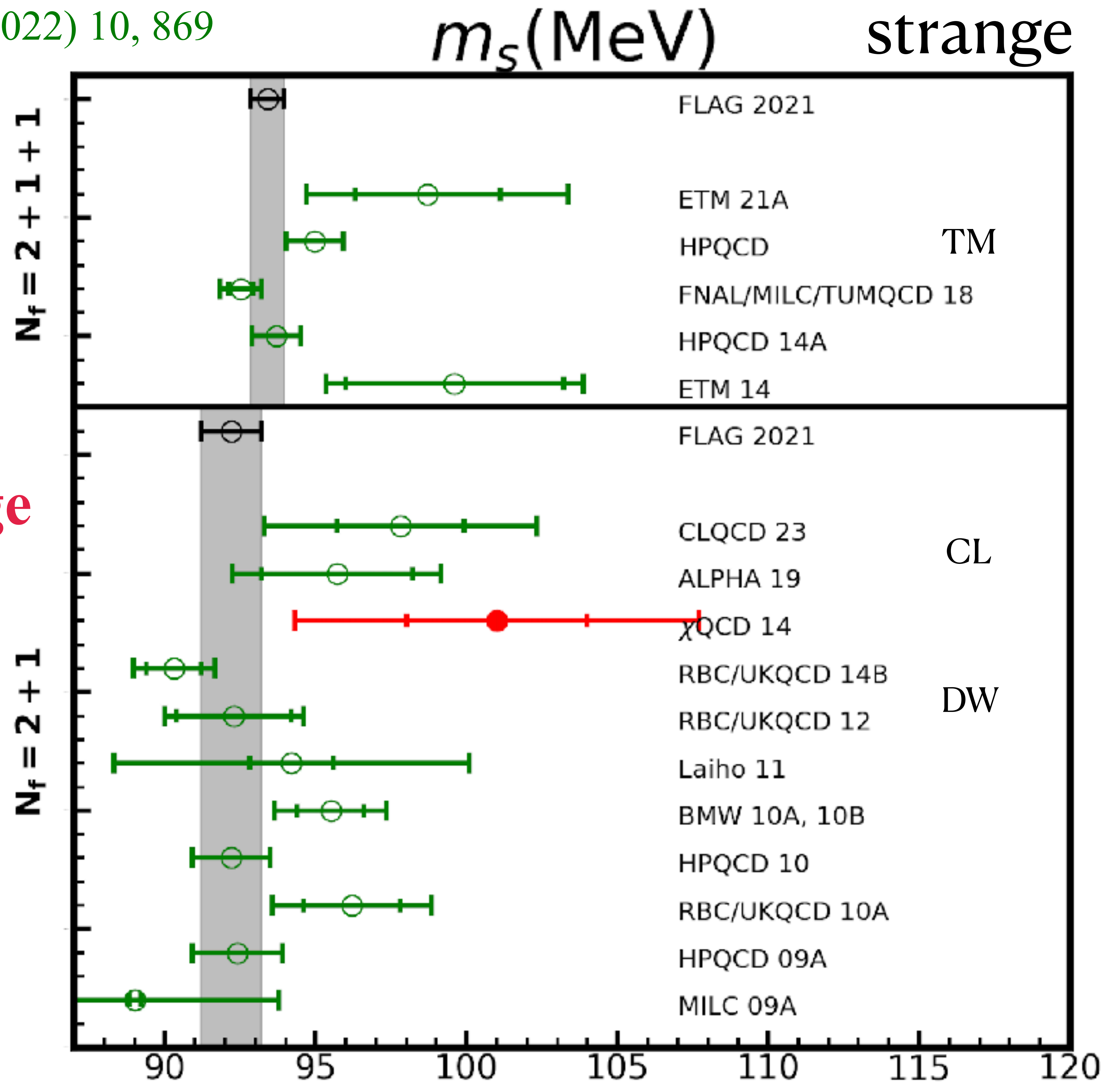
1-2. Preliminary study on quark mass

FLAG 2021, *Eur.Phys.J.C* 82 (2022) 10, 869

light



strange



3 disadvantage 😭

D. Zhao et.al, *PoS* LATTICE2021 (2022) 198
χQCD21 Overlap on DWF $M_\pi \approx 135$MeV

A more detailed analysis on overlap fermions is needed.

Y.B. Yang et.al, *Phys.Rev.D* 92 (2015) 3, 034517
χQCD14 Overlap on DWF $M_\pi \approx 300$MeV

1-2. Preliminary study on quark mass

Low mode substitution

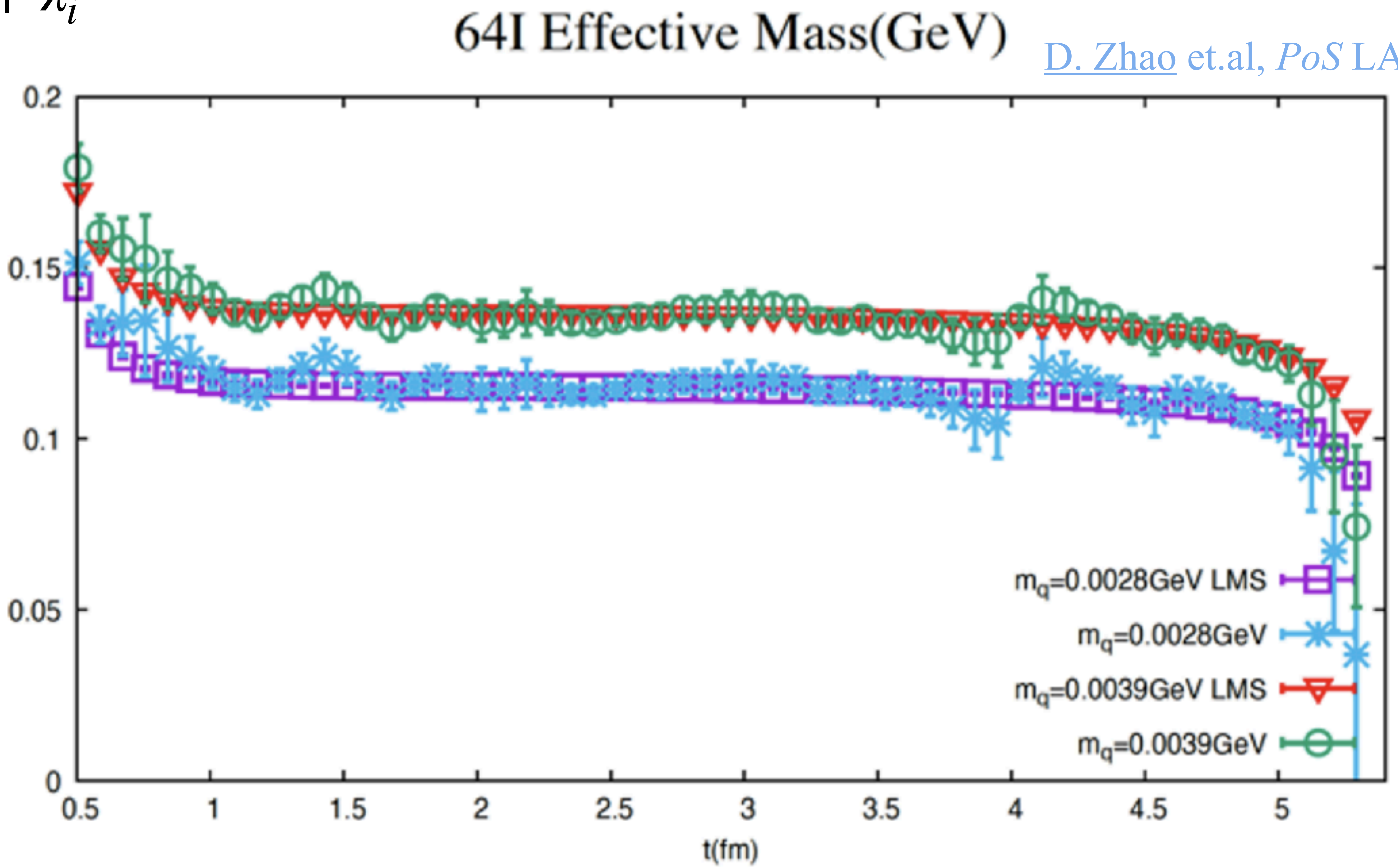
A. Li et.al, *Phys.Rev.D* 82 (2010), 114501

$$S(x,y) = \sum_{\lambda_i} \frac{1}{m+\lambda_i}$$

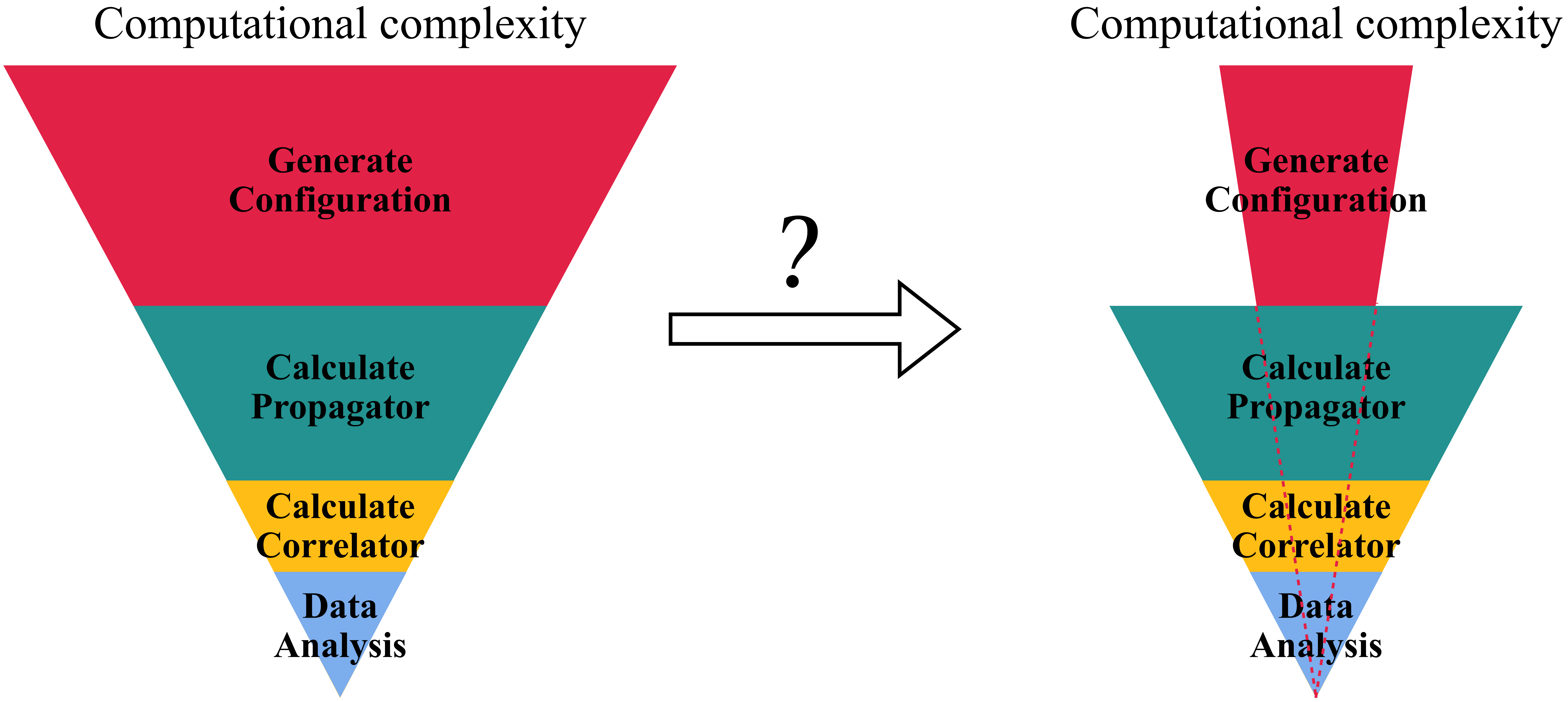
$$C_2(t) = \frac{1}{N_{\mathrm{grid}}} \sum_{\vec{y},\vec{x}_{i,j}\in \mathrm{grid}} \langle S_1(\vec{y},t;\vec{x}_i,0)S_2^\dagger(\vec{y},t;\vec{x}_j,0) - S_{1,\mathrm{L}}(\vec{y},t;\vec{x}_i,0)S_{2,\mathrm{L}}^\dagger(\vec{y},t;\vec{x}_j,0)\rangle$$

$$+\frac{1}{L^3\times T}\sum_{\vec{y},\vec{z},t_0}\langle S_{1,\mathrm{L}}(\vec{y},t+t_0;\vec{z},t_0)S_{2,\mathrm{L}}^\dagger(\vec{y},t+t_0;\vec{z},t_0)\rangle$$

D. Zhao et.al, *PoS* LATTICE2021 (2022) 198



2. Discretized fermion dependence of hadronic spectra in LQCD



2. Discretized fermion dependence of hadronic spectra in LQCD

Definition

MAPQχPT

Val_Val pion mass: $m_{\pi,vv}$

Sea_Sea pion mass: $m_{\pi,ss}$

Val_Sea pion mass: $m_{\pi,vs}$

$$\Delta_{\rm mix}^{v/s}(m_{\pi,\rm vv}, m_{\pi,\rm ss}, a) \equiv m_{\pi,\rm vs}^2 - \frac{m_{\pi,\rm vv}^2 + m_{\pi,\rm ss}^2}{2}$$

Previous works

Valence	Sea	$\delta m_\pi = m_{\pi,vs} - m_{\pi,ss}$ (MeV)	a(fm)	$m_{\pi,ss}$(MeV)	
Overlap	Clover	153	0.09	300	PoS LATTICE2007, 115 (2007)
DW	Staggered	30-60	0.13-0.09	310	Phys. Rev. D86, 014501 (2012)
Overlap	DW	~10	0.11-0.08	300-400	Phys. Rev. D77, 094505 (2008) Phys. Rev. D77, 114501 (2008) Phys. Rev. D96, 054513 (2017)

Innovation

1. based on the calculation at different lattice spacing
2. pion masses $m_{\pi,ss}$ are not limited to the case of ~300MeV
3. mixed action effect with kinds of the valence and sea fermion combinations

2. Discretized fermion dependence of hadronic spectra in LQCD

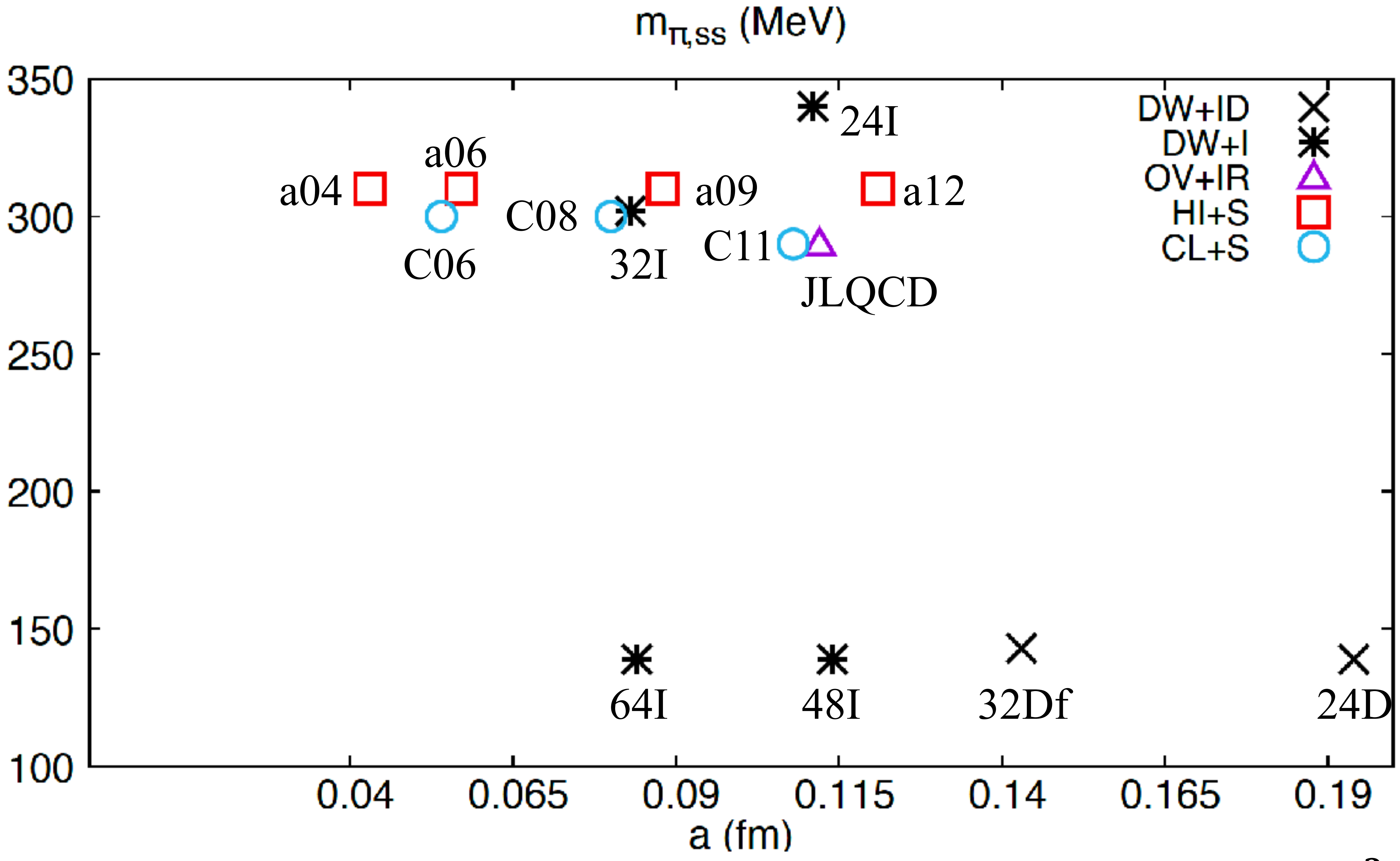
Valence fermion actions

HI: HISQ fermion without any additional smearing on the gauge link;

CL: Clover fermion with 1-step HYP smearing and tree level tadpole improved clover coefficient c_{sw};

OV: Overlap fermion with 1-step HYP smearing and $\rho = 1.5$.

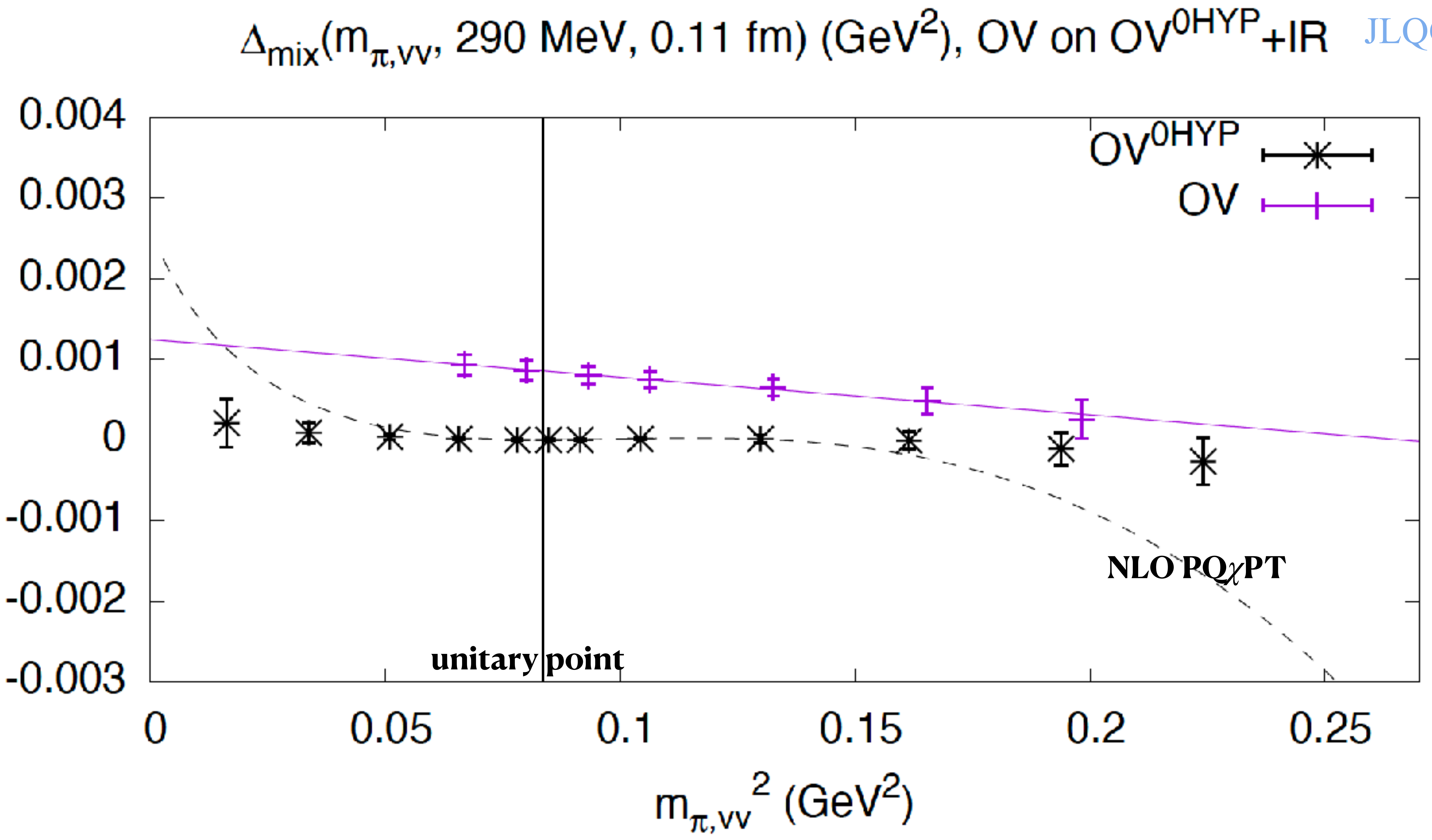
Sea fermion actions



2. Discretized fermion dependence of hadronic spectra in LQCD

Contribution of the delta mix definition to the NLO at $m_{\pi,\mathrm{vv}} \neq m_{\pi,\mathrm{ss}}$

JLQCD H. Fukaya et.al, *Phys.Rev.D* 83, 074501 (2011)



$$\Delta_{\mathrm{mix}}(m_{\pi,\mathrm{vv}}, m_{\pi,\mathrm{ss}}, a) \equiv m_{\pi,\mathrm{vs}}^2 - \frac{m_{\pi,\mathrm{vv}}^2 + m_{\pi,\mathrm{ss}}^2}{2}$$

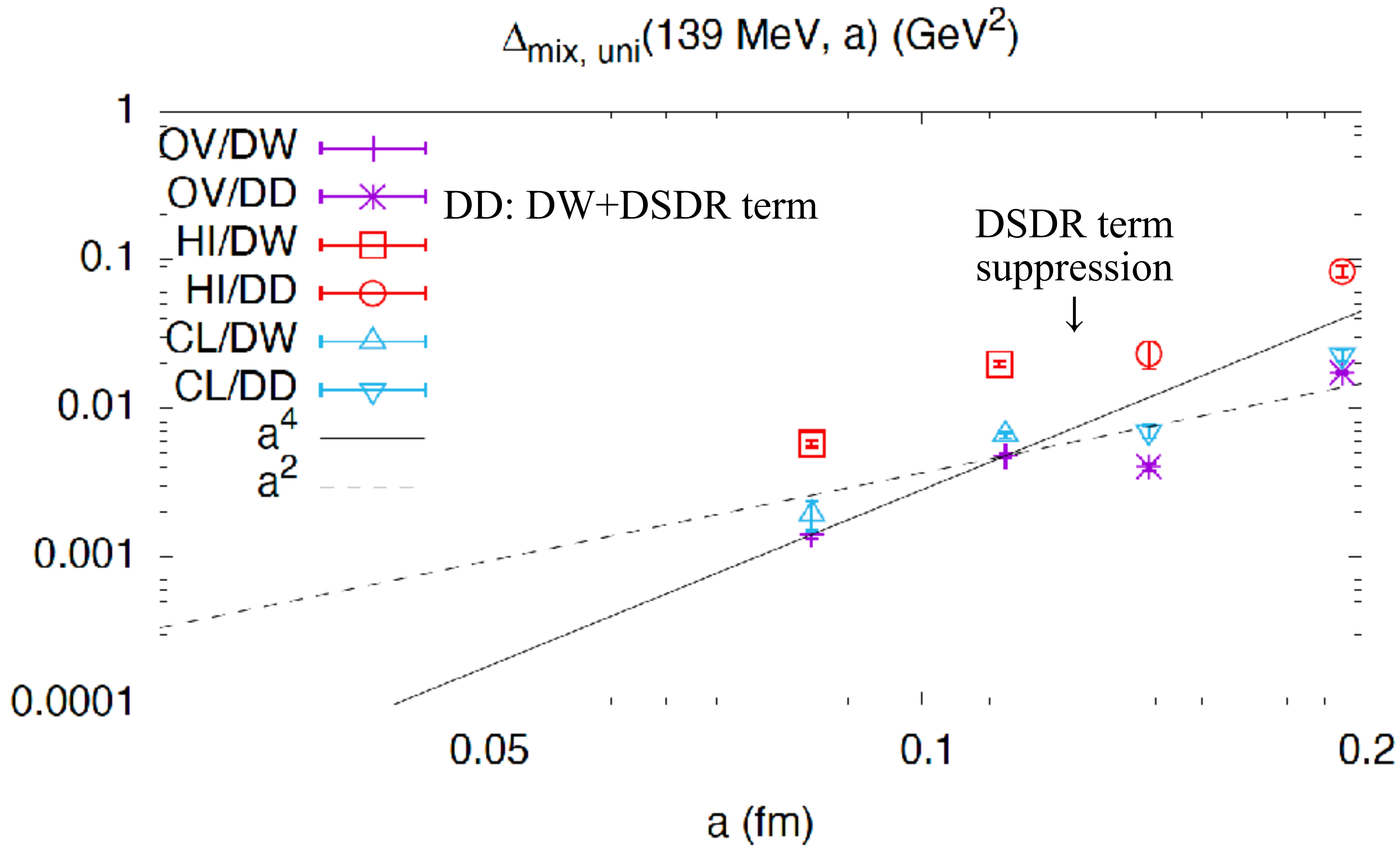
$$\downarrow \; m_{\pi,\mathrm{vv}} = m_{\pi,\mathrm{ss}}$$

$$\Delta_{\mathrm{mix,uni}}(m_\pi, a) \equiv \Delta_{\mathrm{mix}}(m_\pi, m_\pi, a)$$

2. Discretized fermion dependence of hadronic spectra in LQCD

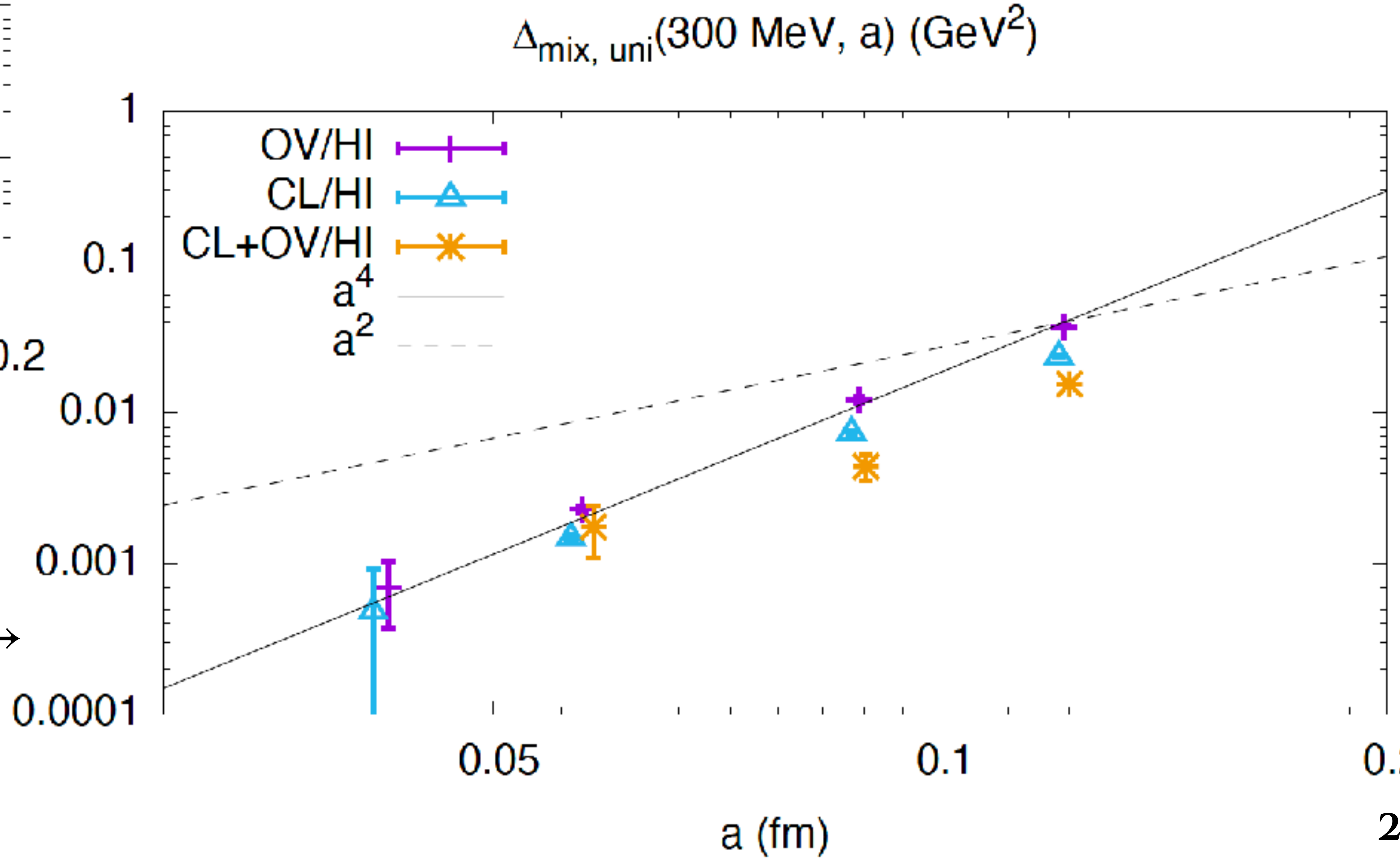
$$\Delta^{B/A}_{\rm mix}(m_\pi, a) \equiv m^2_{\pi,\rm AB} - \frac{m^2_{\pi,\rm AA} + m^2_{\pi,\rm BB}}{2}\Big|_{m_{\pi,\rm BB}=m_{\pi,\rm AA}=m_\pi}$$

$$\bar{\Delta}^{\rm B+C/A}_{\rm mix,uni}(m_\pi, a) \equiv m^2_{\pi,\rm BC} - \frac{m^2_{\pi,\rm BB} + m^2_{\pi,\rm CC}}{2}\Big|_{m_{\pi,\rm BB}=m_{\pi,\rm CC}=m_{\pi,\rm AA}=m_\pi}$$



Delta mix under Domain wall sea

Delta mix under HISQ sea



$$|\,\Delta^{\rm B/A}_{\rm mix,uni} - \Delta^{\rm C/A}_{\rm mix,uni}\,| \le \bar{\Delta}^{\rm B+C/A}_{\rm mix,uni} \le \Delta^{\rm B/A}_{\rm mix,uni} + \Delta^{\rm C/A}_{\rm mix,uni} \longrightarrow$$

Δ_{mix} is less important than other discretization error under small lattice spacing

3. Renormalization scheme in lattice QCD

1-step matching

1. Fit $\frac{Z_S}{Z_V}^*(m_q a) = \frac{A_S}{(m_q a)^2} + \frac{Z_S}{Z_V}^{chiral} + C_S(m_q a)$

2. Times matching factor $\frac{Z_S}{Z_V}^{\overline{\text{MS}}} = C_S^{\overline{\text{MS}},r} \frac{Z_S}{Z_V}^r$

3. Extrapolate $Z_S^{\overline{\text{MS}}}(a, p^2)$ to $a^2p^2 = 0$ as $Z_S^{\overline{\text{MS}}}(0,0)$

4. $m_q^{\overline{\text{MS}}} = m_q^{bare} / Z_s^{\overline{\text{MS}}}$

$r \in \{RI/MOM, RI'/MOM, RI/SMOM\}$

Also see the poster of Mengchu Cai.

Systematic error related to matching

Source	$Z_q^{\overline{\text{MS}}}/Z_V$	$Z_S^{\overline{\text{MS}}}/Z_V$	$Z_P^{\overline{\text{MS}}}/Z_V$	$Z_T^{\overline{\text{MS}}}/Z_V$
Statistical	0.04	0.08	0.21	0.01
Conversion ratio	0.34	2.29	2.15	0.40
Perturbative running	0.03	0.11	0.11	0.03
$\Lambda_{\text{QCD}}^{\overline{\text{MS}}}$	0.02	0.31	0.26	0.04
Lattice spacing	0.01	0.09	0.09	0.03
Fit range of a^2p^2	0.13	0.03	0.27	0.01
Finite-volume effect	0.02	0.07	0.14	0.01
$m_s^{\text{sea}} \neq 0$	0.17	0.46	1.61	0.06
Total uncertainty	0.41	2.36	2.73	0.41

TABLE VI. Summary of uncertainties of RCs in percentage on the 64I ensemble through the intermediate RI/MOM scheme.

systematic error from the fixed order truncation in the **conversion ratio** becomes significantly larger as the lattice spacing increases

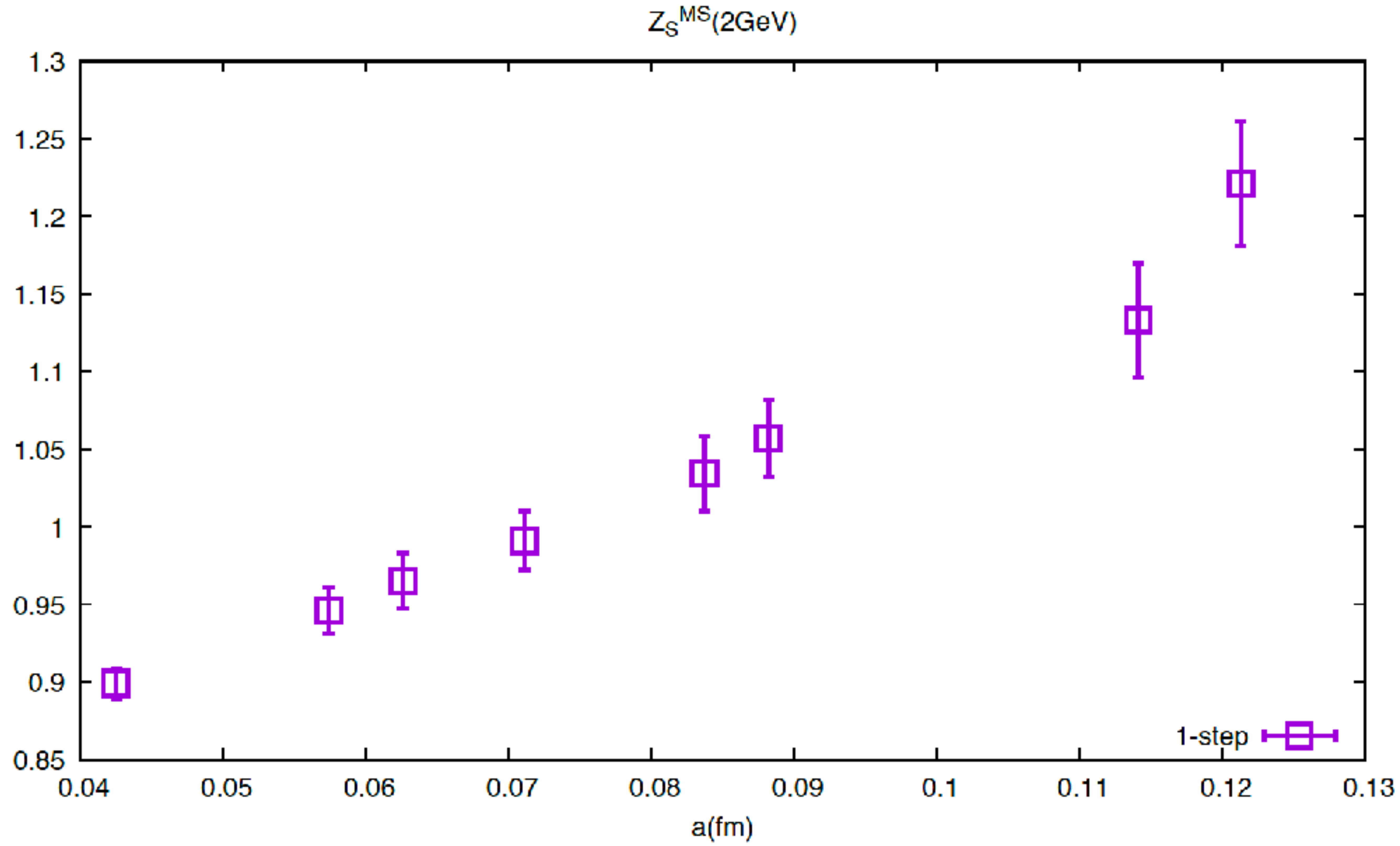
F. He et.al, *Phys.Rev.D* 106 (2022) 11, 114506

3. Renormalization scheme in lattice QCD

Conversion Ratio

$$C_S^{\overline{\text{MS}},\text{RI/MOM}}(\mu) = 1 + 0.4244\alpha_s(\mu) + 1.0068\alpha_s^2(\mu) + 2.7221\alpha_s^3(\mu) + \mathcal{O}(\alpha_s^4)$$

Padè approximation $\mathcal{O}(\alpha_s^4) \approx (2.7221^2/1.0068)\alpha_s^4$



F. He et.al, *Phys.Rev.D* 106 (2022) 11, 114506

2-step matching

$$Z_S^{\overline{\text{MS}}}(\mu_0, 1/a, p^2) = \frac{Z_S^{\overline{\text{MS}}}(\mu_0, 1/a, p^2)}{Z_S^{\overline{\text{MS}}}(\mu_0, 1/a_0, p^2)} Z_S^{\overline{\text{MS}}}(\mu_0, 1/a_0, p^2)$$

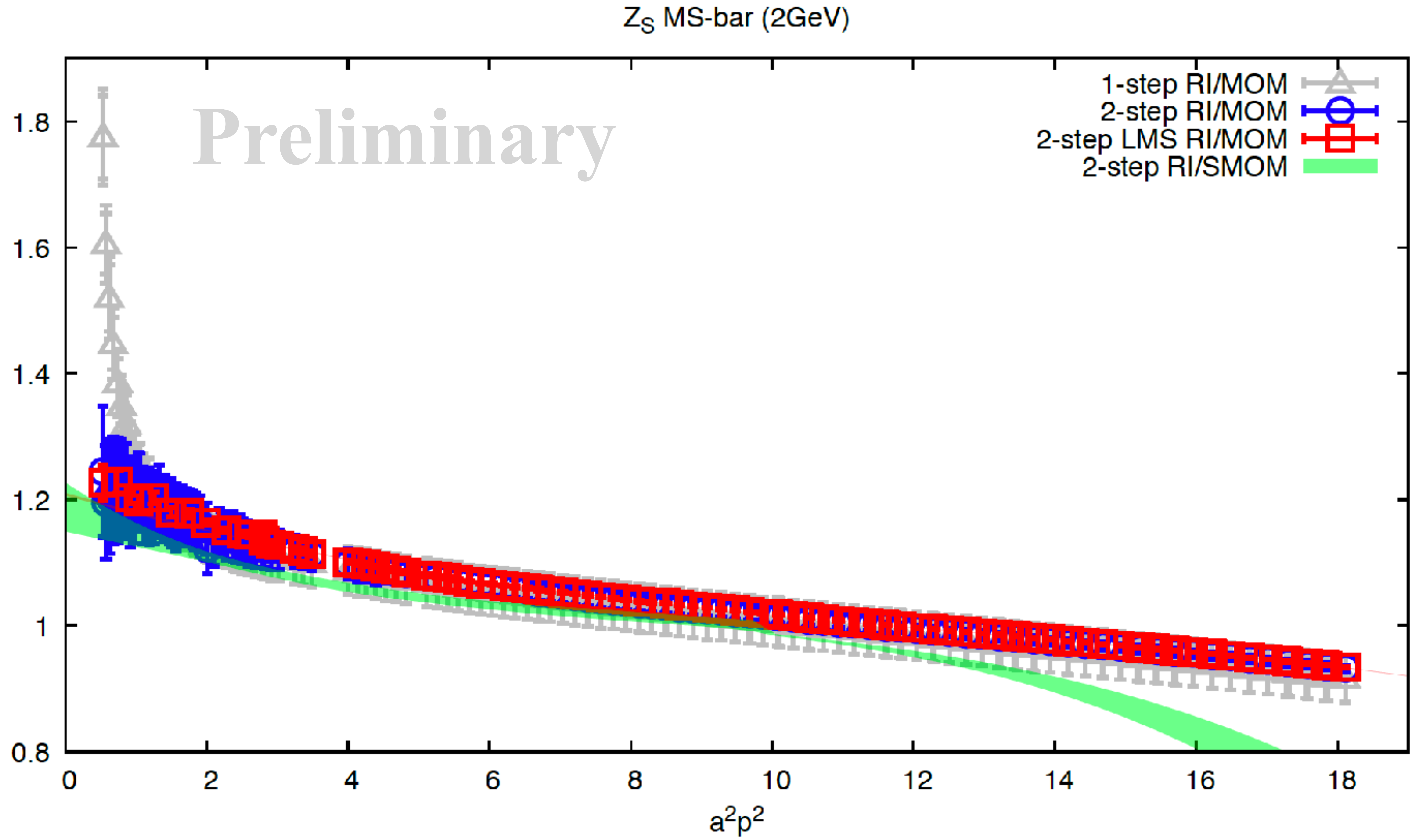
$$= \frac{Z_S^{\text{RI}}(1/a, p^2)}{Z_S^{\text{RI}}(1/a_0, p^2)} [Z_S^{\overline{\text{MS}},1-\text{step}}(\mu_0, 1/a_0) + \mathcal{O}(a_0^2 p^2)],$$

LMS for minimally lattice spacing a_0 ensemble

Systematic errors related to the matching only comes from the statistical error and systematic error of the minimally lattice spacing ensemble

D. Zhao et.al, *in preparation*

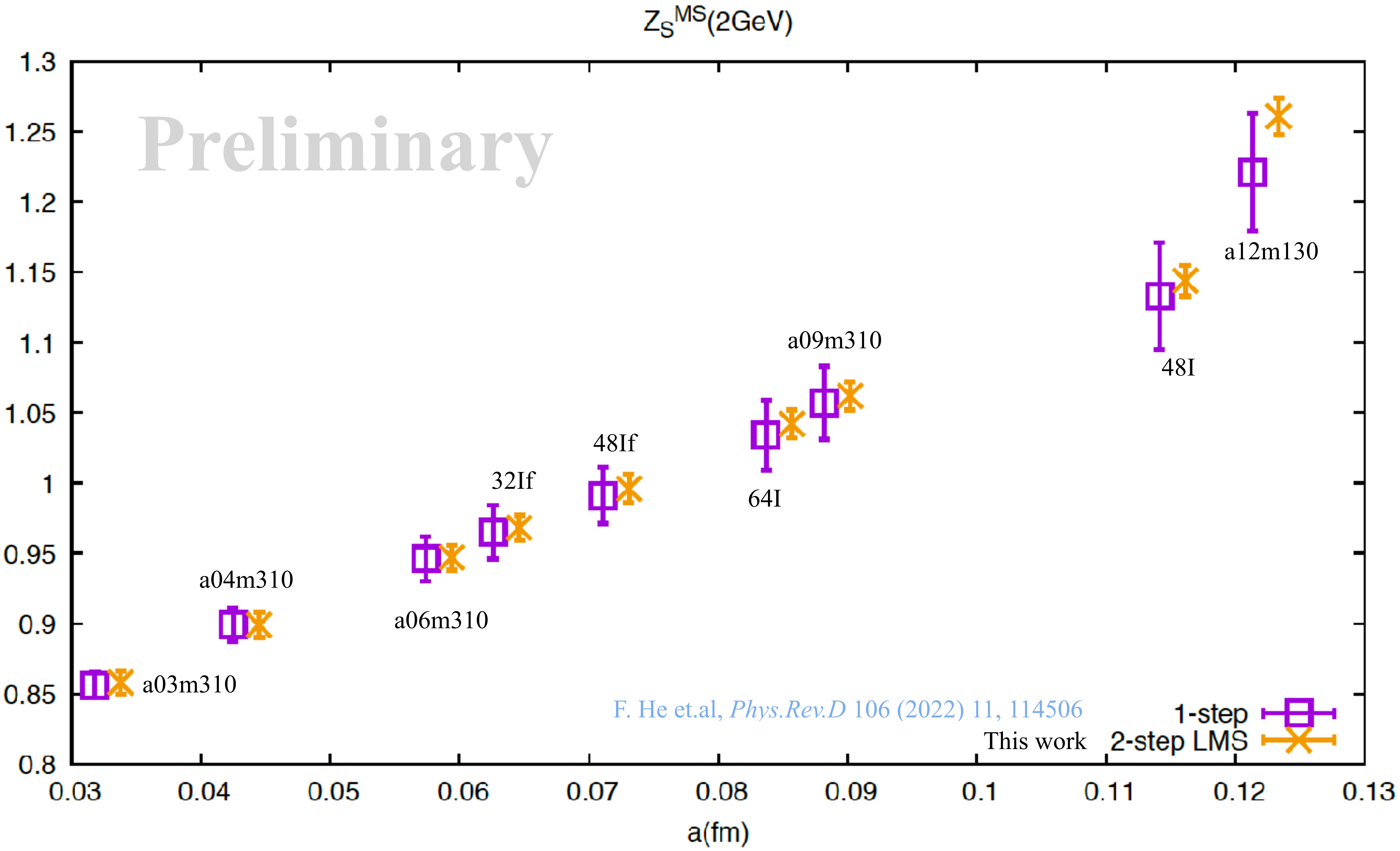
3. Renormalization scheme in lattice QCD



48I(0.1141fm) on a03m310(0.0322fm)

Preliminary result

3. Renormalization scheme in lattice QCD



D. Zhao et.al, *in preparation*

4. Relationship between quark mass and different pseudo-mesons

Valence fermion actions

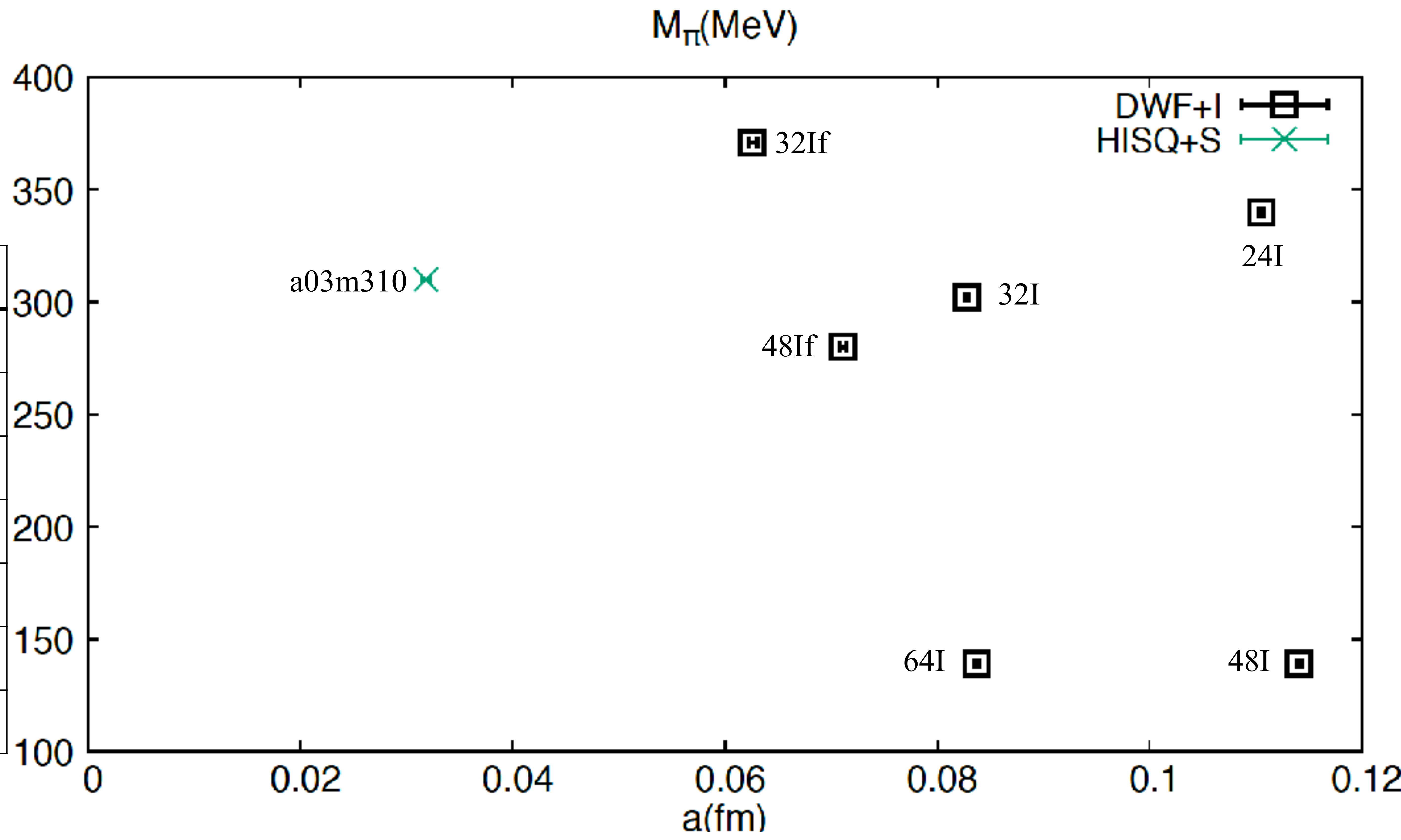
Overlap fermion with 1-step HYP smearing and $\rho = 1.5$.

Choice of χQCD collaboration

Ensemble	L, T	N(glo_fit.)	N(renom.)
48I	48,96	2752	344
24I	24,64	2256	
64I	64,128	1872	312
32I	32,64	1968	
48If	48,96	1424	272
32If	32,64	1446	240
a03m310	96, 288		104

$N = N_{conf} \times N_{source} \times N_{grid}$

Sea fermion actions



4. Relationship between quark mass and different pseudo-mesons

PQχPT

m_l global fit

$$M_\pi^2 = \frac{2\Sigma m_l^v}{F^2}\{1+\frac{2}{N_f}[(2y_v-y_s)\ln(2y_v)+(y_v-y_s)]+2y_v(2\alpha_8-\alpha_5)+2y_sN_f(2\alpha_6-\alpha_4)\}\underline{(1+c_m a^2)(1+c_{ml}e^{-M_\pi L}+c_{ms}(M_{\eta_s}^2-M_{\eta_s,phys}^2))}$$

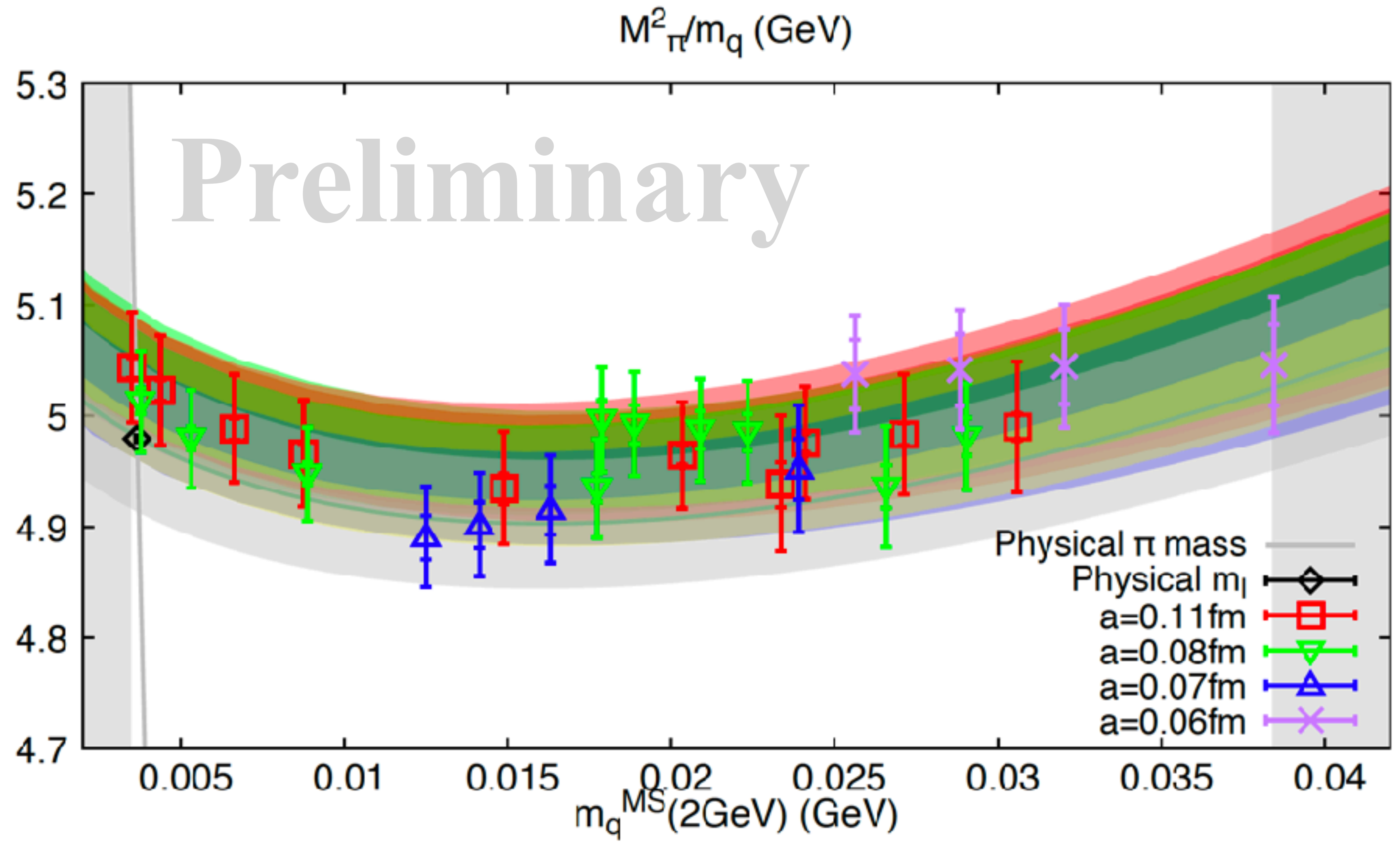
discretization error

$$F_\pi = F(1-\frac{N_f}{2}(y_v+y_s)\ln(y_v+y_s)+\alpha_5 y_v+\alpha_4 N_f y_s)(1+c_f a^2)(1+c_{fl}e^{-M_\pi L}+c_{fs}(M_{\eta_s}^2-M_{\eta_s,phys}^2))$$

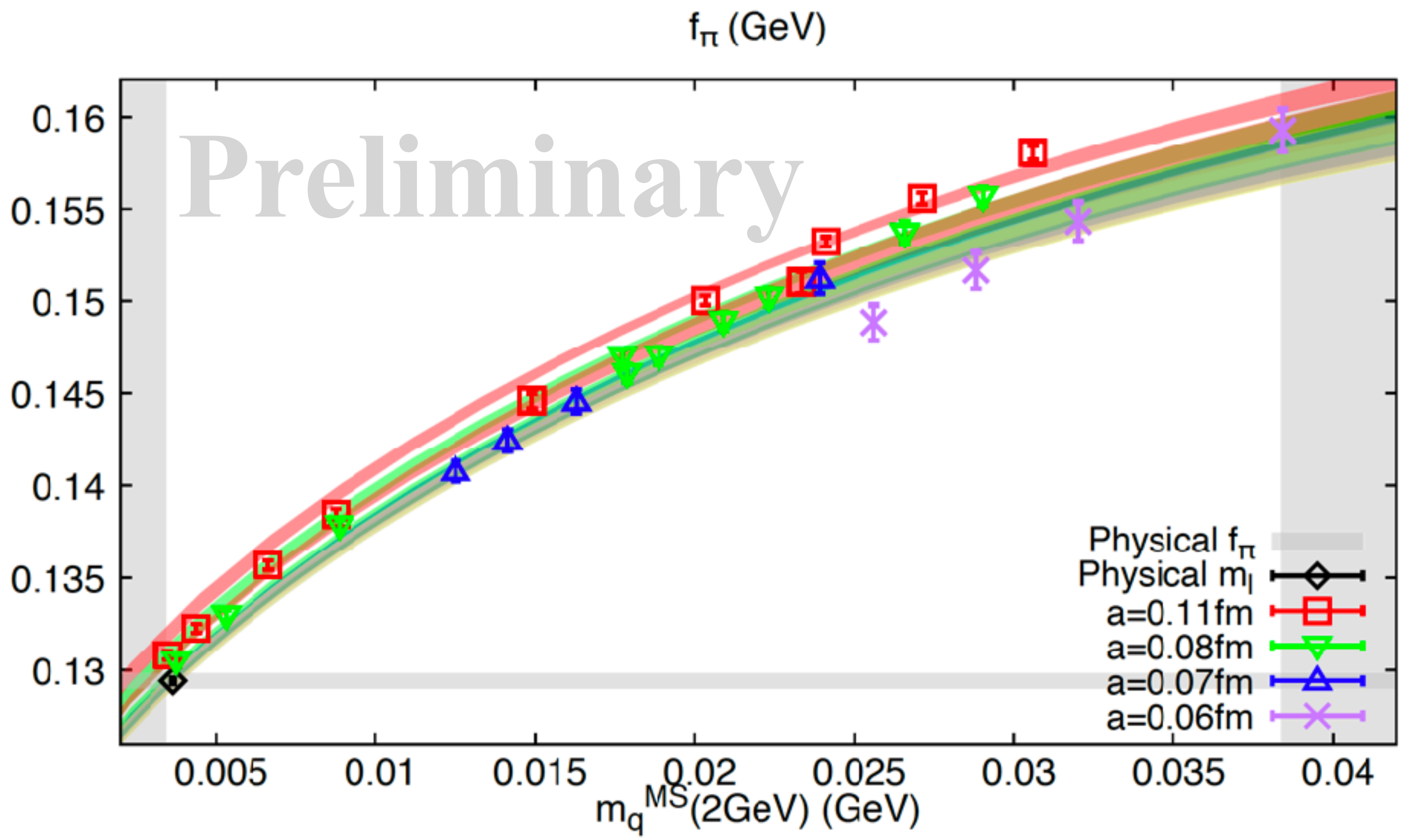
$$y_{v,s} = \frac{\Sigma m_l^{v,s}}{(4\pi F^2)^2}$$

$$M_{\pi,phys}^{isoQCD} = 134.98(5)\text{MeV}$$

P.Zyla et,al, PTEP(2020)083C01(PDG2020)



$M_\pi^2(m_l^v, m_l^v)$



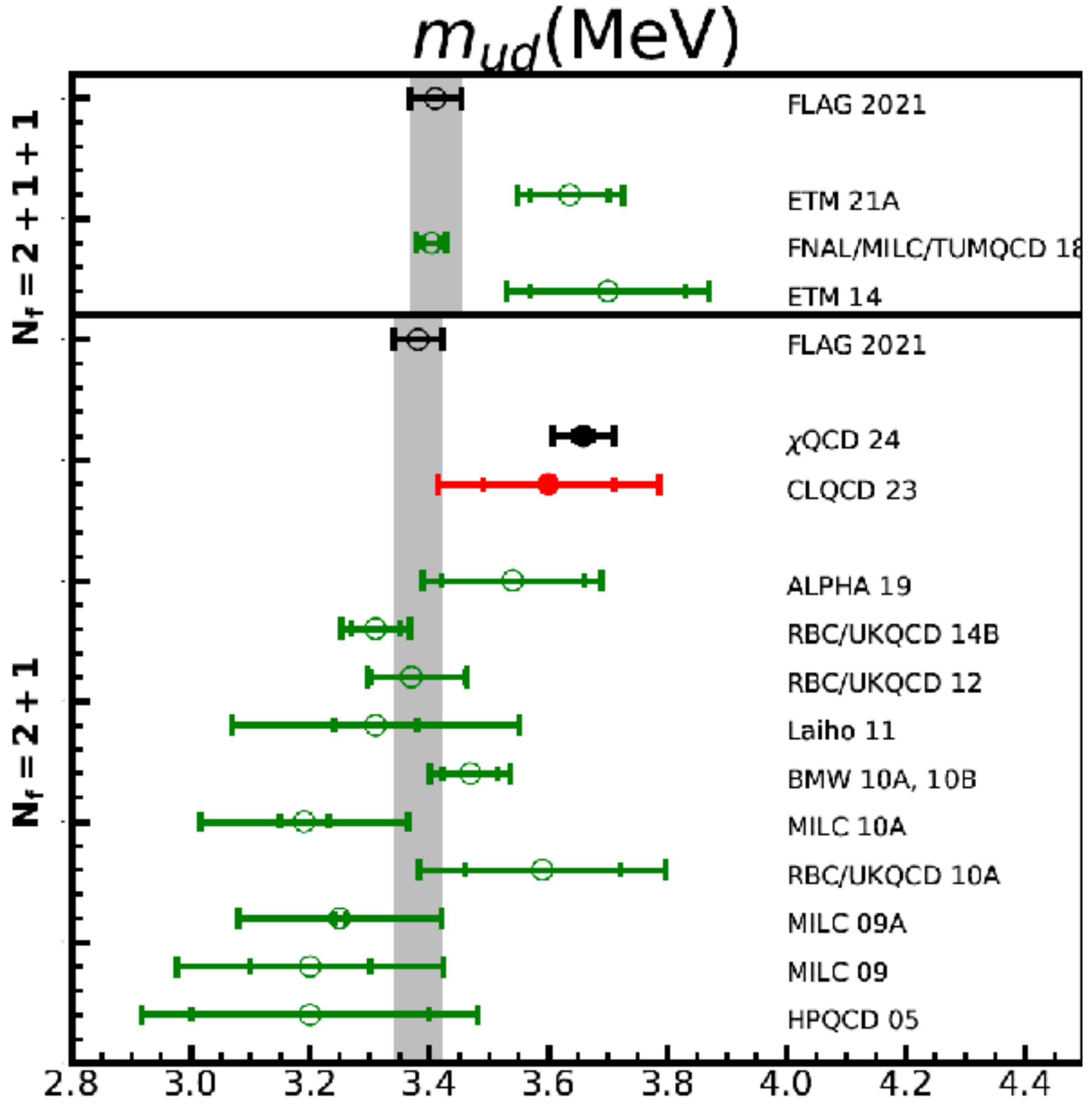
$f_\pi(m_l^v, m_l^v)$

4. Relationship between quark mass and different pseudo-mesons

FLAG 2021, *Eur.Phys.J.C* 82 (2022) 10, 869

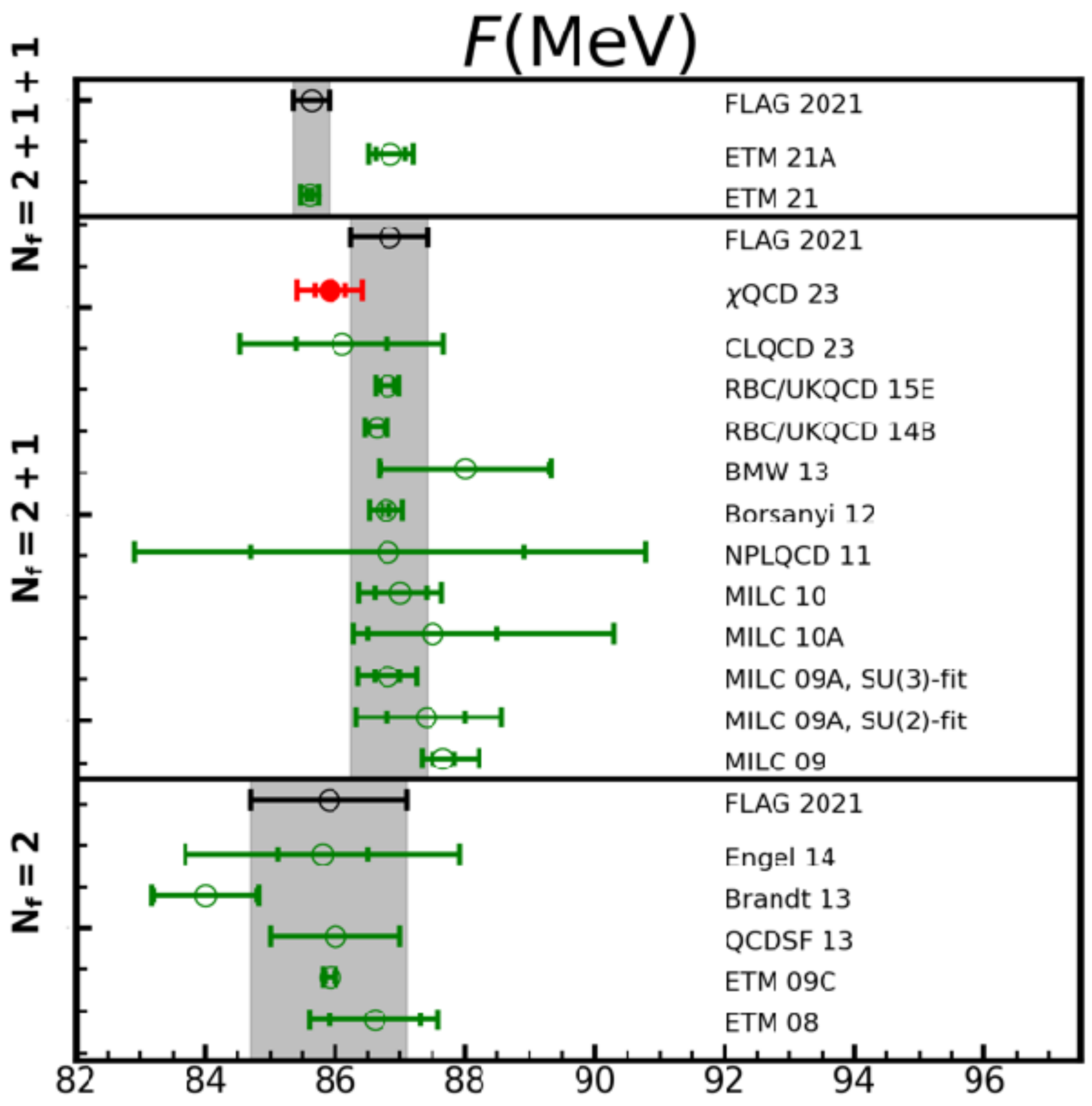
ETM, C. Alexandrou et.al, *Phys.Rev.D* 104 (2021) 074515

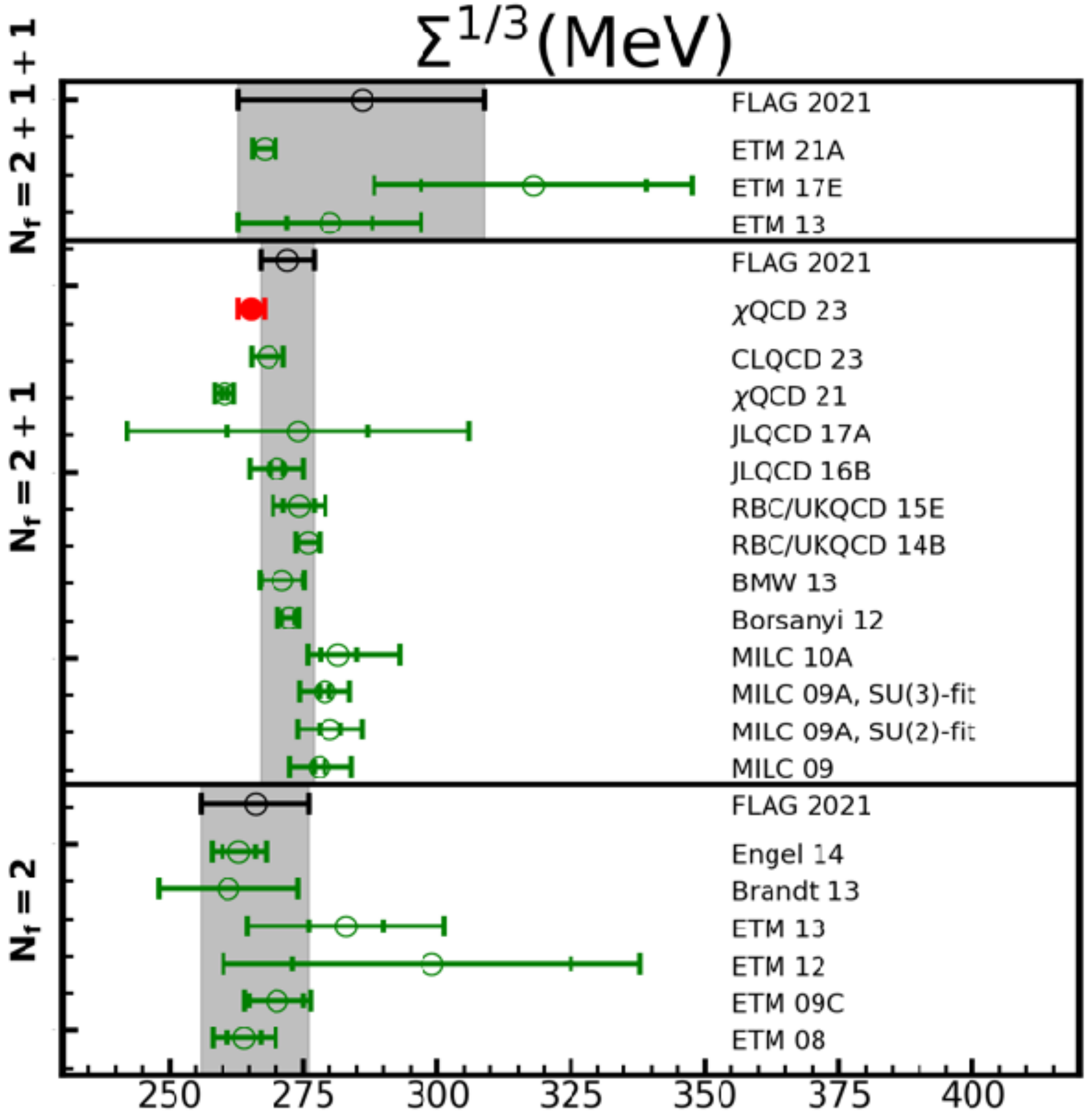
ETM, C. Alexandrou et.al, *Phys.Rev.D* 90 (2014) 074501



9% deviation from FLAG average

Light Quark Mass





LO low energy constants

4. Relationship between quark mass and different pseudo-mesons

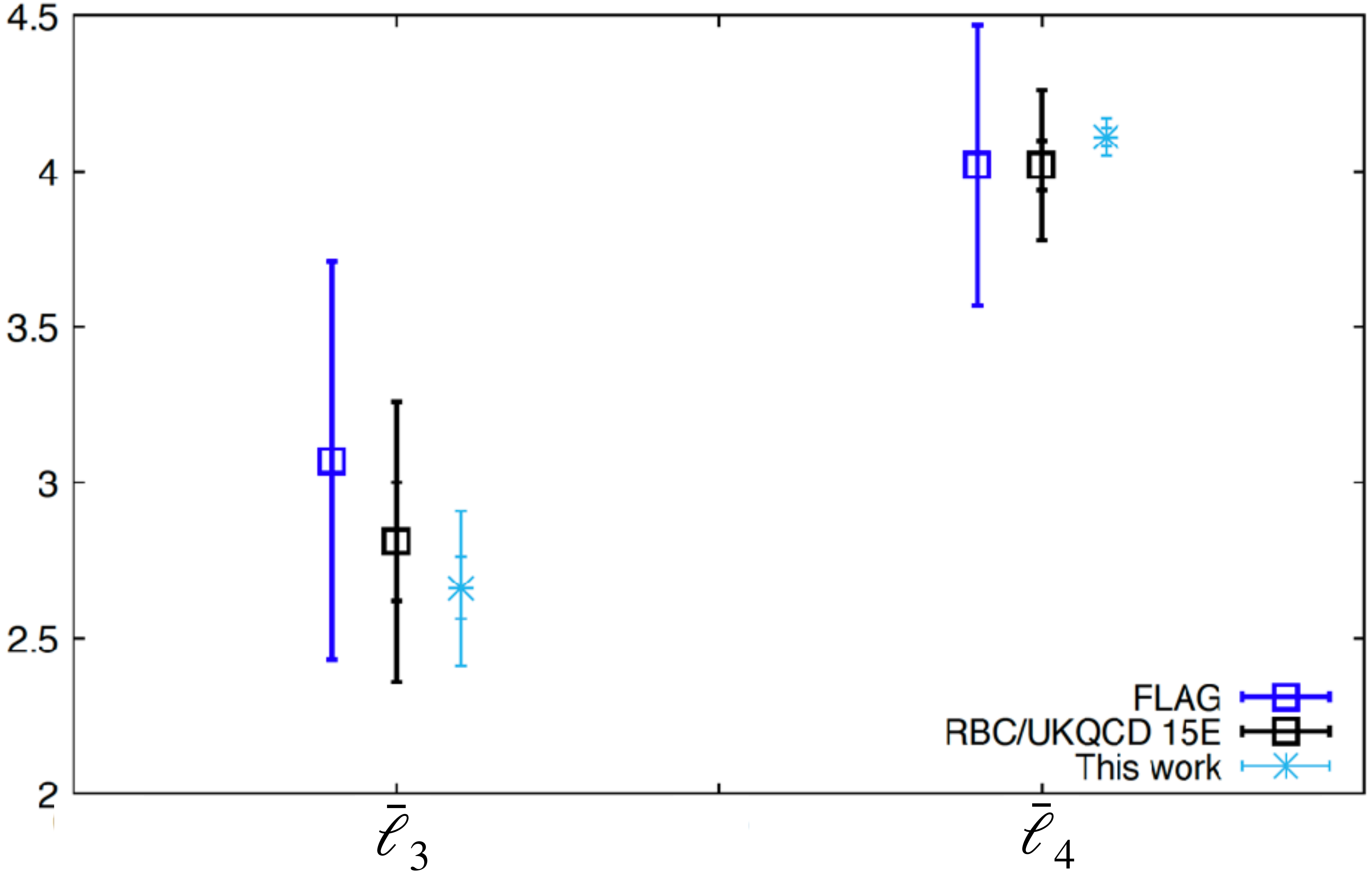
Unquenched SU(2) LECs

$$\bar{\ell}_3 = \ln \frac{(4\pi F)^2}{M^2_{\pi,phys}} - 2[(2\alpha_8 - \alpha_5) + 2(2\alpha_6 - \alpha_4)]$$

$$\bar{\ell}_4 = \ln \frac{(4\pi F)^2}{M^2_{\pi,phys}} + \frac{1}{2}(\alpha_5 + 2\alpha_4)$$

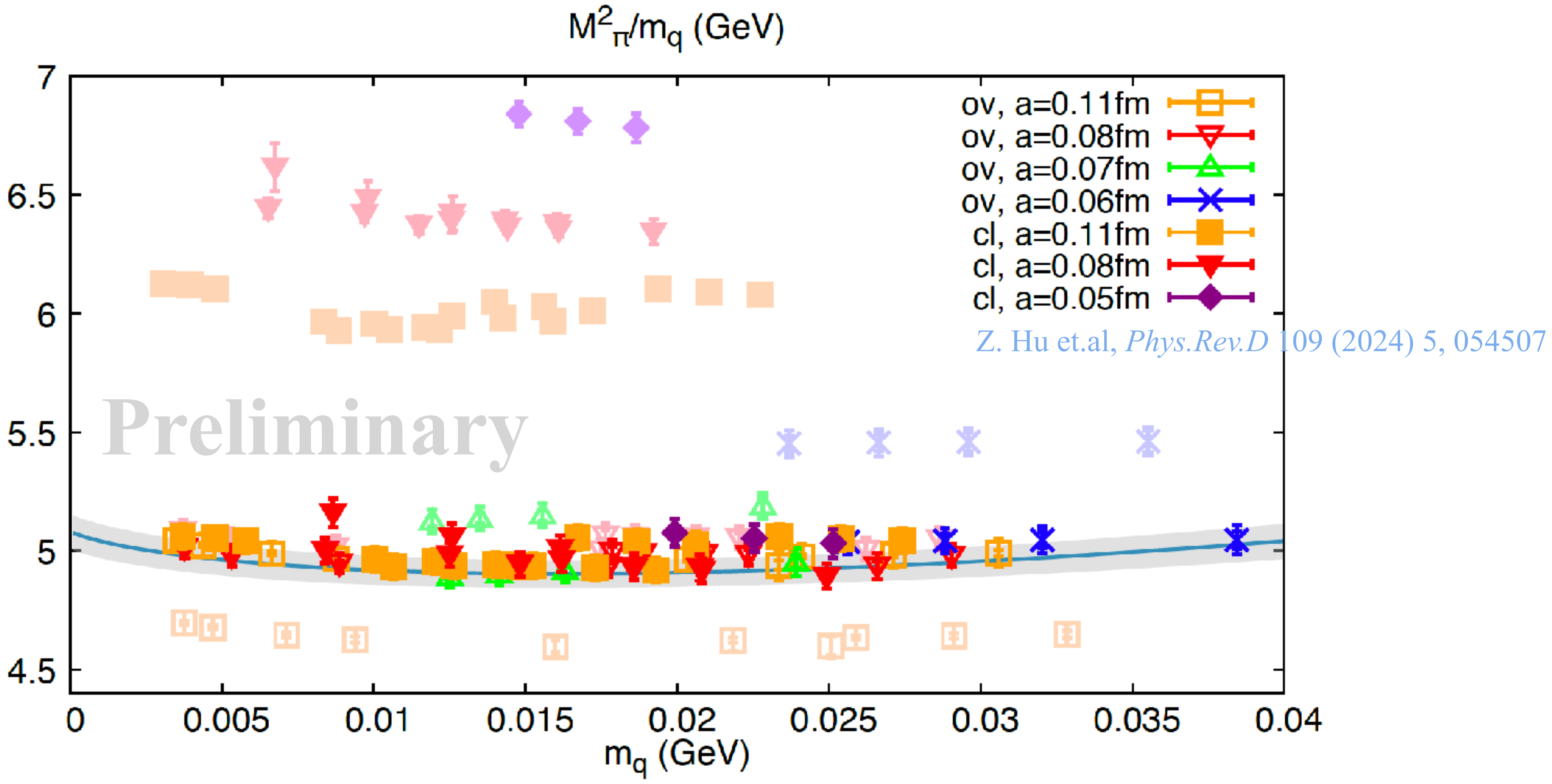
FLAG 2021, *Eur.Phys.J.C* 82 (2022) 10, 869

P.A.Boyle et.al, *Phys.Rev.D* 93 (2016) 5, 054502



NLO low energy constants

4. Relationship between quark mass and different pseudo-mesons



Clover and Overlap are consistent with each other after renormalization.

4. Relationship between quark mass and different pseudo-mesons

$m_{u,d,s}$ **global fit**

ETM, C. Alexandrou et.al, *Phys.Rev.D* 104 (2021) 074515 Eq.49

$$M_K^2 = (c_{1,l}^v m_l^v + c_{1,s}^v m_s^v + c_{1,l}^s m_l^s + c_{1,s}^s m_s^s)(1 + c_{2,l} m_l^v + c_{2,a} a^2 + c_{2,L} e^{-M_\pi L})$$

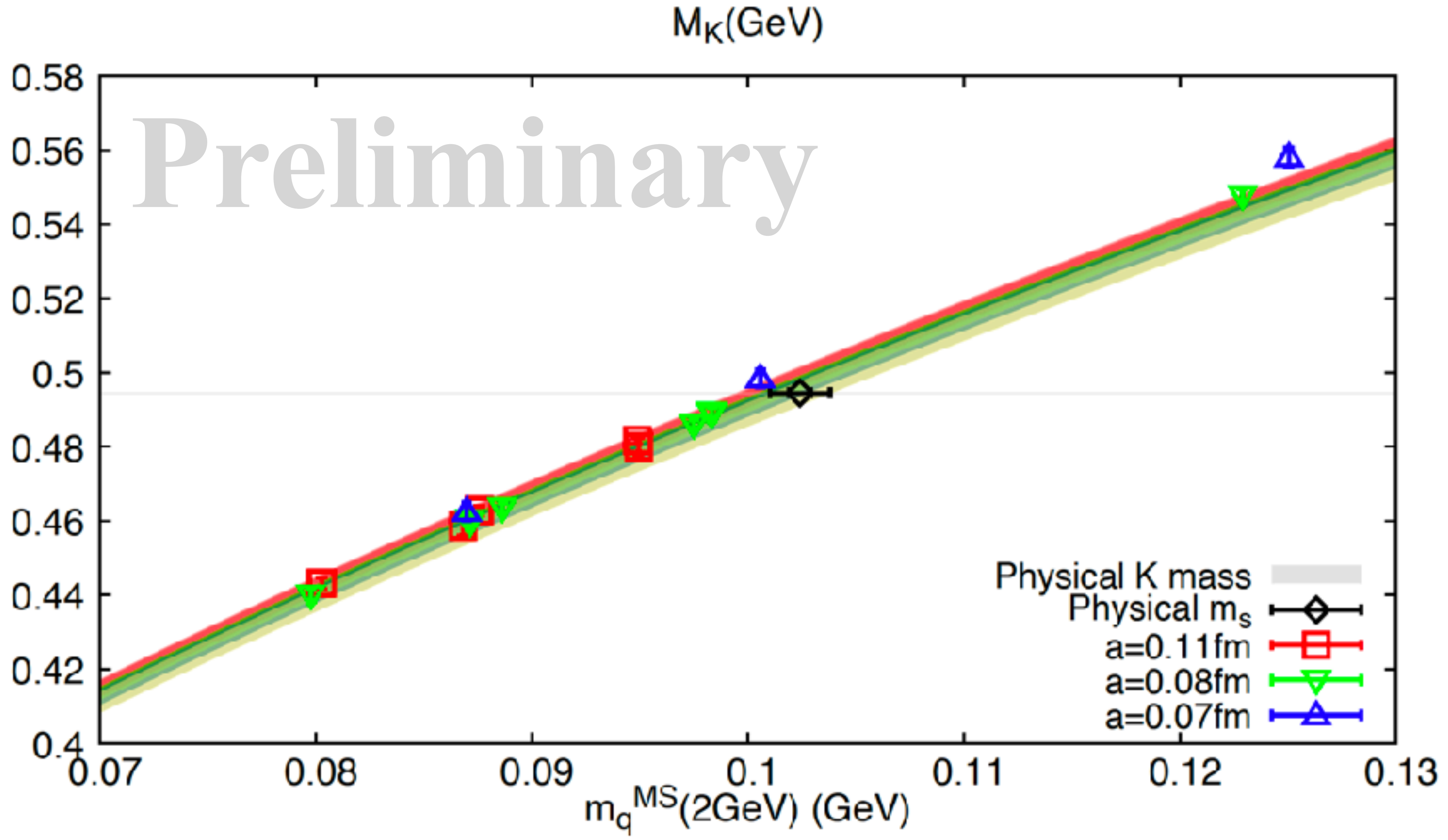
$$M_{K^0,\mathrm{QCD}} = M_{K^0}^{\mathrm{phys}} - \Delta_{\mathrm{QED}} M_{K^0} = 497.44(02)\ \mathrm{MeV}$$

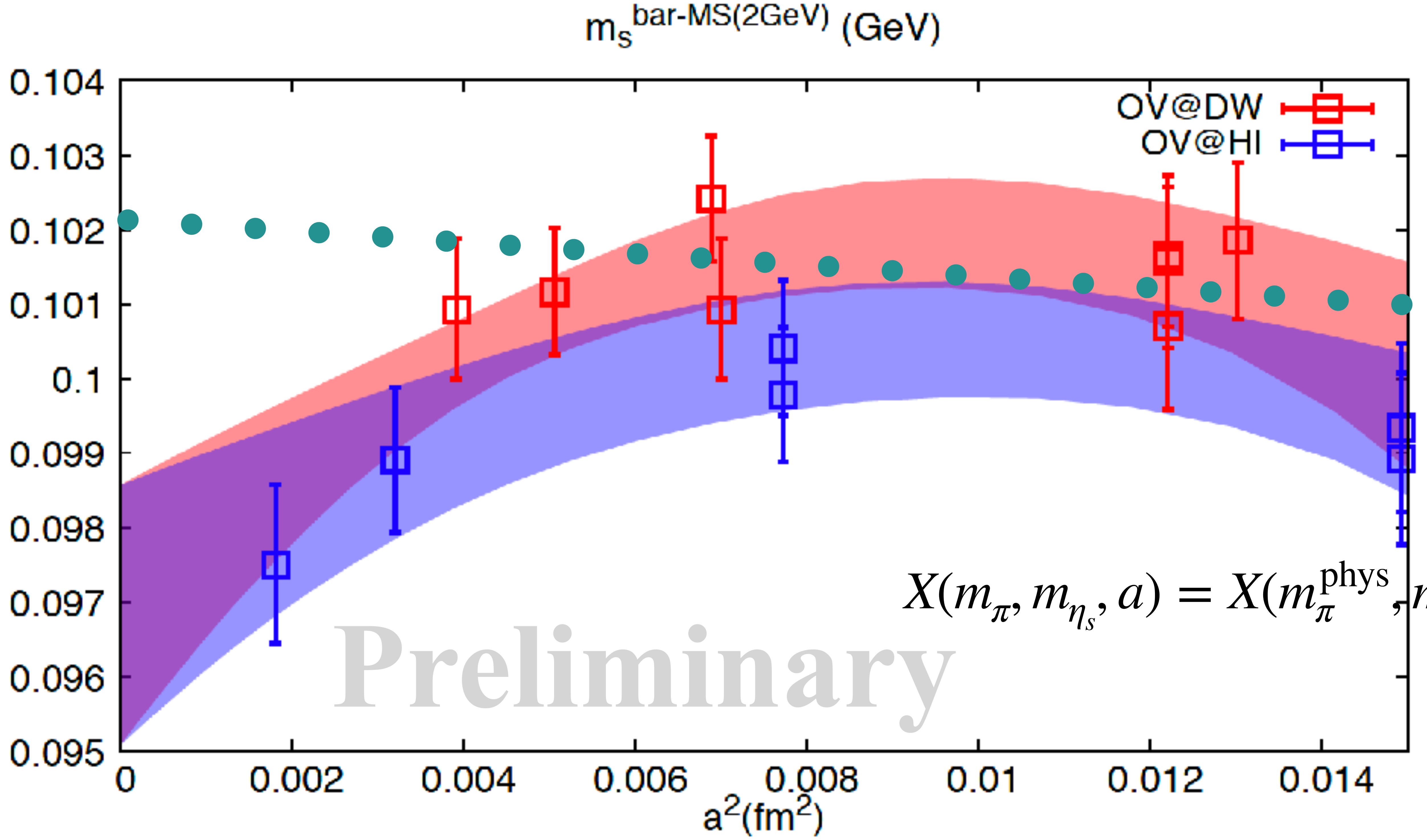
$$m_u^{\mathrm{phys}} + m_d^{\mathrm{phys}} = 2m_l^{\mathrm{phys}}$$

$$M_{K^\pm,\mathrm{QCD}} = M_{K^\pm}^{\mathrm{phys}} - \Delta_{\mathrm{QED}} M_{K^\pm} = 491.44(15)\ \mathrm{MeV}$$

D. Giusti et.al, *Phys.Rev.D.* 95, 114504(2017)

$M_K(m_l^{phys}, m_l^{phys}, m_s^v, m_s^v)$





m_s **extrapolation**

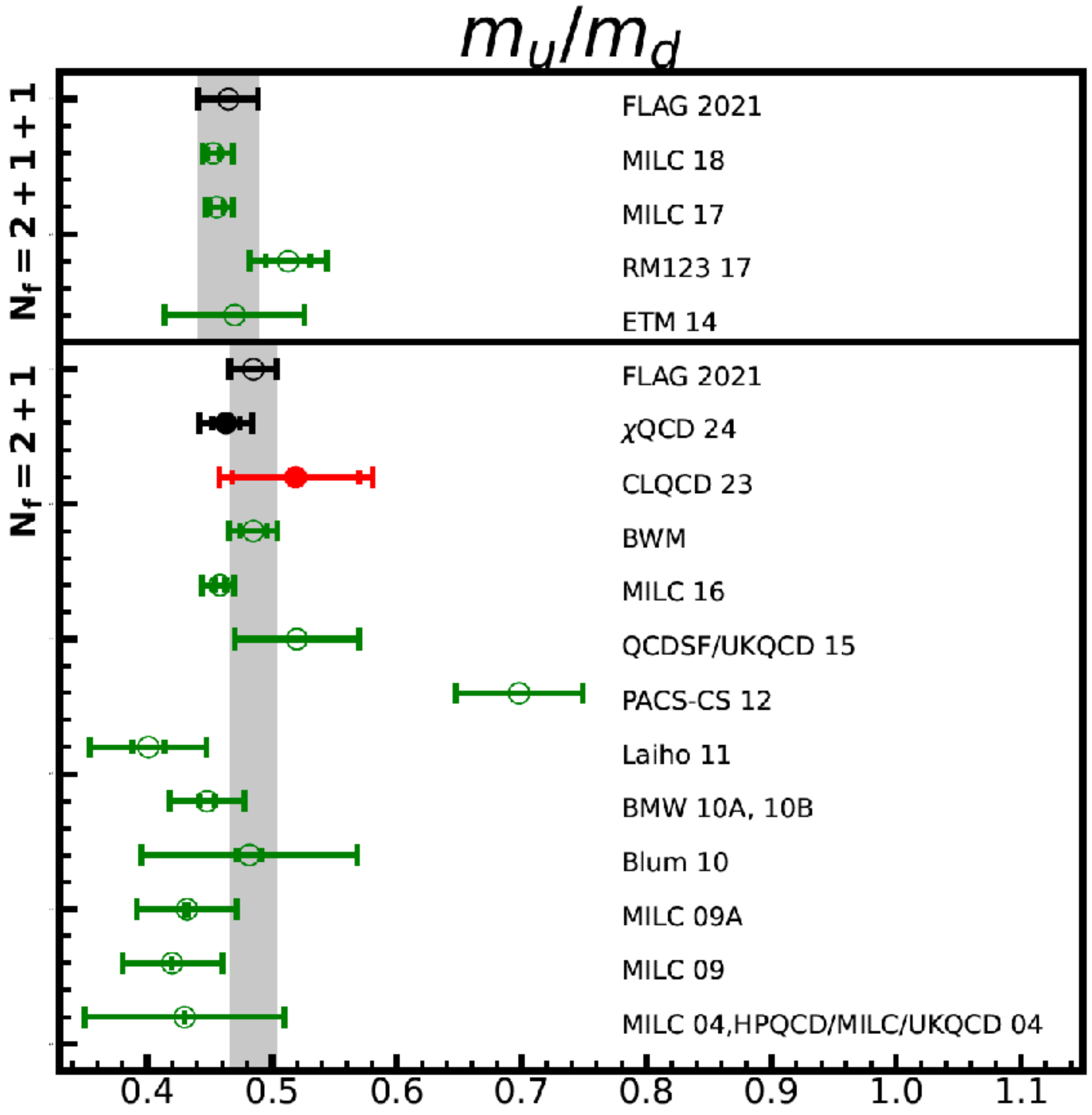
1. Interpolate $M_{\eta_s,simu.}$ to $M_{\eta_s,QCD} = 689.89(49)\mathrm{MeV}$

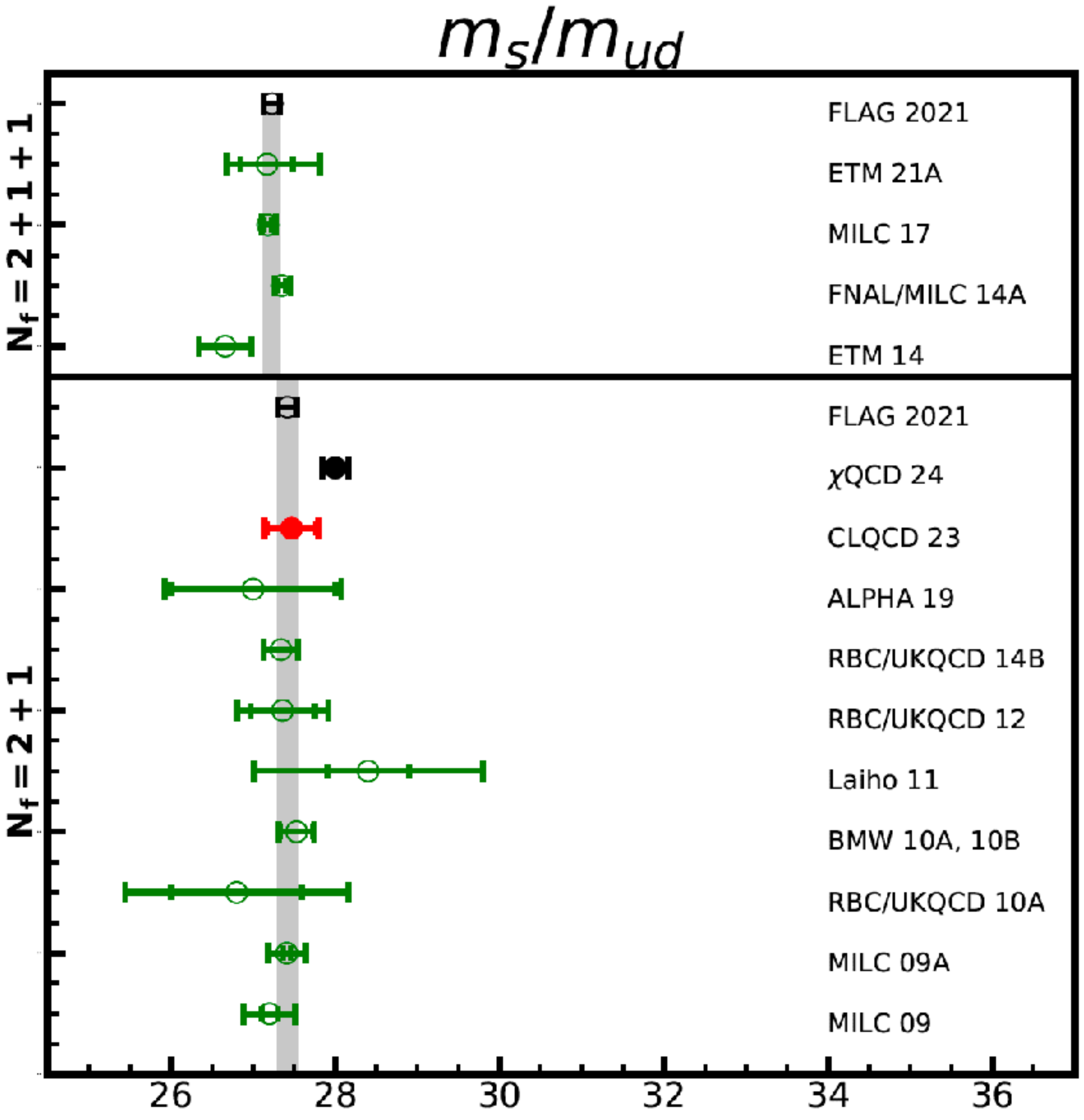
$$X(m_\pi, m_{\eta_s}, a) = X(m_\pi^{\mathrm{phys}}, m_{\eta_s}^{\mathrm{phys}}, 0) + d_1^X(m_\pi^2 - (m_\pi^{\mathrm{phys}})^2) + d_2^X(m_{\eta_s}^2 - (m_{\eta_s}^{\mathrm{phys}})^2) + d_3^X a^2 + d_4^X a^4$$

4. Relationship between quark mass and different pseudo-mesons

FLAG 2021, *Eur.Phys.J.C* 82 (2022) 10, 869

4. Relationship between quark mass and different pseudo-mesons





4. Relationship between quark mass and different pseudo-mesons

m_c interpolation

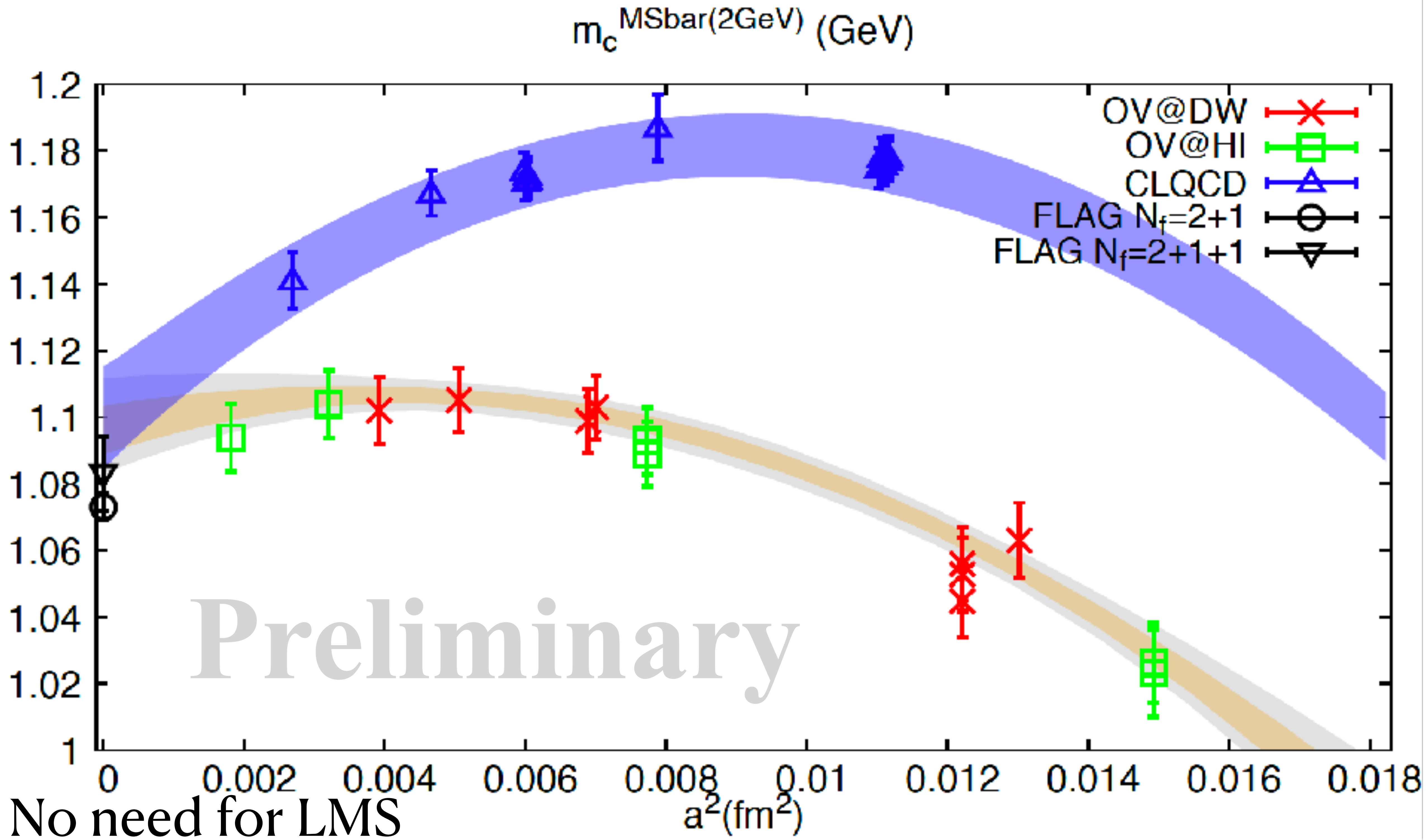
1. Interpolate m_s^v to $m_{s,phys}$
2. Interpolate $M_{D_s,simu.}$ to $M_{D_s,QCD}$

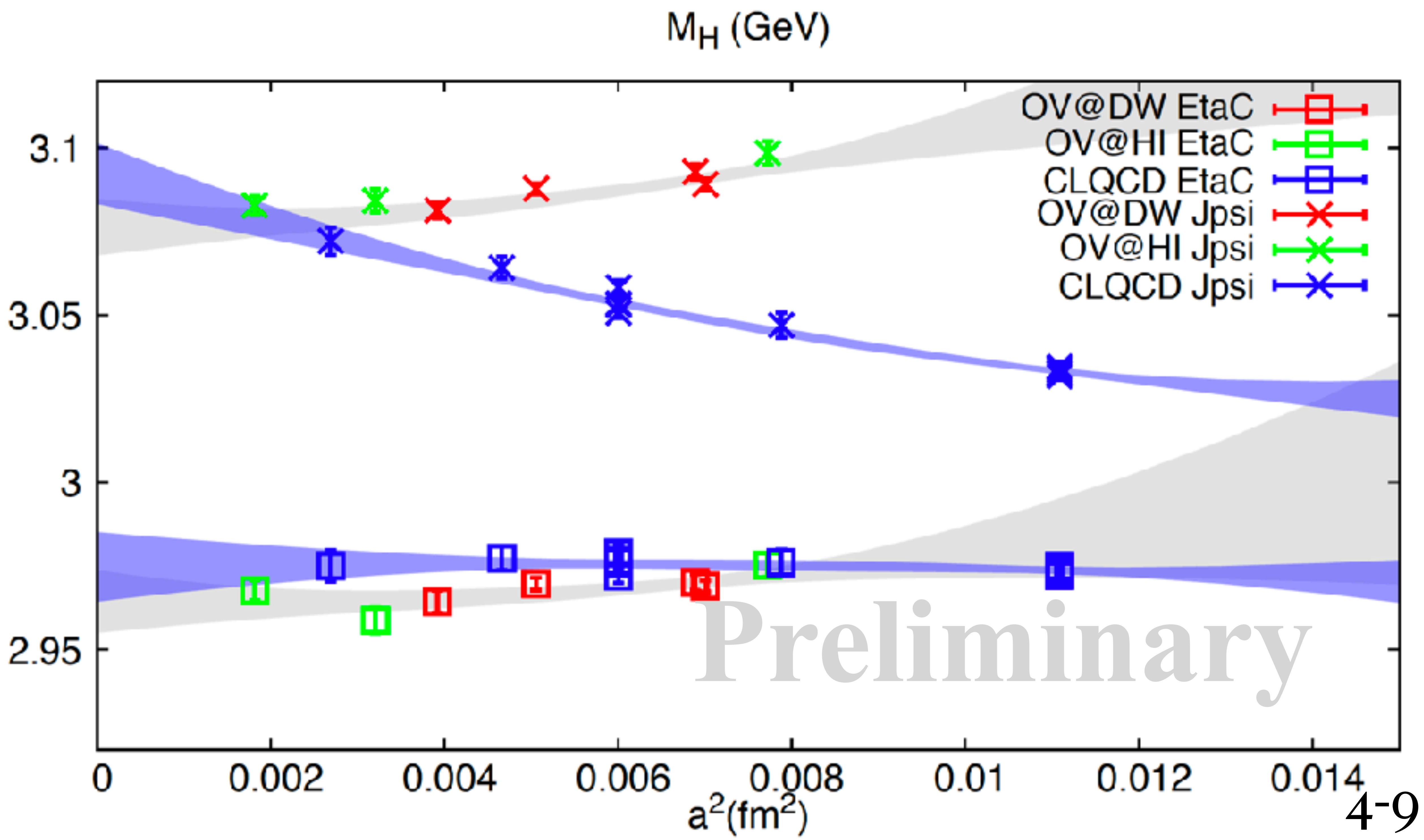
$$M_{D_s^\pm,QCD} = M_{D_s^\pm}^{phys} - \Delta_{QED} M_{D_s^\pm} = 1966.7(1.5)\text{MeV}$$

D. Giusti et.al, *Phys.Rev.D.* 95, 114504(2017)

J/ψ & η_c interpolation

$$X(m_\pi, m_{\eta_s}, a) = X(m_\pi^{\text{phys}}, m_{\eta_s}^{\text{phys}}, 0) + d_1^X(m_\pi^2 - (m_\pi^{\text{phys}})^2) + d_2^X(m_{\eta_s}^2 - (m_{\eta_s}^{\text{phys}})^2) + d_3^X a^2 + d_4^X a^4$$

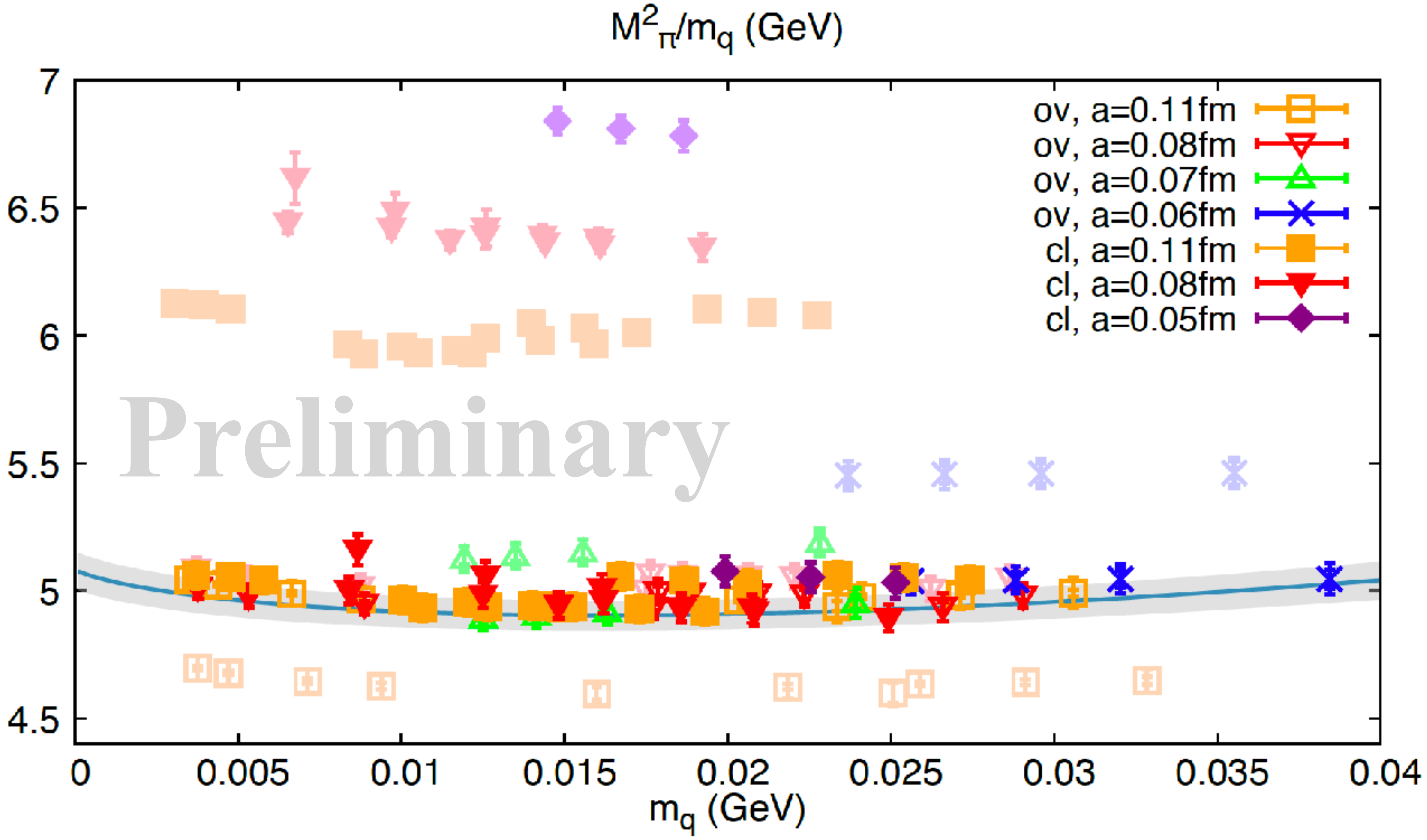


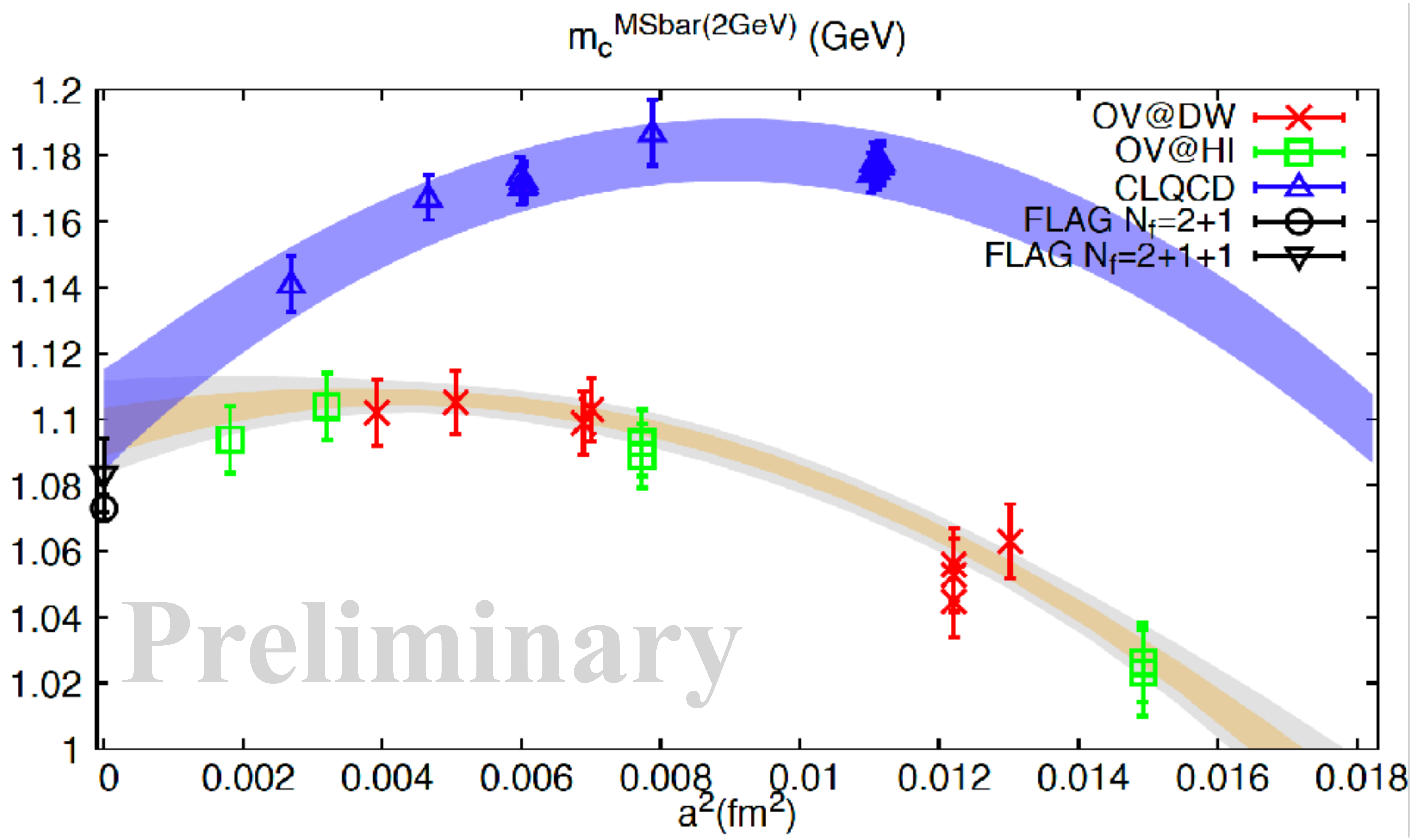


No need for LMS

Summary and Outlook

基于0.03fm-0.12fm的规范组态和低模式替换技术（LMS），使用HYP Smear Overlap Fermion确定了$m_{u,d,s}$, LEC，验证了计算结果在连续外推后与Clover费米子方案基本一致





THANKS FOR YOUR ATTENTION!