Crossing-symmetric dispersive analyses for meson-meson scatterings from lattice QCD data

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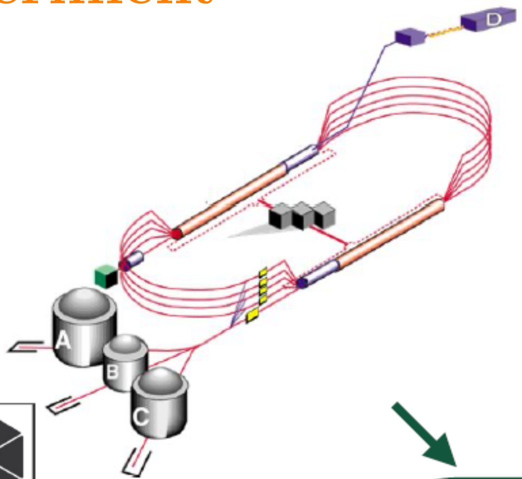
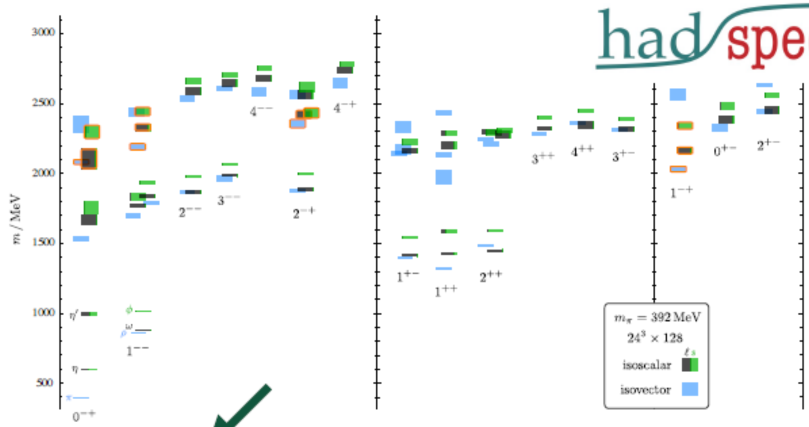
Why lattice QCD?

Experiment



Lattice QCD



Amplitude analyses

- Unitarity
- Analiticity
- Crossing

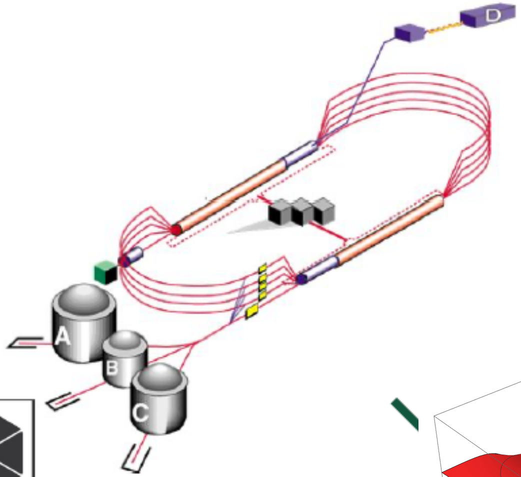
↔ **QCD**

↓

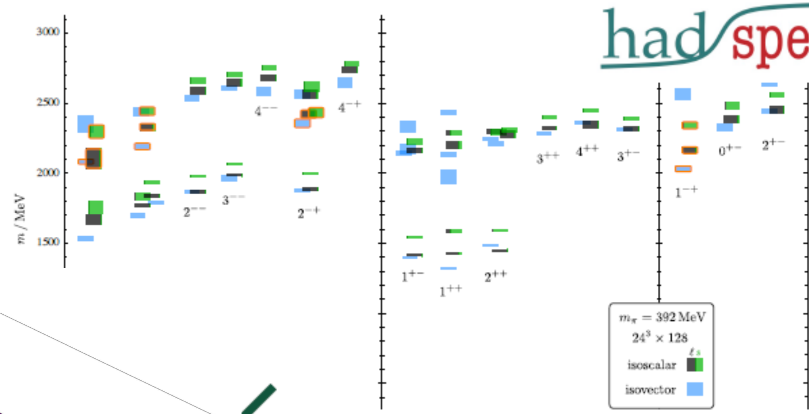
Observables

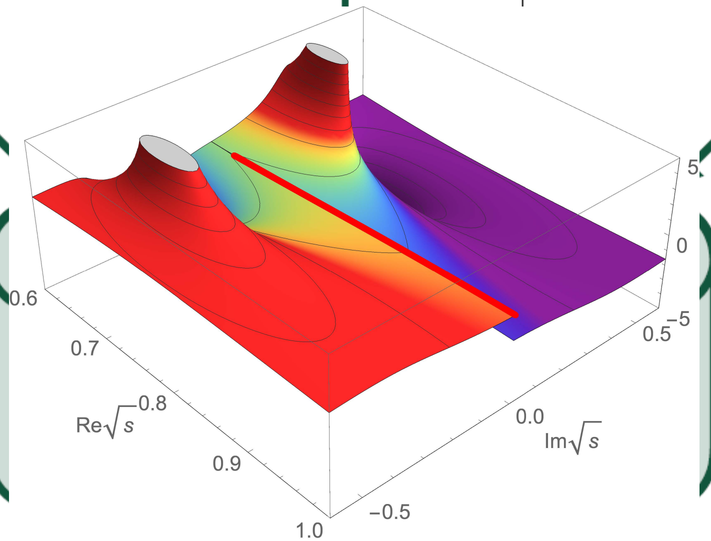
Why lattice QCD?

Experiment



Lattice QCD





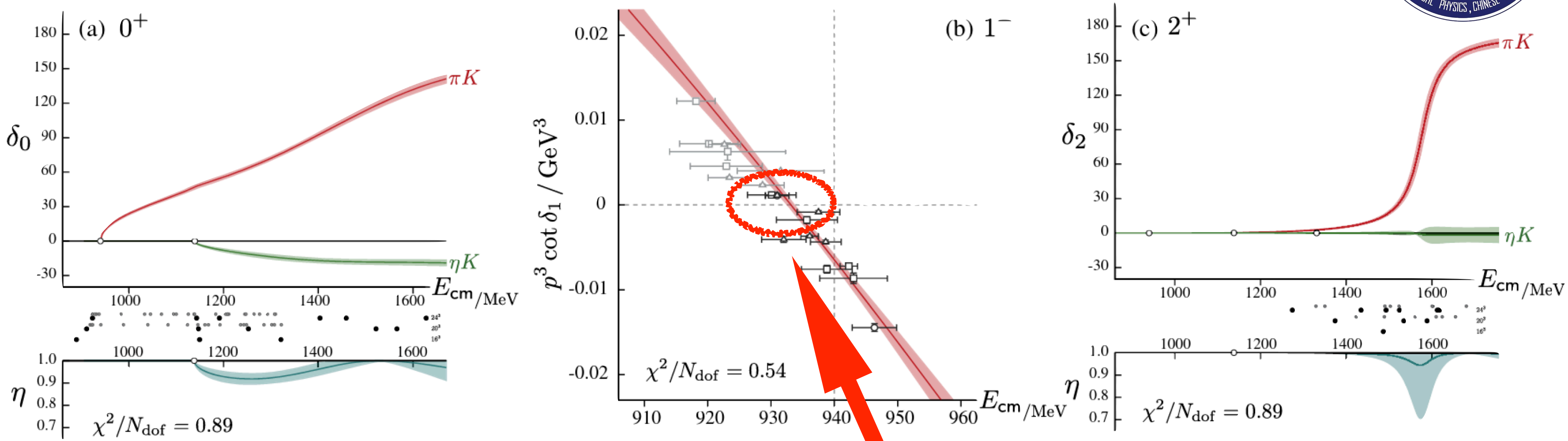
Resonances manifest as the poles of the amplitudes

QCD

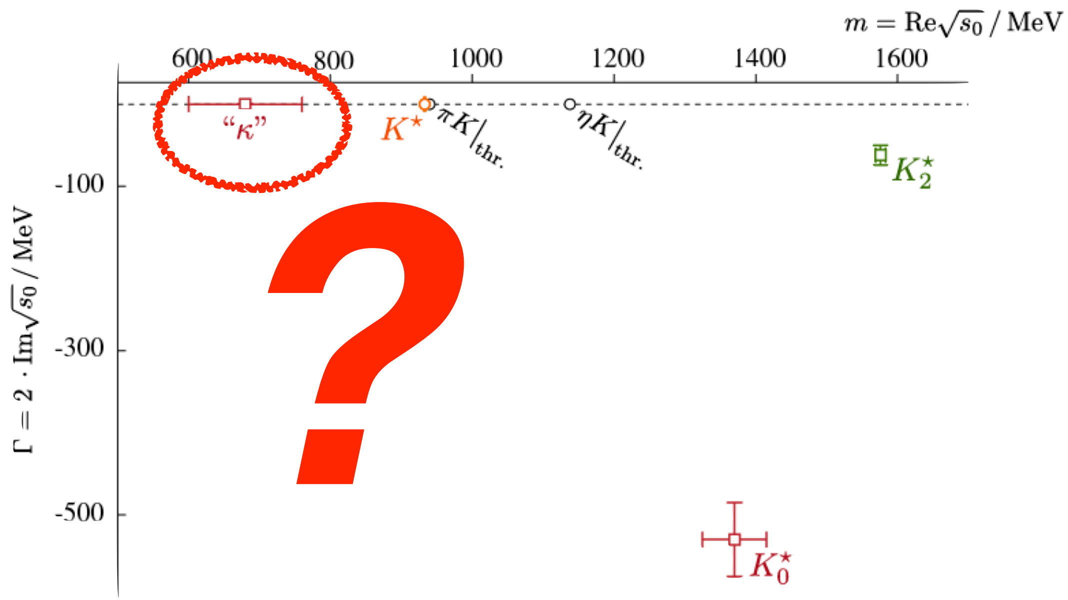
Observables

πK scattering at $m_\pi = 391$ MeV

HSC, PRL (2014); PRD (2015); PRL(2019)



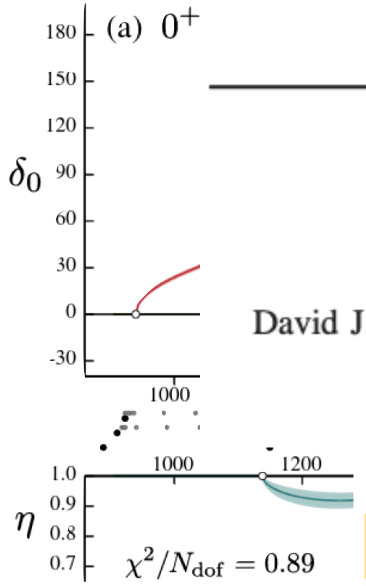
Shallow bound state pole $K^*(892)$

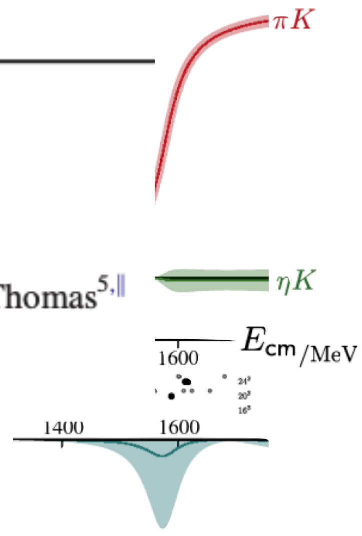


- $m_\pi \sim 390$ MeV, K-matrix fits:
  - $\kappa / K_0^*(700)$: **one virtual state???**
  - $K^*(892)$: **a shallow bound state**

πK scattering at $m_\pi = 391$ MeV

HSC, PRL (2014); PRD (2015); PRL(2019)





PHYSICAL REVIEW LETTERS **123**, 042002 (2019)

Quark-Mass Dependence of Elastic πK Scattering from QCD

David J. Wilson,[1,*] Raúl A. Briceño,[2,3,†] Jozef J. Dudek,[2,4,‡] Robert G. Edwards,[2,§] and Christopher E. Thomas[5,‖]

(for the Hadron Spectrum Collaboration)

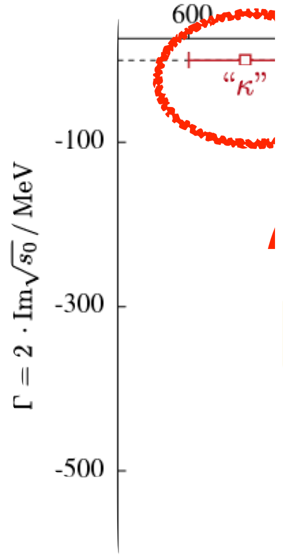
meson-meson continuum. The S-wave amplitudes are well-determined for real energies, however the analytic continuation into the complex plane does not yield a unique result that we can interpret in terms of the κ pole. Along with our previous study of the σ [42], this provides motivation for future analyses that incorporate now-standard lattice QCD analysis techniques, namely *Lüscher-like* analysis of finite-volume spectra, and those in use in the amplitude analysis community, e.g. *Roy-Steiner equations*, which account for the known singularities due to cross-channel physics. In the current case, an input

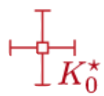
ole

-matrix fits:

e virtual state???

llow bound state





What is Roy or Roy-Steiner type equation?

Roy-Steiner type equations = Analyticity (Causality) + Crossing symmetry + Unitarity

Crossing-symmetric dispersive analyses

S. Roy

F. Steiner

- Renaissance caused by the development of χPT

$\pi\pi$
- G. Colangelo, et al., NPB (2001); B. Ananthanarayan, et. al., Phys. Rept. (2001); **I. Caprini, et al., PRL (2006)**; B. Moussallam, EPJC (2011); Garcia-Martin, et al.,PRD (2011); PRL (2011); I. Caprini, et al., EPJC (2011); J. Pelaez, Phys.Rept. (2016); XHC et.al., PRD (2023); HSC, PRD (2024)…

πK
- P. Buettiker, et al., EPJC (2004); **S. Descotes-Genon, et al., EPJC (2006)**; J. Pelaez and A. Rodas, EPJC(2018); PRL (2020); Phys.Rept. (2022); J. Pelaez et.al., PRL (2023)…

πN
- C. Ditsche, et al., JHEP (2012); M. Hoferichter et.al., JHEP (2012); M. Hoferichter, et al., PRL 115, 092301(2015); PRL 115, 192301 (2015); Phys. Rept. (2016); PLB (2016); EPJA (2016); J. Ruiz de Elvira et.al., JPG (2018); M. Hoferichter, et al., PRL (2018); **XHC, et.al., JHEP (2022)**; **M. Hoferichter, et al., PLB (2024)**…

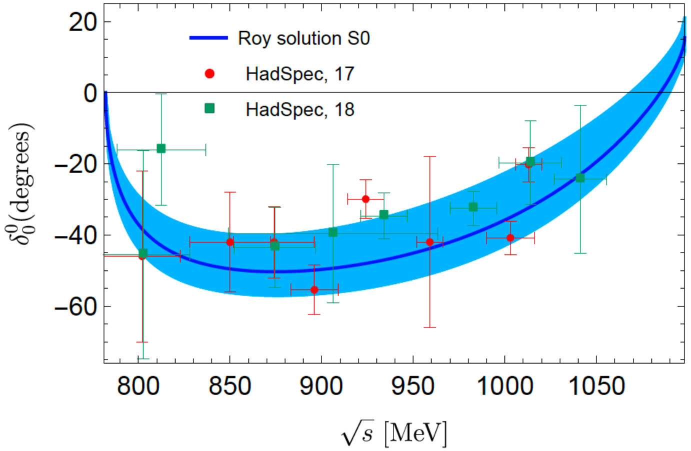
- $\gamma\pi \to \pi\pi$: T. Hannah, NPB (2001); M. Hoferichter et.al., PRD (2012); $\gamma\gamma \to \pi\pi$: M. Hoferichter et.al., EPJC (2011); $\gamma^*\gamma^* \to \pi\pi$: M. Hoferichter and P. Stoffer, JHEP (2019)…

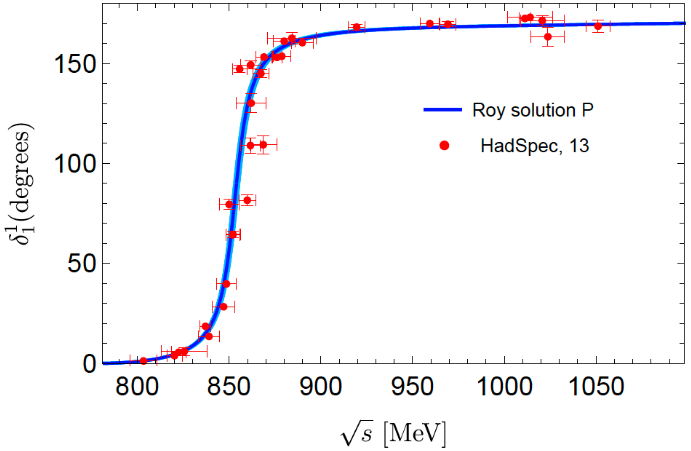
Example: $\pi\pi$ scattering at $m_\pi = 391$ MeV

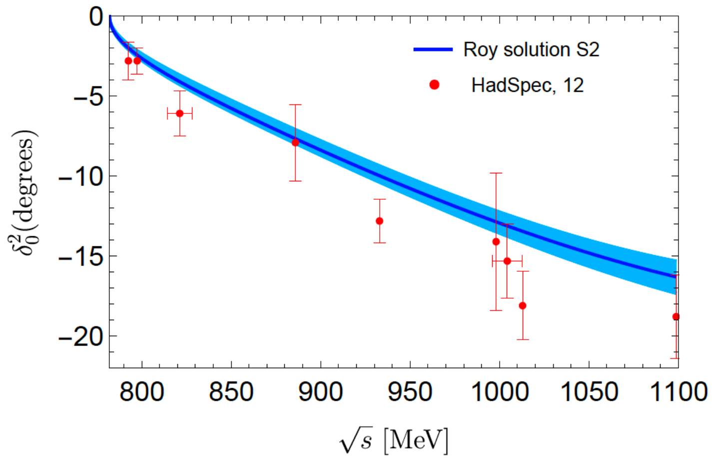
$\sigma/f_0(500)$

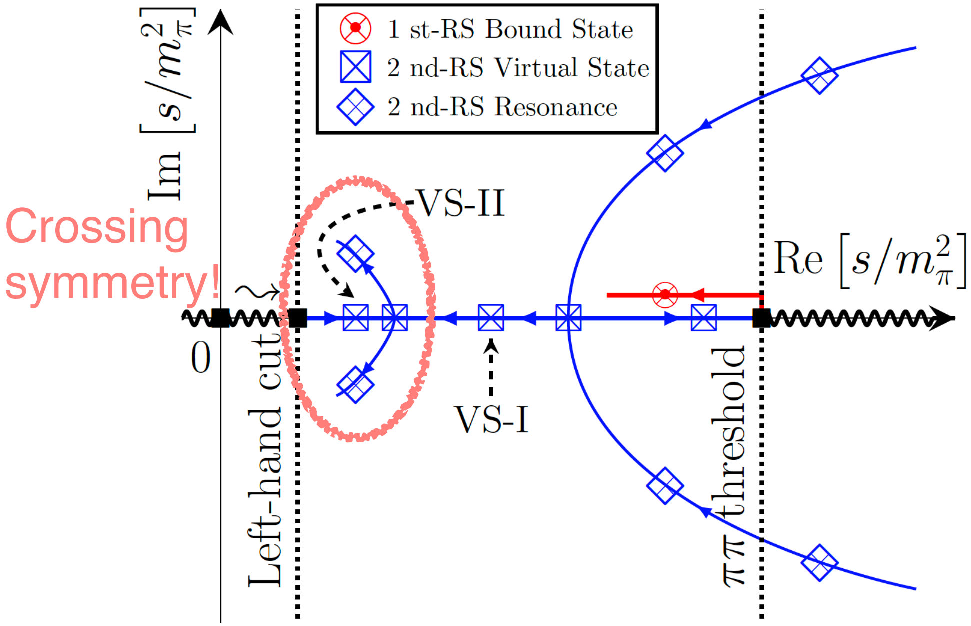
XHC et.al., PRD (2023)

$m_\pi = 391$ MeV $\Rightarrow$ bound state σ pole!







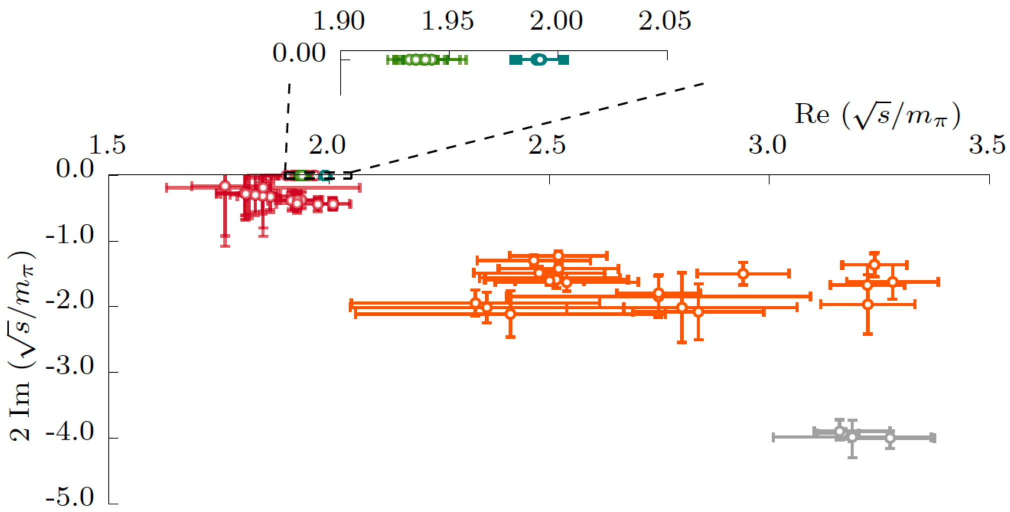


☑ Bound state and virtual state pole trajectories of σ as a function of m_π

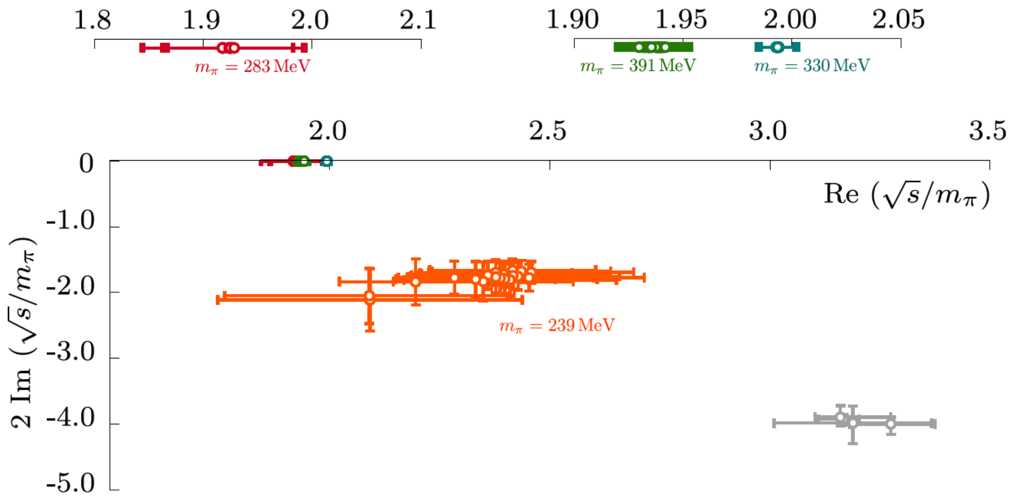
K-matrix analyses v.s. dispersive analyses

$$\sigma/f_0(500)$$

New σ pole positions via preliminary Roy equation analyses **XHC** et.al., PRD (2023); HSC, PRD (2024)



K-Matrix



Roy equation

- Numerical strategy to solve Roy-Steiner type equations

Roy-Steiner type equations

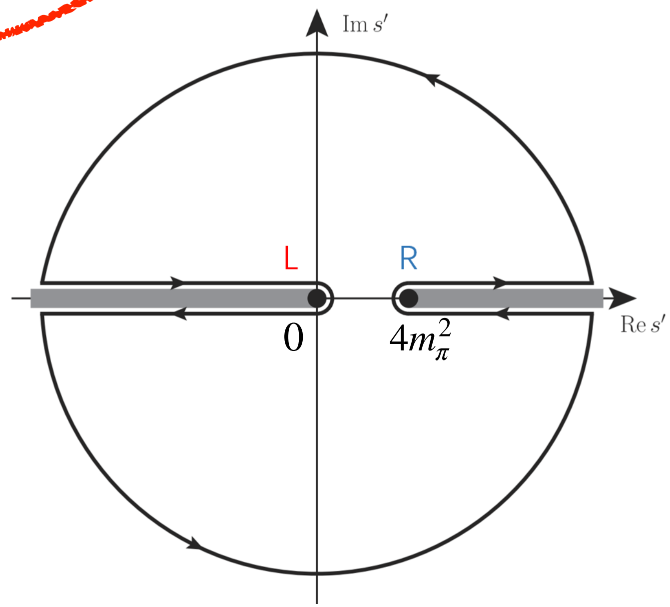
$$\mathrm{Re}\, t_J^I(s) = k_J^I(s) + \sum_{I'} \sum_{J'} \mathscr{P} \int_{4m_\pi^2}^{\infty} \mathrm{d}s' \quad \underbrace{K_{JJ'}^{II'}(s', s)}_{\frac{1}{\pi}\frac{\delta_{JJ'}\delta_{II'}}{s'-s} + \bar{K}_{JJ'}^{II'}(s,s')} \quad \mathrm{Im}\, t_{J'}^{I'}(s')$$

ROY EQUATION

$$K_{00}^{00}(s, s') = \frac{1}{\pi(s'-s)} + \frac{2\ln\left(\frac{s+s'-4M_\pi^2}{s'}\right)}{3\pi\left(s-4M_\pi^2\right)} - \frac{5s'+2s-16M_\pi^2}{3\pi s'\left(s'-4M_\pi^2\right)}$$

$$k_0^0(s) = a_0^0 + \frac{s-4m_\pi^2}{12m_\pi^2}\left(2a_0^0 - 5a_0^2\right)$$

Left-hand cuts



Roy-Steiner type equations

$$\mathrm{Re}\, t_J^I(s) = k_J^I(s) + \sum_{I'} \sum_{J'} \mathscr{P} \int_{4m_\pi^2}^{\infty} \mathrm{d}s' K_{JJ'}^{II'}(s',s)\, \mathrm{Im}\, t_{J'}^{I'}(s')$$

- Phase shifts: $t_J^I(s) = \dfrac{\eta_J^I(s) e^{2i\delta_J^I(s)} - 1}{2i\rho_{\pi\pi}(s)}$ ← *Unitarity*

Inelasticity (input)

$$\frac{\eta(s)\sin 2\delta_J^I(s)}{2\rho(s)} = k_J^I(s) + \sum_{I'} \sum_{J'} \mathscr{P} \int_{4m_\pi^2}^{\infty} \mathrm{d}s' K_{JJ'}^{II'}(s',s) \frac{1 - \eta(s')\cos 2\delta_{J'}^{I'}(s')}{2\rho(s')}$$

- **Infinite-dimensional, nonlinear, inhomogeneous and (Cauchy) singular integral equations!**
- **Nonlinear Fredholm integral equations of the second kind**

T.P. Pool, Nuovo Cim. (1978)
C. Pomponiu and G. Wanders, NPB (1976)
D. Atkinson and R.L. Warnock, PRD (1977)
L. Epele and G. Wanders, NPB (1978), PLB (1978)
……
J. Gasser and G. Wanders, EPJC (1999)
G. Wanders, EPJC (2000)

Validity Domain of the partial wave amplitude

Theorem (Neumann expansion)

If $f(z)$ is an analytic function inside an ellipse $\mathscr{C}$ with the focal points at ± 1. Then the series $f(z) = \sum_{\ell=0}^{\infty} a_\ell P_\ell(z)$ converges at the case of $z \subset \mathscr{C}$

E. T. Whittaker and G. N. Watson, A Course of Modern Analysis

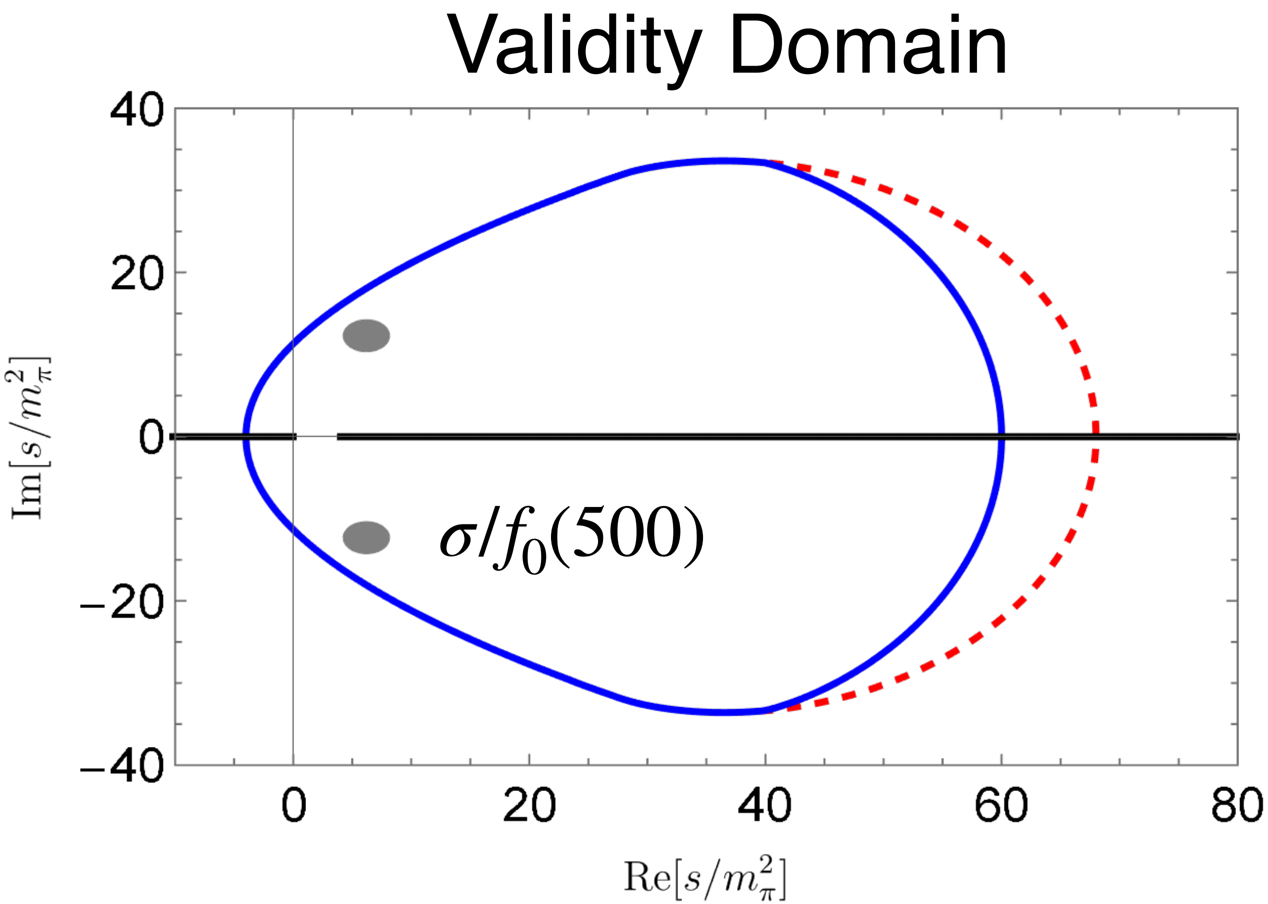
$$T(s, \cos\theta) = 16\pi \sum_{\ell=0}^{\infty} (2\ell + 1) T_\ell(s) P_\ell(\cos\theta)$$

- The concept of "***partial wave amplitude***" only valid in a finite region of the complex s plane—validity domain
- The range of validity domain is determined by the analytic domain of $T(s, \cos\theta)$ in terms of parameter $\cos\theta$

Analytic properties of $T(s, \cos\theta)$

Jost and Lehmann, Nuovo Cim. (1957)
Dyson, Phys. Rev. (1958)
Mandelstam, Phys. Rev. (1958); (1959)
Martin, (1969), Scattering Theory: Unitarity, Analyticity and Crossing

Roy equation for $\pi\pi$ scattering



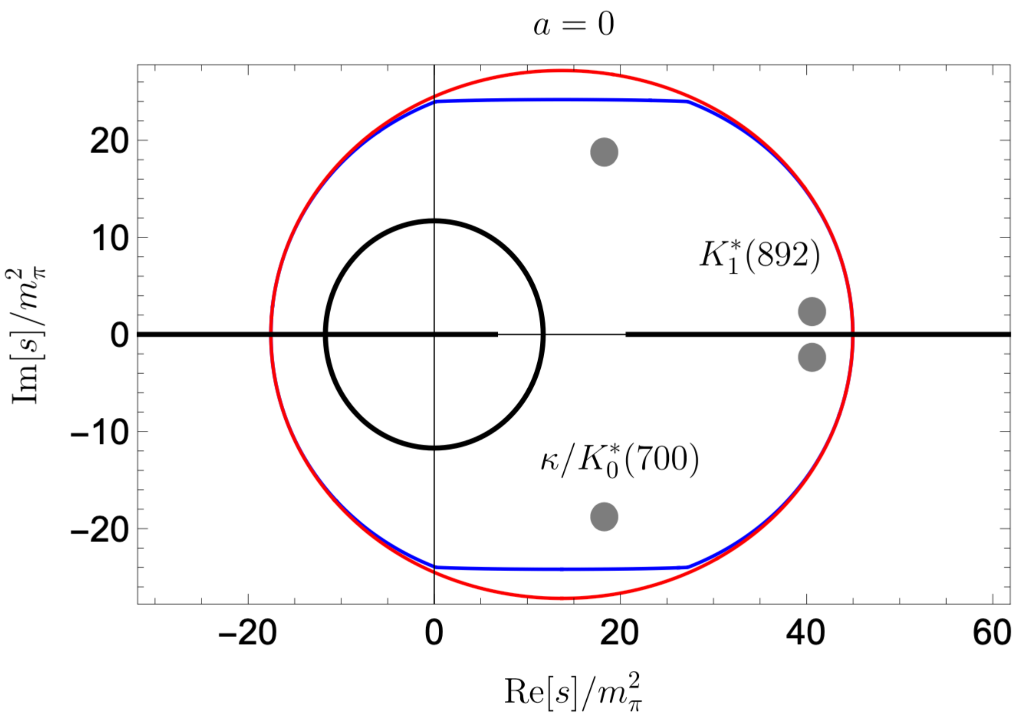
$$m_\sigma = 441^{+16}_{-8}\mathrm{MeV}$$

$$\Gamma_\sigma = 544^{+18}_{-25}\mathrm{MeV}$$

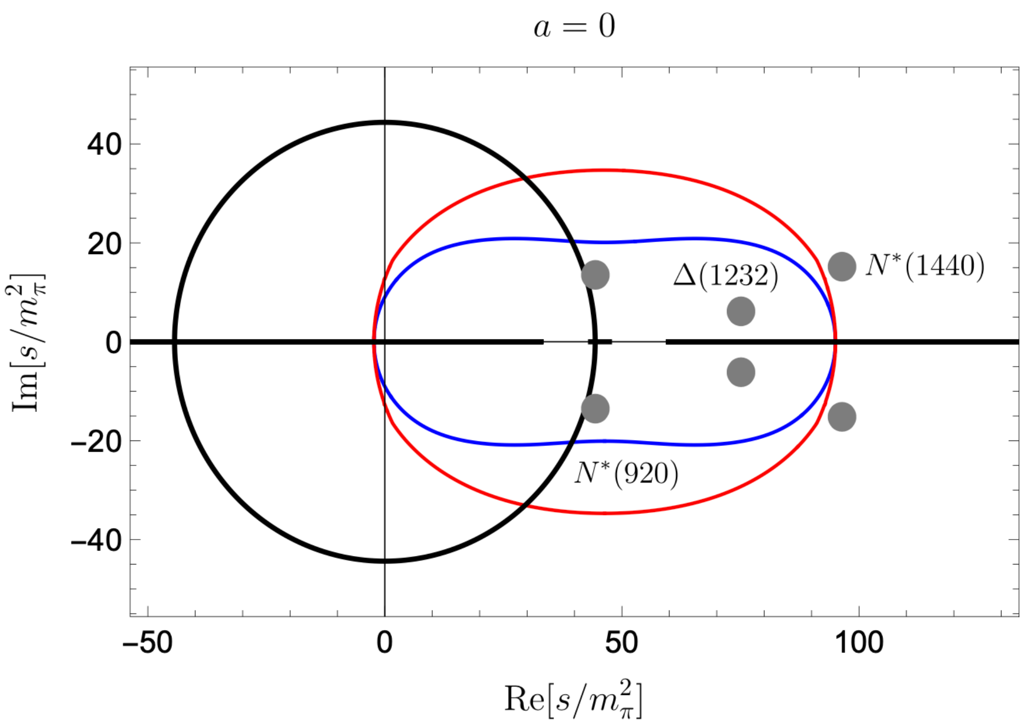
I. Caprini, et al., PRL (2006)

Roy-Steiner type equations

πK



πN



$$m_\kappa = 658 \pm 13\mathrm{MeV}$$
$$\Gamma_\kappa = 557 \pm 24\mathrm{MeV}$$

Descotes-Genon and Moussallam, EPJC (2006)

$$m_{N^*} = 918 \pm 3\mathrm{MeV}$$
$$\Gamma_{N^*} = 326 \pm 18\mathrm{MeV}$$

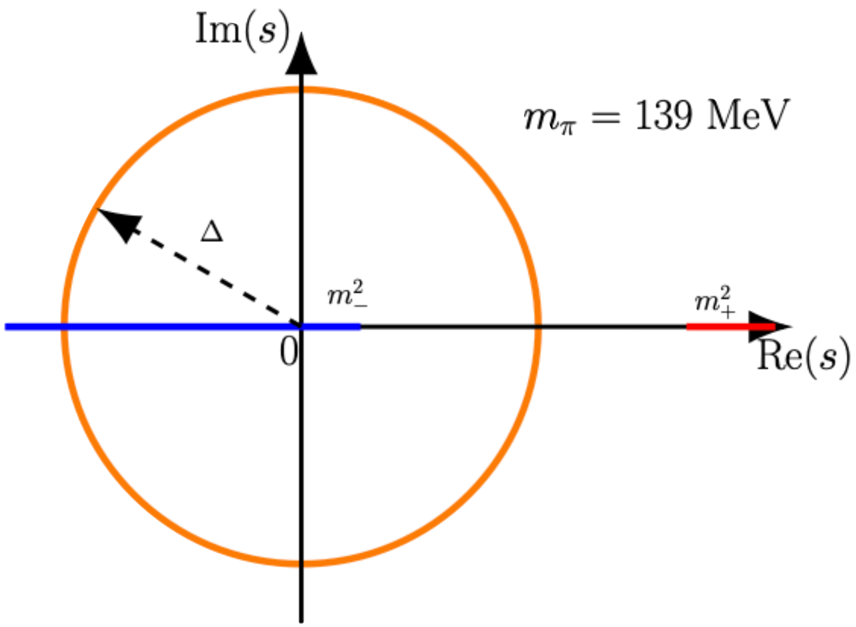
XHC, Li and Zheng, JHEP (2022)

$$m_{N^*} = 913.9 \pm 1.6\mathrm{MeV}$$
$$\Gamma_{N^*} = 337.7 \pm 6.2\mathrm{MeV}$$

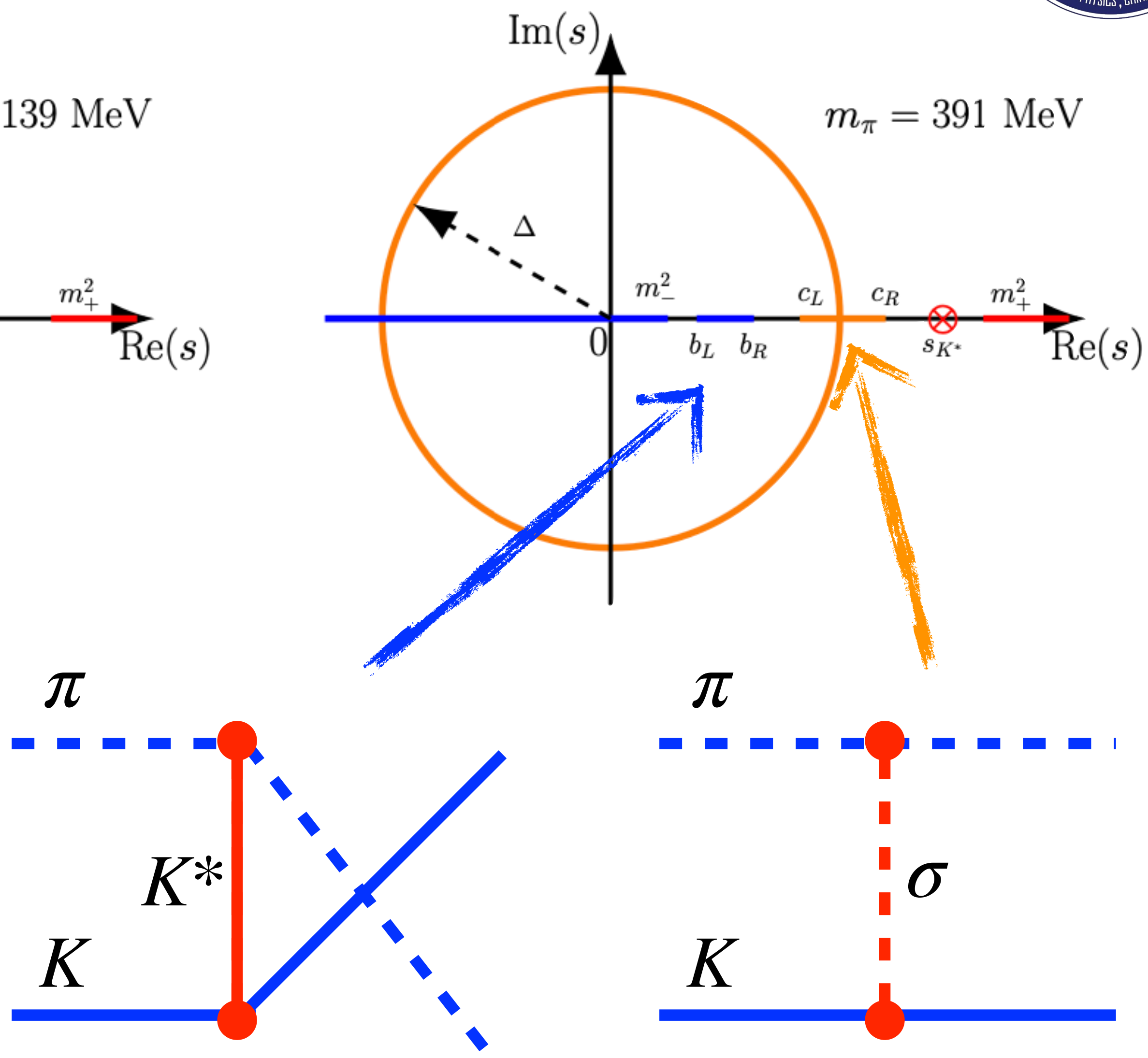
Hoferichter, et al., PLB (2024)

πK scattering at $m_\pi = 391$ MeV

The cut structure of the πK partial-wave amplitudes



Lang, Fortsch. Phys. (1978)

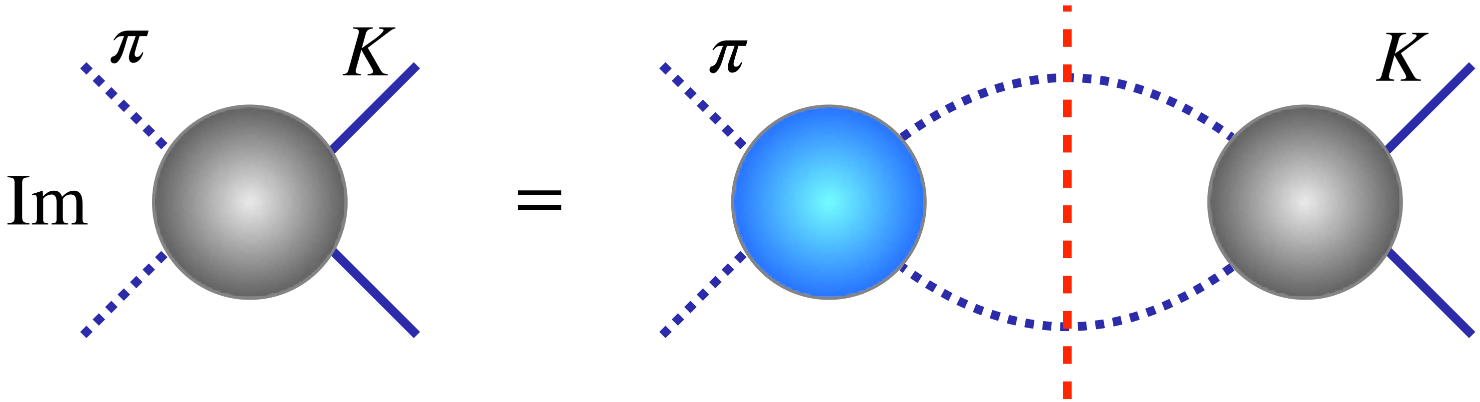


- σ and $K^*(892)$ as the S- and P-wave bound-states
- New left-hand cuts
- No long-range force

t-channel $\pi\pi \to K\bar{K}$ partial wave amplitudes

- t-channel Roy-Steiner equations as a Muskhelishvili-Omnes problem

$$\mathrm{Im}\, g_J^I(t) = \left[t_J^I(s)\right]^* \rho_{\pi\pi}(t) g_J^I(t)$$



t-channel $\pi\pi \to K\bar{K}$ partial wave amplitudes

- t-channel Roy-Steiner equations as a Muskhelishvili-Omnes problem

$$\mathrm{Im}\, g_J^I(t) = \left[t_J^I(s)\right]^* \rho_{\pi\pi}(t) g_J^I(t)$$

- Single-channel: Omnes solution, $\Omega_J^I(t) = \exp\left\{ \frac{t}{\pi} \int_{t_\pi}^{t_m} dt' \, \frac{\phi_J^I(t')}{t'(t'-t)} \right\}$

$$g_J^I(t) = \Delta_J^I(t) + t\Omega_J^I(t) \left[\frac{1}{\pi} \int_{t_\pi}^{t_m} dt' \frac{\Delta_J^I(t') \sin \phi_J^I(t')}{|\Omega_J^I|(t')\, t'(t'-t)} + \frac{1}{\pi} \int_{t_m}^{\infty} dt' \frac{\left|g_J^I(t')\right| \sin \phi_J^I(t')}{|\Omega_J^I|(t')\, t'(t'-t)} \right]$$

Crossing

Unitarity

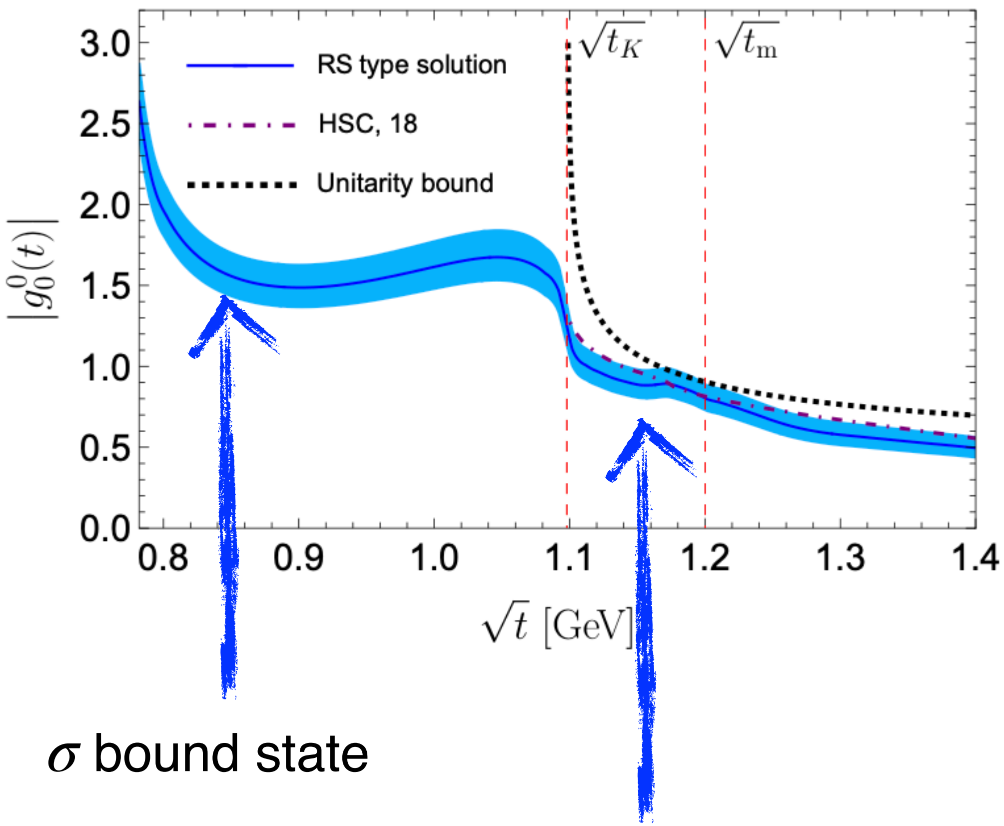
s-channel πK scattering partial waves and t-channel $\pi\pi \to K\bar{K}$ partial waves where $I'J' \neq IJ$

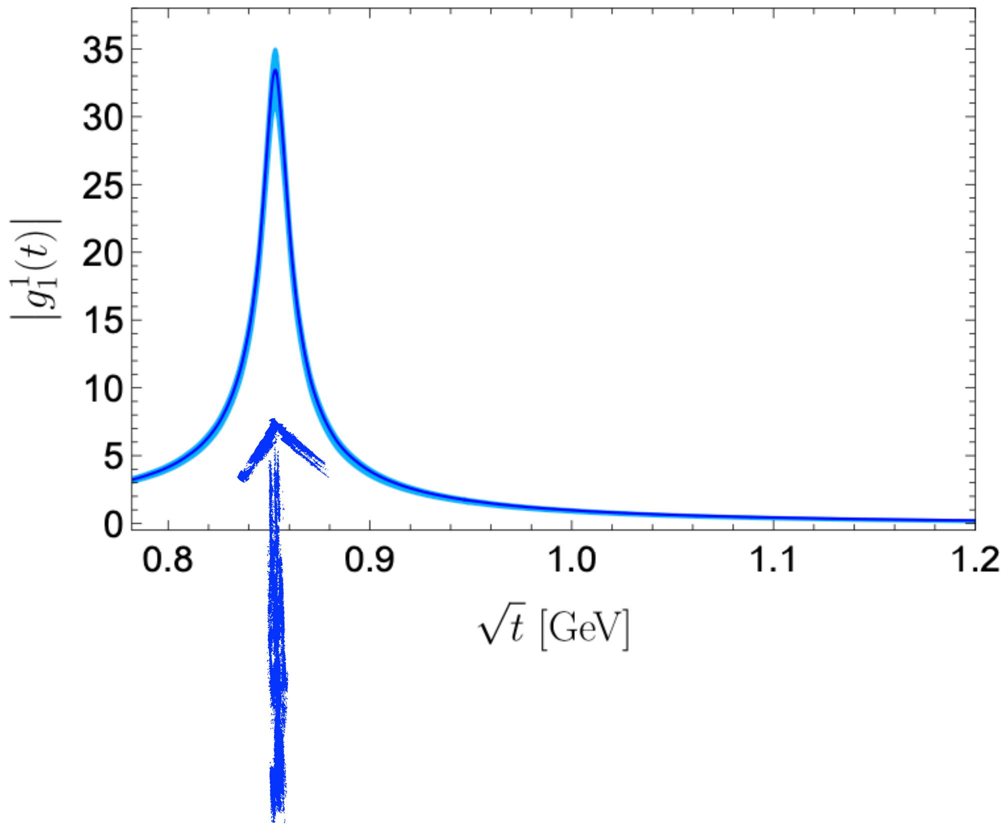
Phase-shifts from $\pi\pi$ scattering: LQCD data + Roy eq. analyses

XHC et.al., PRD (2023)

t-channel $\pi\pi \to K\bar{K}$ partial wave amplitudes

- ☑ t-channel solution from Roy-Steiner equations





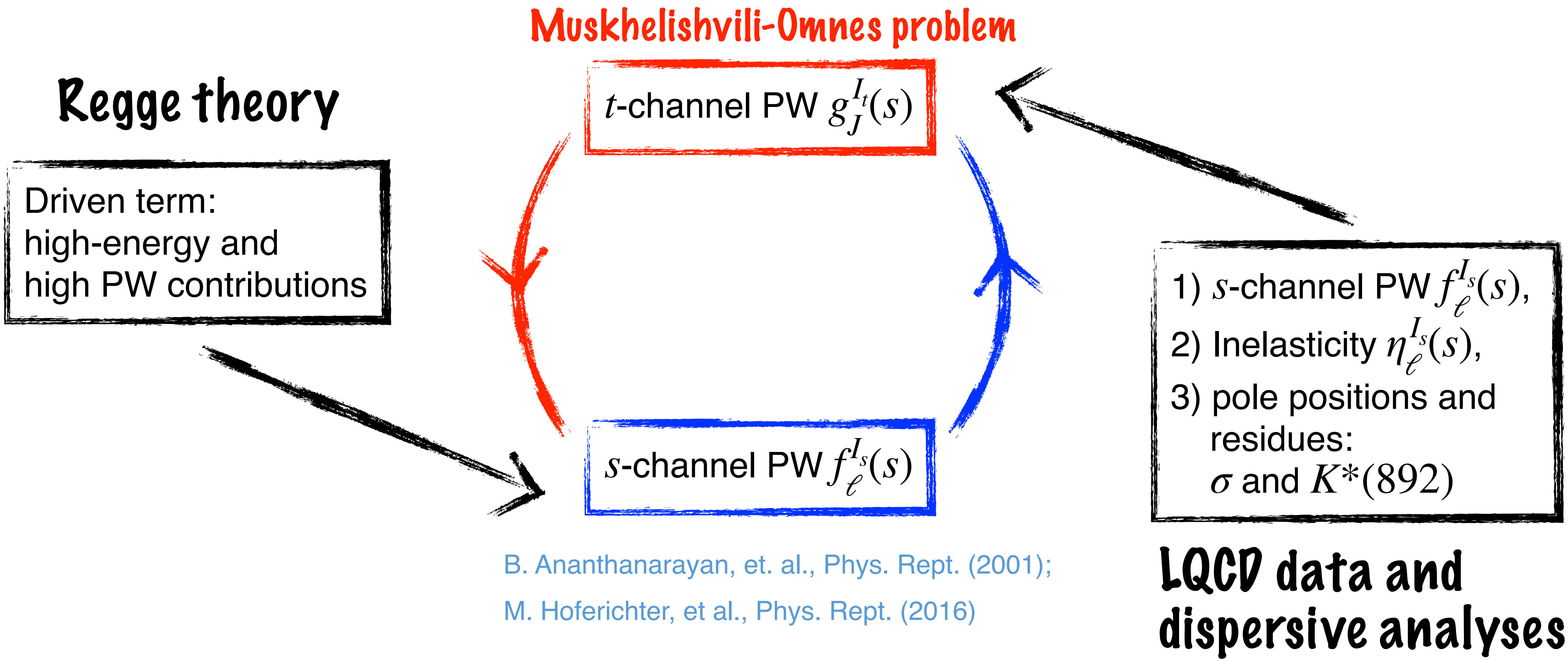
σ bound state

$f_0(980)$

$\rho(770)$ dominance

s-channel $\pi\pi \to K\bar{K}$ partial wave amplitudes

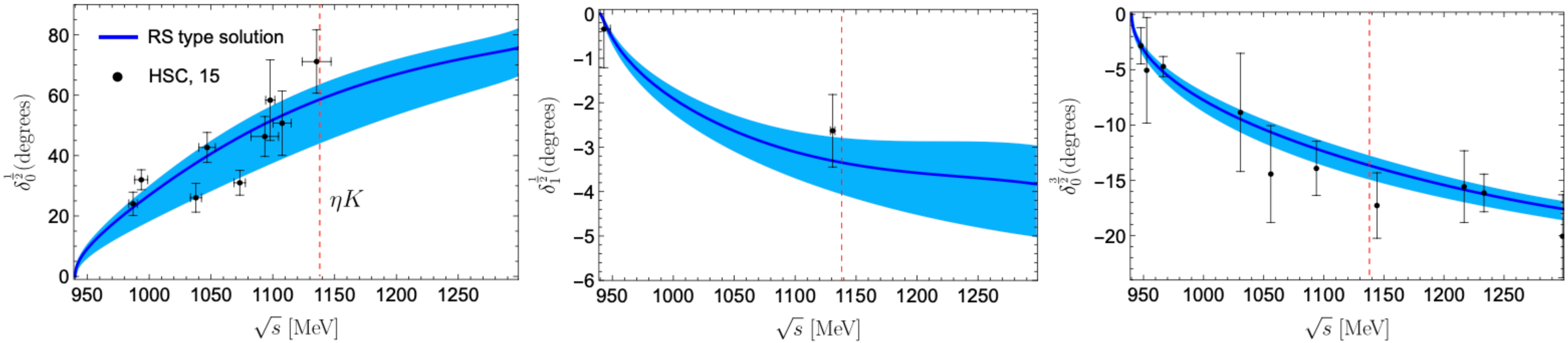
- Numerical strategy to solve Roy-Steiner type equations



B. Ananthanarayan, et. al., Phys. Rept. (2001);
M. Hoferichter, et al., Phys. Rept. (2016)

s-channel $\pi\pi \to K\bar{K}$ partial wave amplitudes

☑ s-channel solution from Roy-Steiner equations

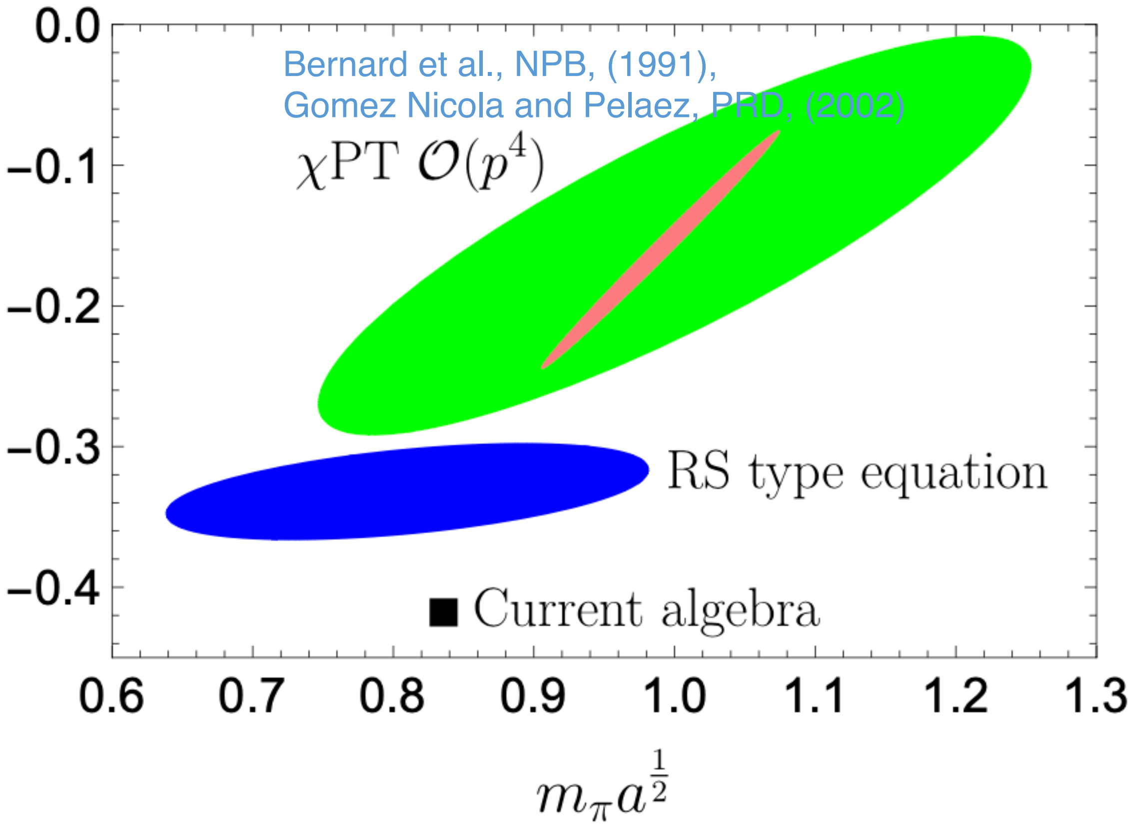


- $I = \dfrac{1}{2}$ S-wave: no sharp features that signal the presence of a nearby pole
- $I = \dfrac{1}{2}$ P-wave: shallow vector bound state $K^*(892)$
- $I = \dfrac{3}{2}$ S-wave: repulsive channel

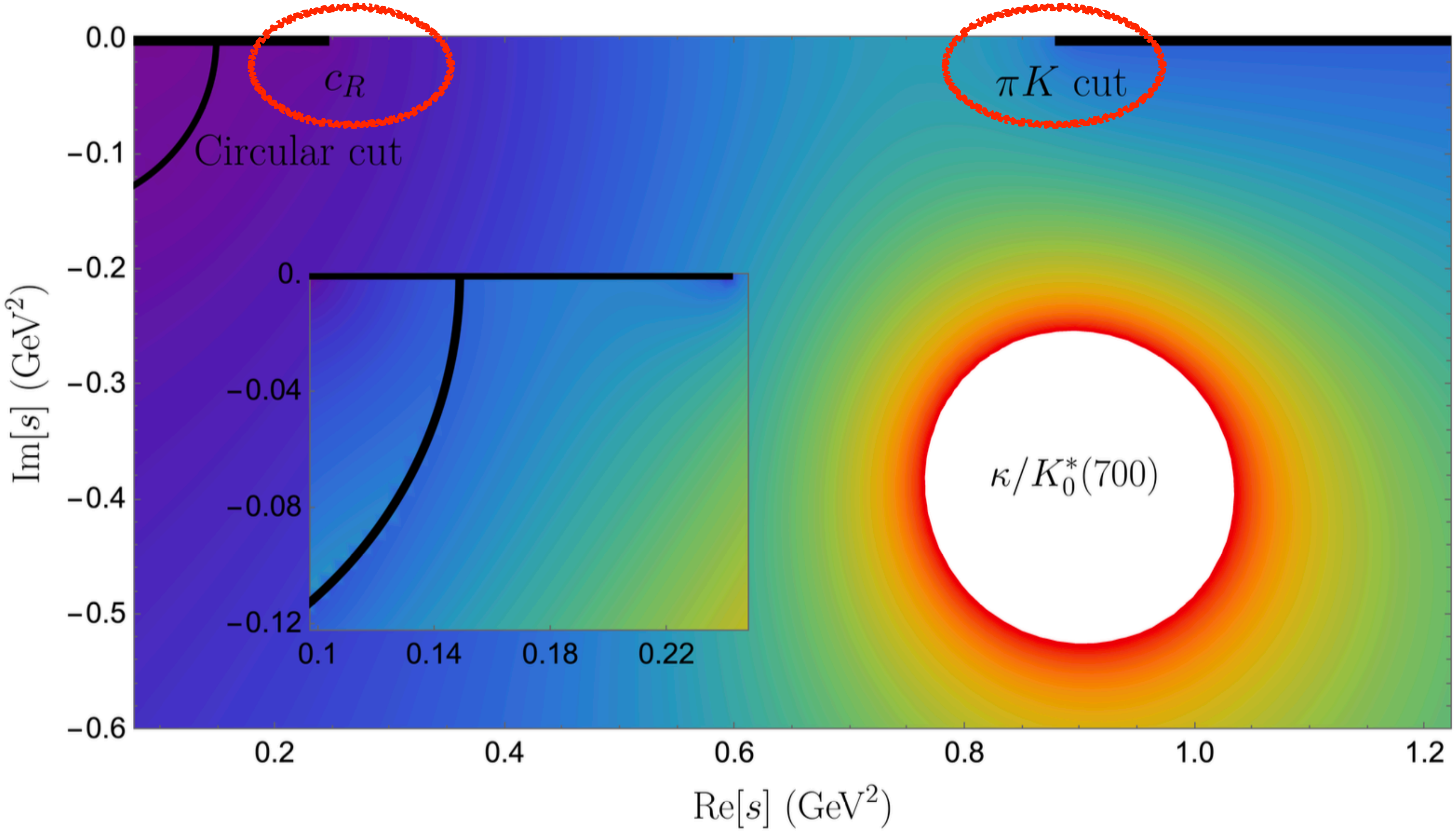
S-wave scattering lengths

- Two scattering lengths:

$$m_\pi a_0^{\frac{1}{2}} = 0.92^{+0.06}_{-0.28}\,, \quad m_\pi a_0^{\frac{3}{2}} = -\left(0.32^{+0.05}_{-0.02}\right)$$



Dispersive determination of $\kappa/K_0^*(700)$ from LQCD data



A broad resonance instead of a deeply bound virtual state pole

Summary and outlook

- ☑ The unity of dispersive techniques and lattice QCD data is powerful to investigate low energy hadron physics
- ☑ Widely-used unitarization methods such as K-matrix, etc., are not good in light meson & baryon studies
- ☑ Dispersive approaches, Muskhelishvili-Omnès formalism, Roy-Steiner type equations, etc. are necessary
- ☐ πD scattering at physical & unphysical m_π: $D_0^*(2300)$, two pole structure
- ☐ KN & $\bar{K}N$ scatterings: $\Lambda(1405)$, two pole structure and strangeness σ term
- ☐ Dispersive determination of three-body resonances?

Thank you for your attention!

Validity Domain