重介子四体半轻衰变的一些探讨

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Overview

Core issues in the semileptonic decays

Dipion LCDAs

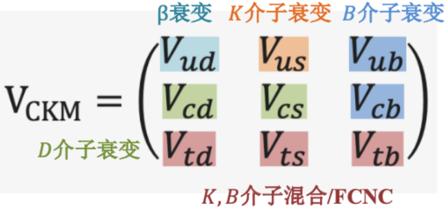
Four-body decays of Heavy mesons

- $B \to [\rho^+ \to] \, \pi^+\pi^0 l\bar{\nu}$
- $D_s \to [f_0 \to] \, [\pi^+\pi^-]_{\rm S} \, e^+\nu_e$

Conclusion and Prospect

Core issues in the semileptonic decays

Semileptonic decays



- $VV^{\dagger} = V^{\dagger}V = I_3$ in the Standard Model
- $VV^{\dagger} \neq V^{\dagger}V \neq I_3$ New Physics
- $|V_{ub}|/|V_{cb}|$ contributes to *CPV* measurement in B decays
- CKM matrix elements are mainly measured via the charged current processes, i.e, $b \to ul^{-}\bar{\nu}, c \to sl^{+}\nu$
- Flavor changing neutral current precesses are sensitive to new physical contributions, i.e, $b \to sl^{+}l^{-}$

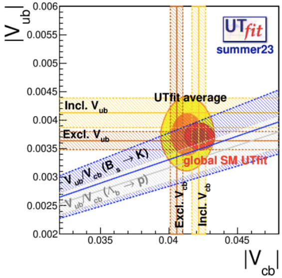
Core issues

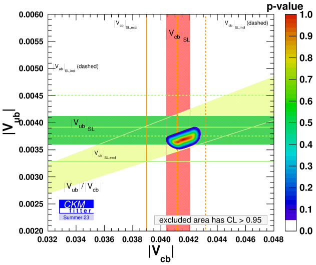
- $|V_{ub}|$ tension $\quad |V_{ub}| = (3.82 \pm 0.20) \times 10^{-3}$ [PDG 2022]

‡ $\sim 2.5\sigma$ tension between $(4.13 \pm 0.25) \times 10^{-3}$ and $(3.70 \pm 0.16) \times 10^{-3}$ measured via the $B \to X_u l^- \bar{\nu}$ and $B \to \pi l^- \bar{\nu}$ processes, respectively.

- $|V_{cb}|$ tension $\quad |V_{ub}| = (40.8 \pm 1.4) \times 10^{-3}$ [PDG 2022]

‡ $\sim 2.5\sigma$ tension between $(42.2 \pm 0.8) \times 10^{-3}$ and $(39.4 \pm 0.8) \times 10^{-3}$ measured via the $B \to X_c l^- \bar{\nu}$ and $B \to D^{(*)} l^- \bar{\nu}$ processes, respectively.





Core issues

- LFU in $b \to c l^- \bar{\nu}$ processes $\quad R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)} \tau^- \bar{\nu})}{\mathcal{B}(B \to D^{(*)} \mu^- \bar{\nu})}$

‡ $R_D = 0.407 \pm 0.046$, $R_{D^*} = 0.306 \pm 0.015$ Average with [Belle PRL124, 161803 (2020)]

‡ $2.1\sigma, 3.0\sigma$ derivations from the SM predictions of
$R_D = 0.298 \pm 0.004$, $R_{D^*} = 0.254 \pm 0.005$ [HFLAV]

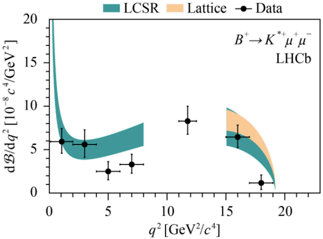
‡ $R_D = 0.441 \pm 0.089$, $R_{D^*} = 0.281 \pm 0.030$ [LHCb PRL131,111802 (2023)]

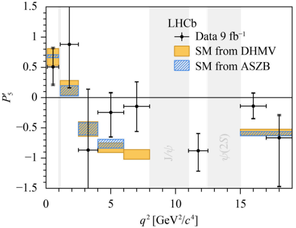
‡ would make the CKM measurements more complicated if $\cdots$

- Anomalies in FCNC processes $B \to K^* \mu^+ \mu^-$

‡ 3.6σ derivation from SM of $d\mathcal{B}(B \to K^* \mu^+ \mu^-)/dq^2$ in $q^2 \in [1, 6]$ GeV^2

‡ 1.9σ derivation from SM of $p_5' = S_5/\sqrt{F_L(1 - F_L)}$ in $q^2 \in [4, 8]$ GeV^2





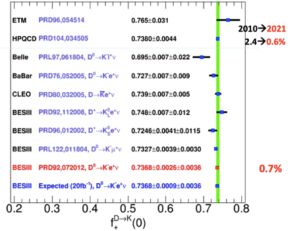
Solution Programmes

- continue to improve the measurements and predictions in the conventional processes

‡ $|V_{cs}|$ issue $|V_{cs}| = 0.975 \pm 0.006$ [PDG 2022]

∗ $\sim 1.5\sigma$ derivation between 0.972 ± 0.007 and 0.984 ± 0.012 measured via the $D \to Kl\nu$ and $D_s \to \mu^+\nu_\mu$ processes, respectively.

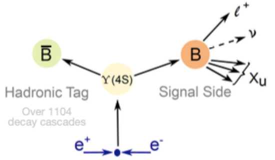
∗ $\sim 3\sigma$ tension two years ago, 0.939 ± 0.038 and 0.992 ± 0.012 [PDG 2020, 2021]



‡ $|V_{ub}|$ result from Belle collaboration with Simultaneous Determination in excl. and incl. processes

∗ $3.78 \pm 0.23 \pm 0.16 \pm 0.14$ and $3.88 \pm 0.20 \pm 0.31 \pm 0.09$ in the exclusive and inclusive processes, respectively [Belle PRL131, 211801 (2023)]

∗ the ration 0.97 ± 0.12 compatible with the world average within 1.2σ, the combined result $(3.84 \pm 0.26) \times 10^{-3}$ is consistent with the CKM fitter within 0.8σ



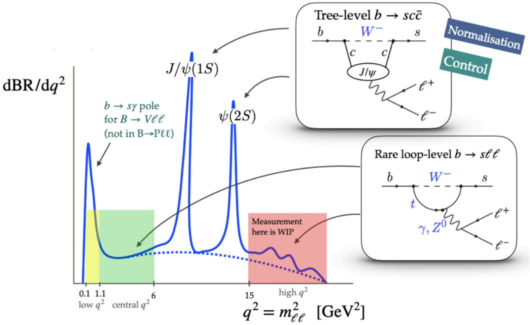
$$\mathcal{B}(B \to \pi^0 \ell\nu) + \mathcal{B}(B \to \pi^+ \ell\nu) + \mathcal{B}(B \to X_u^{\text{other}} \ell\nu) = \mathcal{B}(B \to X_u \ell\nu)$$

Solution Programmes

‡ deep understanding of QCD dynamics in the anomalies

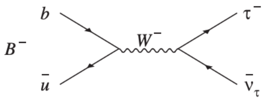
∗ high order QCD corrections

∗ more structures, interactions ⋯



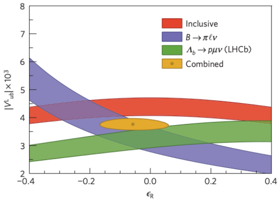
Solution Programmes

- enlarge the set of exclusive processes to study these issues



‡ $|V_{ub}|f_B$ in pure leptonic decay

∗ 0.72 ± 0.09 MeV from Belle, 1.01 ± 0.14 MeV from BABAR, 0.77 ± 0.12 MeV average [FLAG2021]



‡ $|V_{ub}|$ in baryon decay [Nature Physics 11, 743 (2015)]

$$\frac{|V_{ub}|^2}{|V_{cb}|^2} = \frac{\mathcal{B}(\Lambda_b^0 \to p\mu^-\bar{\nu})}{\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+\mu^-\bar{\nu})} R_{\mathrm{FF}} = 0.68 \pm 0.07 \Downarrow$$

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.083 \pm 0.06 \xrightarrow{|V_{cb}|} |V_{ub}| = (3.27 \pm 0.23) \times 10^{-3}$$

∗ consistent with determinations in exclusive $B \to \pi l \bar{\nu}$ decay
confirms the existing incompatibility with the inclusive sample

‡ $|V_{cb}|$ in $B_s \to D_s \mu^+ \nu$

‡ $|V_{ub}|/|V_{cb}|$ in $\mathcal{B}(B_s \to K^- \mu^+ \nu)/\mathcal{B}(B_s \to D_s^- \mu^+ \nu)$

‡ $d\mathcal{B}(B \to K^* \mu^+ \mu^-)/dq^2$ and p_5' in baryon decay $\Lambda_b^0 \to \Lambda \mu^+ \mu^-$

Solution Programmes

- All the above discusses focus on the ground state particles
- continue to enlarge the set of exclusive processes
- Progresses with excited states can also provide independent measurements

- $|V_{ub}|$ in the $B \to \rho l\nu$ process

$*$ ρ is reconstructed by the $\pi\pi$ invariant mass spectral, the underlying consideration is $B \to [\rho^+ \to] \, \pi^+\pi^0 l^- \bar{\nu}_l \, (B_{l4})$ [Faller 2014]

- $|V_{cs}|$ in the $D_s \to [f_0 \to] \, [\pi\pi]_{\mathrm{S}} \, l\nu$ process

$*$ f_0 large uncertainty due to the width and complicate structure

- $|V_{cb}|$ in the $B \to D^* l\nu$ processes
- B anomalies in $B \to K^* l^+ l^-$ processes

$*$ How to calculate the width effect/nonresonant contribution ?

Introduce Dipion LCDAs to demonstrate the width effect and nonresonant background in Heavy Flavor decays

DiPion LCDAs

- Chiral-even LC expansion with gauge factor $[x, 0]$ [Polyakov 1999, Diehl 1998]

$$\langle \pi^a(k_1)\pi^b(k_2)|\bar{q}_f(zn)\gamma_\mu \tau q_{f'}(0)|0\rangle = \kappa_{ab}\, k_\mu \int dx\, e^{iuz(k\cdot n)}\, \Phi_{\parallel}^{ab,ff'}(u,\zeta,k^2)$$

△ $n^2 = 0$, △ index f, f' respects the (anti-)quark flavor, △ a, b indicates the electric charge

△ coefficient $\kappa_{+-/00} = 1$ and $\kappa_{+0} = \sqrt{2}$, △ $k = k_1 + k_2$ is the invariant mass of dipion state

△ $\tau = 1/2, \tau^3/2$ corresponds to the isoscalar and isovector 2πDAs

△ higher twist $\propto 1, \gamma_\mu\gamma_5$ have not been discussed yet, γ_5 vanishes due to P-parity conservation

△ DiPion is not a tetraquark state, but a collinear two pion system with nonlocal $q\bar{q}, \cdots$ operators

† Three independent kinematic variables

△ momentum fraction z carried by anti-quark with respecting to the total momentum of DiPion state,

△ longitudinal momentum fraction carried by one of the pions $\zeta = k_1^+/k^+$, $2q\cdot\bar{k}\,(\propto 2\zeta - 1)$ △ k^2

† Normalization conditions

$$\int_0^1 \Phi_{\parallel}^{I=1}(u,\zeta,k^2) = (2\zeta - 1)F_\pi(k^2)$$

$$\int_0^1 dz\,(2z-1)\Phi_{\parallel}^{I=0}(z,\zeta,k^2) = -2M_2^{(\pi)}\zeta(1-\zeta)F_\pi^{\rm EMT}(k^2)$$

△ $F_\pi^{em}(0) = 1$, △ $F_\pi^{\rm EMT}(0) = 1$

△ $M_2^{(\pi)}$ is the momentum fraction carried by quarks in the pion associated to the usual quark distribution

DiPion LCDAs

- 2πDAs is decomposed in terms of $C_n^{3/2}(2z-1)$ and $C_\ell^{1/2}(2\zeta-1)$

$$\Phi^{I=1}(z,\zeta,k^2,\mu) = 6z(1-z)\sum_{n=0,\text{even}}^{\infty}\sum_{l=1,\text{odd}}^{n+1} B_{n\ell}^{I=1}(k^2,\mu)C_n^{3/2}(2z-1)C_\ell^{1/2}(2\zeta-1)$$

$$\Phi^{I=0}(z,\zeta,k^2,\mu) = 6z(1-z)\sum_{n=1,\text{odd}}^{\infty}\sum_{l=0,\text{even}}^{n+1} B_{n\ell}^{I=0}(k^2,\mu)C_n^{3/2}(2z-1)C_\ell^{1/2}(2\zeta-1)$$

- $B_{n\ell}(k^2,\mu)$ have similar scale dependence as the a_n of π,ρ,f_0 mesons

$$B_{n\ell}(k^2,\mu) = B_{n\ell}(k^2,\mu_0)\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{\frac{\gamma_n^{(0)}-\gamma_0^{(0)}}{2\beta_0}}, \quad \gamma_n^{\perp(\parallel),(0)} = 8C_F\left(\sum_{k=1}^{n+1}\frac{1}{k}-\frac{3}{4}-\frac{1}{2(n+1)(n+2)}\right)$$

- Soft pion theorem relates the chirarlly even coefficients with a_n^π

$$\sum_{\ell=1}^{n+1} B_{n\ell}^{\parallel,I=1}(0) = a_n^\pi, \quad \sum_{\ell=0}^{n+1} B_{n\ell}^{\parallel,I=0}(0) = 0$$

- 2πDAs relate to the skewed parton distributions (SPDs) by crossing

△ express the moments of SPDs in terms of $B_{nl}(k^2)$ in the forward limit as

$$M_{N=\text{odd}}^\pi = \frac{3}{2}\frac{N+1}{2N+1}B_{N-1,N}^{I=1}(0), \quad M_{N=\text{even}}^\pi = 3\frac{N+1}{2N+1}B_{N-1,N}^{I=0}(0)$$

DiPion LCDAs

- In the vicinity of the resonance, 2πDAs reduce to the DAs of ρ/f_0

△ relation between the a_n^ρ and the coefficients $B_{n\ell}$

$$a_n^\rho = B_{n1}(0)\,\mathrm{Exp}\left[\sum_{m=1}^{N-1} c_m^{n1}\, m_\rho^{2m}\right], \quad c_m^{(n1)} = \frac{1}{m!}\frac{d^m}{dk^{2m}}\left[\ln B_{n1}(0) - \ln B_{01}(0)\right]$$

△ f_ρ relates to the imaginary part of $B_{nl}(m_\rho^2)$ by $\quad \langle \pi(k_1)\pi(k_2)|\rho\rangle = g_{\rho\pi\pi}(k_1 - k_2)^\alpha \epsilon_\alpha$

$$f_\rho^{\parallel} = \frac{\sqrt{2}\,\Gamma_\rho\,\mathrm{Im}B_{01}^{\parallel}(m_\rho^2)}{g_{\rho\pi\pi}}, \quad f_\rho^{\perp} = \frac{\sqrt{2}\,\Gamma_\rho\, m_\rho\,\mathrm{Im}B_{01}^{\perp}(m_\rho^2)}{g_{\rho\pi\pi}\, f_{2\pi}^{\perp}}$$

- How to describe **the evolution from $4m_\pi^2$ to large invariant mass** $k^2 \sim \mathcal{O}(m_c^2)$? furtherly to $\mathcal{O}(m_b\lambda_{\mathrm{QCD}})$

‡ Watson theorem of π-π scattering amplitudes

△ implies an intuitive way to express the imaginary part of 2πDAs

△ leads to the Omnés solution of N-subtracted DR for the coefficients

$$B_{n\ell}^{I}(k^2) = B_{n\ell}^{I}(0)\,\mathrm{Exp}\left[\sum_{m=1}^{N-1}\frac{k^{2m}}{m!}\frac{d^m}{dk^{2m}}\ln B_{n\ell}^{I}(0) + \frac{k^{2N}}{\pi}\int_{4m_\pi^2}^{\infty} ds\,\frac{\delta_\ell^{I}(s)}{s^N(s-k^2-i0)}\right]$$

DiPion LCDAs

$$B^I_{n\ell}(k^2) = B^I_{n\ell}(0)\,\mathrm{Exp}\left[\sum_{m=1}^{N-1}\frac{k^{2m}}{m!}\frac{d^m}{dk^{2m}}\ln B^I_{n\ell}(0) + \frac{k^{2N}}{\pi}\int_{4m_\pi^2}^{\infty} ds\,\frac{\delta^I_\ell(s)}{s^N(s-k^2-i0)}\right]$$

- **2πDAs in a wide range of energies is given by δ^I_ℓ and a few subtraction constants**

- The subtraction constants of $B_{n\ell}(s)$ at low s (around the threshold)

(nl)	$B^{\parallel}_{n\ell}(0)$	$c_1^{\parallel,(nl)}$	$\frac{d}{dk^2}\ln B^{\parallel}_{n\ell}(0)$	$B^{\perp}_{n\ell}(0)$	$c_1^{\perp,(nl)}$	$\frac{d}{dk^2}\ln B^{\perp}_{n\ell}(0)$
(01)	1	0	1.46 $\rightarrow$ 1.80	1	0	0.68 $\rightarrow$ 0.60
(21)	-0.113 $\rightarrow$ 0.218	-0.340	0.481	0.113 $\rightarrow$ 0.185	-0.538	-0.153
(23)	0.147 $\rightarrow$ -0.038	0	0.368	0.113 $\rightarrow$ 0.185	0	0.153
(10)	-0.556	-	0.413	-	-	-
(12)	0.556	-	0.413	-	-	-

△ firstly studied in the effective low-energy theory based on instanton vacuum [Polyakov 1999]

△ updated with the kinematical constraints and the new a_2^π, a_2^ρ [**SC** 2019, 2023]

- Above discussions are all at leading twist level, subleading twist LCDAs are not known yet

† Would the chiral EFT help us to set down $B_{nl}(k^2 \sim 4m_\pi^2)$ for both leading and subleading twist LCDAs ?

Four-body decays of Heavy mesons

$$B \to [\rho^+ \to] \, \pi^+ \pi^0 \, l\bar{\nu}$$

$$D_s \to [f_0 \to] \, [\pi\pi]_{\mathrm{S}} \, e^+ \nu_e$$

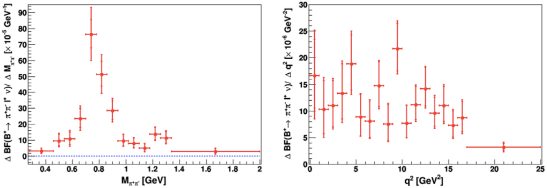
DiPion LCDAs and B_{l4} decays

- B_{l4} decays have rich observables, nontrivial tests of SM [Faller 2014]
- Different exclusive $b \to u$ processes in the $|V_{ub}|$ determination [Faller 2014]
  [Gao, Lü, Shen, Wang, Wei 1902.11092]

$$|V_{ub}| = \left(3.05^{+0.67}_{-0.52}\Big|_{\rm theo} \; {}^{+0.19}_{-0.20}\Big|_{\rm exp}\right) \times 10^{-3}, \quad \text{from } B \to \rho l \nu$$

$$|V_{ub}|_{\rm PDG} = (3.70 \pm 0.12|_{\rm theo} \; \pm 0.10|_{\rm exp}) \times 10^{-3} \quad \text{from } B \to \pi l \nu$$

- $B \to \pi\pi l \bar{\nu}_l$ has already been measured, mainly its resonant part $B \to \rho l \bar{\nu}_l$
  $(1.58 \pm 0.11) \times 10^{-4}$ [CLEO 2000, BABAR 2011, Belle 2013]
- Propose to measure the $B \to \pi^+\pi^0 l^- \bar{\nu}$ decay with $B \to \pi^+\pi^0$ form factor calculation from B meson LCSRs [**SC**, Khodjamirian, Virto 1701,01633]
- First measurement of the branching fraction of $B^+ \to \pi^+\pi^- l^+ \bar{\nu}_l$
  $(2.3 \pm 0.4) \times 10^{-4}$ [Belle 2005.07766] More data on the way from Belle II



$B \to \pi\pi$ form factors

- Dynamics of B_{l4} is governed by the $B \to \pi\pi$ form factors
- A big task for the practitioners of QCD-based methods
- First Lattice QCD study of the $B \to \pi\pi l\bar{\nu}$ transition amplitude in the region of large q^2 and $\pi\pi$ invariant mass near the ρ resonance [Leskovec et.al. 2212.08833[hep-lat]]

$B \to \pi\pi$ **form factors** [Hambrock, Khodjamirian, 1511.02509]

$$
\begin{aligned}
i\langle\pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle &= F_\perp(q^2,k^2,\zeta)\,\frac{2}{\sqrt{k^2}\sqrt{\lambda_B}}\,i\epsilon_{\nu\alpha\beta\gamma}\,q^\alpha\,k^\beta\,\bar{k}^\gamma \\
&+F_t(q^2,k^2,\zeta)\,\frac{q_\nu}{\sqrt{q^2}}+F_0(q^2,k^2,\zeta)\,\frac{2\sqrt{q^2}}{\sqrt{\lambda_B}}\left(k_\nu-\frac{k\cdot q}{q^2}q_\nu\right) \\
&+F_\parallel(q^2,k^2,\zeta)\,\frac{1}{\sqrt{k^2}}\left(\bar{k}_\nu-\frac{4(q\cdot k)(q\cdot\bar{k})}{\lambda_B}\,k_\nu+\frac{4k^2(q\cdot\bar{k})}{\lambda_B}\,q_\nu\right)
\end{aligned}
$$

† $\lambda = \lambda(m_B^2, k^2, q^2)$ is the Källén function

† $q\cdot k = (m_B^2 - q^2 - k^2)/2$ and $q\cdot\bar{k} = \sqrt{\lambda}\beta_\pi(k^2)\cos\theta_\pi/2 = \sqrt{\lambda}\,(2\zeta-1)$

† $\beta_\pi(k^2) = \sqrt{1-4m_\pi^2/k^2}$, θ_π is the angle between the 3-momenta of the neutral pion and the B-meson in the dipion rest frame

$B \to \pi\pi$ form factors

- Starting with the correlation function

$$F_\mu(k_1,k_2,q) = i\int d^4x e^{iq\cdot x}\langle \pi^+(k_1)\pi^0(k_2)|\mathrm{T}\{j_\mu^{V-A}(x), j_5(0)\}|0\rangle$$

$\downarrow$ *Lorentz decomposition*

$$\equiv \varepsilon_{\mu\nu\rho\sigma} q^\nu k_1^\rho k_1^\sigma F^V + q_\mu F^{(A,q)} + k_\mu F^{(A,k)} + \bar{k}_\mu F^{(A,\bar{k})}$$

- Take $F_\perp^l(q^2, k^2, \zeta)$ as an example

$$\frac{F_\perp^l(q^2,k^2,\zeta)}{\sqrt{k^2}\sqrt{\lambda_B}} = \frac{m_b}{\sqrt{2} f_B m_B^2 f_{2\pi}^\perp (2\zeta-1)} \int_{u_0}^1 \frac{du}{u}\, \Phi_\perp^l(u,\zeta,k^2)\, e^{-\frac{s(u)+m_B^2}{M^2}}$$

† Definition of the Chiral-odd DiPion LCDAs

$$\langle \pi^a(k_1)\pi^b(k_2)|\bar{q}_f(xn)\sigma_{\mu\nu}\tau q_{f'}(0)|0\rangle = \kappa_{ab}\frac{2i}{f_{2\pi}^\perp}\frac{k_{1\mu}k_{2\nu}-k_{2\mu}k_{1\nu}}{2\zeta-1}\int dx\, e^{iux(k\cdot n)}\,\Phi_\perp^{ab,ff'}(u,\zeta,k^2)$$

† Partial wave expansion

$$F_{\perp,\parallel}(k^2,q^2,\zeta) = \sum_\ell \sqrt{2\ell+1}\, F_{\perp,\parallel}^{(\ell)}(k^2,q^2)\,\frac{P_\ell^{(1)}(\cos\theta_\pi)}{\sin\theta_\pi}$$

$$F_{0,t}(k^2,q^2,q\cdot\bar{k}) = \sum_\ell \sqrt{2l+1}\, F_{0,t}^{(\ell)}(k^2,q^2)\, P_\ell^{(0)}(\cos\theta_\pi)$$

$B \to \pi\pi$ form factors

- Using the orthogonality relation of the Legender polynomials

$$F_\perp^{(\ell)}(k^2,q^2) = \frac{\sqrt{k^2}}{\sqrt{2}f_{2\pi}^\perp}\frac{\sqrt{\lambda_B}m_b}{m_B^2 f_B}e^{\frac{m_B^2}{M^2}}\sum_{n=0,2,\cdots}\sum_{\ell'=1,3}^{n+1} I_{\ell\ell'}B_{n\ell'}^\perp(k^2,\mu)J_n^\perp(q^2,k^2,M^2,s_0^B).$$

$$I_{\ell\ell'} \equiv -\frac{\sqrt{2\ell+1}(\ell-1!)}{2(\ell+1)!}\int_{-1}^{1}\frac{dz}{z}\sqrt{1-z^2}P_\ell^{(1)}(z)P_{\ell'}^{(0)}(z),$$

$$J_n^\perp(q^2,k^2,M^2,s_0^B) = \int_{u_0}^{1} du e^{\frac{-s}{M^2}}6(1-u)C_n^{3/2}(2u-1).$$

† $I_{\ell\ell'} = 0$ when $\ell > \ell'$, $I_{11} = 1/\sqrt{3}, I_{13} = -1/\sqrt{3}, I_{15} = 4/(5\sqrt{3})$

† $\ell' = 1$, asymptotic DAs, $P-$wave term retains in the DiPion LCDAs

† Short-distance part of the correlation: $\mu = 3$ GeV without NLO correction

† $f_B = 207^{+9}_{-17}$ MeV [P. Gelhausen 2013,2014]

† $M^2 = 16.0 \pm 4.0$ GeV^2 $\leftrightarrow$ $s_0^B = 37.5 \pm 2.5$ GeV^2

- How large of $P-$wave contribution to $B \to \pi\pi$ FFs $(\ell = 1)$?

$$R_\ell \equiv F_\perp^{(\ell>1)}(k^2,q^2)/F_\perp^{(\ell=1)}(k^2,q^2)$$

- How much ρ contained in $P-$wave $B \to \pi\pi$ FFs $(\ell = 1, \ell' = 1)$?

$B \to \pi\pi$ form factors

- How much ρ contained in $P-$wave $B \to \pi\pi$ FFs $(\ell = 1, \ell' = 1)$?

- Hadronic dispersion relation for the P-wave $B \to \pi\pi$ form factors

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle = \langle \pi^+(k_1)\pi^0(k_2)|\rho\rangle\langle\rho|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle + \cdots$$

$$\frac{\sqrt{3}F_\perp^{(\ell=1)}}{\sqrt{k^2}\sqrt{\lambda_B}} = \frac{g_{\rho\pi\pi}}{m_\rho^2 - k^2 - im_\rho\Gamma_\rho(k^2)}\frac{V^{B\to\rho}(q^2)}{m_B + m_\rho} + \cdots$$

△ $\rho \to 2\pi$ strong coupling $g_{\rho\pi\pi} = 5.96 \pm 0.04 \Leftarrow$ the energy dependent total width of ρ

- $B \to \rho$ form factor obtained from leading twist ρ-LCDAs

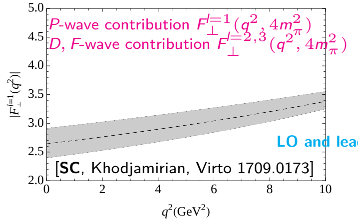
$$V^{B\to\rho}(q^2) = \frac{(m_B + m_\rho)m_b}{2m_B^2 f_B} f_\rho^\perp \int_{u_0}^1 \frac{du}{u}\phi_\perp^{(\rho)}(u)\, e^{m_B^2/M^2} e^{(-q^2\bar{u}+m_\rho^2 u\bar{u})/(uM^2)}$$

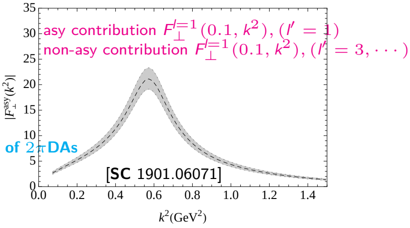
$$i\langle\rho^+(k)|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle = i\varepsilon_{\nu\alpha\beta\gamma}\epsilon^{*\alpha}q^\beta k^\gamma \frac{V(q^2)}{m_B+m_\rho} + \epsilon_\nu^*\,(m_B+m_\rho)\,A_1(q^2)$$

$$-(2k+q)_\nu\,(\epsilon^*\cdot q)\,\frac{A_2(q^2)}{m_B+m_\rho} - q_\nu\,(\epsilon^*\cdot q)\,\frac{2m_\rho^2}{q^2}\left[A_3(q^2) - A_0(q^2)\right]$$

△ ρ meson: $a_2^\perp = 0.2 \pm 0.1$, $a_{n>2} = 0$, $f_\rho^\perp = 160 \pm 10$ MeV

$B \to \pi\pi$ form factors





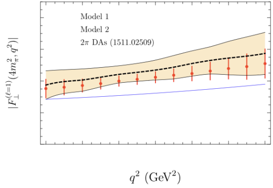
LO and leading twist of 2π DAs

- High partial waves give few percent contributions to $B \to \pi\pi$ form factors
- ρ', ρ'' and NR background contribute $\sim 20\% - 30\%$ to P-wave

- 30% smaller than it obtained from B-meson LCSRs [SC, Khodjamirian and Virto 1701.01663]

† high twist contributions ?

† Uncertainty of B-meson LCDAs ?



$B \to \pi\pi$ form factors

Comparison with the B meson LCSRs

$$F_\mu(k_1, k_2, q) = i \int d^4x e^{iq\cdot x} \langle \pi^+(k_1)\pi^0(k_2)|\mathrm{T}\{j_\mu^{V-A}(x), j_5^B(0)\}|0\rangle$$

$$F_{\mu\nu}(k, q) = i \int d^4x\, e^{ik\cdot x} \langle 0|\mathrm{T}\{\bar{d}(x)\gamma_\mu u(x), \mathrm{T}\{j_\mu^{V-A}(x)\}|\bar{B}^0(q+k)\rangle$$

- At the current accuracy, they give same order plots of $B \to \pi^+\pi^0$ FFs
- For the $P-$wave FFs, they both predict sizable non-ρ contribution $(\sim 10\%)$ $\rho', \rho'', \cdots$ and NR background
- B-meson LCSRs can not predict the contributions from higher partial waves, but it indeed exist while is small in the DiPion LCSRs
- B-meson LCSRs rely on the resonance model (an inverse problem), DiPion LCSRs is currently limited by the poor knowledge of DiPion LCDAs

DiPion LCDAs and D_{l4} decays

Structures of scalar mesons

- $f_0(1370), f_0(1500), a_0(1450), K_0^*(1430)$ form a $SU(3)$ flavor nonet
  $q\bar{q}$ replenished with some possible gluon content
  i.e., $f_0(1370) \to 2\rho \to 4\pi$, $|n\bar{n}\rangle$, $f_0(1500) \to 4\pi, 2\pi$, gluon content

- $f_0(500)/\sigma, f_0(980), a_0(980), K_0^*(700)/\kappa$ form another nonet
  compact tetraquark and $K\bar{K}$ bound state

- the spectral analysis $q\bar{q}$ has one unit of orbital argular momentum which increases the masses, but f_0 and a_0 are mass degeneracy

- in B_s decays $q\bar{q}$ is dominated in the energetic $f_0(980)$
  $q^2\bar{q}^2$ is power suppressed, FSI is also weak [SC, J-M Shen 1907.08401]

- in D_s decays how about the energetic $q\bar{q}$ picture $f_0(980)$?

DiPion LCDAs and D_{l4} decays

- $D_{(s)} \to S l \nu$ decays provide clean environment to study the scalar meson
  $\triangle$ $D_{(s)} \to a_0 e^+ \nu$[BESIII 18, 21], $D^+ \to f_0/\sigma e^+ \nu$[BESIII 19], $D_s \to f_0(\to \pi^+\pi^-)e^+\nu$[CLEO 09]

- $D_s \to f_0(\to \pi^0\pi^0, K_sK_s)e^+\nu$ [BESIII 22], $D_s \to f_0(\to \pi^+\pi^-)e^+\nu$ [BESIII 23]

$$\mathcal{B}(D_s \to f_0(\to \pi^0\pi^0)e^+\nu) = (7.9 \pm 1.4 \pm 0.3) \times 10^{-4}$$
$$\mathcal{B}(D_s \to f_0(\to \pi^+\pi^-)e^+\nu) = (17.2 \pm 1.3 \pm 1.0) \times 10^{-4}$$
$$f_+^{f_0}(0)|V_{cs}| = 0.504 \pm 0.017 \pm 0.035$$

- Theoretical consideration

$$\frac{d\Gamma(D_s^+ \to f_0 l^+ \nu)}{dq^2} = \frac{G_F^2|V_{cs}|^2\lambda^{3/2}(m_{D_s}^2, m_{f_0}^2, q^2)}{192\pi^3 m_{D_s}^3}|f_+(q^2)|^2$$

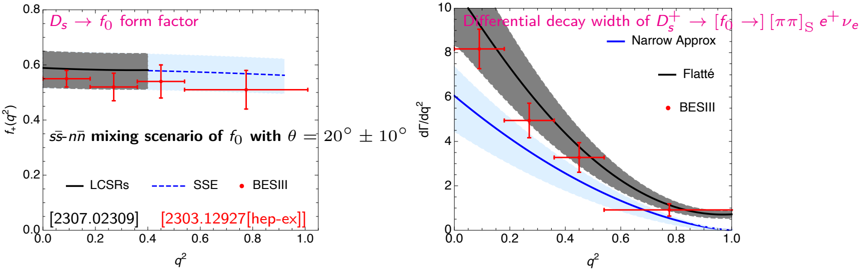
- Improvement with the width effect ($\pi\pi$ invariant mass spectral)

$$\frac{d\Gamma(D_s^+ \to [\pi\pi]_{\rm S}\, l^+ \nu)}{ds dq^2} = \frac{1}{\pi}\frac{G_F^2|V_{cs}|^2}{192\pi^3 m_{D_s}^3}|f_+(q^2)|^2 \frac{\lambda^{3/2}(m_{D_s}^2, s, q^2)\, g_1^2\beta_\pi(s)}{|m_{\rm S}^2 - s + i\left(g_1^2\beta_\pi(s)\right) + g_2^2\beta_K(s)\big)\,|^2}$$

$$\frac{d^2\Gamma(D_s^+ \to [\pi\pi]_{\rm S}\, l^+ \nu)}{dk^2 dq^2} = \frac{G_F^2|V_{cs}|^2}{192\pi^3 m_{D_s}^3}\frac{\beta_{\pi\pi}(k^2)\sqrt{\lambda_{D_s}}q^2}{16\pi}\sum_{\ell=0}^{\infty} 2|F_0^{(\ell)}(q^2, k^2)|^2$$

DiPion LCDAs and D_{l4} decays

$D_s \to f_0$ form factors $\langle f_0(p_1)|\bar{s}\gamma_\mu\gamma_5 c|D_s^+(p)\rangle = -i\left[f_+(q^2)(p+p_1)_\mu + f_-(q^2)q_\mu\right]$



- Twist-3 LCDAs give dominate contribution in $D_s \to f_0, [\pi\pi]_S$ transitions

† does not indicate a breakdown of the twist expansion

† the asymptotic term in the leading twist LCDAs is zero ($a_0 = 0$) due to the charge conjugate invariance

DiPion LCDAs and D_{l4} decays

- QCD description in terms of $\pi\pi$ LCDAs

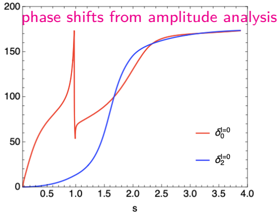
$$\frac{d^2\Gamma(D_s^+ \to [\pi\pi]_S\, l^+\nu)}{dk^2\, dq^2} = \frac{G_F^2|V_{cs}|^2}{192\pi^3 m_{D_s}^3} \frac{\beta_{\pi\pi}(k^2)\sqrt{\lambda_{D_s}}q^2}{16\pi} \sum_{\ell=0}^{\infty} 2|F_0^{(\ell)}(q^2,k^2)|^2$$

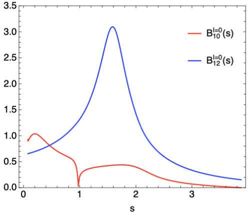
- $D_s \to [\pi\pi]_S$ form factors $\langle[\pi(k_1)\pi(k_2)]_S\,|\bar{s}\gamma_\mu\gamma_5 c|D_s^+(p)\rangle = -iF_0(q^2,s,\zeta)k_\mu^0 + \cdots$

$$\langle\pi^a(k_1)\pi^b(k_2)|\bar{q}_f(zn)\gamma_\mu\tau q_{f'}(0)|0\rangle = \kappa_{ab}\, k_\mu \int dx\, e^{iuz(k\cdot n)}\, \Phi_\parallel^{ab,ff'}(u,\zeta,k^2)$$

$$\Phi^{I=0}(z,\zeta,k^2,\mu) = 6z(1-z) \sum_{n=1,\mathrm{odd}}^{\infty} \sum_{l=0,\mathrm{even}}^{n+1} B_{n\ell}^{I=0}(k^2,\mu)\, C_n^{3/2}(2z-1)\, C_\ell^{1/2}(2\zeta-1)$$

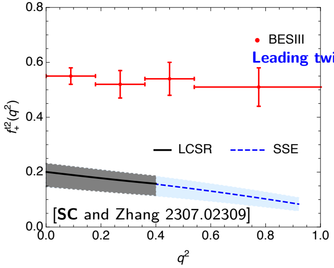
$$B_{n\ell}^I(k^2) = B_{n\ell}^I(0)\,\mathrm{Exp}\left[\sum_{m=1}^{N-1} \frac{k^{2m}}{m!}\frac{d^m}{dk^{2m}} \ln B_{n\ell}^I(0) + \frac{k^{2N}}{\pi}\int_{4m_\pi^2}^{\infty} ds\, \frac{\delta_\ell^I(s)}{s^N(s-k^2-i0)}\right]$$

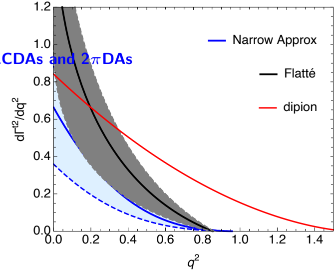




DiPion LCDAs and D_{l4} decays

Leading twist of f_0 LCDAs and 2πDAs





- further measurements would help us to understand DiPion system (f_0)

Conclusion and Prospect

Conclusion and Prospect

- The introduction of DiPion LCDAs provides an opportunity to study the width effects and the structures of nonstable mesons in B_{l4}, D_{l4} processes

- Possible improvement study in the CKM determinations and the flavor anomalies
- A new booster on the accurate calculation in flavor physics ?

Thank you for your patience.

Conclusion and Prospect

Wishlists

- DiPion LCDAs at twist three level
  the QCD definitions and the parameterization in double expansion

† the nonperturbative parameters from low energy effective theory and from the data constraints

- $B \to \pi^+\pi^0 l^- \bar{\nu}$ decay in the full kinematics, Belle II in progress
- Revisit $D_s \to [f_0 \to] [\pi\pi]_{\rm S}\, l^+ \nu$, BESIII
- $B \to [K^* \to] K\pi l^+ l^-$ with $K\pi$ LCDAs
- $B \to [\rho\rho \to] \pi\pi\pi\pi$, LHCb and Belle II

Backup Slides

Pion LCDAs

Study Pion DAs in the light-cone dominated processes

- DA is expressed by the MEs of gauge invariant non-local operators

$$\langle 0|\bar{u}(x)\Gamma[x,-x]d(-x)|\pi^-(P)\rangle$$

$$[x,y] = \mathrm{P}\exp\left[ig\int_0^1 dt(x-y)_\mu A^\mu(tx+\bar{t}y)\right]$$

- Introduce light-like vectors p and z: $P^2 = m_\pi^2,\ p^2 = 0,\ z^2 = 0$

△ $P \to p$ in the limit $m_\pi^2 = 0$ and $x \to z$ for $x^2 = 0$

△ expansion in power of large momentum transfer is governed by contributions from small transversal separations x^2 between constituents

$$z_\mu = x_\mu - \frac{P_\mu}{m_\rho^2}\left[xP - \sqrt{(xP)^2 - x^2 m_\pi^2}\right] = x_\mu\left[1 - \frac{x^2 m_\pi^2}{4(z\cdot p)^2} - \frac{p_\mu}{2}\frac{x^2}{z\cdot p} + \mathcal{O}(x^4)\right]$$

$$p_\mu = P_\mu - \frac{z_\mu}{2}\frac{m_\pi^2}{p\cdot z} \quad \Rightarrow \quad z\cdot P = z\cdot p = \left[(xP)^2 - x^2 m_\pi^2\right]^{1/2}$$

△ Projector onto the directions orthogonal to p and z $\quad g^\perp_{\mu\nu} = g_{\mu\nu} - \frac{1}{p\cdot z}(p_\mu z_\nu + p_\nu z_\mu)$

△ For an arbitrary Lorentz vector a_μ and b_μ, $\quad a_z \equiv a_\mu z^\mu, \quad a_p \equiv a_\mu p^\mu, \quad b_{\mu z} \equiv b_{\mu\nu} z^\nu, \quad \cdots$

Pion LCDAs

- Introduce LCDAs by the MEs of non-local operators on the light-cone

$$\langle 0|\bar{u}(z)\Gamma[z,-z]d(-z)|\pi^-(P)\rangle \propto \phi_t(u,\mu)$$

- Up to (power) twist 3 of lowest Fock wave function $\zeta = 2u-1$, $m_0^\pi = \frac{m_\pi^2}{m_0+m_d}$

$$\langle 0|\bar{u}(z)\gamma_z\gamma_5 d(-z)|\pi^-(P)\rangle = if_\pi p_z \int_0^1 du\, e^{i\zeta p\cdot z}\phi(u,\mu)$$

$$\langle 0|\bar{u}(z)i\gamma_5 d(-z)|\pi^-(P)\rangle = f_\pi m_0^\pi \int_0^1 du\, e^{i\zeta p\cdot z}\phi^p(u,\mu)$$

$$\langle 0|\bar{u}(z)i\sigma_{\mu\nu}\gamma_5 d(-z)|\pi^-(P)\rangle = -\frac{if_\pi m_0^\pi}{3}(p_\mu z_\nu - p_\nu z_\mu)\int_0^1 du\, e^{i\zeta p\cdot z}\phi^\sigma(u,\mu)$$

- LCDAs are dimensionless functions of u and renormalization scale μ

△ describe the probability amplitudes to find the π in a state with minimal number of constitutes and have small transversal separation of order $1/\mu$

△ the nonlocal operators on the lhs are renormalized at scale μ with the factor $Z_2(\mu)$

$\phi_2(u,\mu) = Z_2(\mu)\int^{|k_\perp|<\mu} d^2k_\perp\, \phi_{\rm BS}(u,k_\perp)$

△ decay constant $\langle 0|\bar{u}(0)\gamma_z\gamma_5 d(0)|\pi^-(P)\rangle = if_\pi p_\mu$ △ normalization $\int_0^1 du\,\Phi(u) = 1$

Pion LCDAs

- three sources of the power suppressed contributions to exclusive processes in QCD
  - ‡ bad component (wrong spin projection) in the wave function
  - ‡ transversal motion of q ($\bar{q}$) in the leading twist components
  - ‡ higher Fock states with additional gluons and/or $q\bar{q}$ pairs

- define the LCDAs with the Lorentz and gauge invariant ME

$$\langle 0|\bar{u}(x)\gamma_\mu\gamma_5 d(-x)|\pi^-(P)\rangle = f_\pi \int_0^1 du\, e^{i\zeta P\cdot x}\left[iP_\mu\left(\phi(u) + \frac{x^2}{4} g_1(u,\mu)\right) + \left(x_\mu - \frac{x^2 P_\mu}{2P\cdot x}\right) g_2(u,\mu)\right]$$

$$\langle 0|\bar{u}(x) i\gamma_5 d(-x)|\pi^-(P)\rangle = f_\pi m_0^\pi \int_0^1 du\, e^{i\zeta P\cdot x}\phi^p(u,\mu)$$

$$\langle 0|\bar{u}(x) i\sigma_{\mu\nu}\gamma_5 d(-x)|\pi^-(P)\rangle = -\frac{if_\pi m_0^\pi}{3}\left(P_\mu x_\nu - P_\nu x_\mu\right)\int_0^1 du\, e^{i\zeta P\cdot x}\phi^\sigma(u,\mu)$$

Pion LCDAs

Conformal spin and collinear twist definition

[Braun & Korchemsky & Müller 2003]

- A convenient tool to study DAs is provided by conformal expansion
- the underlying idea of *conformal expansion of LCDAs* is similar to *partial-wave expansion of wave function in quantum mechanism*
- *invariance of massless QCD* *under conformal trans.* *VS rotation symmetry*
- *longitudinal* $\otimes$ *transversal* dofs *VS angular* $\otimes$ *radial* dofs for spherically symmetry potential
- the transversal-momentum dependence (scale dependence of the relevant operators) is governed by the RGE
- the longitudinal-momentum dependence (orthogonal polynomials) is described in terms of irreducible representations of the corresponding symmetry group
  collinear subgroup of conformal group $SL(2,R) \cong SU(1,1) \cong SO(2,1)$

Pion LCDAs

$$\langle 0|\bar{u}_i(0)d_j(z)|\pi^-(p)\rangle = -\frac{i}{4}\int_0^1 du e^{-iup\cdot z}\left[\not{p}\gamma_5\phi_\pi(u) + \gamma_5\phi_\pi^p(u) + \gamma_5(1-\not{n}\not{\bar{n}})\phi_\pi^t(u)\right]_{ji}$$

$$\phi_\pi(u,\mu) = 6u(1-u)\sum_{n=0} a_n^\pi(\mu) C_n^{3/2}(u)$$

$$\phi_\pi^p(u,\mu) = \frac{m_0^\pi(\mu)}{p^+}\left[1 + 30\eta_{3\pi}C_2^{1/2}(u) - 3\eta_{3\pi}\omega_{3\pi}C_4^{1/2}(u)\right]$$

$$\phi_\pi^\sigma(u) = \frac{m_0^\pi(\mu)}{p^+}6u(1-u)\left[1 + 5\eta_{3\pi}C_2^{3/2}(u)\right]$$

- $\phi(x)$ and $\phi^{p,t}(u)$ are the twist two and twist three LCDAs, twist four $\cdots$
- $a_0^\pi = f_\pi$, $a_{n\geqslant 2}^\pi(\mu_0)$ and $m_0^\pi(\mu_0)$ obtained by non-pert. theory/lattice QCD
- μ dependences in a_n^π the integration over the transversal dof

  [Brodsky & Lepage1980, Balitsky & Braun1988]
- $C_n(u)$ are Gegenbauer polynomials $\sim$ Jacobi Polynomials $P_n^{j_1,j_2}\left(\frac{\overleftrightarrow{D}_+}{\overleftrightarrow{\partial}_+}\right)$ in the local collinear conformal expansion longitudinal dof

  [Lepage & Brodsky 1979, 80, Efremov & Radyushkin 1980, Braun & Filyanov 1990]
- Great achievements (high precision) in F_π, $B\to\pi$ et.al processes