北京航空航天大學
BEIHANG UNIVERSITY

Exploring the Heavy Meson Light-Cone Distribution Amplitudes from First-principle

Based on arXiv:2403.17492

In collaboration with: Ji Xu, Wei Wang, et. al.

Qi-An Zhang

Beihang University

Apr. 05, 2024 @ CCNU, Wuhan

Outline

- **Motivation**
- **Methodology: Two-step factorization connecting lattice QCD and HQET**
- **Numerical realization**
- **Results and discussions**
- **Summary and outlook**

Motivation: Why B-meson LCDAs are important?

➢ Weak decays of B meson are critical for:

- **Precise tests of SM**
- **Searching for NP**
- **Understanding the origins of CPV**
- **……**

$B\to\pi\pi$: Phys.Rev.Lett.83,1914(1999), 1422 citations

$B\to\pi K$: Nucl.Phys.B606,245(2001), 1177 citations

$B\to\pi\ell\nu$: Phys.Lett.B633,61(2006), 215 citations

$B\to D\ell\nu$: Phys.Rev.D92,054510(2015), 387 citations

Motivation: Why B-meson LCDAs are important?

- Weak decays of B meson are critical for:
  - **Precise tests of SM**
  - **Searching for NP**
  - **Understanding the origins of CPV**
  - ……

$B\to\pi\pi$: Phys.Rev.Lett.83,1914(1999), 1422 citations
$B\to\pi K$: Nucl.Phys.B606,245(2001), 1177 citations
$B\to\pi\ell\nu$: Phys.Lett.B633,61(2006), 215 citations
$B\to D\ell\nu$: Phys.Rev.D92,054510(2015), 387 citations

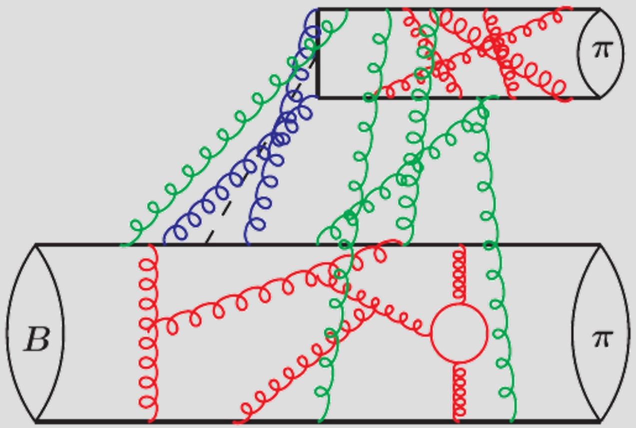
- Factorization: categories by different characteristic scales

$$\langle\pi(p')\pi(q)|Q_i|\bar{B}(p)\rangle = f^{B\to\pi}(q^2)\int_0^1 dx T_i^{\mathrm{I}}(x)\phi_\pi(x) + \int_0^1 d\xi dx dy T_i^{\mathrm{II}}(\xi,x,y)\phi_B(\xi)\phi_\pi(x)\phi_\pi(y)$$

Form factor =
Hard kernel + LCDAs

Hard kernel
(Perturbative)

Meson LCDAs
(Nonperturbative)



Motivation: Research progresses of light meson LCDAs

➢ Light meson LCDAs have been extensively pursued: (1979-2023)

- **Asymptotic LCDAs**

  G. P. Lepage et.al., Phys. Rev. Lett. 43 (1979)
  G. P. Lepage et.al., Phys.Lett.B 87B(1979)

- **Quark model**

  Choi, Phys.Rev.D 75 (2007)

- **Dyson-Schwinger Equation**

  F. Gao, L. Chang et.al. Phys.Rev.D 90 (2014)
  Craig D.et.al., Prog.Part.Nucl.Phys. (2021)

- **QCD Sum rules**

  V.L. Chernyak et. al., Nucl.Phys.B 201 (1982)
  Vladimir M. Braun et. al., Z.Phys.C 44 (1989)
  Patricia Ball et. al., JHEP 08 (2007)

- **Light-cone sum rule**

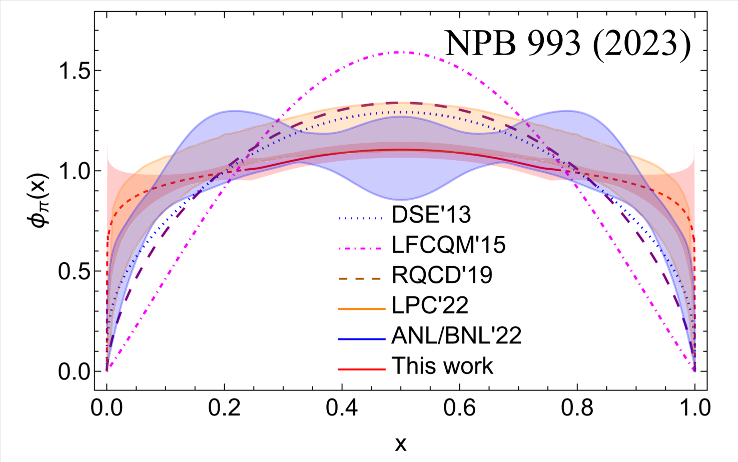
  S. Cheng et.al. Phys.Rev.D 102 (2020)

- **Lattice calculation by OPE**

  G. Martinelli et. al., Phys.Lett.B 190 (1987)
  RQCD Collaboration, JHEP 11 (2020)

- **Lattice calculation by LaMET**

  Zhang, et. al., Phys.Rev.D 95 (2017)
  R. Zhang et.al., Phys.Rev.D 102 (2020)
  J.Hua et.al(LPC)., Pev.Lett.127 (2021)
  J. Holligan et.al., Nucl.Phys.B 993 (2023)



Motivation: Difficulties of heavy meson LCDAs

- **The HQET matrix element of heavy meson** [Grozin, Neubert, 1997; Beneke, Feldmann, 2000]

$$\langle 0 | \bar{q}_\beta(z)[z,0] h_{v\alpha}(0) | \bar{B}(v) \rangle = -\frac{i\tilde{f}_B m_B}{8} \left\{ \left[\varphi_B^+(t,\mu) v_+ \gamma_- + \varphi_B^-(t,\mu) v_- \gamma_+ \right] \gamma_5 \right\}_{\alpha\beta}$$

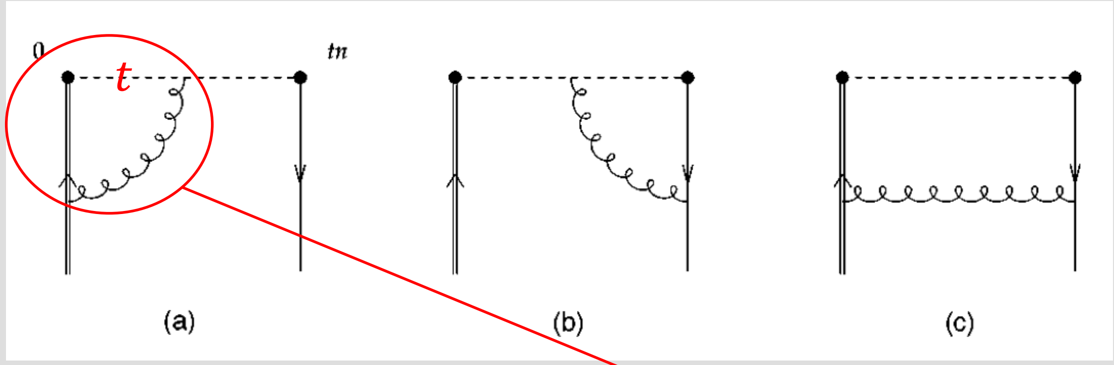
Leading twist　　Sub-leading twist

Motivation: Difficulties of heavy meson LCDAs

- **The HQET matrix element of heavy meson** [Grozin, Neubert, 1997; Beneke, Feldmann, 2000]

$$\langle 0 | \bar{q}_\beta(z)[z,0] h_{v\alpha}(0) | \bar{B}(v) \rangle = -\frac{i\tilde{f}_B m_B}{8} \left\{ \left[\varphi_B^+(t,\mu) v_+ \gamma_- + \varphi_B^-(t,\mu) v_- \gamma_+ \right] \gamma_5 \right\}_{\alpha\beta}$$

Leading twist Sub-leading twist



Diverge at $t \to 0$!

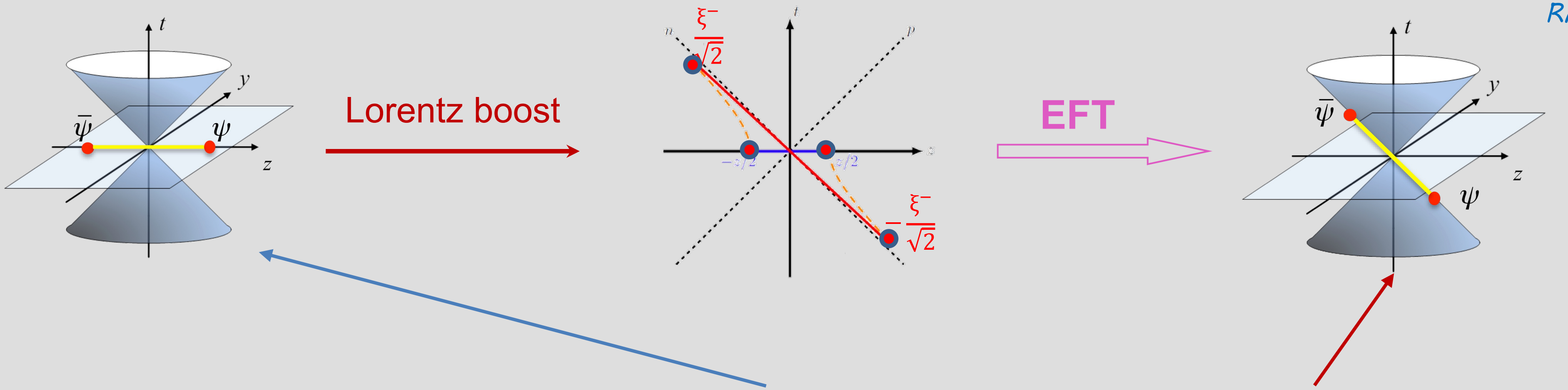
$$O_+^{\text{ren}}(t,\mu) = O_+^{\text{bare}}(t) + \frac{\alpha_s C_F}{4\pi} \left\{ \left(\frac{4}{\hat{\epsilon}^2} + \frac{4}{\hat{\epsilon}} \ln(it\mu) \right) O_+^{\text{bare}}(t) - \frac{4}{\hat{\epsilon}} \int_0^1 du \frac{u}{1-u} \left[O_+^{\text{bare}}(ut) - O_+^{\text{bare}}(t) \right] \right.$$

- Diverge at $t \to 0 \Leftrightarrow$ No local limit
- Non-negative moments $\int dk\, k^n \varphi_+(k)$ for *n=0,1,2,…* are not related to OPE, and actually they diverge
- Cannot obtain φ_B from lattice QCD through their moments.

Motivation: Difficulties of heavy meson LCDAs

➢ How about simulating the **heavy meson quasi DAs** in the framework of **LaMET**?

Ji, PRL110(2013), RMP93(2021), …

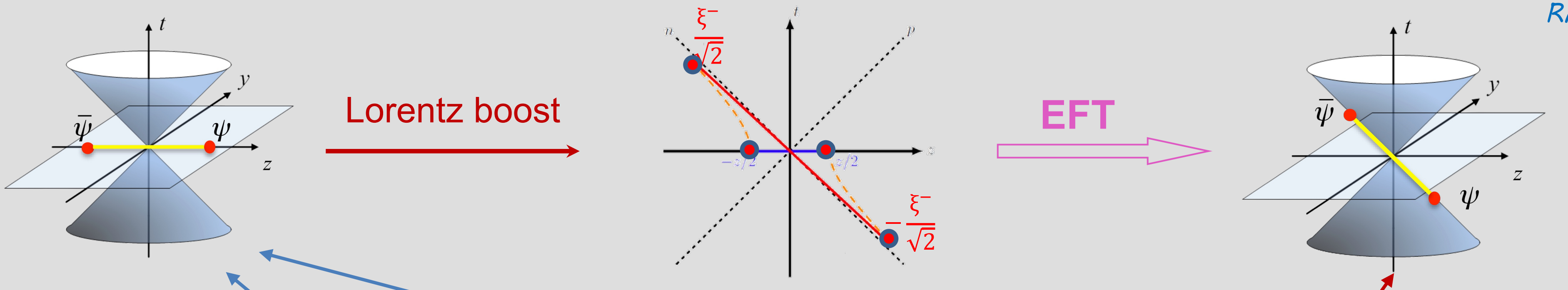


- LaMET provides a connection between equal-time correlator and light-cone one.

Motivation: Difficulties of heavy meson LCDAs

- How about simulating the **heavy meson quasi DAs** in the framework of **LaMET**?

Ji, PRL110(2013), RMP93(2021), …



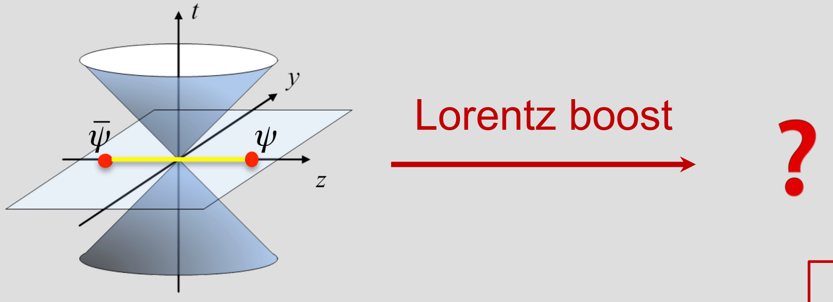
- LaMET provides a connection between equal-time correlator and light-cone one.

- A brute-force way: boost the HQET correlator

$$\varphi_B^+(\xi,\mu) \propto \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi} e^{i n_z\cdot v \xi \tau} \left\langle 0 \left| (\bar{q} W_c)(\tau n_z)\, \eta_z \gamma_5 \left(W_c^\dagger h_v\right)(0) \right| \bar{B}(v) \right\rangle .$$

Xu et. al.,
[PRD102(2020)011502, PRD103(2021)054022, PRD106(2022)114019, PRD106(2022)011503, PRD109(2024)034001, 2401.04291]

Motivation: Difficulties of heavy meson LCDAs



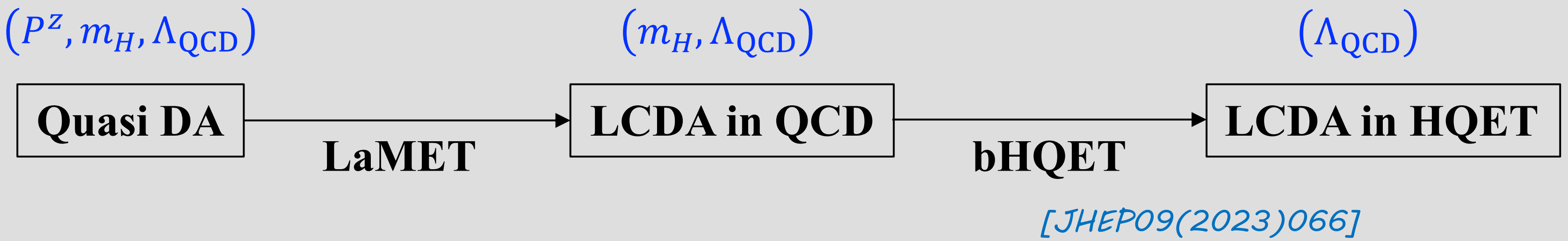
🤔 Difficult to realize the boosted HQET field on lattice QCD.

- A brute-force way: boost the HQET correlator

$$\varphi_B^+(\xi,\mu) \propto \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi} e^{in_z\cdot v\xi\tau} \langle 0 \left| (\bar{q}W_c)(\tau n_z)\,\eta_z\gamma_5 \left(W_c^\dagger h_v\right)(0) \right| \bar{B}(v) \rangle \,.$$

Xu et. al.,
[PRD102(2020)011502, PRD103(2021)054022, PRD106(2022)114019, PRD106(2022)011503, PRD109(2024)034001, 2401.04291]

Methodology: Two-step factorization to access heavy meson LCDA

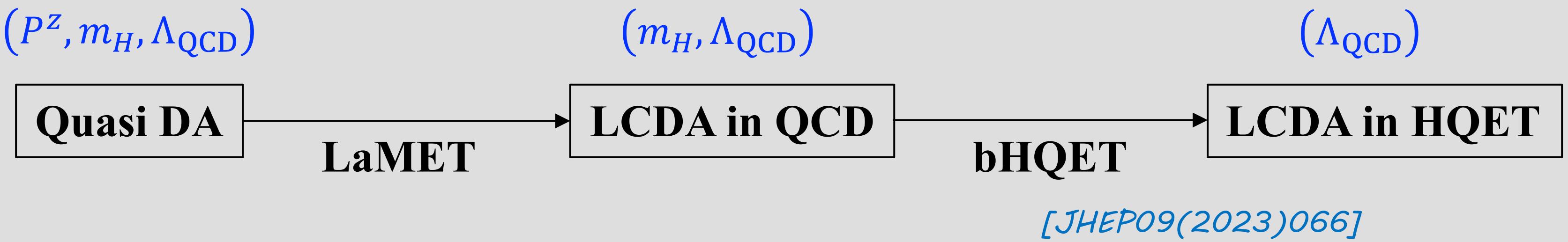


- A multi-scale processes:

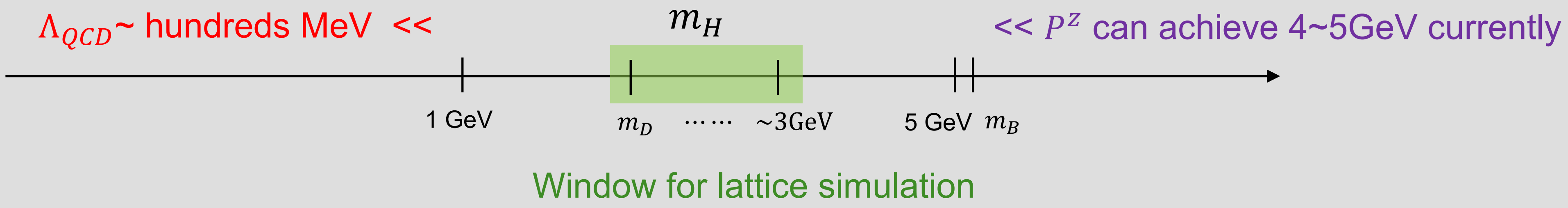
  1. LaMET requires $\Lambda_{\rm QCD}, m_H \ll P^z$ and finally integrate out P^z;
  2. bHQET requires $\Lambda_{\rm QCD} \ll m_H$ and integrate out m_H;

$\Rightarrow$ **Hierarchy $\boldsymbol{\Lambda_{\rm QCD} \ll m_H \ll P^z}$.**

Methodology: Two-step factorization to access heavy meson LCDA



⇒ **Hierarchy $\boldsymbol{\Lambda_{\text{QCD}} \ll m_H \ll P^z}$:** A big challenge for lattice simulation



<u>At this stage, the heavy meson could be D, but by no means be the B meson!</u>

Numerical realization

- A fine CLQCD ensemble for the lattice QCD verification of D meson LCDAs:

  - H48P32, $n_s^3 \times n_t = 48^3 \times 144$, $a = 0.05187\text{fm}$;
  - Coulomb gauge fixed grid source with grid = $1 \times 1 \times n_s$; 549 configurations × 8 measurements;
  - $m_\pi \simeq 317\text{MeV}$, $m_{\eta_s} = 700\text{MeV}$;
  - Determine the charm quark mass by tuning $m_{J/\psi}$ to its physical value, then $m_D \simeq 1.90\text{GeV}$;
  - Boost momenta $P^z = \{2.99, 3.49, 3.98\}\text{GeV}$, spatial separation $z = 0 \sim 12a$.

Quasi DA from lattice QCD calculation

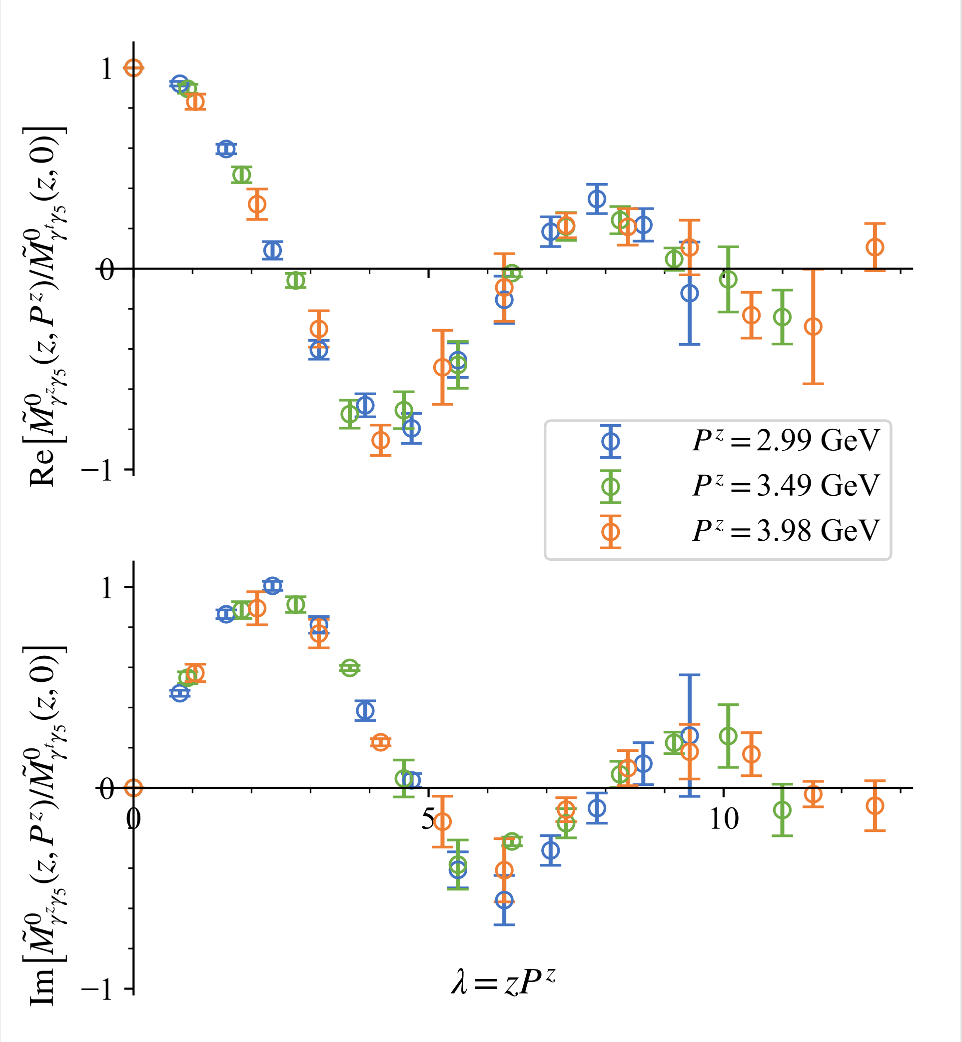
➢ Bare quasi DA matrix elements:

$$\tilde{M}_{\Gamma}^{0}(z, P^z) = \frac{\langle 0 |\bar{q}(z)\Gamma W_c(z,0)Q(0)| H(P^z)\rangle}{\langle 0|\bar{q}(0)\Gamma Q(0)|H(P^z)\rangle}$$

renormalized in ratio scheme

$$\tilde{M}(z, P^z) = \tilde{M}_{\gamma^z\gamma_5}^{0}(z, P^z)/\tilde{M}_{\gamma^t\gamma_5}^{0}(z, 0)$$

- To avoid operator mixing, choose $\Gamma = \gamma^z\gamma_5$ for $\tilde{M}_{\Gamma}^{0}(z, P^z)$ with large P^z.
- Use $\Gamma = \gamma^t\gamma_5$ for the zero-momentum matrix elements.
- Ratio scheme: renormalize the bare matrix elements by corresponding zero-momentum matrix elements.



Matching I: from quasi DAs to LCDAs in QCD

- Quasi DA $\tilde{\phi}(x, P^z)$, include the scales $\Lambda_{\rm QCD} \ll m_H \ll P^z$

$$\tilde{\phi}(x, P^z) = \int \frac{dz}{2\pi} e^{-ixP^z z} \tilde{M}(z, P^z)$$

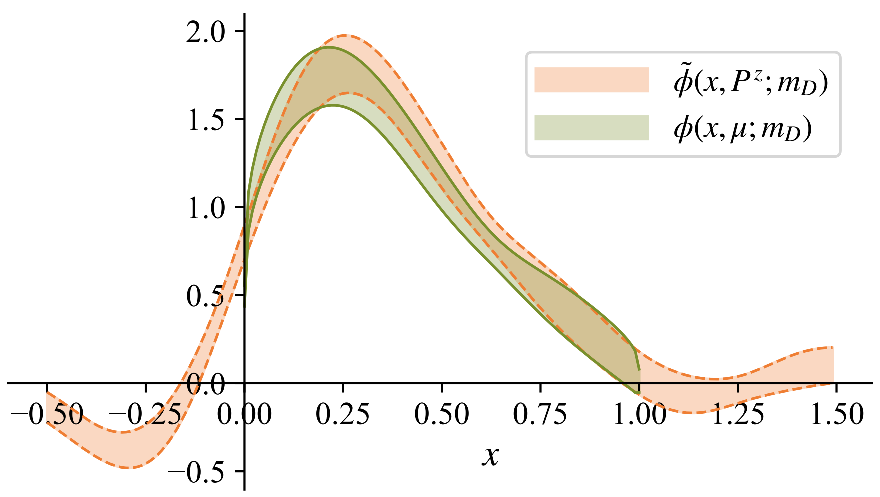
- Matching formula in LaMET: [PRD99(2019)094036, 2403.17492]

$$\tilde{\phi}(x, P^z) = \int_0^1 C\left(x, y, \frac{\mu}{P^z}\right) \phi(y, \mu) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(xP^z, \bar{x}P^z)^2}\right)$$

This matching integrate out P^z, obtain the LCDAs in QCD.

Quasi DA vs QCD LCDA

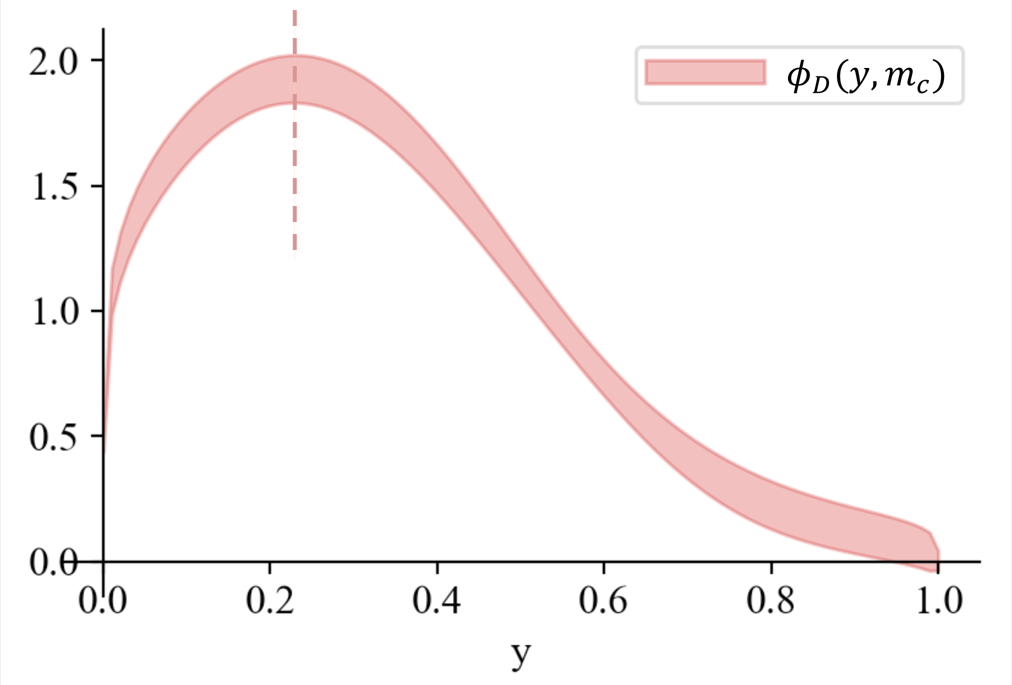
at $P^z = 3.98\text{GeV}$, $\mu = 2\text{GeV}$



LCDAs in QCD

➢ Heavy meson LCDAs in QCD

$$\phi(y,\mu) = \frac{1}{if_H}\int_{-\infty}^{+\infty}\frac{d\tau}{2\pi}e^{iyP_H\tau n_+} \times \langle 0\,|\bar{q}(\tau n_+)\not{n}_+\gamma_5 W_c(\tau n_+,0)Q(0)|\,H(P_H)\rangle$$

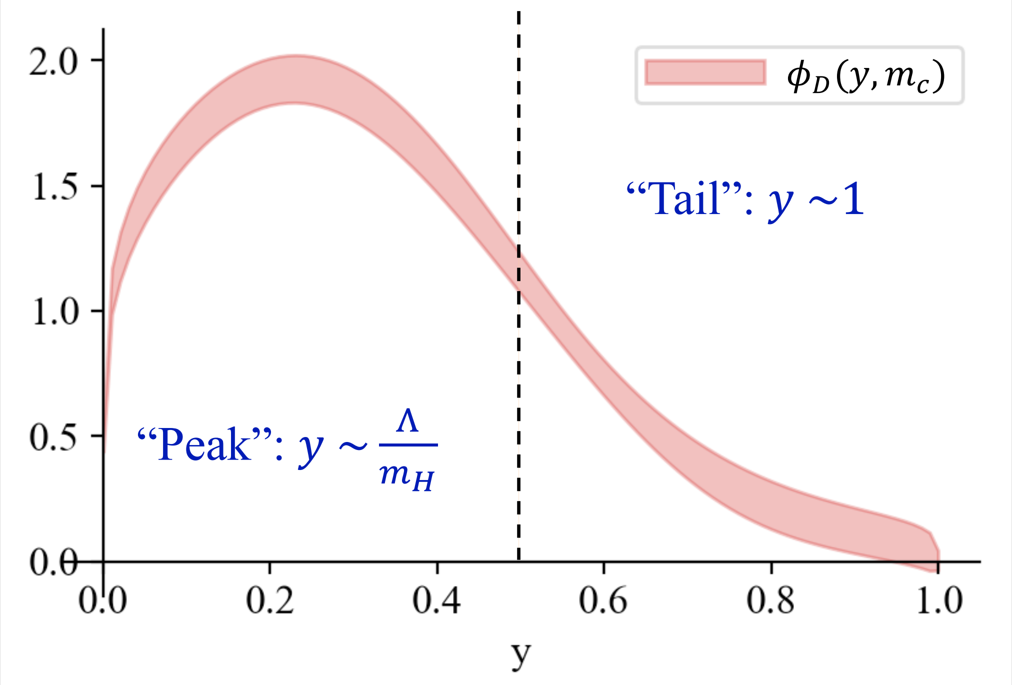


- The peak position dominated by m_H and μ;
- At very large scale $\mu \gg m_H$, asymptotic form;

LCDAs in QCD

➢ Heavy meson LCDAs in QCD

$$\phi(y,\mu) = \frac{1}{if_H}\int_{-\infty}^{+\infty}\frac{d\tau}{2\pi}e^{iyP_H\tau n_+} \times \langle 0\,|\bar{q}(\tau n_+)\not{n}_+\gamma_5 W_c(\tau n_+,0)Q(0)|\,H(P_H)\rangle$$



- The peak position dominated by m_H and μ;
- At very large scale $\mu \gg m_H$, asymptotic form;
- For the scale $\mu \lesssim m_Q$,

➩ Light quark carries small momentum fraction $y\sim\Lambda/m_H$ ⇒ peak region, related to the HQET LCDA;

[JHEP09(2023)066]

➩ $y\sim O(1)$ region be suppressed in LCDA:

P_q is soft-collinear, $\ll P_Q$, only contribute through power corrections;

SCET renormalized matrix element in this region contain only hard-collinear physics, and starts at the one-loop level.

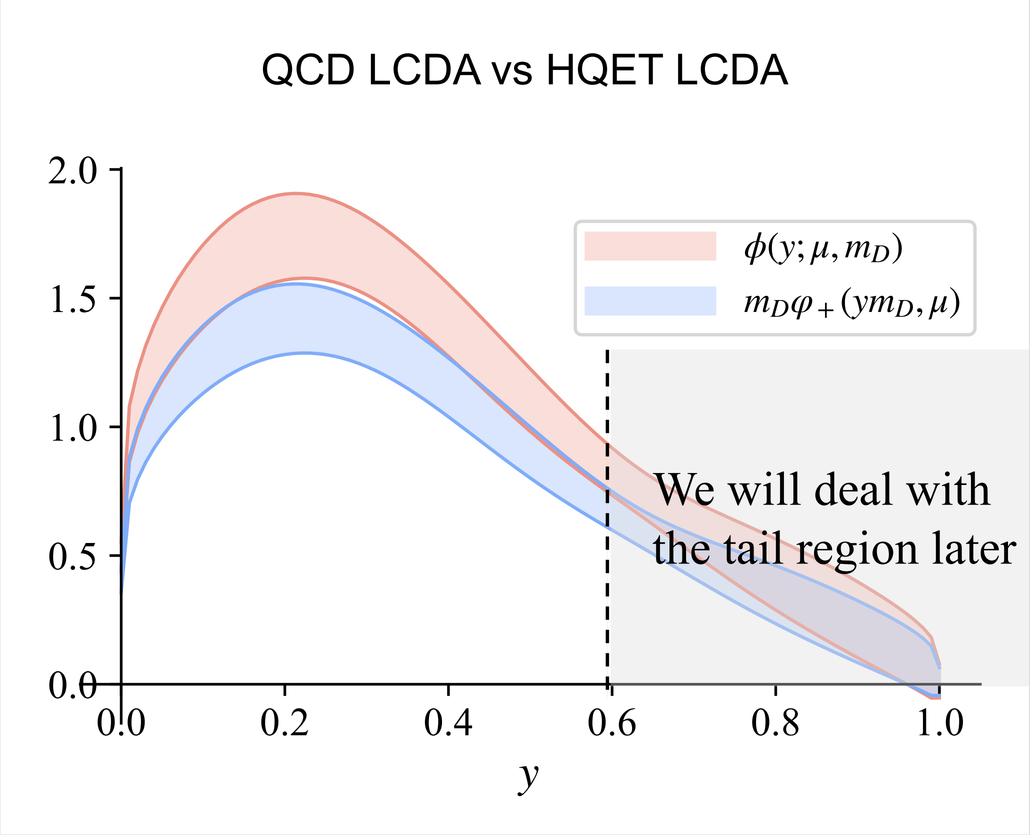
Matching II: connecting LCDAs in QCD and HQET

- Leading twist heavy meson LCDA in HQET

$$\varphi^+(\omega,\mu) = \frac{1}{i\tilde{f}_H(\mu) m_H}\int_{-\infty}^{+\infty}\frac{d\eta}{2\pi}\, e^{i\omega n_+\cdot v\eta} \times \langle 0\left|\bar{q}(\eta n_+)\not{n}_+\gamma_5 W_c(\eta n_+,0) h_v(0)\right| H(v)\rangle$$

connected with the QCD LCDA through a multiplicative factorization in the peak region: [JHEP09(2023)066]

$$\phi\left(y,\mu;m_H\right) = \frac{\tilde{f}_H}{f_H} J_{\rm peak} m_H \varphi^+\left(\omega,\mu\right) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_H}\right)$$



Tails of HQET LCDA

➢ The tail region of HQET LCDA is perturbative: PRD72(2005)094028

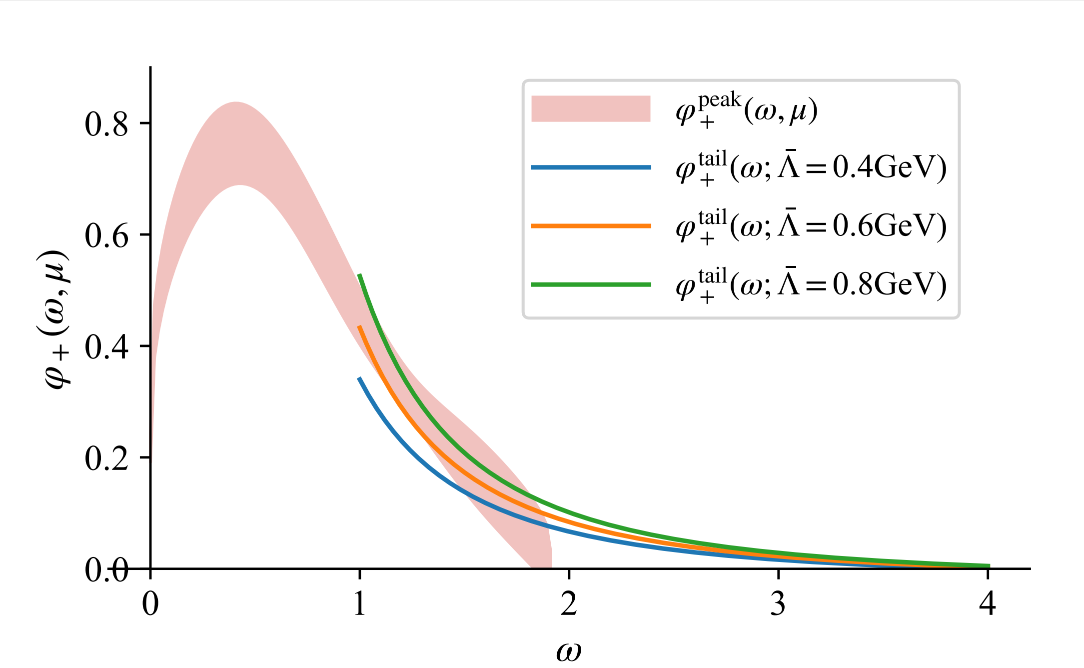
$$\varphi^+_{\text{tail}}(\omega,\mu) = \frac{\alpha_s C_F}{\pi\omega}\left[\left(\frac{1}{2} - \ln\frac{\omega}{\mu}\right) + \frac{4\bar{\Lambda}}{3\omega}\left(2 - \ln\frac{\omega}{\mu}\right)\right]$$

where $\overline{\Lambda} \equiv m_H - m_Q^{\text{pole}}$ reflect the power correction, and usually be chosen as 400~600MeV.

NPB426(1994)301

- We use the difference between the lines to estimate the power correction.

The final results of HQET LCDA will merge the peak (from LQCD) and tail region (from 1-loop calculation).



Comparison with phenomenological models

- Several commonly used models: NPB898,563(2015), JHEP07,154(2018), JHEP05,024(2022)……

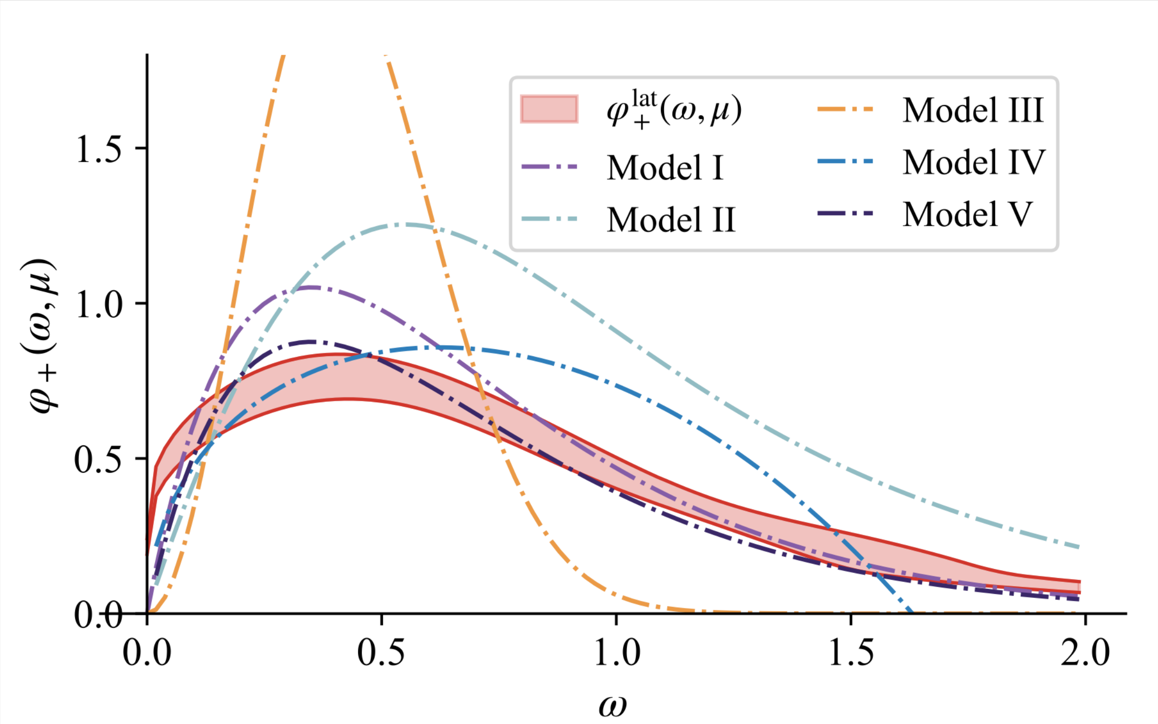
$$\varphi_{\rm I}^{+}(\omega,\mu_0)=\frac{\omega}{\omega_0^2}e^{-\omega/\omega_0}\,,$$

$$\varphi_{\rm II}^{+}(\omega,\mu_0)=\frac{4}{\pi\omega_0}\frac{k}{k^2+1}\left[\frac{1}{k^2+1}-\frac{2\left(\sigma_B^{(1)}-1\right)}{\pi^2}\ln k\right]\,,$$

$$\varphi_{\rm III}^{+}(\omega,\mu_0)=\frac{2\omega^2}{\omega_0\omega_1^2}e^{-(\omega/\omega_1)^2}\,,$$

$$\varphi_{\rm IV}^{+}(\omega,\mu_0)=\frac{\omega}{\omega_0\omega_2}\frac{\omega_2-\omega}{\sqrt{\omega\left(2\omega_2-\omega\right)}}\theta\left(\omega_2-\omega\right)\,,$$

$$\varphi_{\rm V}^{+}(\omega,\mu_0)=\frac{\Gamma(\beta)}{\Gamma(\alpha)}\frac{\omega}{\omega_0^2}e^{-\omega/\omega_0}U(\beta-\alpha,3-\alpha,\omega/\omega_0)\,,$$



First inverse moment

➢ The first inverse moment

$$\lambda_B^{-1}(\mu) = \int_{-\infty}^{\infty} d\omega \frac{\varphi^+(\omega,\mu)}{\omega}$$

- The current numerical results are unable to accomplish the integration over full-ω range;
- We determine the λ_B^{-1} by fitting the parameterization forms of different model.

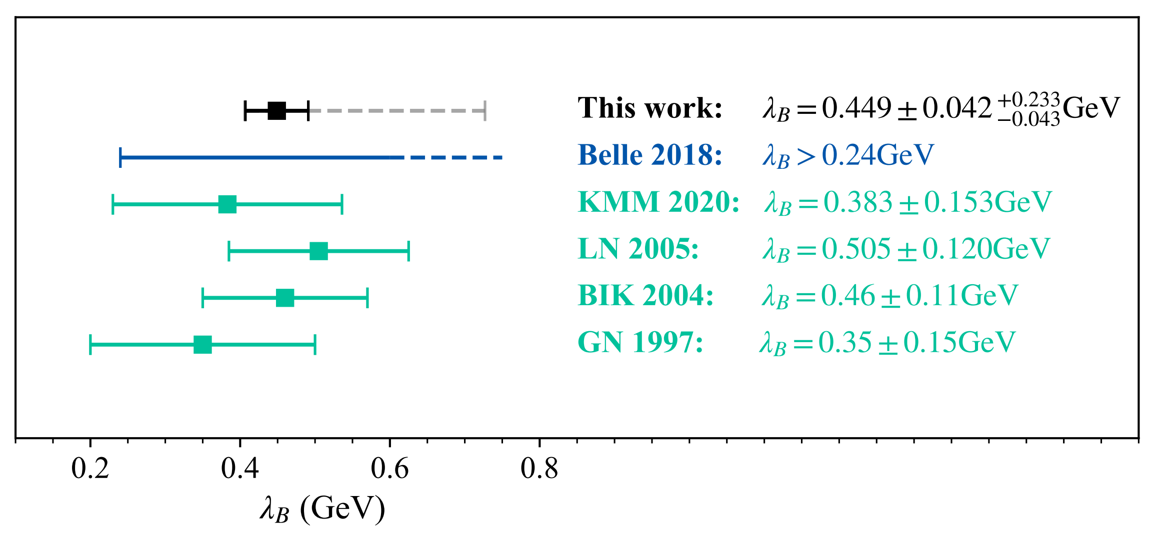
First inverse moment

➢ The first inverse moment

$$\lambda_B^{-1}(\mu) = \int_{-\infty}^{\infty} d\omega \frac{\varphi^+(\omega,\mu)}{\omega}$$

Models	I	II	III	IV	V
Parameters	$\omega_0 = 0.433(23)\mathrm{GeV}$	$\omega_0 = 0.682(45)\mathrm{GeV}$	——	$\omega_0 = 0.427(21)\mathrm{GeV}$	$\omega_0 = 0.449(42)\mathrm{GeV}$
		$\sigma_B^{(1)} = 2.78(48)$			
fit range	$\omega \in [0.2, 1.4]\mathrm{GeV}$	$\omega \in [0.2, 1.4]\mathrm{GeV}$		$\omega \in [0.4, 0.8]\mathrm{GeV}$	$\omega \in [0.2, 1.4]\mathrm{GeV}$
$\chi^2/\mathrm{d.o.f}$	1.4	1.2		2.1	1.0

- The current numerical results are unable to accomplish the integration over full-ω range;
- We determine the λ_B^{-1} by fitting the parameterization forms of different model.



PRD98,112016(2018),
JHEP10,043(2020),
PRD72,094028(2005),
PRD69,034014(2004),
PRD55,272(1997)

Summary and outlook

- ✓ We propose a set of feasible scheme to calculate the heavy meson LCDAs in both QCD and HQET;
- ✓ We use the finest CLQCD ensemble (H48P32) to validate the feasibility of our scheme;
- ✓ We have glanced the heavy LCDAs in HQET from the first principle for the first time.

Of course, the story is just beginning……

- ☐ More systematic lattice QCD calculations:

  Larger P^z and m_H, nonperturbative renormalization, continuum and physical mass extrapolation, operator mixing effects, ……

- ☐ More reliable theoretical frameworks:

  Power corrections, RG resummation, more reliable method to merge the peak and tail regions, ……

Thanks