Topological properties in electromagnetic fields

e-Print: 2312.14660 [hep-lat]

Mamiya

This is proceeding paper in lattice simulation.

Electromagnetic effects on topological observables in QCD

e-Print: 2312.14660 [hep-lat]

Bastian B. Brandt,[a] Gergely Endrődi,[a] José Javier Hernández Hernández,[a,*] Gergely Markó[a] and Laurin Pannullo[a]

[a] *Universität Bielefeld,*
Universitätsstraße 25, 33615 Bielefeld, Germany
E-mail: brandt@physik.uni-bielefeld.de, endrodi@physik.uni-bielefeld.de, hernandez@physik.uni-bielefeld.de, gmarko@physik.uni-bielefeld.de, lpannullo@physik.uni-bielefeld.de

This paper shows preliminary results about…

- Axion-photon coupling (without external electromagnetic fields)
- Magnetic field effect on topological susceptibility at low temperatures.

What are the new findings in this paper?

e-Print: 2312.14660 [hep-lat]

- Axion-photon coupling (without external electromagnetic fields)
- Magnetic field effect on topological susceptibility at low temperatures.

(3 color)

This study is the first attempt to evaluate the electromagnetic effect on topological property using lattice simulation.

Evaluation of Axion-photon coupling based on lattice is first attempt.

What are the new findings in this paper?

e-Print: 2312.14660 [hep-lat]

- Axion-photon coupling (without external electromagnetic fields)
- Magnetic field effect on topological susceptibility at low temperatures.

(3 color)

This study is the first attempt to evaluate the electromagnetic effect on topological property using lattice simulation.

In other situations regarding "topological susceptibility in lattice simulation",

Temperature effect (3 color)	- C. Bonati et al, JHEP 11, 170 (2018), 1807.07954. - S. Borsanyi et al., Nature 539, no. 7627, 69 (2016). - P. Petreczky et al, Phys. Lett. B 762, 498-505 (2016).
Density effect (2 color)	- Itou et al. JHEP01(2020)181 - N. Astrakhantsev et al. PRD 102 (2020) 7, 074507
Magnetic effect	No previous research

QCD Lagrangian with θ term

$$\mathcal{L}_{\rm QCD} = \bar{q}_L i\gamma^\mu D_\mu q_L + \bar{q}_R i\gamma^\mu D_\mu q_R - \bar{q}_L \mathbf{m}_f q_R - \bar{q}_R \mathbf{m}_f q_L + \frac{1}{2}\sum_{a=1}^{8} {\rm tr}\left[(G^a_{\mu\nu}T^a_c)^2\right] + \theta \frac{g^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}$$

U(1) axial anomaly

$$\partial_\mu j_A^{(f)\mu} = 2i\bar{q}^f m_f \gamma_5 q^f + \frac{g^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma} + N_c \frac{e^2[Q^f_{\rm em}]^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

θ is promoted to a dynamical field (axion).

Axion-photon coupling

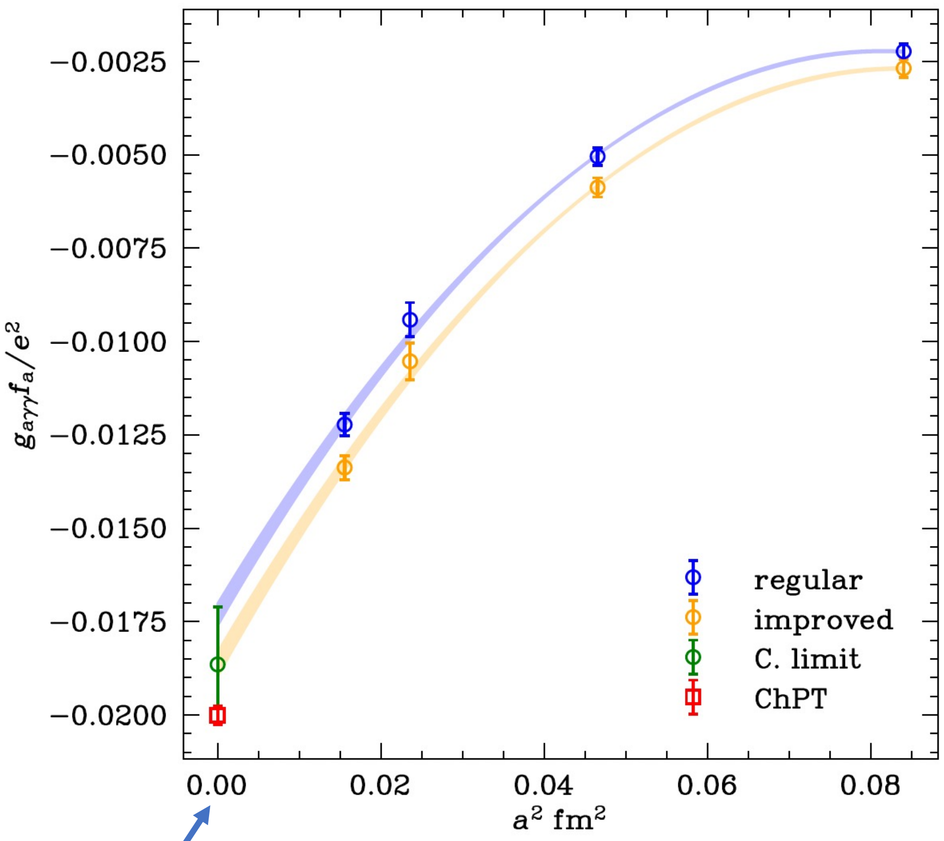
$$g^{\rm QCD}_{a\gamma\gamma} f_a = \frac{i}{\Omega}\frac{\partial^2}{\partial\theta\partial(\vec{E}\cdot\vec{B})}\ln Z_{{\rm QCD}+\theta+{\rm bEM}}\Big|_{\theta=\vec{E}=\vec{B}=0}$$

Ω is space-time volume.

This coupling has been numerically evaluated in e-Print: 2312.14660 [hep-lat].

The coupling has been numerically evaluated in e-Print: 2312.14660 [hep-lat].

- up quark mass = down quark mass



Continuum limit

lattice spacing

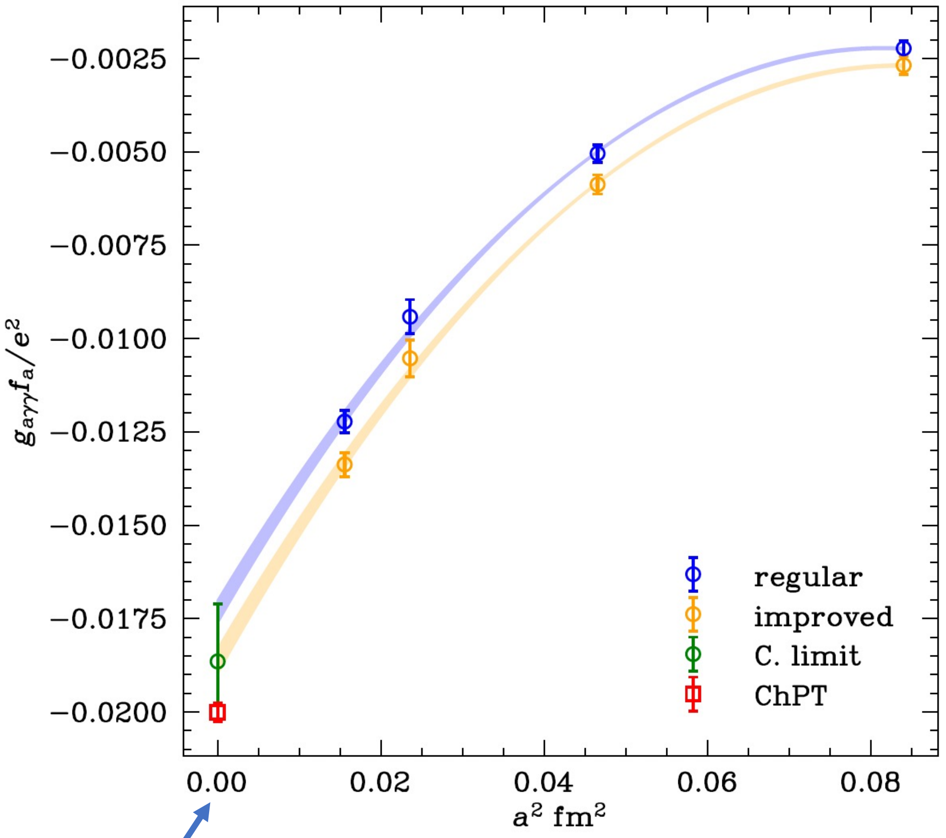
- up quark mass Is not equal to down quark mass

$$g_{a\gamma\gamma}^{\mathrm{QCD}} f_a / e^2 = -0.023(2).$$

at continuum limit with physical quark mass.

The coupling has been numerically evaluated in e-Print: 2312.14660 [hep-lat].

- up quark mass = down quark mass



lattice spacing

Continuum limit

- up quark mass Is not equal to down quark mass

$$g_{a\gamma\gamma}^{\rm QCD} f_a/e^2 = -0.023(2).$$

at continuum limit with physical quark mass.

- Lattice results tell us magnitude/sign of axion-photon coupling:

$$g_{a\gamma\gamma}^{\rm QCD} f_a/e^2 \sim O(-0.02)$$

- Lattice results takes similar value in ChPT.

In this week, I realized that
the coupling constant can be evaluated
based on model independent way.

e-Print: 2312.14660 [hep-lat].

- Lattice results tell us magnitude/sign of axion-photon coupling:

$$g_{a\gamma\gamma}^{\rm QCD} f_a/e^2 \sim O(-0.02)$$

- Lattice results takes similar value in ChPT.

In this week, I realized that the coupling constant can be evaluated based on model independent way.

e-Print: 2312.14660 [hep-lat].

- Lattice results tell us magnitude/sign of axion-photon coupling:

$$g_{a\gamma\gamma}^{\text{QCD}} f_a/e^2 \sim O(-0.02)$$

- Lattice results takes similar value in ChPT.

$$Z_{\text{QCD}} = \int [dqd\bar{q}][dG] \exp\left[i \int d^4x \mathcal{L}_{\text{QCD}}\right]$$

U(1) axial transformation

$$\begin{aligned} Z_{\text{QCD}} \;=\; & \int [dq'd\bar{q}'][dG] \exp\Bigg[i \int d^4x \Bigg(\bar{q}'_L i\gamma^\mu D_\mu q'_L + \bar{q}'_R i\gamma^\mu D_\mu q'_R + \frac{1}{2}\sum_{a=1}^{8} \text{tr}\left[(G^a_{\mu\nu}T^a_c)^2\right] \\ & - \sum_f \left(\bar{q}'^f_L m_f \exp\left(i\frac{\bar{m}}{m_f}\theta\right) q'^f_R + \bar{q}'^f_R m_f \exp\left(-i\frac{\bar{m}}{m_f}\theta\right) q'^f_L \right) \\ & - \left(\sum_f \frac{\bar{m}}{m_f}\theta Q_f^2\right) N_c \frac{e^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \Bigg)\Bigg]. \end{aligned}$$

$$\bar{m} = \left(\sum_f \frac{1}{m_f}\right)^{-1}$$

$$g_{a\gamma\gamma}^{\text{QCD}} f_a/e^2 = -\left[\frac{\bar{m}}{m_u}q_u^2 + \frac{\bar{m}}{m_d}q_d^2 + \frac{\bar{m}}{m_s}q_s^2\right] N_c \frac{1}{32\pi^2} 8$$

I got the analytical expression.

$$m_l = 3.45\text{MeV}$$
$$m_s = 93.4\text{MeV}$$

$$g_{a\gamma\gamma}^{\text{QCD}} f_a/e^2 = -0.0209$$

My model independent result is comparable to the lattice evaluation.

Model independent result
(My result)

$$g_{a\gamma\gamma}^{\mathrm{QCD}} f_a/e^2 = -0.0209$$

e-Print: 2312.14660 [hep-lat].

- Lattice results tell us magnitude/sign of axion-photon coupling:

$$g_{a\gamma\gamma}^{\mathrm{QCD}} f_a/e^2 \sim O(-0.02)$$

- Lattice results takes similar value in ChPT.

The coupling comes from the U(1) anomaly associated with electromagnetic fields.

Such anomaly is the one-loop exact result. → The coupling is exactly fixed. (Anomaly matching)

↓

Axion-photon coupling is model independent.

↓

Is it really necessary to implement lattice simulations/effective model analysis?
→ Maybe no…

e-Print: 2312.14660 [hep-lat]

This paper also shows the magnetic effect on the topological susceptibility.

Definition

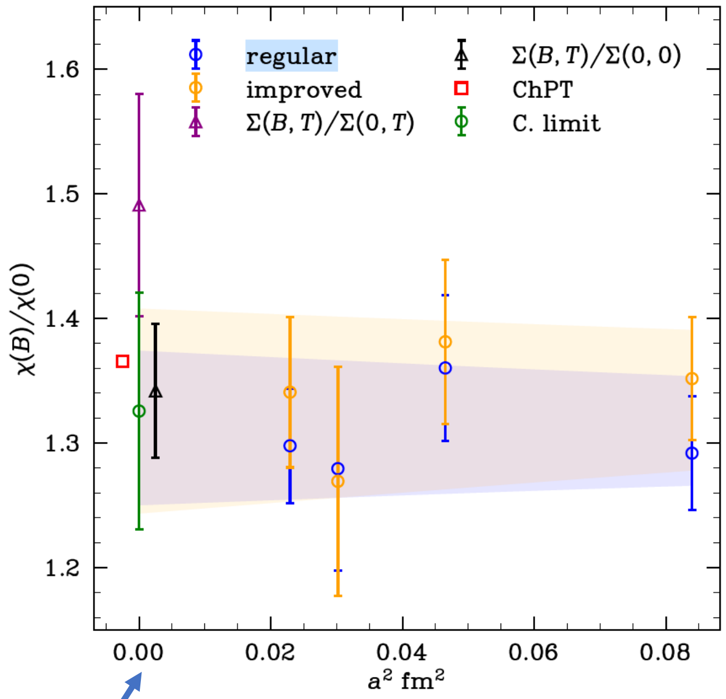
$$\chi_{\text{top}} = -\frac{1}{\Omega} \left. \frac{\partial^2 \ln Z_{\text{QCD}+\theta}}{\partial \theta^2} \right|_{\theta=0} = \frac{\langle Q_{\text{top}}^2 \rangle_{\text{QCD}}}{\Omega}$$

$$\chi_{\text{top}} = -i \int d^4x \left\langle 0 \middle| T \left(\frac{g^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma} \right)(x) \left(\frac{g^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G^b_{\mu\nu} G^b_{\rho\sigma} \right)(0) \middle| 0 \right\rangle$$

$$= -\bar{m}^2 \left[\sum_f \frac{1}{m_f} \langle 0 | \bar{q}^f q^f | 0 \rangle + i \int d^4x \left\langle 0 \middle| T \left(\sum_f \bar{q}^f i\gamma_5 q^f \right)(x) \left(\sum_{f'} \bar{q}^{f'} i\gamma_5 q^{f'} \right)(0) \middle| 0 \right\rangle \right]$$

e-Print: 2312.14660 [hep-lat]

Lattice observation



lattice spacing

Continuum limit

$$eB = 0.5\,\mathrm{GeV}^2$$

$$T = 113\,\mathrm{MeV}$$

$$\frac{\chi_{\mathrm{top}}(eB = 0.5\mathrm{GeV}^2)}{\chi_{\mathrm{top}}(B=0)} > 1$$

Magnetic field enhances the topological susceptibility.

It seems…

Magnetic field is “catalyzer” for the topological property?

But

The lattice data is too limited. → More results are needed to clarify the magnetic property.

Summary

e-Print: 2312.14660 [hep-lat]

Recent lattice paper shows preliminary results,

- Axion-photon coupling
- Magnetic field effect on topological susceptibility

e-Print: 2312.14660 [hep-lat].

- Lattice results tell us magnitude/sign of axion-photon coupling:

$$g_{a\gamma\gamma}^{\rm QCD} f_a/e^2 \sim O(-0.02)$$

$$\frac{\chi_{\rm top}(eB = 0.5{\rm GeV}^2)}{\chi_{\rm top}(B=0)} > 1$$

Magnetic field enhances the topological susceptibility.

Summary

e-Print: 2312.14660 [hep-lat]

Recent lattice paper shows preliminary results,

- Axion-photon coupling
- Magnetic field effect on topological susceptibility

e-Print: 2312.14660 [hep-lat].

- Lattice results tell us magnitude/sign of axion-photon coupling:

$$g_{a\gamma\gamma}^{\rm QCD} f_a/e^2 \sim O(-0.02)$$

$$\frac{\chi_{\rm top}(eB = 0.5{\rm GeV}^2)}{\chi_{\rm top}(B=0)} > 1$$

Magnetic field enhances the topological susceptibility.

I also obtained the comping based on model independent way.

$$g_{a\gamma\gamma}^{\rm QCD} f_a/e^2 = -0.0209$$

In my future work,
I will address this topic using NJL model with Irfan.