Planet formation I: dust dynamics and coagulation

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Outline

- Dust dynamics
- Dust coagulation
- Planetesimal formation

Main references:

- Armitage, P. J. 2020, Astrophysics of planet formation, Second Edition
- Lesur et al. 2023, PPVII, ch.13

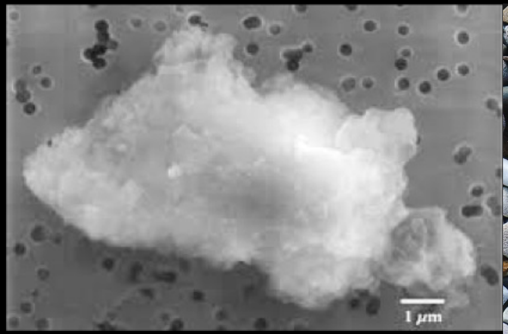
Part I: dust dynamics

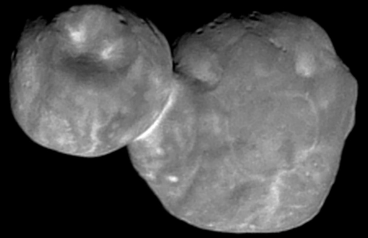
Planet formation: evolutions of solids

Aerodynamic coupling

Gravitational coupling

ISM dust | pebble | planetesimal | planet





$\sim 1\ \mu$m | ~ 10 cm | ~ 1 km | $\sim 10^4$ km

Opt/IR

mm: e.g. ALMA, ngVLA…

Kepler/TESS/…

Dust size growth → Accretion/migration

Planetesimal formation?

Aerodynamic drag on solid particles

Aerodynamic force for the particle with a velocity Δv relative to gas

Usually in the subsonic regime ($\Delta v < c_s$)

- Epstein drag (particle size $a \lesssim \lambda$, mean free path of gas; via collision):

  Collision rate from front side: $f_+ \simeq \pi a^2 (v_{th} + \Delta v) \frac{\rho_g}{\mu m_H}$

  Collision rate from back side: $f_- \simeq \pi a^2 (v_{th} - \Delta v) \frac{\rho_g}{\mu m_H}$

  Net drag force: $F_D \propto -a^2 \rho_g \, v_{th} \Delta v$

- Stokes drag ($a \gtrsim \lambda$; via molecular viscosity):

  $F_D \propto -\frac{C_D}{2} \pi a^2 \rho_g \Delta v \Delta \mathrm{v},$

$C_D \simeq 24 \mathrm{Re}^{-1}$,	$\mathrm{Re} < 1$,
$C_D \simeq 24 \mathrm{Re}^{-0.6}$,	$1 < \mathrm{Re} < 800$,
$C_D \simeq 0.44$,	$\mathrm{Re} > 800$.

  C_D depends on the Reynolds number $\mathrm{Re} = 2a\Delta v / \nu_m$ with $\nu_m = 1/2 v_{th} \lambda$

Epstein and Stokes drag transition around $a = 9/4\lambda$

Supersonic regime ($\Delta v > c_s$): $F_D \propto -a^2 \rho_g \Delta v^2$

Stopping time and Stokes number

Stopping time $t_{\mathrm{stop}} = mv/|F_D|$

Epstein drag regime: $t_{stop} = \frac{\rho_s}{\rho_g} \frac{a}{v_{th}}$; for $a < 9/4\lambda$

Stokes drag regime: $t_{stop} = \begin{cases} \frac{2*\rho_s\, a^2}{9 \nu_m \rho_g}; \ \mathrm{Re} < 1 \\ \frac{2^{0.6} \rho_s a^{1.6}}{9 \nu_m^{0.6} \rho_g^{1.4} v_{th}^{0.4}}; \ 1 < \mathrm{Re} < 800 \\ \frac{6 \rho_s a}{\rho_g v_{th}}; \ \mathrm{Re} > 800 \end{cases}$

Dimensionless stopping time $\tau_{\mathrm{stop}} \equiv t_{\mathrm{stop}} \Omega_{\mathrm{K}}$

Stokes number: $\mathrm{St} = t_{\mathrm{stop}}/t_{\mathrm{eddy}}$

For Epstein regime ($a < 9/4\lambda$), $\mathrm{St} = \frac{\pi \rho_s a}{2 \Sigma_g}$

Coupling strength between dust and gas

$\tau_{\rm stop} \ll 1$, strong coupling;

$\tau_{\rm stop} \gg 1$, decoupling;

$\tau_{\rm stop} \sim 1$, marginally coupling;

$$t_{\rm stop} = \frac{\rho_s}{\rho_g}\frac{a}{v_{th}}$$

Typical value: $t_{\rm stop} \simeq 3$ s, $\tau_{\rm stop} \simeq 6\times10^{-7}(a/1\mu m)$

($\rho_g = 10^{-9}$g cm^{-3}, $\rho_m = 3$ g cm^{-3}, $v_{th} = 10^5$cm s^{-1}, $a = 1$µm at 1au)

For mm dust, $\tau_{\rm stop} \simeq 10^{-3}$ at 1 au, $\tau_{\rm stop} \simeq 0.1$ at 30 au

Dust settling

- Vertical component of stellar gravity balanced by aerodynamic drag

$$|F_{\rm grav}| = m\Omega^2 z, \quad \Leftrightarrow \quad |F_{\rm D}| = \frac{4\pi}{3}\rho s^2 v_{\rm th} v$$

- Dust settling velocity and timescale ($t_{\rm settle} = z/|v_{\rm settle}|$)

$$v_{\rm settle} = \frac{\rho_{\rm m}}{\rho}\frac{s}{v_{\rm th}}\Omega^2 z \qquad t_{\rm settle} = \frac{2}{\pi}\frac{\Sigma}{\rho_{\rm m} s \Omega}\exp\left[-\frac{z^2}{2h^2}\right]$$

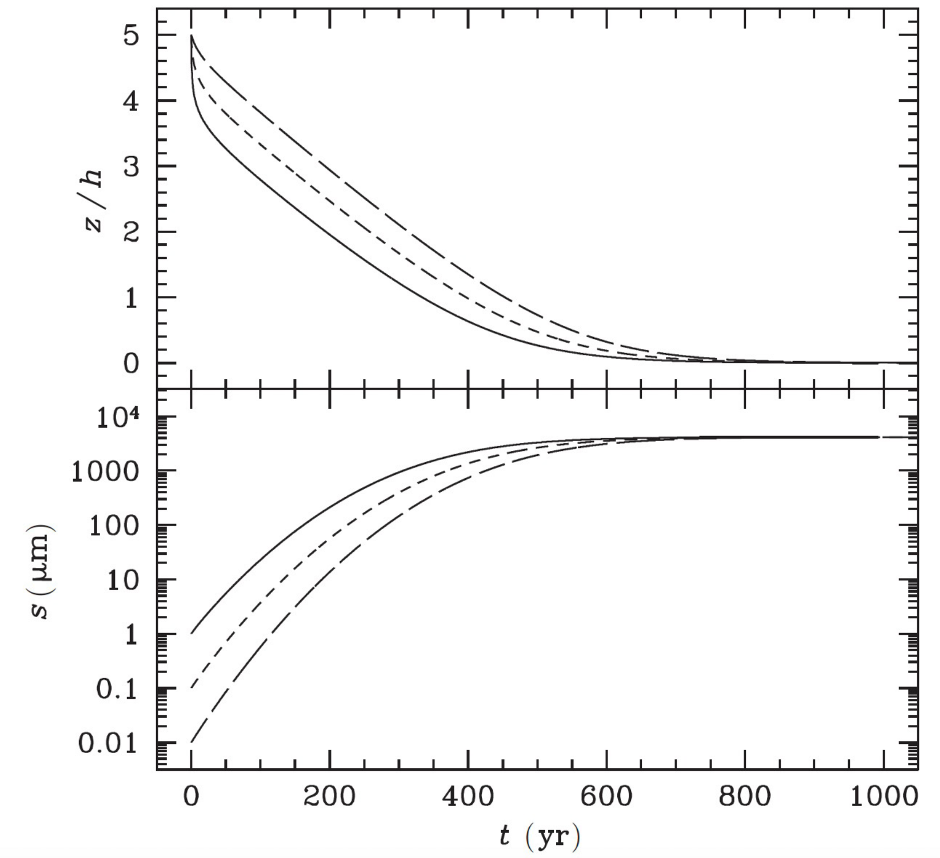
- For $1\mu m$ dust particle at $z \sim h$ at 1 au, $v_{\rm settle} \simeq 0.06$ cm s^{-1}, and $t_{\rm settle} \simeq 1.5\times10^5$ yr $\ll$ disk life time
- Settling is much faster at higher z (dependence on gas density $\rho_{\rm g}$)

Dust settling with coagulation

Dust settling depends on its size, and the size will grow during settling.

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \pi s^2 |v_{\mathrm{settle}}| f \rho(z),$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = -\frac{\rho_{\mathrm{m}}}{\rho} \frac{s}{v_{\mathrm{th}}} \Omega^2 z.$$



With coagulation, particles settle to the disk mid-plane on $\sim 10^3$ yr – at least 10^2 faster than non-coagulation case.

Caveats: no turbulence and fragmentation

Dust settling with turbulence

Dust remain suspended in the presence of vigorous air currents

- Turbulence stirs up small solid particles and prevents them from settling into a thin layer at the disk mid-plane.
- Conditions for turbulence to stir up the dust -- dust diffusion time shorter than settling time

$$t_{\rm diffuse} = \frac{z^2}{D}, \qquad D \sim \nu = \frac{\alpha c_s^2}{\Omega}$$

Minimum α for turbulence to oppose settling: $\alpha \gtrsim \frac{\pi e^{1/2}}{2} \frac{\rho_s a}{\Sigma_g}$

with $\Sigma_g = 10^2 \mathrm{g\ cm^{-2}}, \rho_m = 3\ \mathrm{g\ cm^{-3}}, a = 1\mu\mathrm{m}, \alpha \gtrsim 10^{-5}$; a larger α needed to stir up larger particles

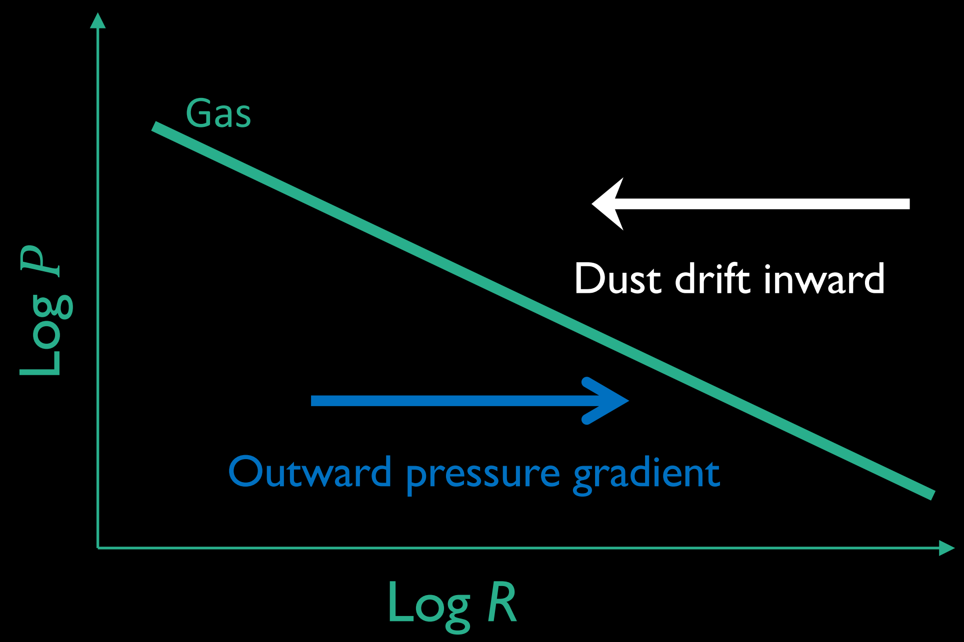
- More formal condition (by solving advection–diffusion equation), $\frac{h_{\rm d}}{h_{\rm g}} \simeq \sqrt{\frac{\alpha}{t_{\rm stop}\Omega}}$

Dust radial drift (I)

- Gas rotates sub-Keplerian velocity due to pressure gradient

$$v_{\phi,g} = v_K(1-\eta)^{1/2},$$
$$\text{with } \eta = nc_s^2/v_K^2$$

$$\frac{v_{\phi,\mathrm{gas}}^2}{r} = \frac{GM_*}{r^2} + \frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r}$$



- Dust experiences head wind→lost angular momentum→drift inward

(Weidenschilling1977)

$$v_{\mathrm{d},r} = \frac{v_{\mathrm{g},r} + 2\mathrm{St}\Delta v_{\mathrm{g},\phi}}{1+\mathrm{St}^2}$$

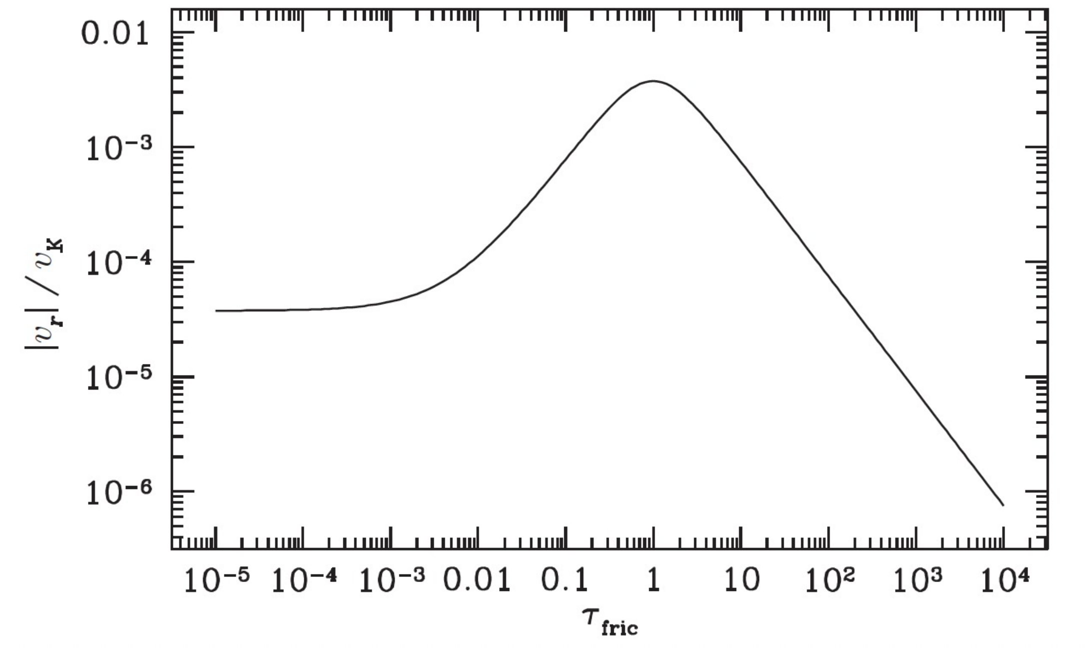
Dust drift towards higher pressure regions.

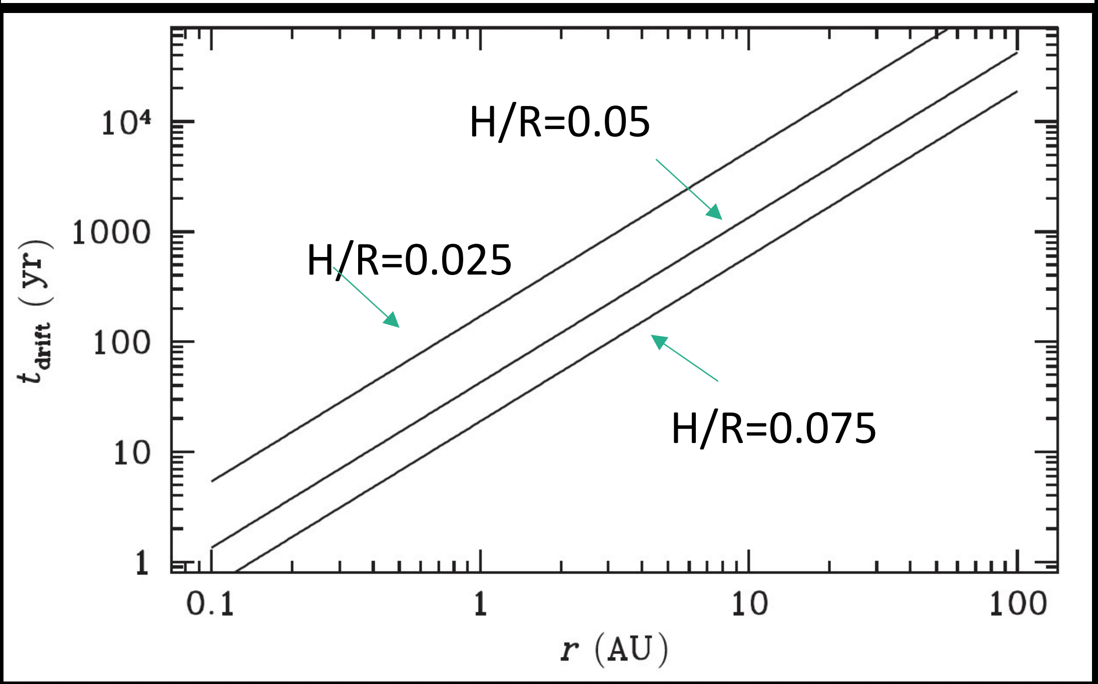
Dust radial drift (II)

- Drift is most efficient for particles with $St \simeq 1$

$$v_{\mathrm{d},r} = \frac{v_{\mathrm{g},r} + 2\mathrm{St}\Delta v_{\mathrm{g},\phi}}{1 + \mathrm{St}^2}$$

- At 5 au, with $\Sigma_{\mathrm{g}} = 10^2\ \mathrm{g\ cm}^{-2}$, $H/R = 0.05$, $St \simeq 1 \rightarrow a \simeq 20\ cm$; typically in the range of $\sim 10\ cm$ – a few m range
  meter-size barrier





Implications

- Planetesimal formation must be rapid
- Radial redistribution of dust is very likely to occur (e.g., dust disk size smaller than gas disk)

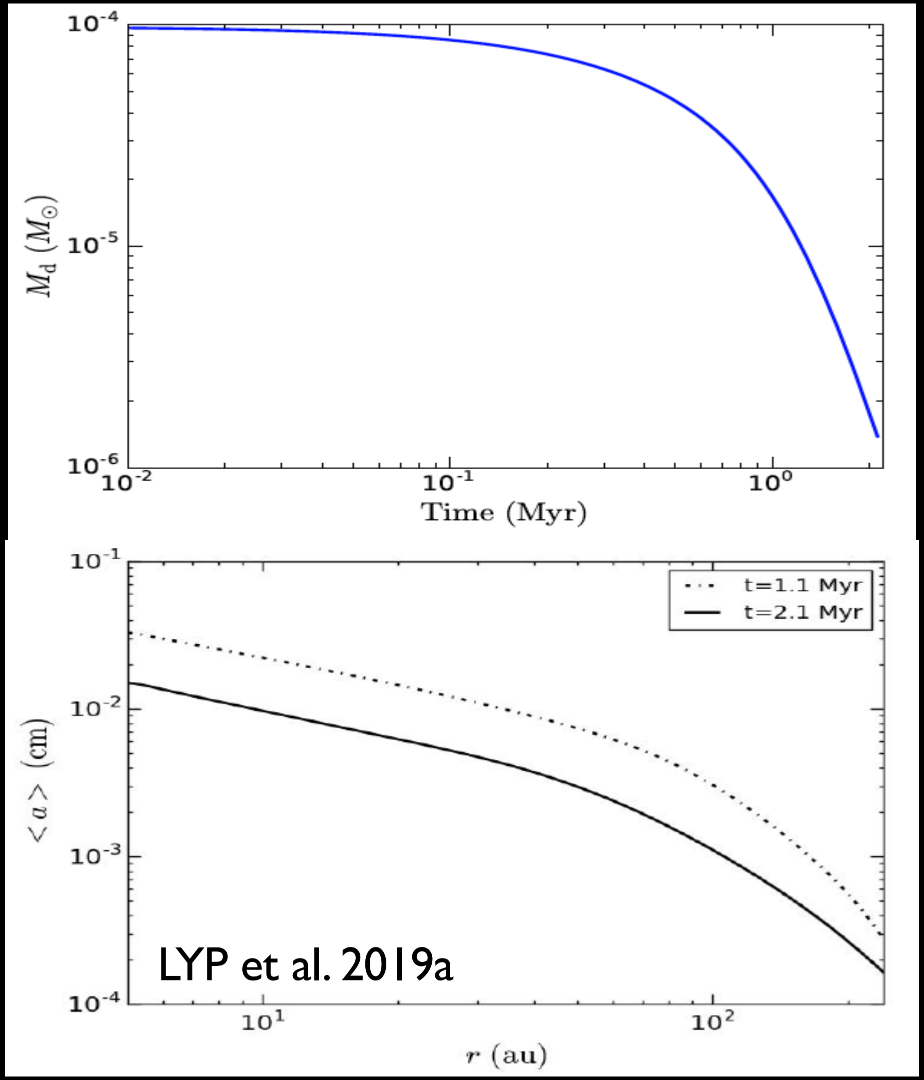
Radial drift with coagulation/fragmentation

- With coagulation (only consider radial drift velocity): $$\frac{\mathrm{d}m}{\mathrm{d}t} = \pi s^2 |v_r| f \rho_0,$$
- Particles can grow before drift ($t_{\mathrm{grow}} < t_{\mathrm{drift}}$) $$s \lesssim \frac{3f}{4\sqrt{2\pi}} \left(\frac{h}{r}\right)^{-1} \frac{\Sigma}{\rho_{\mathrm{m}}}.$$

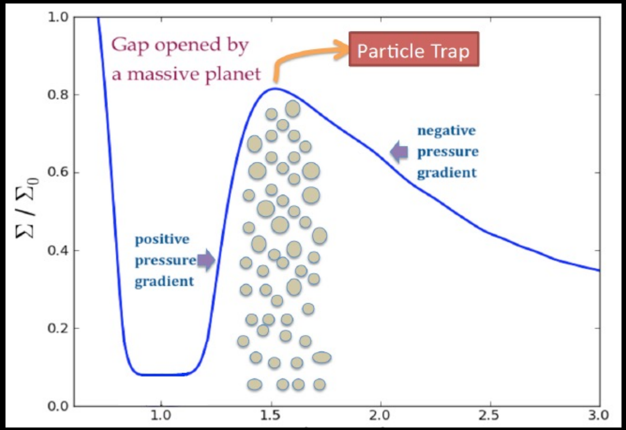
This can be as large as 2 m ($\Sigma_g = 10^3$ g cm^{-2}, $H/R = 0.05, \rho_m = 3$ g cm^{-3}, $f = 0.1$).

Caveats: no fragmentation

- With coagulation/fragmentation:
  Most (~99%) of dust is depleted at ~Myr due to radial drift!
  Dust size cannot grow efficiently!



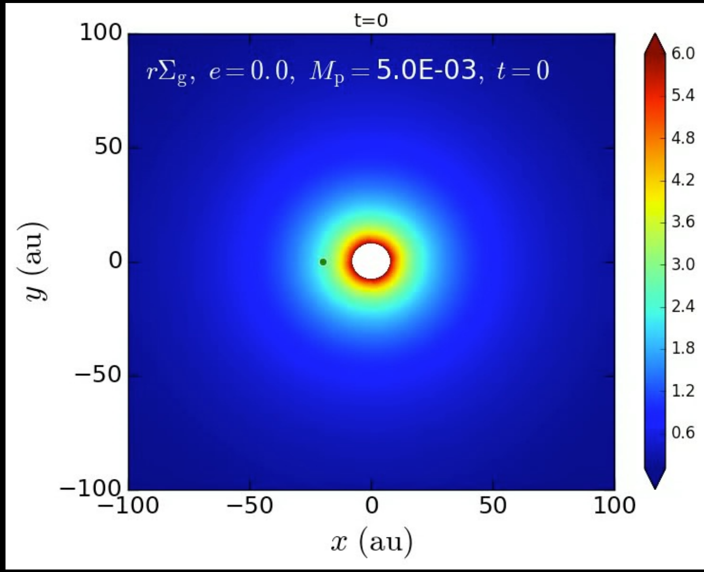
Dust trap by gaseous vortex/ring

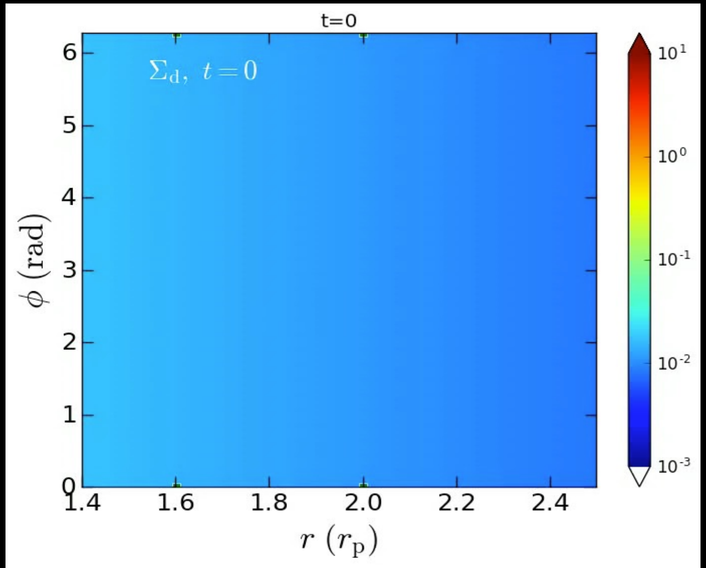


$r\ (r_p)$

A pressure bump at the edge of a planet gap can act as dust trap.

Low viscosity (7×10^{-5}) and High planet mass ($5\ M_J$)
→steep density gradient
→Rossby wave unstable





2D simulations of dust-gas dynamics

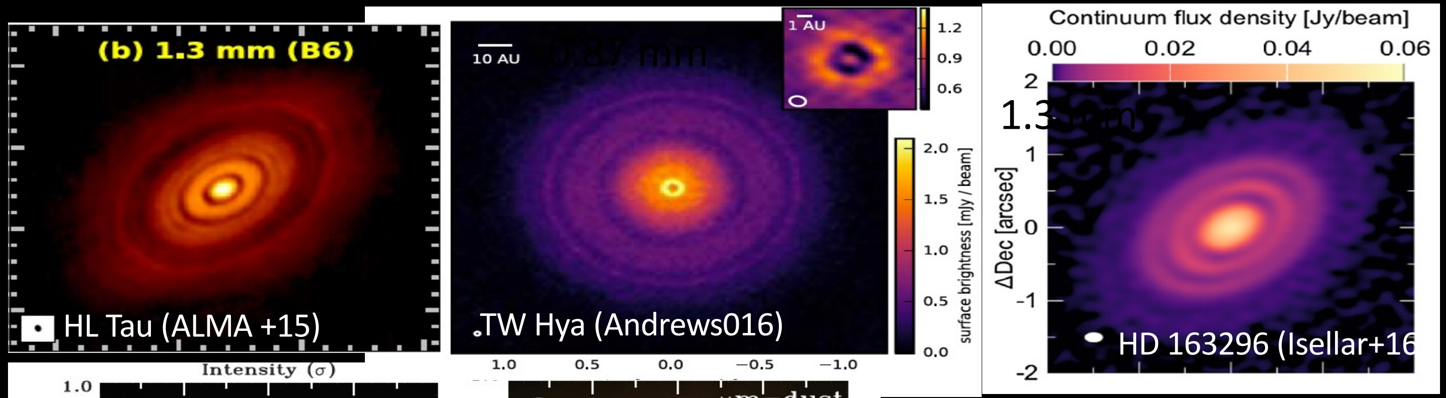
0.2 mm dust
dust/gas = 0.01
With dust feedback

Gas Surface Density

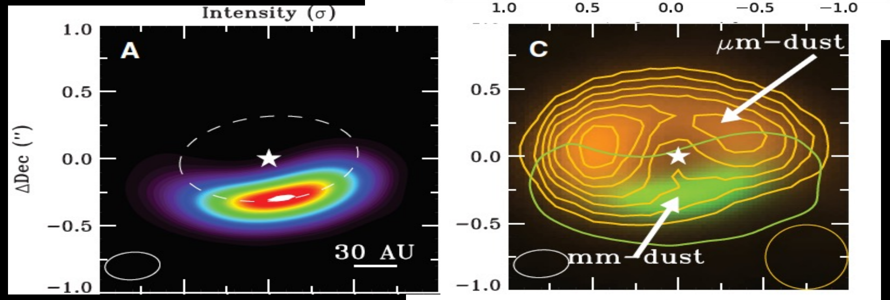
Dust Surface Density

Observational implications

Ringed structures

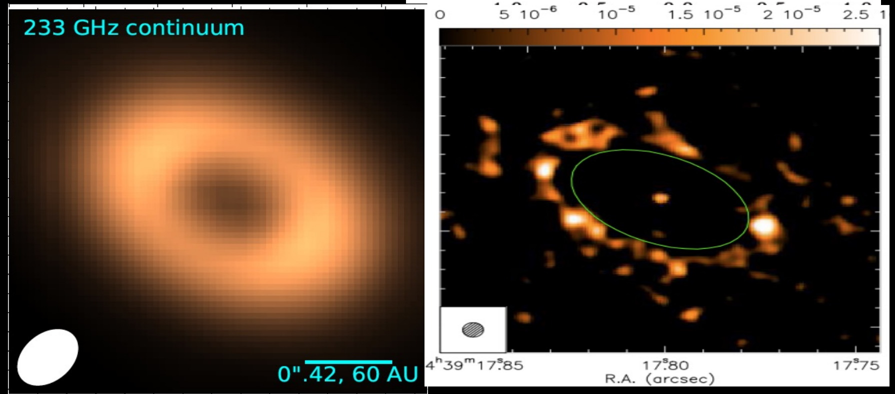


Asymmetry



IRS 48
(van der Marel+2013)

Clumpy



LkCa 15
(Isella+2014)

See also many other: AA Tau (Loomis+17), Elias 24 (Cieza+17; Cox+17; Dipierro+18), AS 209 (Fedele+18), GY 91 (Sheehan +18), V1094 Sco (Ansdell+18; van Terwisga+18), MWC 758 (Boehler+18, Dong+18), 16 disks (van der Marel+19), Long et al. (2018) ,DSHARP (Andrew et al. 2018)

Turbulent radial drift

- Turbulence does not alter the mean sub-Keplerian flow that is responsible for radial drift.

$$\frac{\partial \Sigma_{\rm d}}{\partial t} + \nabla \cdot \mathbf{F}_{\rm d} = 0, \qquad \mathbf{F}_{\rm d} = \Sigma_{\rm d}\mathbf{v} - D\Sigma\nabla\left(\frac{\Sigma_{\rm d}}{\Sigma}\right).$$

$f = \frac{\Sigma_d}{\Sigma}$, with continuity equation for gas

$$\frac{\partial f}{\partial t} = \frac{1}{r\Sigma}\frac{\partial}{\partial r}\left(Dr\Sigma\frac{\partial f}{\partial r}\right) - v_r\frac{\partial f}{\partial r}.$$

- A competion between diffusion and radial advection, described by Schmidt number: $Sc \equiv \frac{\nu}{D}$,

ν: gas kinematic viscosity, D: (particles) diffusion coefficient

Upstream diffusion is important for smaller Schmidt number (Sc<0.33)

Diffusion of large particles

For small particles ($\tau_{\rm stop} \ll 1$), $D_{\rm p} \simeq D_{\rm g}$ and $Sc \simeq 1$.

For large particles ($\tau_{\rm stop} \gg 1$), $\frac{D_{\rm g}}{D_p} \sim \tau_{\rm stop}^2$

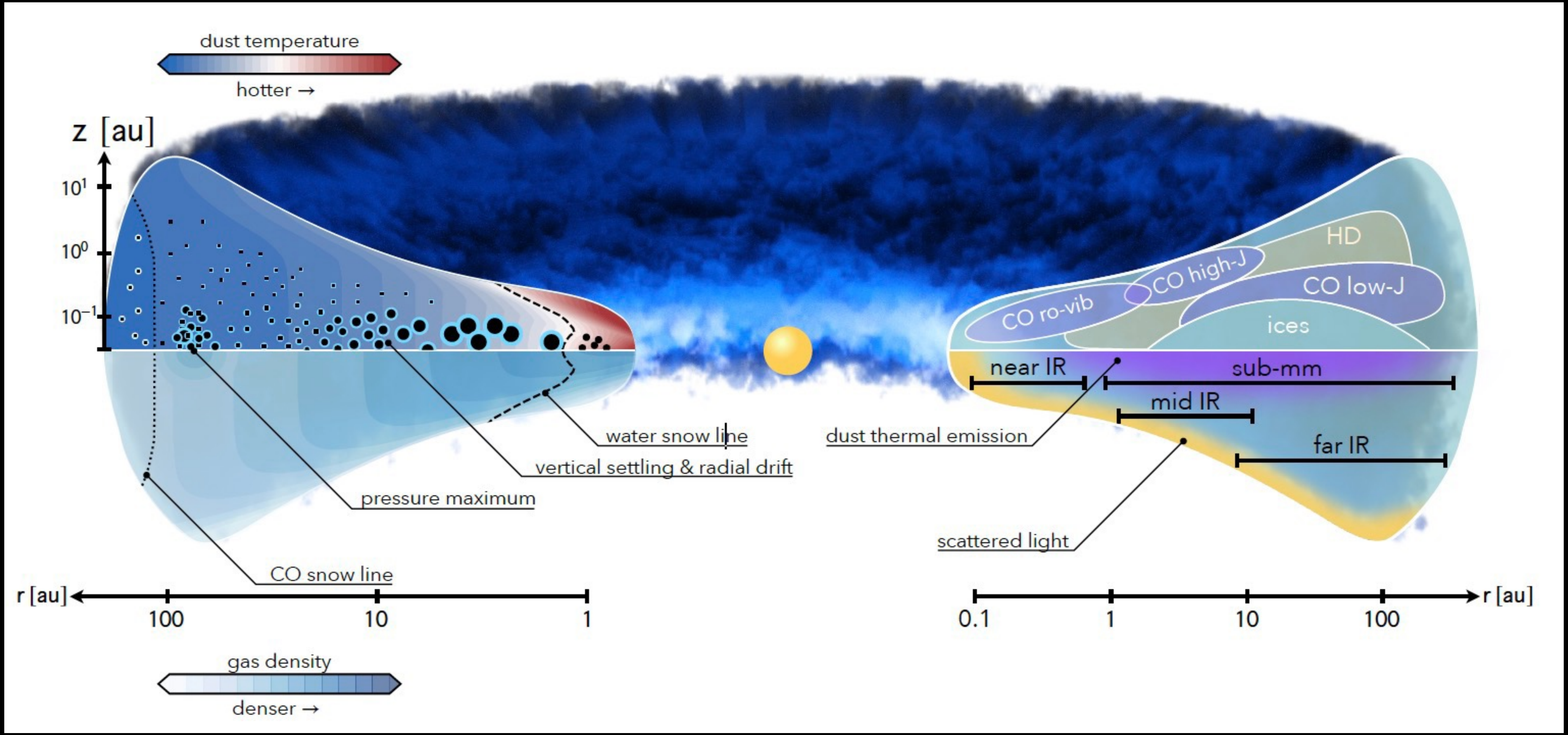
$\frac{D_{\rm g}}{D_p} \simeq 1 + \tau_{\rm stop}^2$ (Youdin & Lithwick 2007)

($D_{\rm g} \sim \nu \sim \alpha c_s H$)

Implications:

The radial diffusion is only important for relatively small particles

Dust distribution in protoplanetary disk



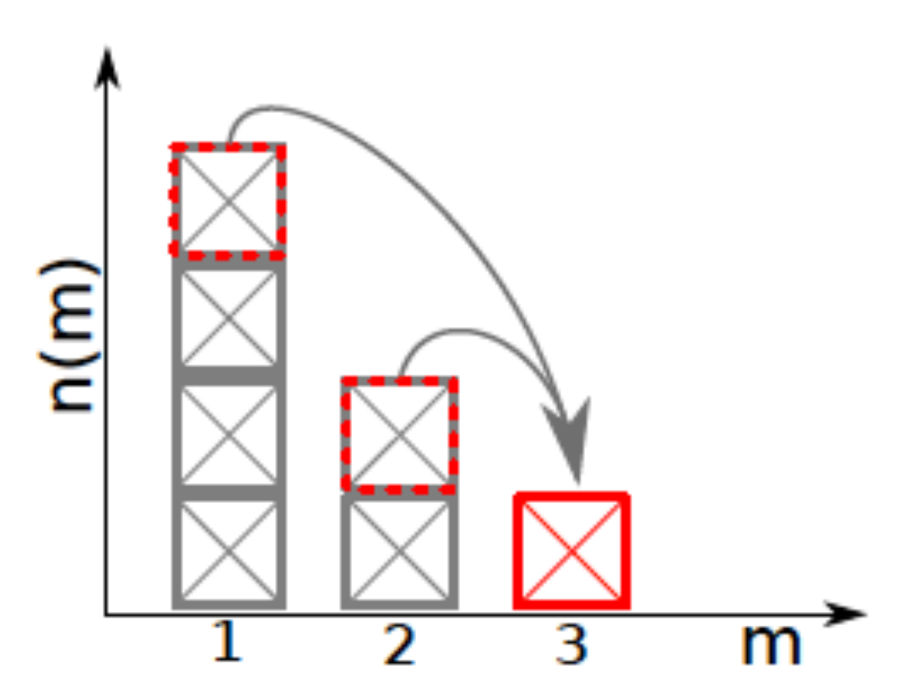
Miotello et al. 2023, PPVII, ch.14

Part II: dust coagulation

Particles growth via coagulation

- Grains stick together or fragment , depending on relative velocity.
- Dust size evolution governed by Smoluchowski equation: *(Smoluchowski 1916)*

$$\frac{\partial n(m)}{\partial t} = \frac{1}{2}\int_0^m A(m', m-m')n(m')n(m-m')\mathrm{d}m' - n(m)\int_0^\infty A(m', m)n(m')\mathrm{d}m',$$



Reaction kernel: $P(m_1, m_2, \Delta v)$ probability for adhesion, Δv relative velocity, $\sigma(m_1, m_2)$ cross section

$$A(m_1, m_2) = P(m_1, m_2, \Delta v)\Delta v(m_1, m_2)\sigma(m_1, m_2).$$

Relative velocities

- Brownian motion

$$v_{ij}^{\mathrm{rel,brown}} = \min\left(\sqrt{\frac{8k_\mathrm{B}T(m_i+m_j)}{\pi m_i m_j}},\, c_\mathrm{s}\right)$$

- Vertical settling

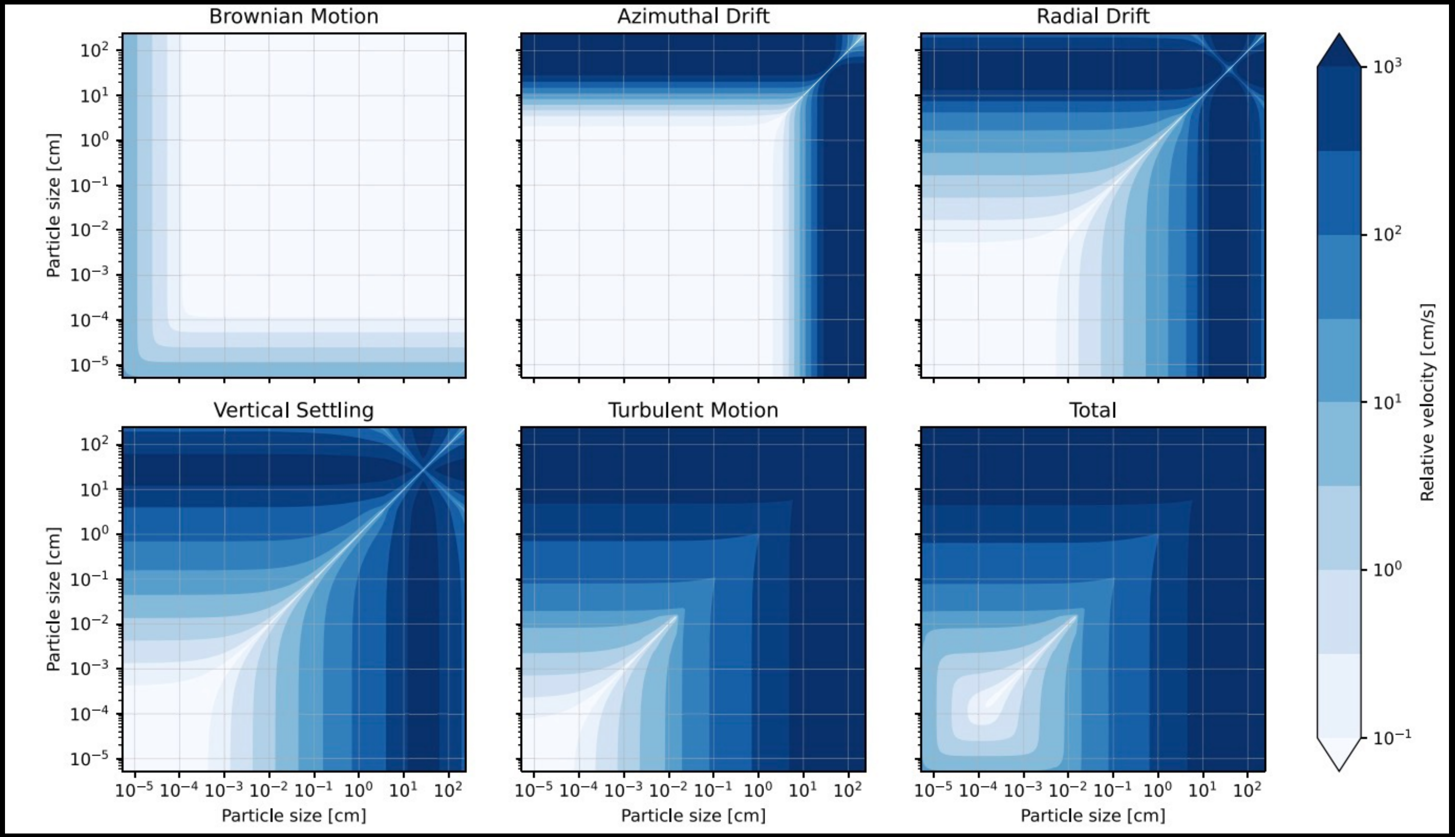
$$v_{ij}^{\mathrm{rel,sett}} = \left| h_i \min\left(\mathrm{St}_i, \frac{1}{2}\right) - h_j \min\left(\mathrm{St}_j, \frac{1}{2}\right) \right| \Omega_\mathrm{K}$$

- Radial drift $v_{ij}^{\mathrm{rel,rad}} = |v_{\mathrm{d},i} - v_{\mathrm{d},j}|$
- Azimuthal drift

$$v_{ij}^{\mathrm{rel,azi}} = \left| v_{\mathrm{drift}}^{\mathrm{max}} \left(\frac{1}{1+\mathrm{St}_i^2} - \frac{1}{1+\mathrm{St}_j^2} \right) \right|$$

- Turbulence motion (Ormel&Cuzzi, 2007; dominant)

$$\Delta v^2 \simeq 3/2\alpha\tau_{\mathrm{stop}}c_s^2$$



Different relative velocity at 1 au

Birnstiel et al. 2010; Stammler & Birnstiel 2022

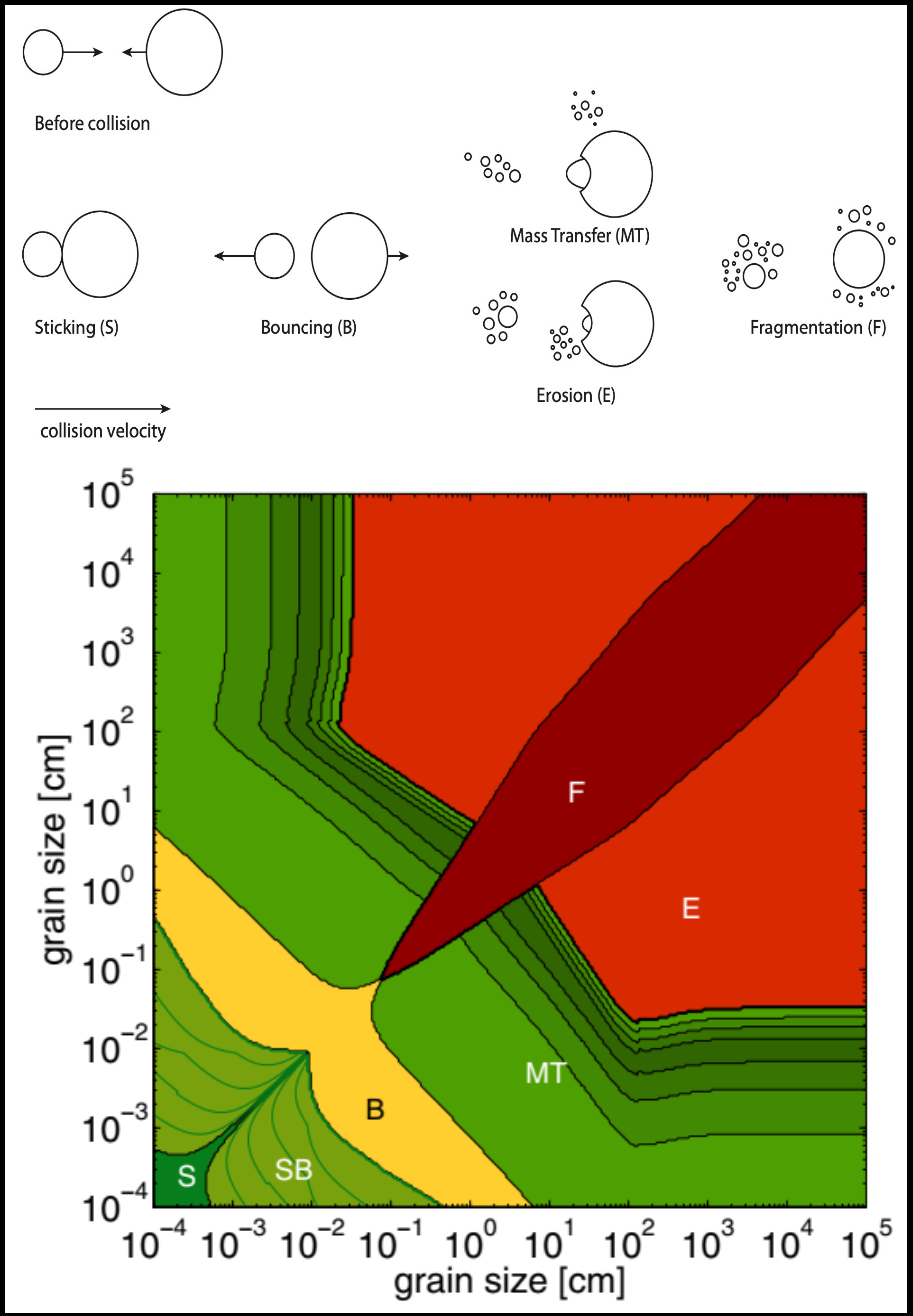
Dust coagulation

Relative velocity $\Delta v > v_{\rm frag}$ (fragmentation velocity) → fragmentation; Otherwise, sticking

Many complications

- Outcome: Sticking, bouncing, erosion, mass transfer, fragmentation
- Composition of dust particles (silicates within snow line ($v_{\rm frag} \sim 1{\rm m/s}$), outer region: water and CO ices ($v_{\rm frag} \sim 10\ {\rm m/s}$))
- Geometry of particles (porosity etc.),
- Fragment distribution, and temperature dependence of $v_{\rm frag}$, coupled with disk evolution, etc…

Open code: DustPy by Stammler & Birnstiel (2022)



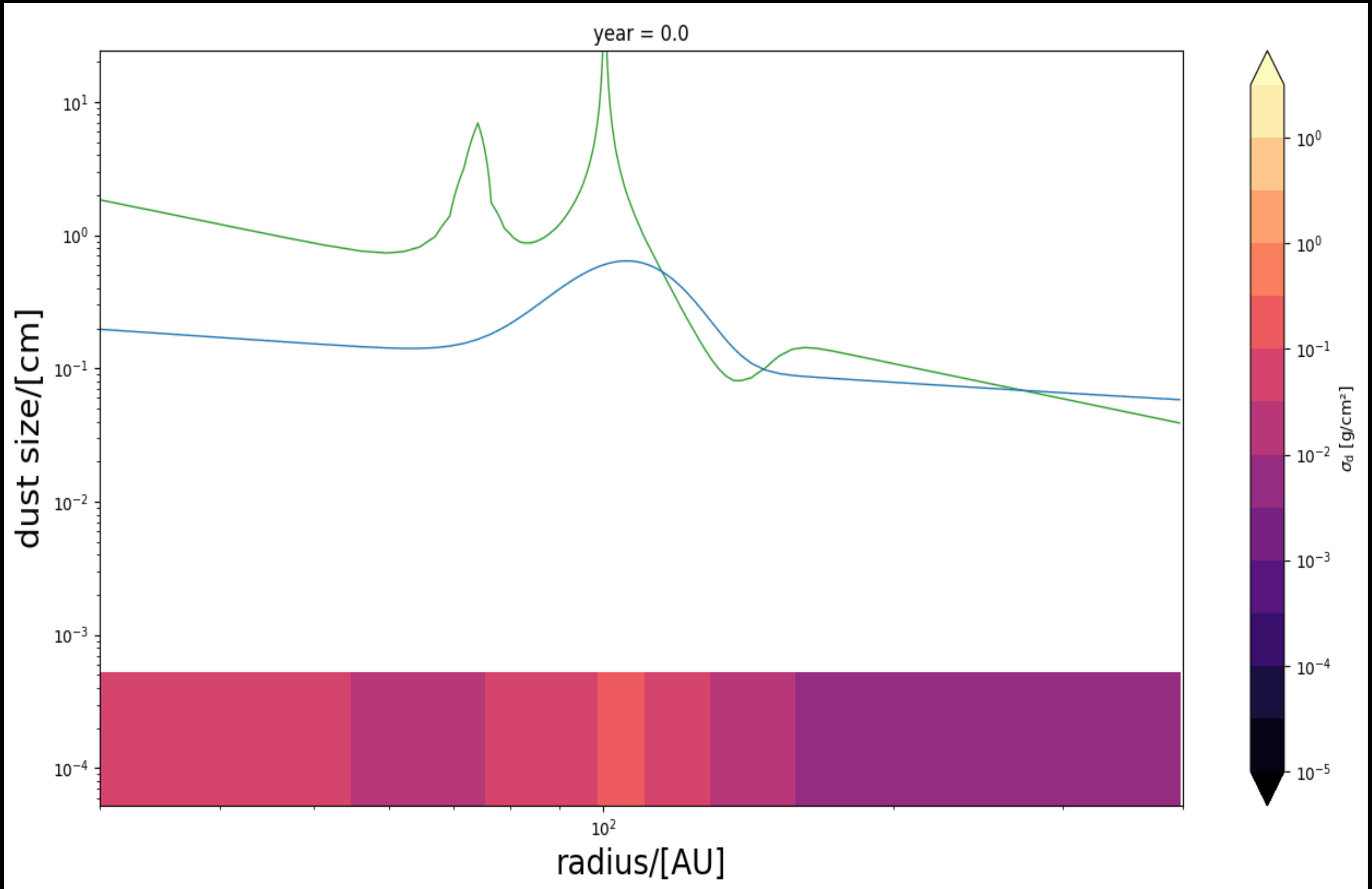
Windmark+12

Dust coagulation simulations

Blue line
fragmention barrier

Green line:
drift barrier

White line: $St = 1$

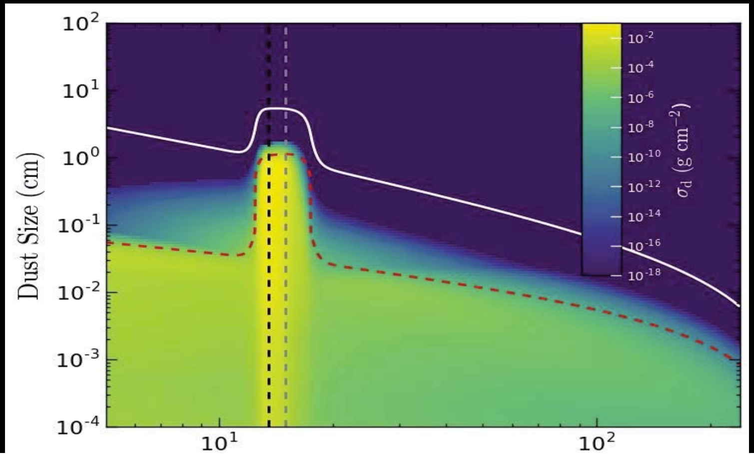


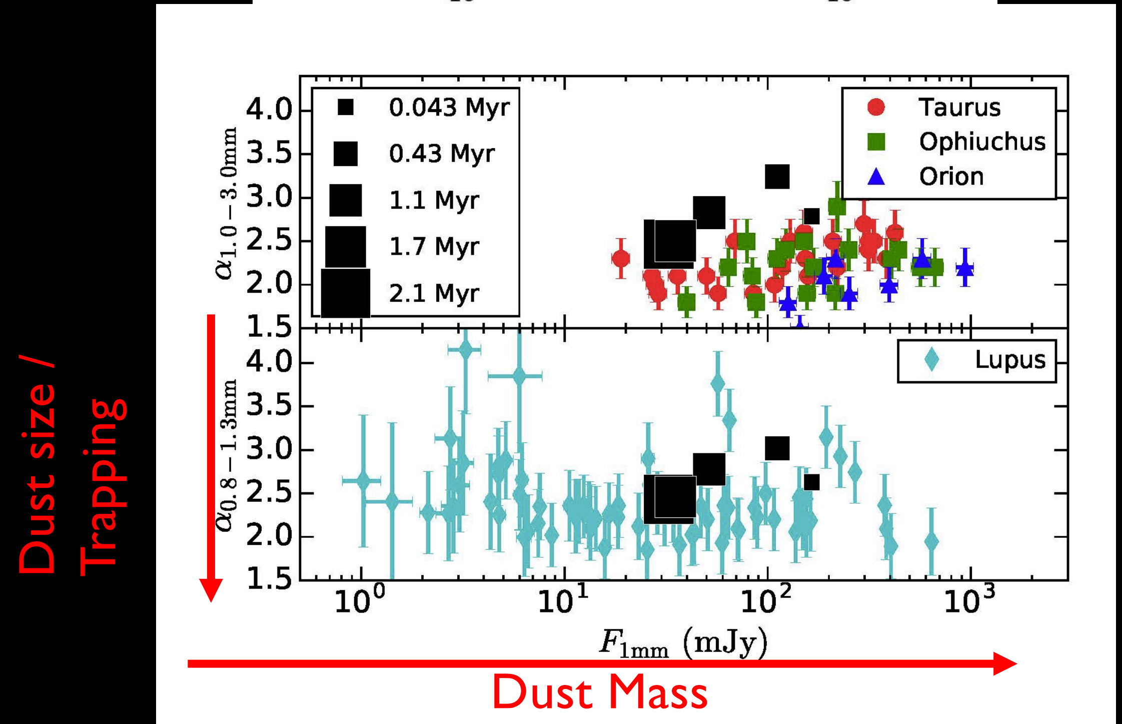
Credit: Linhan Yang using DustPy

Dust coagulation in ringed structures

- Most of dust (95%) and gas trapped in bump
  → essential to retain dust mass/ size growth.
- A power-law distribution for dust in bump [$n(a) \propto a^{-3.5}$]

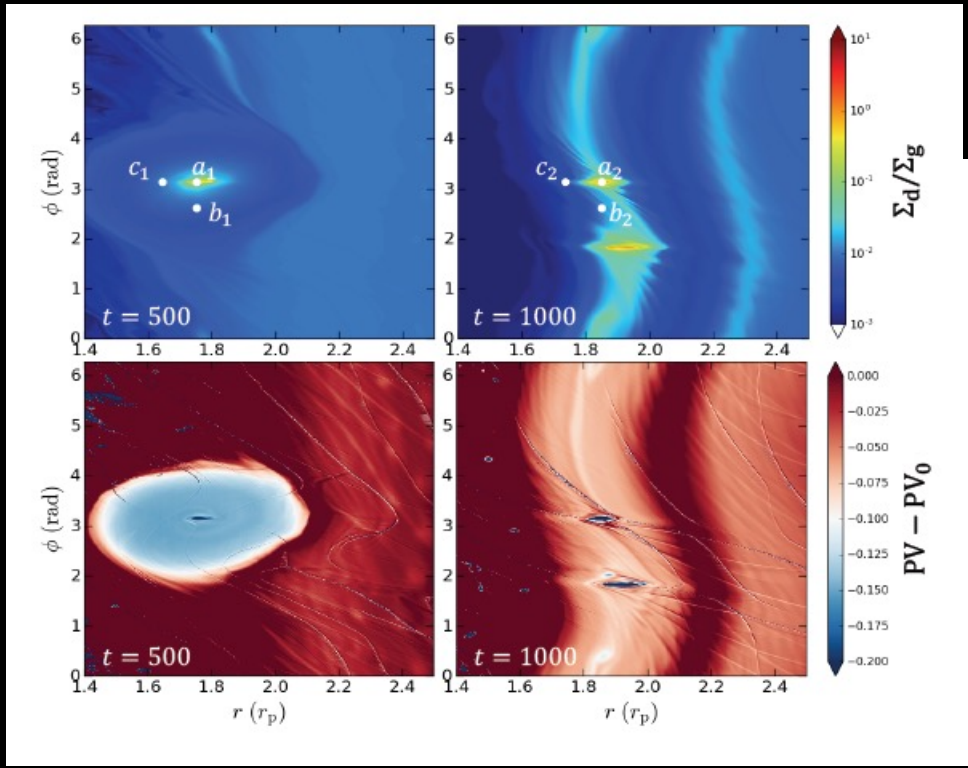
- **Efficient size growth in bump**, $a_{\rm max} \sim 1$ cm.
- **With dust trap**: global spectral index close to $\alpha_{\rm mm} \sim 2.5$.
- Need fragmentation velocity ~ a few m/s.



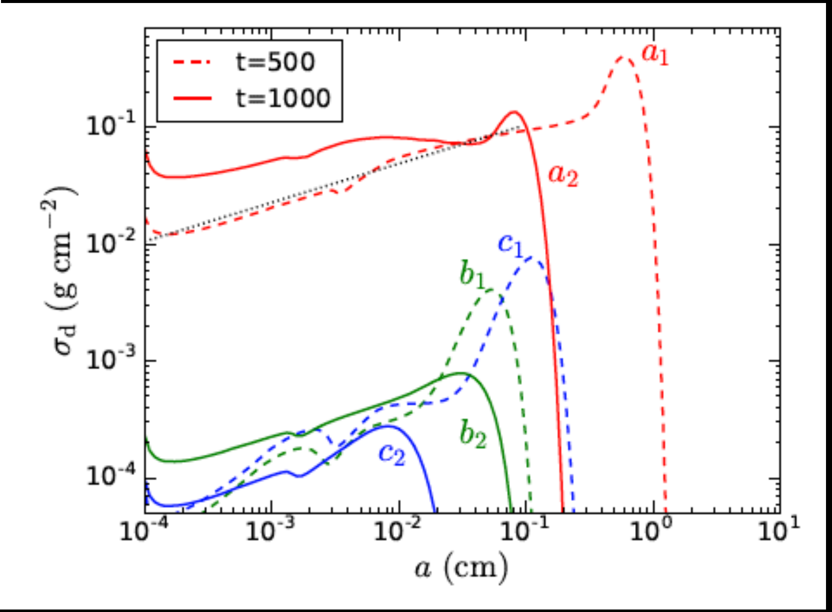


LYP et al. 2019a

Planet-induced vortices with dust coagulation



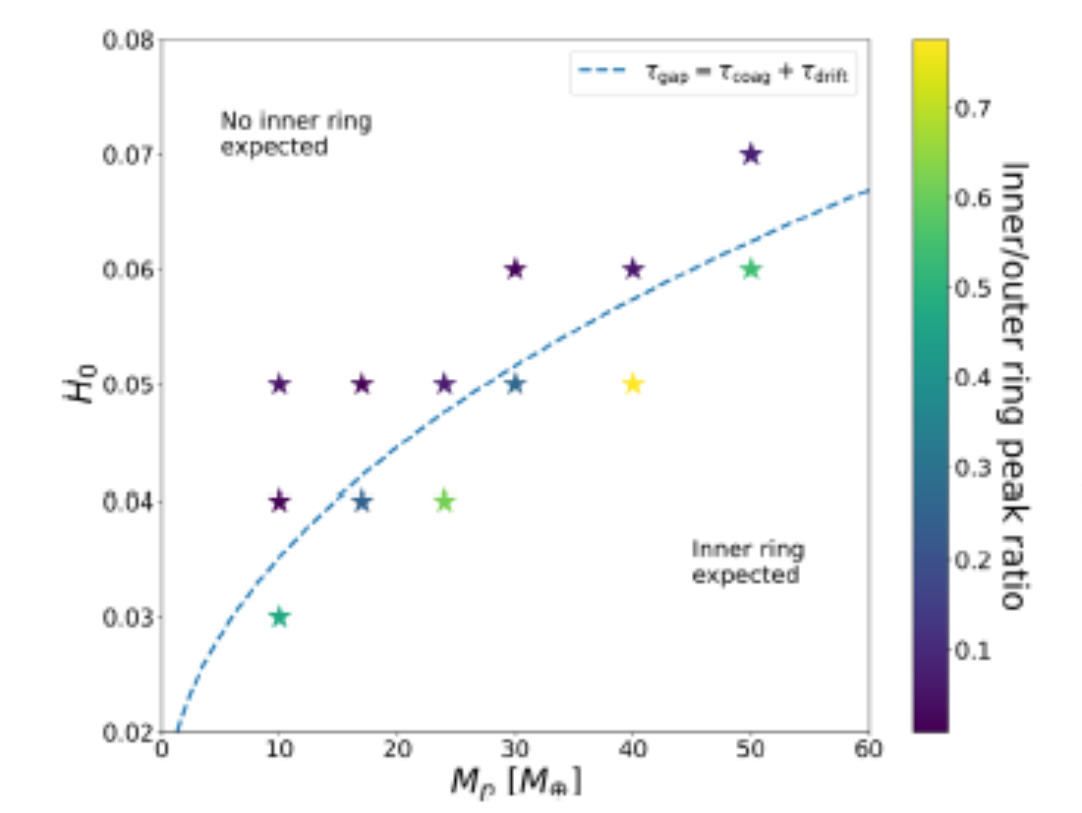
low α,high m_p



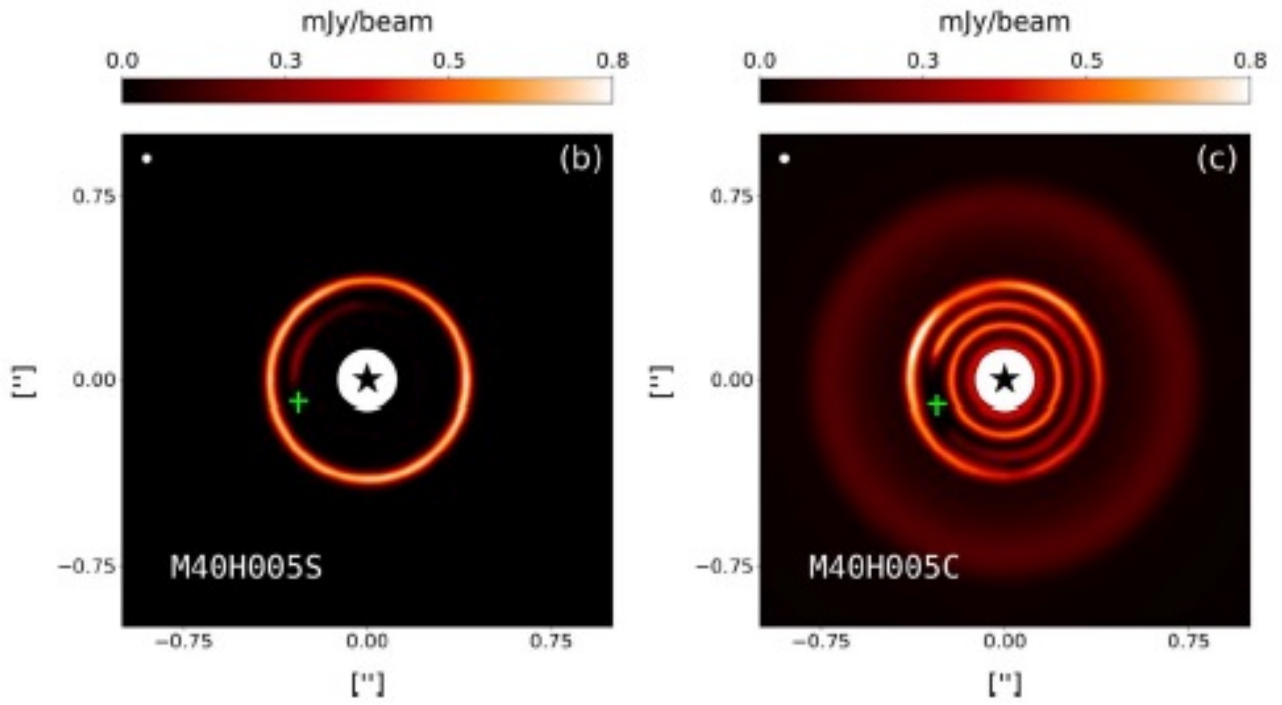
- Dust size is quite non-uniform within vortex region
- Dust size growth facilizes the increase of dust/gas ratio.
- Vortex lifetime can be significantly impacted by dust feedback when coagulation included.

LYP et al. 2020a

Ring morphology with dust coagulation



low α,low m_p



inner rings disappears due to dust coagulation and subsequent radial drift (if faster than gap opening timescale)

Laune et al. 2020

Part III: planetesimal formation

Gravitational collapse of planetesimals

- Particles grow via collisions
- Large particles settle to midplane
- Gravitional unstable, and collapse into planetesimals

Dispersion relation between ω and k

(c_s: velocity dispersion for particles; κ: epicyclic frequency)

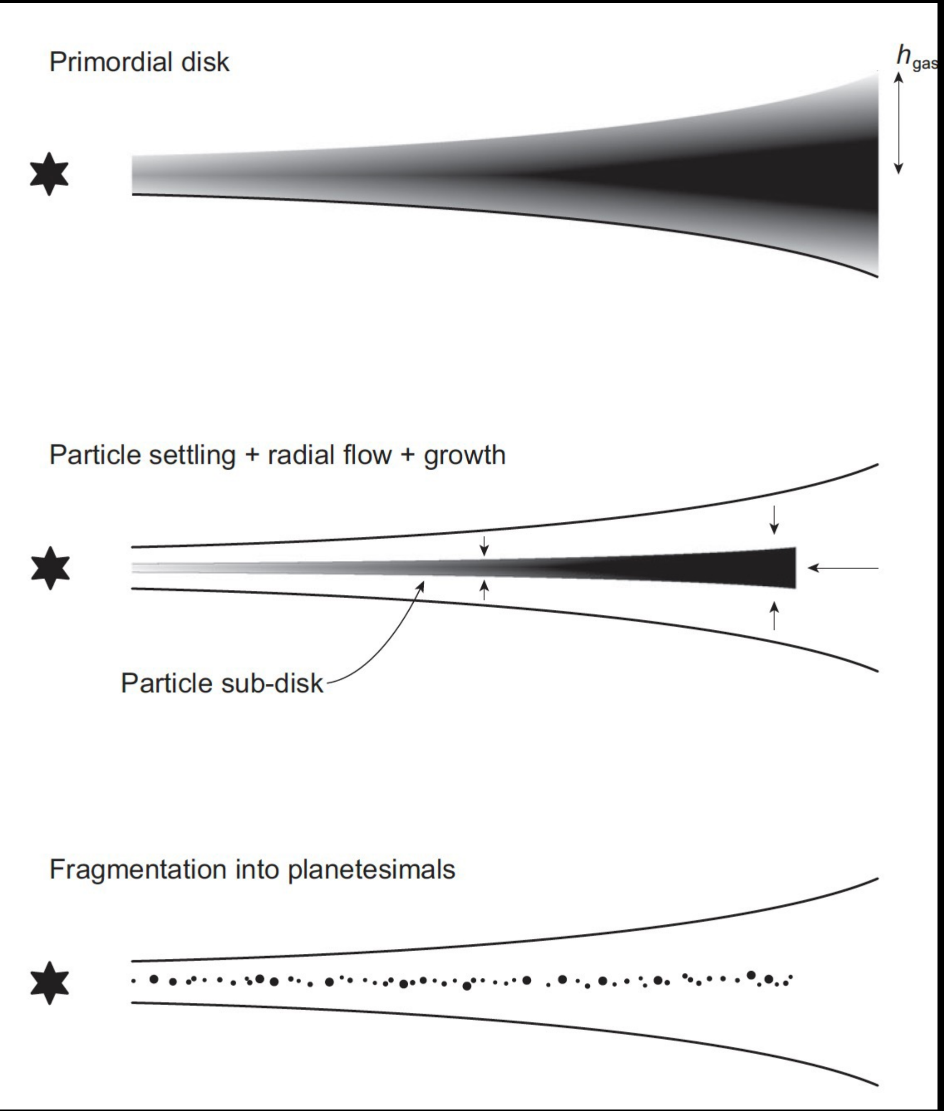
$$\omega^2 = \kappa^2 + c_s^2 k^2 - 2\pi G \Sigma_0 |k|$$

$$\frac{\partial \Sigma}{\partial t} + \nabla \cdot (\Sigma \mathbf{v}) = 0,$$
$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{\nabla p}{\Sigma} - \nabla \Phi,$$
$$\nabla^2 \Phi = 4\pi G \rho.$$

$$\Sigma_1(r,t) \propto \exp[\mathrm{i}(kr - \omega t)]$$

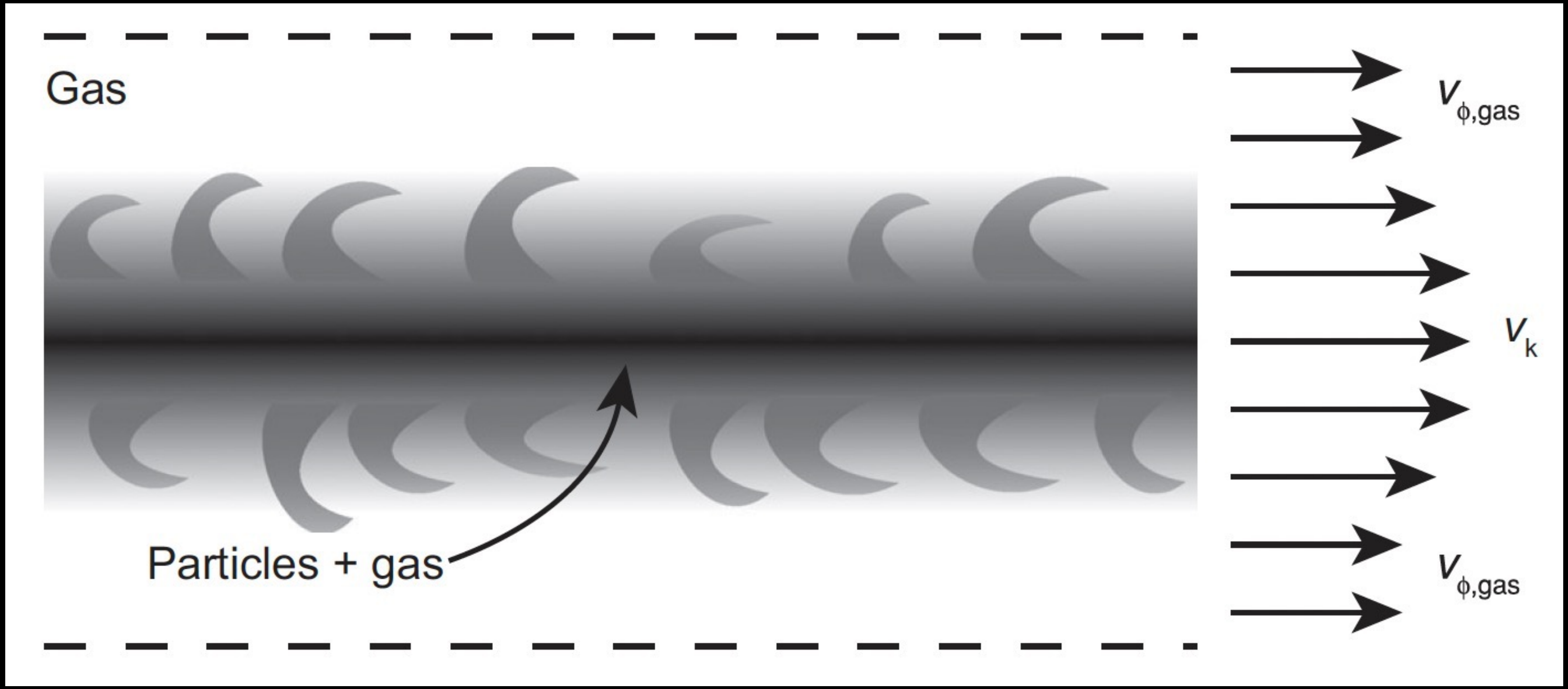
- Most unstable wavelength: $\lambda \sim \frac{2\pi}{k_{\min}} = \frac{2\sigma^2}{G\Sigma_s}$
- Mass of clumps: $m_p \sim \pi \lambda^2 \Sigma_s \sim 4\pi^5 G^2 Q_{\rm crit}^4 \frac{\Sigma_s^3}{\Omega^4}$

At 1 au, $m_p(1\mathrm{au}) \sim 3\times 10^{18}\mathrm{g}$ (size ~ 10 km)



Safronov 1969; Goldreich, Ward 1973

Self-excited turbulence



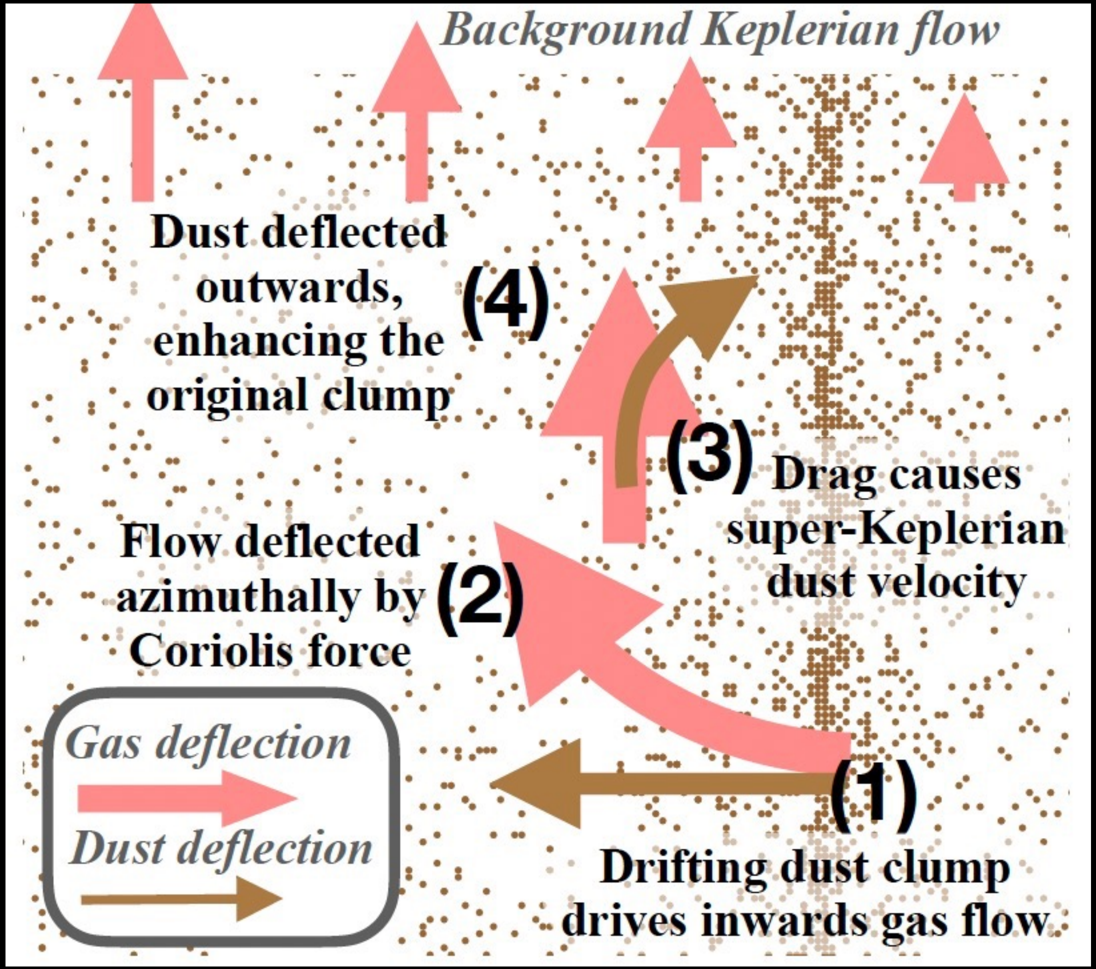
But secular gravitational instability (e.g., Shariff & Cuzzi 2011; Youdin2011)

- Settling of dust increases the dust-to-gas ratio at the midplane
- Backreaction becomes stronger, modify gas v_ϕ for different z
- Vertical shear can be unstable to Kelvin-Helmholtz instability (KHI)
- Particles cannot settle into a very thin layer

Streaming instability (SI)

- Streaming instability can happen when two fluids (gas and solid particles) have a mutual velocity and interact via aerodynamic forces.

Dust concentration due to radial drift/settling → backreaction to the gas stronger → weaken the headwind → reduce the radial drift, enhance the concentration

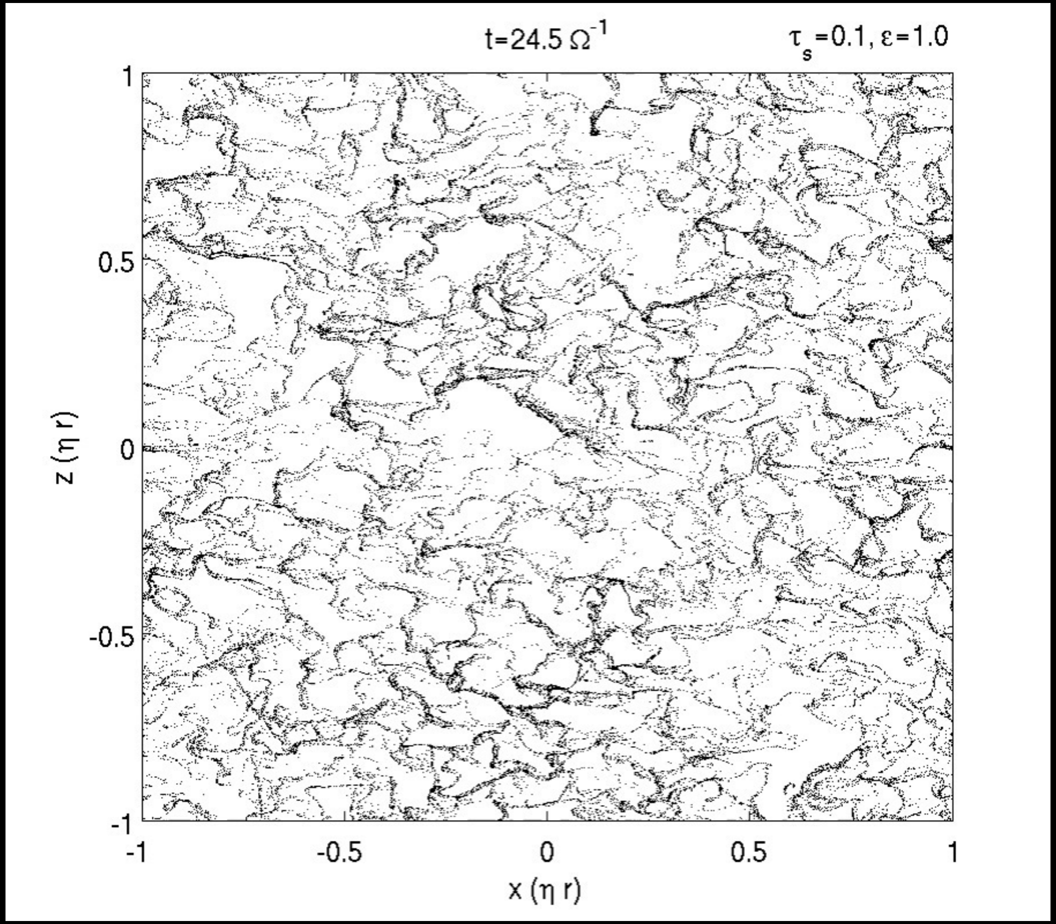


- Growth rate: on a time scale intermediate between dynamical and radial drift time scales.
- Consequence: small-scale particle concentration ($\ll h$), part of a broad class of resonant drag instabilities (Squire & Hopkins 2018).

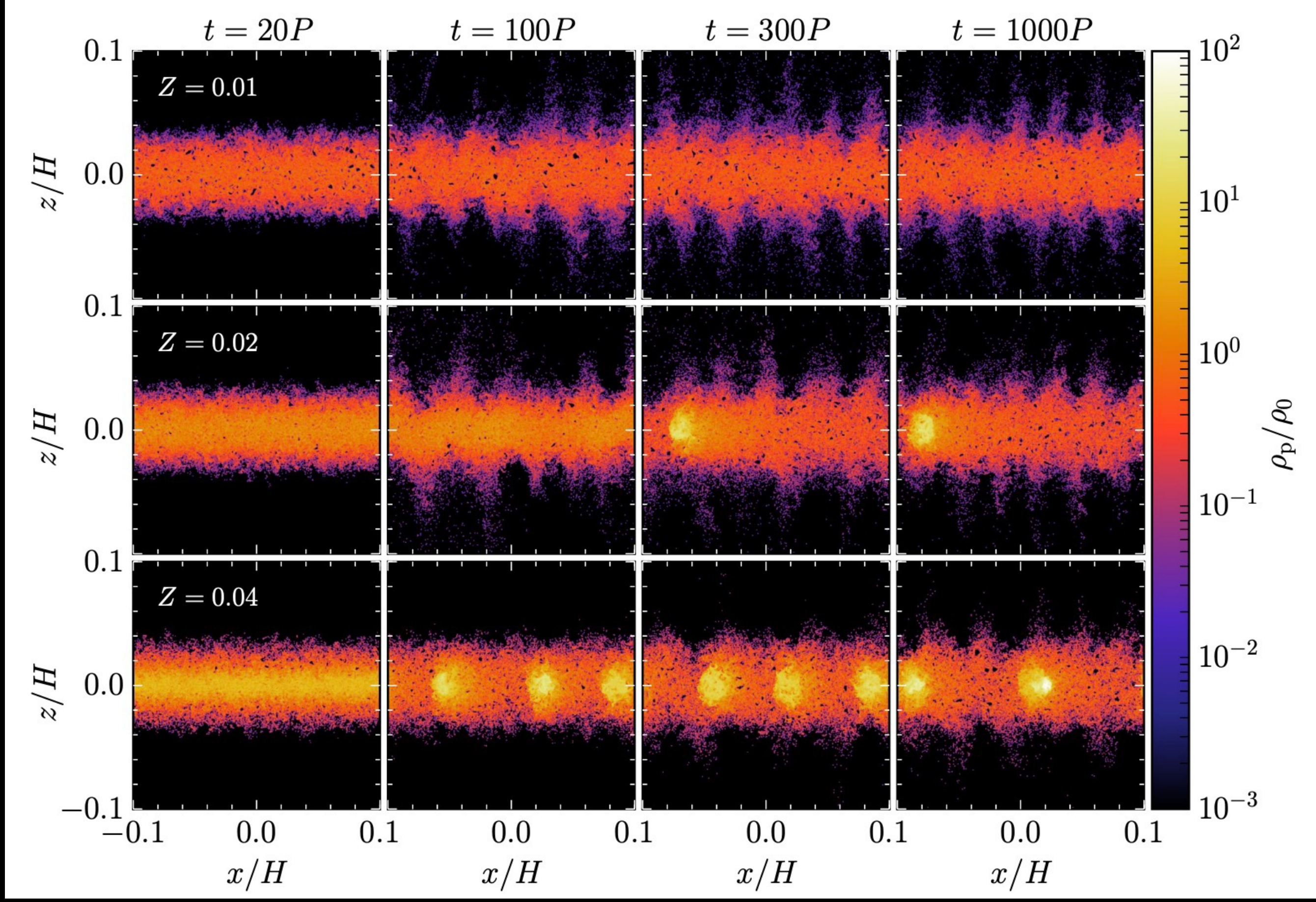
Youdin & Goodman, 2005; review by Lesur et al. 2023

Basic properties

- Efficiently concentration up to $10^3 \rho_g$
- Particle rings and spacings have width $<$ 10% gas scale height



Bai & Stone, 2010a



Yang et al. 2017

Condition for clumping

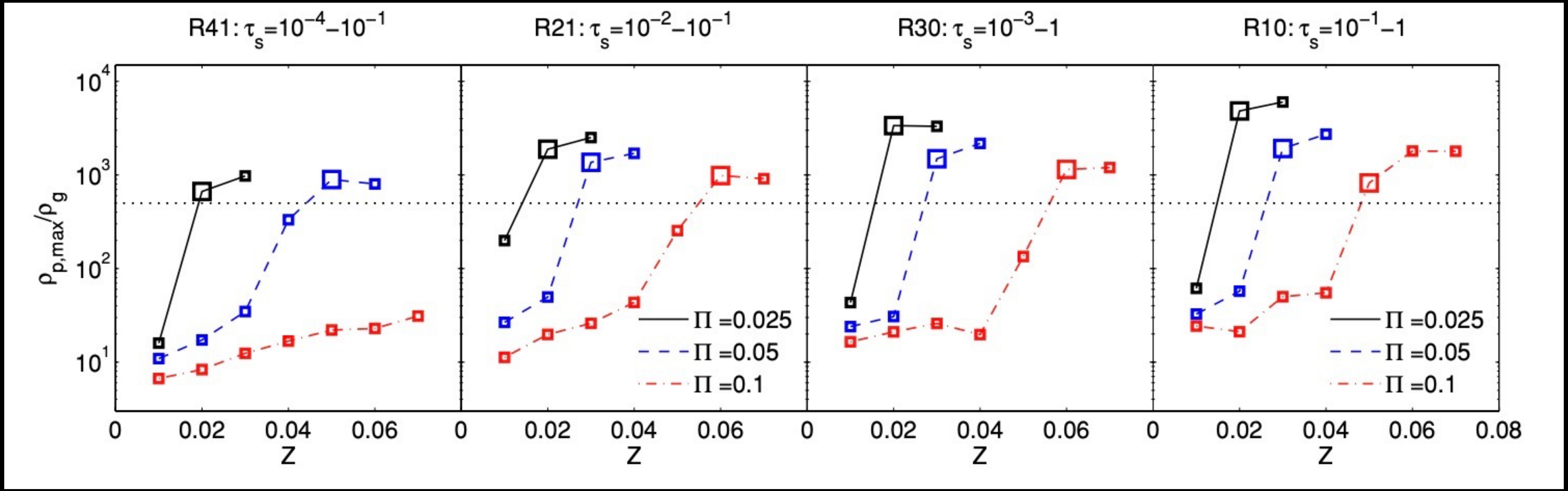
Condition for gravitational collapse: clump density exceeds Roche density ρ_{R}.

$$a_{\mathrm{tidal}} = \frac{GM_*}{a^2} - \frac{GM_*}{(a+r)^2} \simeq \frac{2GM_*}{a^3}r \qquad a_{\mathrm{grav}} \sim \frac{Gm}{r^2}$$

$$\Rightarrow \quad r \lesssim \left(\frac{m}{M_*}\right)^{1/3} a \qquad \Rightarrow \quad \rho > \rho_R \sim \frac{M_*}{a^3} = 6\times10^{-7}\left(\frac{M_*}{M_\odot}\right)\left(\frac{a}{\mathrm{au}}\right)^{-3}\ \mathrm{g\ cm^{-3}}$$

Enhancement of $\frac{\delta\rho_{\mathrm{d}}}{\rho_{\mathrm{d,0}}} \sim 10^4$ are needed before gravitational collapse.

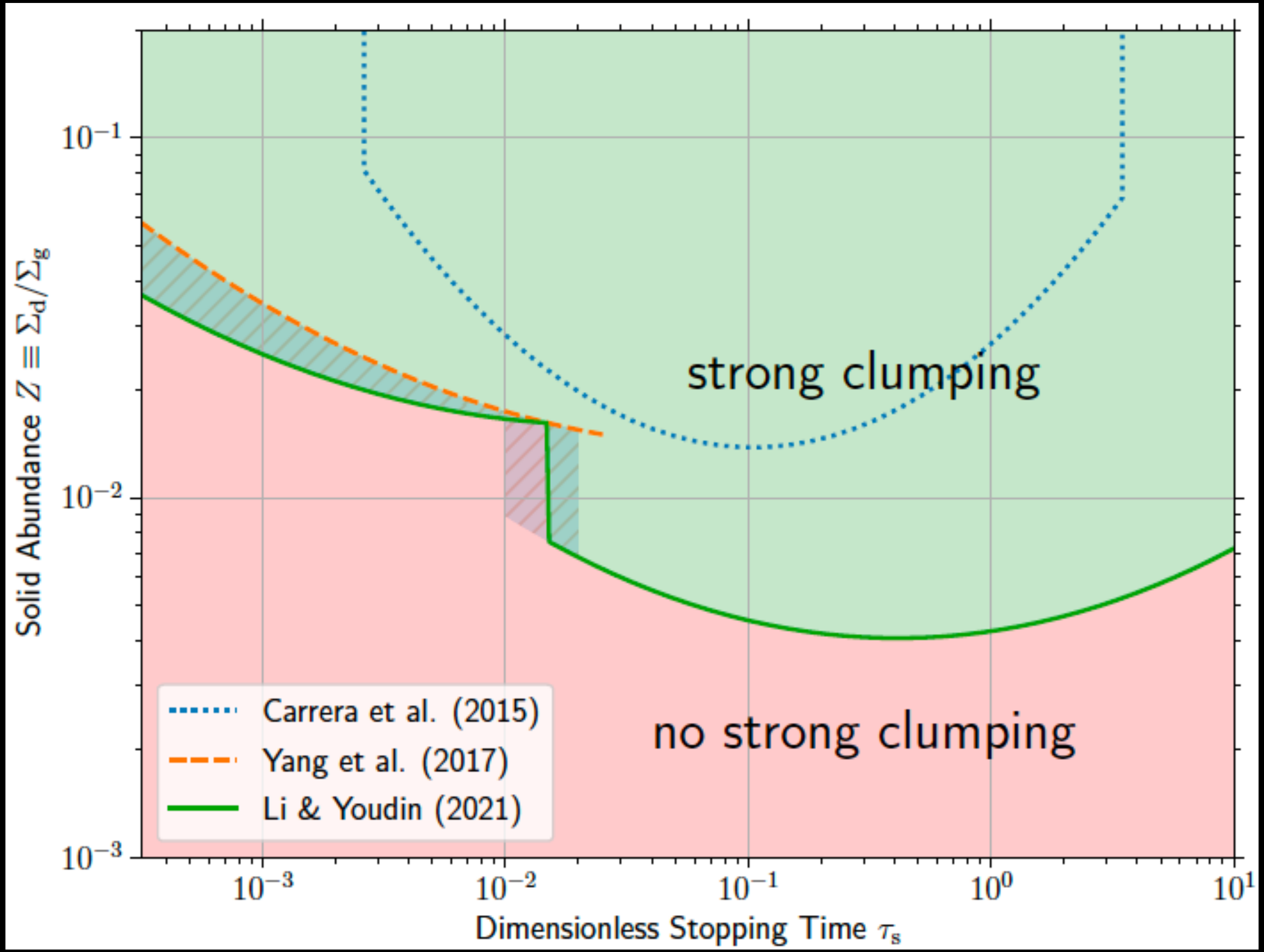
Planetesimal formation condition



- Need super-solar metallicity (Z) and marginally coupled dust (moderate τ_s)
- Smaller pressure gradient (smaller Π) is favored for clumping

Bai & Stone 2010c;

Streaming instability: clumping condition

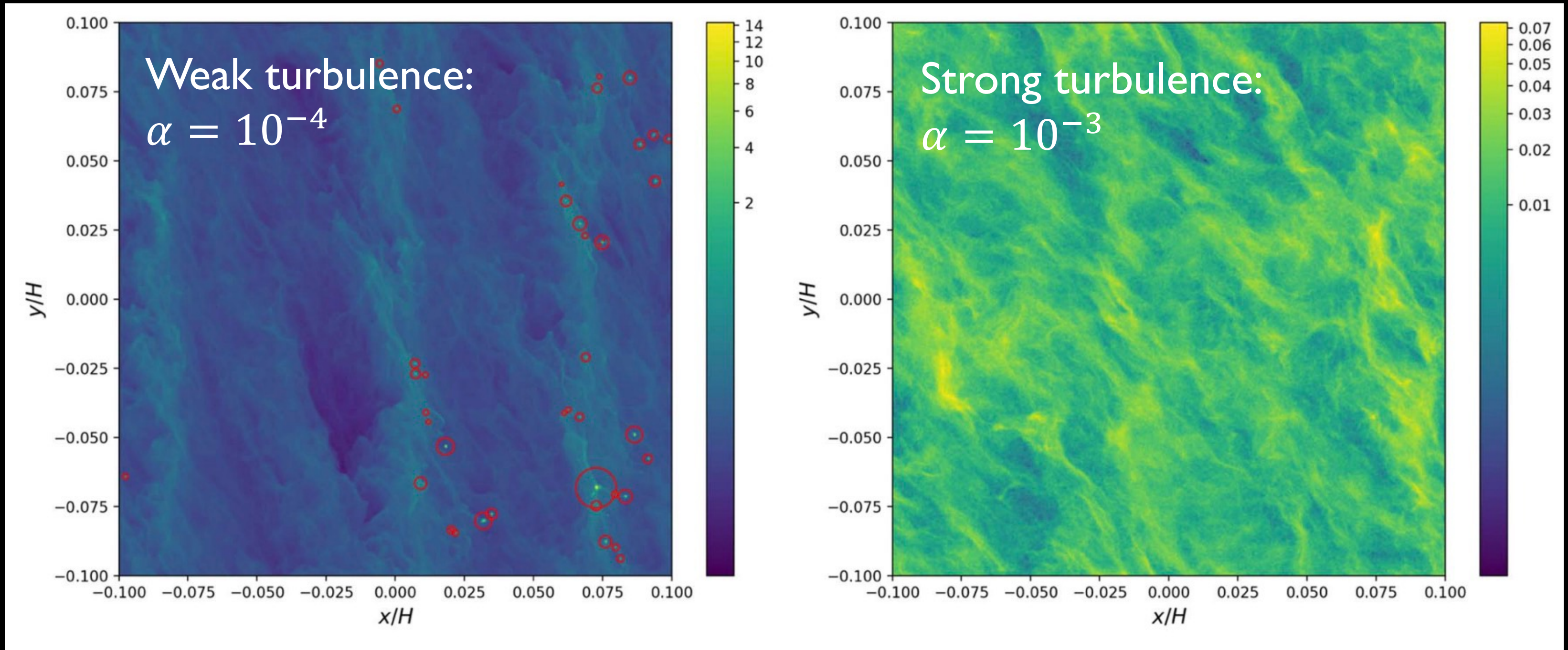


Caveats:
No turbulence
Single dust species

Carrera et al. 2015; Yang et al. 2017; Li R. et al. 2021; review by Lesur et al. 2023

SI with external turbulence

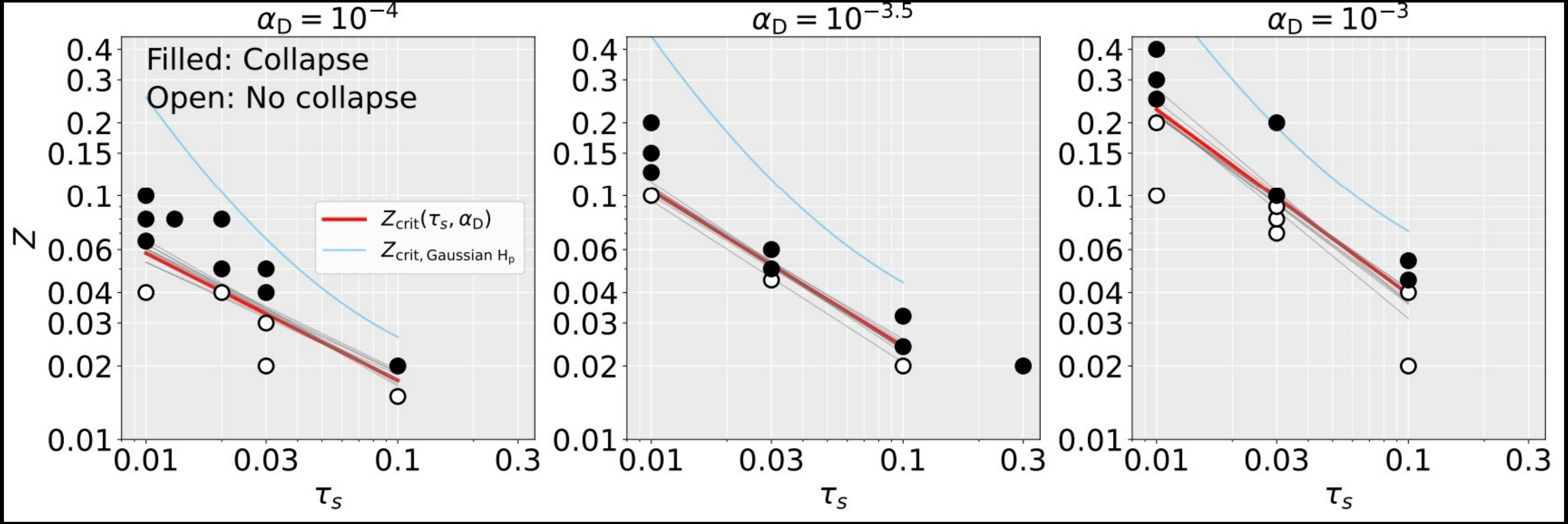
- Linear analysis: turbulence suppresses linear growth of SI (Chen & Lin 2020; Umurhan et al. 2020)
- Confirmed by non-linear simulations (Gole et al. 2020)



External turbulence suppress planetesimal formation if $\alpha \gtrsim 10^{-3.5}$

Gole et al. 2020

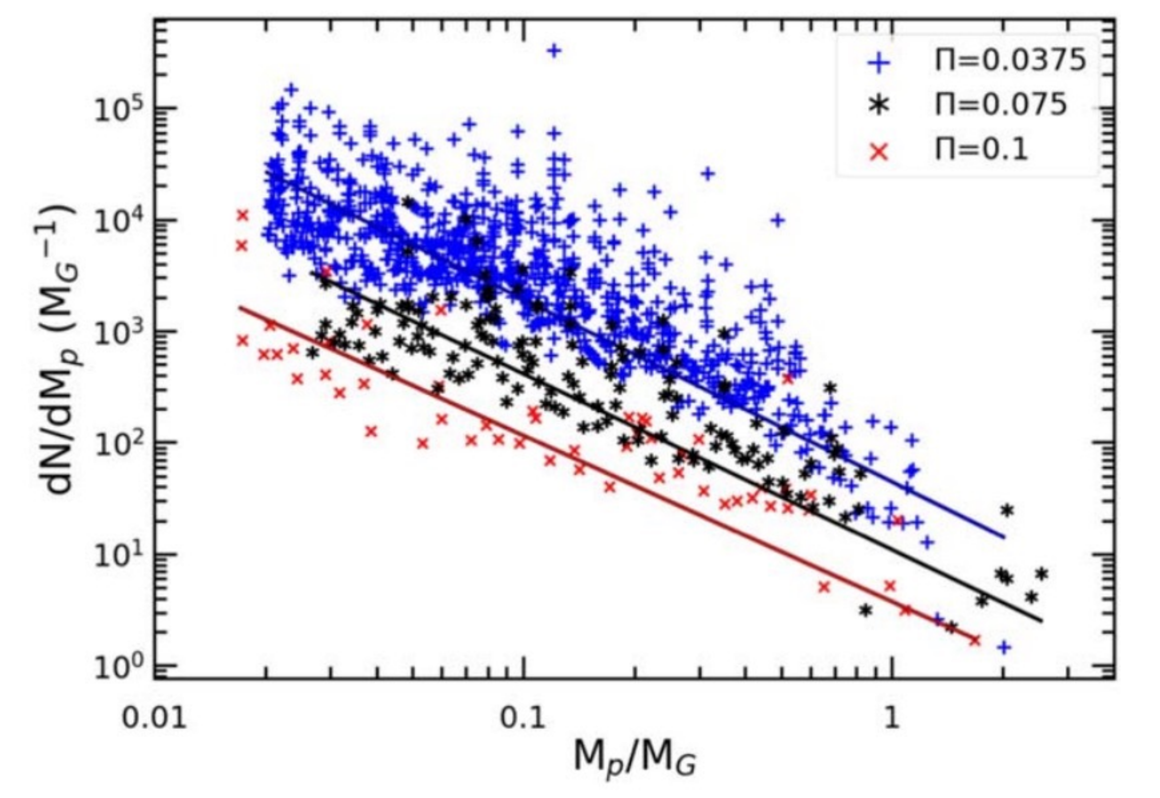
SI with external turbulence: parameter survey



External turbulence significantly increases the critical value z_{crit}

Lim et al. 2024

Initial mass function of planetesimals



- Power-law distribution $\frac{dN}{dM_p} \propto M_p^{-p}$ with $p \simeq 1.6$
- Not sensitive to pebble size (τ_s) or solid abundance (Z)

Johansen+ 2015; Simon et al. 2017; Abod et al. 2019;

Summary

- Dust particles experience radial drift.
- Dust size growth via coagulation: radial drift and fragmention barrier
- Planetesmial formation
  - Gravitational instability
  - Dust coagulation
  - Streaming instability