Grid Based Method for Gas and Dust Dynamics

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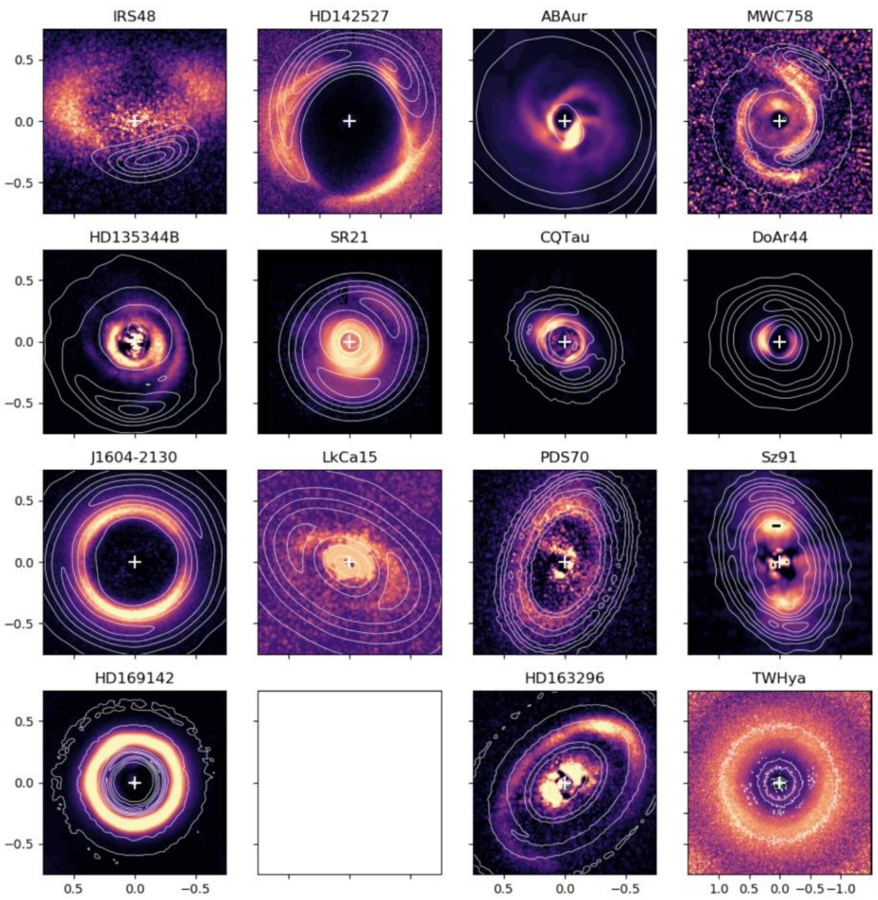
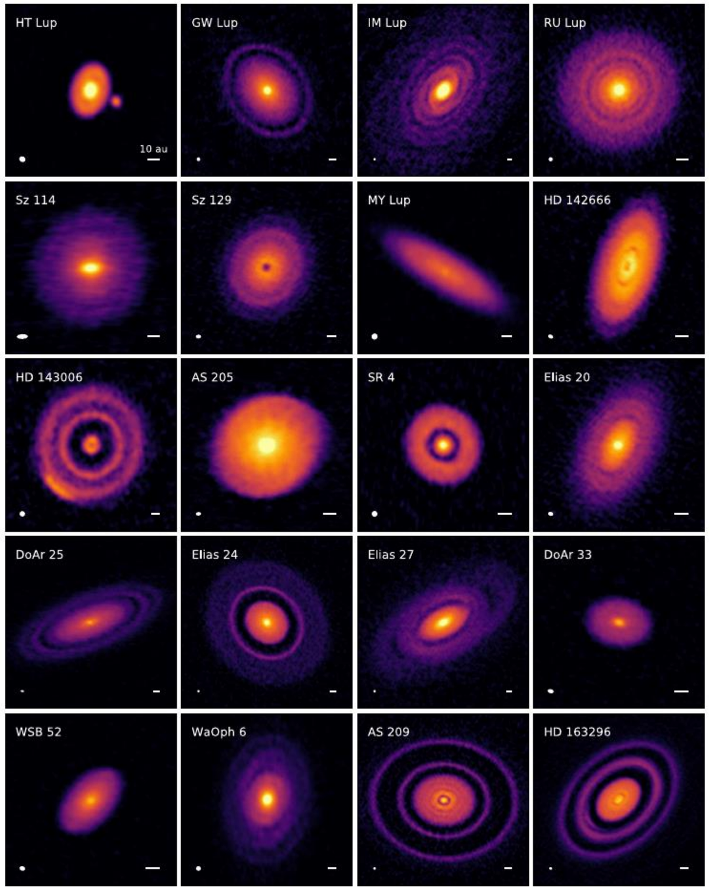
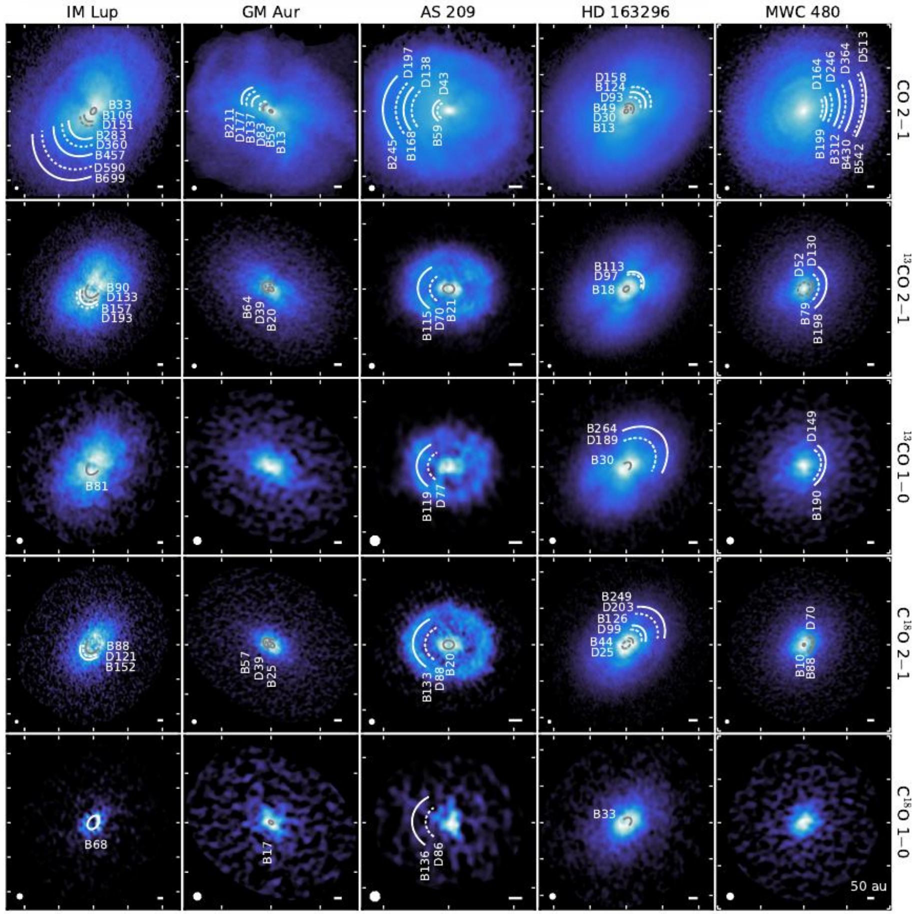
Observational Tracers & Substructures

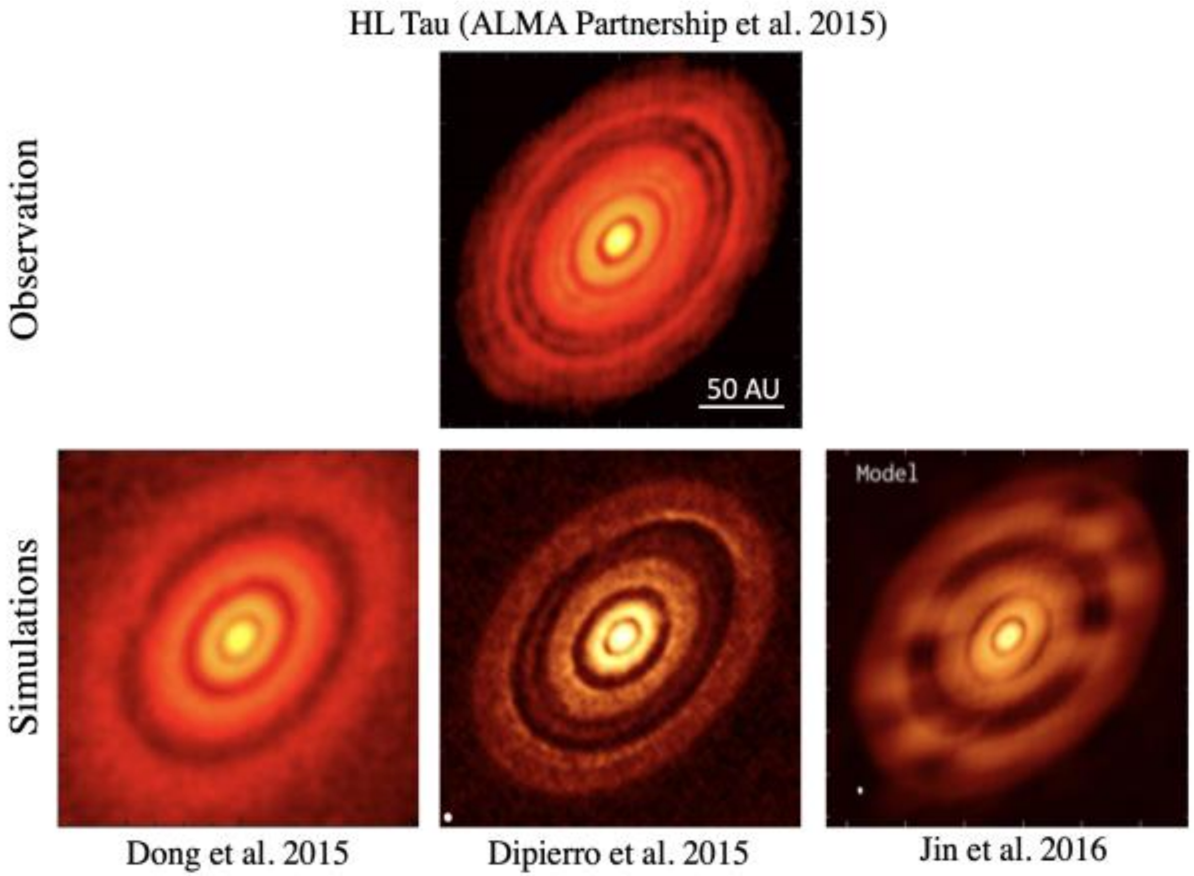


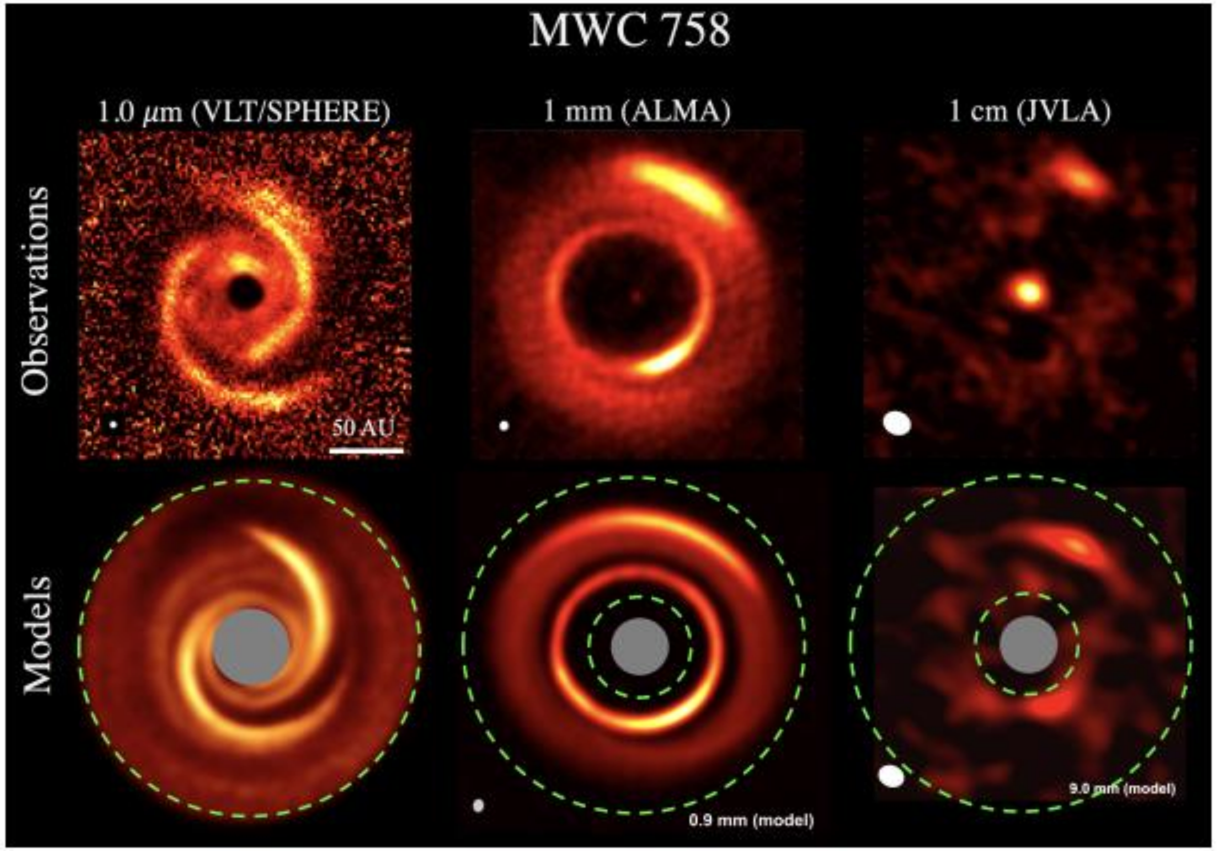




van der Marel et al. 2020; Andrews & DSHARP 2018; Öberg & MAPS 2021

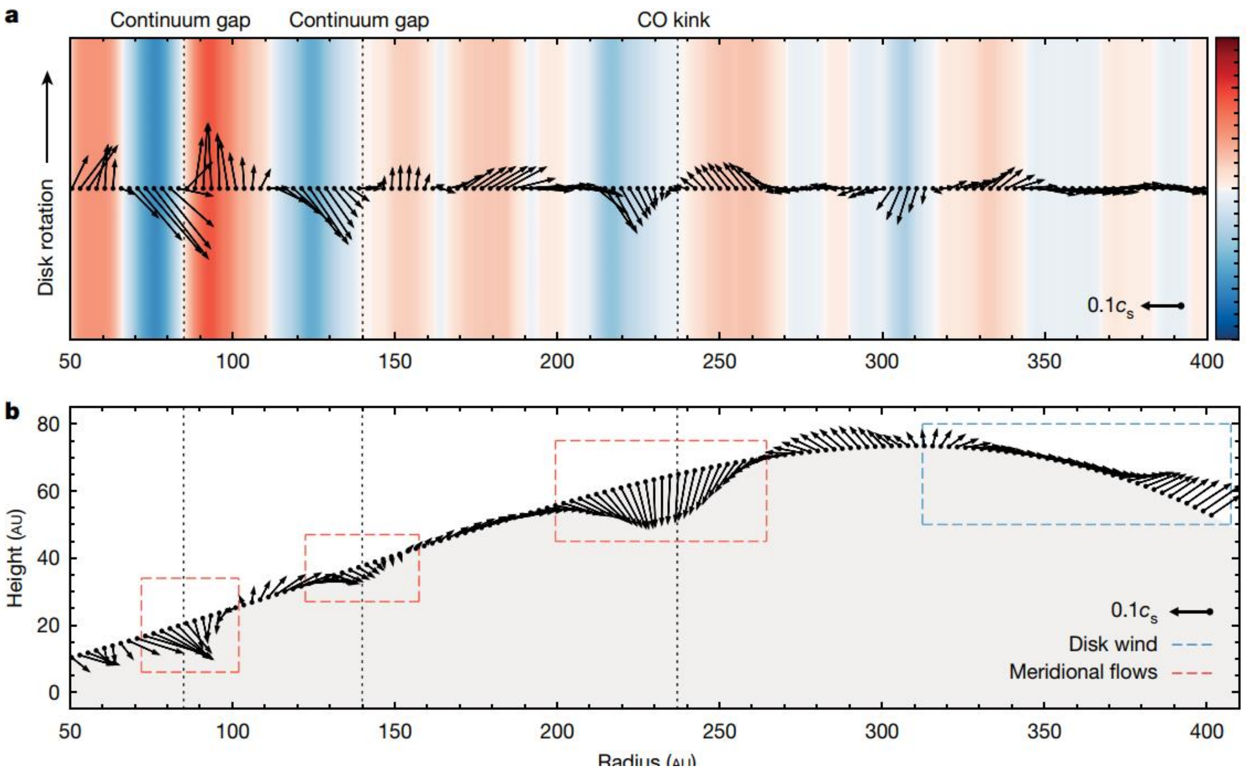
Observation & Simulation

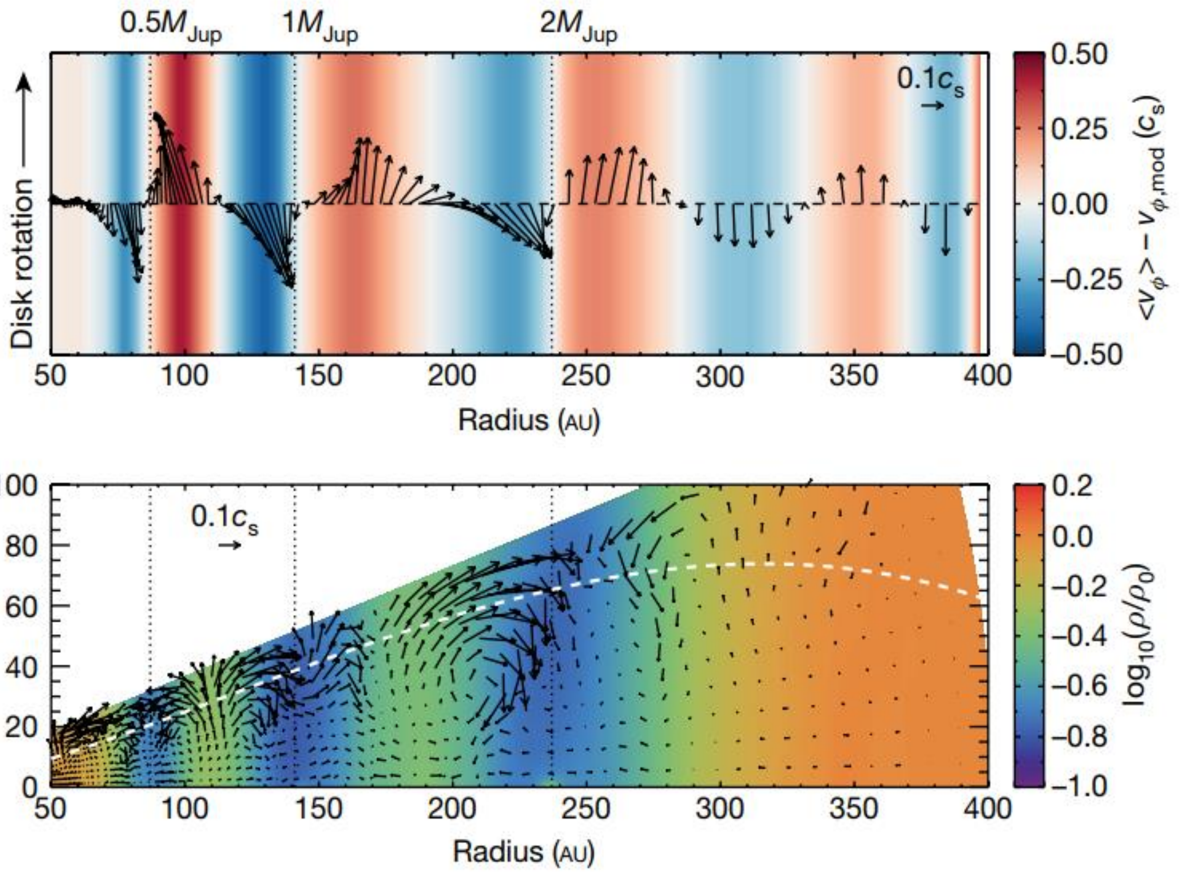




Observational dust images and hydrodynamic simulations of HL Tau (Left) and MWC758 (right)

Observation & Simulation





Teague et al. 2019 combined the ALMA observational ^{12}CO gas line emission (left) and 3D hydrodynamic simulation (right) to confirm the existence of **meridional flows** of HD 163296.

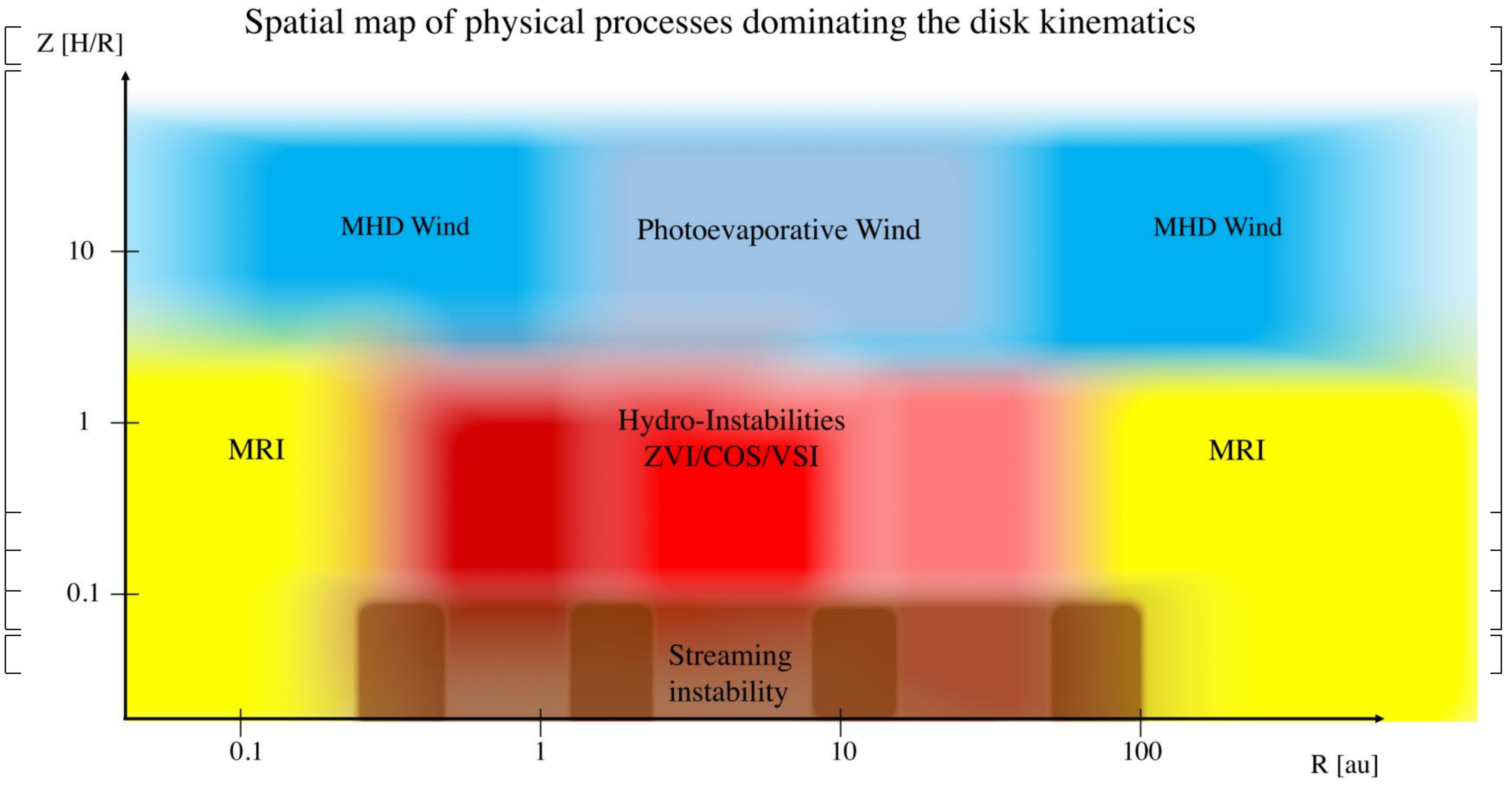
Three Fundamental Problems in Protoplanetary Disks

What are the mechanisms of accretion and dispersal; how is angular momentum transferred in protoplanetary disks?

Instabilities (Turbulence)
Magnetorotational Instability
Gravitational Instability
Hydrodynamic Instabilities
(Zombie Vortex Instability,
Convective Overstability,
Vertical Shear Instability)

Disk Winds
Photoevapation, Magneto-Centrifugal/Thermal Wind

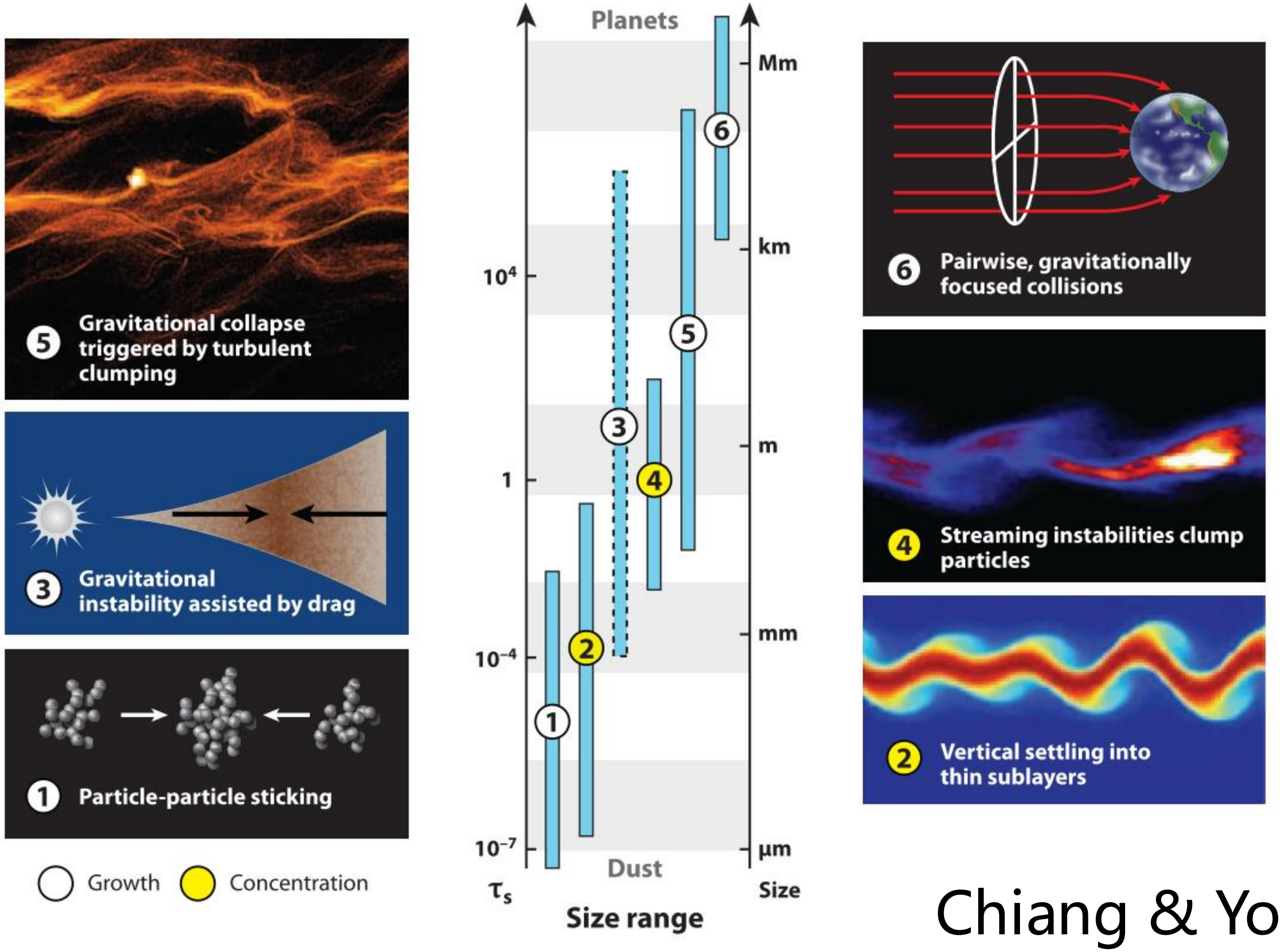
Density Waves
Planets, Brown Dwarfs, Binaries, Vortices, Gravitational Instability



Lesur et al. 2022, PPVII chapter

Three Fundamental Problems in Protoplanetary Disks

How is dust accumulated and aggregated; how do the formations of planetsimals and planets start from dust?



Chiang & Youdin 2010

From dust to pebble

Dust Coagulation, Snow Lines, Pressure Maxima
(planetary gap edges, dead zones edges, zonal flows, vortices, density waves)

From pebble to planetesimals

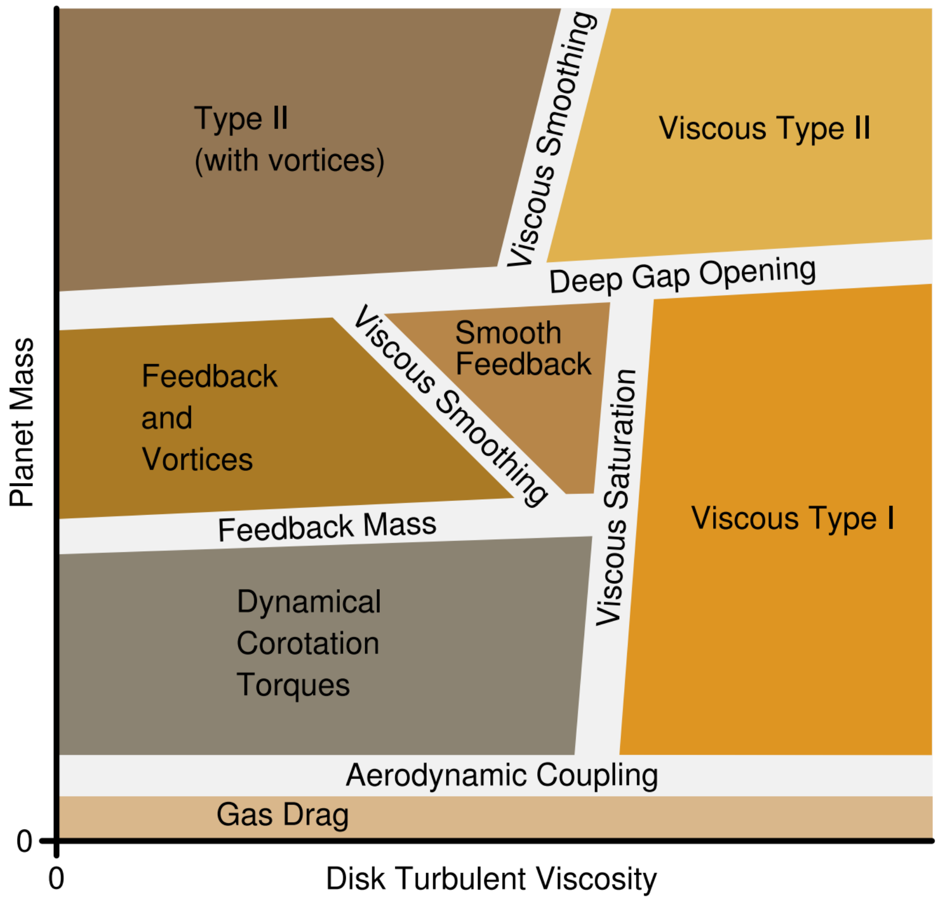
Streaming Instability, Vertical/Self Gravity

From planetesimals to planets

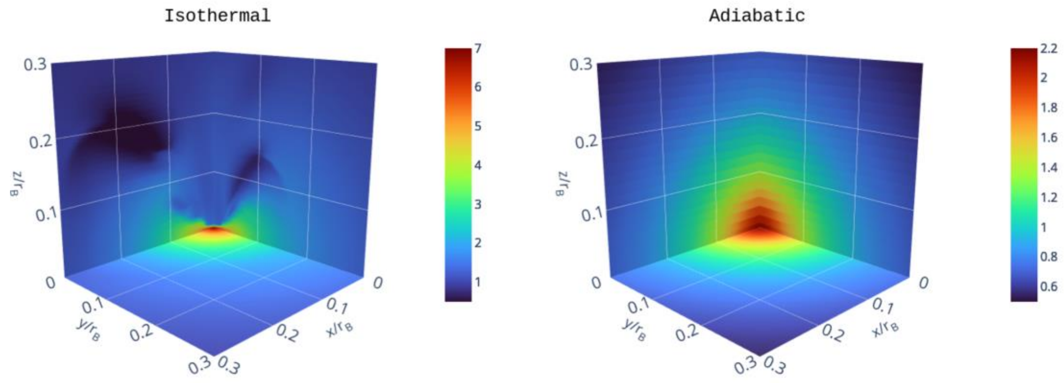
Planetesimal/Pebble Accretion, Core Accretion

Three Fundamental Problems in Protoplanetary Disks

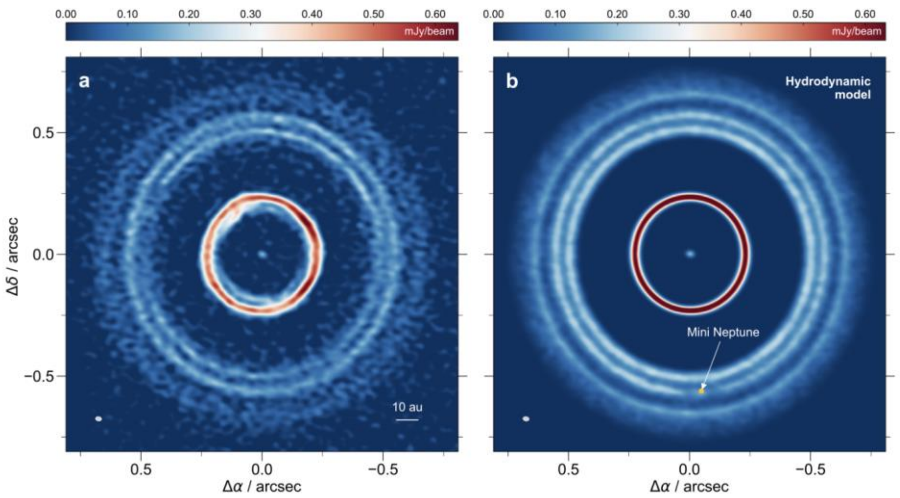
How do planets interact with disk (gas and dust) and other planets to end up in their final configurations?



Planet Migration (McNally et al. 2019a)



Circumplanetary Disks (Fung et al. 2019)



Substructures
(HD 169142,
Pérez et al. 2019)

Numerical Methods to Study Dust-Gas Mixture

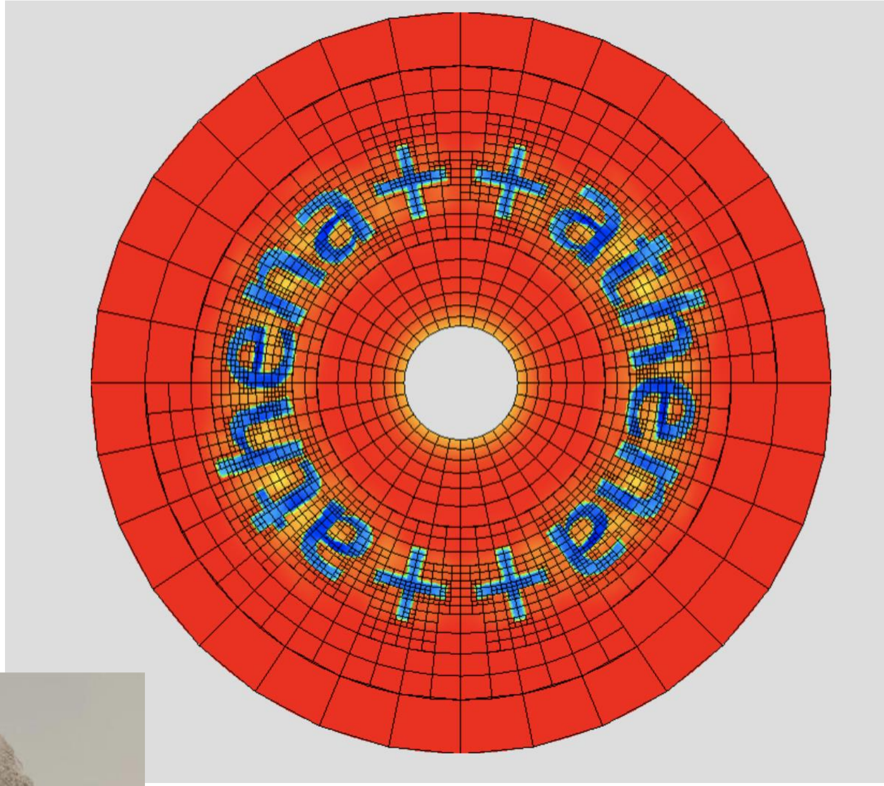
Particles-Mesh Method (Pencil; FLASH; PLUTO; FARGO-ADSG; Athena; **Athena++, Chao-Chin Yang in prep.**)

- Catches the dynamics of dust more rigorously, particles can cross each other
- Code design and load unbalancing are more complex
- Challenging to implement two-way gas drag in a fully implicit manner, and to incorporate dust coagulation

Multi-Fluids Approach (PIERNIK; MPI-AMRVAC; LA-COMPASS; FARGO3D; **Athena++, Huang & Bai, ApJS 2022**)

- Applicable primarily for small and intermediate sized dust (tightly and marginally coupled dust), but will fail for crossing trajectories
- Easier for parallelization and load balancing
- Straightforward to be implemented fully implicit methods and dust coagulation
- Flexible sub-grid physics: can treat gas turbulence as dust diffusion when needed

Athena++



Athena++ is a complete re-write of the Athena astrophysical MHD code in C++ (Stone et al. 2020). It is a **shock-capturing Godunov code** implemented by **Finite Volume Method (FVM)**.

- **Flexible Mesh Strutures and Coordinates**
- **Including more Capabilities and Physics**
- **Excellent Performance and Scalability**

Athena, an ancient Greek goddess with wisdom, warfare and handicraft.
(This photo was taken in Metropolitan Museum of Art)

The Astrophysical Journal Supplement Series, 249:4 (40pp), 2020 July https://doi.org/10.3847/1538-4365/ab929b

The Athena++ Adaptive Mesh Refinement Framework: Design and Magnetohydrodynamic Solvers

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Equations of Gas and Dust

$4N_d$+5 Equations

Integration
Advection
Diffusion
External Force
Drag (Relaxation)

$$\frac{\partial \rho_g}{\partial t} + \nabla \cdot (\rho_g \boldsymbol{v}_g) = 0 \ ,$$

$$\frac{\partial (\rho_g \boldsymbol{v}_g)}{\partial t} + \nabla \cdot (\rho_g \boldsymbol{v}_g \boldsymbol{v}_g + P_g \boldsymbol{I} + \boldsymbol{\Pi}_\nu) = \rho_g \boldsymbol{f}_{g,src} + \sum_{n=1}^{N_d} \rho_{d,n} \frac{\boldsymbol{v}_{d,n} - \boldsymbol{v}_g}{T_{s,n}} \ ,$$

$$\frac{\partial E_g}{\partial t} + \nabla \cdot [(E_g + P_g) \boldsymbol{v}_g + \boldsymbol{\Pi}_\nu \cdot \boldsymbol{v}_g] = \rho_g \boldsymbol{f}_{g,src} \cdot \boldsymbol{v}_g + \sum_{n=1}^{N_d} \rho_{d,n} \frac{\boldsymbol{v}_{d,n} - \boldsymbol{v}_g}{T_{s,n}} \cdot \boldsymbol{v}_g + \omega \sum_{n=1}^{N_d} \rho_{d,n} \frac{(\boldsymbol{v}_{d,n} - \boldsymbol{v}_g)^2}{T_{s,n}} \ ,$$

$$\frac{\partial \rho_{d,n}}{\partial t} + \nabla \cdot (\rho_{d,n} \boldsymbol{v}_{d,n} + \mathscr{F}_{dif,n}) = 0 \ ,$$

$$\frac{\partial \rho_{d,n} (\boldsymbol{v}_{d,n} + \boldsymbol{v}_{d,dif,n})}{\partial t} + \nabla \cdot (\rho_{d,n} \boldsymbol{v}_{d,n} \boldsymbol{v}_{d,n} + \boldsymbol{\Pi}_{dif,n}) = \rho_{d,n} \boldsymbol{f}_{d,src,n} + \rho_{d,n} \frac{\boldsymbol{v}_g - \boldsymbol{v}_{d,n}}{T_{s,n}} \ .$$

Aerodynamic Drags and Stiff Regimes

The aerodynamic drag force (**velocity relaxation**) is:

$$\dot{v}(t) \propto \frac{\rho_d}{\rho_g} \frac{v_g - v_d}{T_s}$$

The analytical solution is:

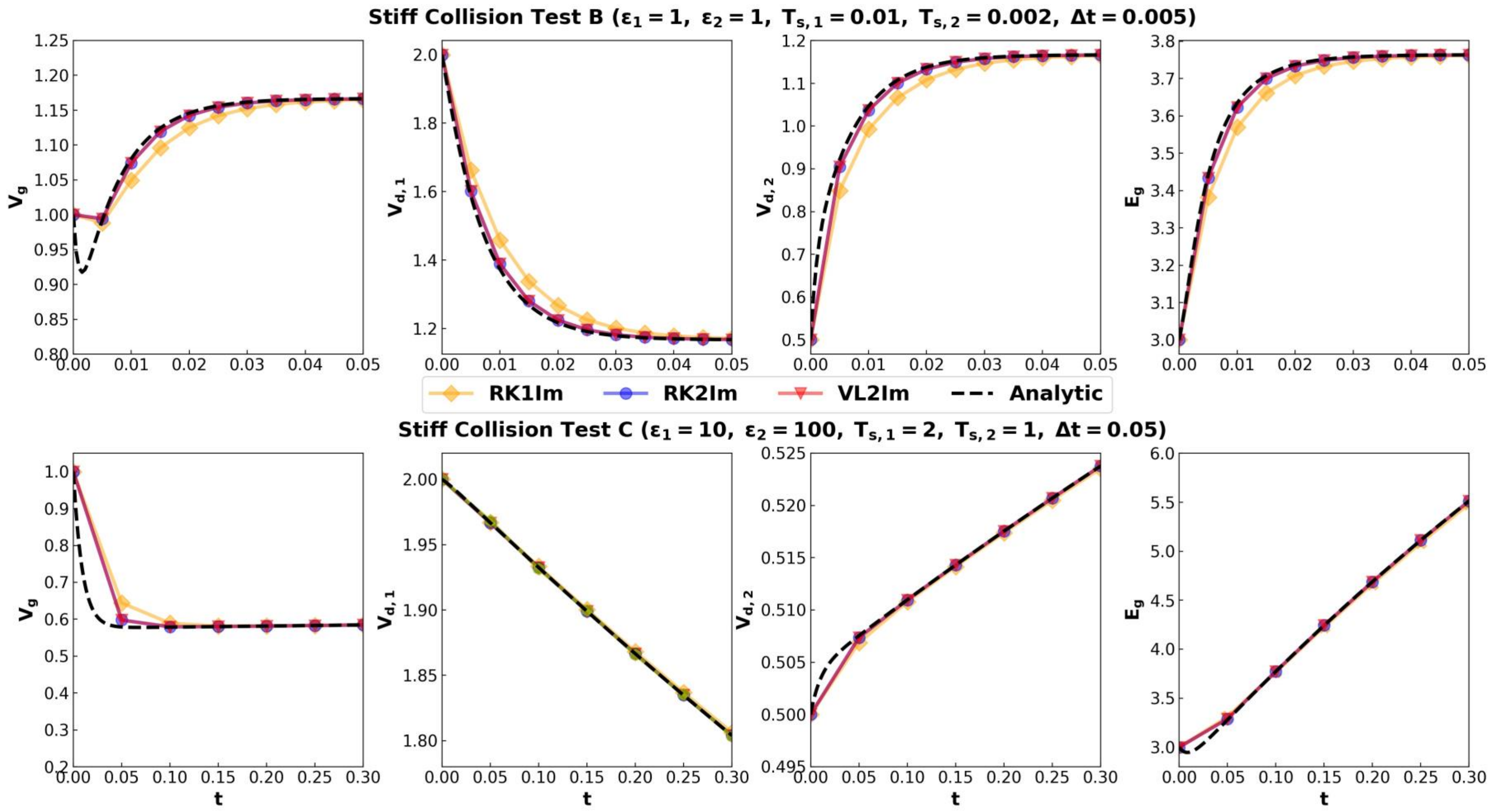
$$v(t) \propto exp(\lambda\, t)$$

The **stiffness** is caused by a Jacobian that possesses **eigenvalues λ with large negative real parts**. The solution has eigen modes varied with time dramatically.

$$|\Re e(\lambda)|\Delta t \geq 1$$

As a rule of thumb, the **drag will become stiff** when you have a small stopping time $T_s \lesssim \Delta t$ and a large dust-to-gas ratio $\rho_d/\rho_g \gg 1$.

Stiff Drags and Drag Integrators



Explicit drag integrators are **unstable** to the stiff drags.
Semi-implicit drag integrators create **artificial oscillations**.
Only **fully implicit** drag integrators are **robust** in stiff regimes.

Momentum Correction of Dust Diffusion

$$\mathscr{F}_{\mathrm{dif},n} \equiv -\rho_{\mathrm{g}} D_{\mathrm{d},n} \nabla \left(\frac{\rho_{\mathrm{d},n}}{\rho_{\mathrm{g}}} \right) = \rho_{\mathrm{d}} \boldsymbol{v}_{\mathrm{d,dif},n} \ ,$$

(Cuzzi et al. 1993)

The gas turbulent term on the dust continuity equation will **violate the Galilean invariance** of the system.

We add three terms on the momentum equations of dust:

$$\frac{\partial \rho_{\mathrm{d},n} (\boldsymbol{v}_{\mathrm{d},n} + \boldsymbol{v}_{\mathrm{d,dif},n})}{\partial t} + \nabla \cdot (\rho_{\mathrm{d},n} \boldsymbol{v}_{\mathrm{d},n} \boldsymbol{v}_{\mathrm{d},n} + \boldsymbol{\Pi}_{\mathrm{dif},n}) = \rho_{\mathrm{d},n} \boldsymbol{f}_{\mathrm{d,src},n} + \rho_{\mathrm{d},n} \frac{\boldsymbol{v}_{\mathrm{g}} - \boldsymbol{v}_{\mathrm{d},n}}{T_{\mathrm{s},n}} \ .$$

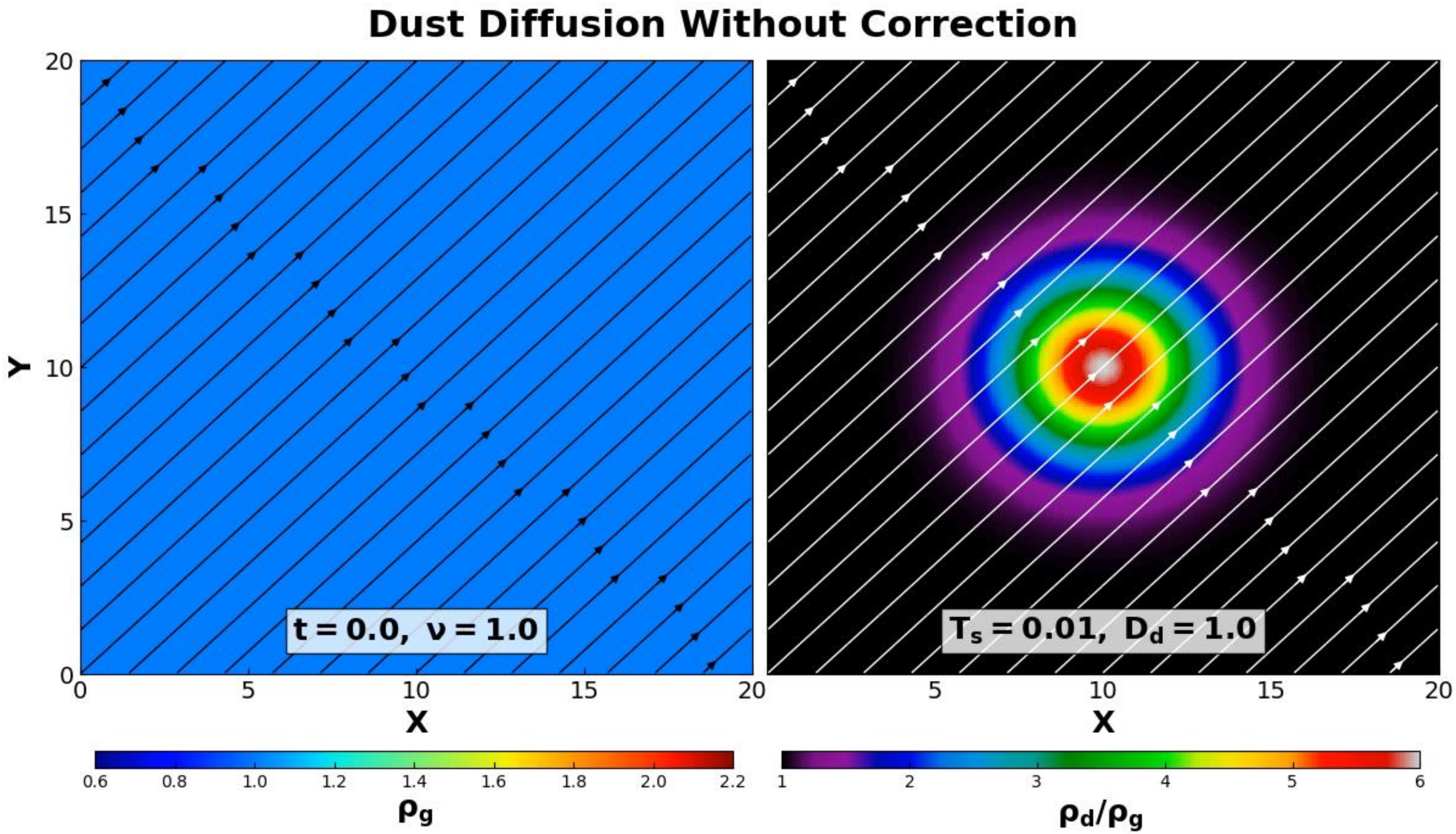
Diffusive Velocity

$$\Pi_{\mathrm{dif},n,ij} = v_{\mathrm{d},n,j} \mathscr{F}_{\mathrm{dif},n,i} + v_{\mathrm{d},n,i} \mathscr{F}_{\mathrm{dif},n,j} \ .$$

Diffusion of i-momentum in j-direction

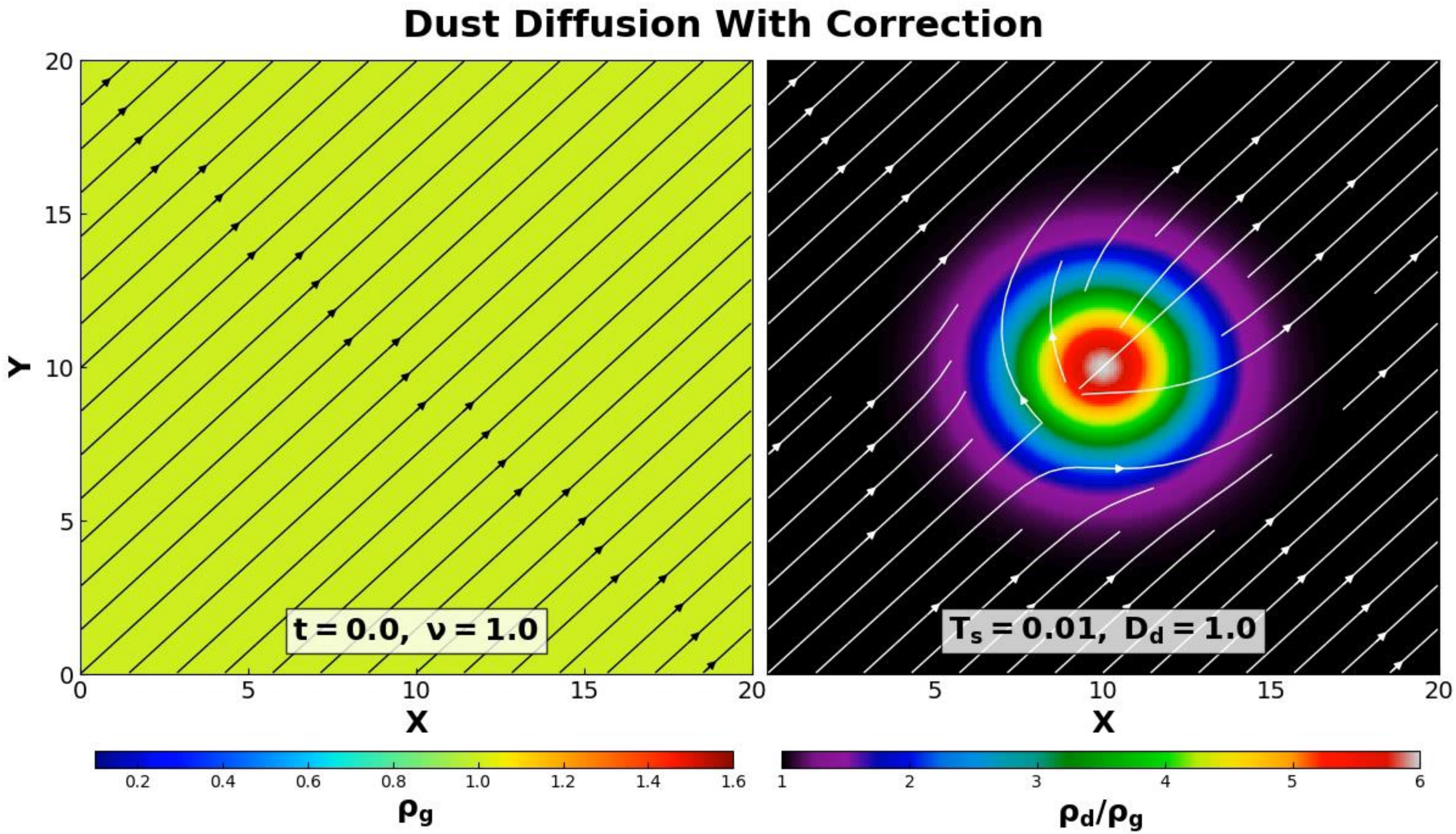
Advection of i-diffusive flux in j-direction

2D Gaussian Dust Diffusion



Without momentum correction, it creates **an asymmetric numerical artifact** in gas density. The **dust-gas system violates the Galilean invariance**.

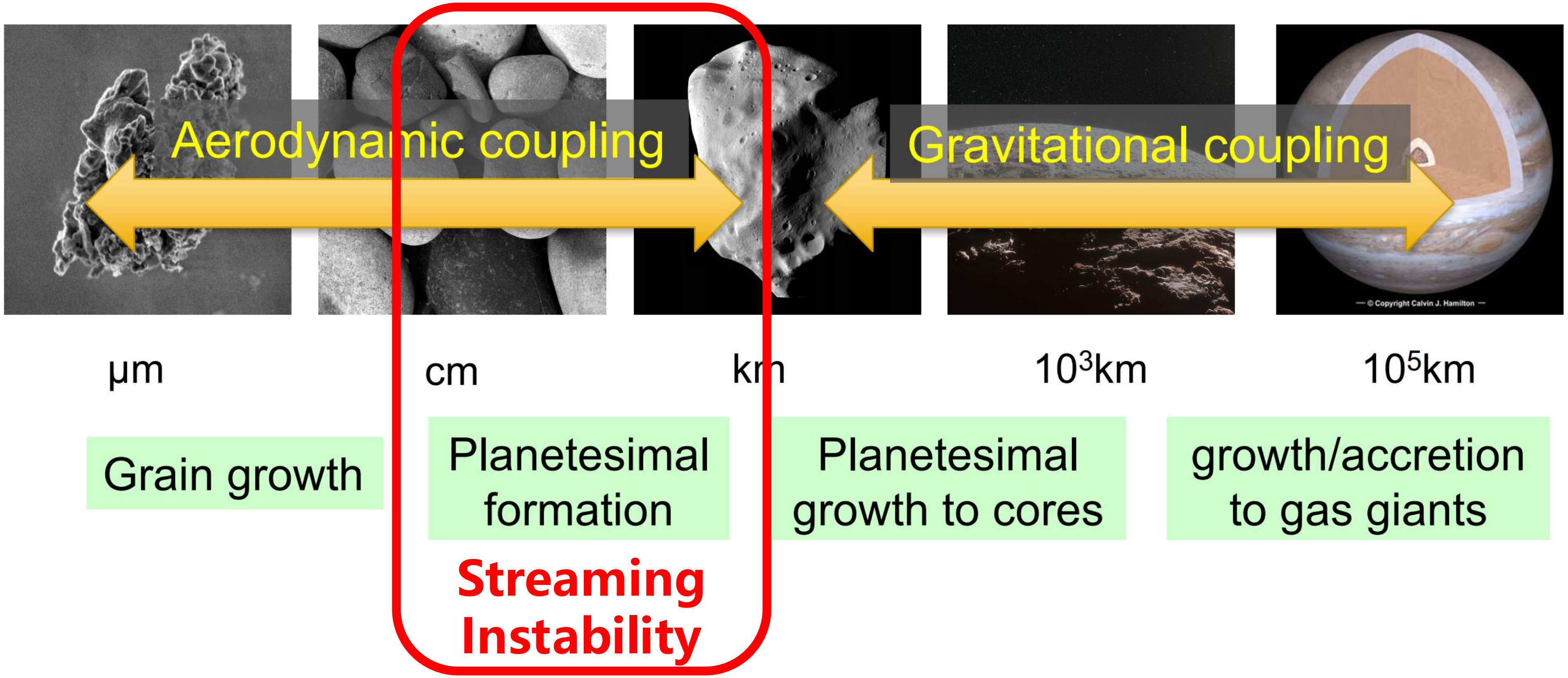
2D Gaussian Dust Diffusion



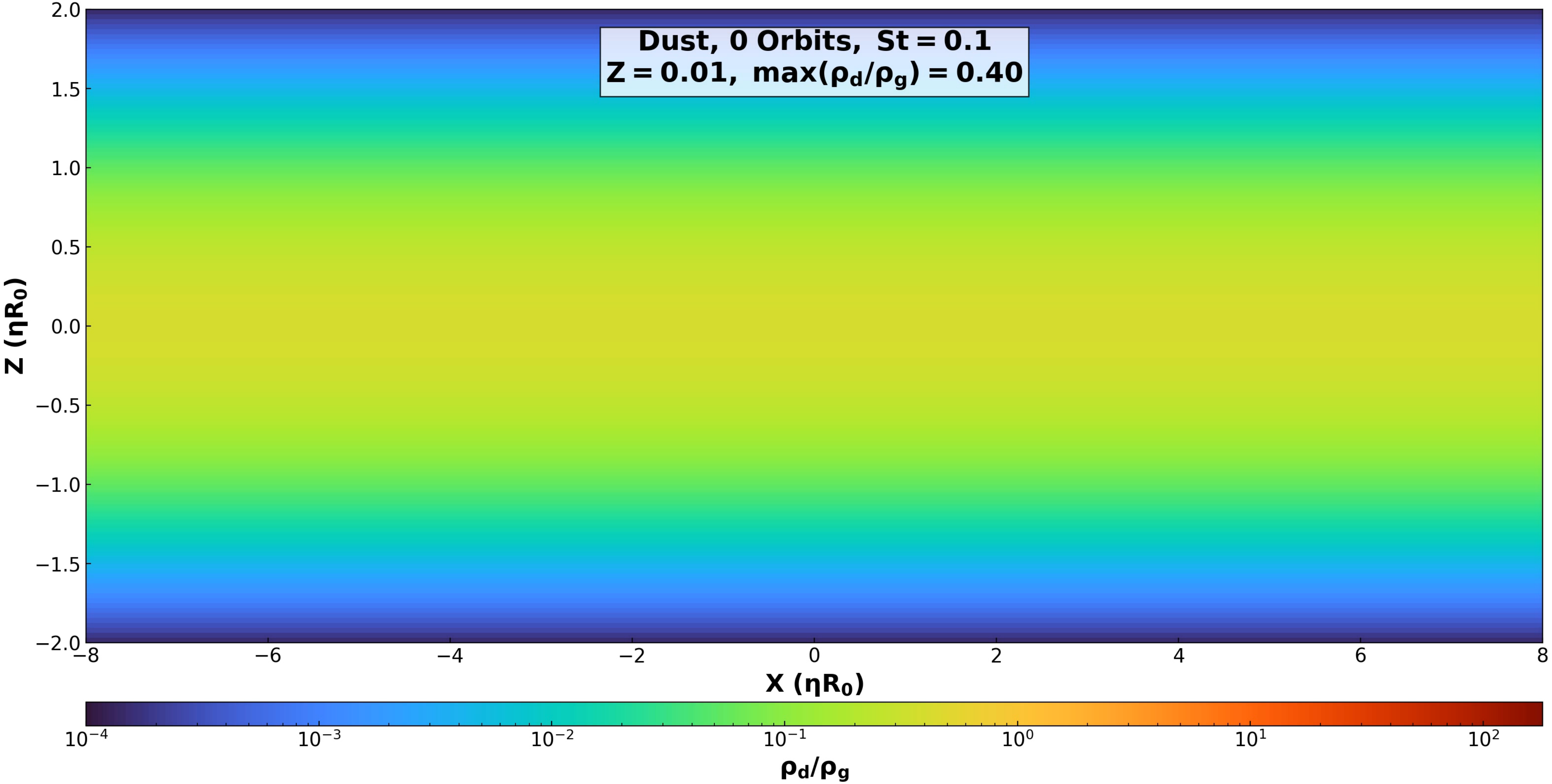
With momentum correction, the circular gas and dust density patterns are well preserved. Dust diffusion **does not depend on the choice of reference frame**, so **it ensures the Galilean invariance**.

Streaming Instability (SI)

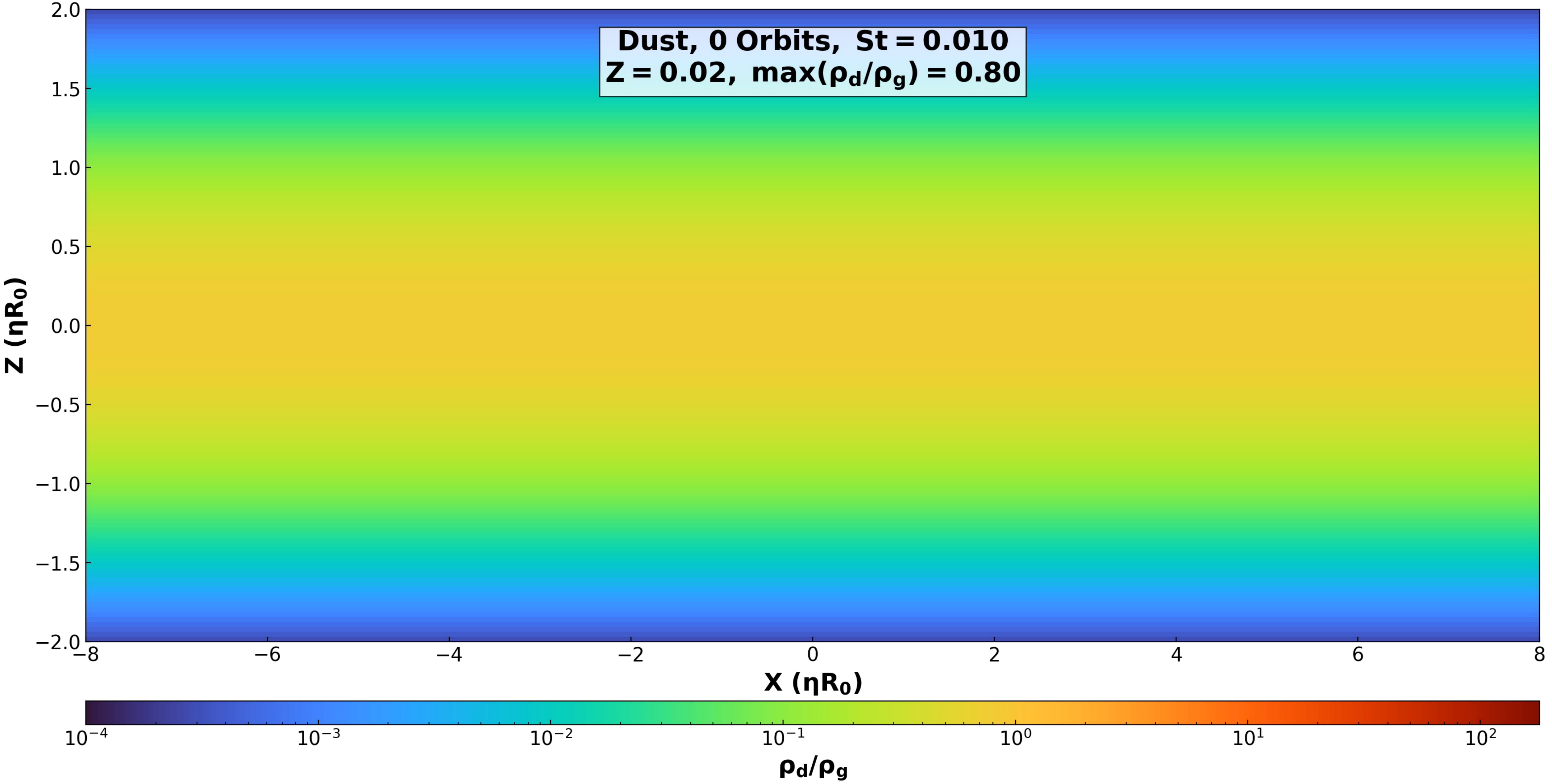
Streaming Instability is **an axisymmetric dust-gas resonance drag instability**. It creates **strong dust concentration** and is tightly related to the **planetesimal formation** (Youdin & Johansen; Johansen & Youdin 2007). The free energy of SI comes from the **radial gas pressure gradient** $\eta \equiv \frac{1}{2}\left(\frac{c_s}{v_K}\right)^2 \frac{d \ln P}{d \ln r}$.



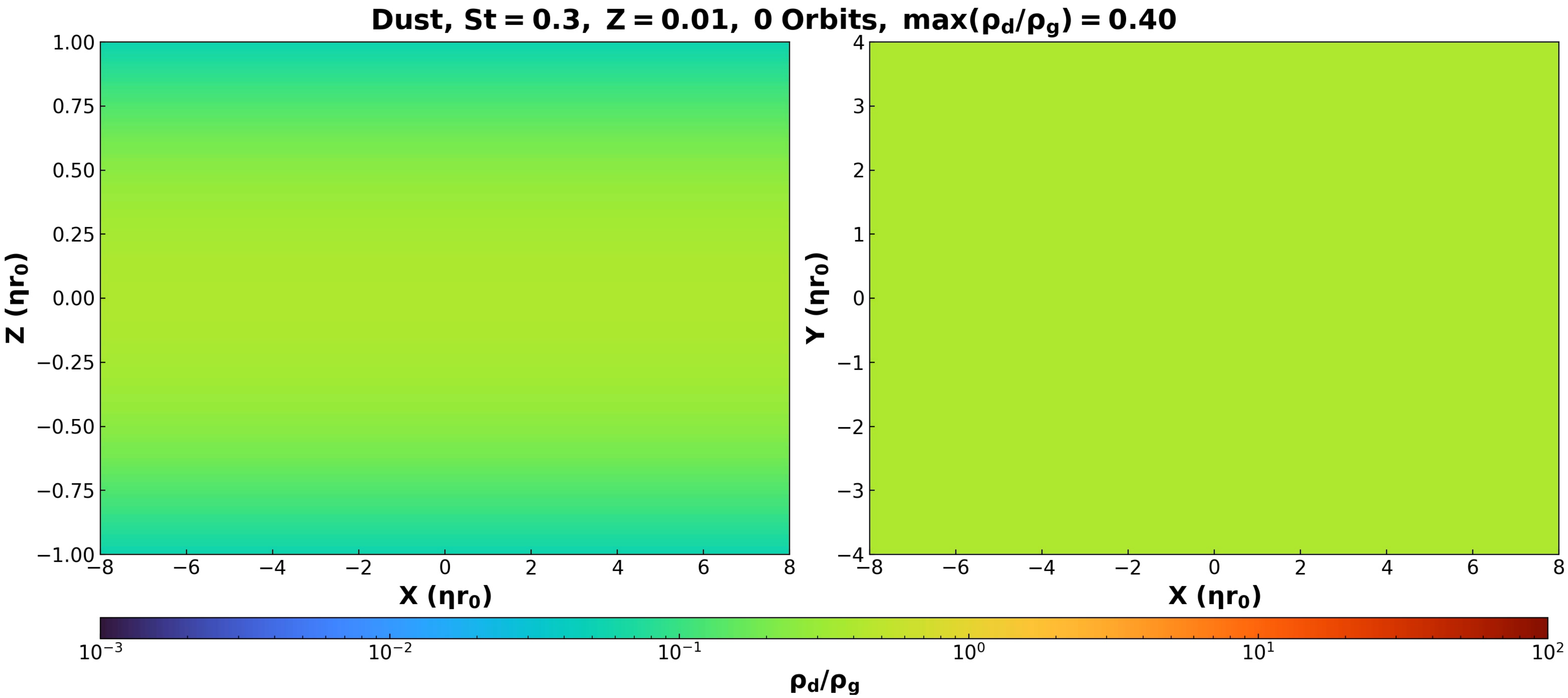
SI and Dust Clumping



SI and Dust Clumping



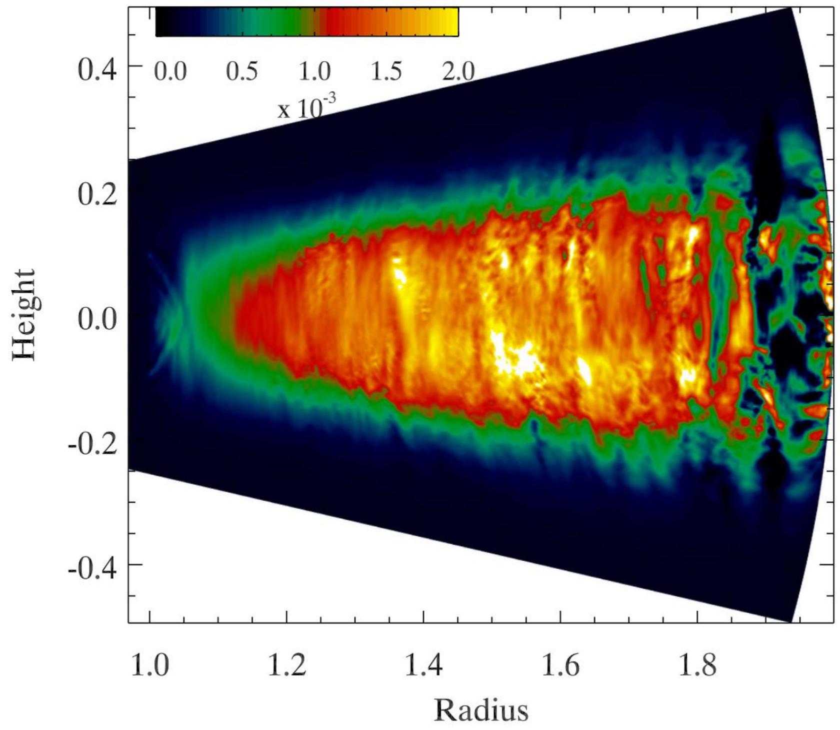
SI and Dust Clumping



Vertical Shear Instability (VSI)

In hydrostatic equilibrium, protoplanetary disks generally have **vertical shear in rotation velocity** $\frac{\partial \Omega}{\partial z} \neq 0$, it can trigger **Vertical Shear Instability** (with **short thermal cooling,** $\tau_{cool} \ll \Omega_K^{-1}$). VSI is a disk variant of the **GSF Instability** in differentially rotating stars (Goldreich & Schubert 1967, Fricke 1968).

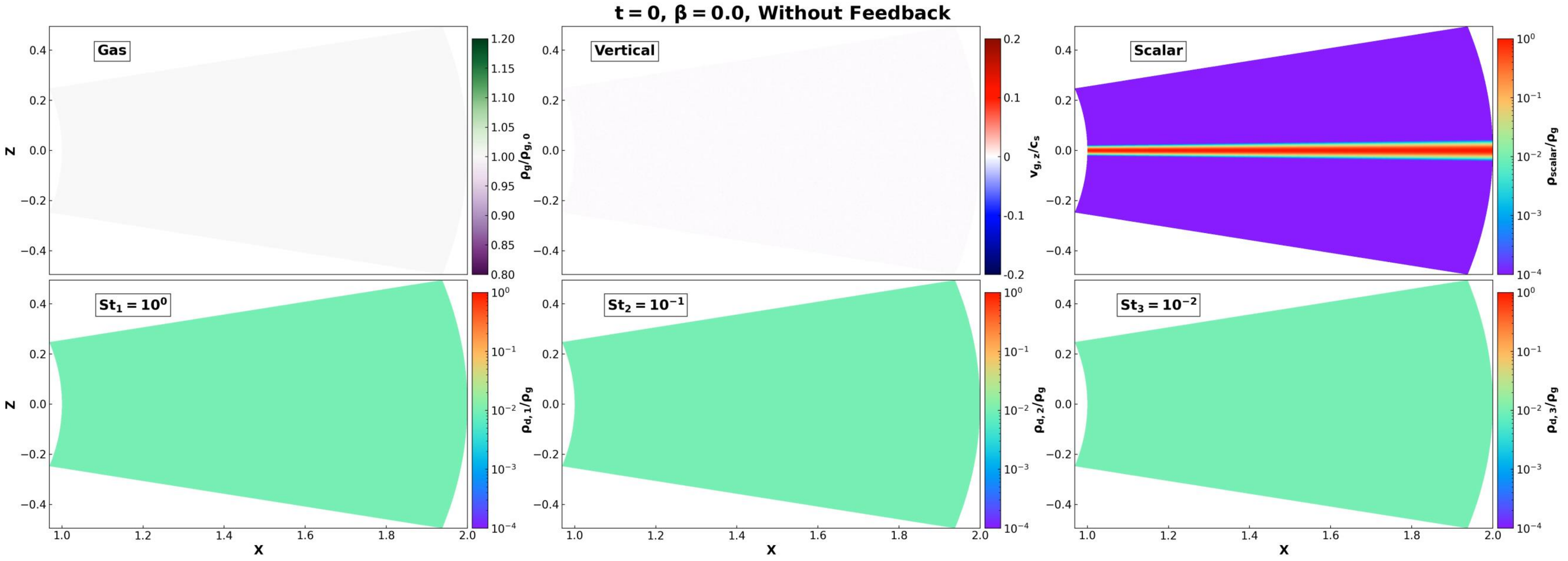
$$\frac{\partial L^2}{\partial R} - \frac{k_R}{k_z}\frac{\partial L^2}{\partial z} < 0 \qquad \text{for } k_R/k_z \gg 1$$



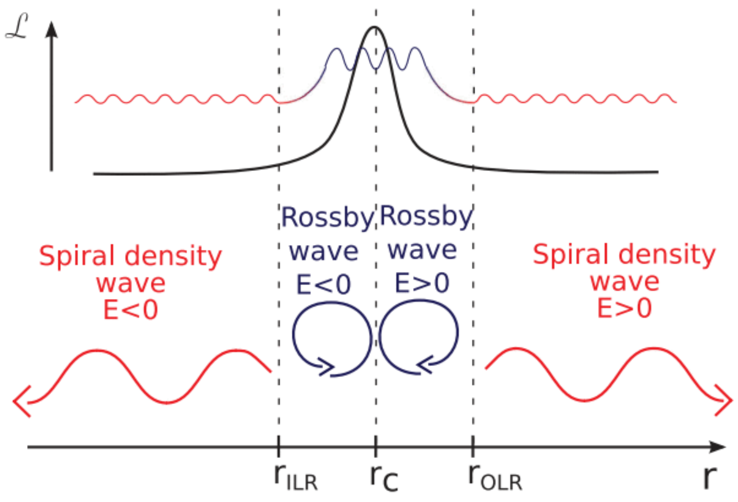
Nelson et al. 2013

- Creates **modest turbulence** to drive the **accretion** of disks.
- Leads to prominent **vertical motion (body modes)** and hence **dust stirring**.
- In 3D, produces **vortices** by secondary instability **(Rossby Wave Instability)**.

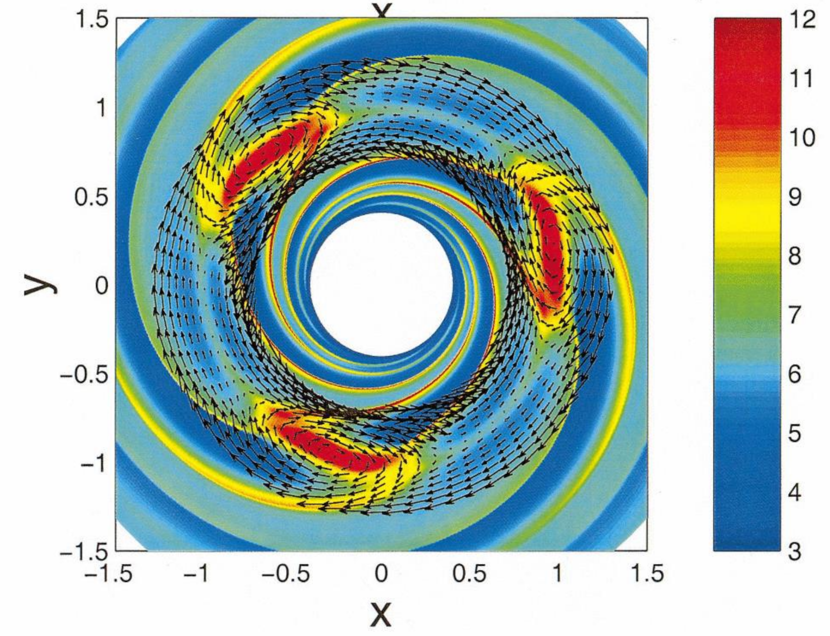
VSI with Dust Stirring



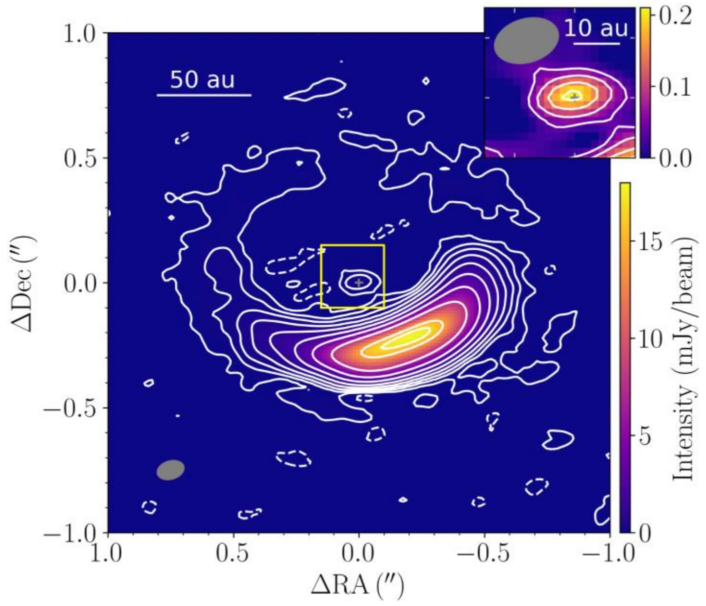
Rossby Wave Instability (RWI)



Leaking Rossby waves excite spiral density waves (Meheut et al. 2013)



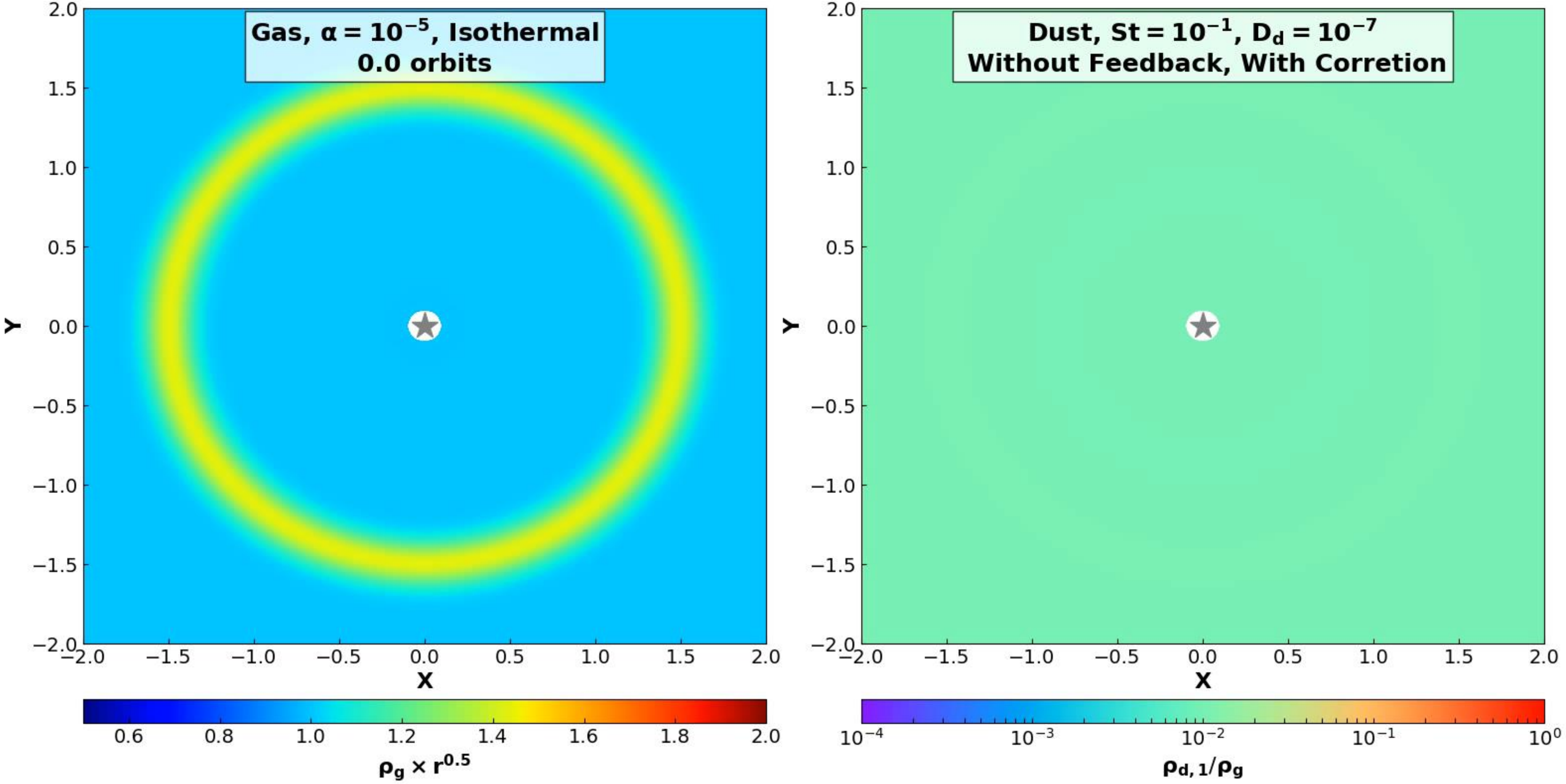
RWI generates large scale vortices (Li et al. 2001)



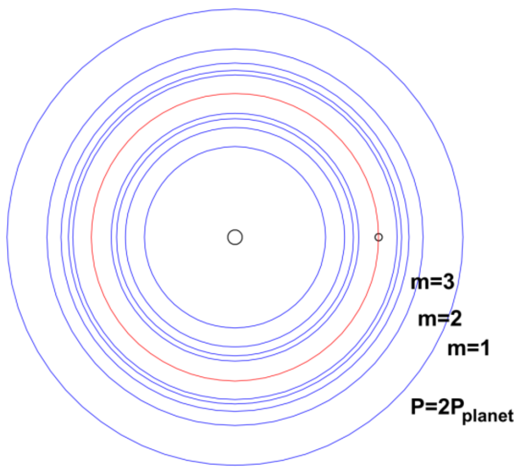
ALMA dust continuum observation of IRS 48 (Yang et al. 2023a)

Rossby Wave Instability is **a local non-axisymmetric instability**, and is tightly related to the nonlinear regimes of VSI and Zombie Vortex Instability (ZVI). RWI happens in disks where there is **a local maximum of $\mathcal{L}(r) = \frac{\Sigma S^{2/\gamma}}{2(\nabla\times u)\cdot\hat{z}}$**, i.e., **a local minimum of vortensity $\frac{(\nabla\times u)\cdot\hat{z}}{\Sigma}$** in barotropic disks. RWI creates **large scale vortices** to trap dust.

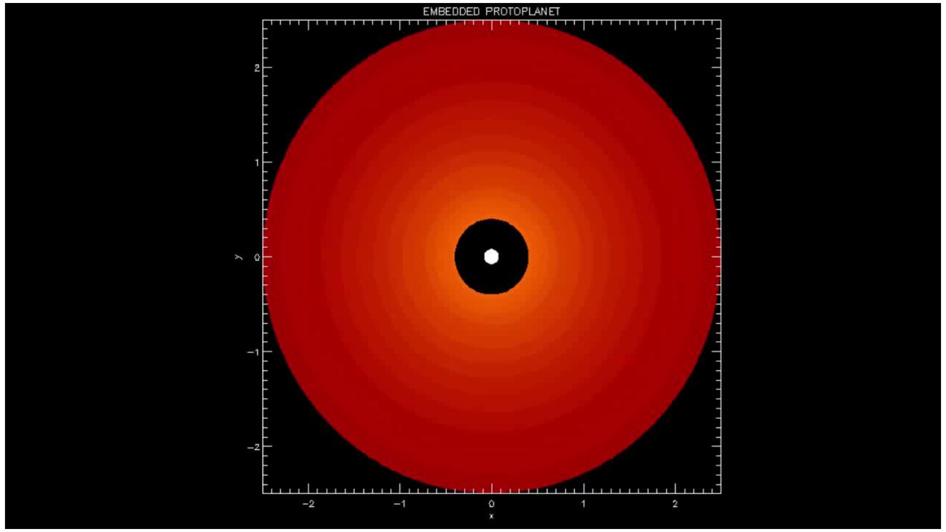
RWI with Dust Trapping



Planet-Disk Interaction

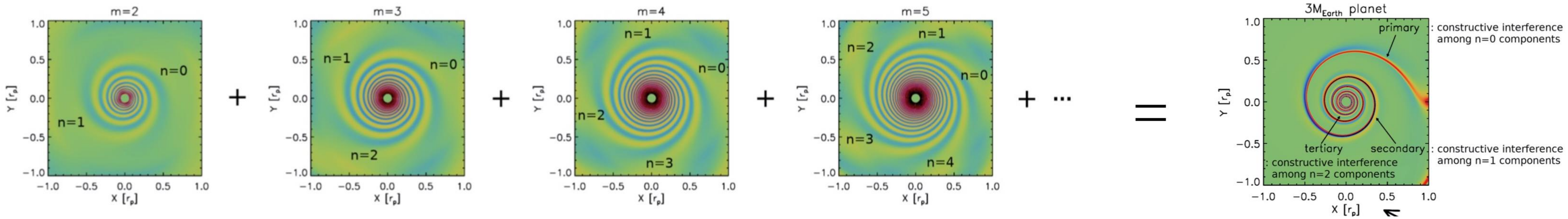


$$r_L = \left(1 \pm \frac{1}{m}\right)^{2/3} r_p$$



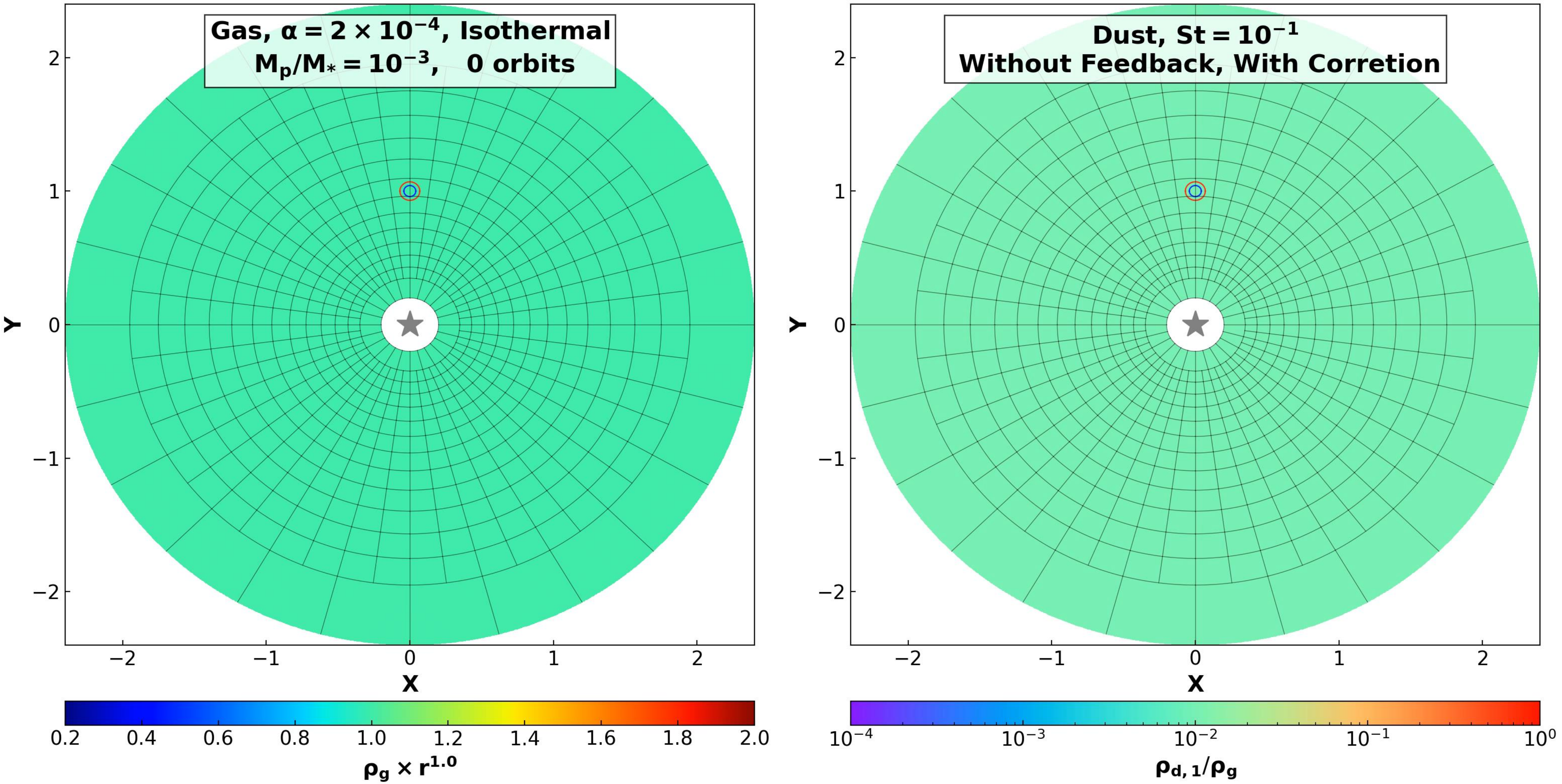
Planet excites **density waves** at **Lindblad Resonance locations** (Armitage 2007).

Asymmetries between inner and outer waves **lead planets to migrate** (F. Messet).



Combine different Fourier azimuthal modes, then we get **coherent spiral density waves induced by planets** (Bae & Zhu 2018).

3D Planet-Disk Interaction with Dust



2D Planet-Disk Interaction with Dust