Planet formation lecture II

Planetesimal accretion & pebble accretion

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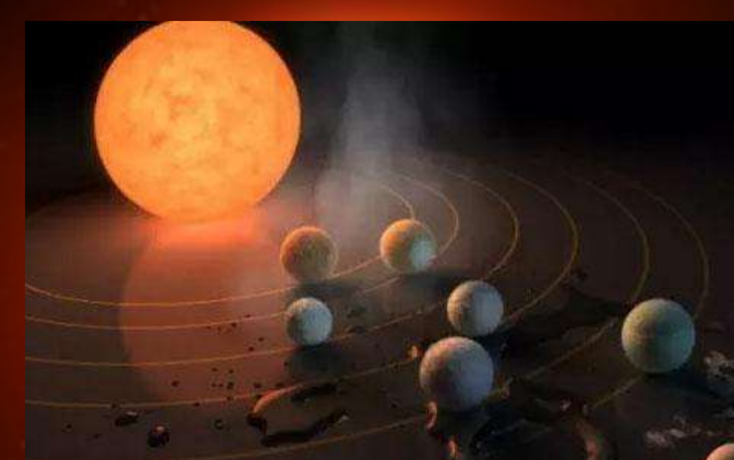


References

1.Lecture notes on the formation and early evolution of planetary system

Armitage Chapter III https://arxiv.org/abs/astro-ph/0701485v6

2. Planet formation theory in the era of ALMA and Kepler

Drazkowska et al. PPVII chapter, 2023

3. A tale of planet formation: from dust to planets

Liu & Ji, 2020

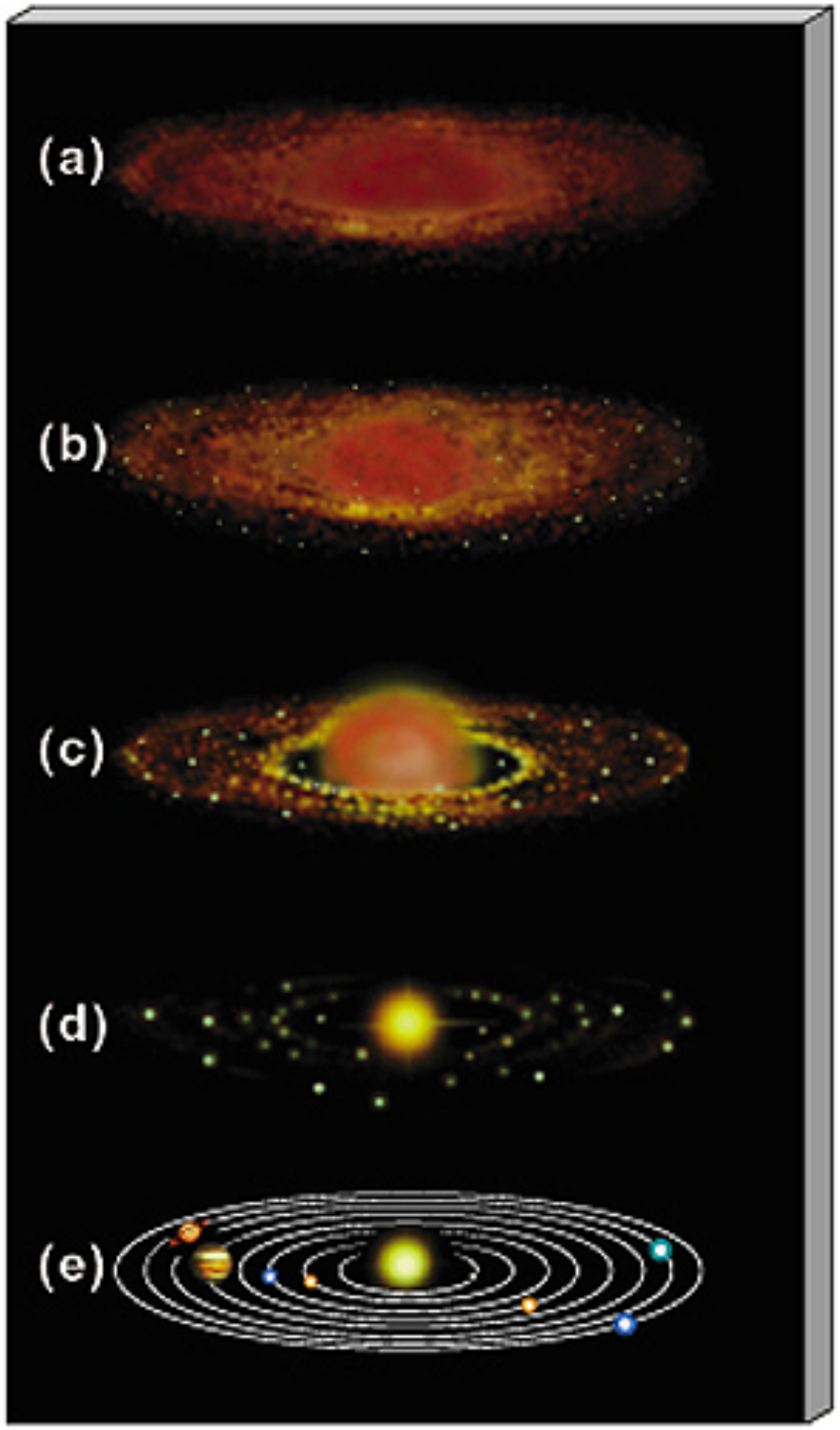
4. Planet formation: key mechanisms and global models

Raymond & Morbidelli, 2020

Outline

1. Introduction
2. Planetesimal accretion & pebble accretion
3. Single birth site + hybrid accretion model

Nebular hypothesis



➡ cloud collapse

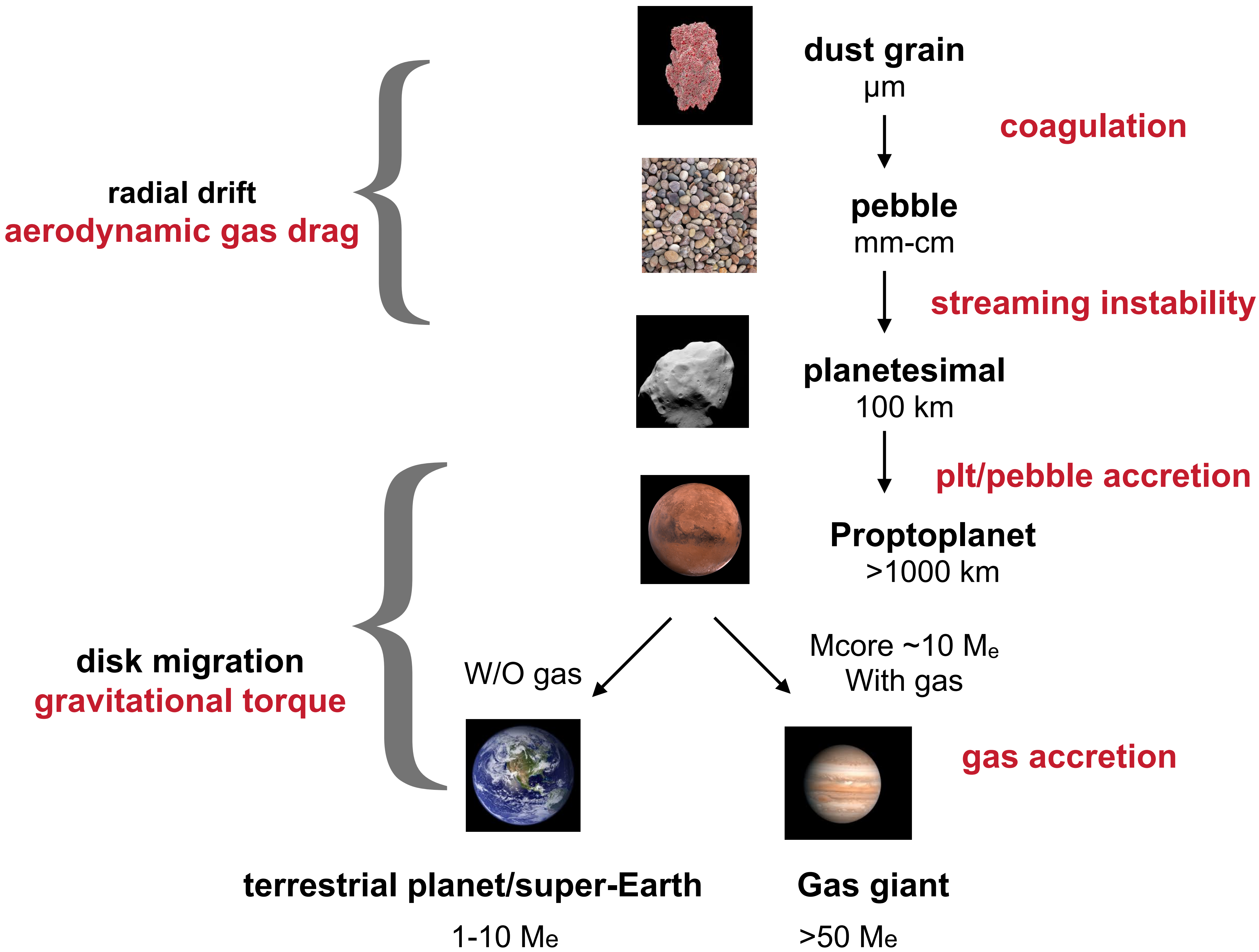
➡ proto-star and protoplanetary disk

➡ dust growth and planet formation

➡ protoplanetary disk dispersal (a few Myr)

➡ final planetary system

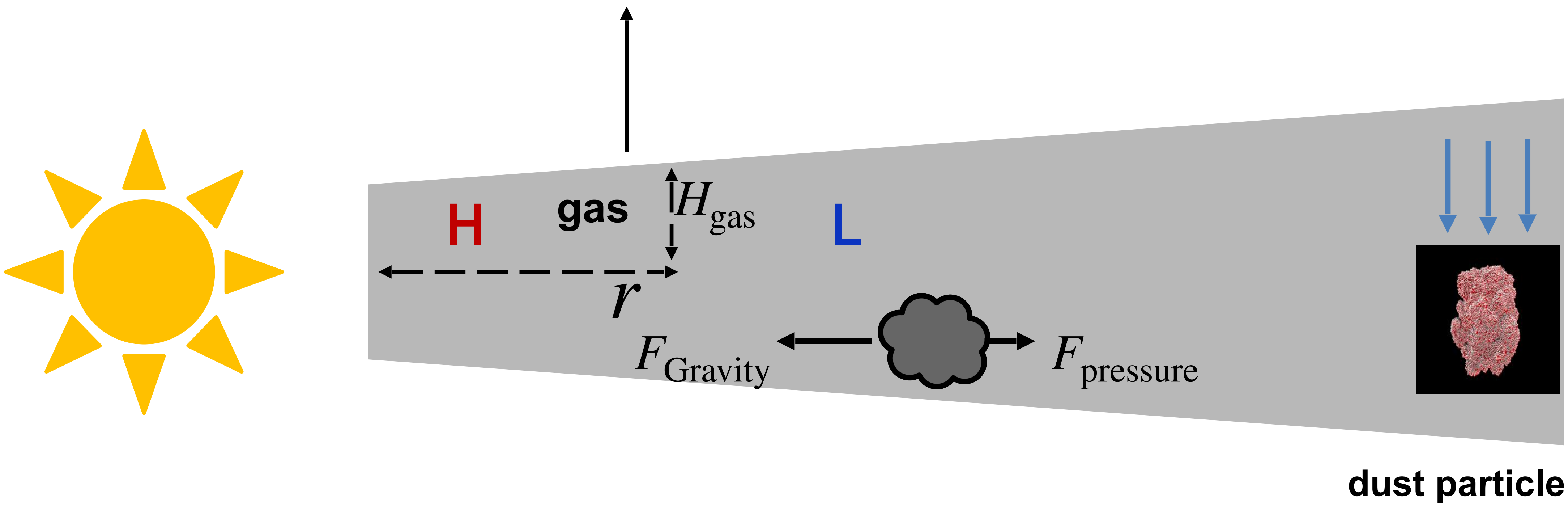
Sketch of planet formation



Radial drift of dust

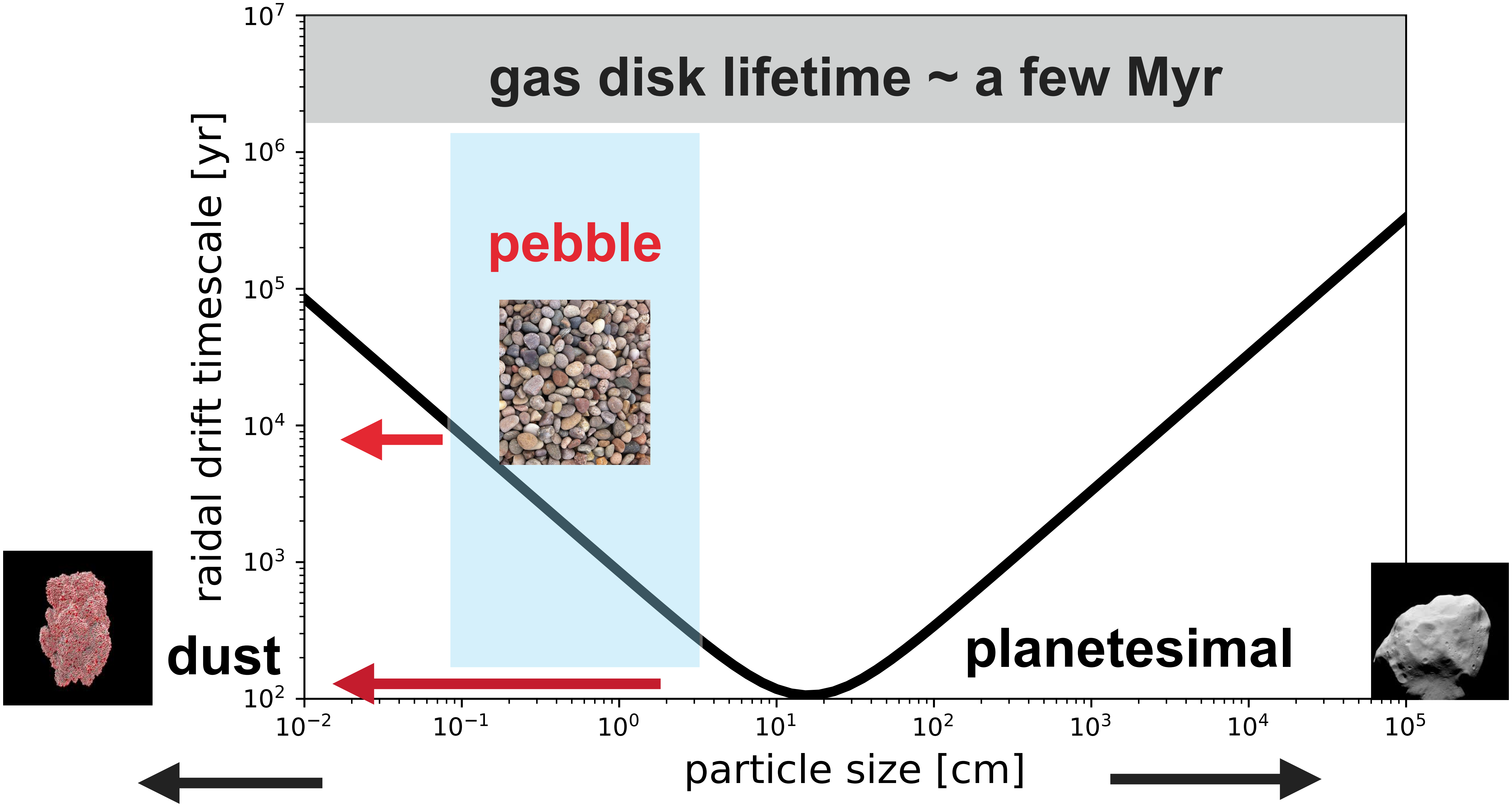
$$V_{gas} = (1 - \eta)V_K$$

$$\eta = -\frac{1}{2}\left(\frac{H_{gas}}{r}\right)^2 \frac{\partial lnP}{\partial lnr} \sim 10^{-3}$$



Dust particles drift into the **high pressure region of the disk.**

Pebbles drift too fast



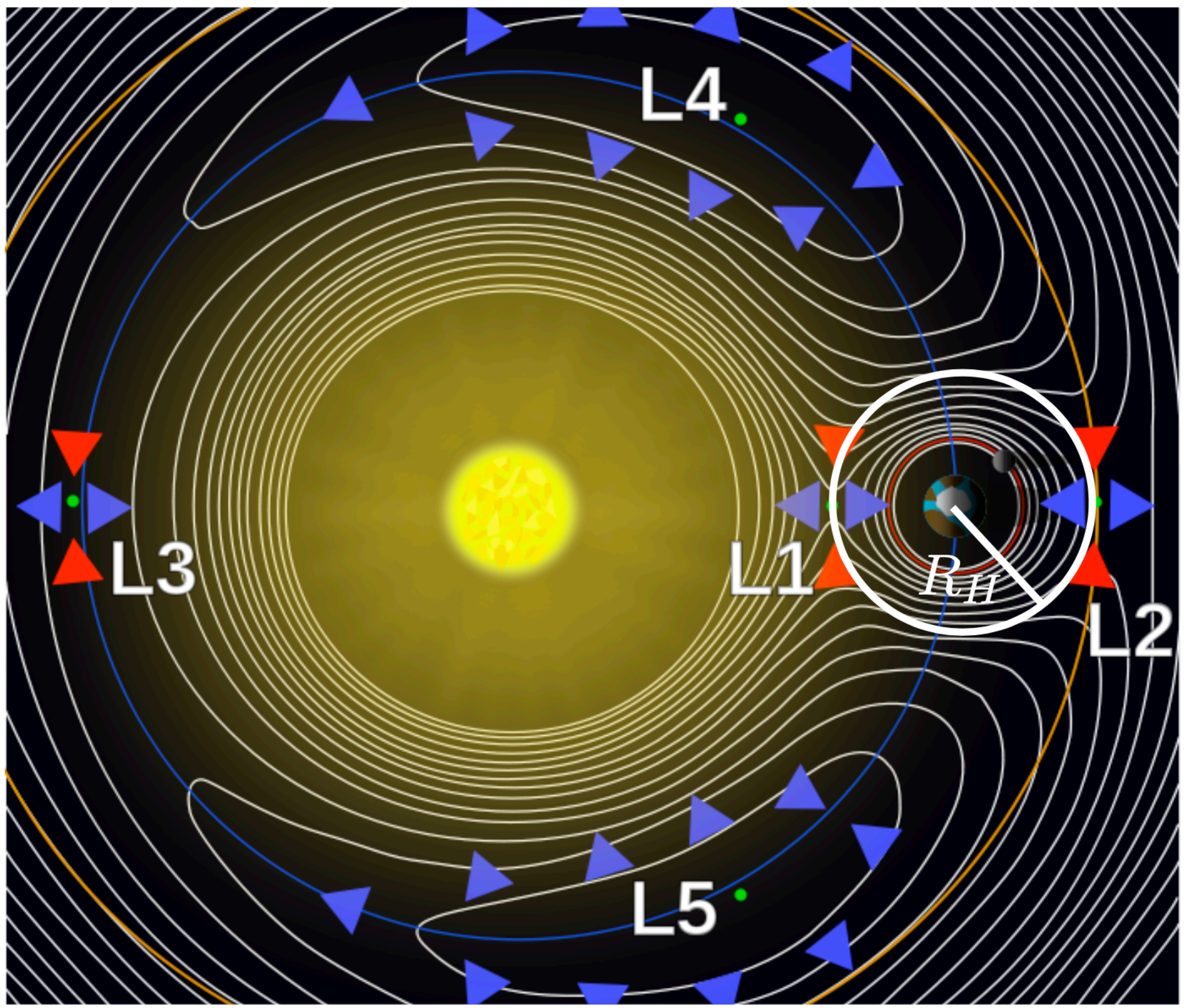
$$v_r = -\frac{2\tau_s}{1+\tau_s^2}\eta v_K \qquad \tau_s \approx \frac{R_p\rho_\bullet}{c_s\rho_g}\Omega_K \sim R_p\rho_\bullet/\Sigma_g$$

Planetesimals accretion

- Hill radius
- Gravitational focusing factor
- Runaway and oligarchic growth
- Isolation mass

Hill radius

Hill sphere (Roche radius): the region where the gravity of the planet is dominant over the gravity of the star.



$$\left(\frac{Gm}{R_H^3}\right)^{1/2} \sim \left(\frac{GM}{a^3}\right)^{1/2} \sim \Omega$$

$$R_H = \left(\frac{m}{3M}\right)^{1/3} a$$

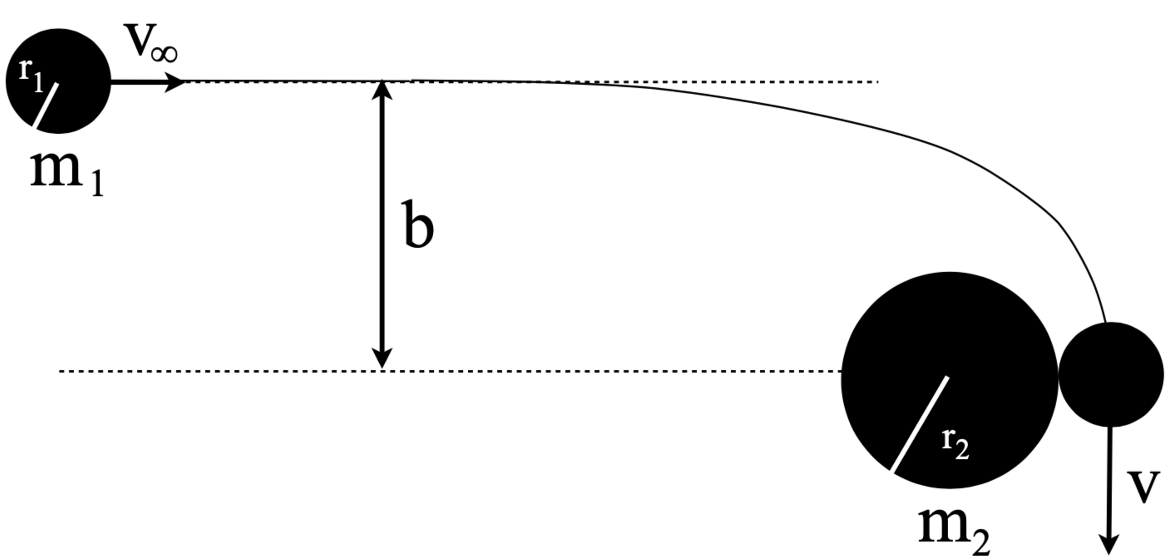
Earth $a = 1\ \mathrm{AU},\ \mathrm{R_H} = 0.01\ \mathrm{AU}$

Jupiter $a = 5.2\ \mathrm{AU},\ \mathrm{R_H} = 0.36\ \mathrm{AU}$

Two body interaction: gravitational focusing

Geometrical cross section of two bodies $\sigma_{\text{geo}} = \pi(r_1 + r_2)^2$

Gravitational focussing: the attracting nature of the gravity leads to an increased collisional cross section.



Energy conservation: $E = \frac{1}{2} m_1 v_\infty^2 = \frac{1}{2} m_1 v^2 - \frac{G m_1 m_2}{r_1 + r_2}$

angular momentum conservation: $J = m_1 b v_\infty = m_1 (r_1 + r_2) v$

Two body interaction: gravitational focusing

$$b^2 = (r_1 + r_2)^2 \left(1 + \frac{v_{\rm esc}^2}{v_\infty^2} \right) \qquad v_{\rm esc} = \left(\frac{2Gm_2}{r_1 + r_2} \right)^{1/2}$$

It provides the collisional cross section:

$$\sigma_{\rm col} = \pi (r_1 + r_2)^2 \left(1 + \frac{v_{\rm esc}^2}{v_\infty^2} \right)$$

geometrical cross section — gravitational focusing factor $\Gamma_{\rm g}$

$$\theta = \frac{v_{\rm esc}^2}{2 v_\infty^2}$$ Safronov number (1969)

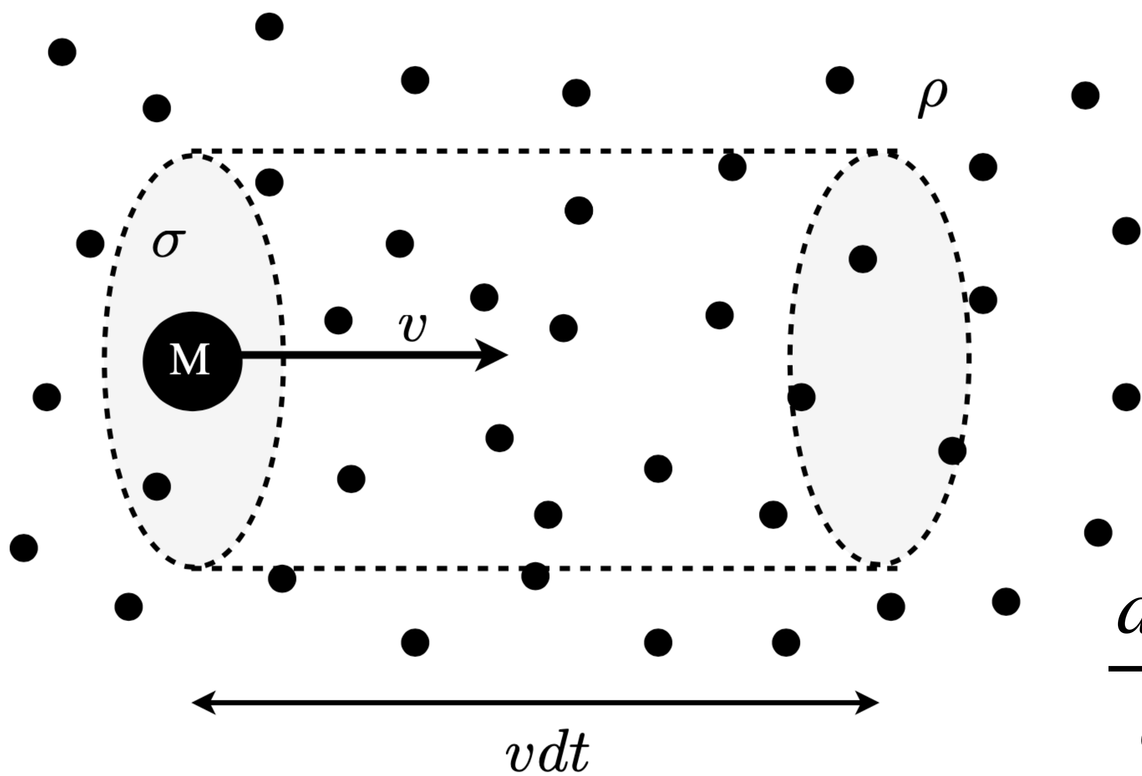
$\theta \ll 1, \ \sigma \propto r_2^2$ Geometrical regime

$\theta \gg 1, \ \sigma \propto r_2^4$ Gravitational focusing regime

$r_1 \ll r_2$

Growth rates

One big body accreting from a sea of small planetesimals.



$$\rho_s = \frac{\Sigma_s}{2H_s} \qquad \frac{H_s}{a} \sim \frac{v_\infty}{v_\mathrm{K}}$$

Collision rate: $n\sigma\Delta v$

Growth rate of the big body M

$$\frac{dM}{dt} = (n\sigma\Delta v)m_s = \rho_s \sigma v_\infty = \rho_s v_\infty \pi r^2 (1 + \frac{v_\mathrm{esc}}{v_\infty})^2$$

$$\frac{dM}{dt} \sim \Sigma_s \Omega \pi r^2 (1 + \frac{v_\mathrm{esc}^2}{v_\infty^2})$$

Growth rates

One big body accreting from a sea of small planetesimals.

$$\frac{dM}{dt} \sim \Sigma_s \Omega \pi r^2 \left(1 + \frac{v_{\rm esc}^2}{v_\infty^2} \right)$$

- The velocity dispersion is the key factor. The gravitational focusing factor can be more complex in three-body case.
- The growth rate is higher in disks with larger planetesimal surface density.
- Since both surface density and angular frequency generally decreases with distance, the planet grows slower at large orbital distances.

Growth rates

$$\frac{dM}{dt} \sim \Sigma_s \Omega \pi r^2 (1 + \frac{v_{\rm esc}^2}{v_\infty^2}) \qquad \sim \frac{GM\Sigma_s\Omega r}{v_\infty^2} \propto M^{4/3} \qquad when\ \mathrm{v}_\infty \ll \mathrm{v}_{\rm esc}$$

$$\sim \Sigma_s \Omega r^2 \propto M^{2/3} \qquad when\ \mathrm{v}_\infty \gtrsim \mathrm{v}_{\rm esc}$$

Runaway growth:
the bigger body, the faster it grows

$$\frac{1}{M}\frac{dM}{dt} \propto M^{1/3}$$

If M1>M2, then runaway means
d(M1/M2)/dt>0

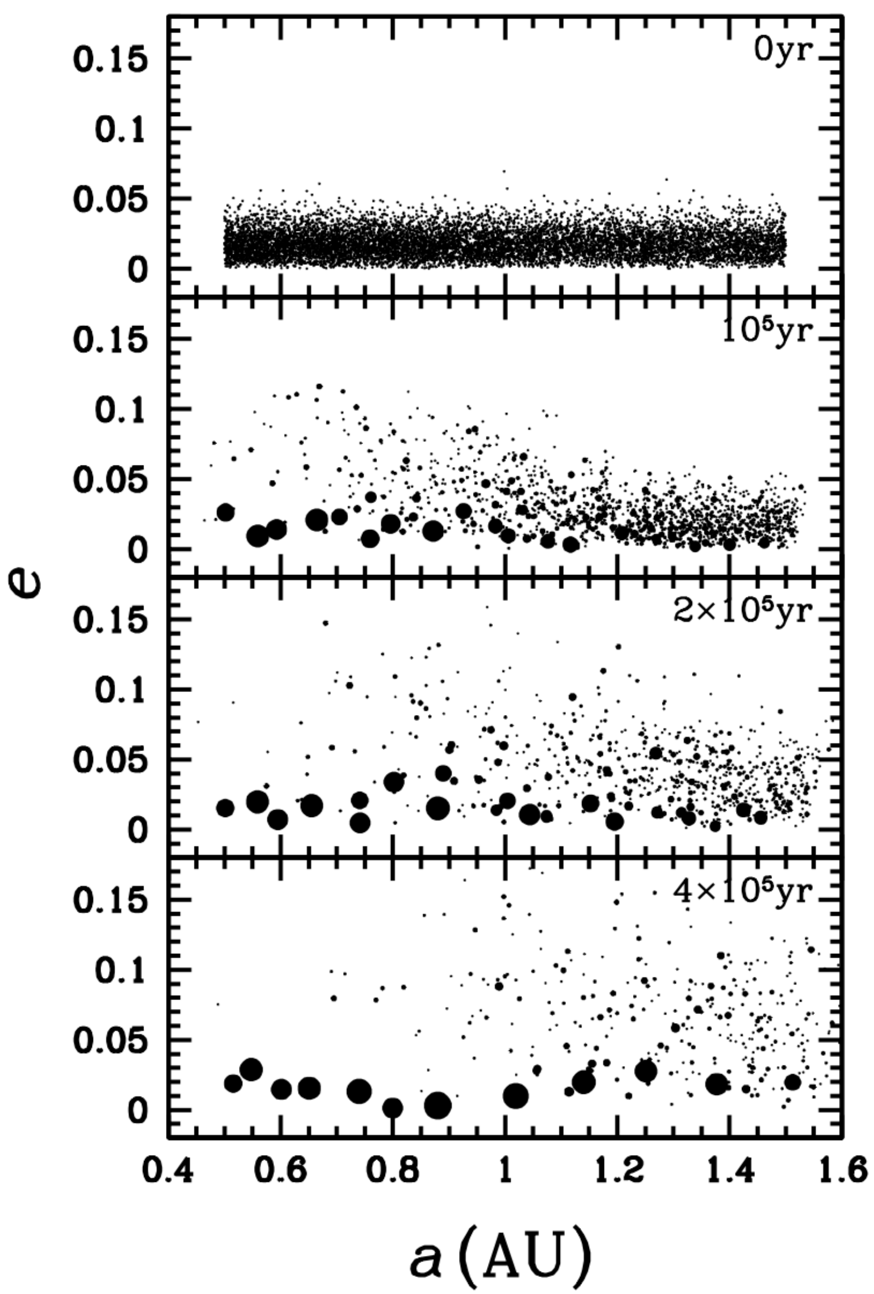
- bigger bodies + small planetesimal population.
- dynamical friction/equipartition of energy indicates that e and i of big body remain small, while e and I of small planetesimals are not affected by the growth of the big one.
- the escape velocity of big body increases due to mass growth, the gravitational focusing attracts more planetesimals and the big body grows faster.
- a positive feedback->runaway growth

Oligarchic growth

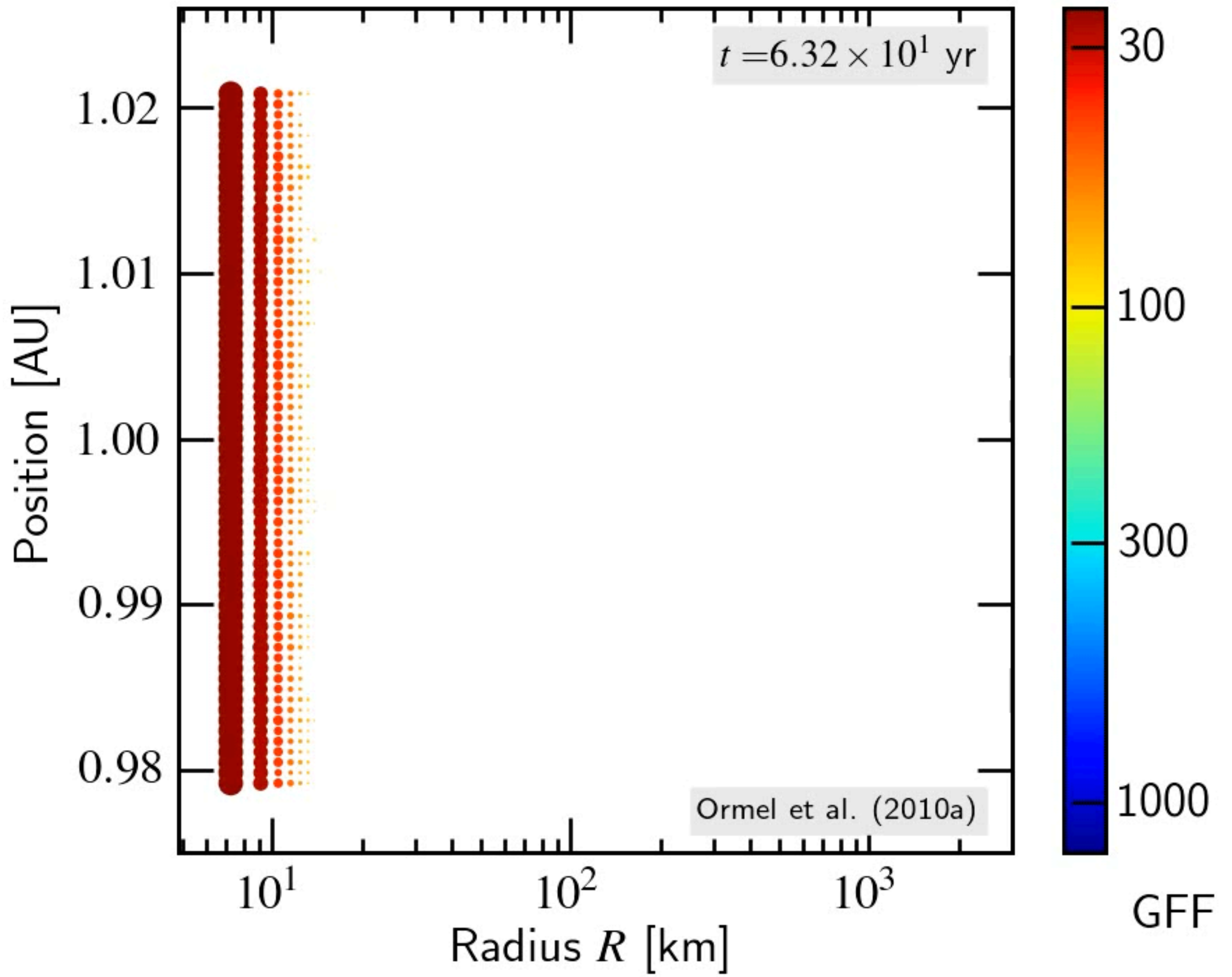
The growing bigger bodies have a feedback onto the random velocities of small ones. It gradually reduces the gravitational focusing factor.

The massive bodies grow slower than less massive bodies (still faster than planetesimals). Therefore, these embryos tend to grow towards similar masses (so called “oligarchy”).

$$\frac{1}{M}\frac{dM}{dt} \propto \frac{1}{M^{1/3}}$$


0yr
10⁵yr
2×10⁵yr
4×10⁵yr
0.15
0.1
0.05
0
e
0.4
0.6
0.8
1
1.2
1.4
1.6
a (AU)

Kokubo & Ida

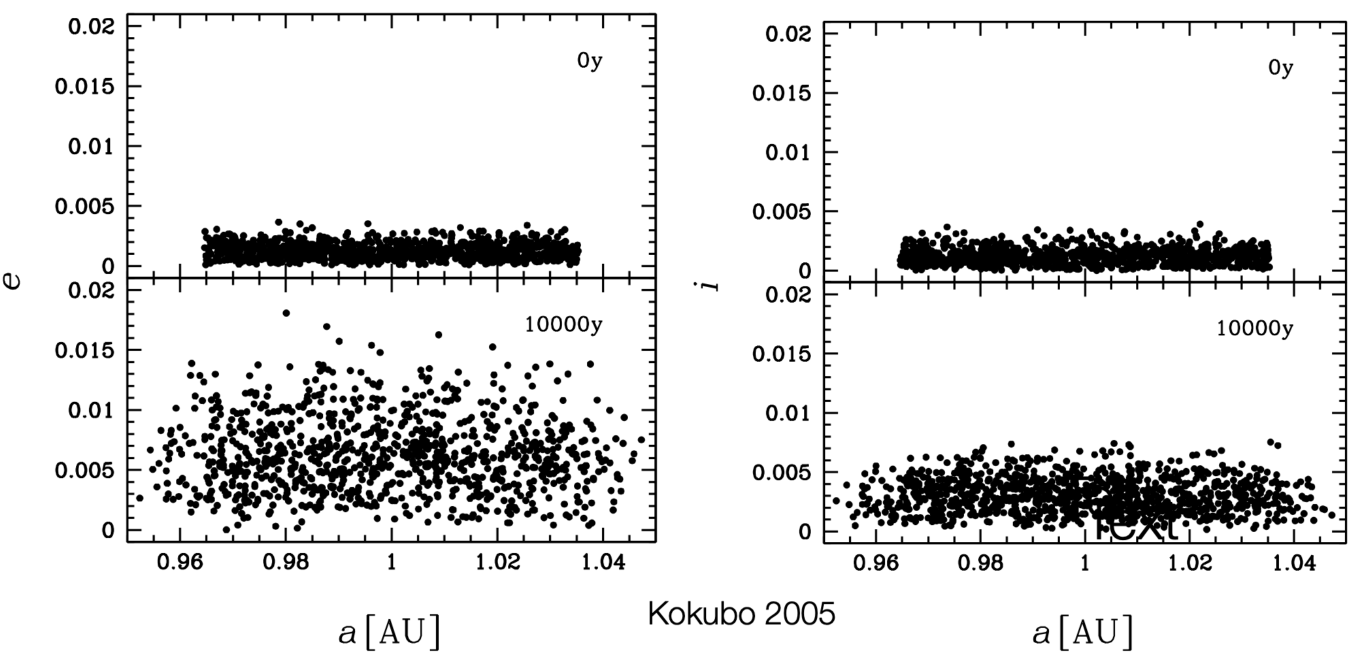

t = 6.32 × 10¹ yr
1.02
1.01
1.00
0.99
0.98
Position [AU]
10¹
10²
10³
Radius R [km]
Ormel et al. (2010a)
30
100
300
1000
GFF

QA: What determines the random velocity of the small planetesimals?

Viscous stirring through gravitational scattering among planetesimals and embryos.

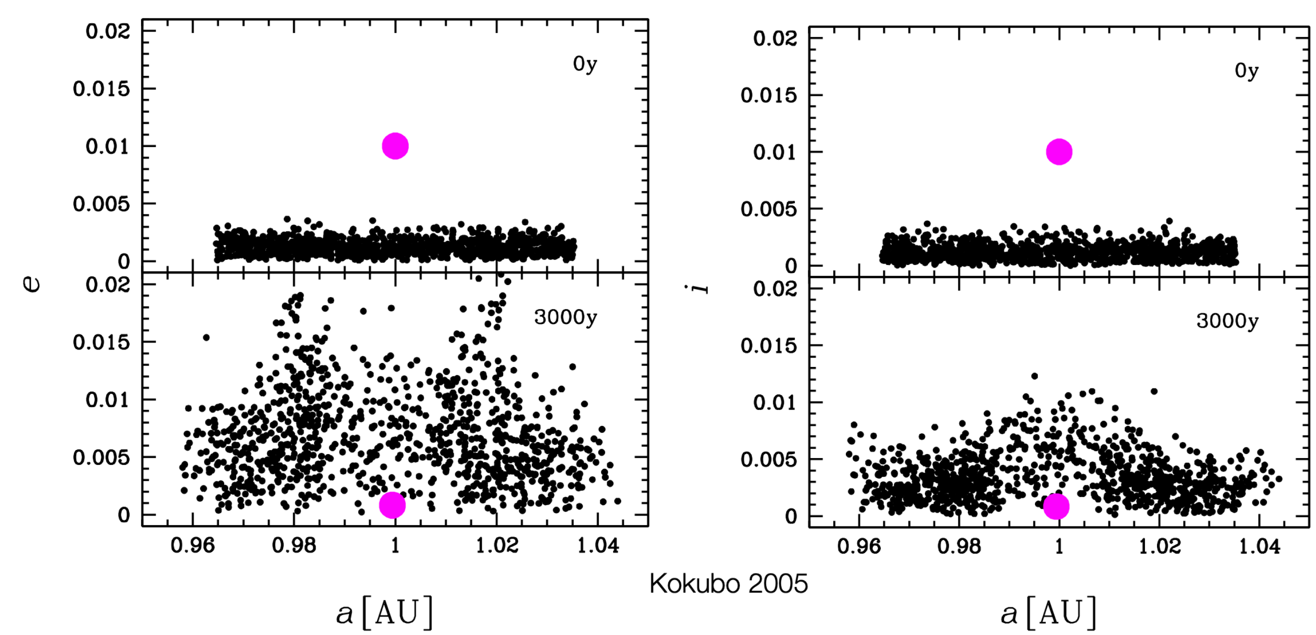
Damping by gas drag.

Dynamical friction: energy transfer from large bodies to small bodies.



Kokubo 2005

Direct N-body simulation of 1000 equal-mass (m = 10^{24}g) planetesimals distributed in a ring at a = 1AU with width Δa = 0.07AU.



Kokubo 2005

Note the dynamical “cooling” of the body objects and the “heating” of small ones.

Planetesimal isolation mass

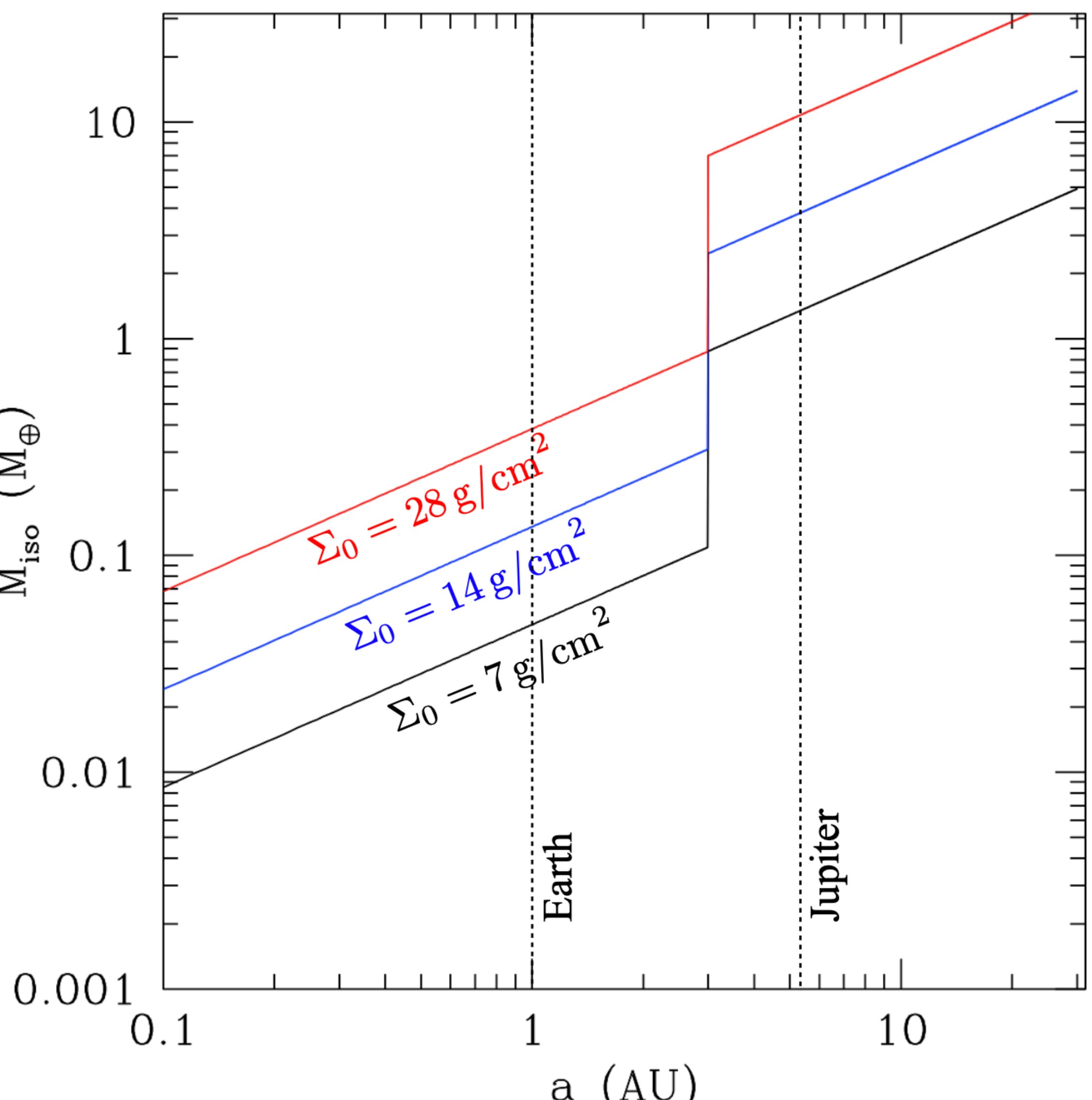
feeding zone of the planet $$\Delta a = fR_{\mathrm{H}} \propto M^{1/3}$$

The planet clears all the material in its feeding zone

$$M_{\mathrm{iso}} = \int_{a-\Delta a}^{a+\Delta a} 2\pi\Sigma_p(r) r dr$$

$$M_{\mathrm{iso}} = \frac{(4\pi f a^2 \Sigma_p)^{3/2}}{(3M_\star)^{1/2}}$$

$$\Sigma_p(a) = \eta_{ice}\Sigma_0 \left(\frac{a}{1\mathrm{AU}}\right)^{\alpha} \ \mathrm{g/cm^2} \ \text{with} \quad \eta_{ice} = \begin{cases} 1 & \text{if } a < a_{ice} \\ 4 & \text{if } a \geq a_{ice} \end{cases}$$

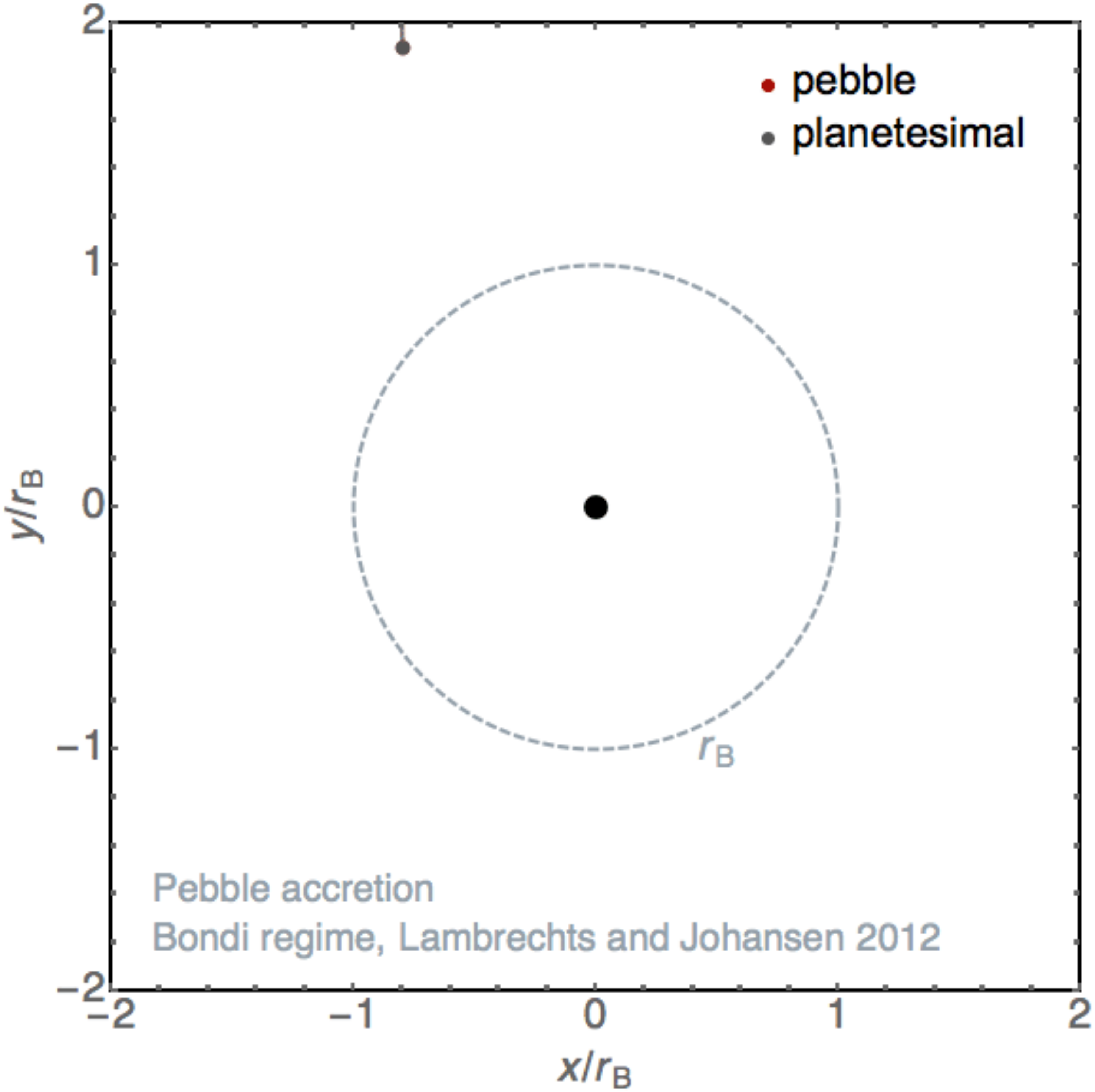


For the MMSN, we have isolation mass of 0.05 M_{earth} at 1 AU and 2 M_{earth} at 5.2 AU.

This holds for circular orbits without orbital migration!

Pebble accretion

Pebble accretion: larger accretion radius

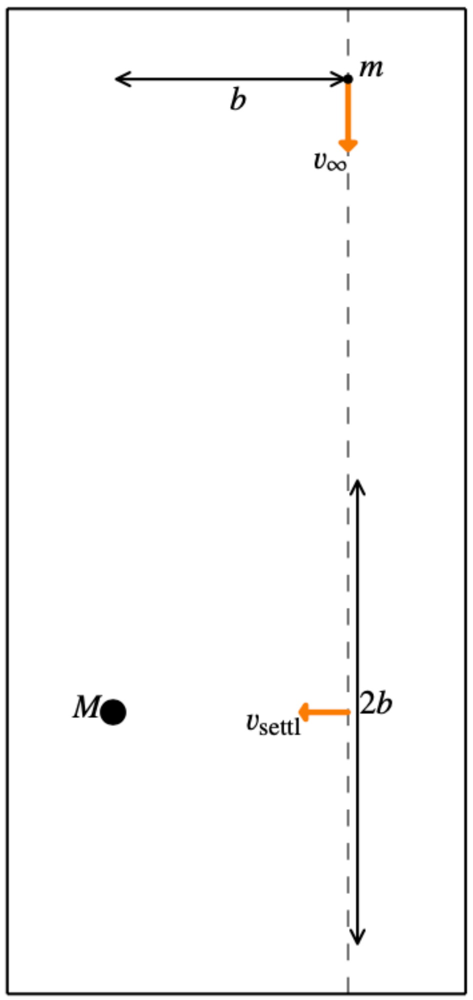


Pebbles
mm-cm

gas drag +
gravitational force

Ormel & Klahr 2010, Lambrechts & Johansen 2012, Morbidelli+2015, Levison+2015, Ida+2016, Xu & Bai 2017, Liu & Ormel 2018, Ormel & Liu 2018

Pebble accretion: larger accretion radius



$t_{\rm stop}$ How fast particles adjust the moment to the gas

$t_{\rm enc} \sim b/v_\infty$ The duration of particle-planet encounter

By the force balance, the particle would settle towards to the planet at a velocity $v_{\rm set} = \frac{GM}{b^2} t_{\rm stop}$

$$t_{\rm set} \sim \frac{b}{v_{\rm set}} \sim \frac{b^3}{GMt_{\rm stop}}$$

Pebble accretion occurs

1) $t_{\rm set} < t_{\rm enc}$ particles can settle during encounter

2) $t_{\rm stop} < t_{\rm enc}$ drag is important during encounter

Equation the settle time and encounter time can get maximum accretion radius

$$b_{\rm PA} \sim \sqrt{\frac{GMt_{\rm stop}}{v_\infty}}$$

Pebble accretion: larger accretion radius

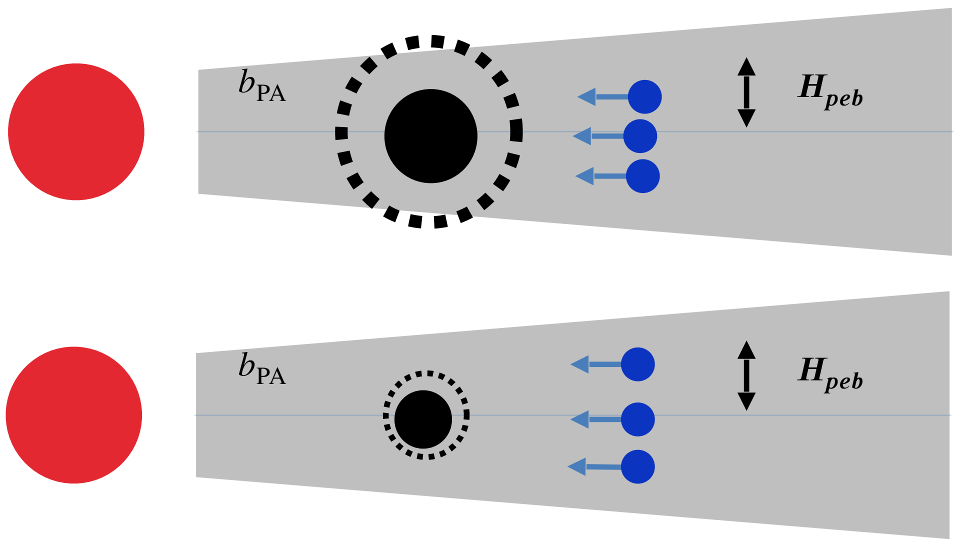
v_∞ matters in pebble accretion

Headwind regime (Bondi regime): $v_\infty \sim \eta v_K, \ \ b_{PA} \sim \sqrt{\frac{GMt_{stop}}{\eta v_K}}$

shear regime (Hill regime): $v_\infty \sim b\Omega_K, \ \ b_{PA}^3 \sim \frac{GMt_{stop}}{\Omega}, \ \ b_{PA} \sim R_H \tau_s^{1/3}$

QA: compare the maximum planetesimal accretion radius and pebble accretion radius

Pebble accretion: 2D vs 3D



$b_{\rm PA} > H_{\rm peb}$ 2D $\dot{M}_{\rm PA,2D} = 2b_{\rm PA}\Delta v \Sigma_{\rm p} \sim 2\sqrt{GMt_{\rm stop}\Delta v}\Sigma_{\rm p}$

$b_{\rm PA} < H_{\rm peb}$ 3D $\dot{M}_{\rm PA,3D} = b_{\rm PA}^2 \Delta v \rho_{\rm p} \sim GMt_{\rm stop}\rho_{\rm p}$

QA: What's the role of disk turbulence on pebble accretion?

What's the mass for the onset of pebble accretio

Pebble accretion occurs 1) $t_{set} < t_{enc}$ particles can settle during encounter

2) $t_{stop} < t_{enc}$ drag is important during encounter

$$t_{enc} \sim b/v_\infty \qquad b_{PA} \sim \sqrt{\frac{GM_p t_{stop}}{v_\infty}}$$

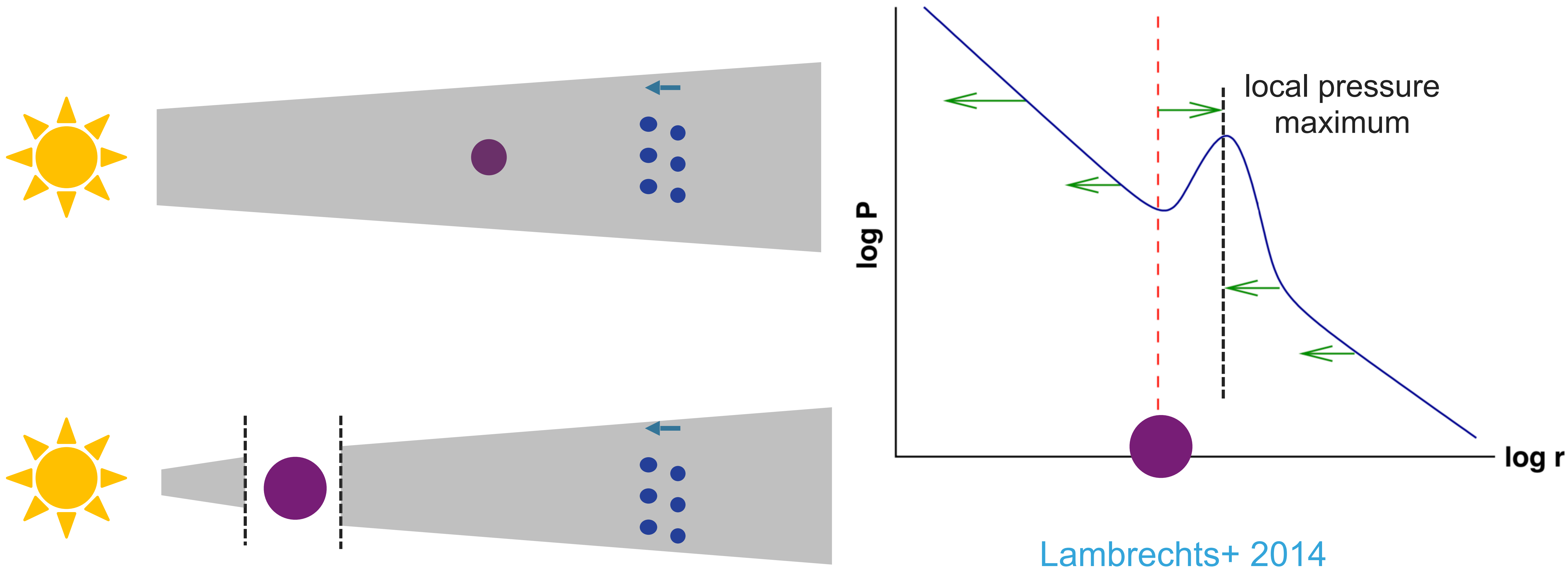
$$v_{\infty,crit} \sim \left(\frac{M_p}{M_\star \tau_s}\right)^{1/3} v_K \sim \eta v_K$$

$$M_{onset} = \tau_s \eta^3 M_\star$$

$$\begin{aligned} M_{onset} &= \tau_s \eta^3 M_\star \\ &= 2.5 \times 10^{-4} \left(\frac{\tau_s}{0.1}\right) \left(\frac{\eta}{2 \times 10^{-3}}\right)^3 \left(\frac{M_\star}{M_\odot}\right) M_\oplus \end{aligned}$$

Massive planets stop accreting pebbles

Pebble isolation mass



Lambrechts+ 2014

$$M_{\rm iso} \simeq 20\ M_\oplus \left(\frac{h_{\rm gas}}{0.05}\right)^3 \left(\frac{M_\star}{M_\odot}\right)$$

$\approx 1\ M_\oplus$ for 0.1 $M_\odot$ stars

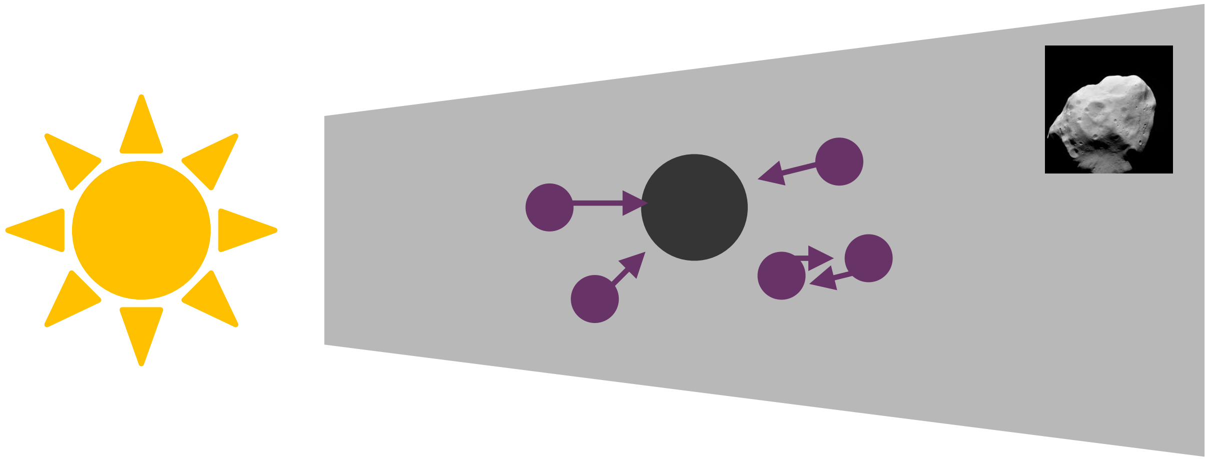
$\approx 20\ M_\oplus$ for 1 $M_\odot$ stars

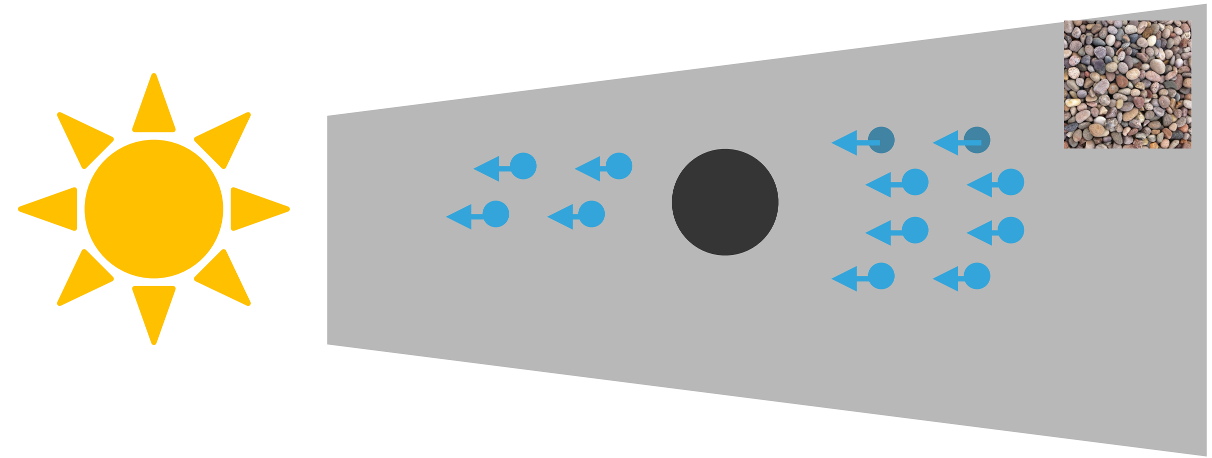
Planetesimals accretion

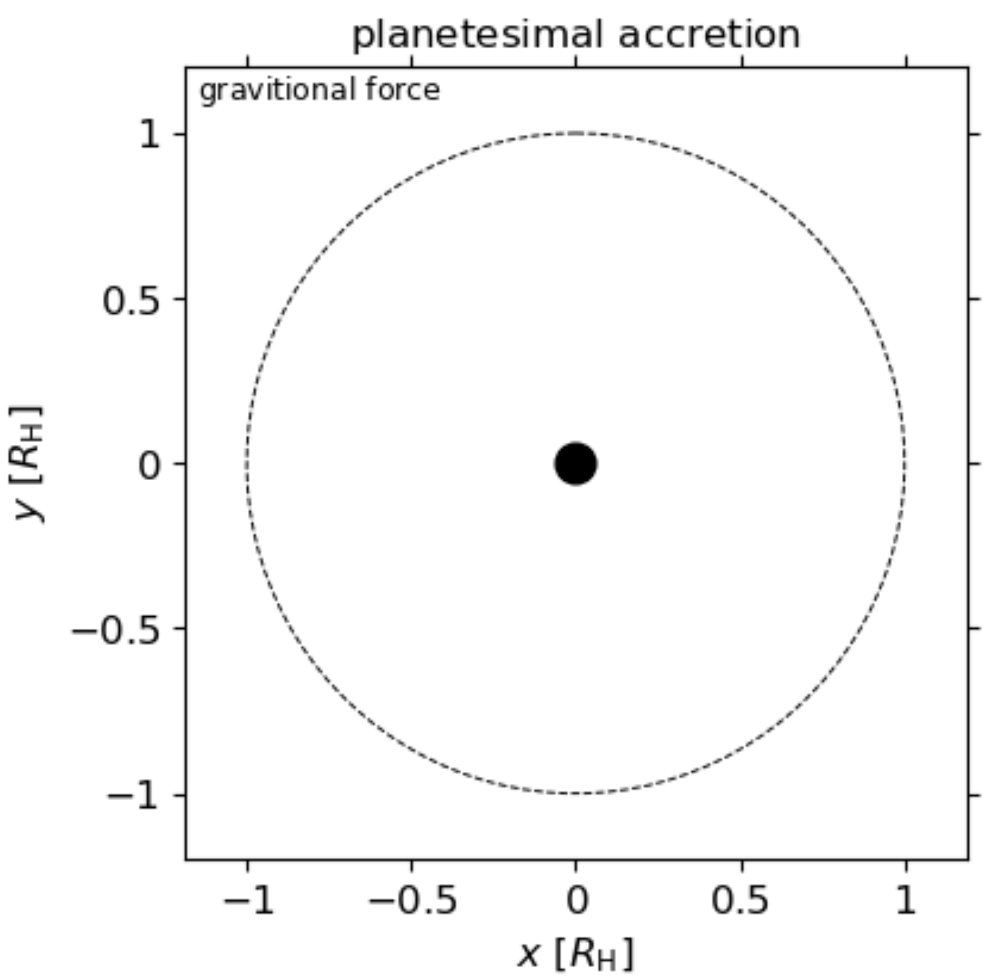
Safronov1969; Wetherill & Stewart 1989,Kokubo & Ida 1996,1998

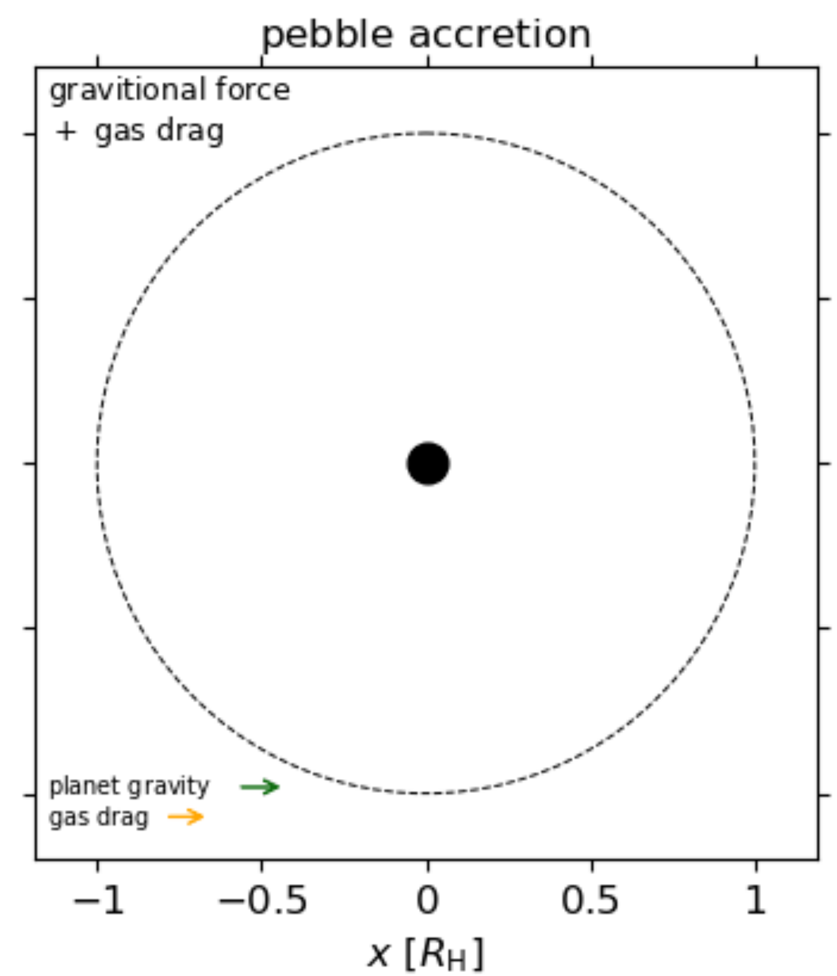
Pebble accretion

Ormel & Klahr 2010; Lambrechts & Johansen2012



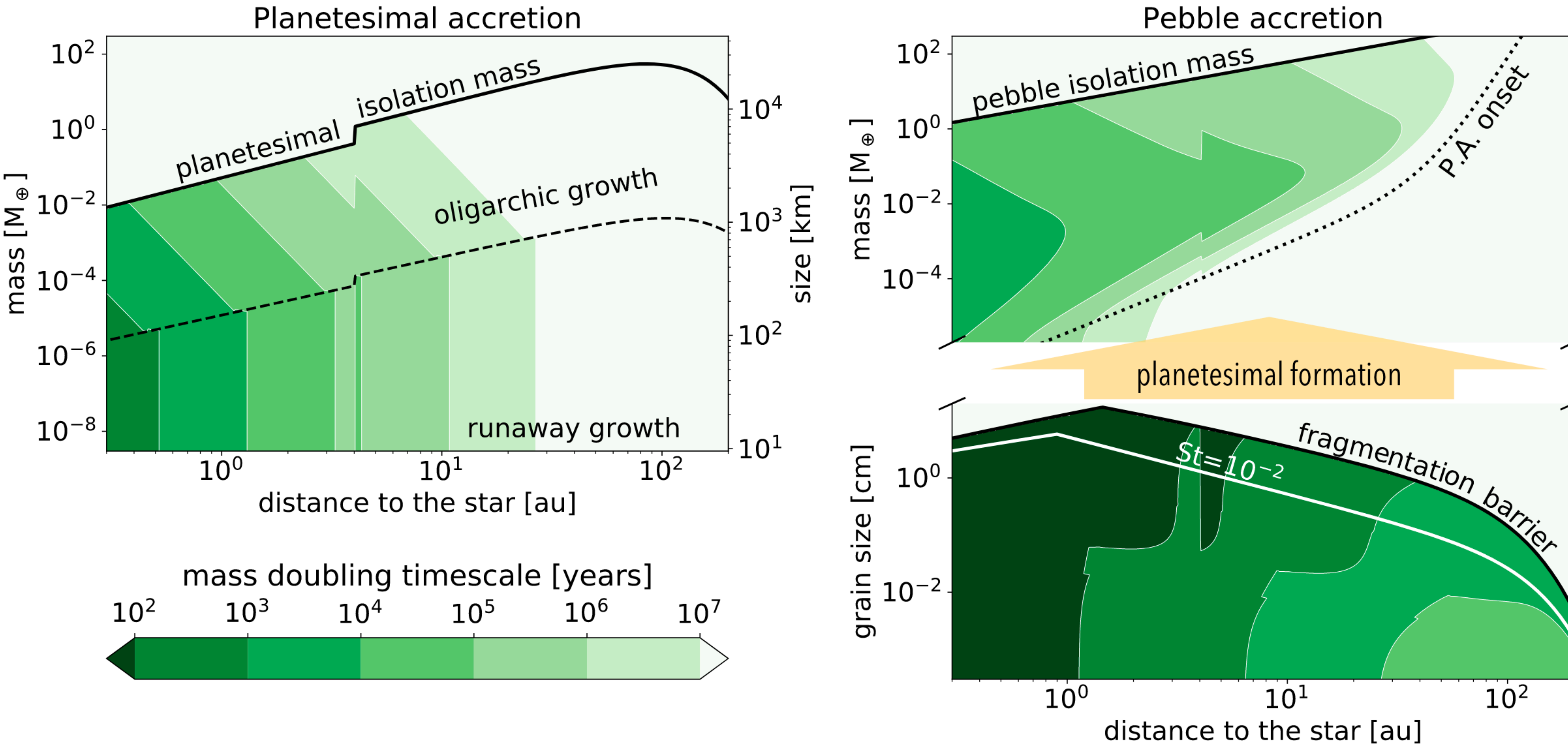






Viewed in the co-moving frame

Credit:Liu & Ji 2020

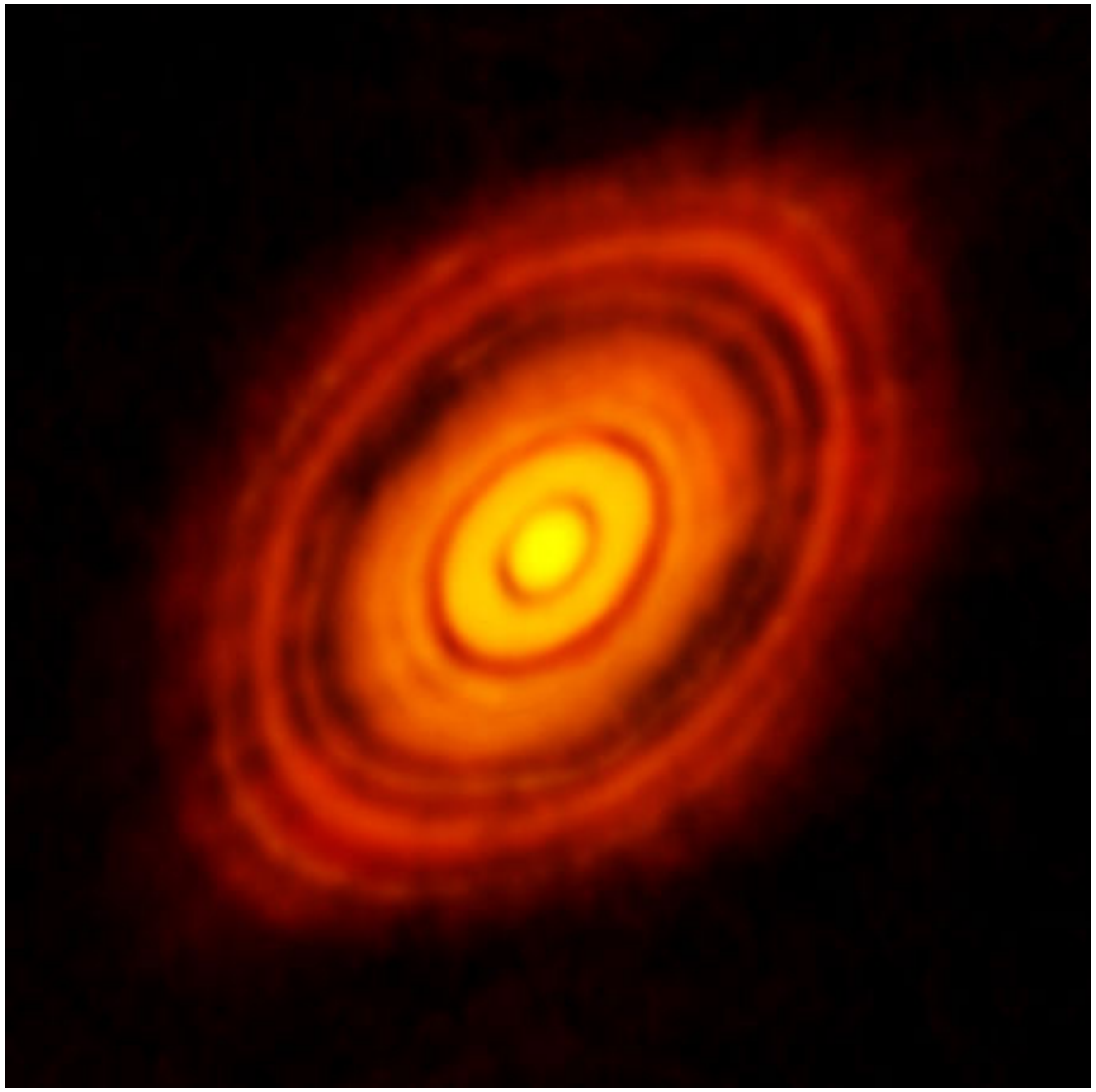


Drazkowska+2022

Single site planetesimal formation + hybrid accretion

Classical view: planetesimals forms everywhere

Think a bit more: solid concentration and planetesimal formation might not happen everywhere

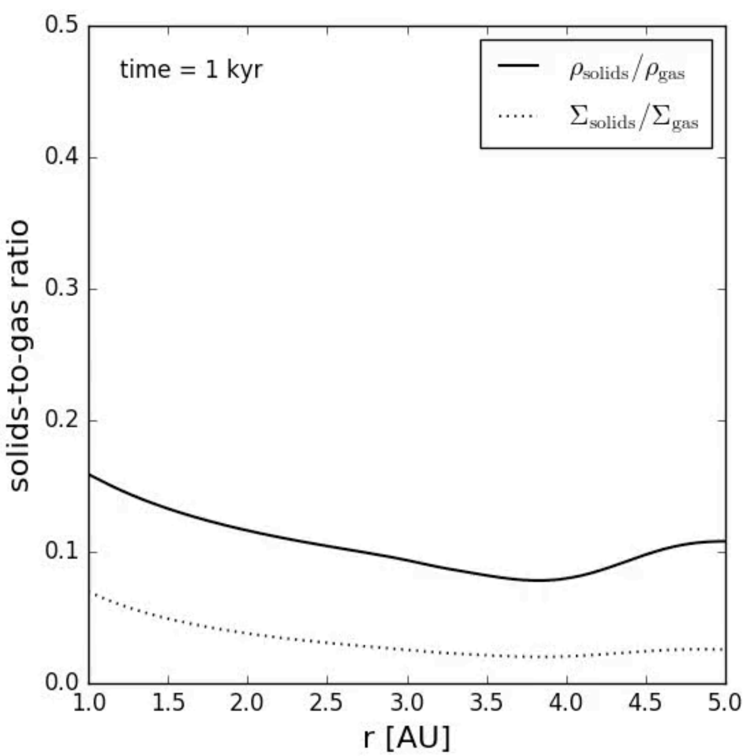


Where planetesimals can form

SI condition:enhanced solid density $\rho_{\rm peb} \simeq \rho_{\rm gas}$

1.ice lines

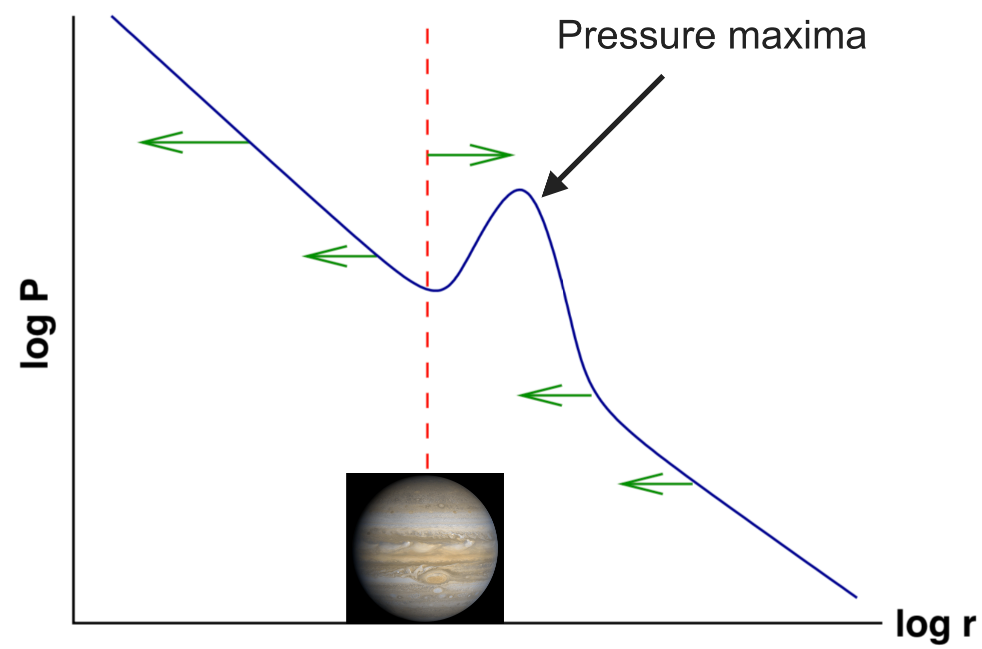
Ros & Johansen 2013
Ida & Guillot 2016
Schoonenberg & Ormel 2017
Drazkowska & Alibert 2017



2. edge of planetary-induced gap

Stammler+2019

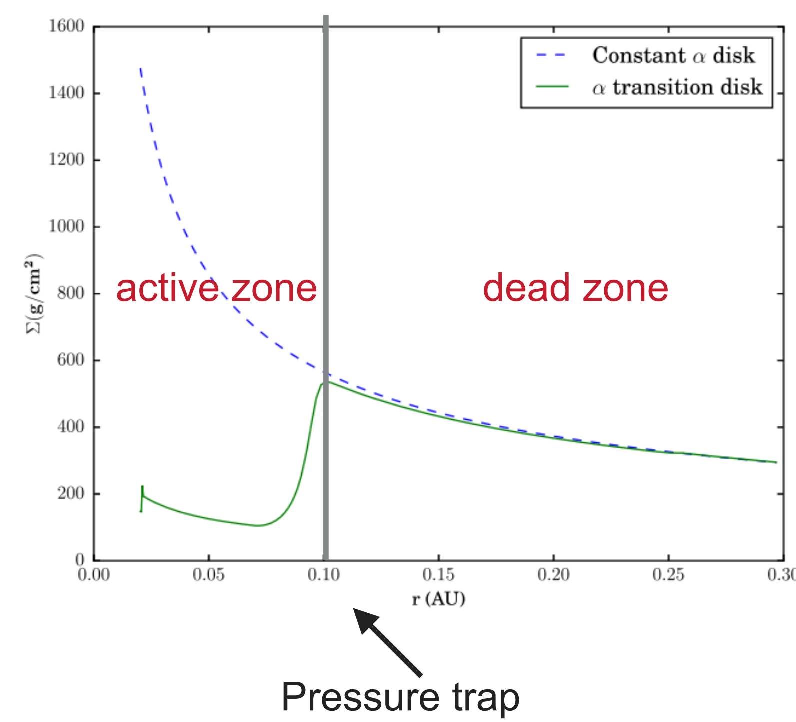
Eriksson, Johansen & Liu 2020



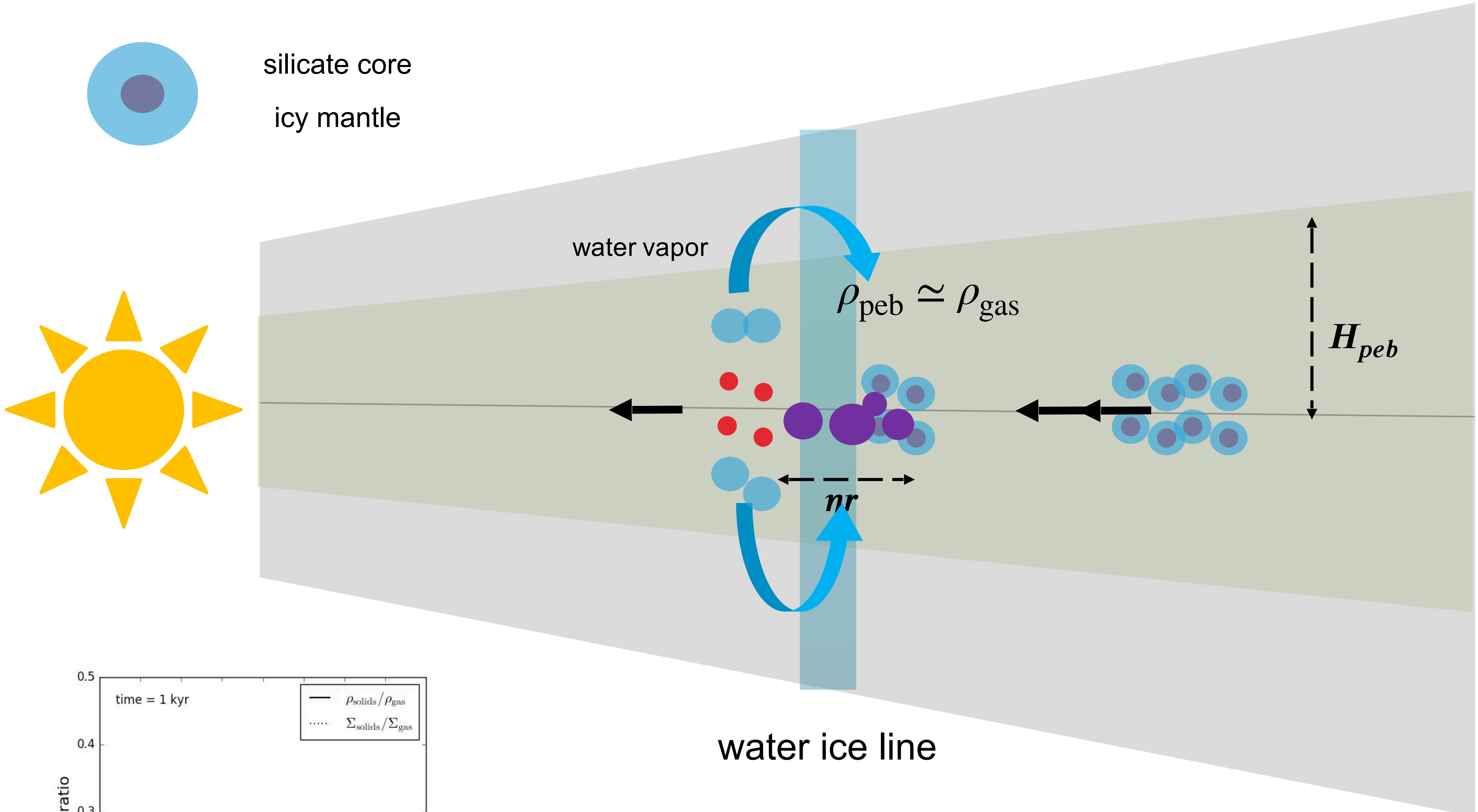
3. deadzone boundary

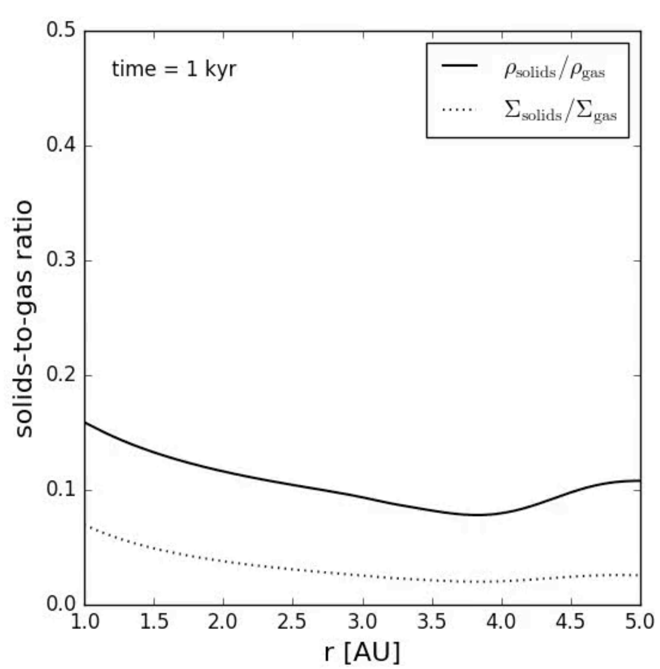
Chatterjee & Tan 2014,2015

Hu, Zhu & Tan+ 2016,2018



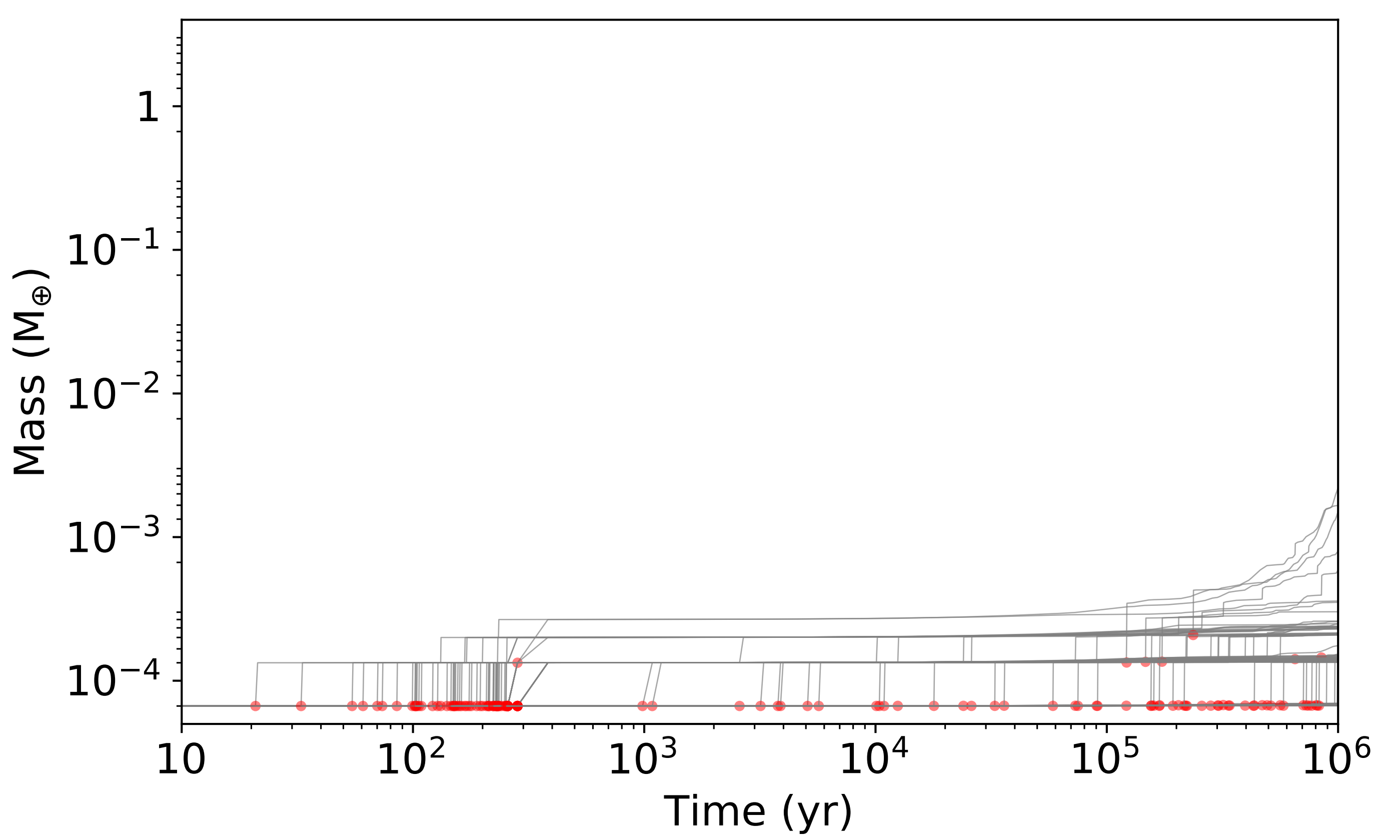
Streaming instability at ice line

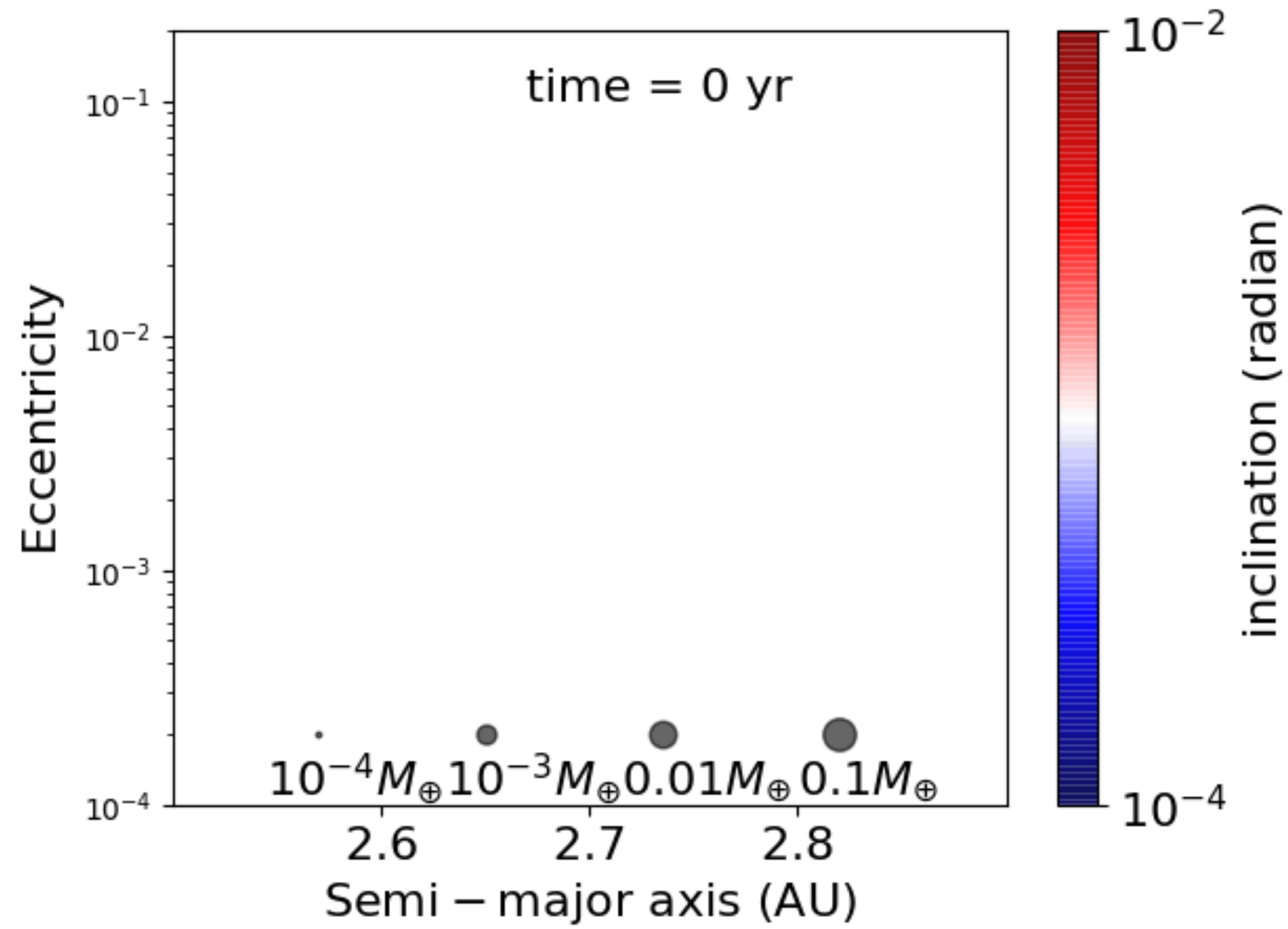




1. Mono-dispersed population: limited growth

R_{pl}=400 km (7e-5 M_E), N_{pl}=580, pebble flux:100 M_E/Myr

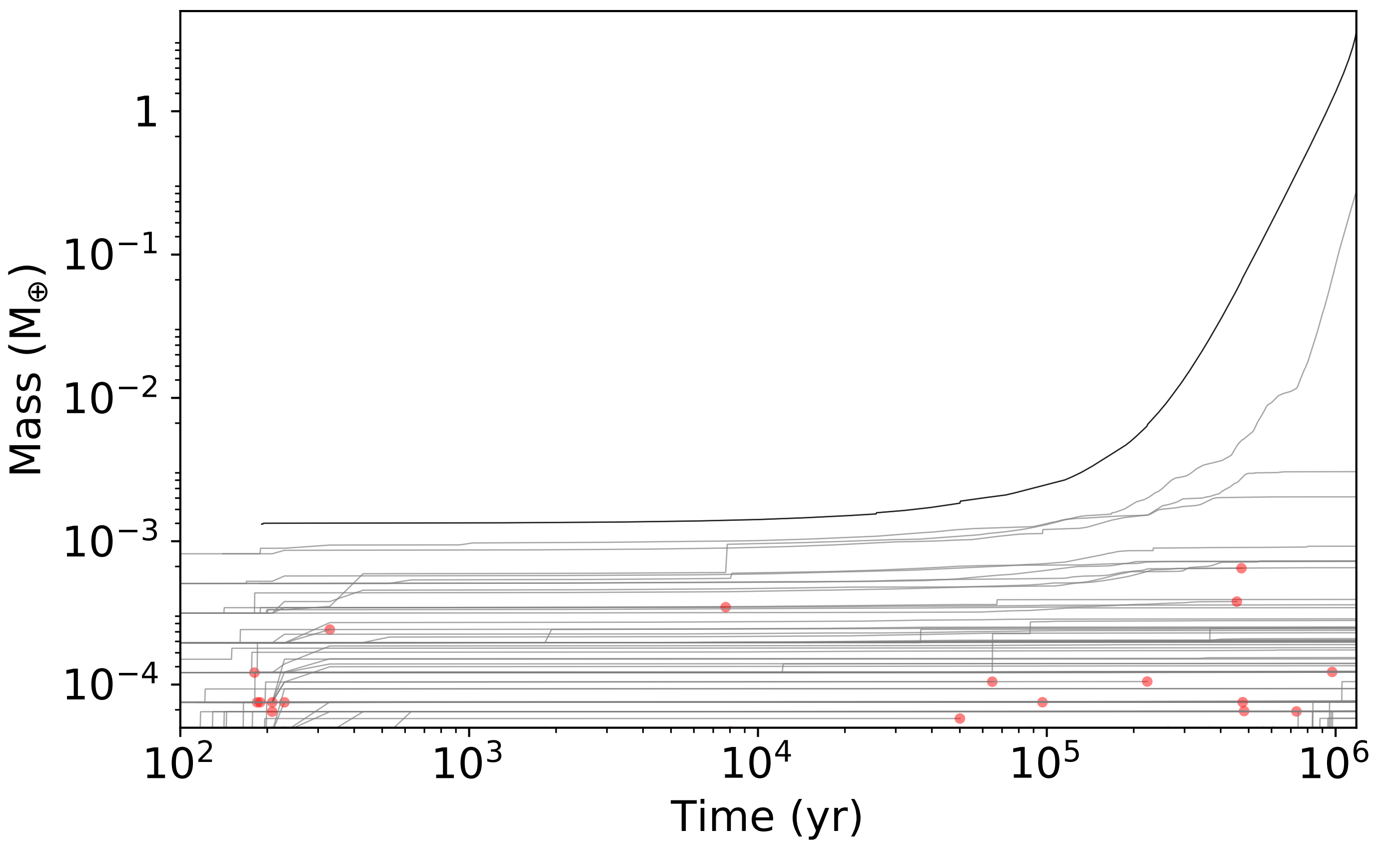


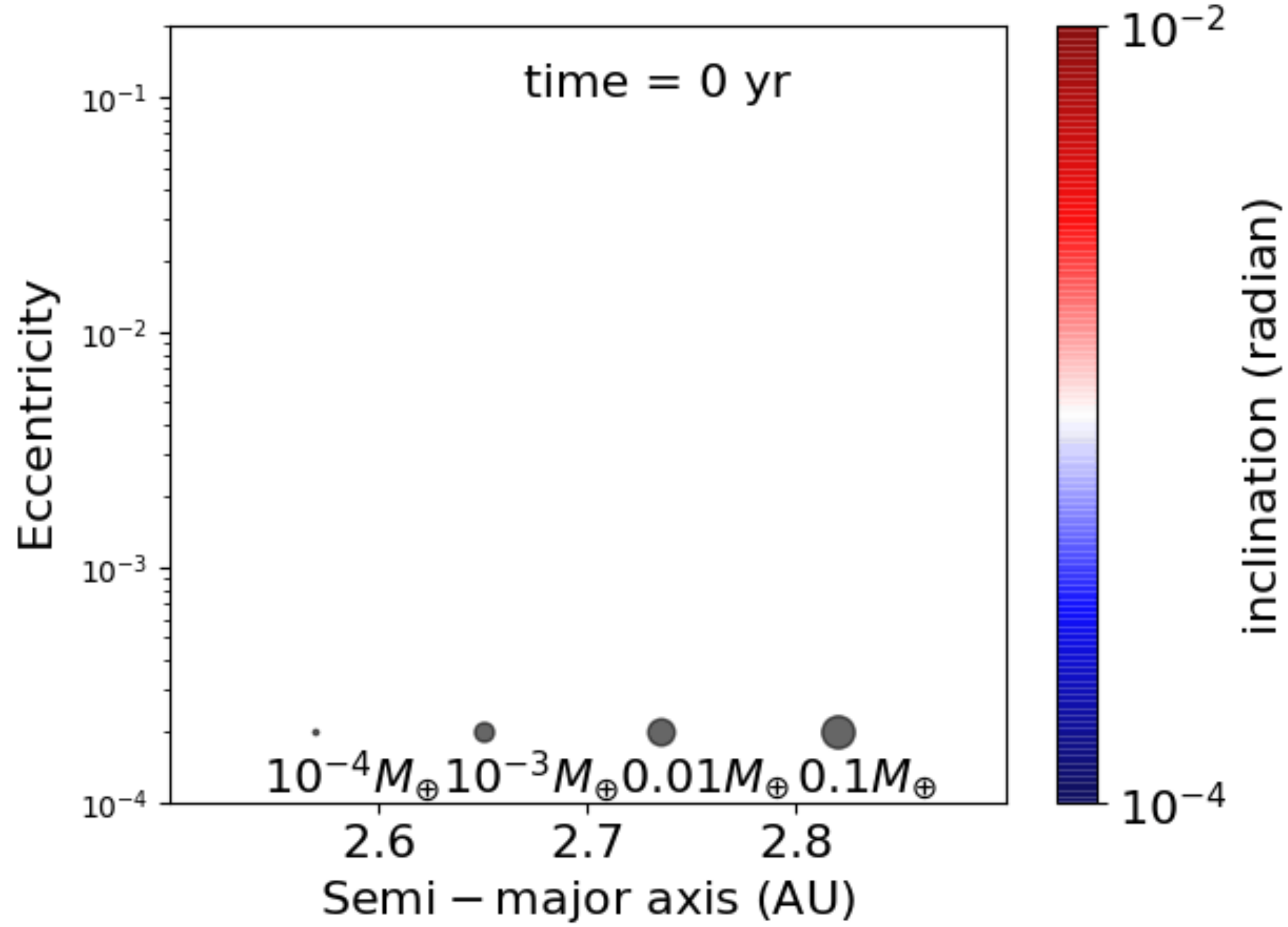


eccentricities and inclinations get excited

2. Poly-dispersed population: promote planet growth

R_{pl}=200-1000 km followed by Schafer+2017, pebble flux:100 M_E/Myr



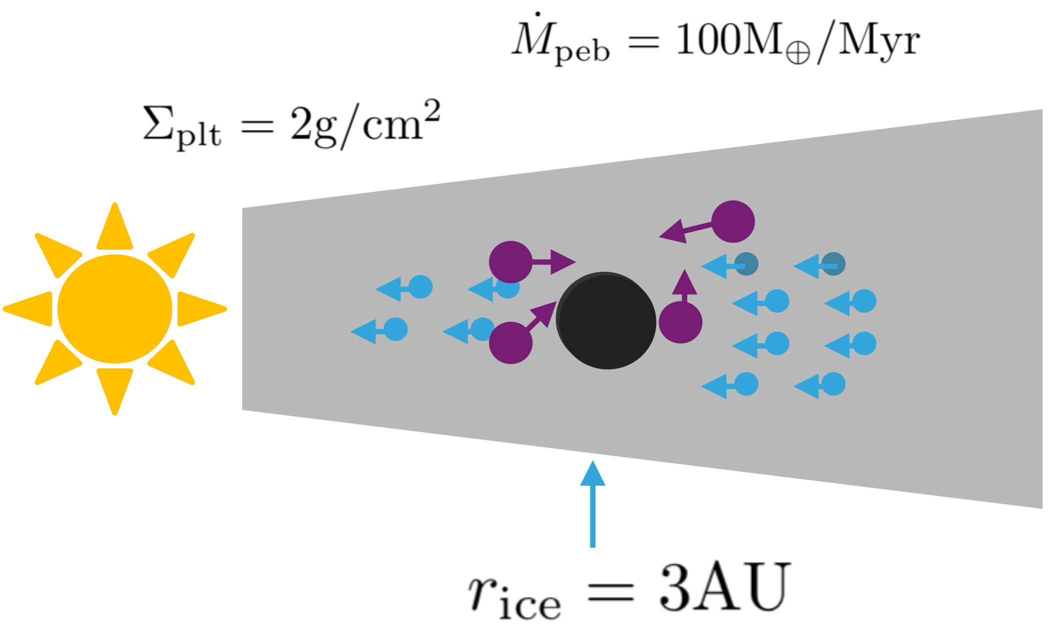


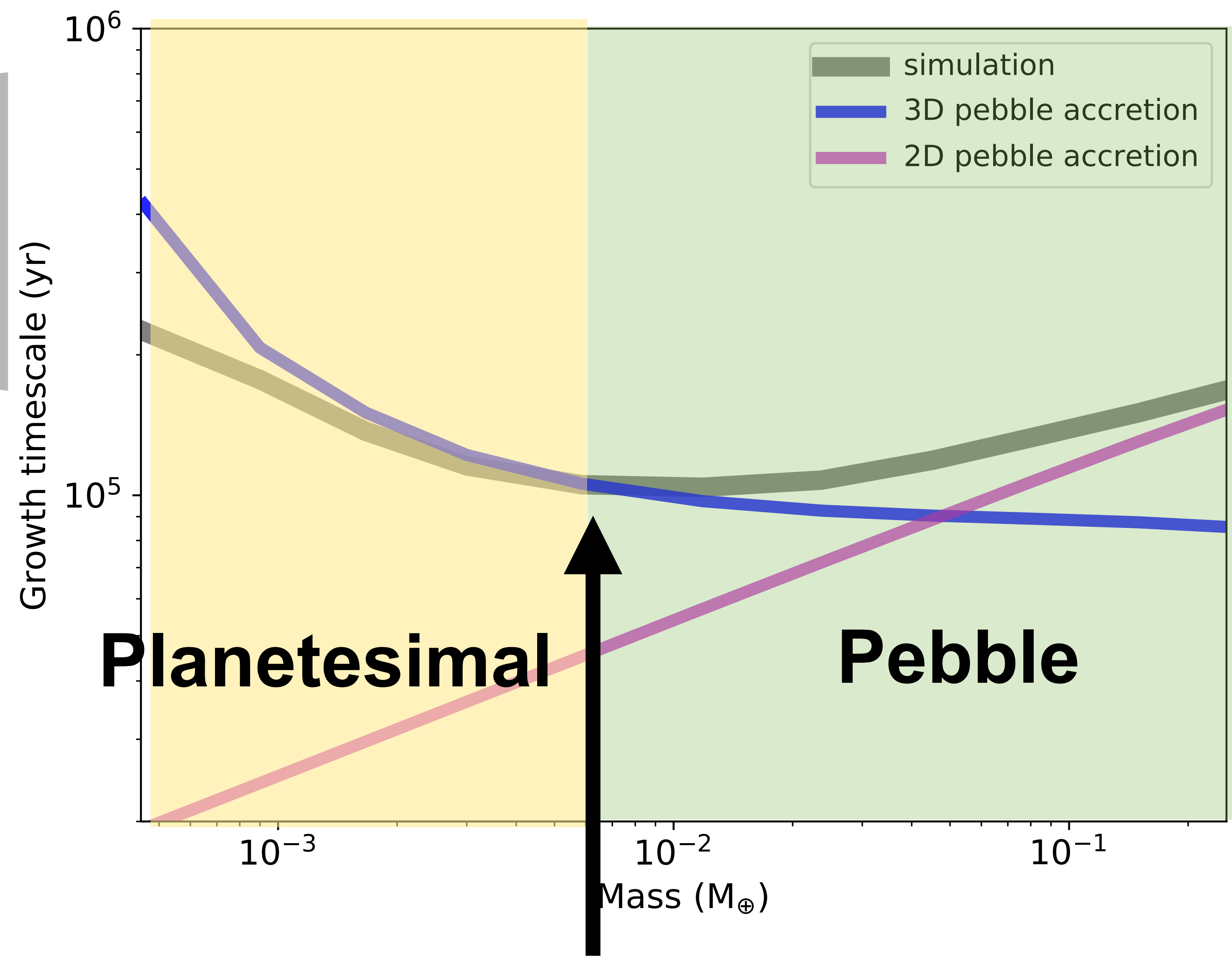
Largest body accretes most planetesimals and pebbles

Small planetesimals damp large body's random velocity

Liu, Ormel & Johansen 2019

From Planetesimal Accretion to Pebble Accretion



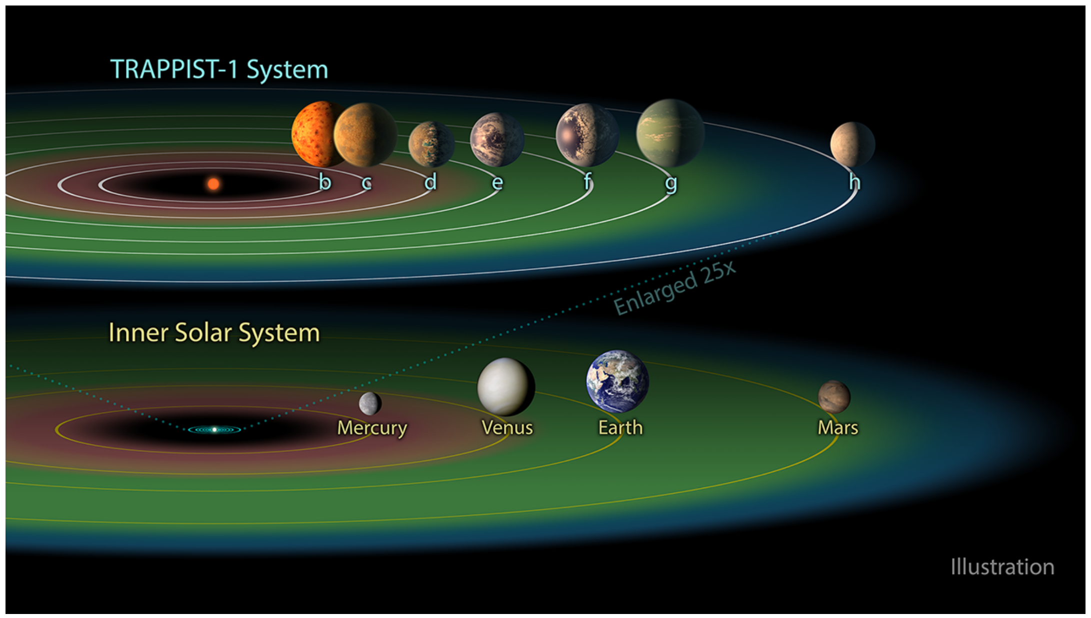


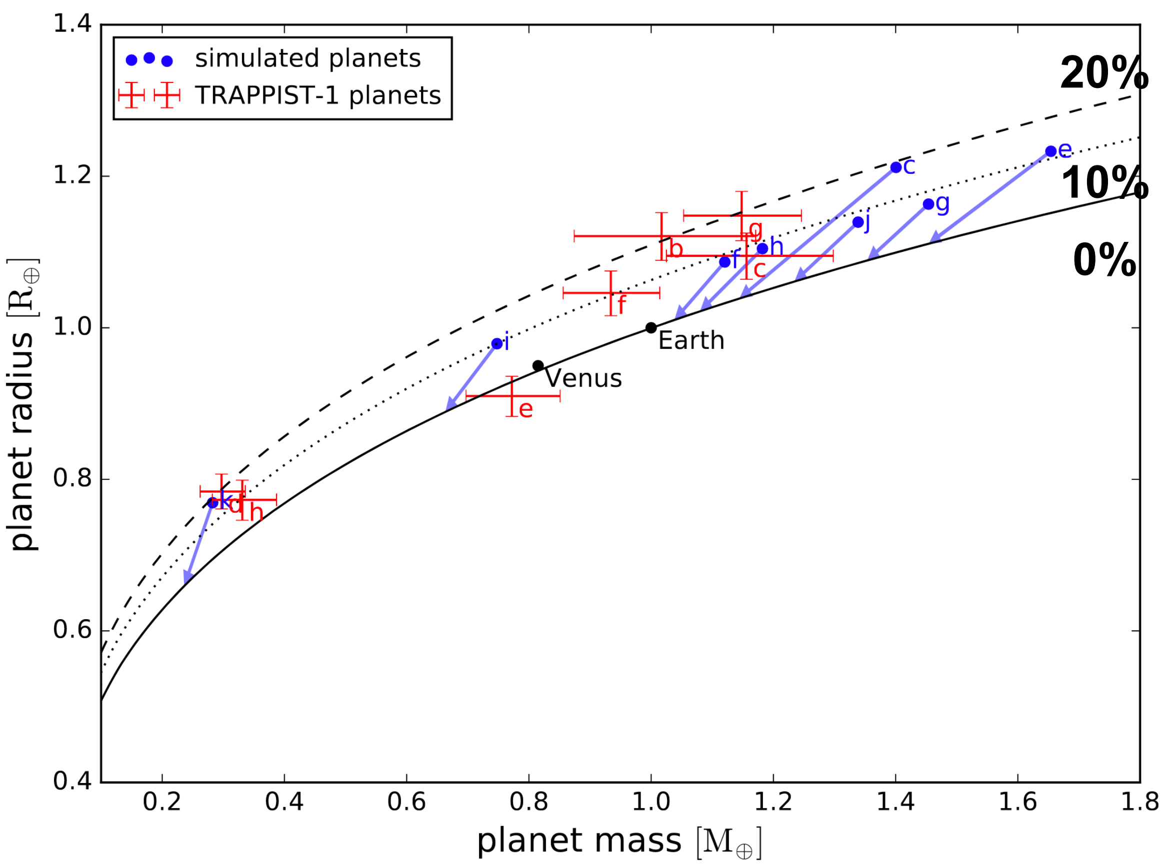
$$M_{\rm trans} = 10^{-3} - 10^{-2}\ M_\oplus$$

Liu, Ormel & Johansen 2019

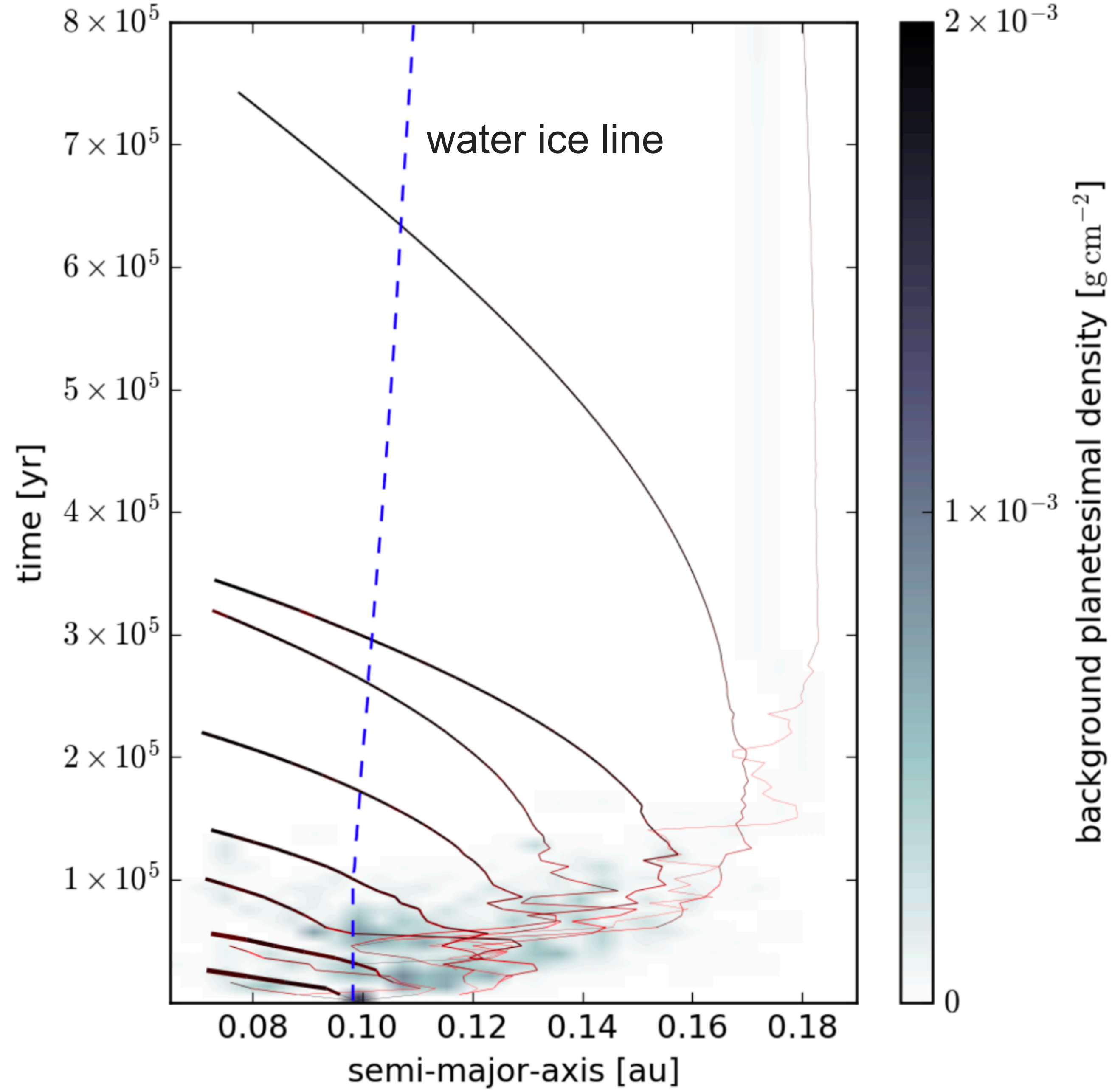
Formation of Trappist-1 system

7 Earth-mass planets around 0.08 solar-mass dwarf





planetesimal water ice line formation + **pebble accretion**



Ormel, Liu & Schoonenberg, 2017

Schoonenberg, Liu, Ormel & Dorn, 2019