原行星盘与行星形成暑期学校, 北京, 2024/07/24

Protoplanetary Disks: Gas Dynamics

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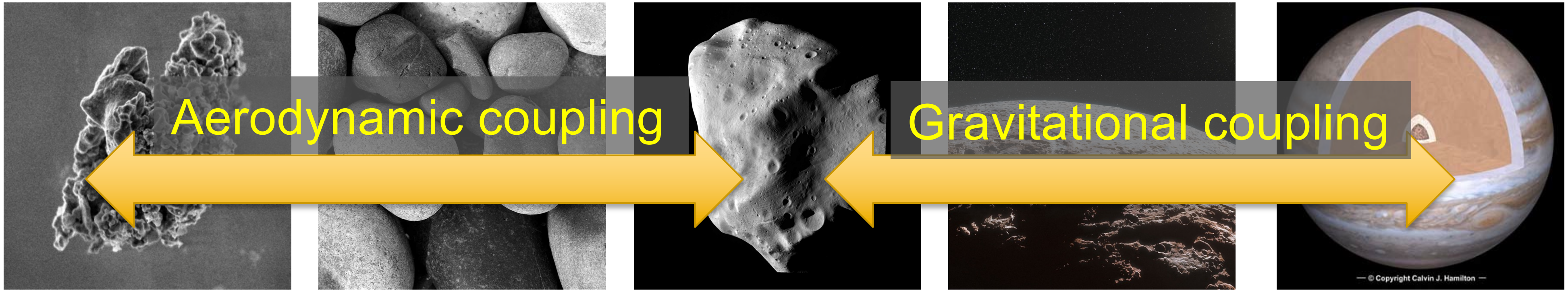
Why do we care about gas dynamics?



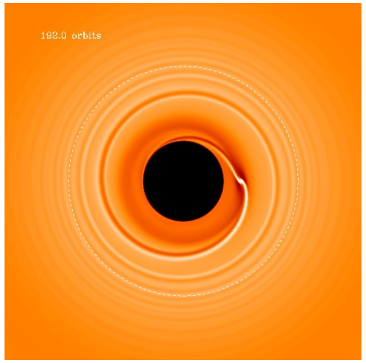
μm cm km 10^3km 10^5km

Grain growth

Planetesimal formation

Planetesimal growth to cores

growth/accretion to gas giants



Planet migration

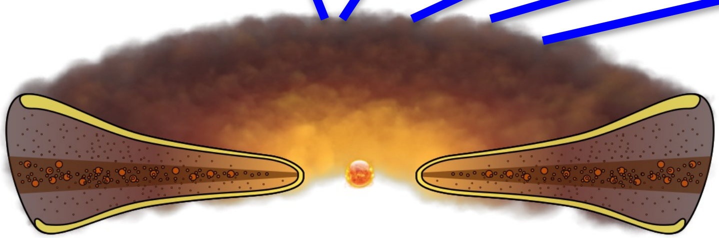
Essentially all processes depend on the gas dynamics of protoplanetary disks.

Most importantly:

- Global structure and disk evolution
- Level of turbulence

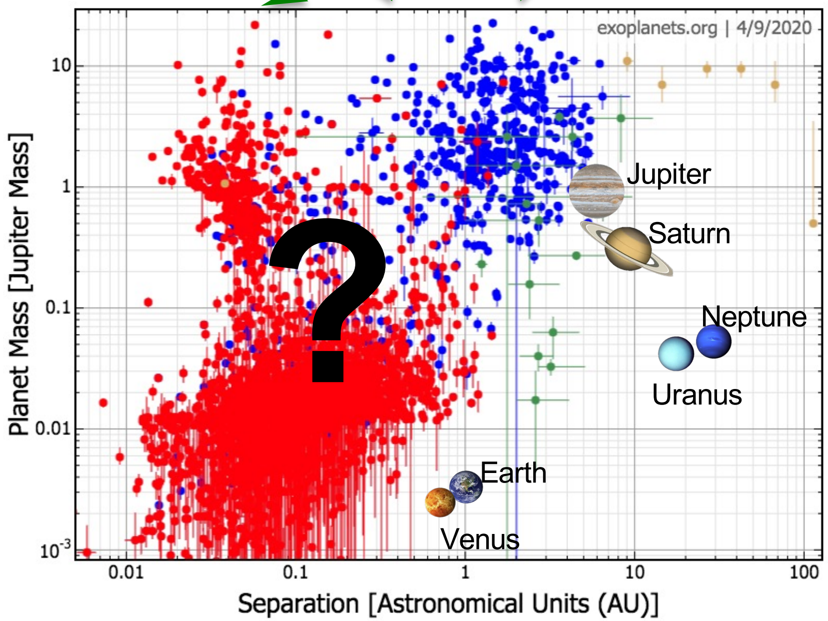
Theory confronting exoplanet populations

Grain growth | Planetesimal formation | Core growth | Accretion of gas | Planet migration



Disk structure
(density/temperature/chemical)

Flow structure
(turbulence, wind, circulation…)



Outline

- Angular momentum transport: general
- Radial transport: instabilities and turbulence
- Vertical transport: magnetized wind
- Recent development

Useful references:

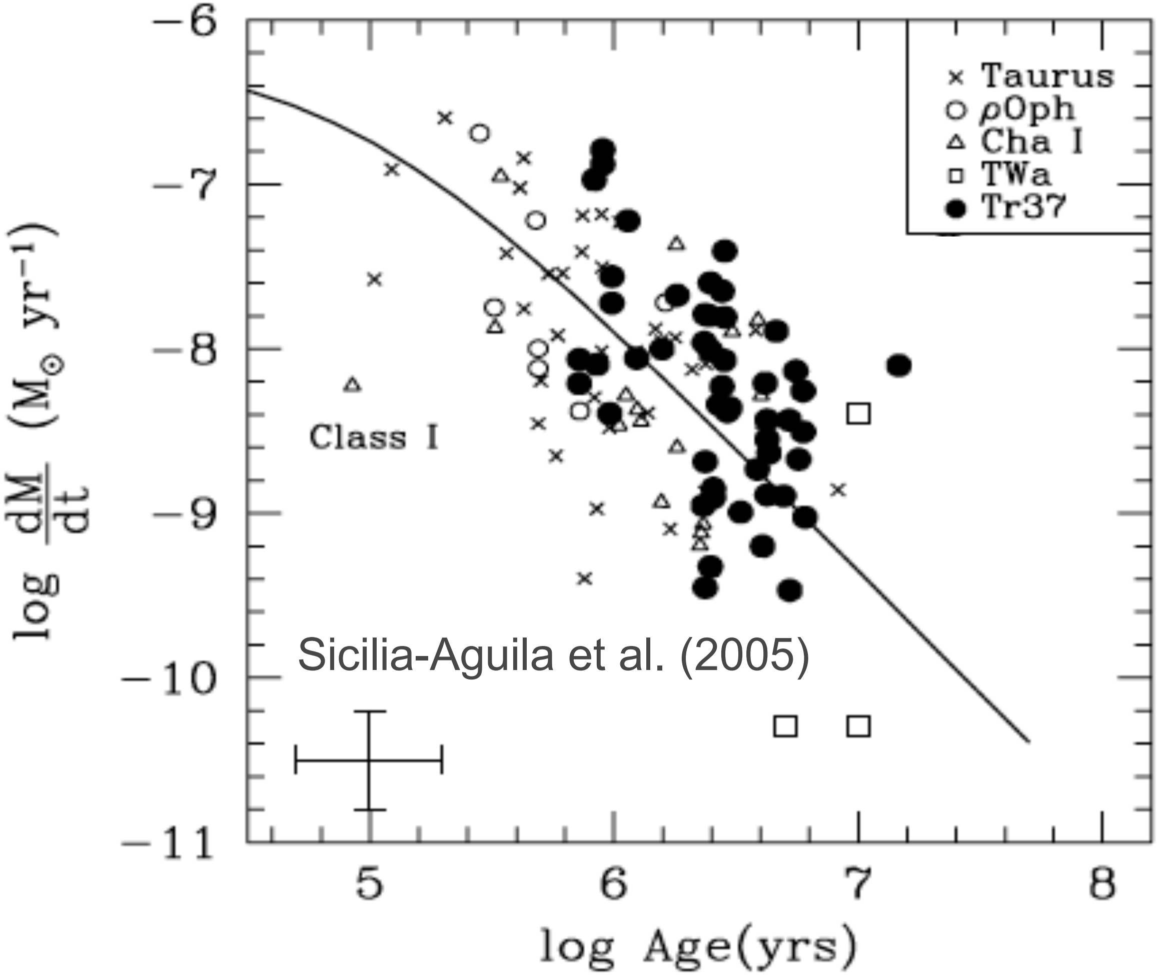
Physical Processes in Protoplanetary Disks by Armitage (2019)

PPVII review chapter by Lesur et al. (2023)

Review in the solar system context by Weiss, Bai & Fu (2021)

Review on hydrodynamic turbulence by Lyra & Umurhan (2019)

PPDs as accretion disks

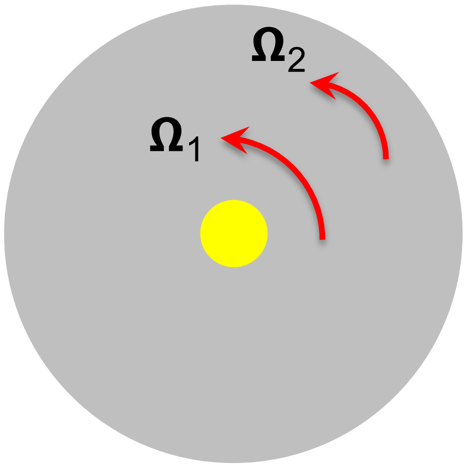


Note: stellar age can be approximately estimated based on stellar luminosity and photospheric temperature.

- Typical accretion rate ~ $10^{-8}\ M_\odot$ yr $^{-1}$ with large scatter.
- Accretion rate decreases with age.

They provide crucial constraints for understanding the gas dynamics of PPDs.

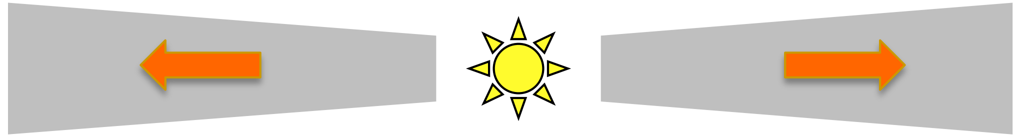
Angular momentum transport: viscosity



Viscosity tries to make $\mathbf{\Omega}_1=\mathbf{\Omega}_2$

Inner disk lose A.M to outer disk

Transport A.M. radially outward



viscosity / turbulence

By the gravito-/hydro/magnetic stresses

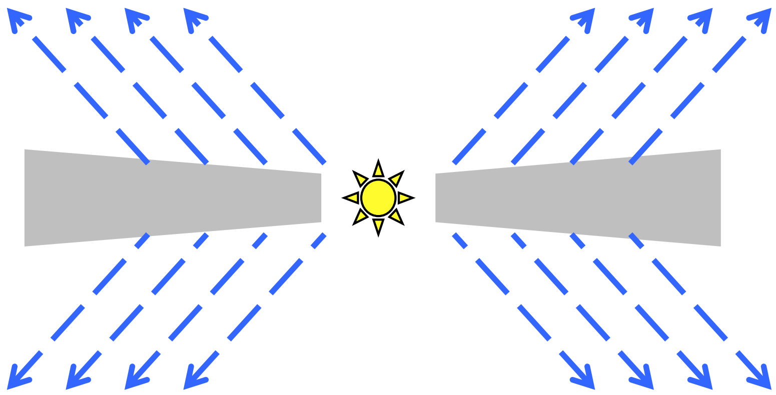
MHD mechanisms are sensitive to ionization

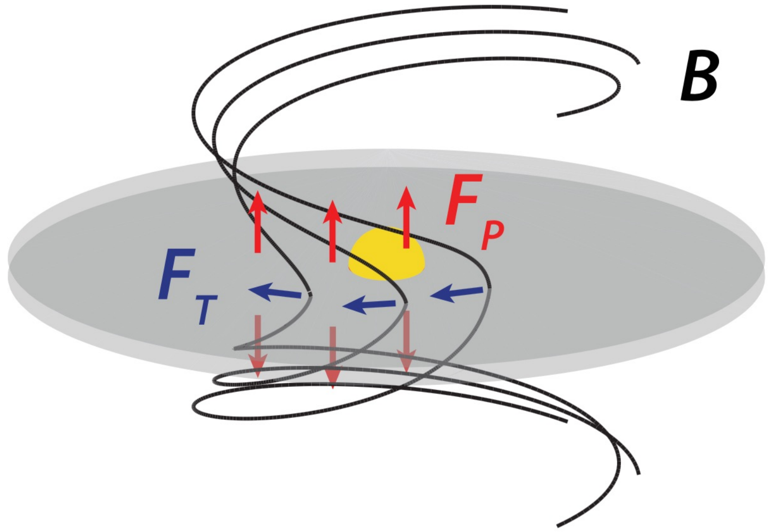
Hydro mechanisms are sensitive to thermodynamics

Gravitational stresses require massive disks

Angular momentum transport: magnetized wind

Transport A.M. vertically





Need large-scale poloidal field threading the disk.

(Adopted from Weiss, Bai & Fu, 2021)

Wind properties are sensitive to disk physics.

Angular momentum transport

Recall the momentum equation in conservative form:

$$\partial_t(\rho \boldsymbol{v}) + \nabla \cdot \mathsf{M} = -\rho \nabla \Phi$$

where the momentum flux tensor is: $$\mathsf{M} \equiv \rho \boldsymbol{v}\boldsymbol{v} - \frac{\boldsymbol{B}\boldsymbol{B}}{4\pi} + \left(P + \frac{B^2}{8\pi}\right)\mathsf{I} \ .$$

In cylindrical coordinates, the R and ϕ components become:

$$\begin{aligned} \partial_t(\rho v_R) + \frac{1}{R}\partial_R(R\mathsf{M}_{RR}) + \frac{1}{R}\partial_\phi \mathsf{M}_{\phi R} + \partial_z \mathsf{M}_{zR} &= \frac{1}{R}\mathsf{M}_{\phi\phi} - \rho \partial_R \Phi \ , \\ \partial_t(\rho v_\phi) + \frac{1}{R^2}\partial_R(R^2\mathsf{M}_{R\phi}) + \frac{1}{R}\partial_\phi \mathsf{M}_{\phi\phi} + \partial_z \mathsf{M}_{z\phi} &= -\frac{\rho}{R}\partial_\phi \Phi \ , \end{aligned}$$

The ϕ component can be rewritten to express angular momentum conservation:

$$\partial_t(\rho R v_\phi) + \frac{1}{R}\partial_R(R^2\mathsf{M}_{R\phi}) + \cancel{\frac{1}{R}\partial_\phi(R\mathsf{M}_{\phi\phi})} + \partial_z(R\mathsf{M}_{z\phi}) = \cancel{-\rho\partial_\phi\Phi}$$

Angular momentum transport

Integrating over z and ϕ, we obtain:

$$\frac{\partial(2\pi R\Sigma j_z)}{\partial t} + \frac{\partial}{\partial R}\left[2\pi R^2 \int_{-\infty}^{\infty} dz \left(\overline{\rho v_R v_\phi} - \frac{\overline{B_R B_\phi}}{4\pi}\right)\right] + 2\pi R^2 \left(\overline{\rho v_z v_\phi} - \frac{\overline{B_z B_\phi}}{4\pi}\right)\Bigg|_{-\infty}^{\infty} = 0$$

Accounting for the accretion flow, this becomes

$$\frac{\partial J_z}{\partial t} + \frac{\partial}{\partial R}\left(-\dot{M}_a j_z + 2\pi R^2 \int_{-\infty}^{\infty} T_{R\phi} dz\right) + 2\pi R^2 T_{z\phi}\Bigg|_{z=-z_s}^{z_s} = 0$$

AM flux due to accretion

AM transport (radial)

AM extraction (vertical)

$$J_z \equiv 2\pi R \Sigma j_z \ , \quad j_z \equiv R v_{\phi,0}$$

(specific AM)

$$T_{z\phi} \equiv \overline{\rho \delta v_z \delta v_\phi} - \frac{\overline{B_z B_\phi}}{4\pi}$$

$$T_{R\phi} \equiv \overline{\rho \delta v_R \delta v_\phi} - \frac{\overline{B_R B_\phi}}{4\pi}$$

Accretion rate (steady state)

Accounting for the accretion flow, this becomes

$$\frac{\partial J_z}{\partial t} + \frac{\partial}{\partial R}\left(-\dot{M}_a j_z + 2\pi R^2 \int_{-\infty}^{\infty} T_{R\phi} dz \right) + 2\pi R^2 T_{z\phi}\Big|_{z=-z_s}^{z_s} = 0$$

AM flux due to accretion

AM transport (radial)

AM extraction (vertical)

Radial transport

$$\dot{M}_a \approx \frac{2\pi}{\Omega} \int_{-\infty}^{\infty} T_{R\phi} dz$$

Vertical transport

$$\dot{M}_a \approx \frac{8\pi R}{\Omega} |T_{z\phi}|_{z_s}$$

(Assuming $T_{z\phi}(z_s)=-T_{z\phi}(-z_s)$).

If $T_{R\phi}$ and $T_{z\phi}$ are similar, then vertical transport (by wind) is more efficient than radial transport by a factor of ~R/H >>1.

The α-disk model

- Radial transport of angular momentum by viscosity:

$$T_{R\phi} = \alpha P$$

(Shakura & Sunyaev, 1973)

For N-S viscosity:

$$T_{R\phi} = -\sigma_{R\phi} = -\rho\nu R\frac{d\Omega}{dR} = \frac{3}{2}\rho\nu\Omega = \left(\frac{3}{2}\frac{\nu}{c_s H}\right)P$$

With microscopic viscosity: $\alpha \sim \lambda_{\rm mfp}/H \ll 1$

Required value of α to explain PPD accretion rate: $\sim 10^{-3}$-10^{-2}

Need anomalous viscosity (e.g., turbulence) to boost α.

Viscous evolution based on the α-disk model

Angular momentum conservation (ignore wind):

$$\frac{\partial J_z}{\partial t} + \frac{\partial}{\partial R}\left(- \dot{M}_a j_z + 2\pi R^2 \int_{-\infty}^{\infty} T_{R\phi} dz \right) + 2\pi R^2 T_{z\phi}\Big|_{z=-z_s}^{z_s} = 0$$

$$\Rightarrow \quad 2\pi R j_z \frac{\partial \Sigma}{\partial t} + \frac{\partial}{\partial R}\left(- \dot{M}_a j_z + 2\pi R^2 \alpha c_s^2 \Sigma \right) = 0$$

Mass conservation: $$2\pi R \frac{\partial \Sigma}{\partial t} - \frac{\partial \dot{M}_a}{\partial R} = 0$$

Viscous evolution equation:

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{R} \frac{\partial}{\partial R}\left[\frac{dR}{dj_z} \frac{\partial}{\partial R}\left(\alpha c_s^2 \Sigma R^2 \right) \right]$$

Note: $j_z(R) = \Omega R^2$

Gas is assumed to be locally isothermal.

Viscous evolution based on the α-disk model

Viscous evolution equation:

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{R}\frac{\partial}{\partial R}\left[\frac{dR}{dj_z}\frac{\partial}{\partial R}\left(\alpha c_s^2 \Sigma R^2\right)\right]$$

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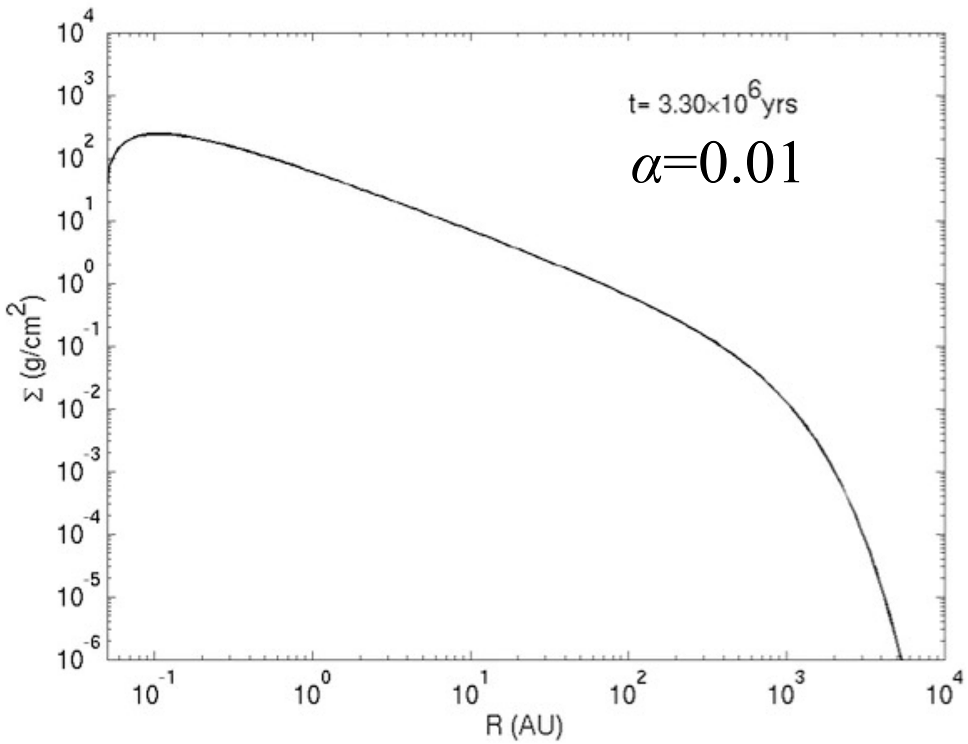
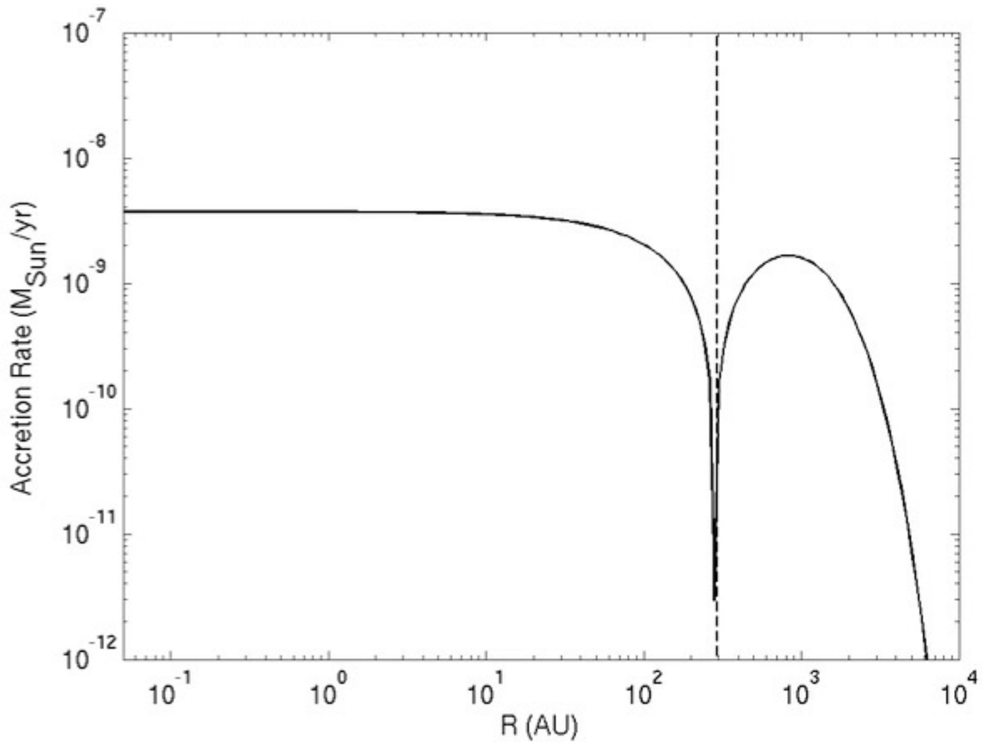




Wind-driven accretion

General expectation: wind extracts angular momentum => the entire disk shrinks

Armitage 2013, Bai 2016, Suzuki+2016, Tabone+2022

Governing equations:

$$2\pi R j_z \frac{\partial \Sigma}{\partial t} + \frac{\partial}{\partial R}\left(- \dot{M}_a j_z + 2\pi R^2 \int_{-z_b}^{z_b} T_{R\phi} dz \right) + 2\pi R^2 T_{z\phi}\Big|_{-z_b}^{z_b} = 0$$

$$2\pi R \frac{\partial \Sigma}{\partial t} - \frac{\partial \dot{M}_a}{\partial R} + \frac{\partial \dot{M}_{\rm loss}}{\partial R} = 0$$

Generalized equation for disk evolution:

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{R}\frac{\partial}{\partial R}\left[\frac{dR}{dj_z}\left(\frac{\partial}{\partial R}\alpha c_s^2 \Sigma R^2 + R^2 T_{z\phi}|_{-z_b}^{z_b}\right)\right] - \frac{\partial \dot{M}_{\rm loss}}{\partial R}$$

(diffusion) (advection) (loss)

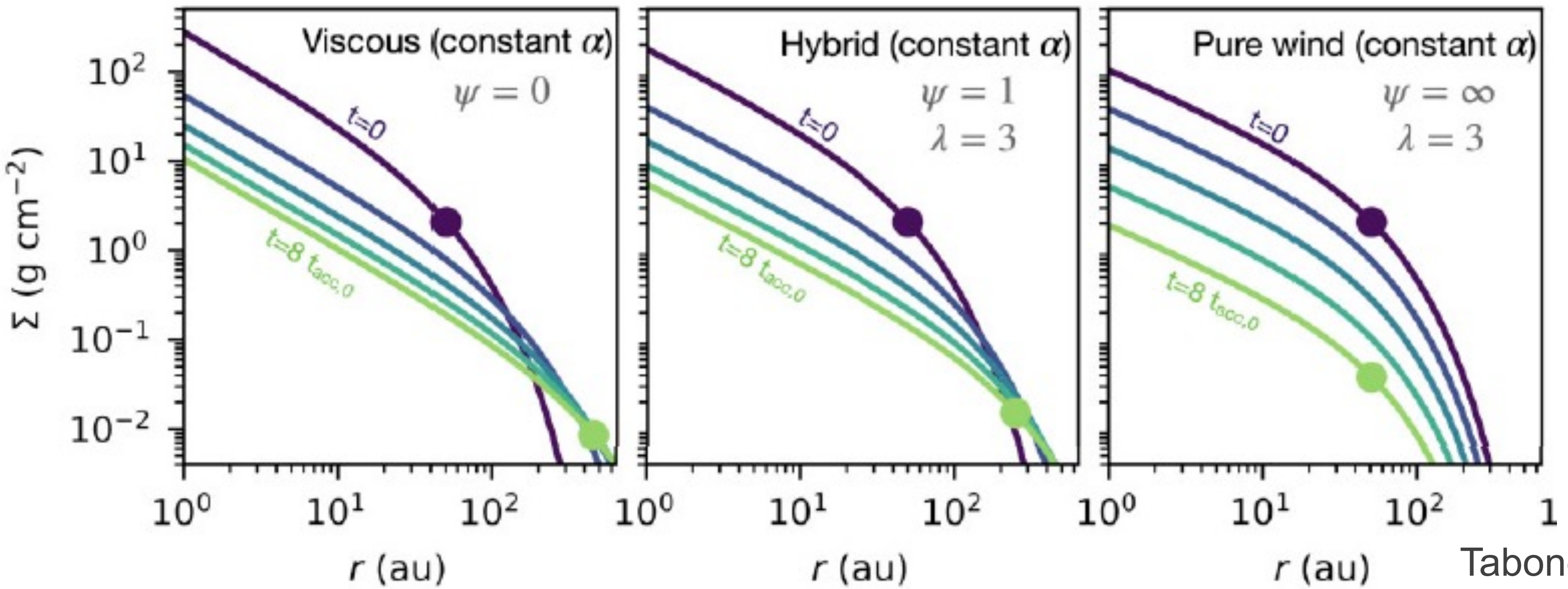
Wind-driven accretion

Disk evolution equation:

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{R}\frac{\partial}{\partial R}\left[\frac{dR}{dj_z}\left(\frac{\partial}{\partial R}\alpha c_s^2 \Sigma R^2 + R^2 T_{z\phi}|_{-z_b}^{z_b}\right)\right] - \frac{\partial \dot{M}_{\rm loss}}{\partial R}$$

(diffusion) (advection) (loss)

If we prescribe $T_{z\phi}$ in a similar way as $T_{R\phi}$, the results should qualitatively look like:



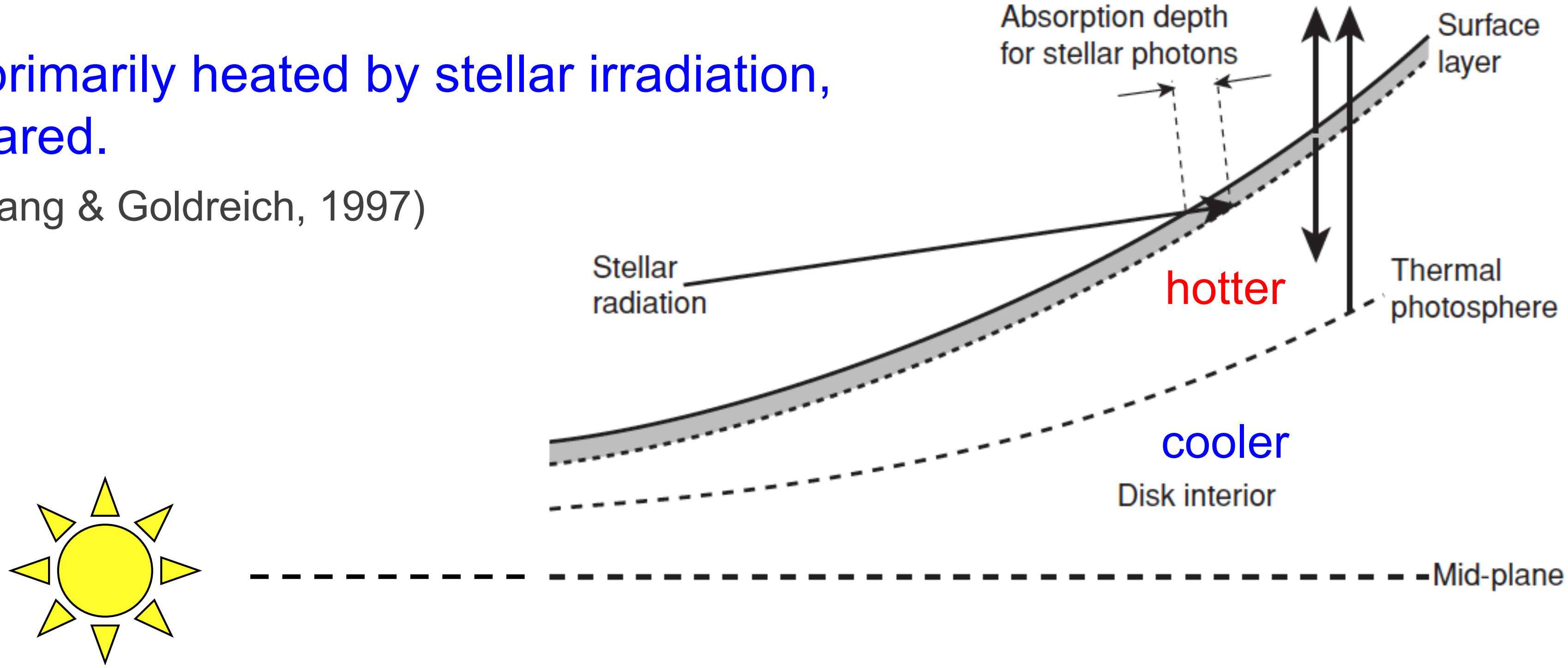
Tabone+2022

We will come back to this with new development.

Disk temperature structure: irradiation

Disk is primarily heated by stellar irradiation, and is flared.

(Chiang & Goldreich, 1997)



To 0th order, may approximate the disk equilibrium temperature as

$$\alpha \frac{L_*}{4\pi R^2} \approx \sigma T_{\rm disk}^4 \quad \Rightarrow \quad T_{\rm disk} \sim R^{-1/2}$$

(~R^{-1} for a flat disk)

=> Study disk gas dynamics on top of this temperature background.

Energy dissipation

Stress from radial transport of angular momentum:

$$T_{R\phi} \equiv \overline{\rho\delta v_R \delta v_\phi} - \frac{\overline{B_R B_\phi}}{4\pi}$$

Rate of energy generation (≈dissipation):

$$Q^+ = -\frac{d\Omega}{d\ln R} T_{R\phi} = \frac{3}{2}\Omega T_{R\phi}$$

Radial transport of A.M. is usually accompanied by (local) heating.

In general, viscous heating only dominates the inner disk region (<1AU).

Question: why?

Wind-driven accretion is accompanied by Joule heating (i.e., dissipation of magnetic field), but it is generally insignificant. (Mori+2019)

Outline

- Angular momentum transport: general
- Radial transport: instabilities and turbulence
- Vertical transport: magnetized wind
- Recent development

Stability criterion for Keplerian disks

Are Keplerian disks hydrodynamically stable?

- Rayleigh criterion: differentially rotating fluid is stable when specific angular momentum increases outward (Rayleigh, 1916):

$$\frac{d(R^2\Omega)}{dR} > 0$$

In other words, the epicyclic frequency:

$$\kappa^2 = \frac{1}{R^3}\frac{\partial}{\partial R}(R^4\Omega^2) > 0$$

- The more general Solberg-Høiland criterion for stability (e.g., Tassoul 1978):

$$\kappa^2 + N_R^2 + N_z^2 > 0$$

where

$$N_R^2 = \frac{1}{\rho C_p}\left|\frac{\partial P}{\partial R}\right|\frac{\partial S}{\partial R}\ , \quad N_z^2 = \frac{1}{\rho C_p}\left|\frac{\partial P}{\partial z}\right|\frac{\partial S}{\partial z}\ ,$$

Brunt-Väisälä frequency (buoyancy)

Note: this criterion applies to adiabatic perturbations.

Keplerian disks are generally considered to be hydrodynamically stable!

Hydrodynamic instabilities

- Gravitational instability

Toomre's Q parameter:

$$Q \equiv \frac{c_s \Omega}{\pi G \Sigma}$$

Disk is gravitationally unstable if $Q<1$.

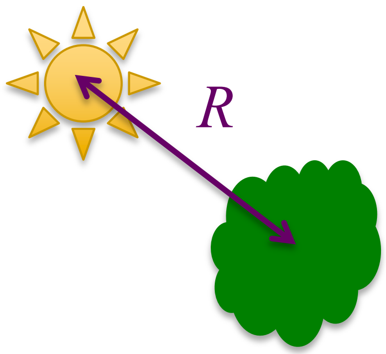
It is likely achieved in early stages of PPDs when it is massive, but not over most of their lifetime.

- Rossby-wave instability
- Convective overstability
- Vertical shear instability
- Zombie vortex instability

Many of these instabilities require special thermodynamic conditions to operate, and are not sufficiently powerful to drive rapid accretion.

Gravitational instability

Consider a clump of gas in the disk with size L.



$$M_{\rm clump} \sim \Sigma \pi L^2$$

Self-gravity, tidal force, pressure

Rotation (tidal force) stabilizes against gravitational collapse if:

$$\frac{GM_*}{R^2}\frac{L}{R} > \frac{GM_{\rm clump}}{L^2}$$

$$\Rightarrow \quad L > \frac{\pi G \Sigma}{\Omega^2}$$

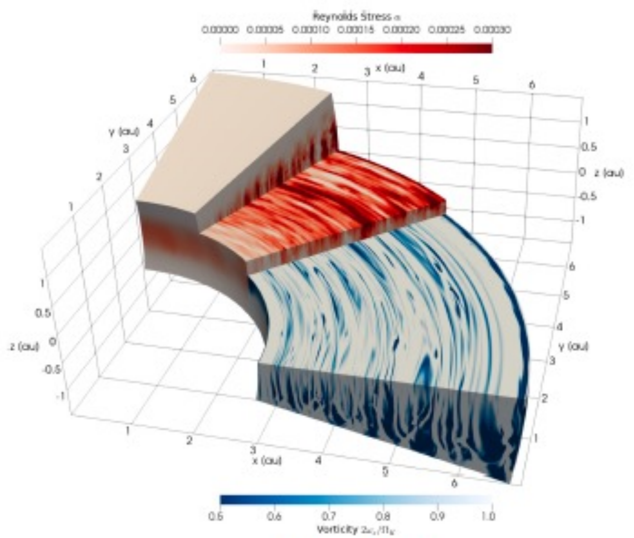
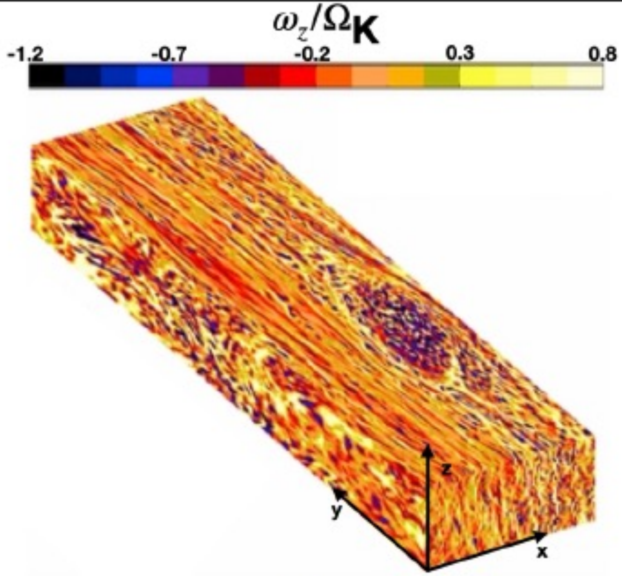
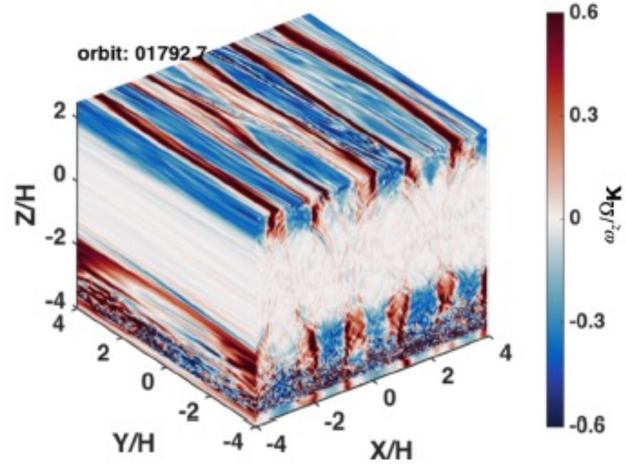
Gas pressure stabilizes against gravitational collapse if:

$$\frac{1}{\rho}\frac{P}{L} > \frac{GM_{\rm clump}}{L^2} \quad \Rightarrow \quad L < \frac{c_s^2}{\pi G \Sigma}$$

Thermo-hydrodynamical instabilities

PPDs are not adiabatic, and perturbations can be non-linear.

From Lesur+2023 review:

<table>
<tr><td>Vertical Shear Instability</td><td>Convective Overstability</td><td>Zombie Vortex Instability</td></tr>
<tr><td>
</td><td>
</td><td>
</td></tr>
<tr><td>$\tau_{\rm cool}\Omega_{\rm K} \ll 1$</td><td>$\tau_{\rm cool}\Omega_{\rm K} \sim 1$</td><td>$\tau_{\rm cool}\Omega_{\rm K} \gg 1$</td></tr>
<tr><td>$q \neq 0$</td><td>$-1 < p/q < 1/(\gamma-1)$</td><td>$|z| \gtrsim \sqrt{\gamma/(\gamma-1)}H$</td></tr>
<tr><td>$\alpha_{\rm SS} \sim 10^{-4}$</td><td>$\alpha_{\rm SS} \sim 10^{-3}$</td><td>$\alpha_{\rm SS} \sim (10^{-5}\text{–}10^{-4})^{\dagger}$</td></tr>
<tr><td colspan="3">Outcome: turbulence & vortices</td></tr>
</table>

Nelson+2013 (Nearly isothermal)

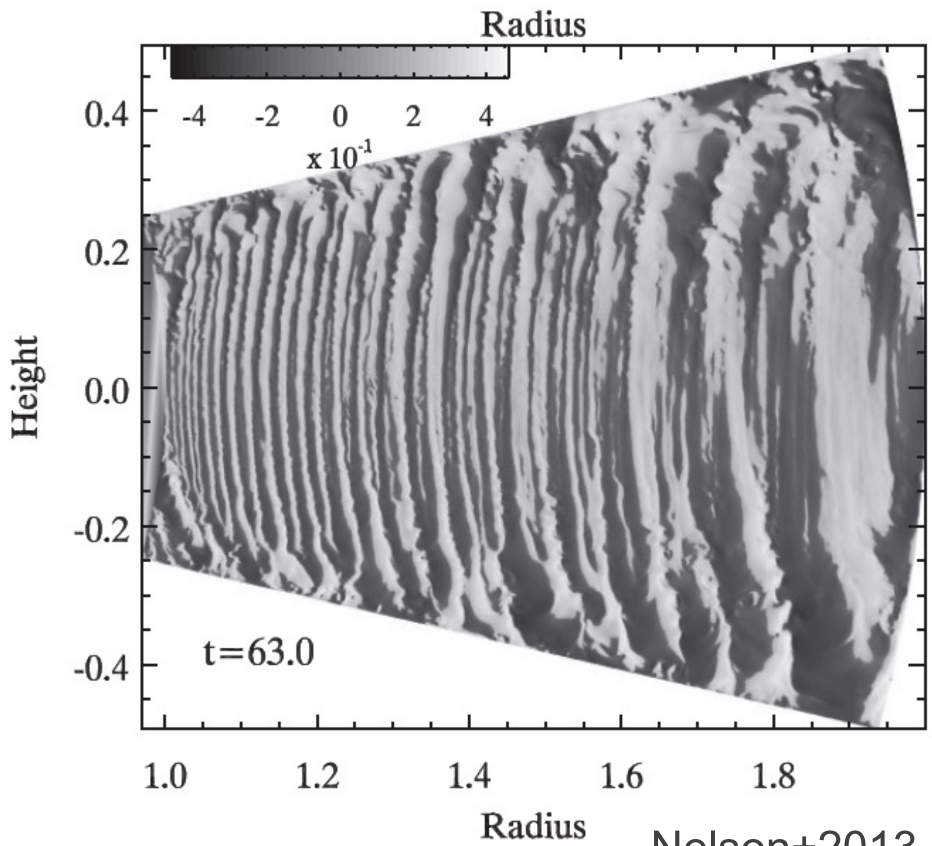
Klahr & Habbard 2014, Lyra 2014 (cooling time ~ dynamical time)

Marcus et al. 2013 (adiabatic but nonlinear)

The vertical shear instability (VSI)

In hydrostatic equilibrium, PPDs generally have vertical shear in rotation velocity => source of free energy to power the VSI (with short coolilng).

(variant of the GSF instability in differentially rotating stars (Goldreich & Schubert 1967, Fricke 1968)



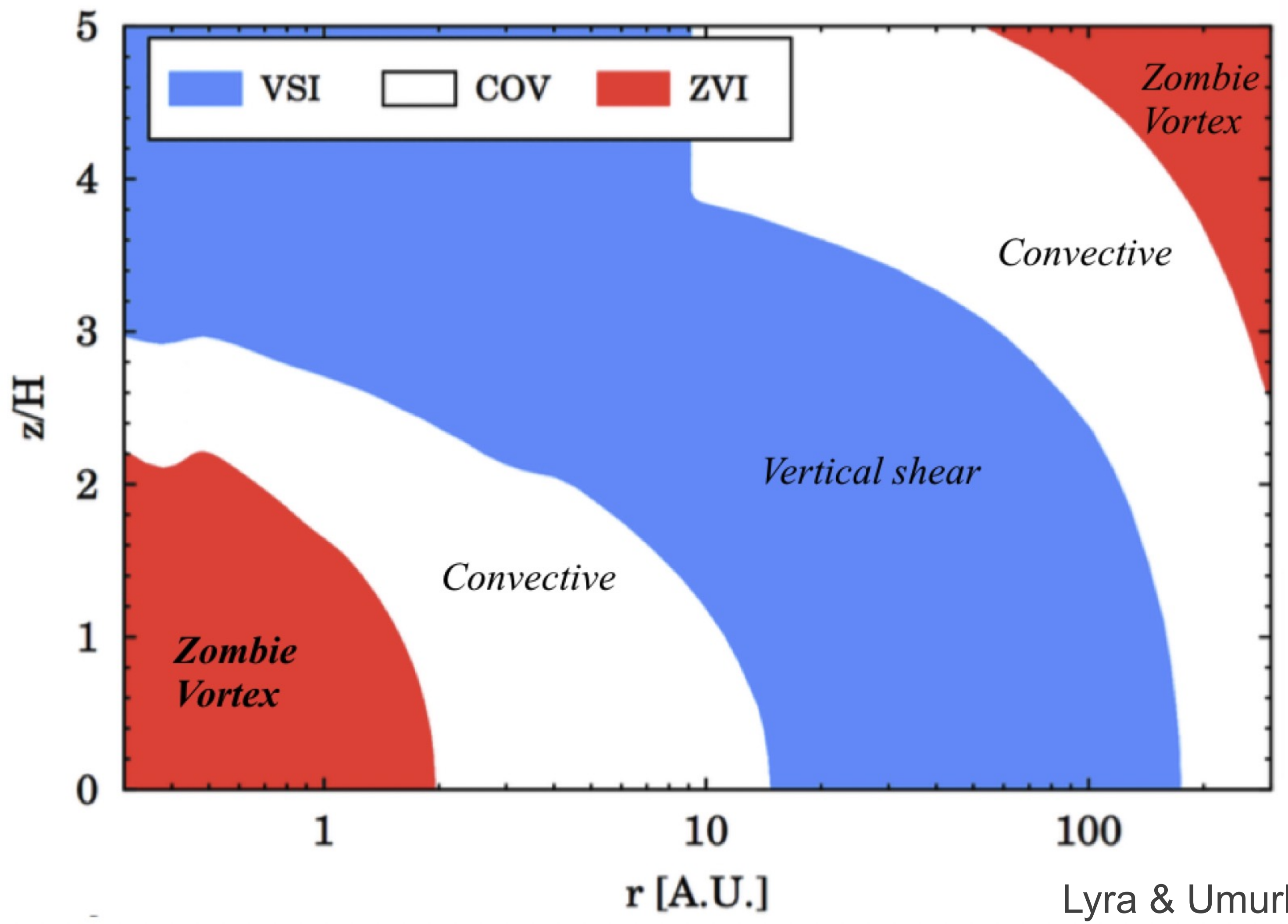
Nelson+2013

Leads to prominent vertical motion (body modes) and hence dust stirring.

In 3D, will produce turbulence by secondary instabilities.

(Richard+2016, Manger & Klahr 2018)

Occurrence of thermo-hydro-instabilities



Lyra & Umurhan 2019

Based on estimated cooling rates with standard disk model and dust opacities.

The Rossby-wave instability

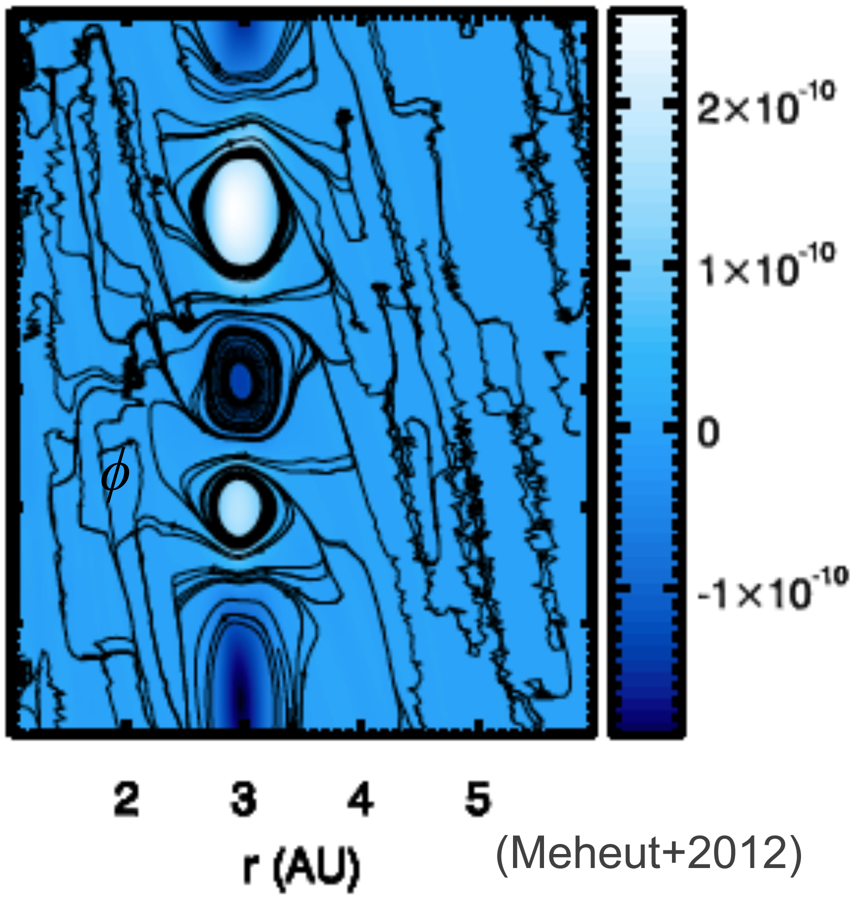
Rossby waves occur in rotating fluids under a gradient in potential vorticity.

In the disk context, it is defined as $\Omega_{\rm PV} \equiv \dfrac{(\nabla \times \boldsymbol{v})_z}{\Sigma}$

The Rossby wave instability (RWI) occurs when there is a non-smooth radial variation in potential vorticity.
(Lovelace+ 1999, Ono+16)

Lead to the production of multiple vortices, eventually merging into one giant vortex.
(e.g., Li+ 2000,2001,Meheut+2010, 2012)

Usually results from a radial pressure bump, e.g., from planet-disk interaction
(e.g., de Val-boro+ 2007, Lyra+2009, Zhu+ 2014)



(Meheut+2012)

More later (Ruobing/Pinghui's lectures).

Magnetorotational instability (MRI)

- Rayleigh criterion for unmagnetized rotating disks:

Unstable if: $$\frac{d(\Omega R^2)}{dR} < 0$$ (Rayleigh, 1916)

Confirmed experimentally (Ji et al. 2006).

All astrophysical disks should be stable against this criterion.

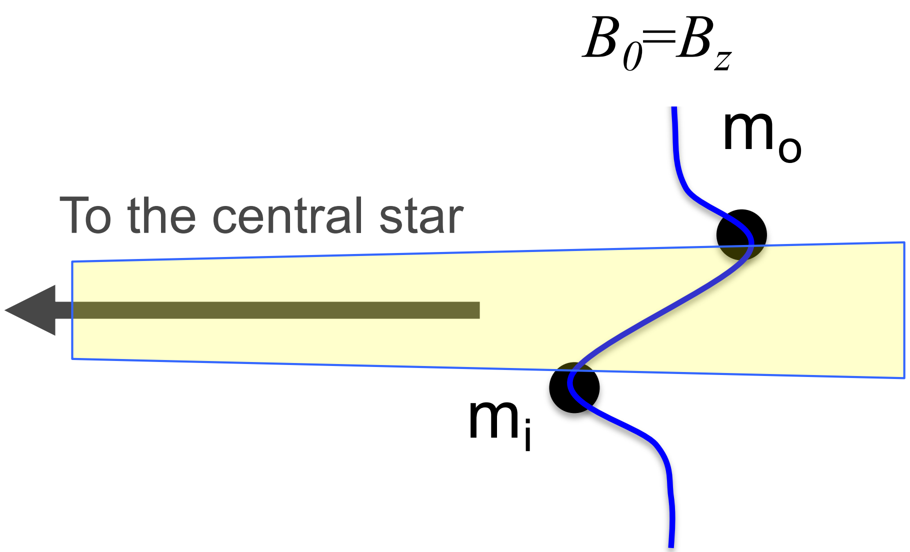
- Including (a vertical, well-coupled) magnetic field qualitatively changes the criterion (even as B->0):

Unstable if: $$\frac{d\Omega}{dR} < 0$$ Velikhov (1959), Chandrasekhar (1960), Balbus & Hawley (1991)

All astrophysical disks should be unstable!

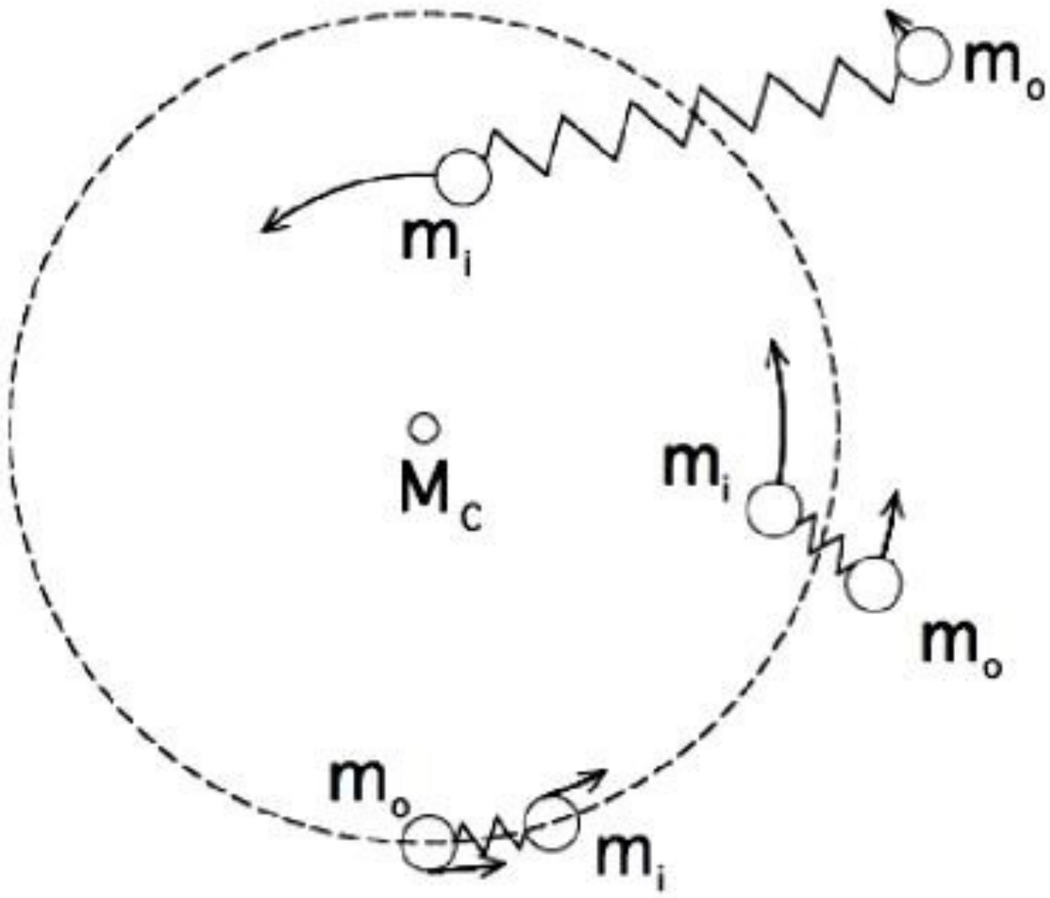
Magnetorotational instability (MRI)

Edge on view:

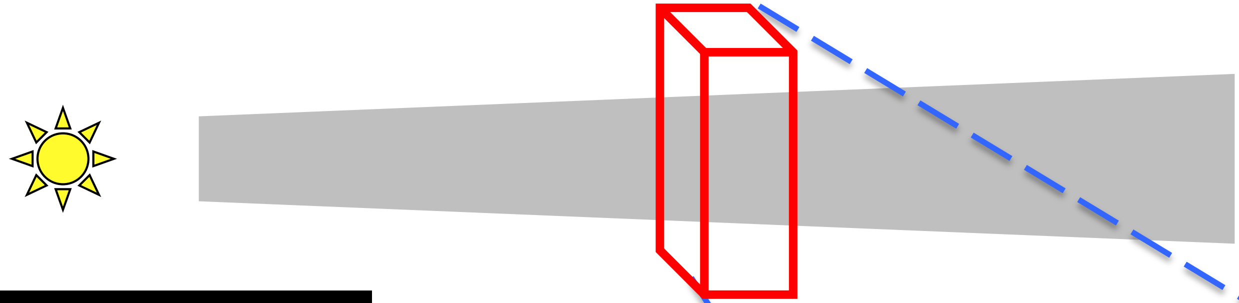


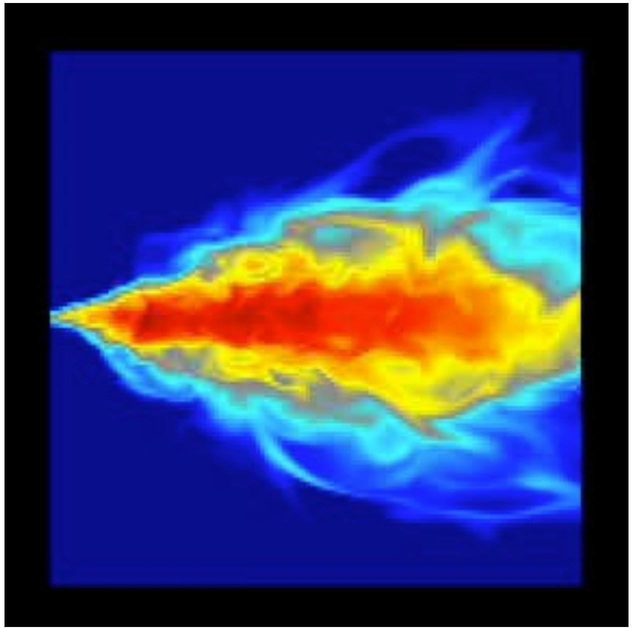
Magnetic tension force behaves like a spring.

Face on view:



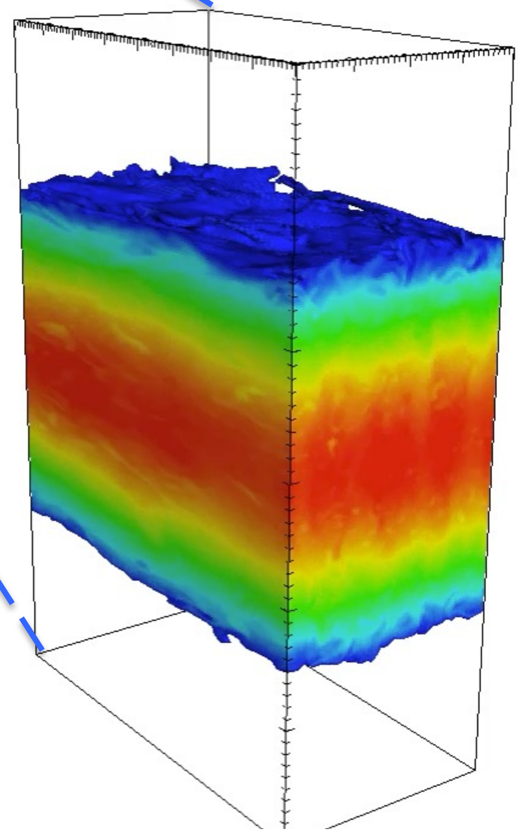
Local and global simulations of the MRI





from John Hawley

For a black hole accretion disk, but physics is similar.

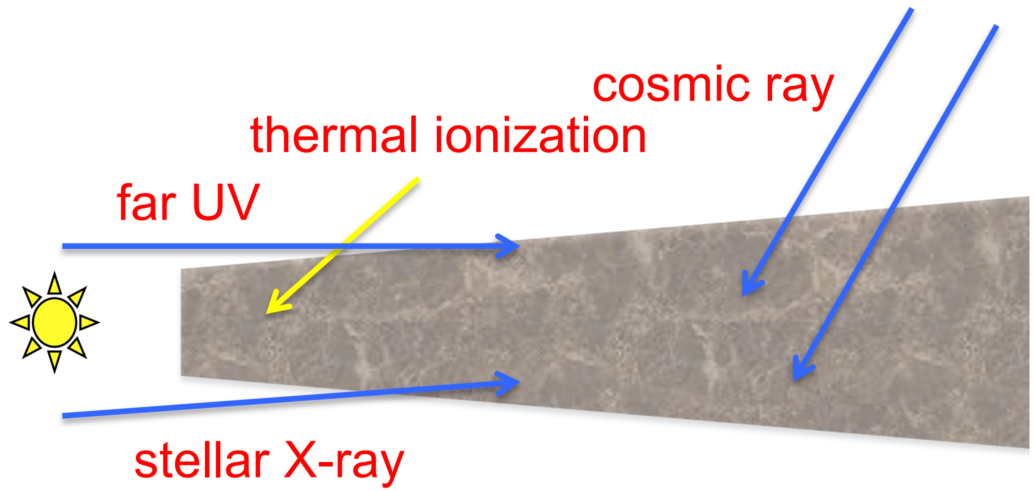


$$\alpha \sim (\delta v / c_s)^2$$

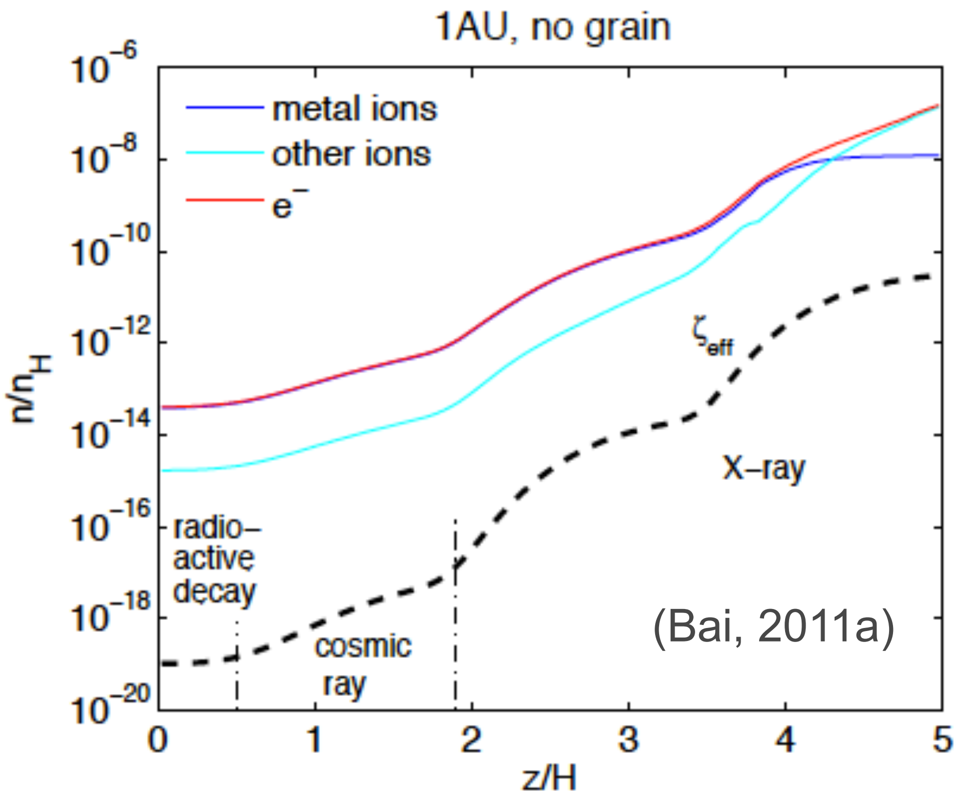
Typically, α ~0.01-0.1

Sufficient to explain PPD accretion rates, **BUT**…

PPDs are extremely weakly ionized



Umibayashi & Nakano (1981)
Igea & Glassgold (1999)
Perez-Becker & Chiang (2011b)

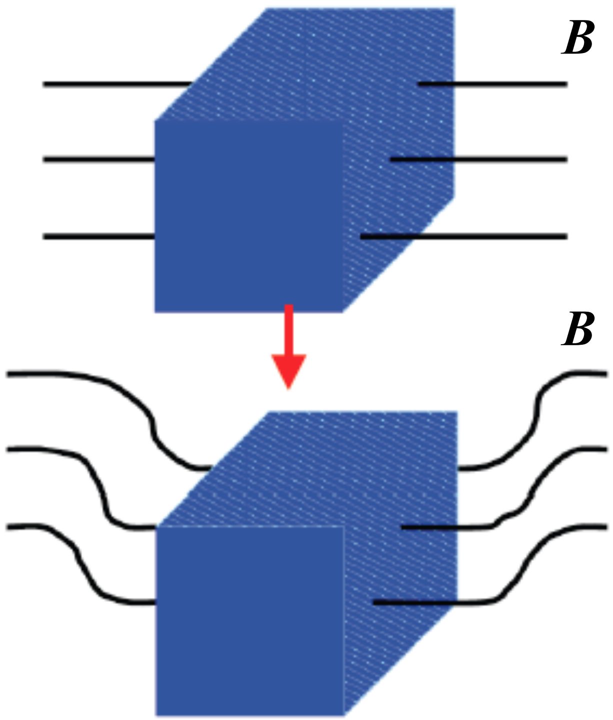


Very weak ionization in the gas substantially reduces the coupling between gas and magnetic fields, which can suppress the MRI!

Gas-B field coupling in weakly ionized gas

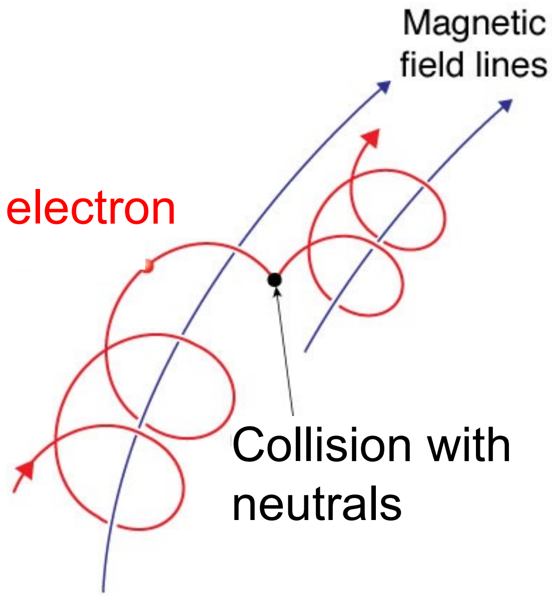
Ionized gas: well coupled

B field is frozen in the gas
(flux freezing)



Ideal Magnetohydrodynamics (MHD)

Weakly ionized gas: poorly coupled



B field is frozen in the electrons
(most mobile charged species)

+

Resistivity: B field slips through the electrons due to collisions.

Non-ideal MHD

Disk microphysics: non-ideal MHD effects

Induction equation (grain-free):

resistivity

$$\frac{\partial \boldsymbol{B}}{\partial t} = c\nabla \times \boldsymbol{E} = \nabla \times (\boldsymbol{v}_e \times \boldsymbol{B})$$

$$\boldsymbol{v}_e = \boldsymbol{v} + (\boldsymbol{v}_e - \boldsymbol{v}_i) + (\boldsymbol{v}_i - \boldsymbol{v})$$

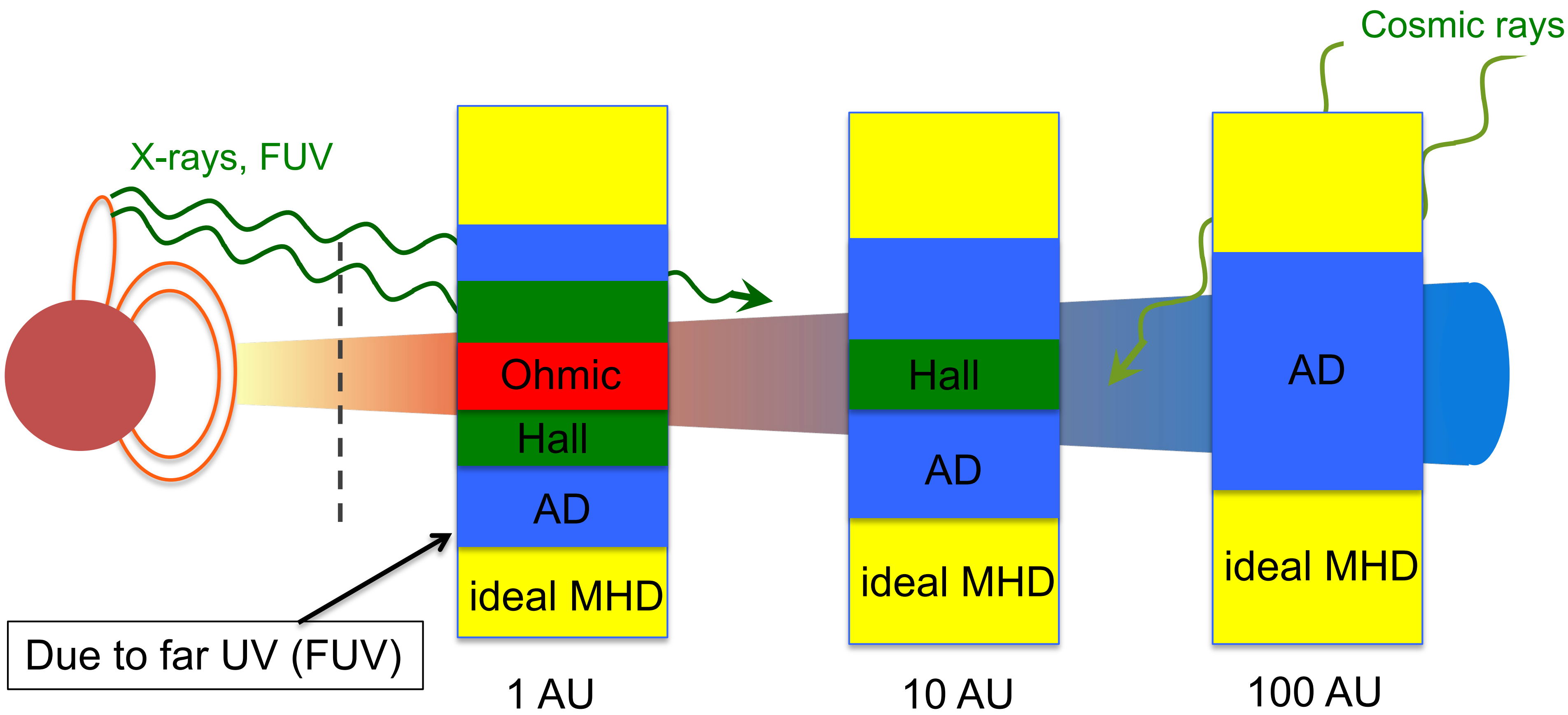
$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) - \nabla \times \left[\frac{4\pi\eta}{c}\boldsymbol{J} + \frac{\boldsymbol{J} \times \boldsymbol{B}}{en_e} - \frac{(\boldsymbol{J} \times \boldsymbol{B}) \times \boldsymbol{B}}{c\gamma\rho\rho_i} \right]$$

Non-ideal MHD terms

inductive — Ohmic — Hall — Ambipolar diffusion (AD)

$$\sim \frac{n}{n_e} \qquad \sim \frac{n}{n_e}\frac{B}{\rho} \qquad \sim \frac{n}{n_e}\frac{B^2}{\rho^2}$$

Disk microphysics: non-ideal MHD effects

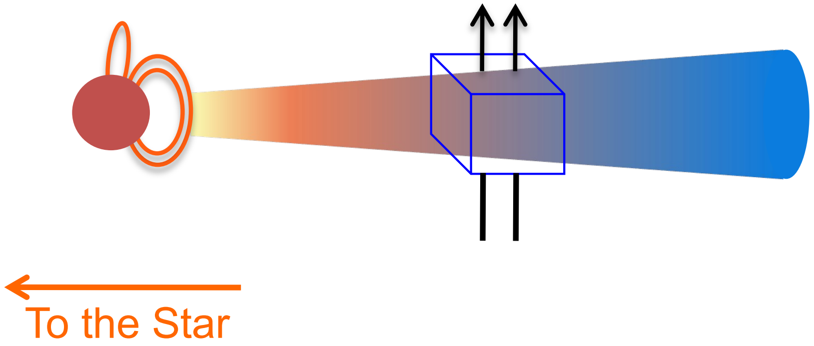


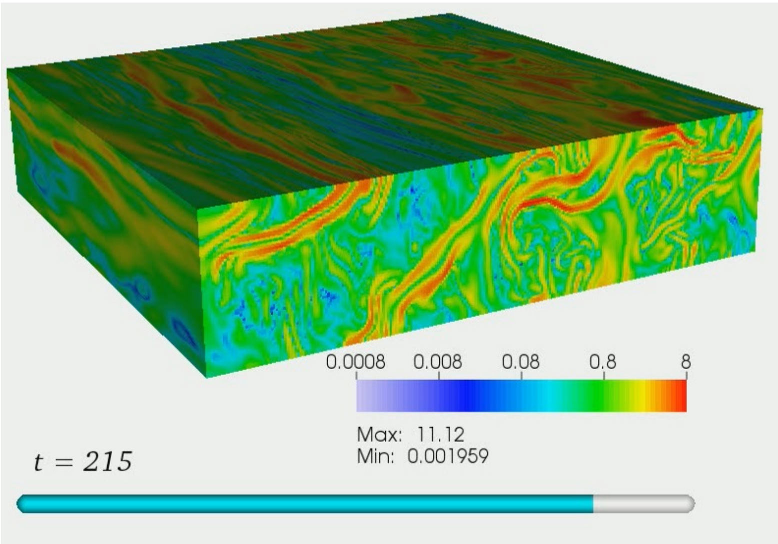
Elsasser numbers: $\Lambda_{O,H,A} = \frac{v_A^2}{\eta_{O,H,A}\Omega}$

Characterizes the importance of these effects. Unimportant when >>1.

MRI with ambipolar diffusion (AD)

Early studies in shearing box:

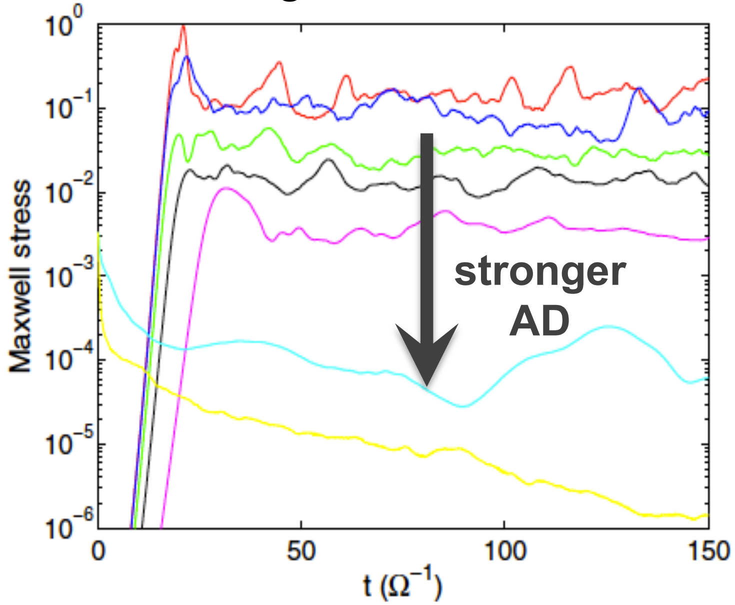


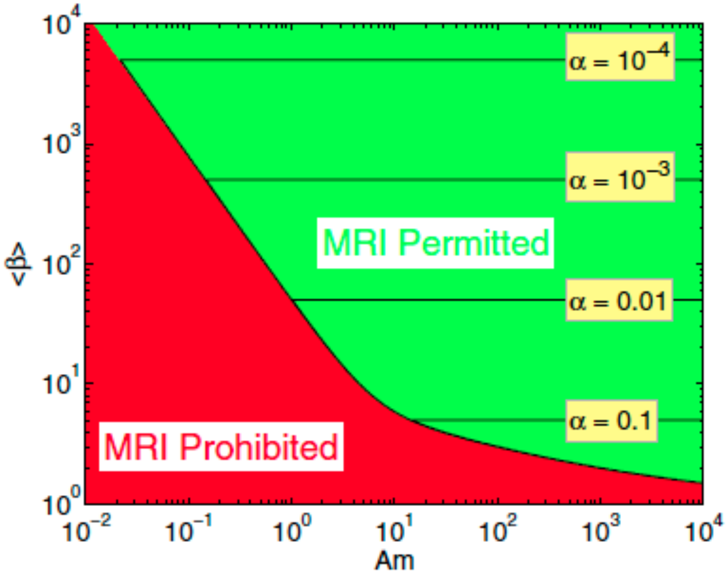


The MRI is suppressed/ damped with strong AD, making it insufficient to drive rapid accretion.

(Bai 11a,b, Perez-Becker & Chiang 11a,b)

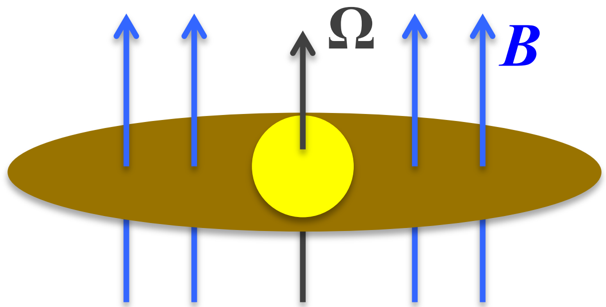
strength of turbulence

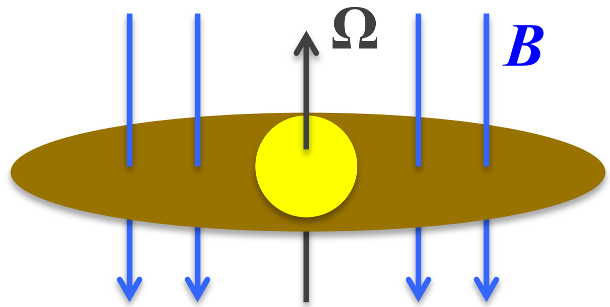




(Bai & Stone 2011)

The Hall effect: what happens if we flip B?





Lorentz force: $\sim \boldsymbol{J} \times \boldsymbol{B}$ is unaffected.

Note $\boldsymbol{J} = \frac{c}{4\pi} \nabla \times \boldsymbol{B}$

Induction equation (no grain):

$$\frac{\partial B}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) - \nabla \times \left[\frac{4\pi\eta}{c} \boldsymbol{J} + \frac{\boldsymbol{J} \times \boldsymbol{B}}{e n_e} - \frac{(\boldsymbol{J} \times \boldsymbol{B}) \times \boldsymbol{B}}{c \gamma \rho \rho_i} \right]$$

	inductive	Ohmic	Hall	AD
-	-	-	$(-)^2 = +$	$(-)^3 = -$

The Hall term is Polarity Dependent!

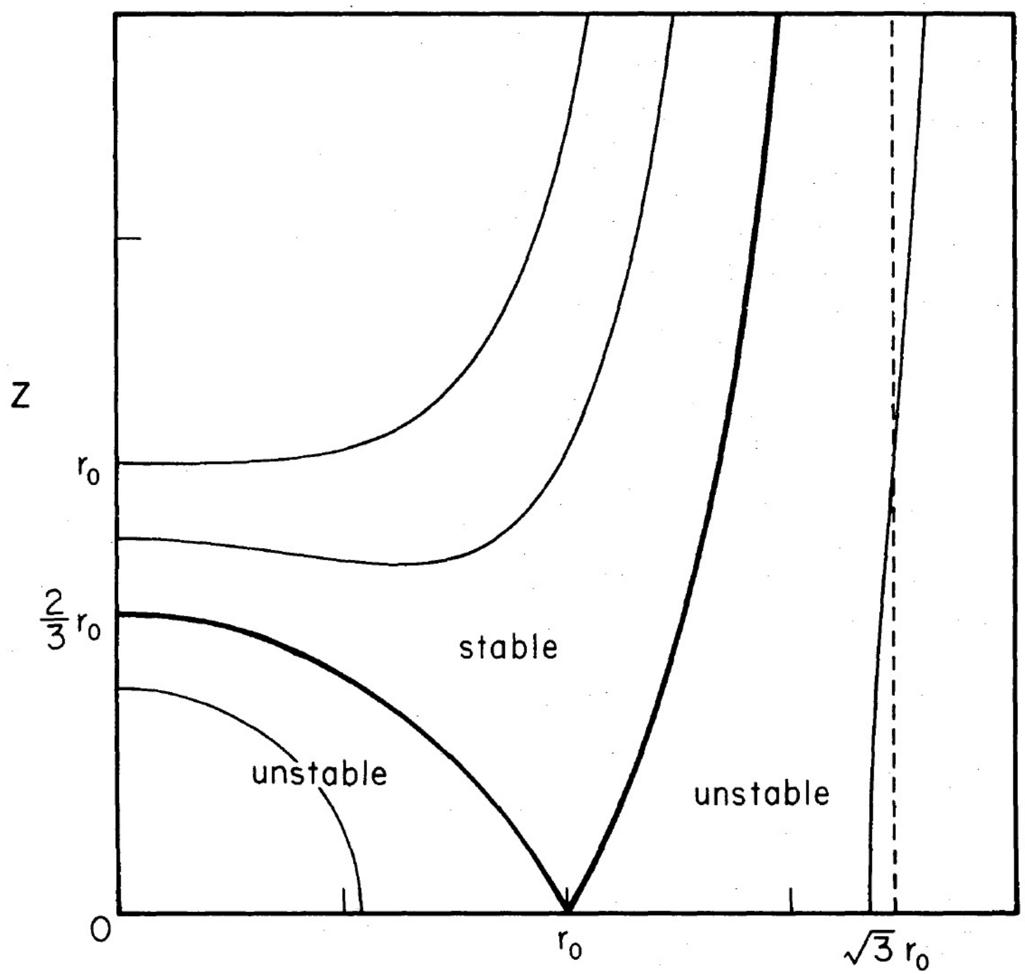
Local simulations if PPDs that include the Hall effect: Bai 14,15, Lesur+14, Simon+15

Outline

- Angular momentum transport: general
- Radial transport: instabilities and turbulence
- Vertical transport: magnetized wind
- Recent development

Magnetized winds: strong poloidal field

Magneto-centrifugal wind:



(Blandford & Payne 1982)

B field is anchored to a razor-thin disk.

For sufficiently strong B field, it remains poloidal and enforces corotation with the disk at the footpoint.

Effective potential (corotation with r_0):

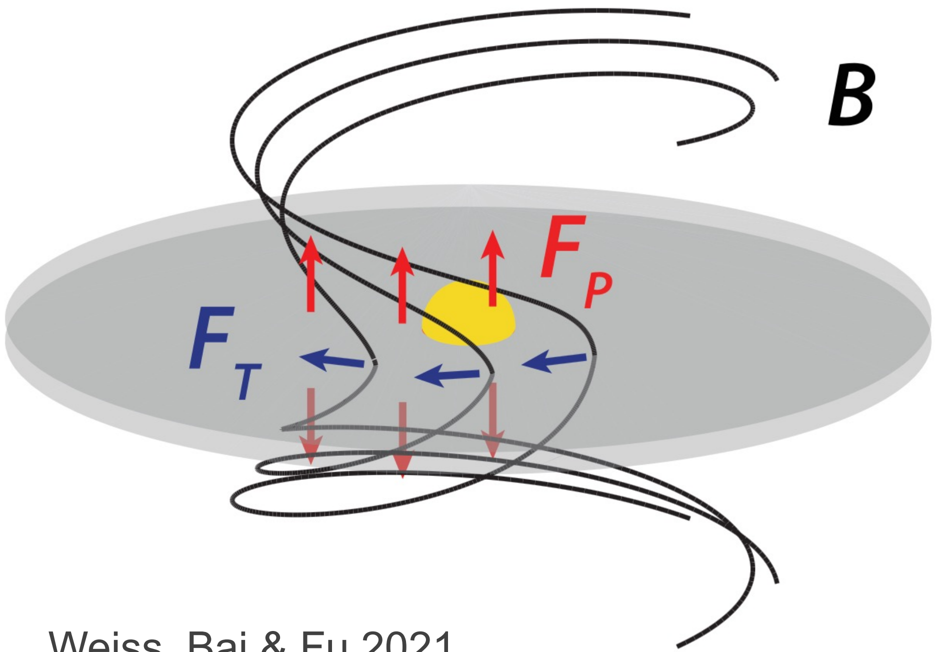
$$\phi(r,z) = -\frac{GM}{r_0}\left[\tfrac{1}{2}\left(\frac{r}{r_0}\right)^2 + \frac{r_0}{(r^2+z^2)^{1/2}}\right]$$

centrifugal central star

<u>Centrifugal fling when inclination about the disk is less than 60deg.</u>

Magnetized winds: weak poloidal field

Magnetic pressure gradient:



Weiss, Bai & Fu 2021

B field is anchored to the disk.

For weak B field, it can no longer enforce corotation with footpoint and develops strong toroidal component, with two consequences:

1. Magnetic pressure (from B_ϕ) builds up and pushes the gas up (i.e. outflow).

2. Field lines are bent => the restoring force from magnetic tension extracts disk angular momentum.

See Spruit (1996)

Magnetized winds: conservation laws

See Spruit (1996)

Assumptions: steady state, ideal MHD and axisymmetry.

Field and velocity can be decomposed by

$$\boldsymbol{B} = \boldsymbol{B}_p + B_\phi \boldsymbol{e}_\phi \ , \qquad \boldsymbol{v} = \boldsymbol{v}_p + R\Omega(R)\boldsymbol{e}_\phi \ ,$$

poloidal toroidal

=> $\boldsymbol{v}_p \parallel \boldsymbol{B}_p$

The following quantities are conserved along poloidal field lines (launched from R_0):

Mass flux: $k \equiv \dfrac{4\pi\rho v_p}{B_p}$

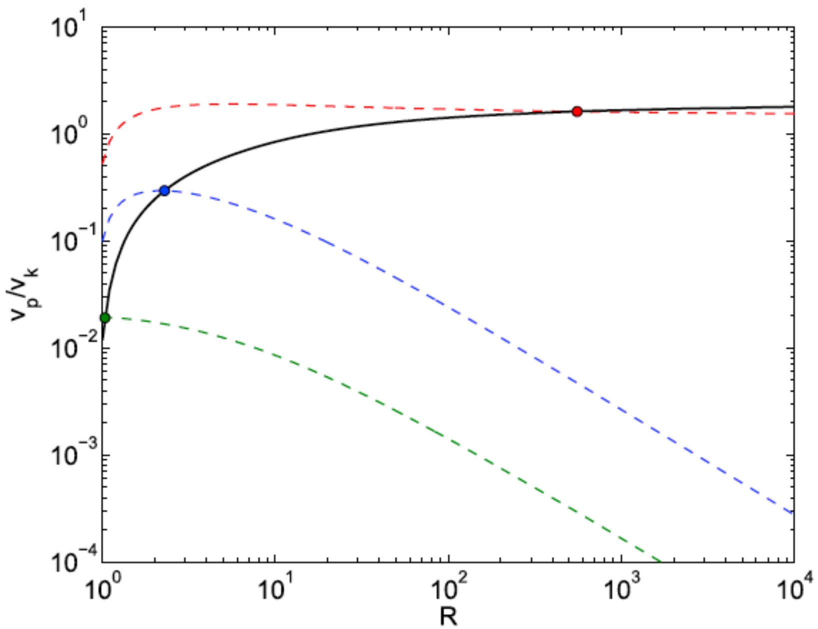
Angular velocity of magnetic flux surface: $\omega \equiv \Omega - \dfrac{kB_\phi}{4\pi\rho R}$

Angular momentum flux: $l \equiv \Omega R^2 - \dfrac{RB_\phi}{k} = \omega R_A^2$ ← Alfven radius

Energy flux: $e \equiv \dfrac{v^2}{2} + h + \Phi - \dfrac{\omega R B_\phi}{k}$

Bernoulli: $E \equiv e - \omega l = \dfrac{v^2}{2} - \omega R v_\phi + h + \Phi = \dfrac{v_p^2 + (v_\phi - \omega R)^2}{2} + h + \Phi_{\rm eff}$

Magnetized winds: Alfven radius



Bai et al. 2016

Critical points:

v_p= slow/Alfven/fast magnetosonic speed

At the Alfvén point: $v_p^2 = B_p^2/4\pi\rho$

Lever arm:

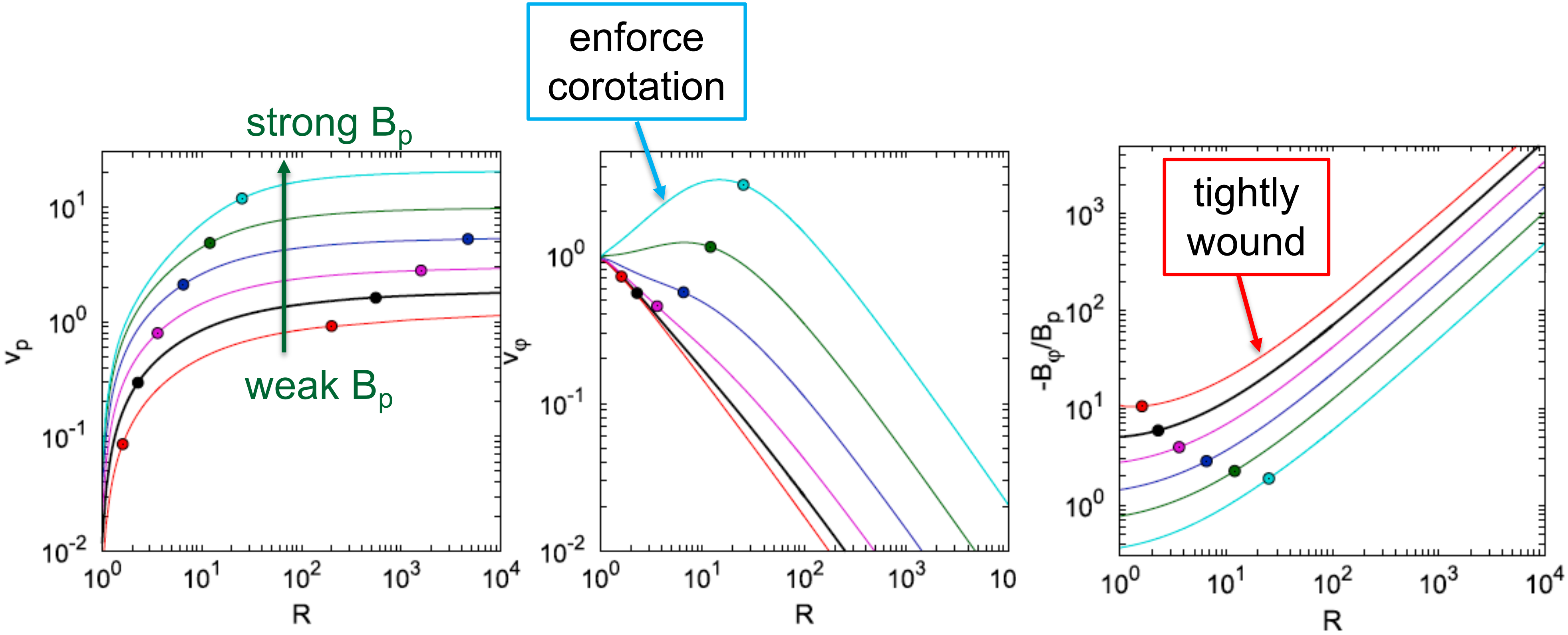
$$\lambda \equiv (R_A/R_0)^2$$

Accretion vs mass loss: $$\dot{M}_{\rm acc}\frac{dj}{dR} = \frac{d\dot{M}_{\rm wind}}{dR}\Omega_K(R_A^2 - R_0^2)$$

Ejection index: $$\xi \equiv \frac{d\dot{M}_{\rm wind}/d\ln R}{\dot{M}_{\rm acc}}\bigg|_{R=R_0} = \frac{1}{2}\frac{1}{(R_A/R_0)^2 - 1}$$

Transition between the two regimes

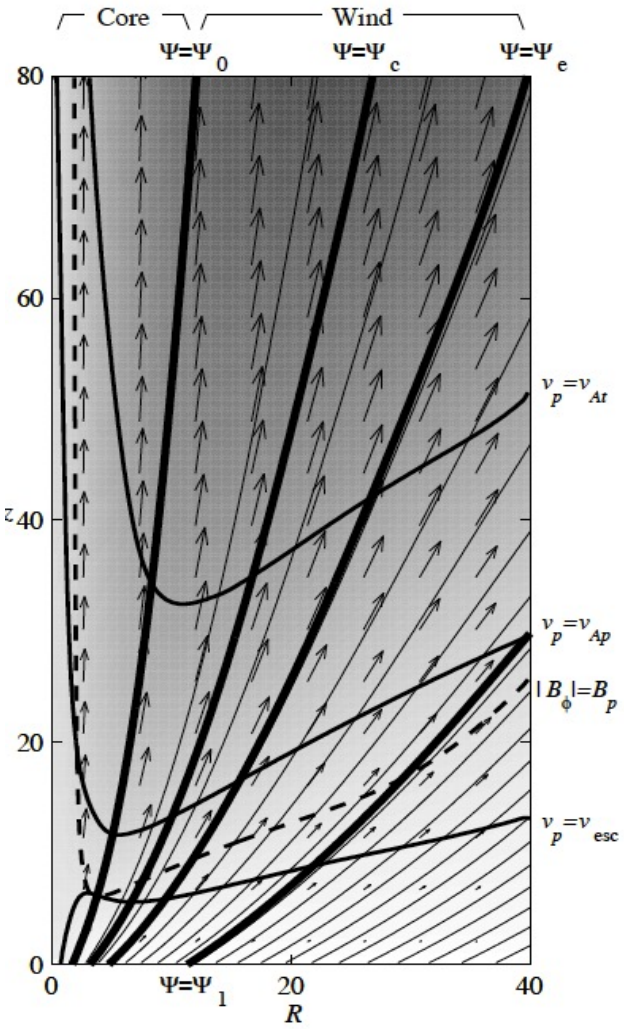
Prescribe the shape of B_p, solve the conservation laws and match critical points.



Bai et al. 2016

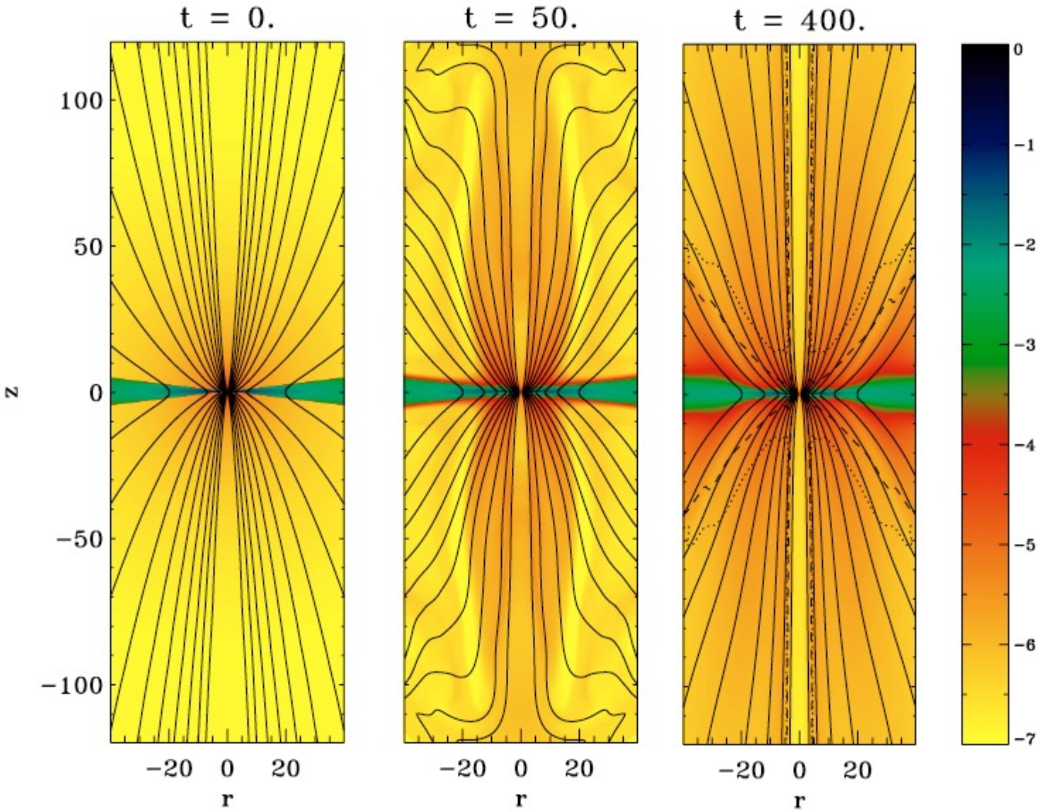
More self-consistent treatment needs simulations: cross field force balance.

Early studies



(Krasnopolsky et al. 1999)

Treat the disk as boundary condition.



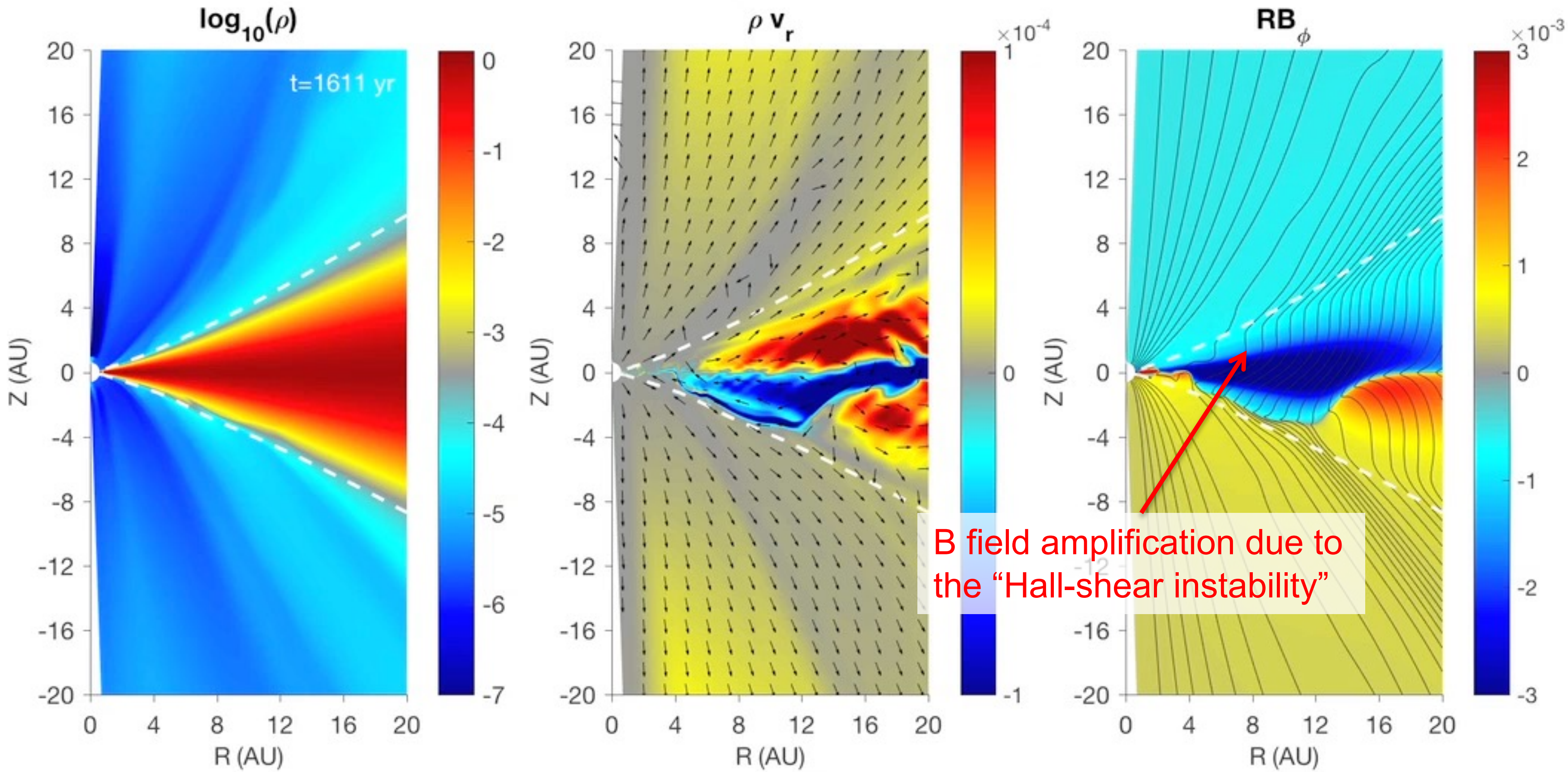
Zanni+2007

Resolve the disk while artificially add resistivity (to suppress the MRI).

Full disk microphysics

2D axisymmetric, all 3 non-ideal MHD effects included.

Bai, 2017

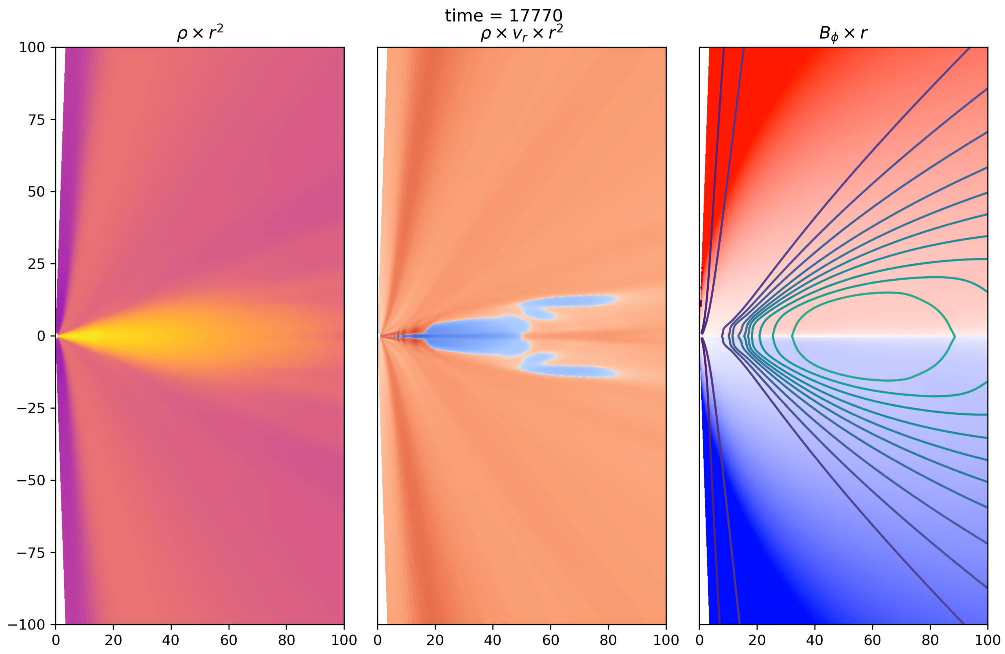


Complex flow structures with major implications for planet formation.

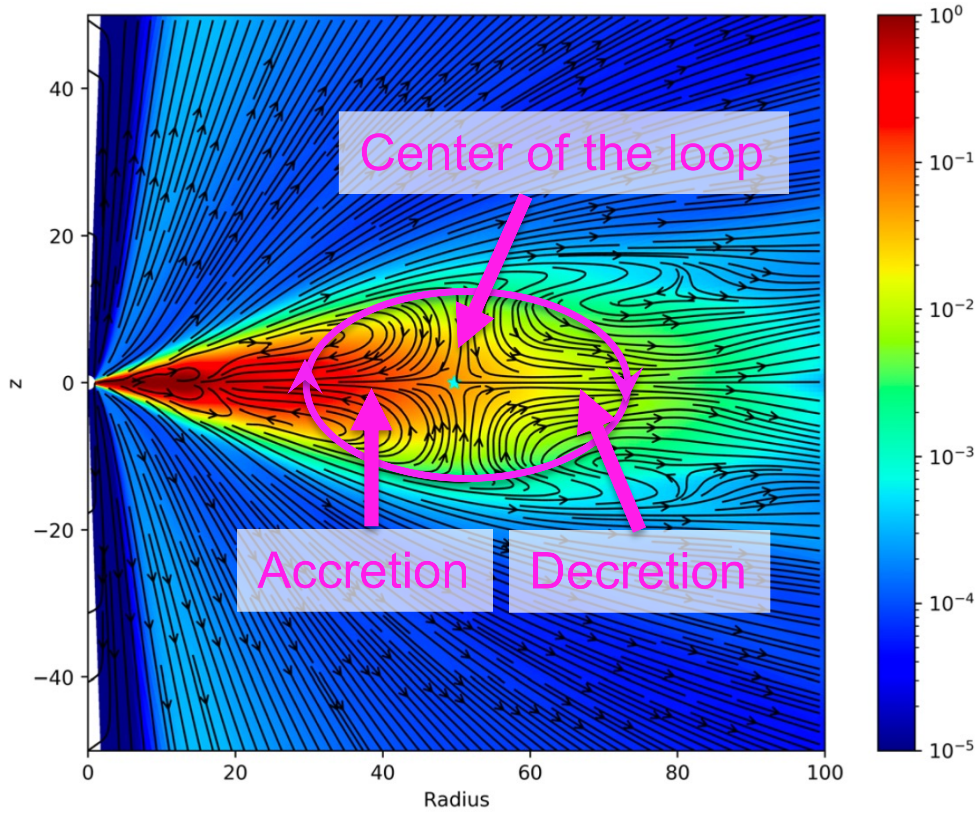
Outline

- Angular momentum transport: general
- Radial transport: instabilities and turbulence
- Vertical transport: magnetized wind
- Recent development

Does wind-driven accretion disk always shrink?



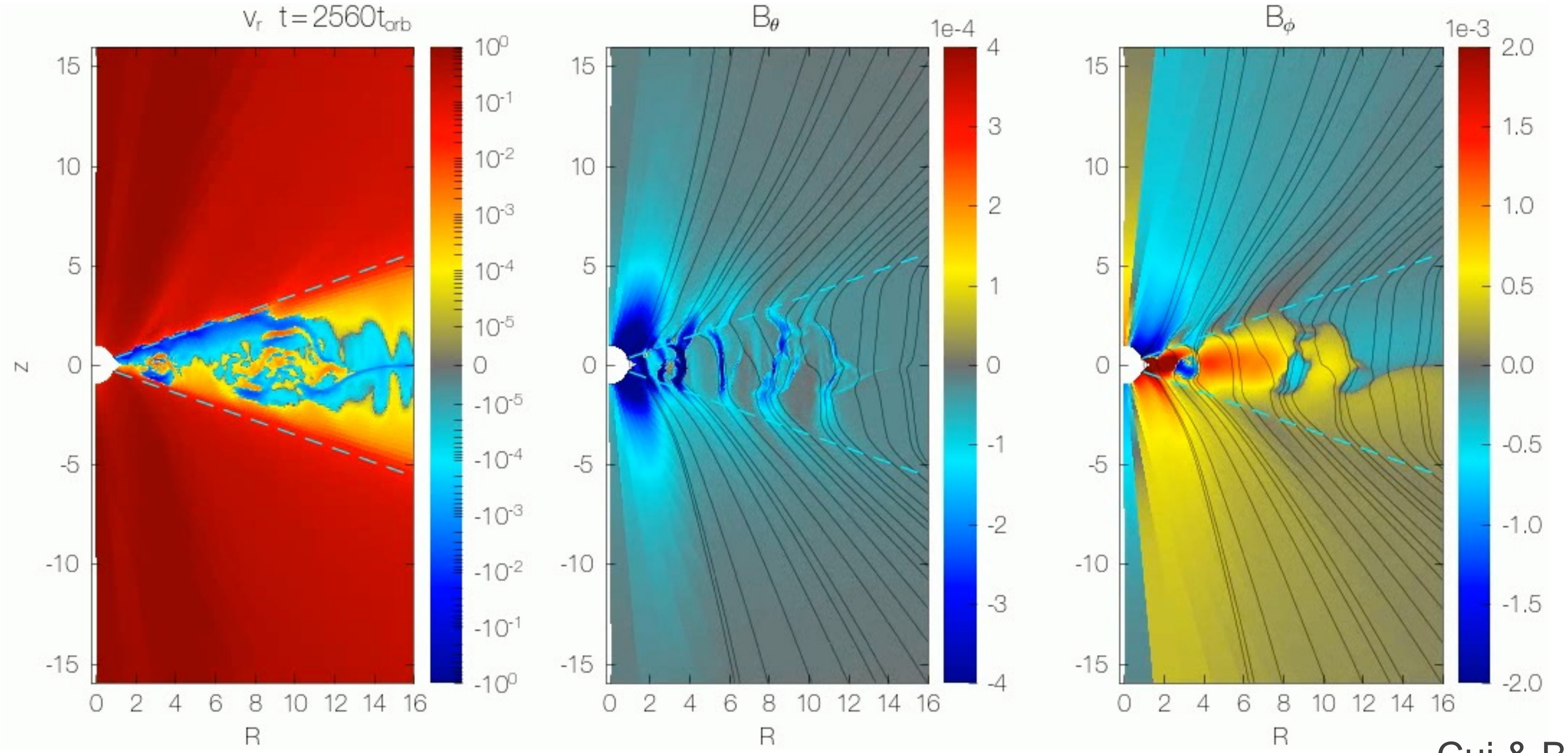
(Yang & Bai 2021)



Formation of poloidal B field loops beyond disk truncation radius could make the disk expand!

How exactly disk would evolve is more complex (we don't know yet).

Co-existence of turbulence and winds



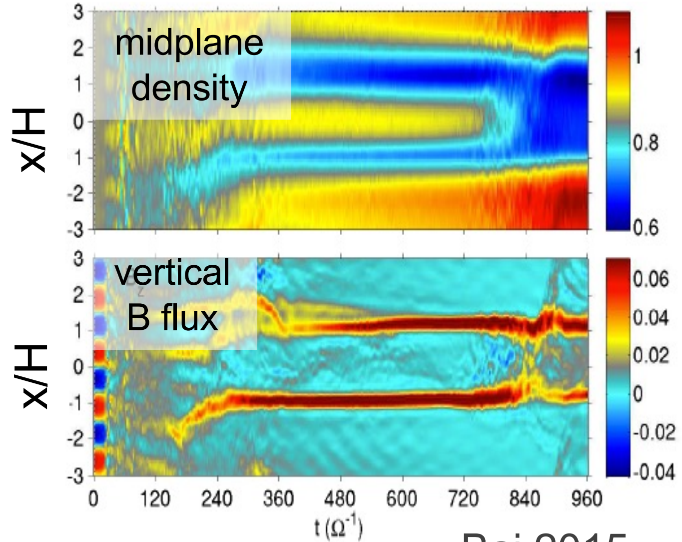
Cui & Bai (2021)

The MRI co-exists with wind, but wind dominates angular momentum transport.

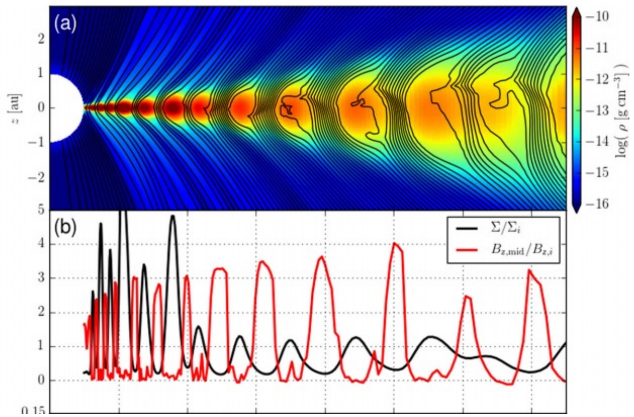
Co-existence of the VSI and MHD winds (Cui & Bai 2020)

Co-existence of the VSI and the MRI (+MHD winds) (Cui & Bai 2022)

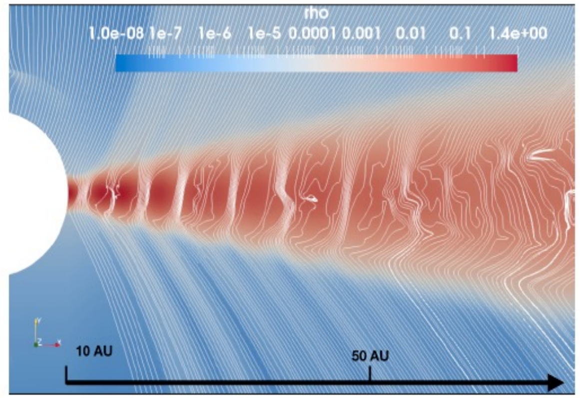
Formation of disk substructures



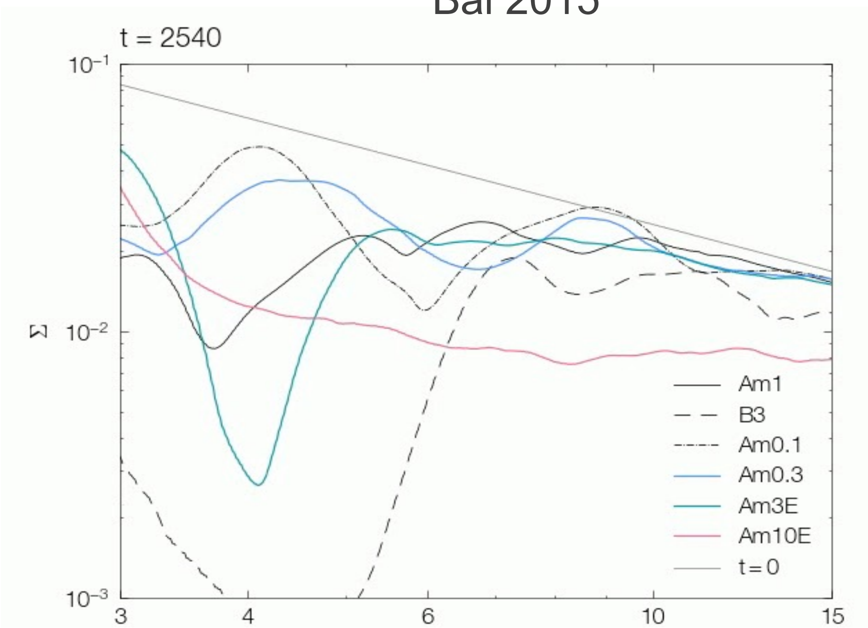
Bai 2015



Suriano+2018



Riols+2020

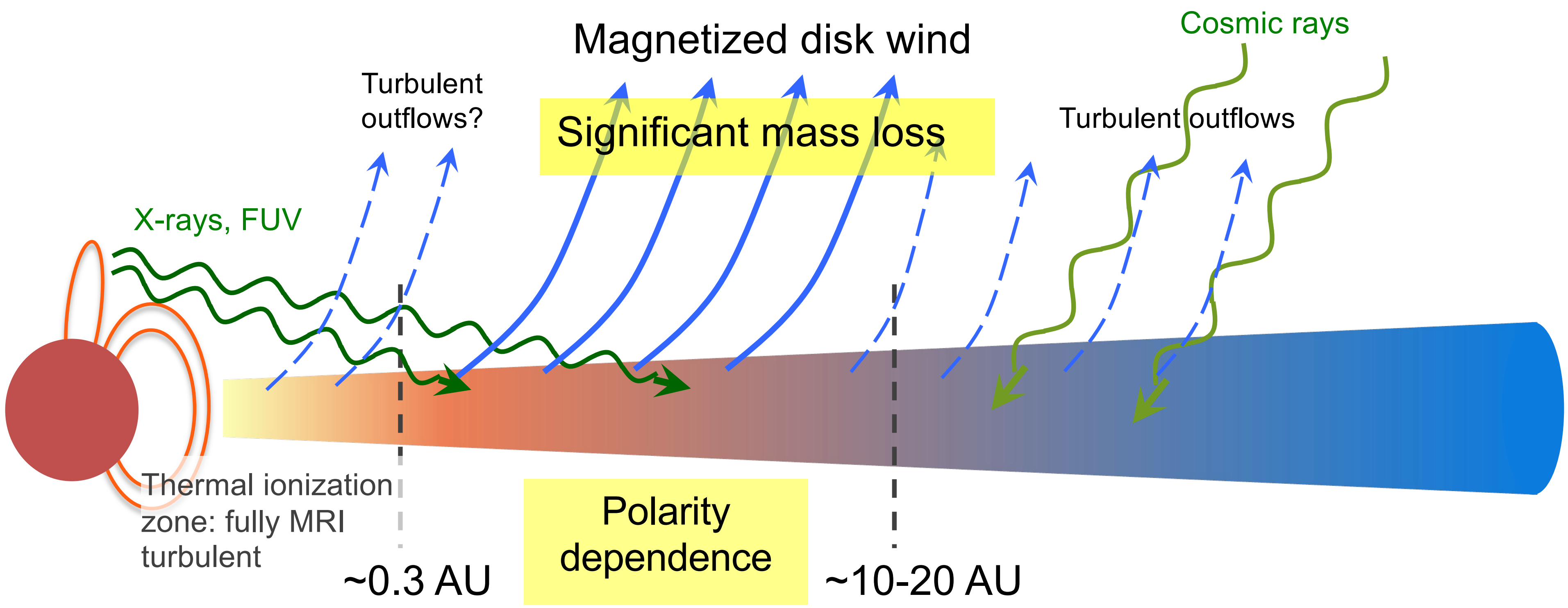


Cui & Bai 2021

Magnetic flux concentration => forming ring-like substructures

The process is generic but stochastic.

Current understandings

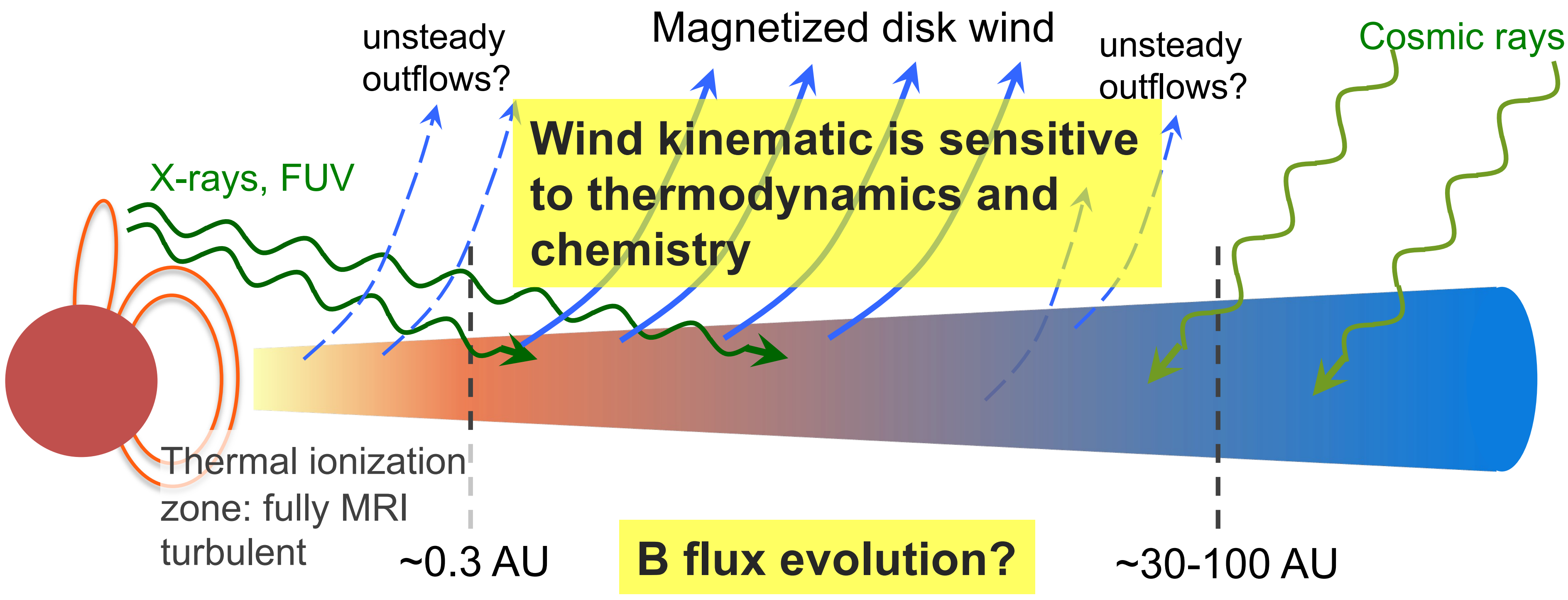


MRI suppressed by Ohmic +AD, disk is largely laminar.

MRI damped by AD

Additional action of various hydro instabilities

Future directions



Innermost region: transition to full MRI turbulence?

Outer region: influence from environment

Implications for planet formation? Grain growth/transport, planetesimal formation, planetary growth, pebble accretion, planet migration

Summary

- Angular momentum in PPDs determines disk structure and evolution, fundamental to understand planet formation.
- Two modes: radial transport (by “viscosity”) and vertical transport (by magnetized wind).
- Several thermo-hydrodynamic mechanisms can be viable to produce weak-to-modest level of turbulence.
- The MRI is a powerful instability to drive viscous accretion, but requires the B field to be coupled to the gas.
- PPDs are extremely weakly ionized, resulting in non-ideal MHD effects => suppress or damp the MRI.
- Wind-driven accretion dominates (other turbulence co-exists), gas dynamics depends on the polarity of B field.