Quantum Entanglement & Bell In-equality Violation @ Colliders

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TH, M. Low, A. Wu, arXiv:2310.17696;
TH, K. Cheng, M. Low, arXiv: 2311.09166; 2407.01672

Motivation

"If you think you understand quantum mechanics, you don't understand quantum mechanics."

-- Richard P. Feynman

"… it is my task to convince you not to turn a way because you don't understand it. You see my physics students don't understand it. That's because I don't understand it. Nobody does."

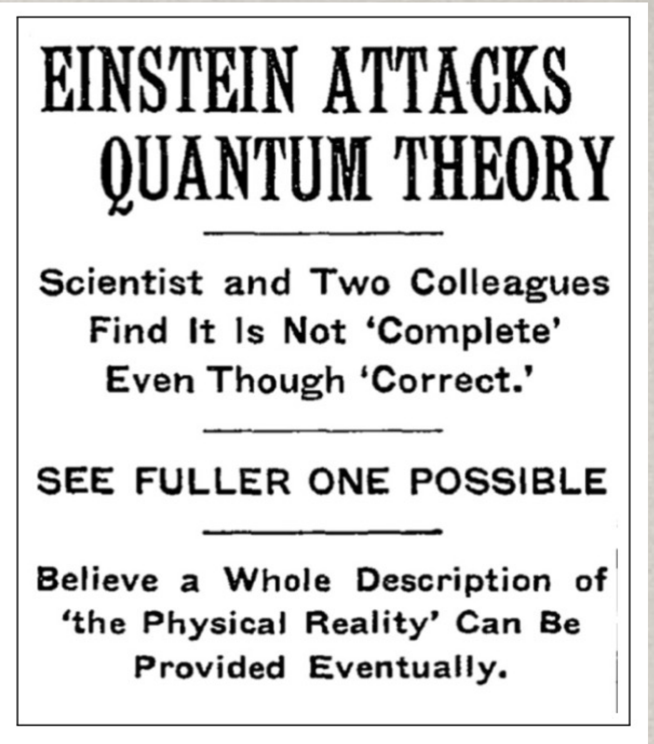
Study QM in the HE relativistic regime!

Einstein-Podolsky-Rosen Paradox

(Phys. Rev. 1935)

"Can quantum-mechanical description of physical reality be considered complete?"

"Local Hidden Variable Theory"



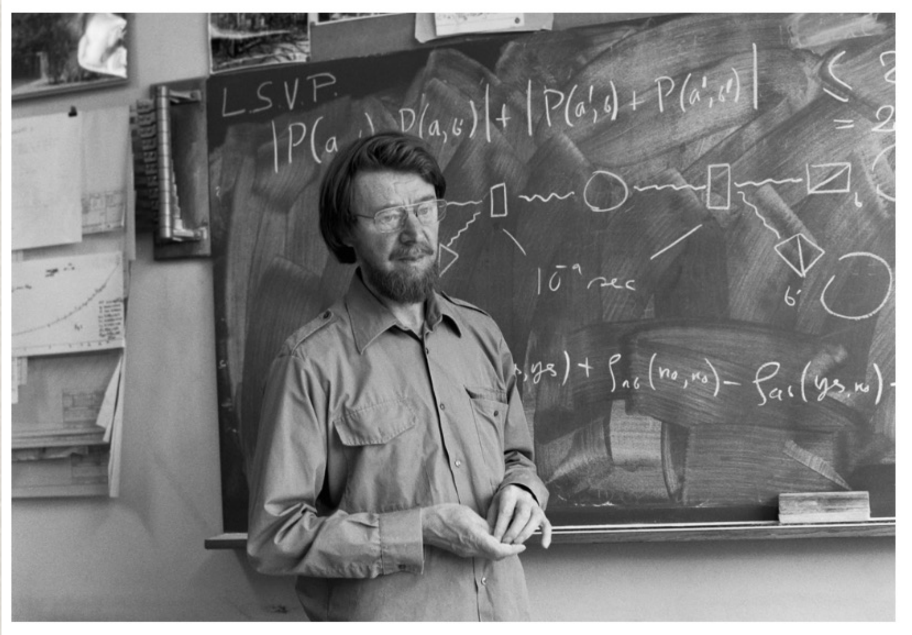
EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

John S. Bell's Inequality



"On the Einstein-Podolsky-Rosen paradox" (1964)

Alice & Bob's individual measurements:

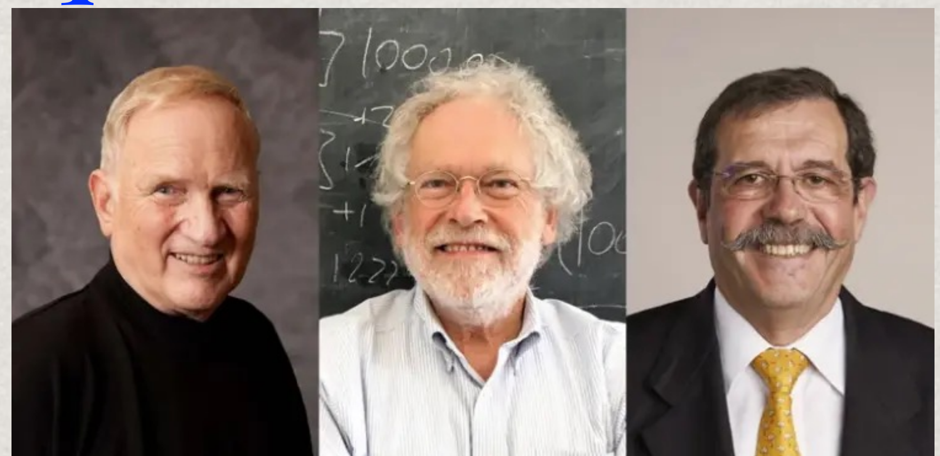
$$1 + P(\mathbf{b}, \mathbf{c}) \geq |P(\mathbf{a}, \mathbf{b}) - P(\mathbf{a}, \mathbf{c})|.$$

Non-Communitivity is the key:

Bell's Inequality CAN BE violated by QM measurements; but NOT by an EPR's Local Hidden Variable Theory .

→ **"Quantum Information"**

2022 Nobel Prize for physics:
"pioneering quantum information science”



Clauser Zeilinger Aspect

What we don’t do:

We DO NOT question QM !

We DO NOT test QM against the “Hidden Variable Theories”.

What we do:

In the framework of QFT, in the HE regime at colliders,

- We lay out the QM predictions / information.
- We calculate the QM correlations.
- Hope to establish the quantum tomography.
- Seek for BSM effects.

Quantum State

For a state vector $|\phi_i\rangle$

Density matrix

a state

$$\rho = \sum_i n_i \left|\phi_i\right\rangle \left\langle\phi_i\right|$$

an observable

$$\langle \mathcal{O} \rangle = \mathrm{Tr}(\mathcal{O}\rho)$$

For a pure state: $n_i = 1$; for a mixed state: $\boldsymbol{\Sigma}_i n_i = 1$.

For a **single qubit** (*i.e.*, a doublet of spin, iso-spin etc.):

$$\rho = \frac{1}{2}\Big(\mathbb{I}_2 + \sum_i B_i \sigma_i\Big)$$

For a bipartite system (*i.e.*, ½⊗½)

$$\rho = \frac{1}{4}\Big(\mathbb{I}_4 + \sum_i \left(B_i^{\mathcal{A}}\left(\sigma_i \otimes \mathbb{I}_2\right) + B_i^{\mathcal{B}}\left(\mathbb{I}_2 \otimes \sigma_i\right)\right) + \sum_{i,j} C_{ij}\left(\sigma_i \otimes \sigma_j\right)\Big)$$

$B_i^{A,B}$ the polarizations, C_{ij} the spin-correlation matrix

The 15 coefficients → **Quantum Tomography** for the bipartite.

Quantum Entanglement

For a bipartite system, *i.e.*, $\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$:

Singlet:

entangled → $|0,0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$

Triplet:

$$\begin{aligned} |1,1\rangle &= \uparrow\uparrow \\ |1,0\rangle &= \tfrac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) \\ |1,-1\rangle &= \downarrow\downarrow \end{aligned}$$

← entangled

Separable

Non-Separable

$$\rho \neq \sum_{a=1}^{N} p_a \, \rho_a^{\mathcal{A}} \otimes \rho_a^{\mathcal{B}}$$

Quantum entanglement → sub-states inseparable

Peres-Horodecki criterion:
a necessary condition for entanglement

A state is entangled (inseparable) if a partial transpose

$$\rho^{T_2} = \sum_n p_n \rho_n^a \otimes \left(\rho_n^b\right)^T$$

is not non-negative.

Quantum Entanglement

Peres-Horodecki criterion leads to several inequalities
→ Quantitative measure of entanglement

It has been a customary to introduce the concurrence, that can be written in C_i, the eigenvalues of C_{ij}:

Concurrence

$$\mathcal{C}(\rho) = \begin{cases} \frac{1}{2}\max(|C_1 + C_2| - 1 - C_3, 0), & C_3 \le 0 \\ \frac{1}{2}\max(|C_1 - C_2| - 1 + C_3, 0), & C_3 \ge 0 \end{cases}$$

It is shown that :

$$0 \le \mathcal{C}(\rho) \le 1$$

No entanglement

Maximumly entangled

→ Quantum information even in space-like separation

Afik and Munoz de Nova, arXiv: 2003.02280

John Bell's Inequality

In our setting, Alice & Bob's two correlated measurements can be cast to a Glauser-Horne-Shimony-Holt form.

Classical/LHVT should satisfy

$$\langle A_1 B_1 \rangle - \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle + \langle A_2 B_2 \rangle \leq 2$$

Or: $$\left| \vec{a}_1 \cdot C \cdot (\vec{b}_1 - \vec{b}_2) + \vec{a}_2 \cdot C \cdot (\vec{b}_1 + \vec{b}_2) \right| \leq 2$$

QM may violate this in certain phase space !

E.g., choosing

$$A_1 = \sigma_1, \quad A_2 = \sigma_3, \quad B_1 = \pm \frac{1}{\sqrt{2}} (\sigma_1 + \sigma_3), \quad B_2 = \pm \frac{1}{\sqrt{2}} (-\sigma_1 + \sigma_3)$$

$$\Rightarrow \ |C_{11} \pm C_{33}| \leq \sqrt{2}$$

Top-pair & spin correlation

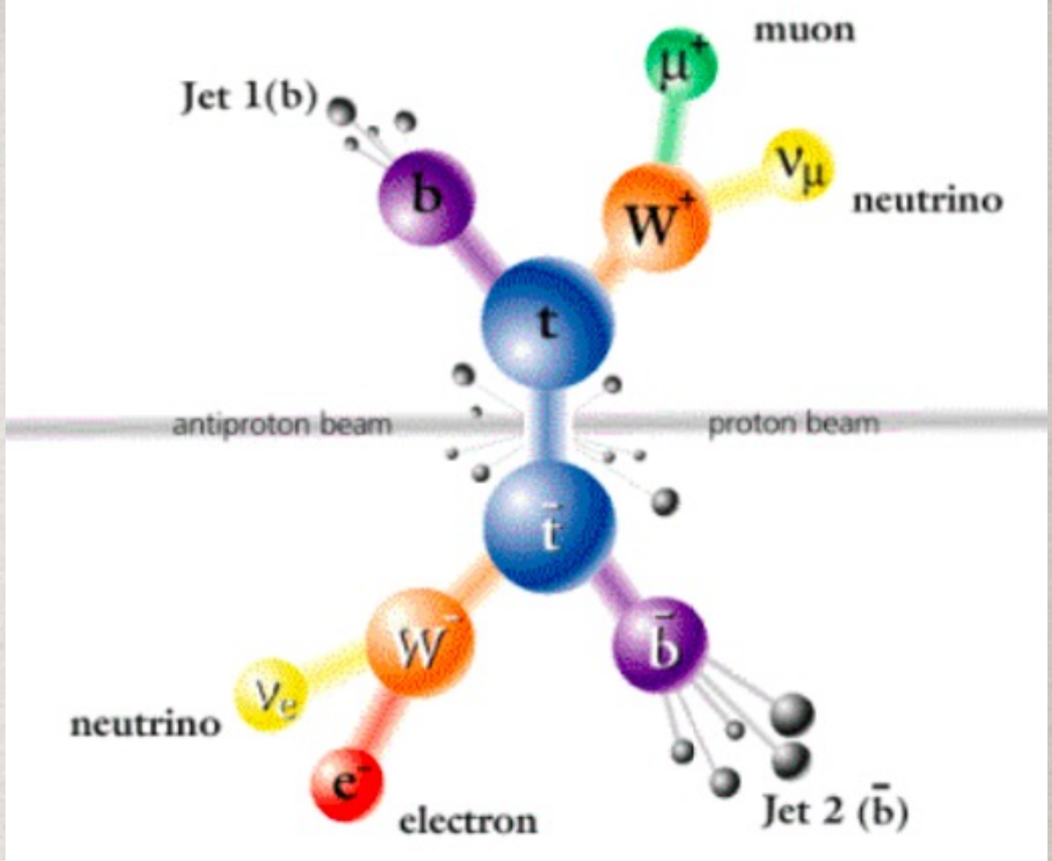
decaying to $A_{1,2,3}$, $B_{1,2,3}$

$$\sigma(XY \to t\bar{t} \to (A_1A_2A_3)(B_1B_2B_3)) =$$

$$\int d\Omega^A d\Omega^B \left(\frac{d\Gamma_{ab}}{d\Omega^A}\right) R_{ab,\bar{a}\bar{b}} \left(\frac{d\bar{\Gamma}_{\bar{a}\bar{b}}}{d\Omega^B}\right)$$

$$\frac{d\Gamma_{ab}}{d\Omega} \propto \delta_{ab} + \kappa\, \sigma^i_{ab}\, \Omega^i$$

Spin analyzing power



$$\Rightarrow \frac{1}{\sigma}\frac{d\sigma}{d\Omega^A d\Omega^B} = \frac{1}{(4\pi)^2}\left(1 + \kappa^A P^A_i \Omega^A_i + \kappa^B P^B_i \Omega^B_i + \kappa^A\kappa^B \Omega^A_i C_{ij} \Omega^B_j\right)$$

Direction of A, B

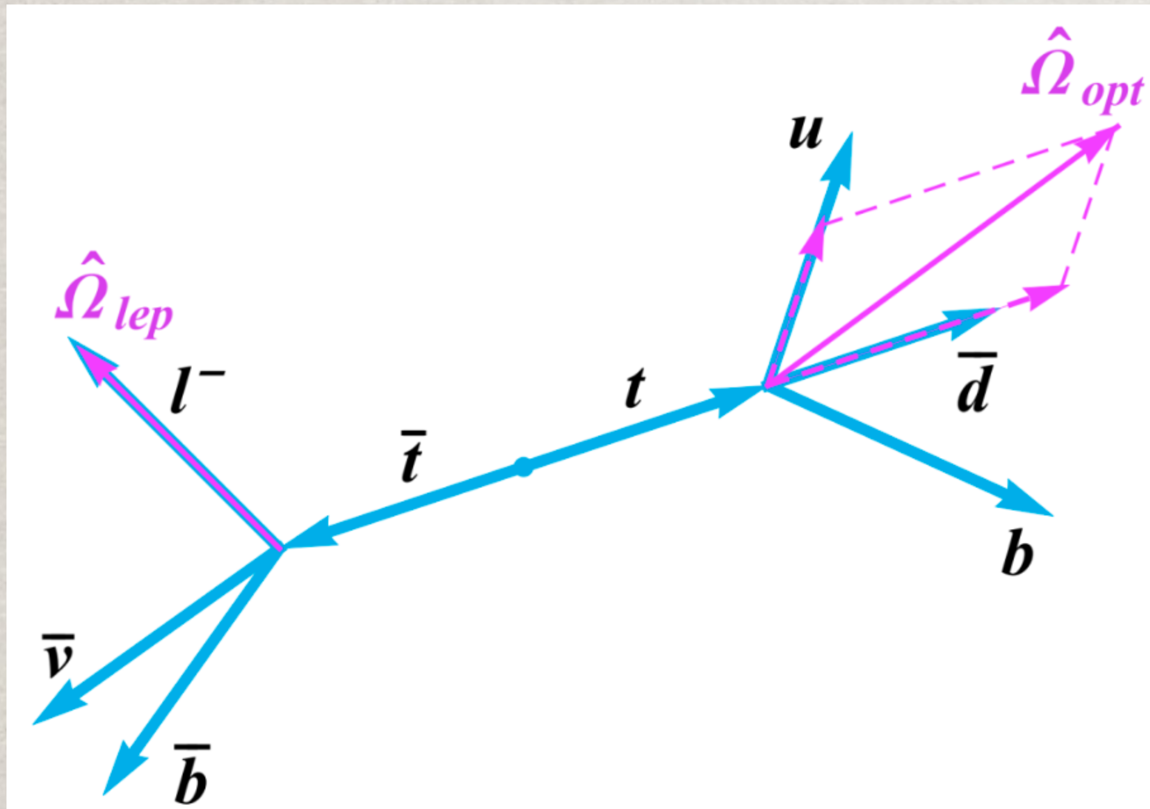
$$\Rightarrow \frac{1}{\sigma}\frac{d\sigma}{d(\cos\theta^A_i \cos\theta^B_j)} = -\frac{1 + \kappa^A\kappa^B C_{ij}\cos\theta^A_i \cos\theta^B_j}{2} \log|\cos\theta^A_i \cos\theta^B_j|$$

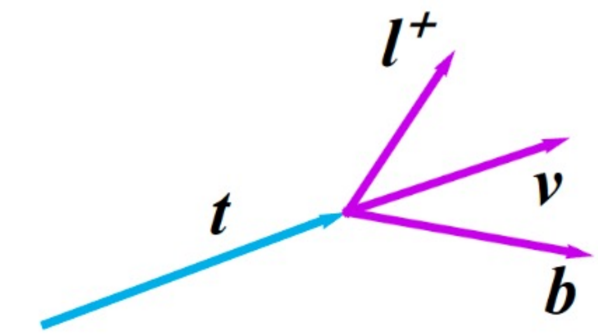
Polar angle of A with respect to the i-th axis

$$\Rightarrow C_{ij} = \frac{4}{\kappa^A\kappa^B}\frac{N(\cos\theta^A_i \cos\theta^B_j > 0) - N(\cos\theta^A_i \cos\theta^B_j < 0)}{N(\cos\theta^A_i \cos\theta^B_j > 0) + N(\cos\theta^A_i \cos\theta^B_j < 0)}$$

Top-pair leptonic + hadronic decays

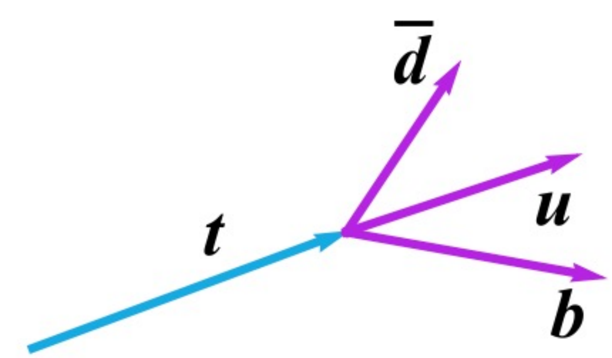
Z. Dong, Dorival Goncalves, et al., arXiv:2305.07075
TH, M. Low, A. Wu, arXiv:2310.17696





Leptonic decay:

$$\kappa_{\ell+} = 1.0$$



Hadronic decay:

$$\kappa_{\bar{d}} = 1.0$$
$$\kappa_u = 0.34$$

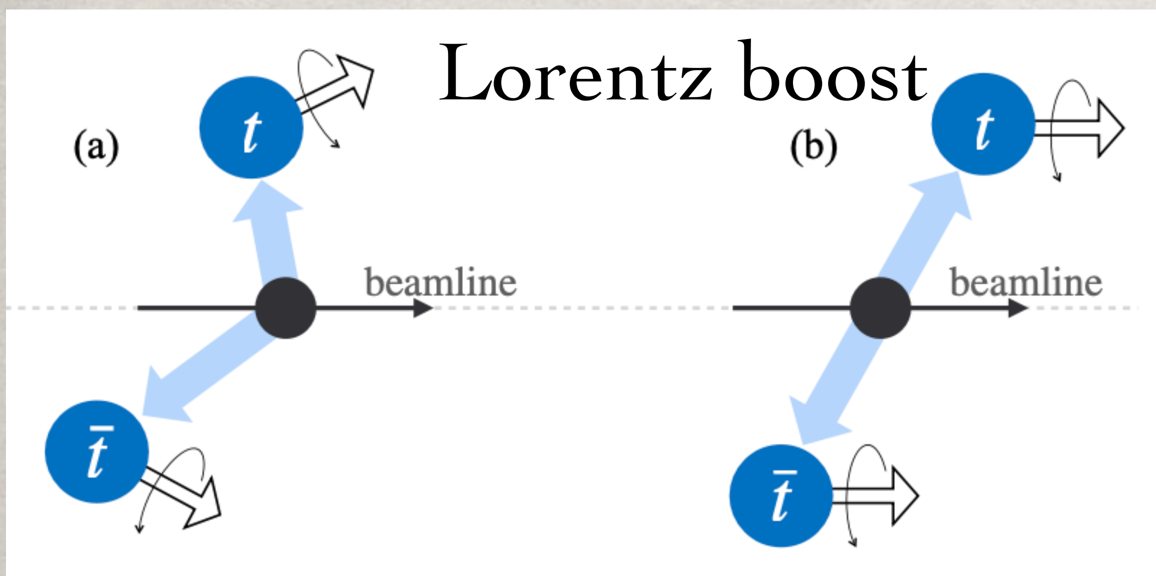
optimized direction:

$$\vec{\Omega}_{\rm opt}(\cos\theta_W) = P_{d\to p_{\rm soft}}(\cos\theta_W)\,\hat{p}_{\rm soft} + P_{d\to p_{\rm hard}}(\cos\theta_W)\,\hat{p}_{\rm hard}$$

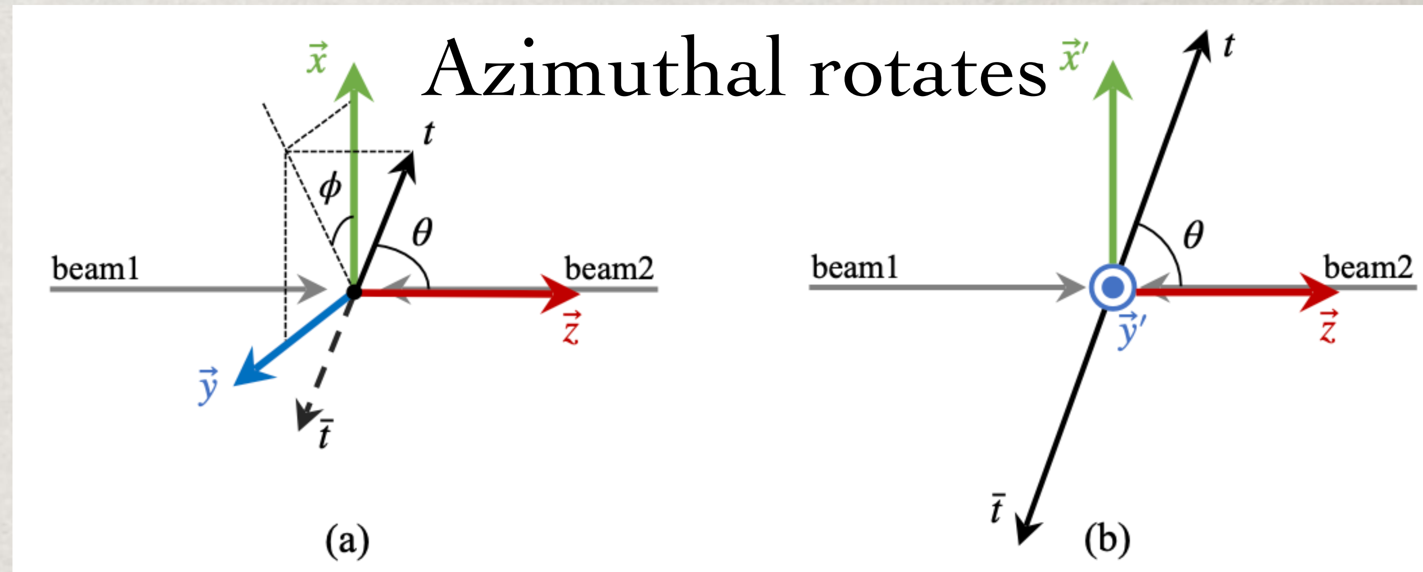
$\Rightarrow$ $\kappa_{\rm opt} = 0.64$ (arXiv:1401.3021)

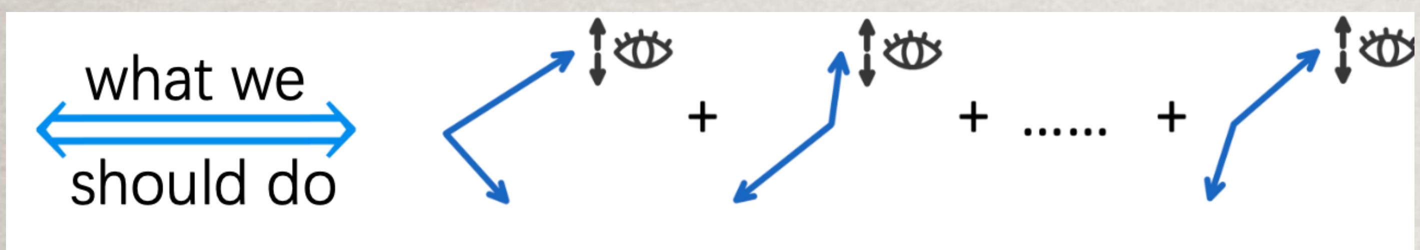
Quantum entanglement in high collisions: Fictitious states

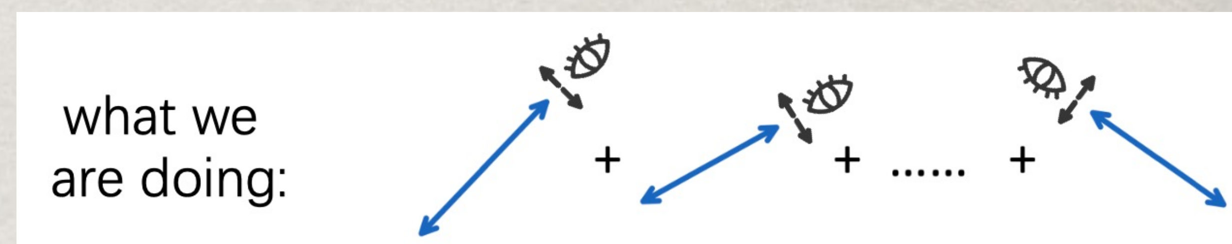
Lorentz boost



Azimuthal rotates







$$\rho = \ldots\ldots + \sum_{i,j} C_{ij}(\sigma_i \otimes \sigma_j)$$

$$C_{ij} = \langle S_i^t S_j^{\bar{t}} \rangle$$

$$\bar{\rho} = \ldots\ldots + \sum_{\bar{i},\bar{j}} C_{\bar{i}\bar{j}}(\sigma_{\bar{i}} \otimes \sigma_{\bar{j}})$$

Thus the Wigner rotation: $\vec{p}^{\,\prime}_{\ell+} = R(\Lambda, p_t^{\text{lab}})\vec{p}_{\ell+}, \qquad \vec{p}^{\,\prime}_{\ell-} = R(\Lambda, p_{\bar{t}}^{\text{lab}})\vec{p}_{\ell-}.$

$$C'(\Omega) = R_t^T(\Lambda_\Omega)C(\Omega)R_{\bar{t}}(\Lambda_\Omega).$$

Afik and Munoz de Nova, arXiv: 2003.02280

TH, K. Cheng, M. Low, arXiv: 2311.09166; 2407,01672

Quantum entanglement at high energies: Fictitious states

From a well-prepared quantum state to a fictitious state:

$$\bar{\rho} \to \sum_{a \in \text{events}} U_a^\dagger \rho_a U_a \neq U^\dagger \bar{\rho} U \,.$$

Thus, a measurement on a fictitious state depends on the frame/base choice of each measurement!

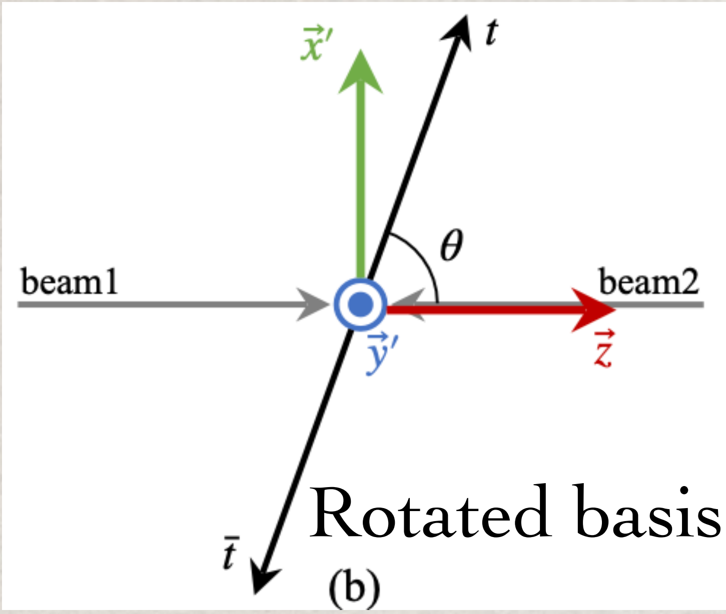
We showed: TH, K. Cheng, M. Low, arXiv: 2311.09166

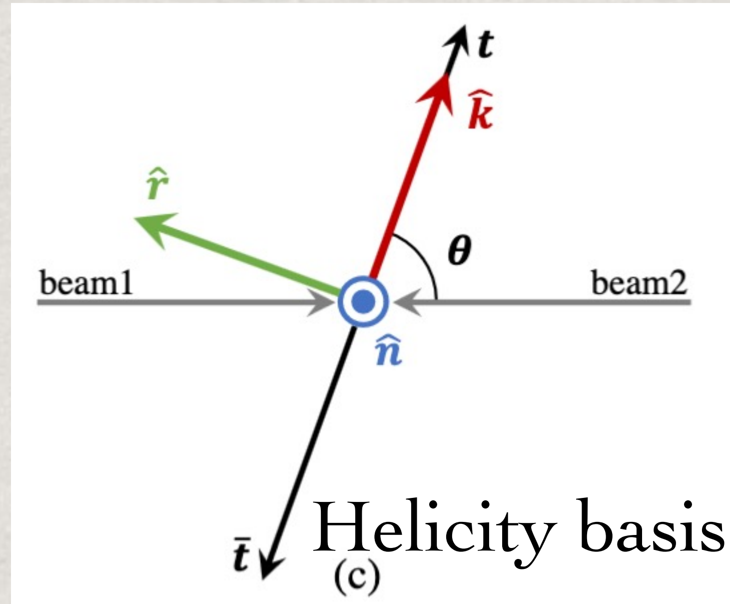
$$\mathcal{C}(\rho_{\text{fictitious}}) > 0 \quad \Longrightarrow \quad \mathcal{C}(\rho_{\text{sub}} \in \rho) > 0$$

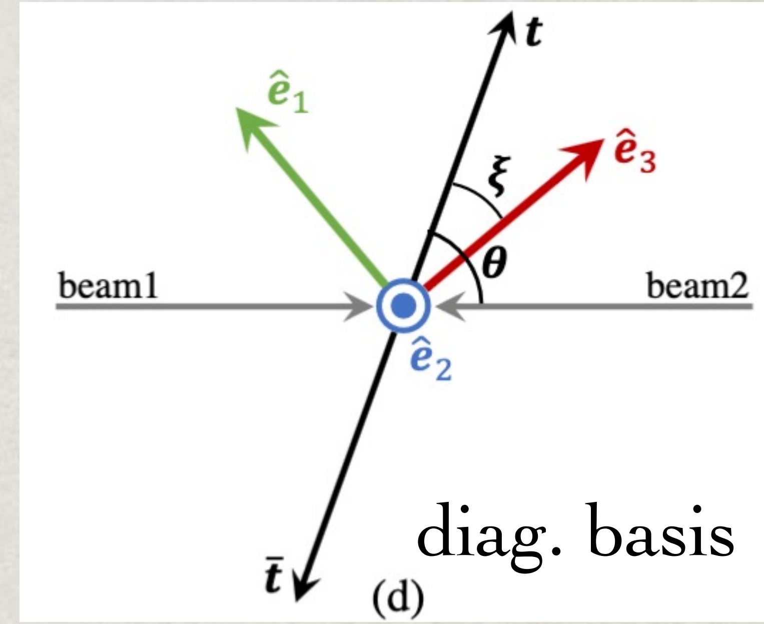
$$\text{Bell}(\rho_{\text{fictitious}}) > \sqrt{2} \quad \Longrightarrow \quad \text{Bell}(\rho_{\text{sub}} \in \rho) > \sqrt{2}$$

Fictious states carry the system quantum information!

Basis optimization



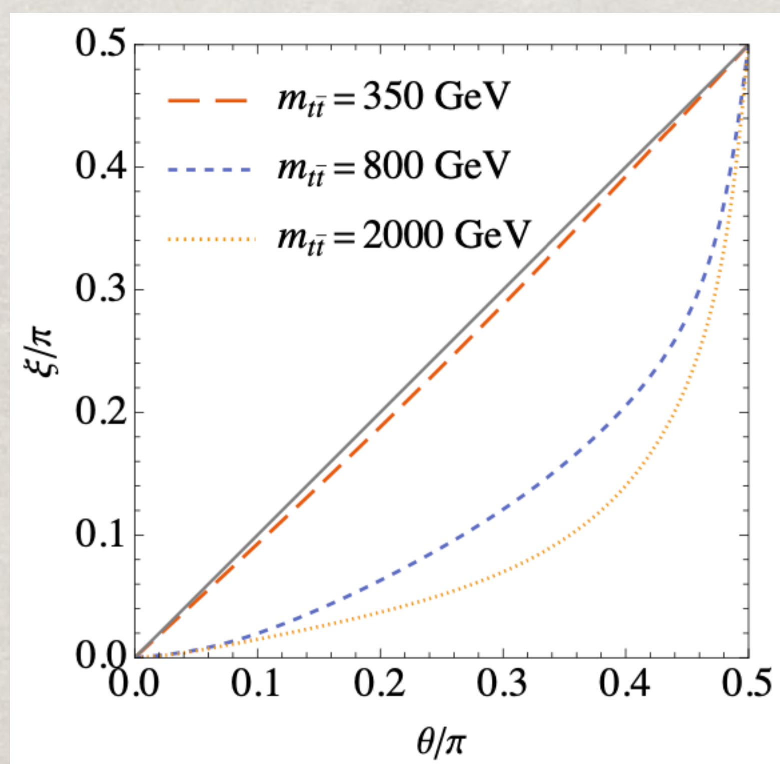


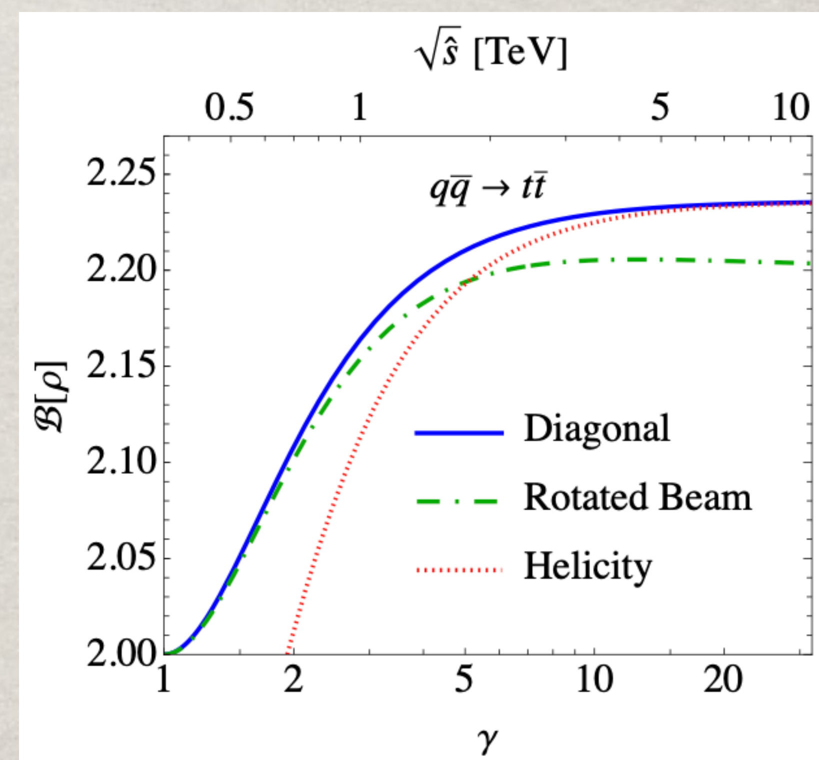


The frame that diagonalizes C_{ij} leads to the maximum sensitivity – advantage of a fictitious state!

$$\bar{C}_{ij} = \frac{1}{\sigma}\int \mathrm{d}\Omega \, \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} C_{ij,\Omega},$$

$$C_\Omega^{\rm diag} = R_\Omega C_\Omega R_\Omega^T = \begin{pmatrix} \mu_{1,\Omega} & 0 & 0 \\ 0 & \mu_{2,\Omega} & 0 \\ 0 & 0 & \mu_{3,\Omega} \end{pmatrix}$$

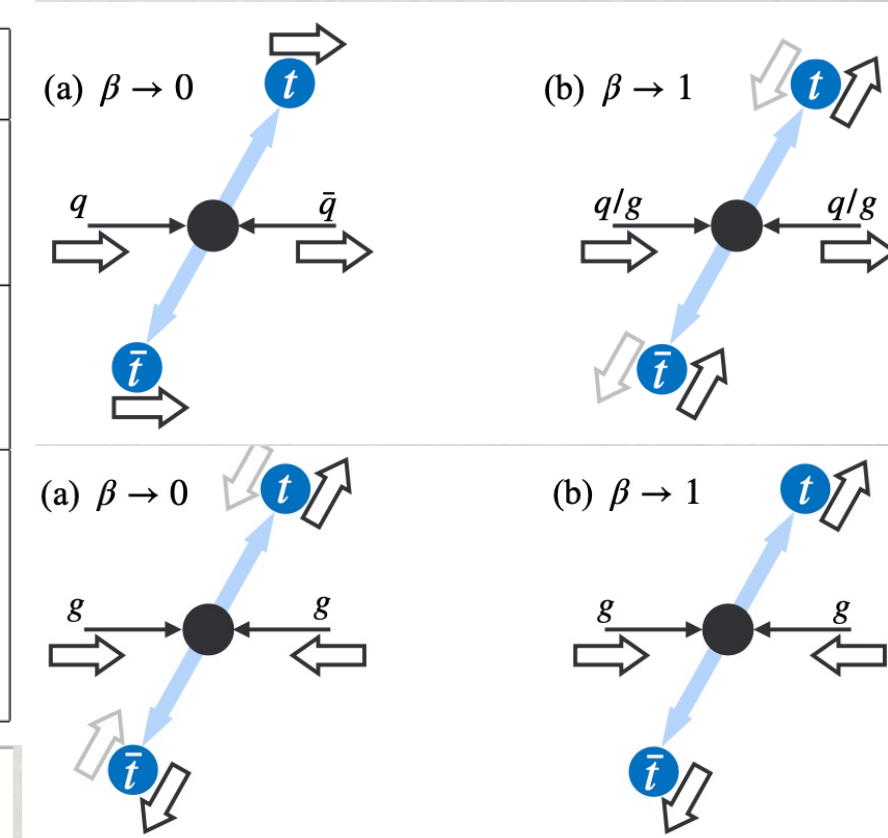


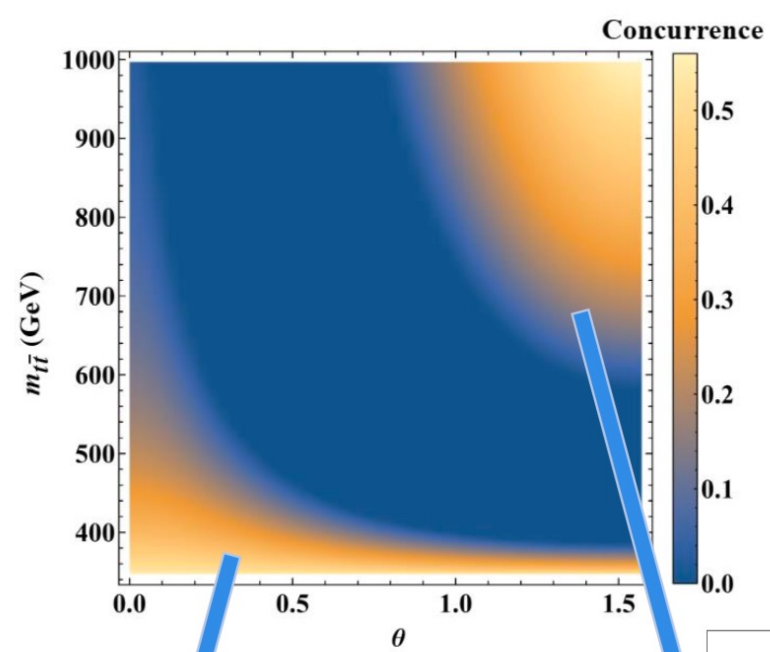


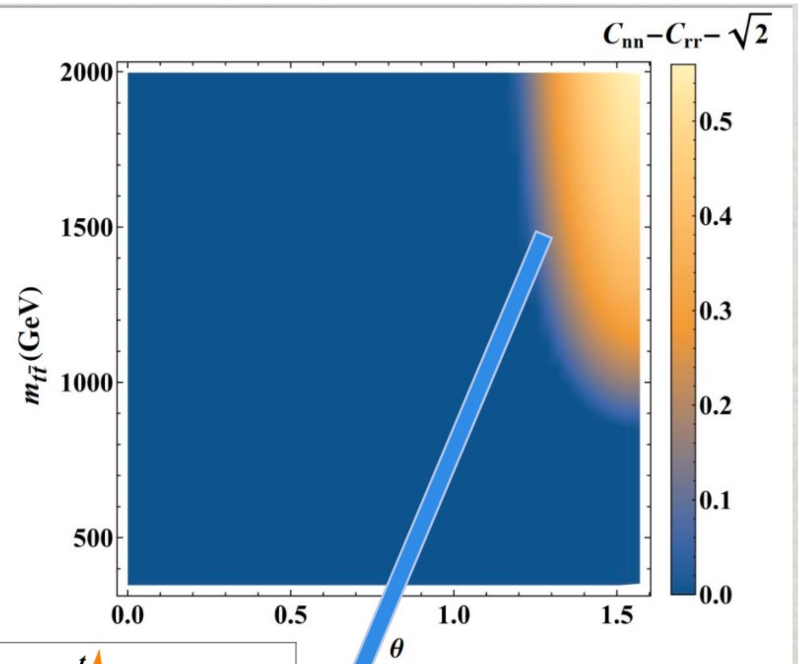
TH, M. Low, A. Wu, arXiv:2310.17696; TH, Cheng, Low, arXiv: 2311.09166; arXiv:2407.01672.

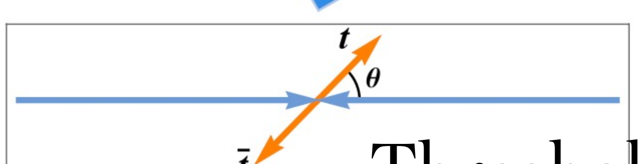
Partonic level results

QCD production	$\overline{\sum}\lvert\mathcal{M}\rvert^2$	spin correlation matrix C_{ij}	ξ
$q\bar{q} \to t\bar{t}$	$\kappa_q\left(2-\beta^2 s_\theta^2\right)$	$\begin{pmatrix} \frac{(2-\beta^2)s_\theta^2}{2-\beta^2 s_\theta^2} & 0 & -\frac{2c_\theta s_\theta\sqrt{1-\beta^2}}{2-\beta^2 s_\theta^2} \\ 0 & \frac{-\beta^2 s_\theta^2}{2-\beta^2 s_\theta^2} & 0 \\ -\frac{2c_\theta s_\theta\sqrt{1-\beta^2}}{2-\beta^2 s_\theta^2} & 0 & \frac{2c_\theta^2+\beta^2 s_\theta^2}{2-\beta^2 s_\theta^2} \end{pmatrix}$	$\tan\xi = \frac{1}{\gamma}\tan\theta$
$g_L g_R \to t\bar{t}$	$\kappa_g \beta^2 s_\theta^2(2-\beta^2 s_\theta^2)$		$\tan\xi = \frac{1}{\gamma}\tan\theta$
$g_L g_L / g_R g_R \to t\bar{t}$	$\kappa_g(1-\beta^4)$	$\begin{pmatrix} \frac{\beta^2-1}{\beta^2+1} & 0 & 0 \\ 0 & \frac{\beta^2-1}{\beta^2+1} & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\xi = 0$

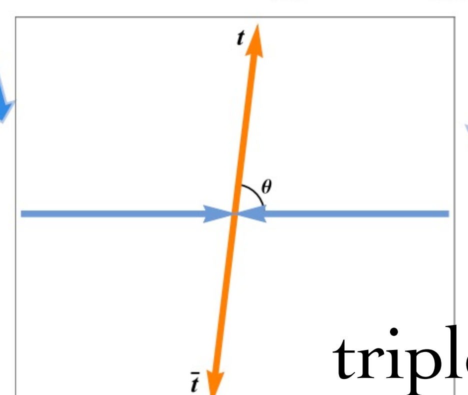








Threshold:
singlet dominance



Boosted:
triplet dominance

- Threshold:
high rate, low sensitivity
- Highly boosted:
Low rate, high sensitivity

Simulation results

Realistic simulations:

- Top-pair semi-leptonic channel
- MadGraph 5+Pythia 8+Delphes 3
- Detector effects by "parametric fit"

Entanglement $C > 0$

	Result(139fb^{-1})	Precision
Boosted	0.276 ± 0.026	**9.5%**
Threshold	0.261 ± 0.008	**3.0%**

Bell's inequality violation

$|C_{11} \pm C_{33}| \leq \sqrt{2}$ $\qquad \mathcal{S} = \frac{\mathcal{B} - 2}{\delta \mathcal{B}}$

Result(3ab^{-1})	Significance
0.23 ± 0.06	$\mathbf{4.1\sigma}$

TH, M. Low, A. Wu, arXiv:2310.17696;
Recent LHC studies for top leptonic/semi-leptonic decays:
ATLAS: arXiv:2311.07288; CMS: arXiv:2406.03976

For a recent review, see e.g., arXiv: 2402.07972.

Lepton colliders: $e^+ e^- \to t\bar{t}$ (other fermion pair)

In Helicity Basis C_{ij}:

$$C_{ij} = \frac{1}{f_V^2(2-\beta^2 s_\theta^2) + f_A^2\beta^2(1+c_\theta^2) \pm 4 f_V f_A \beta c_\theta} \times \tag{75}$$
$$\begin{pmatrix} s_\theta^2\left(f_V^2(2-\beta^2) - f_A^2\beta^2\right) & 0 & -2(f_V^2 c_\theta \pm f_V f_A \beta) s_\theta \sqrt{1-\beta^2} \\ 0 & (f_A^2 - f_V^2)\beta^2 s_\theta^2 & 0 \\ -2(f_V^2 c_\theta \pm f_V f_A \beta) s_\theta \sqrt{1-\beta^2} & 0 & f_V^2(2c_\theta^2 + \beta^2 s_\theta^2) + f_A^2\beta^2(1+c_\theta^2) \pm 4 f_V f_A \beta \end{pmatrix}.$$

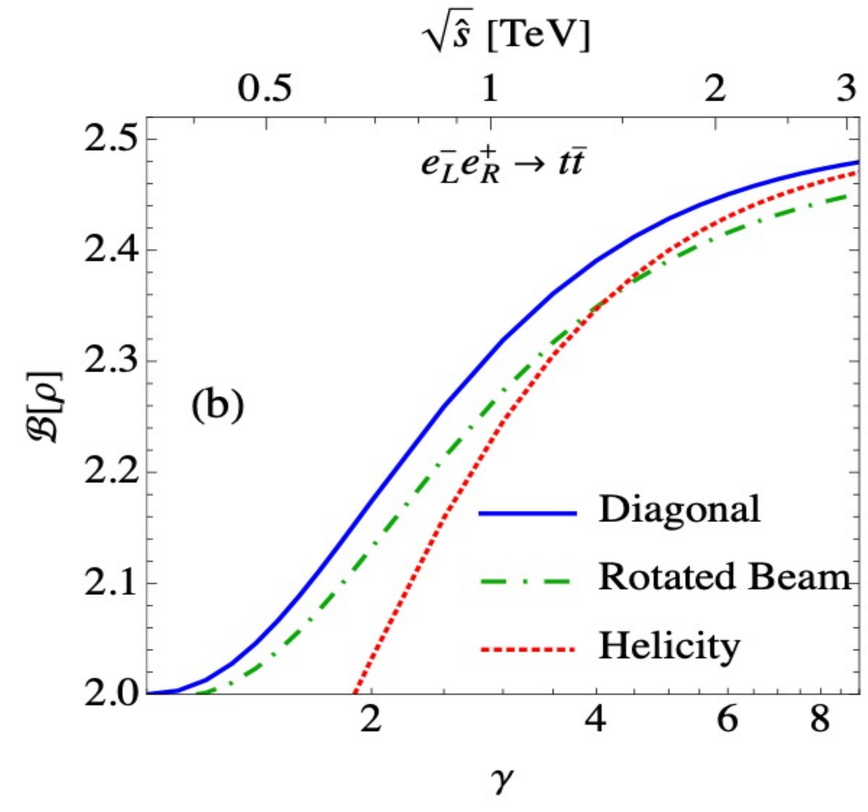
Eigenvalues:

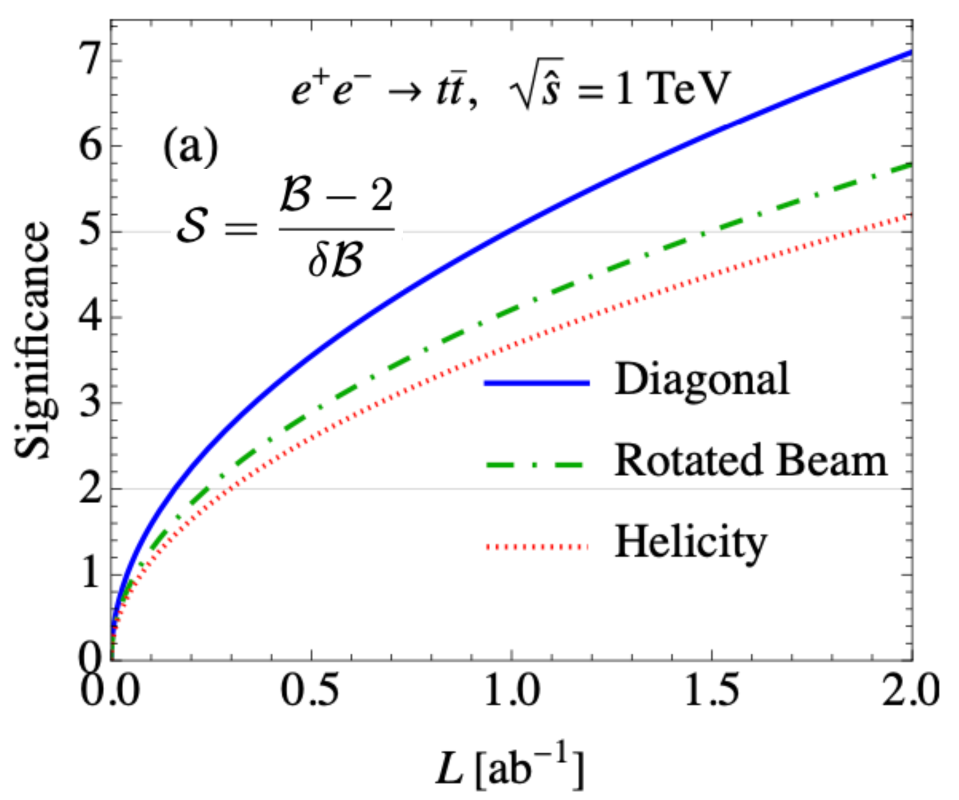
$$\mu_1 = -\mu_2 = \frac{\beta^2 s_\theta^2 (f_V^2 - f_A^2)}{f_V^2(2-\beta^2 s_\theta^2) \pm 4\beta f_V f_A c_\theta + \beta^2 f_A^2 (1+c_\theta^2)}, \qquad \mu_3 = 1$$

$$f_V^L = Q_e Q_t + \frac{(I_e^3 - Q_e s_W^2)(I_t^3 - 2Q_t s_W^2)}{2c_W^2 s_W^2} \frac{\hat{s}}{\hat{s} - m_Z^2}, \quad f_V^R = Q_e Q_t - \frac{Q_e(I_t^3 - 2Q_t s_W^2)}{2c_W^2} \frac{\hat{s}}{\hat{s} - m_Z^2},$$
$$f_A^L = -\frac{(I_e^3 - Q_e s_W^2) I_t^3}{2c_W^2 s_W^2} \frac{\hat{s}}{\hat{s} - m_Z^2}, \qquad f_A^R = \frac{Q_e I_t^3}{2c_W^2} \frac{\hat{s}}{\hat{s} - m_Z^2},$$

Diagonal basis:

$$\tan\xi = \frac{1}{\gamma} \frac{f_V s_\theta}{f_V c_\theta \pm f_A \beta},$$





Further remarks & Conclusions

- Collider experiments produce a vast data sample with rich combinations of quantum numbers: spin, flavor …
- We clarify the "fictitious states", and propose observables.
- We identify the optimal axis choice to enhance the sensitivity.

→ encouraging results for entanglement & Bell inequality measrements.

- Our methodology is applicable to other colliders, other quantum systems: Qubits, Qutrits … multiple particles …

The World is quantum-mechanical !

QIS is NOW !