Institute of Theoretical Physics
Chinese Academy of Sciences

Effective Field Theories from IR Amplitudes to UV Physics

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New Opportunities for Particle Physics (NOPP)

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Outline

- Why, what and how EFTs for new physics?

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2201.04639]

- EFT operators from on-shell amplitudes

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2012.09188]

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2007.07899]

[Hao-Lin Li, Jing Shu, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2005.00008]

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2305.10481]

- EFTs at Broken Phase

[Hao Sun, Ming-Lei Xiao, **J.H.Yu**, 2206.07722]

[Hao Sun, Ming-Lei Xiao, **J.H.Yu**, 2210.14939]

[Huayang Song, Hao Sun, **J.H.Yu**, 2305.16770]

[Huayang Song, Hao Sun, **J.H.Yu**, 2306.05999]

[Chuan-Qiang Song, Hao Sun, **J.H.Yu**, 2404.15047]

- UV Completion of EFT Operators

[Xuan-He Li, Hao Sun, Feng-Jie Tang, **J.H.Yu**, 2404.14152]

[Hao Sun, Yi-Ning Wang, **J.H.Yu**, in préparation]

[Hao-Lin Li, Yu-Han Ni, Ming-Lei Xiao, **J.H.Yu**, 2204.03660]

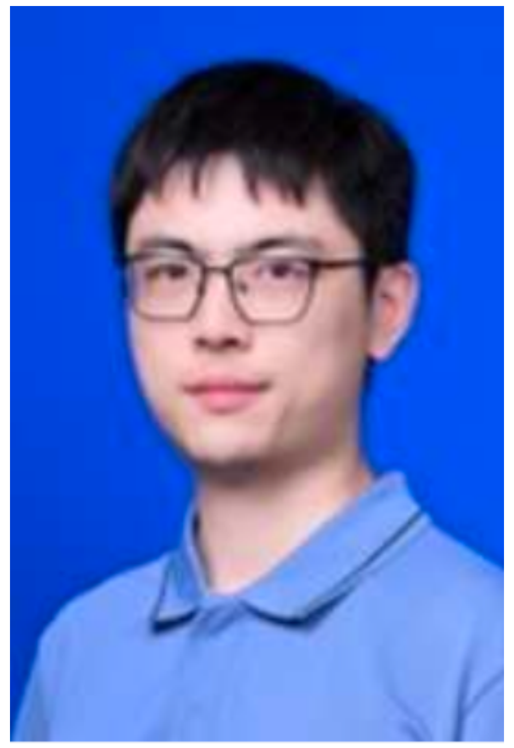
- Summary and outlook

[Xu-Xiang Li, Zhe Ren, **J.H.Yu,** 2307.10380]

[Hao-Lin Li, Yu-Han Ni, Ming-Lei Xiao, **J.H.Yu**, 2309.15933]

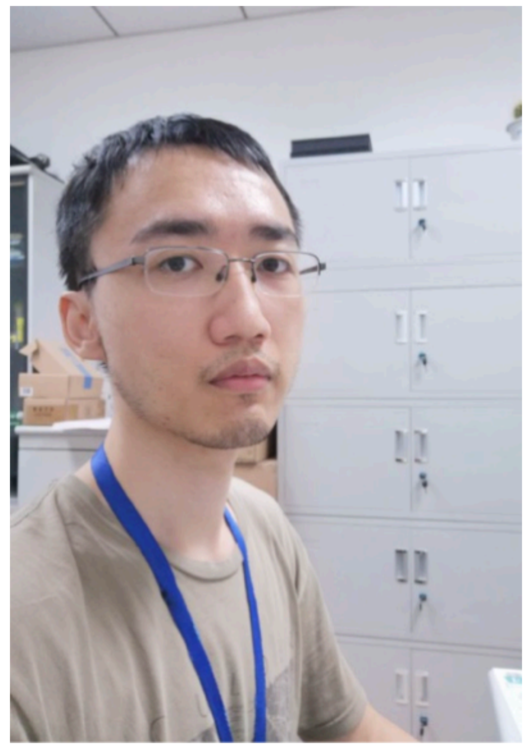
Collaborators

Previous and current postdoc and students at ITP-CAS (2019 - now)



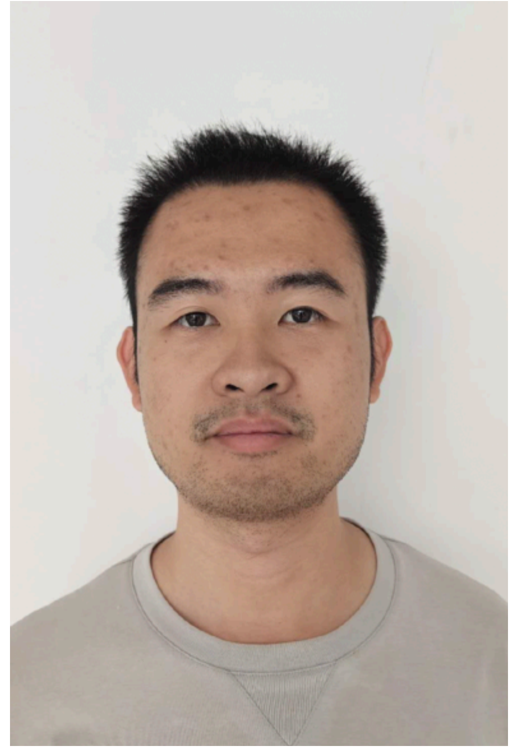
Ming-Lei Xiao

SYSU Faculty



Hao-Lin Li

UCLouven Postdoc

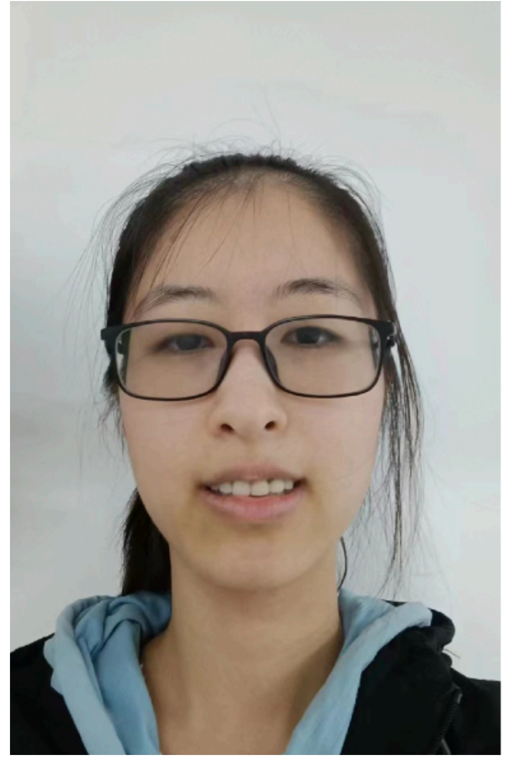


Yu-Hui Zheng

KIAS Postdoc

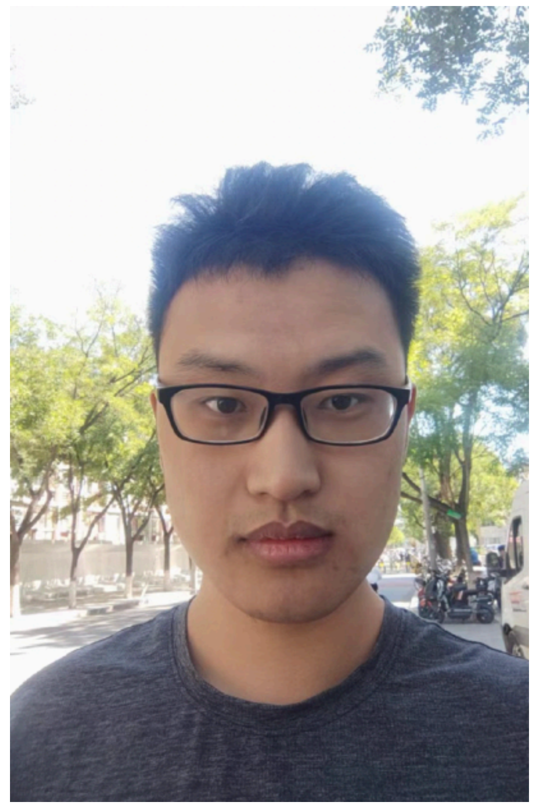
Zhe Ren

UGranada Postdoc



Yu-Han Ni

ITP-CAS Ph.D.



Hao Sun

ITP-CAS Ph.D.

Yi-Ning Wang

ITP-CAS Ph.D.

Huayang Song

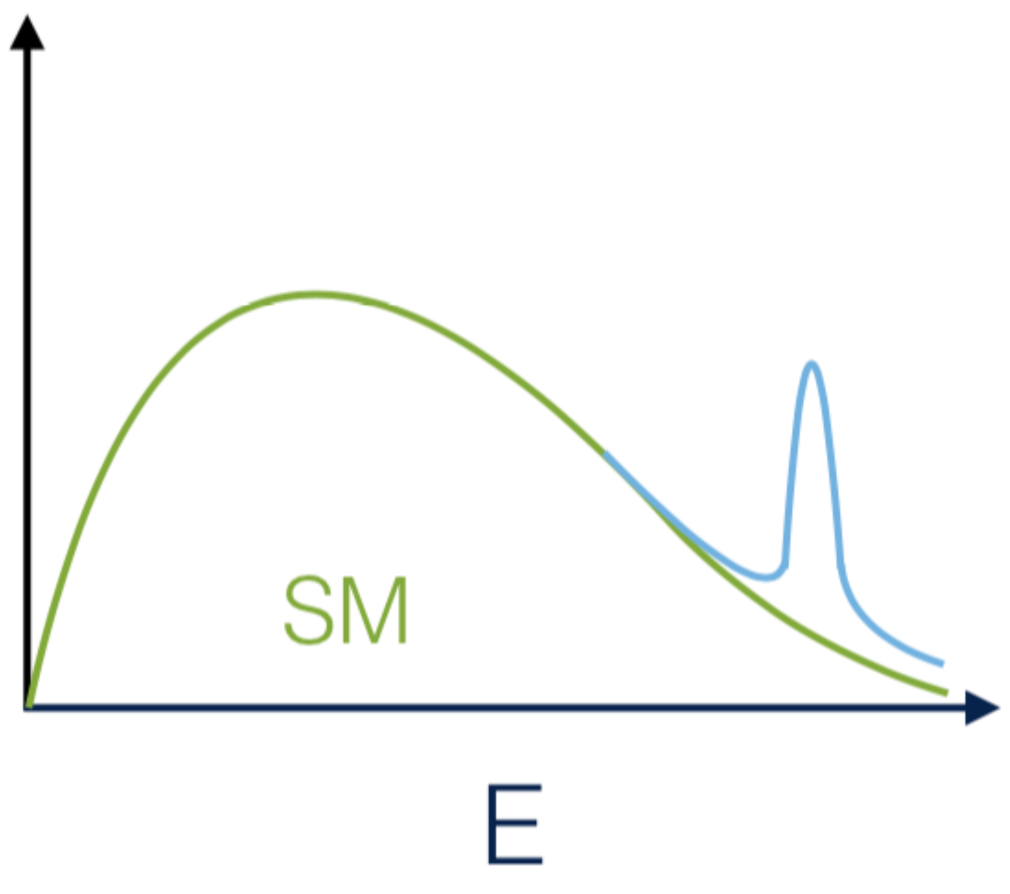
ITP-CAS Postdoc

Why, what, how EFTs for New Physics?

Search for new physics

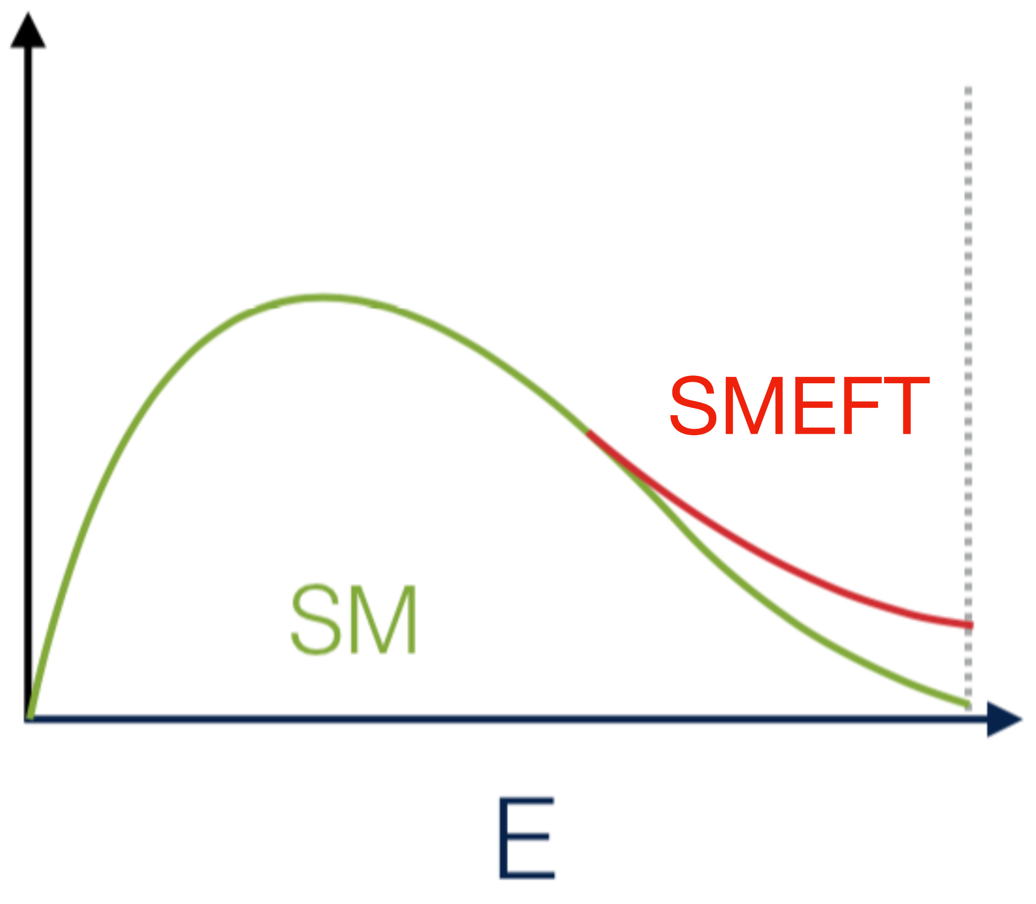
Two ways of searching new physics: top-down vs bottom-up

Direct searches



Null result at the LHC

Precision measurements



Experiments: Resonance bump hunting at the LHC

Theories: **top-down model building**

Experiments: distribution tails at the LHC

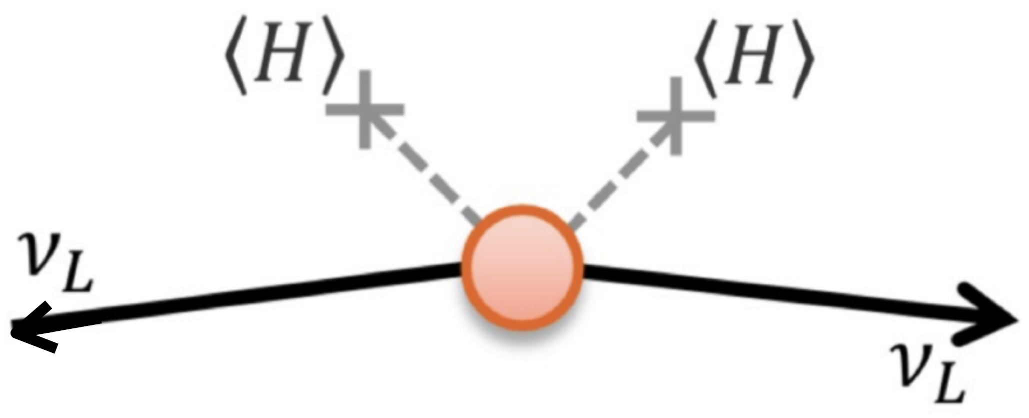
Theories: **Effective field theory (EFT)** description

New physics without new particle

New physics without new particle

The existence of neutrino masses is the first evidence of new physics beyond standard model

Measurements of neutrino oscillation

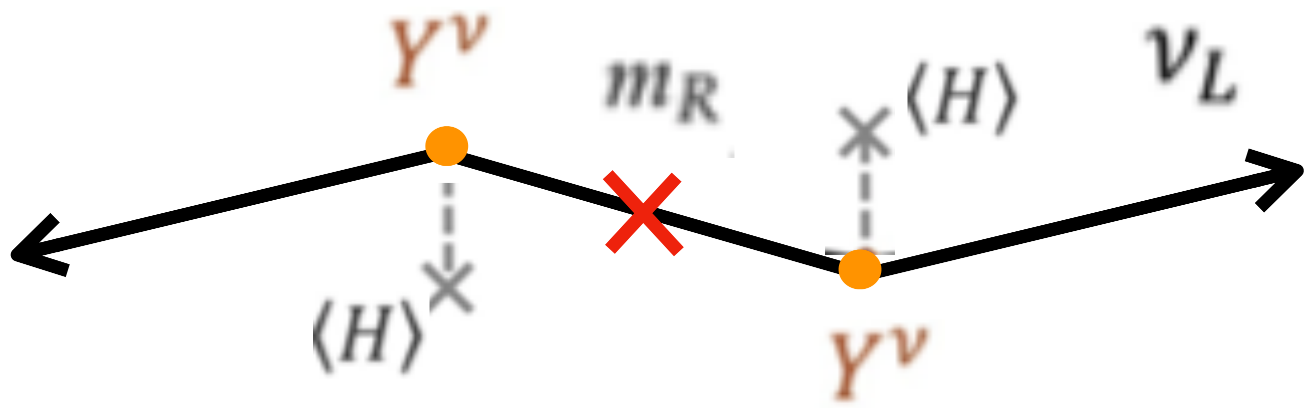


$$\frac{c_{ij}}{\Lambda}(L_iH)(L_jH) + \text{h.c.}$$

[Weinberg, 1979]

$$\to c_{ij}\frac{v^2}{\Lambda}\nu_i\nu_j + \text{h.c.}$$

Beyond the current experimental searches



for Majorana neutrino

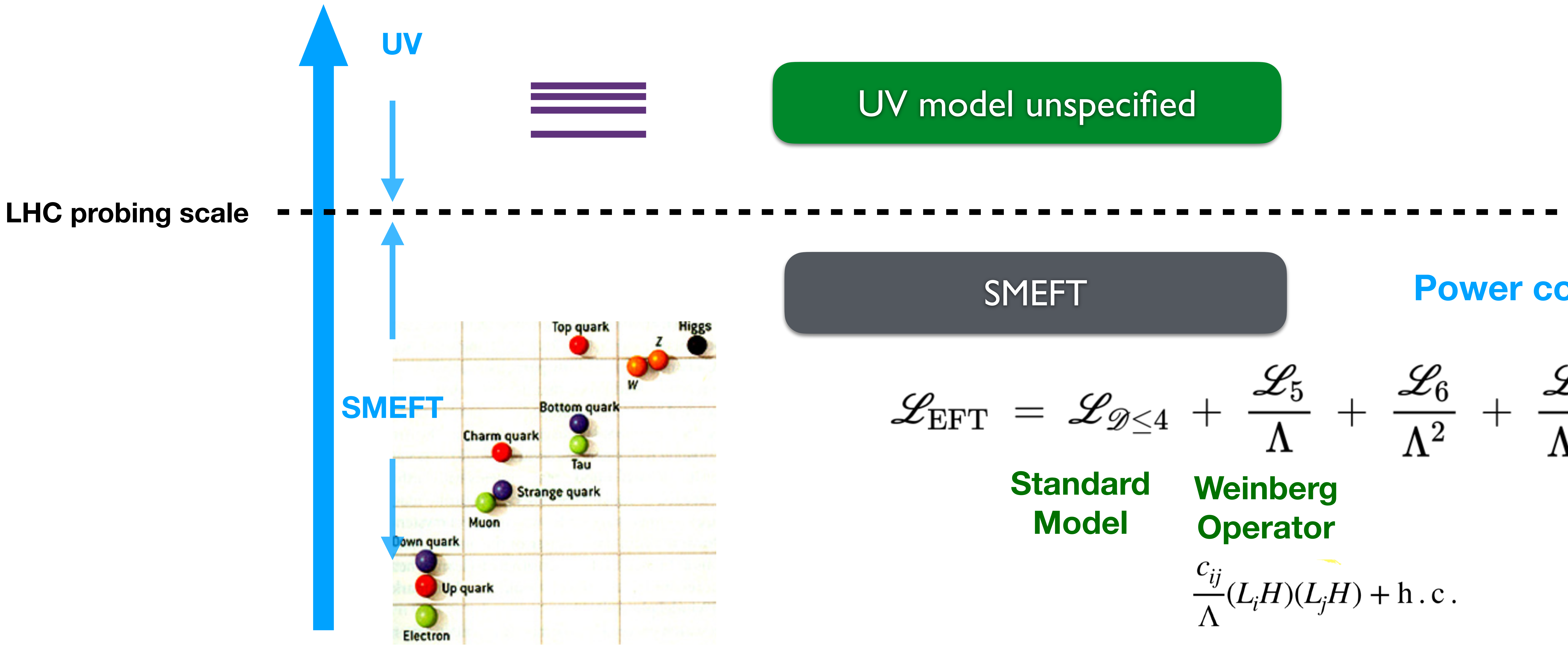
$$m_\nu = \frac{(Y^\nu v_{\rm EW})^2}{m_R}$$

"… the effective field theory point of view had predicted the neutrino masses"

[Weinberg, 2021]

Standard model effective field theory

The bottom-up approach: write down the most general Lagrangian with only SM dof and gauge symmetry



UV model unspecified

SMEFT

Power counting: canonical dimension

$$\mathscr{L}_{\text{EFT}} = \mathscr{L}_{\mathscr{D}\leq 4} + \frac{\mathscr{L}_5}{\Lambda} + \frac{\mathscr{L}_6}{\Lambda^2} + \frac{\mathscr{L}_7}{\Lambda^3} + \frac{\mathscr{L}_8}{\Lambda^4} + \frac{\mathscr{L}_9}{\Lambda^5} + \ldots$$

Standard Model

Weinberg Operator

$$\frac{c_{ij}}{\Lambda}(L_i H)(L_j H) + \text{h.c.}$$

Standard Model Effective Field Theory (SMEFT) provides systematic parameterization of all possible Lorentz-inv. new physics

Dim-6 operators

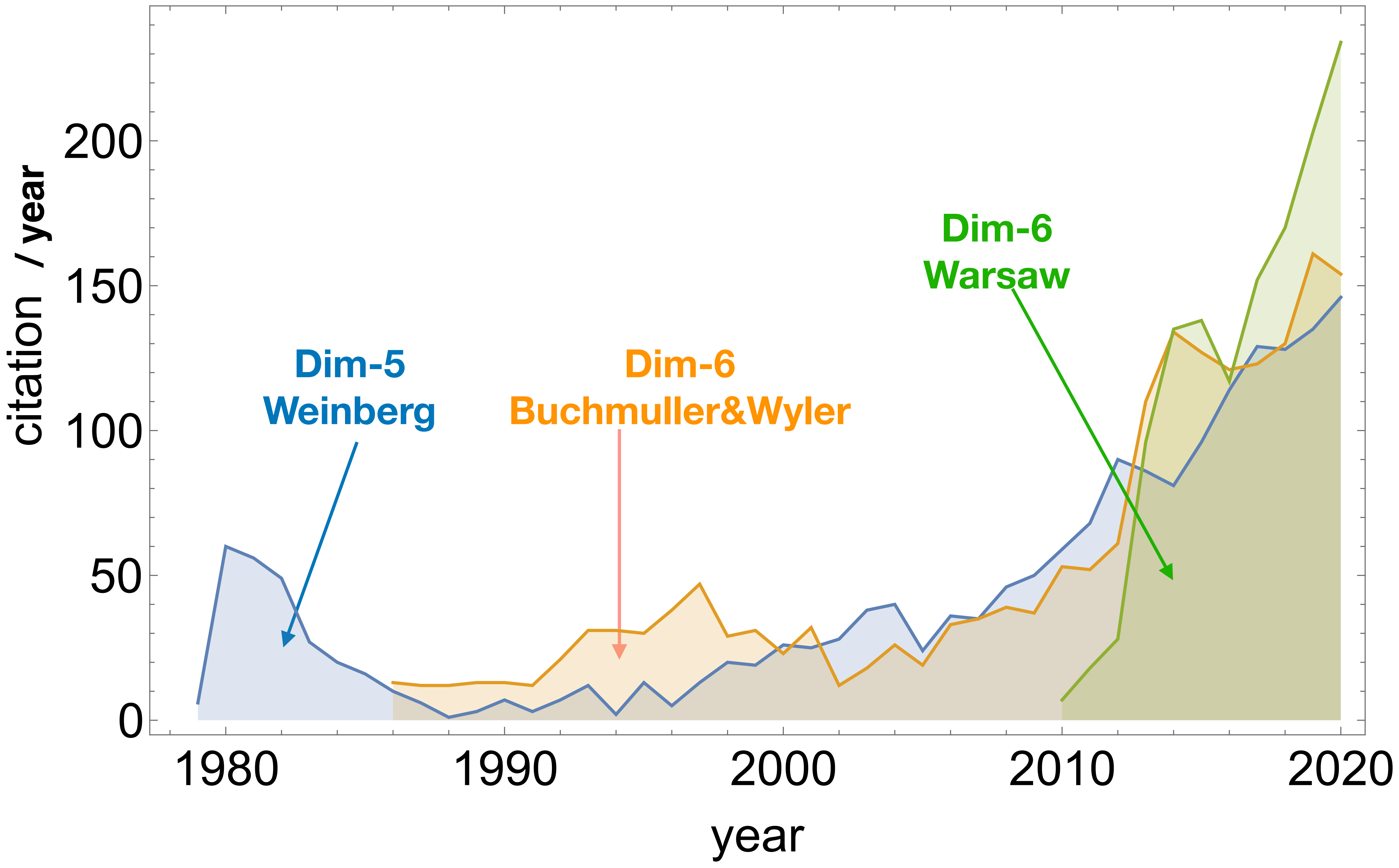
Dimension-6 operators parametrize new physics effects at low energy, electroweak precision, and Higgs boson physics, etc

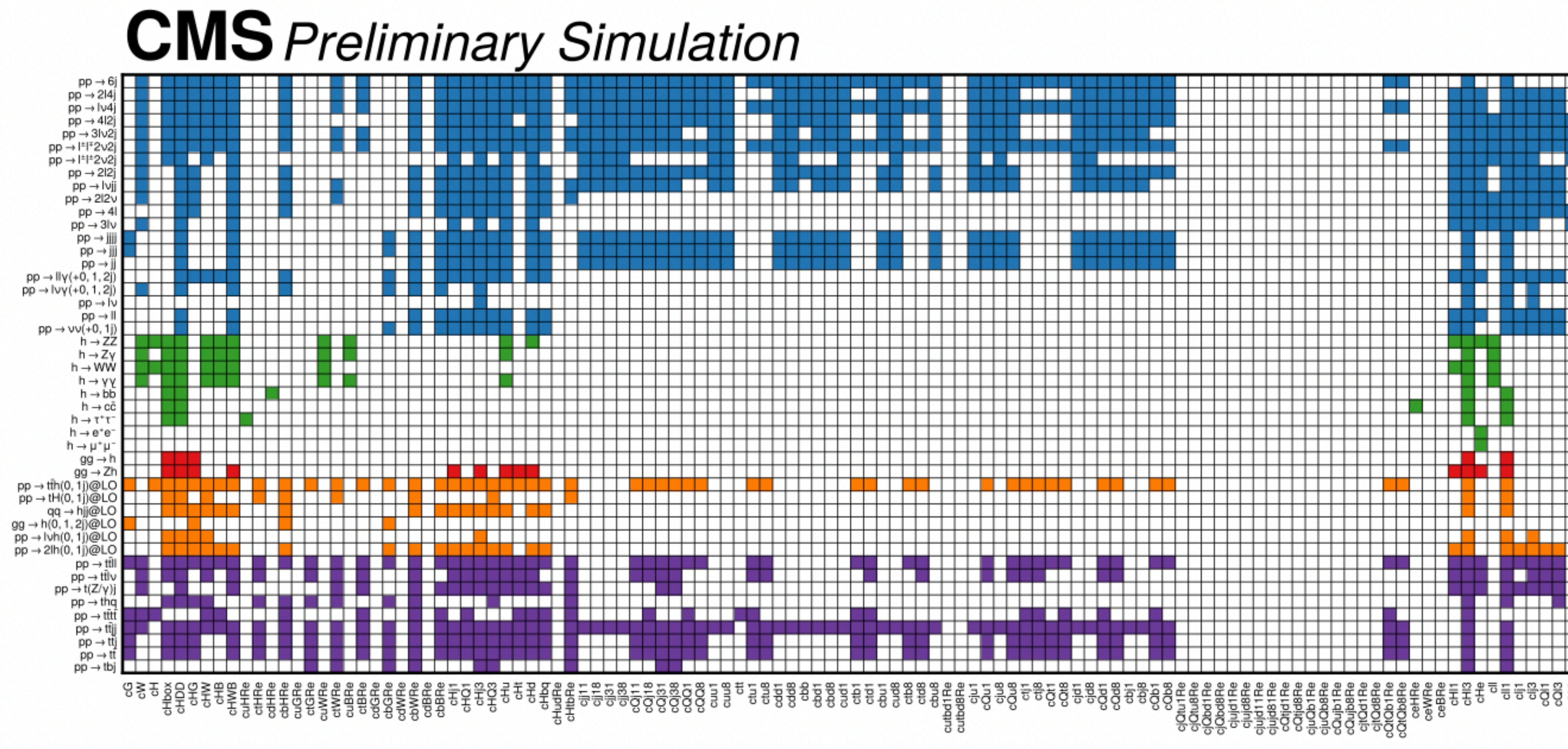
$$\mathscr{L}_{\rm EFT} = \mathscr{L}_{\mathscr{D}\le 4} + \frac{\mathscr{L}_5}{\Lambda} + \frac{\mathscr{L}_6}{\Lambda^2} + \frac{\mathscr{L}_7}{\Lambda^3} + \frac{\mathscr{L}_8}{\Lambda^4} + \frac{\mathscr{L}_9}{\Lambda^5} + \dots$$

[Buchmuller and Wyler, 1986]

[Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]

One important task of LHC run-3 : dim-6 operator Wilson coefficients





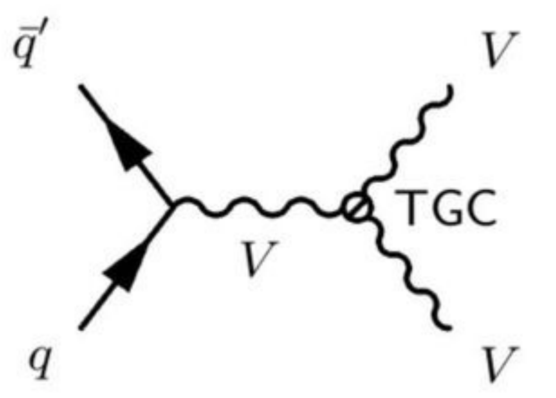
[See Jiayin Gu's talk for global fit]

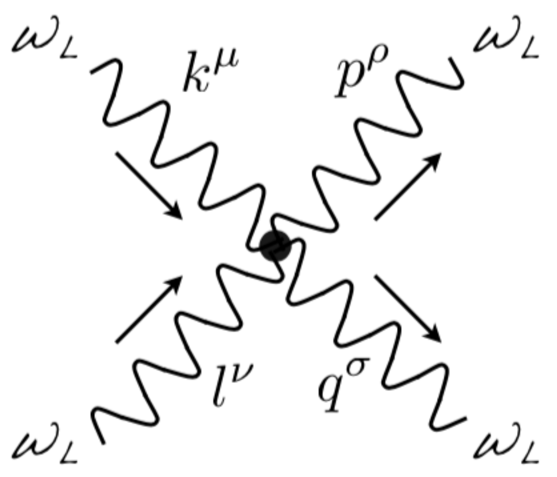
Higher dimensional operators

According to power counting rules, typically the higher dim operators are suppressed

$$|\mathcal{A}|^2 \sim \left| A_{\rm SM} + \frac{A_{\rm dim-6}}{\Lambda^2} + \frac{A_{\rm dim-8}}{\Lambda^4} + \ldots \right|^2 \sim |A_{\rm SM}|^2 + \frac{2}{\Lambda^2} A_{\rm dim-6} A^*_{\rm SM} + \frac{1}{\Lambda^4} |A_{\rm dim-6}|^2 + \frac{2}{\Lambda^4} A_{\rm dim-8} A^*_{\rm SM}$$

Collider searches





nTGC at dim-8

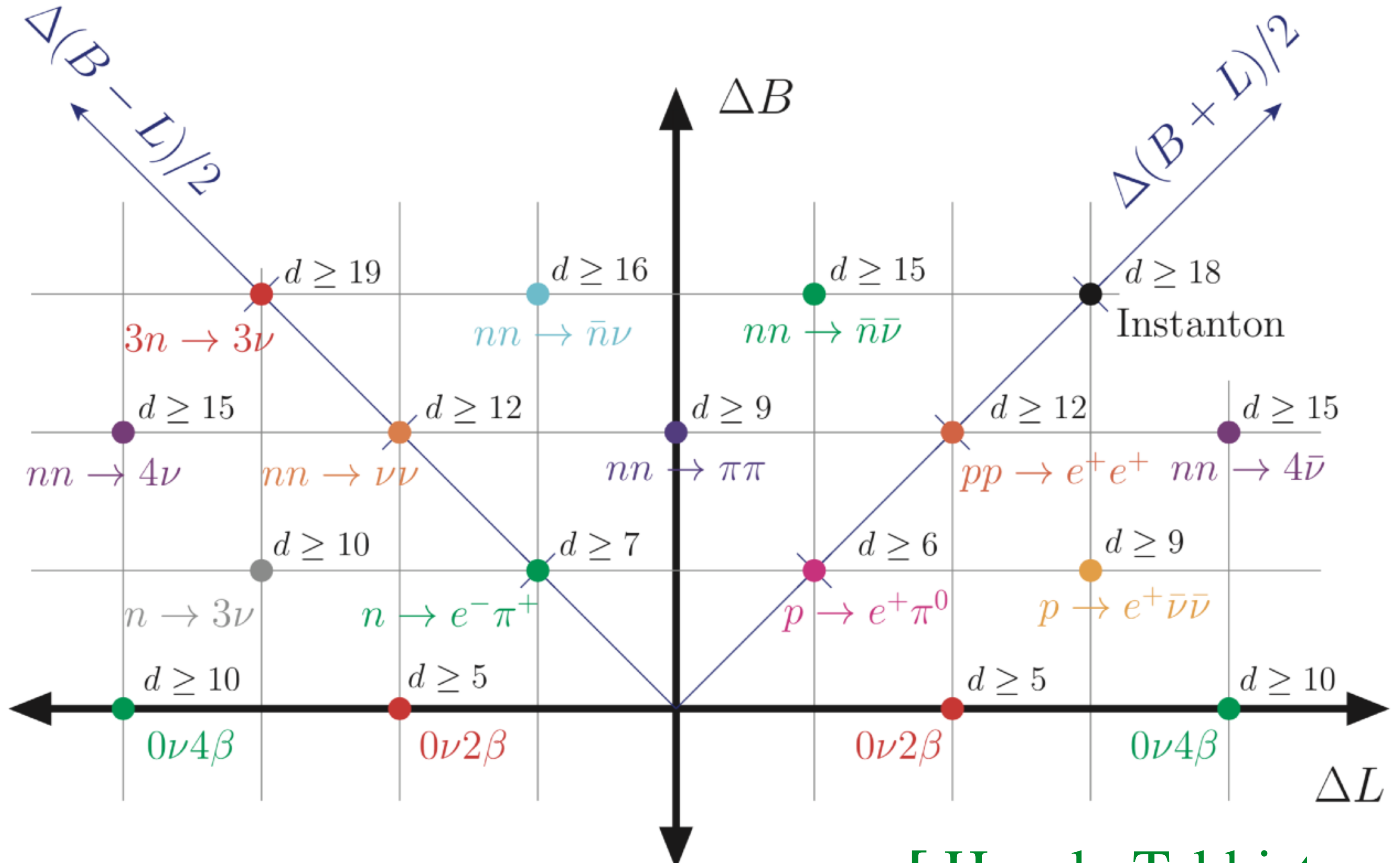
WW scattering

[Ellis, He, Ge, Xiao, 2020]

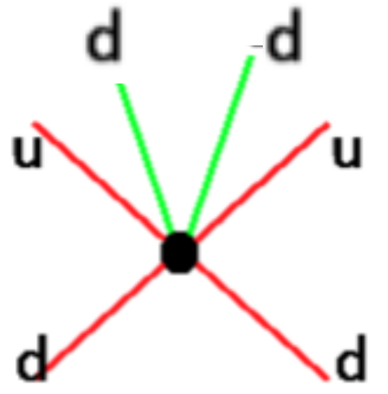
$$\Delta\sigma_{\rm dim=8}(q\bar{q} \to V_T V_T) \sim \frac{g^4_{\rm SM}}{E^2}\Big[\overbrace{c_8 \frac{E^4}{\Lambda^4}}^{\rm BSM_8 \times SM} + \overbrace{c_8^2 \frac{E^8}{\Lambda^8}}^{\rm BSM_8{}^2} + \ldots \Big]$$

[Degrande, Li, 2023]

Low energy precision



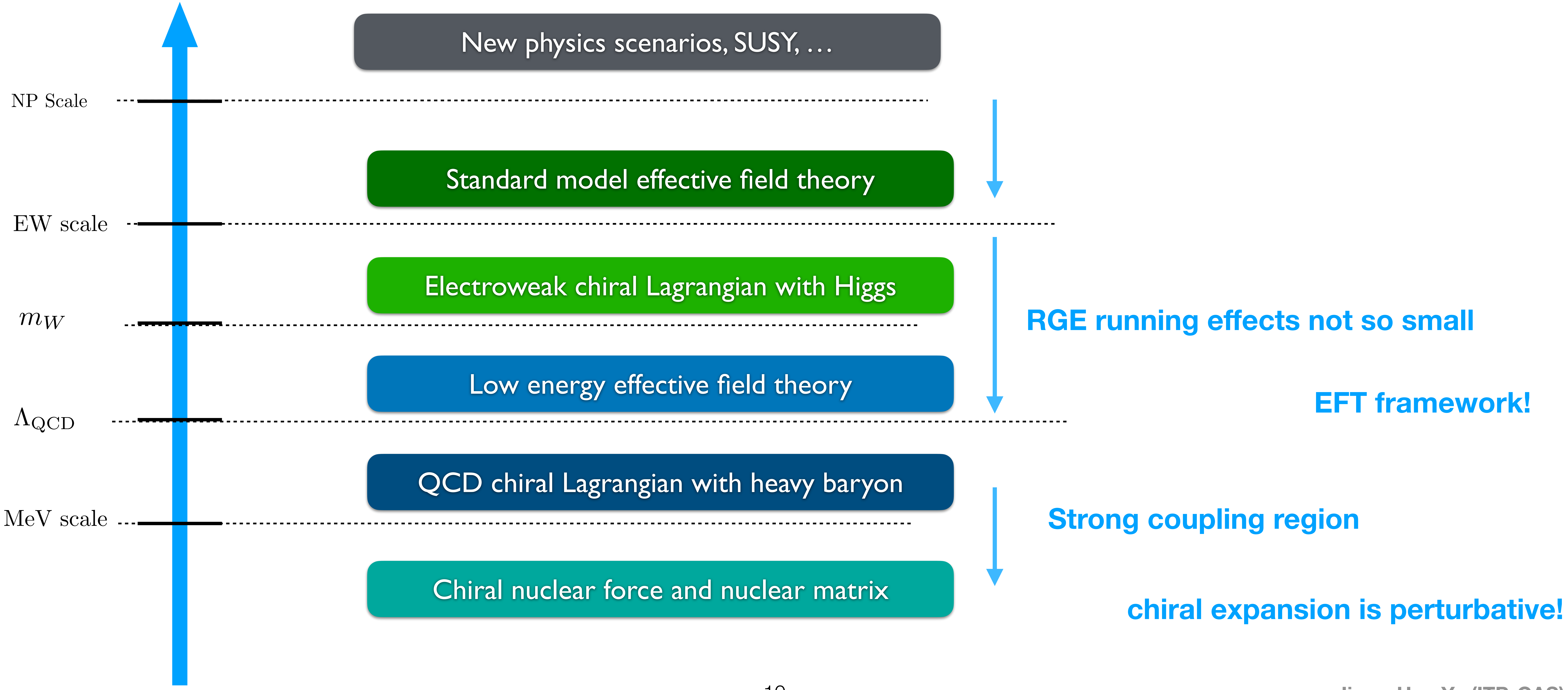
[Heeck, Takhistov, 2020]



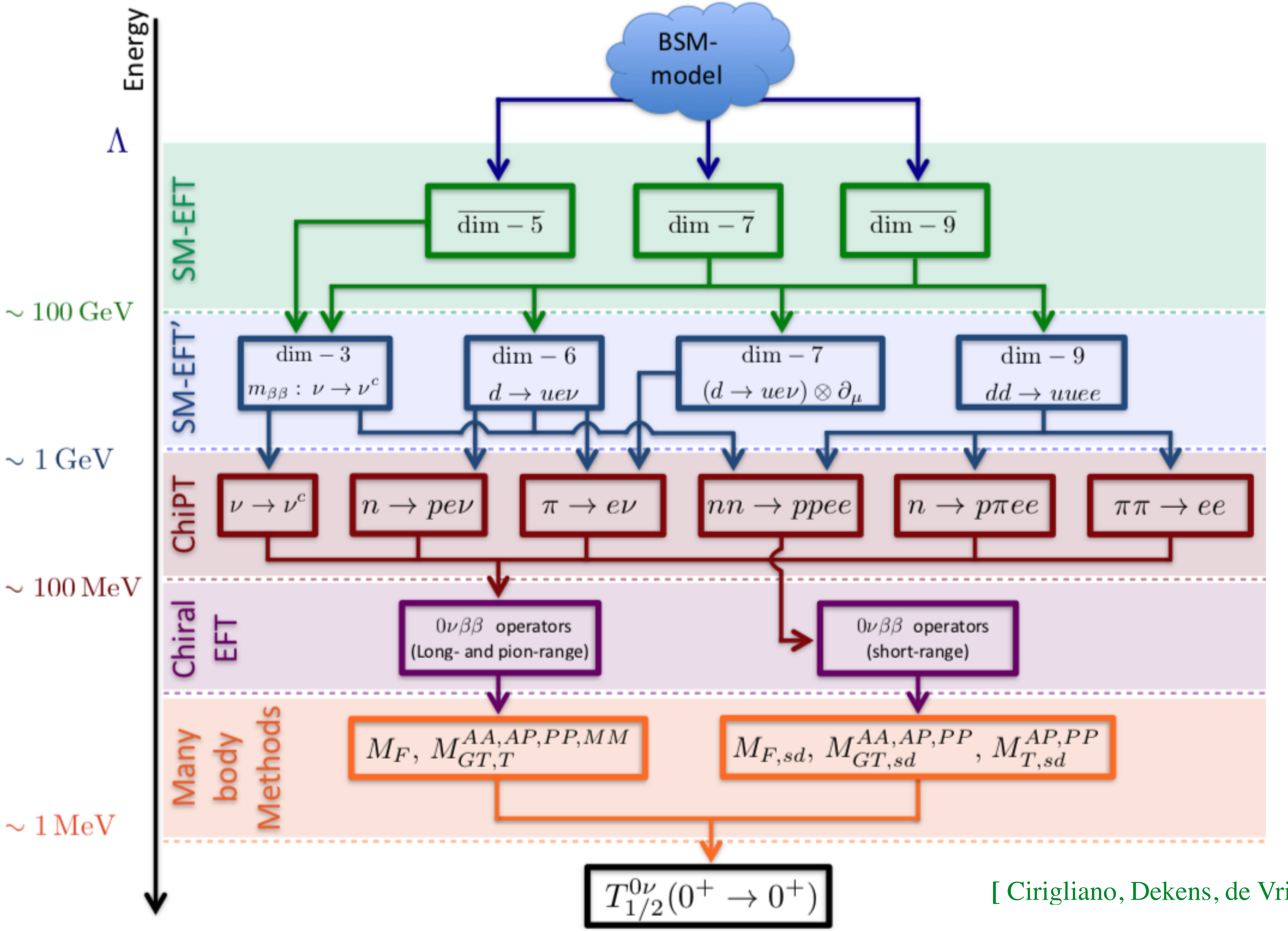
Neutron-antineutron Oscillation at dim-9

Tower of effective field theories

To investigate various precision processes, it is natural to consider different EFTs to avoid large Logs



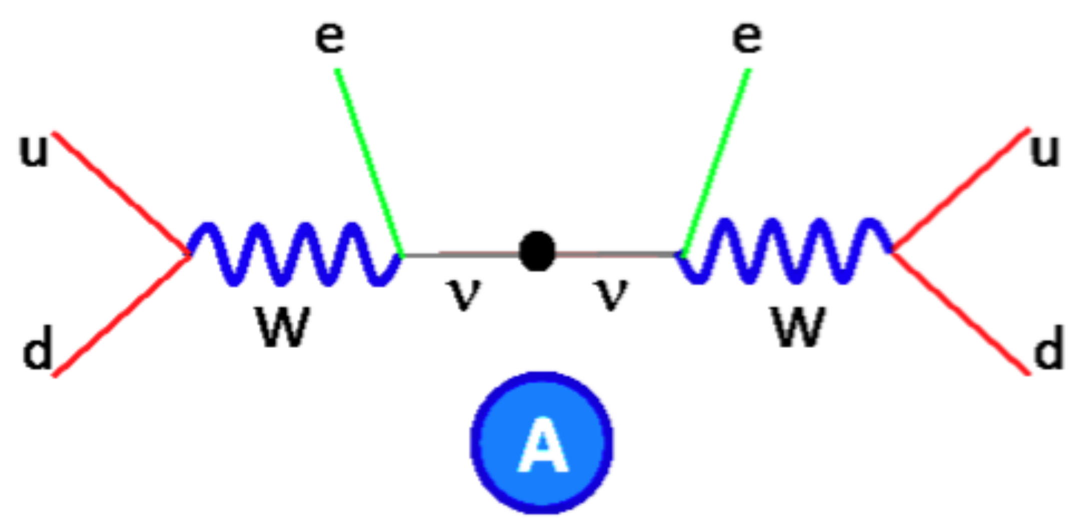
Neutrinoless double beta decay (0vbb)

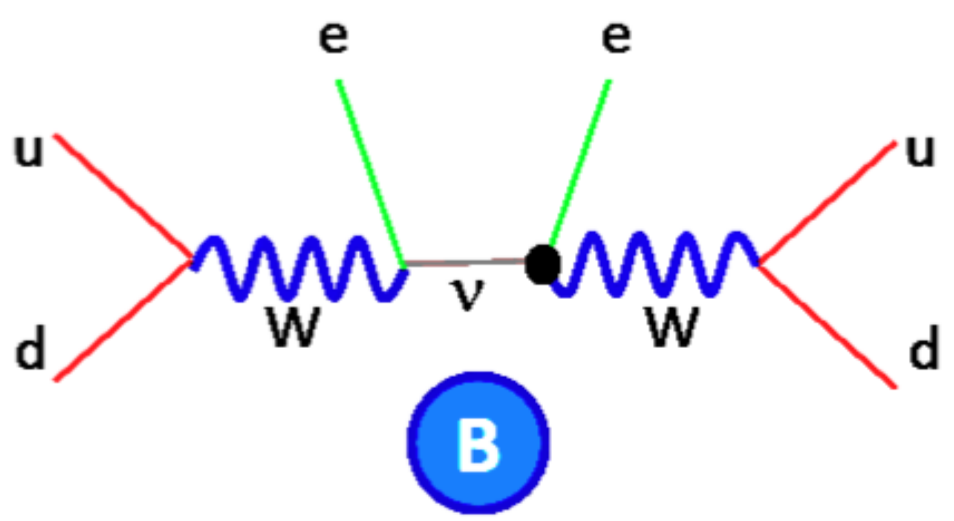


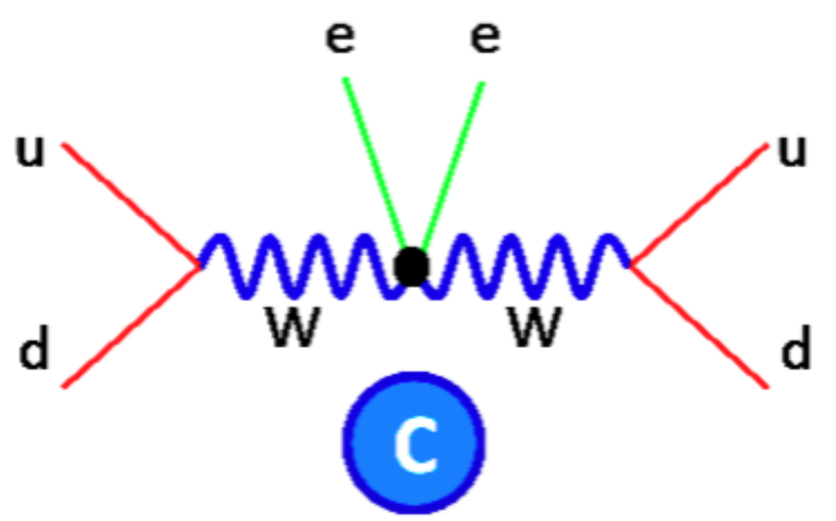
[Cirigliano, Dekens, de Vries, Graesser, Mereghetti, 2017]

Operators for 0vbb

The higher dim operators may play the leading roles on describing 0vbb process







Dim-5, 7

$\mathcal{O}_5$ $\quad \epsilon^{ik}\epsilon^{jl}(\ell_i^T C \ell_j) H_k H_l$

$\mathcal{O}_{LH}$ $\quad \epsilon^{ik}\epsilon^{jl}(\ell_i^T C \ell_j) H_k H_l (H^\dagger H)$

Dim-7

$\mathcal{O}_{LeHD}$ $\quad \epsilon^{ij}\epsilon^{kl}(\ell_i^T C \gamma^\mu e) H_j H_k (i D_\mu H_l)$

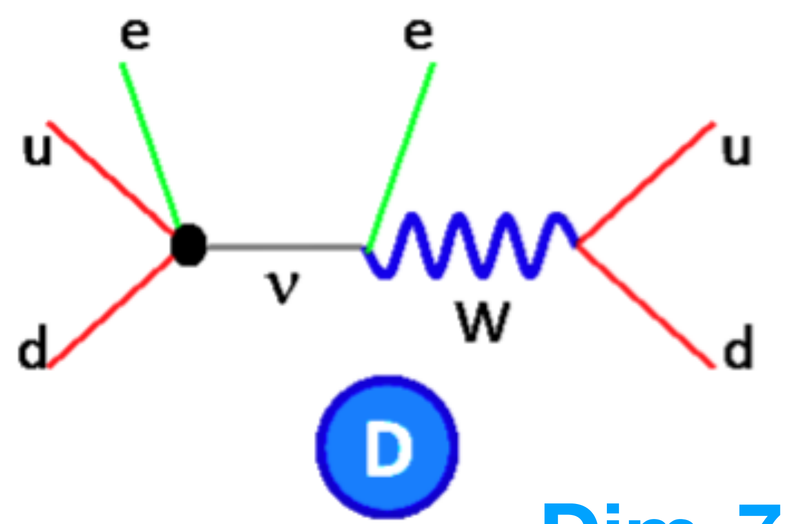
$\mathcal{O}_{LHW}$ $\quad -\epsilon^{ik}(\epsilon\tau^I)^{jl}(\ell_i^T C i \sigma^{\mu\nu} \ell_j) H_k H_l W^I_{\mu\nu}$

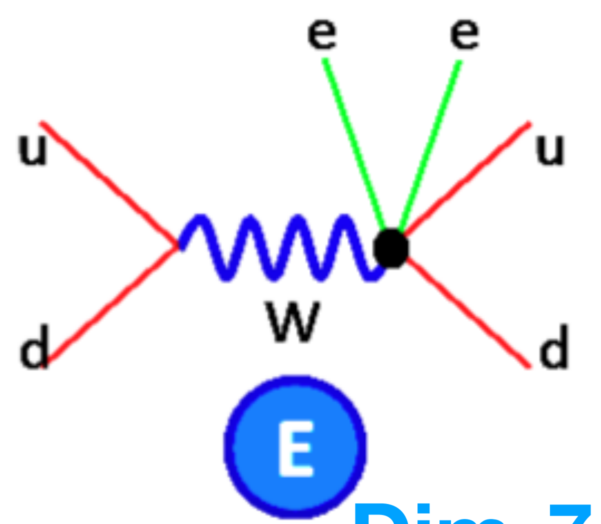
Dim-7, 9

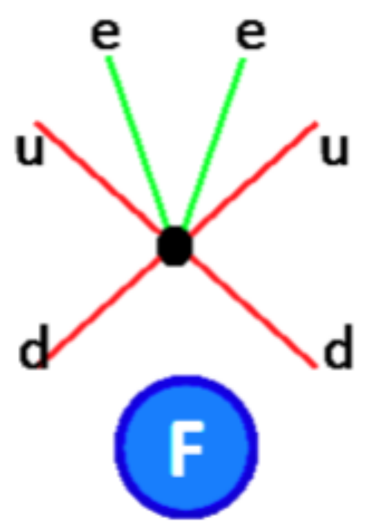
$D^2\, H^{\dagger 2}\, L^{\dagger 2}$

$\mathcal{O}_{LHD1}$ $\quad \epsilon^{ij}\epsilon^{kl}(\ell_i^T C D_\mu \ell_j)(H_k D^\nu H_l)$

$D^2\, H^{\dagger 2}\, L^{\dagger 2}\, W_L$







Dim-7

$\mathcal{O}_{dLQLH1}$ $\quad \epsilon^{ij}\epsilon^{kl}(\overline{d}^a \ell_i)(q^T_{aj} C \ell_k) H_l$

$\mathcal{O}_{dLQLH2}$ $\quad \epsilon^{ik}\epsilon^{jl}(\overline{d}^a \ell_i)(q^T_{aj} C \ell_k) H_l$

$\mathcal{O}_{dLueH}$ $\quad \epsilon^{ij}(\overline{d}^a \ell_i)(u^T_a C e) H_j$

$\mathcal{O}_{QuLLH}$ $\quad \epsilon^{ij}(\overline{q}^{ak} u_a)(\ell^T_k C \ell_i) H_j$

Dim-7, 9

$\mathcal{O}_{duLLD}$ $\quad \epsilon^{ij}(\overline{d}^a \gamma^\mu u_a)(\ell_i^T C i D_\mu \ell_j)$

$D\, d_c^\dagger\, L^{\dagger 2}\, u_c$

$d_c^\dagger\, e_c\, H^\dagger\, L^\dagger\, u_c\, W_L$

Dim-9

$d_c^2\, L^2\, Q^2$, $d_c^2\, d_c^\dagger\, L^2\, u_c^\dagger$, $d_c\, L^2\, u_c\, u_c^{\dagger 2}$, $d_c^2\, e_c^\dagger\, L\, Q\, u_c^\dagger$,
$d_c^{\dagger 2}\, e_c^2\, u_c^2$, $d_c\, L^2\, Q\, Q^\dagger\, u_c^\dagger$, $d_c^\dagger\, e_c\, L^\dagger\, Q\, u_c^2$, $L^{\dagger 2}\, Q^2\, u_c^2$

How to obtain complete and independent effective operators?

Dim-6 Operators

[Buchmuller and Wyler, 1986]

$O_\varphi = \frac{1}{3}(\varphi^\dagger\varphi)^3$,

$O_{\partial\varphi} = \frac{1}{2}\partial_\mu(\varphi^\dagger\varphi)\,\partial^\mu(\varphi^\dagger\varphi)$.

$O_{e\varphi} = (\varphi^\dagger\varphi)(\bar{\ell}e\varphi)$,

$O_{u\varphi} = (\varphi^\dagger\varphi)(\bar{q}u\tilde{\varphi})$,

$O_{d\varphi} = (\varphi^\dagger\varphi)(\bar{q}d\varphi)$,

$O_G = f_{ABC}G_\mu^{A\nu}G_\nu^{B\lambda}G_\lambda^{C\mu}$,

$O_{\tilde{G}} = f_{ABC}\tilde{G}_\mu^{A\nu}G_\nu^{B\lambda}G_\lambda^{C\mu}$,

$O_W = \varepsilon_{IJK}W_\mu^{I\nu}W_\nu^{J\lambda}W_\lambda^{K\mu}$,

$O_{\tilde{W}} = \varepsilon_{IJK}\tilde{W}_\mu^{I\nu}W_\nu^{J\lambda}W_\lambda^{K\mu}$.

$O_{\ell B} = i\bar{\ell}\gamma_\mu D_\nu \ell B^{\mu\nu}$,

$O_{qB} = i\bar{q}\gamma_\mu D_\nu q B^{\mu\nu}$,

$O_{\varphi G} = \frac{1}{2}(\varphi^\dagger\varphi)G_{\mu\nu}^A G^{A\mu\nu}$,

$O_{\varphi W} = \frac{1}{2}(\varphi^\dagger\varphi)W_{\mu\nu}^I W^{I\mu\nu}$,

$O_{\varphi B} = \frac{1}{2}(\varphi^\dagger\varphi)B_{\mu\nu}B^{\mu\nu}$,

$O_{WB} = (\varphi^\dagger\tau^I\varphi)W_{\mu\nu}^I B^{\mu\nu}$,

$O_\varphi^{(1)} = (\varphi^\dagger\varphi)(D_\mu\varphi^\dagger D^\mu\varphi)$,

$O_{\varphi\tilde{G}} = (\varphi^\dagger\varphi)\tilde{G}_{\mu\nu}^A G^{A\mu\nu}$,

$O_{\varphi\tilde{W}} = (\varphi^\dagger\varphi)\tilde{W}_{\mu\nu}^I W^{I\mu\nu}$,

$O_{\varphi\tilde{B}} = (\varphi^\dagger\varphi)\tilde{B}_{\mu\nu}B^{\mu\nu}$,

$O_{\tilde{W}B} = (\varphi^\dagger\tau^I\varphi)\tilde{W}_{\mu\nu}^I B^{\mu\nu}$,

$O_\varphi^{(3)} = (\varphi^\dagger D^\mu\varphi)(D_\mu\varphi^\dagger\varphi)$.

$O_{\ell W} = i\bar{\ell}\tau^I\gamma_\mu D_\nu \ell W^{I\mu\nu}$,

$O_{eB} = i\bar{e}\gamma_\mu D_\nu e B^{\mu\nu}$,

$O_{qG} = i\bar{q}\lambda^A\gamma_\mu D_\nu q G^{A\mu\nu}$,

$O_{qW} = i\bar{q}\tau^I\gamma_\mu D_\nu q W^{I\mu\nu}$,

$O_{uG} = i\bar{u}\lambda^A\gamma_\mu D_\nu u G^{A\mu\nu}$,

$O_{uB} = i\bar{u}\gamma_\mu D_\nu u B^{\mu\nu}$,

$O_{dG} = i\bar{d}\lambda^A\gamma_\mu D_\nu d G^{A\mu\nu}$,

$O_{dB} = i\bar{d}\gamma_\mu D_\nu d B^{\mu\nu}$.

$O_{\varphi\ell}^{(1)} = i(\varphi^\dagger D_\mu\varphi)(\bar{\ell}\gamma^\mu\ell)$,

$O_{\varphi\ell}^{(3)} = i(\varphi^\dagger D_\mu\tau^I\varphi)(\bar{\ell}\gamma^\mu\tau^I\ell)$,

$O_{\varphi e} = i(\varphi^\dagger D_\mu\varphi)(\bar{e}\gamma^\mu e)$,

$O_{\varphi q}^{(1)} = i(\varphi^\dagger D_\mu\varphi)(\bar{q}\gamma^\mu q)$,

$O_{\varphi q}^{(3)} = i(\varphi^\dagger D_\mu\tau^I\varphi)(\bar{q}\gamma^\mu\tau^I q)$,

$O_{\varphi_u} = i(\varphi^\dagger D_\mu\varphi)(\bar{u}\gamma^\mu u)$,

$O_{\varphi_d} = i(\varphi^\dagger D_\mu\varphi)(\bar{d}\gamma^\mu d)$,

$O_{D_e} = (\bar{\ell}D_\mu e)D^\mu\varphi$, $O_{\bar{D}_e} = (D_\mu\bar{\ell}e)D^\mu\varphi$, $^\dagger\varepsilon D_\mu\varphi)(\bar{u}\gamma^\mu d)$.

$O_{Du} = (\bar{q}D_\mu u)D^\mu\tilde{\varphi}$, $O_{\bar{D}u} = (D_\mu\bar{q}u)D^\mu\tilde{\varphi}$,

$O_{Dd} = (\bar{q}D_\mu d)D^\mu\varphi$, $O_{\bar{D}d} = (D_\mu\bar{q}d)D^\mu\varphi$,

$O_{eW} = (\bar{\ell}\sigma^{\mu\nu}\tau^I e)\varphi W_{\mu\nu}^I$, $O_{eB} = (\bar{\ell}\sigma^{\mu\nu}e)\varphi B_{\mu\nu}$,

$O_{uG} = (\bar{q}\sigma^{\mu\nu}\lambda^A u)\tilde{\varphi}G_{\mu\nu}^A$,

$O_{uW} = (\bar{q}\sigma^{\mu\nu}\tau^I u)\tilde{\varphi}W_{\mu\nu}^I$, $O_{uB} = (\bar{q}\sigma^{\mu\nu}u)\tilde{\varphi}B_{\mu\nu}$,

$O_{dG} = (\bar{q}\sigma^{\mu\nu}\lambda^A d)\varphi G_{\mu\nu}^A$,

$O_{dW} = (\bar{q}\sigma^{\mu\nu}\tau^I d)\varphi W_{\mu\nu}^I$, $O_{dW} = (\bar{q}\sigma^{\mu\nu}d)\varphi B_{\mu\nu}$.

$O_{\ell\ell}^{(1)} = \frac{1}{2}(\bar{\ell}\gamma_\mu\ell)(\bar{\ell}\gamma^\mu\ell)$, $O_{\ell\ell}^{(3)} = \frac{1}{2}(\bar{\ell}\gamma_\mu\tau^I\ell)(\bar{\ell}\gamma^\mu\tau^I\ell)$,

$O_{qq}^{(1,1)} = \frac{1}{2}(\bar{q}\gamma_\mu q)(\bar{q}\gamma^\mu q)$, $O_{qq}^{(8,1)} = \frac{1}{2}(\bar{q}\gamma_\mu\lambda^A q)(\bar{q}\gamma^\mu\lambda^A q)$,

$O_{qq}^{(1,3)} = \frac{1}{2}(\bar{q}\gamma_\mu\tau^I q)(\bar{q}\gamma^\mu\tau^I q)$, $O_{qq}^{(8,3)} = \frac{1}{2}(\bar{q}\gamma_\mu\lambda^A\tau^I q)(\bar{q}\gamma^\mu\lambda^A\tau^I q)$,

$O_{\ell q}^{(1)} = (\bar{\ell}\gamma_\mu\ell)(\bar{q}\gamma^\mu q)$, $O_{\ell q}^{(3)} = (\bar{\ell}\gamma_\mu\tau^I\ell)(\bar{q}\gamma^\mu\tau^I q)$.

$O_{ee} = \frac{1}{2}(\bar{e}\gamma_\mu e)(\bar{e}\gamma^\mu e)$,

$O_{uu}^{(1)} = \frac{1}{2}(\bar{u}\gamma_\mu u)(\bar{u}\gamma^\mu u)$, $O_{uu}^{(8)} = \frac{1}{2}(\bar{u}\gamma_\mu\lambda^A u)(\bar{u}\gamma^\mu\lambda^A u)$,

$O_{dd}^{(1)} = \frac{1}{2}(\bar{d}\gamma_\mu d)(\bar{d}\gamma^\mu d)$, $O_{dd}^{(8)} = \frac{1}{2}(\bar{d}\gamma_\mu\lambda^A d)(\bar{d}\gamma^\mu\lambda^A d)$,

$O_{eu} = (\bar{e}\gamma_\mu e)(\bar{u}\gamma^\mu u)$,

$O_{ed} = (\bar{e}\gamma_\mu e)(\bar{d}\gamma^\mu d)$,

$O_{ud}^{(1)} = (\bar{u}\gamma_\mu u)(\bar{d}\gamma^\mu d)$, $O_{ud}^{(8)} = (\bar{u}\gamma_\mu\lambda^A u)(\bar{d}\gamma^\mu\lambda^A d)$.

$O_{\ell e} = (\bar{\ell}e)(\bar{e}\ell)$,

$O_{\ell u} = (\bar{\ell}u)(\bar{u}\ell)$,

$O_{\ell d} = (\bar{\ell}d)(\bar{d}\ell)$,

$O_{qe} = (\bar{q}e)(\bar{e}q)$,

$O_{qu}^{(1)} = (\bar{q}u)(\bar{u}q)$,

$O_{qd}^{(1)} = (\bar{q}d)(\bar{d}q)$,

$O_{qde} = (\bar{\ell}e)(\bar{d}q)$.

$O_{qq}^{(1)} = (\bar{q}u)(\bar{q}d)$,

$O_{qq}^{(8)} = (\bar{q}\lambda^A u)(\bar{q}\lambda^A d)$,

$O_{\ell q} = (\bar{\ell}e)(\bar{q}u)$.

$O_{qu}^{(8)} = (\bar{q}\lambda^A u)(\bar{u}\lambda^A q)$,

$O_{qd}^{(8)} = (\bar{q}\lambda^A d)(\bar{d}\lambda^A q)$,

Equation of motion (field redefinition)

$$(D^\mu D_\mu\varphi)^j = m^2\varphi^j - \lambda(\varphi^\dagger\varphi)\varphi^j - \bar{e}\,\Gamma_e^\dagger l^j + \varepsilon_{jk}\bar{q}^k\,\Gamma_u u - \bar{d}\,\Gamma_d^\dagger q^j$$

$$i\not{D}l = \Gamma_e e\varphi, \quad i\not{D}e = \Gamma_e^\dagger\varphi^\dagger l, \quad i\not{D}q = \Gamma_u u\tilde{\varphi} + \Gamma_d d\varphi, \quad i\not{D}u = \Gamma_u^\dagger\tilde{\varphi}^\dagger q,$$

$$(D^\rho W_{\rho\mu})^I = \frac{g}{2}\left(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi + \bar{l}\gamma_\mu\tau^I l + \bar{q}\gamma_\mu\tau^I q\right),$$

Covariant derivative commutator

$$[D_\rho, D_\alpha] \sim X_{\rho\alpha}$$

80 → 59

Bianchi identity $D_{[\rho}X_{\mu\nu]} = 0$

Integration by part (total derivatives)

$$(D^n\varphi)^\dagger(D^m\varphi) = -(D^{n+1}\varphi)^\dagger(D^{m-1}\varphi) + \partial\left[(D^n\varphi)^\dagger(D^{m-1}\varphi)\right]$$

Fierz identity

$$T_{\alpha\beta}^A T_{\kappa\lambda}^A = \frac{1}{2}\delta_{\alpha\lambda}\delta_{\kappa\beta} - \frac{1}{6}\delta_{\alpha\beta}\delta_{\kappa\lambda}$$

$$\tau_{jk}^I\tau_{mn}^I = 2\delta_{jn}\delta_{mk} - \delta_{jk}\delta_{mn}$$

80-1-16-5+1 = 59

[Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]

X^3		φ^6 and φ^4D^2		$\psi^2\varphi^3$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{l}_pe_r\varphi)$
$Q_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_pu_r\widetilde{\varphi})$
Q_W	$\varepsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star\left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_pd_r\varphi)$
$Q_{\widetilde{W}}$	$\varepsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$				

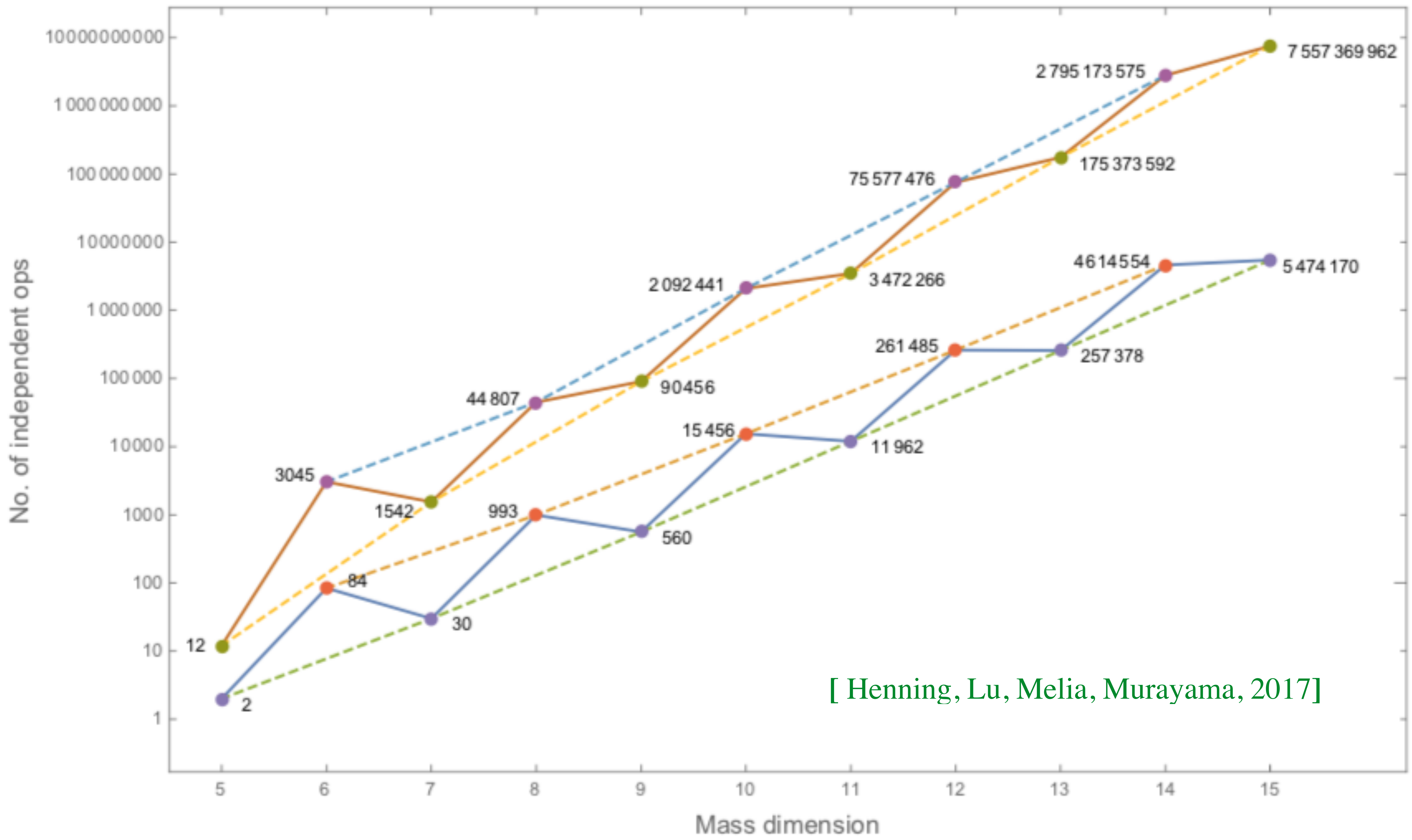
$X^2\varphi^2$		$\psi^2X\varphi$		$\psi^2\varphi^2D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\,G_{\mu\nu}^AG^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I\varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{l}_p\gamma^\mu l_r)$
$Q_{\varphi\widetilde{G}}$	$\varphi^\dagger\varphi\,\widetilde{G}_{\mu\nu}^AG^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi)(\bar{l}_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\,W_{\mu\nu}^IW^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p\sigma^{\mu\nu}T^Au_r)\widetilde{\varphi}\,G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{e}_p\gamma^\mu e_r)$
$Q_{\varphi\widetilde{W}}$	$\varphi^\dagger\varphi\,\widetilde{W}_{\mu\nu}^IW^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\widetilde{\varphi}\,W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{q}_p\gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\,B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{\varphi}\,B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi)(\bar{q}_p\tau^I\gamma^\mu q_r)$
$Q_{\varphi\widetilde{B}}$	$\varphi^\dagger\varphi\,\widetilde{B}_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p\sigma^{\mu\nu}T^Ad_r)\varphi\,G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{u}_p\gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\,W_{\mu\nu}^IB^{\mu\nu}$	Q_{dW}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I\varphi\,W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{d}_p\gamma^\mu d_r)$
$Q_{\varphi\widetilde{W}B}$	$\varphi^\dagger\tau^I\varphi\,\widetilde{W}_{\mu\nu}^IB^{\mu\nu}$	Q_{dB}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\varphi\,B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde{\varphi}^\dagger D_\mu\varphi)(\bar{u}_p\gamma^\mu d_r)$

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^Iq_r)(\bar{q}_s\gamma^\mu\tau^Iq_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^Il_r)(\bar{q}_s\gamma^\mu\tau^Iq_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^Aq_r)(\bar{u}_s\gamma^\mu T^Au_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^Au_r)(\bar{d}_s\gamma^\mu T^Ad_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^Aq_r)(\bar{d}_s\gamma^\mu T^Ad_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^je_r)(\bar{d}_sq_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^TCu_r^\beta\right]\left[(q_s^{\gamma j})^TCl_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^ju_r)\varepsilon_{jk}(\bar{q}_s^kd_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^TCq_r^{\beta k}\right]\left[(u_s^\gamma)^TCe_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^jT^Au_r)\varepsilon_{jk}(\bar{q}_s^kT^Ad_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^TCq_r^{\beta k}\right]\left[(q_s^{\gamma m})^TCl_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^je_r)\varepsilon_{jk}(\bar{q}_s^ku_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^TCu_r^\beta\right]\left[(u_s^\gamma)^TCe_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu}e_r)\varepsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu}u_t)$		

Moore’s Law on EFT Operators

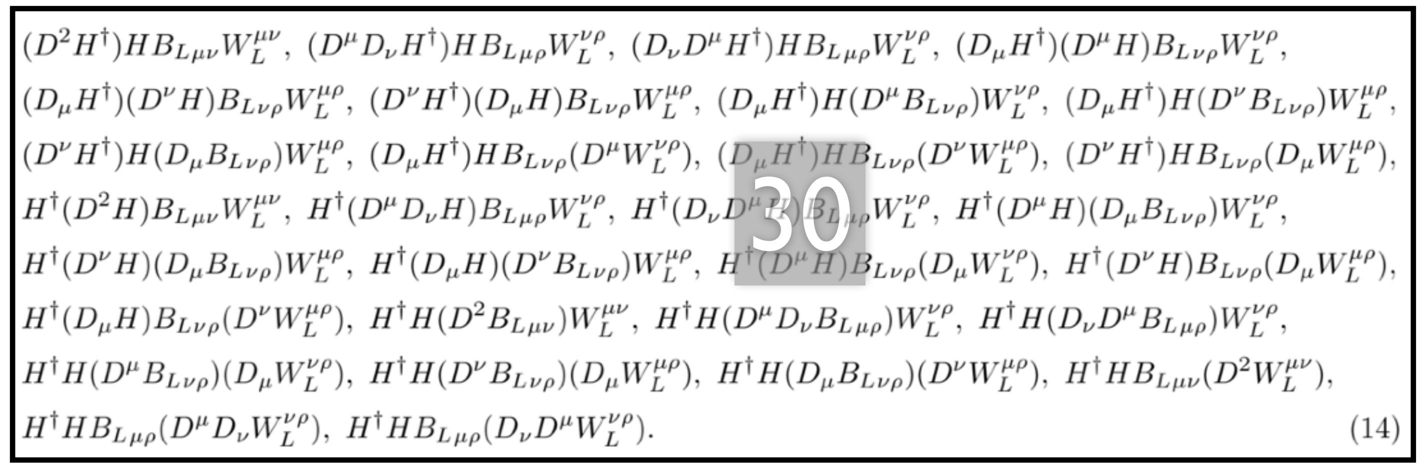
Number of EFT operators grows very fast for higher dim



Derivatives

$BWHH^\dagger D^2$

2

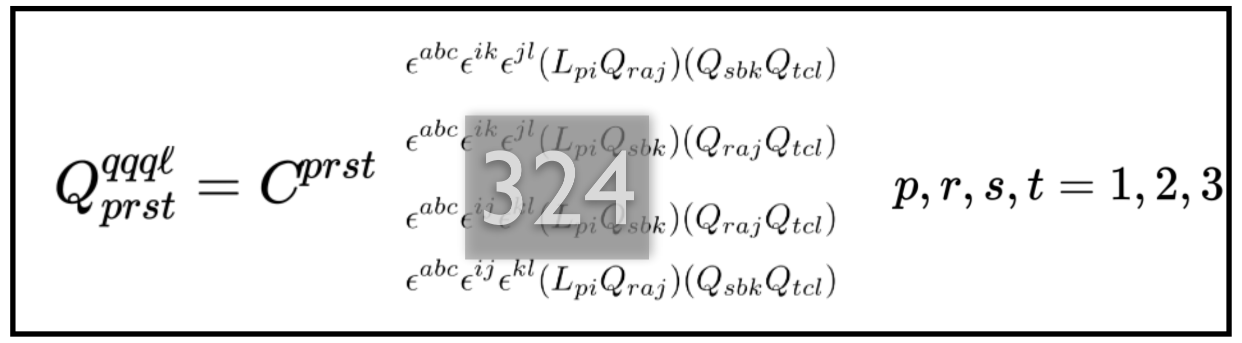


Which 2 should be picked up?

Repeated fields

$QQQL$

57



What flavor relations should be imposed?

Operators from On-shell Amplitudes

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2201.04639]

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2012.09188]

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2007.07899]

[Hao-Lin Li, Jing Shu, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2005.00008]

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2305.10481]

Weinberg's legacy

[Weinberg 1979]

Weinberg's Folk theorem on bottom-up EFT

a folk theorem: "if one writes down the most general possible Lagrangian, including *all* terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix consistent with perturbative unitarity, analyticity, cluster decomposition, and the assumed symmetry properties."

Weinberg's QFT book on constructive QFT

Poincare invariance of S matrix tells that scattering amplitudes for n particles

$$\mathcal{M}(p_a,\sigma_a) \to \mathcal{M}^{\Lambda}(p_a,\sigma_a) = \prod_a \left(D_{\sigma_a\sigma'_a}(W)\right)\mathcal{M}\left((\Lambda p)_a,\sigma'_a\right) \qquad (p_a,\sigma_a) \text{ for } a=1,\cdots,n.$$

Local Fields	Particles
Lagrangian	**Poincare + locality + unitarity**
Feynman amplitudes (off-shell, Lorentz transformation)	**On-shell amplitudes** (on-shell, little group transformation)
Field redefinition/gauge redundancies	**Advantage: no such redundancies**

On-shell scattering amplitude

The spinor helicity formalism has manifest little group covariance with only physical dof

$$p^2 = \det(p_{\alpha\dot\alpha}) = 0 \;\to\; p_{\alpha\dot\alpha} \equiv \lambda_\alpha \tilde\lambda_{\dot\alpha} \equiv |p\rangle_\alpha [p|_{\dot\alpha}$$

Spinor-helicity

$$A(1^{h_1}2^{h_2}3^{h_3}) = \begin{cases} \langle 12\rangle^{h_3-h_1-h_2}\langle 23\rangle^{h_1-h_2-h_3}\langle 31\rangle^{h_2-h_3-h_1}, & h \le 0 \\ [12]^{h_1+h_2-h_3}[23]^{h_2+h_3-h_1}[31]^{h_3+h_1-h_2}, & h \ge 0 \end{cases}$$

$$A(1^{h_1}\cdots n^{h_n}) \to \prod_i t_i^{-2h_i} A(1^{h_1}\cdots n^{h_n})$$

Amplitude-operator correspondence

$$A(1_a^- 2_b^- 3_c^+) = f_{abc}\frac{\langle 12\rangle^3}{\langle 13\rangle\langle 32\rangle}$$

$$A(1_a^+ 2_b^+ 3_c^-) = f_{abc}\frac{[12]^3}{[13][32]}$$

$$e^+_{\alpha\dot\alpha} = \frac{\eta_\alpha\tilde\lambda_{\dot\alpha}}{\langle\eta\lambda\rangle} \quad \text{and} \quad e^-_{\alpha\dot\alpha} = \frac{\lambda_\alpha\tilde\eta_{\dot\alpha}}{[\tilde\lambda\tilde\eta]}$$

$$\eta_\alpha \to \eta'_\alpha = a\eta_\alpha + b\lambda_\alpha$$

$$\longrightarrow \; f_{abc}\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)A^{\mu b}A^{\nu c}$$

$$A(1_a^- 2_b^- 3_c^-) = f_{abc}\langle 12\rangle\langle 23\rangle\langle 31\rangle$$

$$A(1_a^+ 2_b^+ 3_c^+) = f_{abc}[12][23][31],$$

$$\lambda_\alpha\lambda_\beta \to F_{L\alpha\beta}, \quad \tilde\lambda_{\dot\alpha}\tilde\lambda_{\dot\beta} \to F_{R\dot\alpha\dot\beta}$$

$$\longrightarrow \; f_{abc}F^{a\nu}_{\mu}F^{b\rho}_{\nu}F^{c\mu}_{\rho}$$

$$A(1^{--}2^{--}3^{--}) = (\langle 12\rangle\langle 23\rangle\langle 31\rangle)^2$$

$$A(1^{++}2^{++}3^{++}) = ([12][23][31])^2$$

$$\lambda_\alpha\lambda_\beta\lambda_\gamma\lambda_\delta \to \mathcal{C}_{\alpha\beta\gamma\delta}$$

$$\longrightarrow \; R_{\mu\nu}{}^{\rho\sigma}R_{\rho\sigma}{}^{\alpha\beta}R_{\alpha\beta}{}^{\mu\nu}$$

$$A(1^{-\frac12}2^{+\frac12}3^{+}) = \frac{\langle 13\rangle^2}{\langle 12\rangle}$$

$$A(1^{+\frac12}2^{-\frac12}3^{-}) = \frac{[13]^2}{[12]}$$

$$\lambda_\alpha \to \psi_\alpha, \quad \tilde\lambda_{\dot\alpha} \to \psi^\dagger_{\dot\alpha}$$

$$\longrightarrow \; \bar\psi\gamma^\mu\psi A_\mu$$

$$A(1^{+\frac12}2^{+\frac12}3^{+}) = \langle 13\rangle\langle 23\rangle$$

$$A(1^{-\frac12}2^{-\frac12}3^{-}) = [13][23]$$

$$\longrightarrow \; \bar\psi\sigma^{\mu\nu}\psi F_{\mu\nu}$$

Amplitude and operator correspondence

Spinor-helicity	SL(2,C)	SO(3,1)
	$\phi \in (0,0)$	ϕ
λ_α	$\psi_\alpha \in (1/2,0)$ $\quad \psi^\dagger_{\dot\alpha} \in (0,1/2),$	ψ
$\lambda_\alpha\lambda_\beta$	$F_{\mathrm{L}\alpha\beta} = \frac{i}{2}F_{\mu\nu}\sigma^{\mu\nu}_{\alpha\beta} \in (1,0)$ $\quad F_{\mathrm{R}\dot\alpha\dot\beta} = -\frac{i}{2}F_{\mu\nu}\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta} \in (0,1).$	$F_{\mu\nu}$
$\lambda_\alpha\lambda_\beta\lambda_\gamma\lambda_\delta$	$\mathcal{C}_{\alpha\beta\gamma\delta} = C_{\mu\nu\rho\sigma}\sigma^{\mu\nu}_{\alpha\beta}\sigma^{\rho\sigma}_{\gamma\delta} \in (2,0)$	$R_{\mu\nu\rho\sigma}$
$\lambda_\alpha\tilde\lambda_{\dot\alpha}$	$D_{\alpha\dot\alpha} = D_\mu\sigma^\mu_{\alpha\dot\alpha} \in (1/2,1/2).$	D_μ

4-pt contact amplitudes:

Little group scaling + 4-pt kinematics + Schouten

$$\langle ij\rangle\lambda_k + \langle ki\rangle\lambda_j + \langle jk\rangle\lambda_i = 0$$

5-pt or more?

… + momentum conservation + Gram determinant

$$\sum_{i=1}^{6} p_i = 0 \;\rightarrow\; \mathrm{Gram}(p_1,p_2,p_3,p_4,p_5) = 0 \;\rightarrow\; 4\ p_i \text{ independent in 4-dim}$$

mult.	min. dim.	helicity conf.	spinor structures
3-pt	dim-3	sss	constant
	dim-4	f^+f^+s	$[12]$
	dim-5	v^+v^+s	$[12]^2$
		$v^+f^+f^+$	$[12][13]$
	dim-6	$v^+v^+v^+$	$[12][13][23]$
4-pt	dim-4	$ssss$	constant; s_{ij}; $s_{ij}s_{kl}$
	dim-5	f^+f^+ss	$[12](s_{ij})$
	dim-6	v^+v^+ss	$[12]^2(s_{ij})$
		$v^+f^+f^+s$	$[12][13](s_{ij})$
		$f^+f^+f^+f^+$	$[12][34](s_{ij})$, $[13][24](s_{ij})$
		$f^+f^+f^-f^-$	$[12]\langle 34\rangle(s_{ij})$
		f^+f^-ss	$[1(3-4)2\rangle(s_{ij})$
3-pt	dim-5	t^+t^+s	$[12]^4$
	dim-6	$t^+t^+t^+$	$[12]^2[13]^2[23]^2$
		$t^+t^+v^+$	$[12]^3[23][13]$
		$t^+v^+v^+$	$[12]^2[13]^2$
4-pt	dim-6	t^+t^+ss	$[12]^4$; $[12]^4 s_{12}$

[Shadmi, Weiss, 2018]
[Ma, Shu, Xiao, 2019]
[Durieux, Kitahara, Shadmi, Weiss, 2019]
[Durieux, Machado 2019]
[Falkowski, Machado, 2019]

Operator as spinor Young tensor

[Li, Ren, Xiao, **Yu**, Zheng, 2201.04639]
[Li, Ren, Shu, Xiao, **Yu**, Zheng, 2005.00008]
[Li, Ren, Xiao, **Yu**, Zheng, 2007.07899]
[Li, Ren, Xiao, **Yu**, Zheng, 2012.09188]

Operator

$$\mathcal{O}_N^{(d)} = (\epsilon^{\alpha_i\alpha_j})^{\otimes n}(\tilde{\epsilon}_{\dot\alpha_i\dot\alpha_j})^{\otimes\tilde n}\prod_{i=1}^{N}(D^{r_i-|h_i|}\Psi_{i,a_i})^{\dot\alpha_i^{r_i+h_i}}_{\alpha_i^{r_i-h_i}}$$

Symmetrize indices

$$D_{[\alpha\dot\alpha}D_{\beta]\dot\beta} = D_\mu D_\nu \sigma^\mu_{[\alpha\dot\alpha}\sigma^\nu_{\beta]\dot\beta} = -D^2\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta} + \frac{i}{2}[D_\mu, D_\nu]\epsilon_{\alpha\beta}(\bar\sigma^{\mu\nu})_{\dot\alpha\dot\beta},$$

$$D_{[\alpha\dot\alpha}\psi_{\beta]} = D_\mu\sigma^\mu_{[\alpha\dot\alpha}\psi_{\beta]} = -\epsilon_{\alpha\beta}(D_\mu\sigma^\mu\psi)_{\dot\alpha},$$

$$D_{[\alpha\dot\alpha}F_{\mathrm{L}\beta]\gamma} = \frac{i}{2}D_\mu F_{\mathrm{L}\nu\rho}\sigma^\mu_{[\alpha\dot\alpha}\sigma^{\nu\rho}_{\beta]\gamma} = iD^\mu F_{\mathrm{L}\mu\nu}\epsilon_{\alpha\beta}\sigma^\nu_{\gamma\dot\alpha},$$

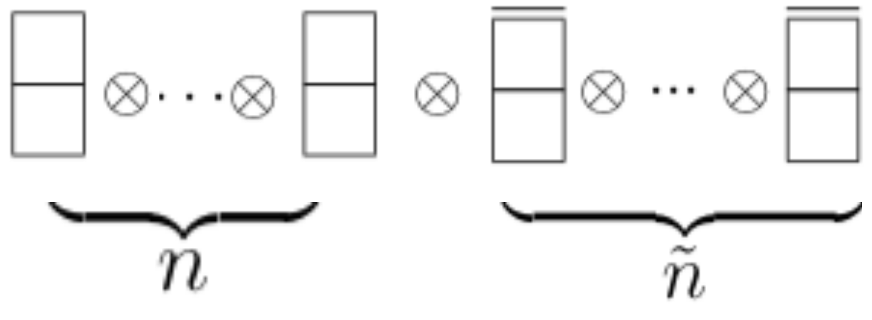
$$(D\psi)_{\alpha\beta\dot\alpha} = -\frac{1}{2}\epsilon_{\alpha\beta}(\not{D}\psi)_{\dot\alpha} + \frac{1}{2}(D\psi)_{(\alpha\beta)\dot\alpha}$$

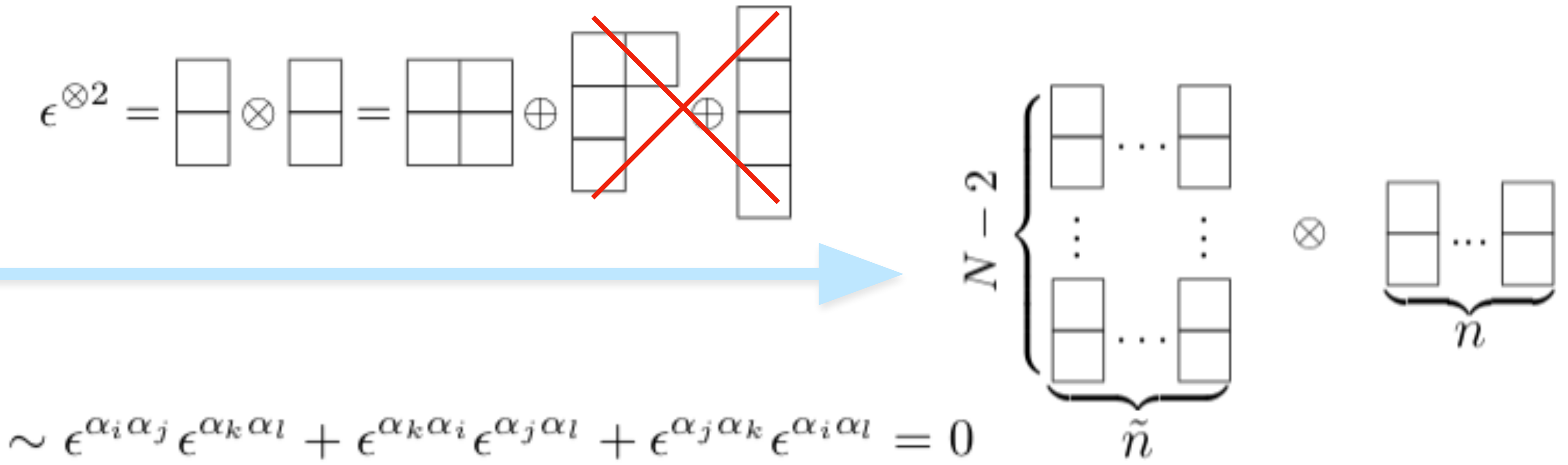
EOM CDC

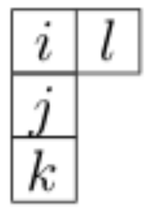
$$(\epsilon^{\alpha_i\alpha_j})^{\otimes n}(\tilde\epsilon_{\dot\alpha_i\dot\alpha_j})^{\otimes\tilde n}\prod_{i=1}^{N}\lambda_i^{r_i-h_i}\tilde\lambda^{i,r_i+h_i}$$

SL(2,C) x SU(N)

$$\lambda_i \to \sum_j U_i^j\lambda_j, \quad \tilde\lambda^i \to \sum_k U^{\dagger i}{}_k\tilde\lambda^k,$$

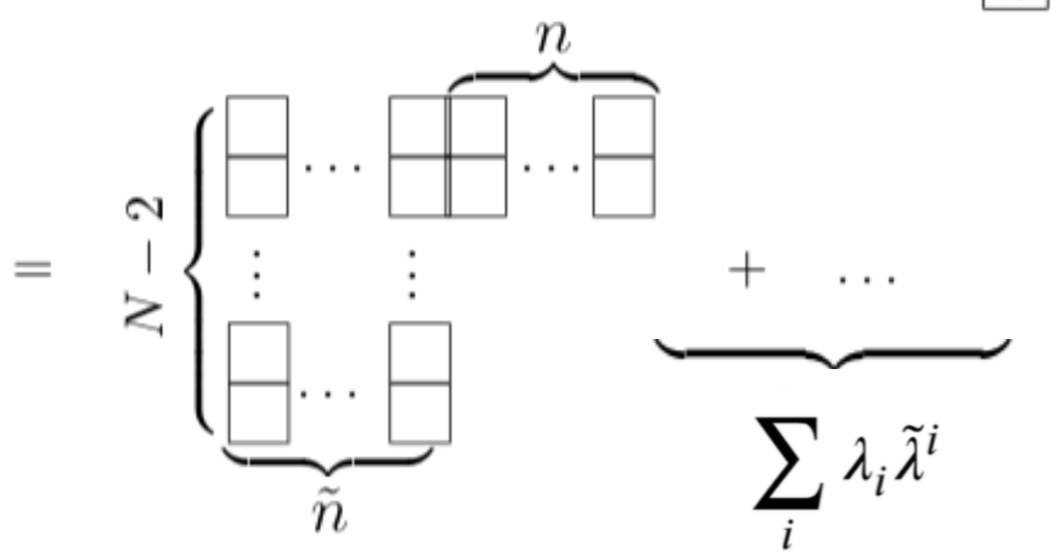






$$\sim \epsilon^{\alpha_i\alpha_j}\epsilon^{\alpha_k\alpha_l} + \epsilon^{\alpha_k\alpha_i}\epsilon^{\alpha_j\alpha_l} + \epsilon^{\alpha_j\alpha_k}\epsilon^{\alpha_i\alpha_l} = 0$$

Schouten identity [Henning, Melia, 2019]

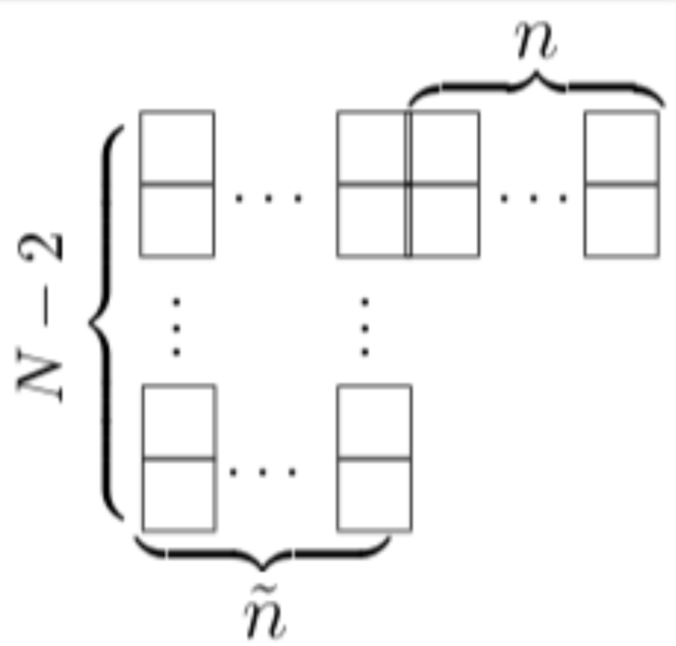


total derivatives

$$\delta^{(4)}\left(\sum_{i=1}^N \lambda_i\tilde\lambda_i\right)$$

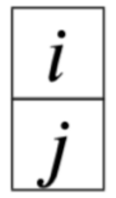
$$\sum_i \lambda_i\tilde\lambda^i \to \sum_i\sum_j\sum_k U_i^j U^{\dagger i}{}_k\lambda_j\tilde\lambda^k = \sum_j \lambda_j\tilde\lambda^j$$

SSYT



$$\{\overbrace{1,\ldots,1}^{\#1},\overbrace{2,\ldots,2}^{\#2},\ldots,\overbrace{N,\ldots,N}^{\#N}\}$$

$$\#i = \tilde n - 2h_i$$



$\Leftrightarrow \langle ij\rangle$

On-shell Amplitude

1	1	1	2
2	3	3	4

$\langle 13\rangle\langle 13\rangle\langle 24\rangle[34]$ $\quad F_{\mathrm{L}1}^{\alpha\beta}\psi_2^\gamma(D\psi_3)_{\alpha\beta\dot\alpha}(D\phi_4)_\gamma{}^{\dot\alpha}$

1	1	1	3
2	2	3	4

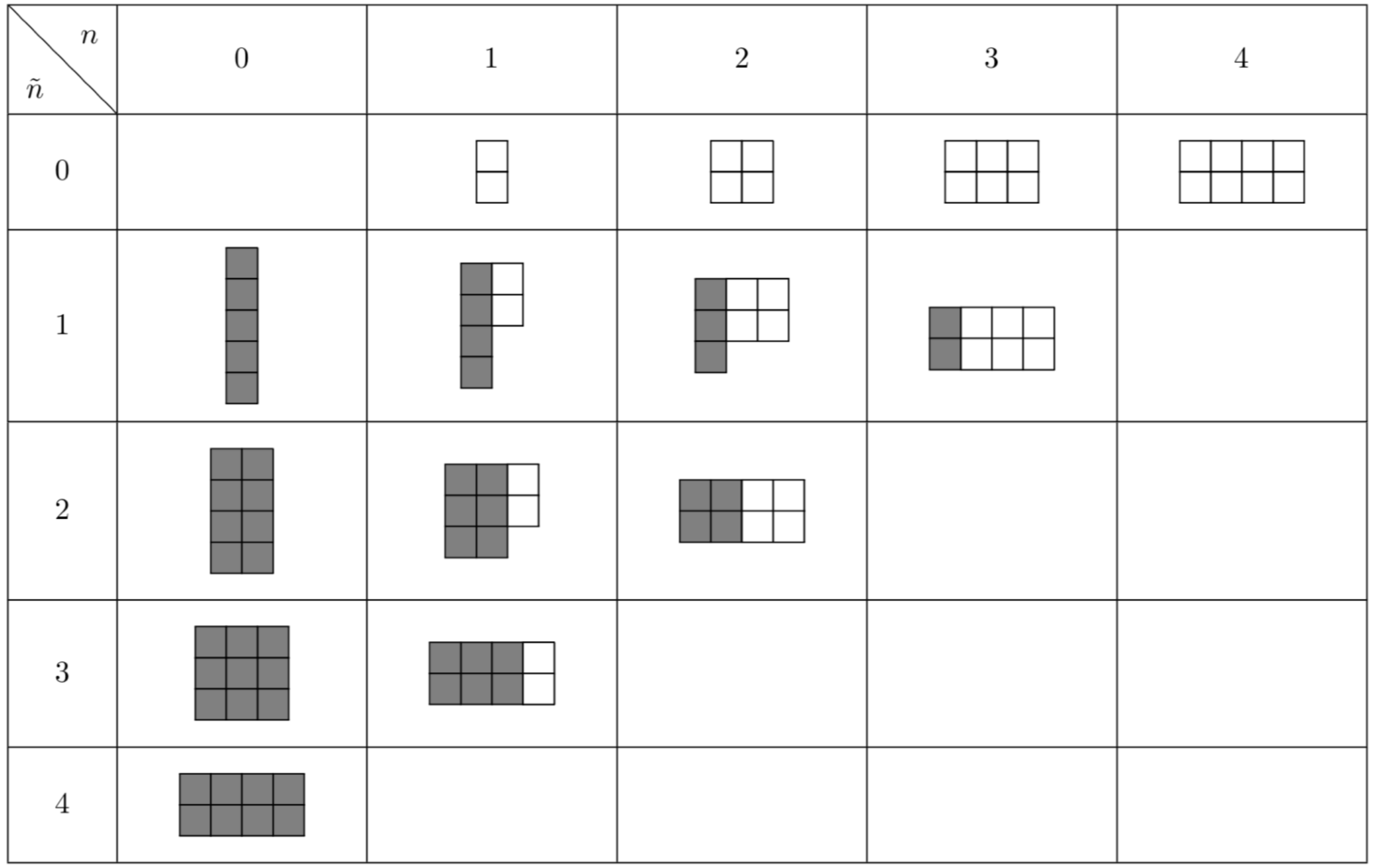
$\langle 12\rangle\langle 13\rangle\langle 34\rangle[34]$ $\quad F_{\mathrm{L}1}^{\alpha\beta}\psi_{2\alpha}(D\psi_3)_\beta{}^\gamma{}_{\dot\alpha}(D\phi_4)_\gamma{}^{\dot\alpha}$

Procedure and comparison

Dim-8 operators: 993 (44807) operators for 1 (3) generations

Step-1

$\tilde{n}$ \ n	0	1	2	3	4
0	ϕ^8	$\psi^2\phi^5$	$\psi^4\phi^2,\ F_L\psi^2\phi^3,\ F_L^2\phi^4$	$F_L\psi^4,\ F_L^2\psi^2\phi,\ F_L^3\phi^2$	F_L^4
1	$\psi^{\dagger 2}\phi^5$	$\psi^{\dagger 2}\psi^2\phi^2,\ \psi^\dagger\psi\phi^4 D,\ \phi^6 D^2$	$F_L\psi^{\dagger 2}\psi^2,\ F_L^2\psi^{\dagger 2}\phi,\ \psi^\dagger\psi^3\phi D,\ F_L\psi^\dagger\psi\phi^2 D,\ \psi^2\phi^3 D^2,\ F_L\phi^4 D^2$	$F_L^2\psi^\dagger\psi D,\ \psi^4 D^2,\ F_L\psi^2\phi D^2,\ F_L^2\phi^2 D^2$	
2	$\psi^{\dagger 4}\phi^2,\ F_R\psi^{\dagger 2}\phi^3,\ F_R^2\phi^4$	$F_R\psi^{\dagger 2}\psi^2,\ F_R^2\psi^2\phi,\ \psi^{\dagger 3}\psi\phi D,\ F_R\psi^\dagger\psi\phi^2 D,\ \psi^{\dagger 2}\phi^3 D^2,\ F_R\phi^4 D^2$	$F_R^2F_L^2,\ F_RF_L\psi^\dagger\psi D,\ \psi^{\dagger 2}\psi^2 D^2,\ F_R\psi^2\phi D^2,\ F_L\psi^{\dagger 2}\phi D^2,\ F_RF_L\phi^2 D^2,\ \phi^4 D^4,\ \psi^\dagger\psi\phi^2 D^3$		
3	$F_R\psi^{\dagger 4},\ F_R^2\psi^{\dagger 2}\phi,\ F_R^3\phi^2$	$F_R^2\psi^\dagger\psi D,\ \psi^{\dagger 4}D^2,\ F_R\psi^{\dagger 2}\phi D^2,\ F_R^2\phi^2 D^2$			
4	F_R^4				



Step-2

$BWHH^\dagger D^2$ $\quad \#1 = 3, \#2 = 3, \#3 = 1, \#4 = 1$

Step-3

1	1	1	3
2	2	2	4

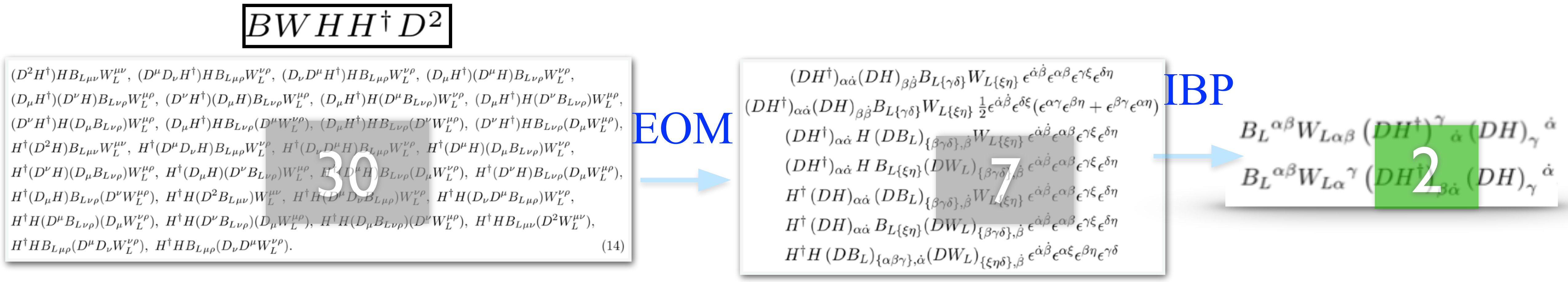
1	1	1	2
2	2	3	4

2

$\langle 13\rangle\langle 13\rangle\langle 24\rangle[34]$ $\quad$ $B_L{}^{\alpha\beta}W_{L\alpha\beta}(DH^\dagger)^\gamma{}_{\dot\alpha}(DH)_\gamma{}^{\dot\alpha},$

$\langle 12\rangle\langle 13\rangle\langle 34\rangle[34]$ $\quad$ $B_L{}^{\alpha\beta}W_{L\alpha}{}^\gamma(DH^\dagger)_{\beta\dot\alpha}(DH)_\gamma{}^{\dot\alpha}$

Traditional method [Hays, Martin, Sanz, Setford, 2018]



Jiang-Hao Yu (ITP-CAS)

SMEFT operator bases up to dim-9

$$\mathscr{L}_{\text{EFT}} = \mathscr{L}_{\mathscr{D}\leq 4} + \frac{\mathscr{L}_5}{\Lambda} + \frac{\mathscr{L}_6}{\Lambda^2} + \frac{\mathscr{L}_7}{\Lambda^3} + \frac{\mathscr{L}_8}{\Lambda^4} + \frac{\mathscr{L}_9}{\Lambda^5} + \ldots$$

Standard Model Effective Field Theory

SU(3) x SU(2) x U(1) gauge symmetry

Dim-5: [Weinberg, 1979]

Dim-6: [Buchmuller, Wyler, 1986]
[Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]

Dim-7: [Lehman, 2014]
[Henning, Lu, Melia, Murayama, 2015]
[Liao, Ma, 2016]

Dim-8: [Li, Ren, Shu, Xiao, **Yu**, Zheng, **2020**]
[Murphy, 2020]

Dim-9: [Li, Ren, Xiao, **Yu**, Zheng, **2020**]
[Liao, Ma, 2020]

Low Energy Effective Field Theory

SU(3) x U(1) gauge symmetry

Dim-5: [Dirac, 1932]

Dim-6: [Fermi, 1934]
[Lee, Yang, 1956]
[Jenkins, Manohar, Stoffer, 2017]

Dim-7: [Liao, Ma, Wang, 2020]
[Li, Ren, Xiao, **Yu**, Zheng, 2020]

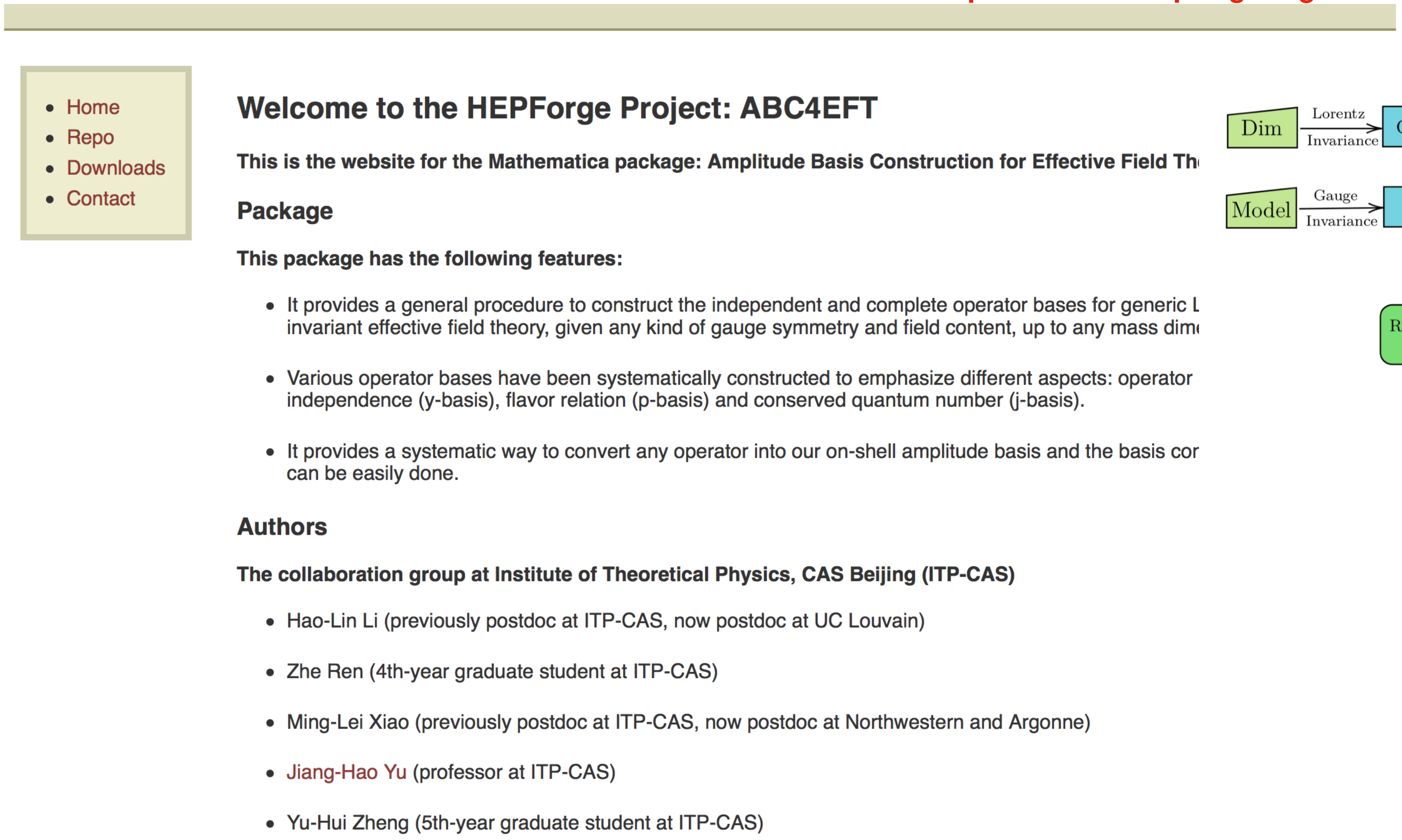
Dim-8: [Li, Ren, Xiao, **Yu**, Zheng, 2020]
[Murphy, 2020]

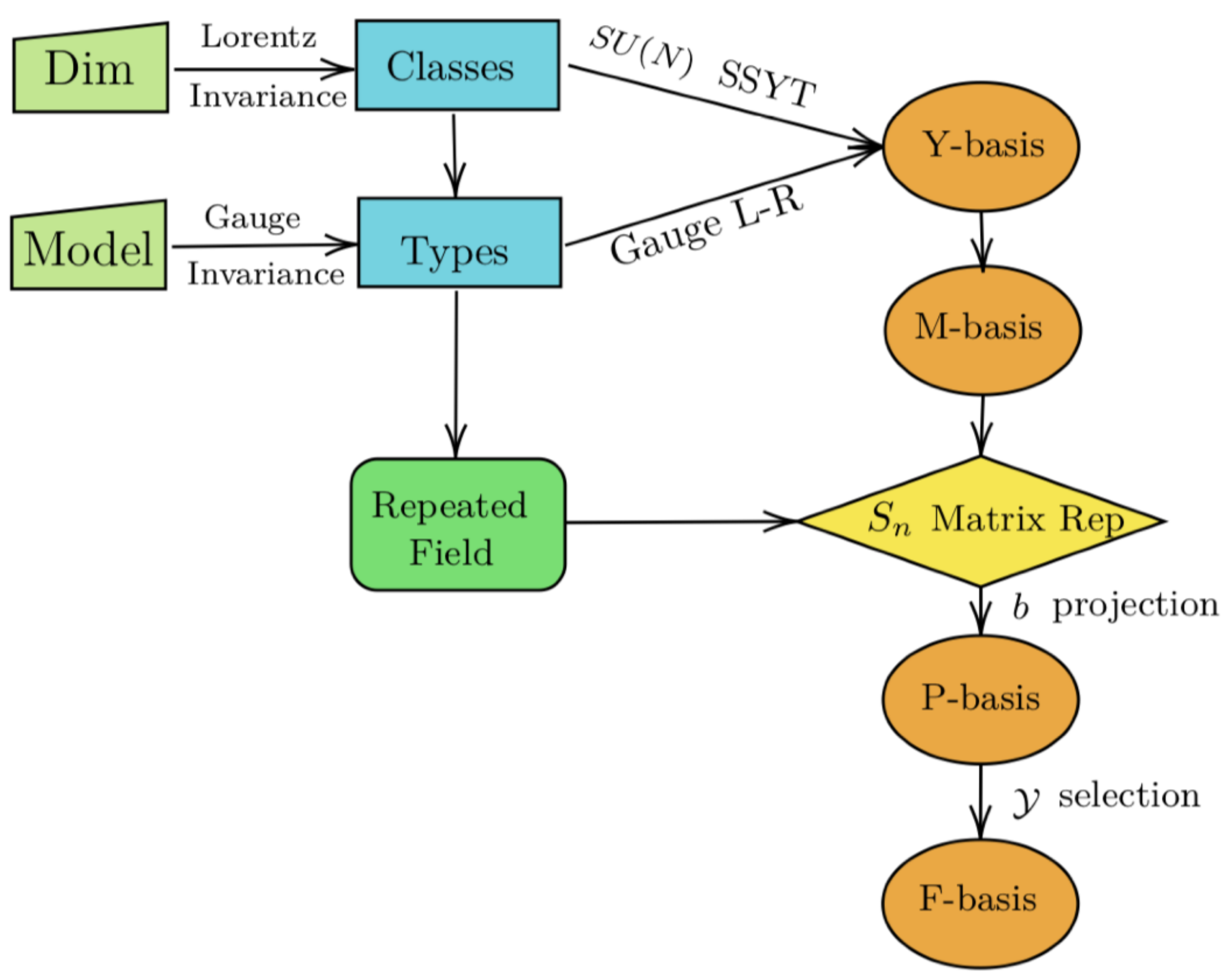
Dim-9: [Li, Ren, Xiao, **Yu**, Zheng, 2020]

Operator Bases for Generic EFT up to All Order

Amplitude Basis Construction for Effective Field Theory [Li, Ren, Xiao, **Yu**, Zheng, 2201.04639]

https://abc4eft.hepforge.org/





Fully Automatic

Dark matter EFT
Sterile neutrino EFT
Gravity EFT
Axion EFT
...

Gravity EFT and Dark Matter EFT

Gravity Effective Field Theory

[Donoghue, 1994]

Curved space **Tangent space**

$$g^{\mu\nu}(x) = e^{\mu}{}_{a}(x)\, e^{\nu}{}_{b}(x)\, \eta^{ab}$$

$\sigma^{\mu} = e^{\nu}{}_{a}(x)\, \sigma^{a}$

$$\sigma^{\mu}_{\alpha\dot\alpha}\tilde\sigma^{\nu\dot\alpha\beta} + \sigma^{\nu}{}_{\alpha\dot\alpha}\tilde\sigma^{\mu\dot\alpha\beta} = -2g^{\mu\nu}(x)\mathbf{1}_{\alpha}{}^{\beta}$$

$$\sigma^{a}_{\alpha\dot\alpha}\tilde\sigma^{b\dot\alpha\beta} + \sigma^{b}{}_{\alpha\dot\alpha}\tilde\sigma^{a\dot\alpha\beta} = -2\eta^{ab}\mathbf{1}_{\alpha}{}^{\beta}$$

$$\mathcal{R}_{\alpha\beta\gamma\delta} = R_{\mu\nu\rho\sigma}\sigma^{\mu\nu}_{\alpha\beta}\sigma^{\rho\sigma}_{\gamma\delta} = R_{abcd}\sigma^{ab}_{\alpha\beta}\sigma^{cd}_{\gamma\delta}$$

$$R_{\mu\nu\rho\sigma} = g_{\mu[\rho}R_{\sigma]\nu} - g_{\nu[\rho}R_{\sigma]\mu} - \frac{1}{2}g_{\mu[\rho}g_{\sigma]\nu}R + C_{\mathrm{L}\mu\nu\rho\sigma} + C_{\mathrm{R}\mu\nu\rho\sigma} + \frac{1}{6}g_{\mu[\rho}g_{\sigma]\nu}R.$$

(1,1) **(2,0)** **(0,2)** **(0,0)**

dimension	class	amplitude	operator
dim-6	C_{L}^3	$\langle 12\rangle^2 \langle 13\rangle^2 \langle 23\rangle^2$	$C_{\mathrm{L}\mu\nu\rho\sigma}C_{\mathrm{L}}^{\mu\nu\lambda\delta}C_{\mathrm{L}\lambda\delta}{}^{\rho\sigma}$
dim-8	C_{L}^4	$\langle 12\rangle^4 \langle 34\rangle^4$	$C_{\mathrm{L}\mu\nu\rho\sigma}C_{\mathrm{L}}^{\mu\nu\rho\sigma}C_{\mathrm{L}\lambda\eta\xi\tau}C_{\mathrm{L}}^{\lambda\eta\xi\tau}$
	$C_{\mathrm{L}}^2C_{\mathrm{R}}^2$	$\langle 12\rangle^4 [34]^4$	$C_{\mathrm{L}\mu\nu\rho\sigma}C_{\mathrm{L}}^{\mu\nu\rho\sigma}C_{\mathrm{R}\lambda\eta\xi\tau}C_{\mathrm{R}}^{\lambda\eta\xi\tau}$
dim-10	$C_{\mathrm{L}}^4D^2$	$\langle 12\rangle^4 \langle 34\rangle^4 s_{34}$	$C_{\mathrm{L}\mu\nu\rho\sigma}C_{\mathrm{L}}^{\mu\nu\rho\sigma}D_{\zeta}C_{\mathrm{L}\lambda\eta\xi\tau}D^{\zeta}C_{\mathrm{L}}^{\lambda\eta\xi\tau}$
	$C_{\mathrm{L}}^2C_{\mathrm{R}}^2D^2$	$\langle 12\rangle^2 [34]^2 s_{34}$	$C_{\mathrm{L}\mu\nu\rho\sigma}C_{\mathrm{L}}^{\mu\nu\rho\sigma}D_{\zeta}C_{\mathrm{R}\lambda\eta\xi\tau}D^{\zeta}C_{\mathrm{R}}^{\lambda\eta\xi\tau}$
	C_{L}^5	$\langle 12\rangle^4 \langle 34\rangle^2 \langle 35\rangle^2 \langle 45\rangle^2$	$C_{\mathrm{L}\mu\nu\lambda\rho}C_{\mathrm{L}\tau\upsilon\phi\chi}C_{\mathrm{L}}^{\mu\nu\eta\xi}C_{\mathrm{L}}^{\tau\upsilon\phi\chi}C_{\mathrm{L}\eta\xi}{}^{\lambda\rho}$
	$C_{\mathrm{L}}^3C_{\mathrm{R}}^2$	$\langle 12\rangle^2 \langle 13\rangle^2 \langle 23\rangle^2 [34]^4$	$C_{\mathrm{L}\mu\nu\lambda\rho}C_{\mathrm{R}\tau\upsilon\phi\chi}C_{\mathrm{L}}^{\mu\nu\eta\xi}C_{\mathrm{R}}^{\tau\upsilon\phi\chi}C_{\mathrm{L}\eta\xi}{}^{\lambda\rho}$

dimension	class	amplitude	operator
dim-6	$C_{\mathrm{L}}F_{\mathrm{L}}^2$	$\delta^{A_2A_3}\langle 12\rangle^2\langle 13\rangle^2$	$C_{\mathrm{L}\mu\nu\rho\sigma}G_{\mathrm{L}}^{A\mu\nu}G_{\mathrm{L}}^{A\rho\sigma}$
dim-8	$C_{\mathrm{L}}^2F_{\mathrm{L}}^2$	$\delta^{A_3A_4}\langle 12\rangle^4\langle 34\rangle^2$	$C_{\mathrm{L}\mu\nu\rho\sigma}C_{\mathrm{L}\mu\nu\rho\sigma}G^{A}_{\mathrm{L}\lambda\zeta}G_{\mathrm{L}}^{A\lambda\zeta}$
		$\delta^{A_3A_4}\langle 12\rangle^2\langle 13\rangle^2\langle 24\rangle^2$	$C_{\mathrm{L}\mu\nu\rho\sigma}C_{\mathrm{L}\mu\nu\rho\sigma}G^{A}_{\mathrm{L}\lambda\zeta}G_{\mathrm{L}}^{A\lambda\zeta}$
	$C_{\mathrm{R}}^2F_{\mathrm{L}}^2$	$\delta^{A_1A_2}\langle 12\rangle^2[34]^4$	$C_{\mathrm{R}\mu\nu\rho\sigma}C_{\mathrm{R}\mu\nu\rho\sigma}G^{A}_{\mathrm{L}\lambda\zeta}G_{\mathrm{L}}^{A\lambda\zeta}$

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2305.10481]

Dark Matter EFT

xSMEFT, Scalar dark matter

Sterile neutrino, Fermion dark matter

Vector Dark matter

Singlet			*dim*-4	*dim*-5	*dim*-6	*dim*-7	*dim*-8
Scalar	Real	w/o $\mathbf{Z_2}$	-	$9+6n_f^2$	$10+n_f+7n_f^2$	$30+n_f+\frac{965}{12}n_f^2 +\frac{3}{2}n_f^3+\frac{397}{12}n_f^4$	$\frac{1}{12}(516+36n_f+1241n_f^2 +42n_f^3+661n_f^4)$
		w/ $\mathbf{Z_2}$	-	-	$10+6n_f^2$	$n_f+n_f^2$	$\frac{1}{12}(516+1085n_f^2 +18n_f^3+397n_f^4)$
	Complex		-	-	$12+11n_f^2$	$n_f+n_f^2$	$58+\frac{1745}{12}n_f^2 +\frac{3}{2}n_f^3+\frac{397}{12}n_f^4$
Fermion	Majorana	w/o $\mathbf{Z_2}$	$2n_f$	2	$2+8n_f +6n_f^2+13n_f^3$	$\frac{4}{3}(12+14n_f +9n_f^2+25n_f^3)$	$18+68n_f +56n_f^2+181n_f^3$
		w $\mathbf{Z_2}$	-	2	$2+5n_f^2$	$4(4+3n_f^2)$	$18+2n_f+57n_f^2$
	Dirac		-	4	$7+10n_f^2$	$22+28n_f^2$	$43+n_f+113n_f^2$
Vector	Real	w/o $\mathbf{Z_2}$	$4+5n_f^2$	-	$37+71n_f^2$	$\frac{2}{3}(8n_f^2+7n_f^4)$	$\frac{1}{12}(4836+13529n_f^2 +6n_f^3+4477n_f^4)$
		w/ $\mathbf{Z_2}$	2	-	$22+21n_f^2$	$n_f+n_f^2$	$\frac{1}{6}(1152+2503n_f^2 +231n_f^4)$
	Complex		3	-	$51+42n_f^2$	n_f^2	$\frac{1}{3}(1617+2579n_f^2 +3n_f^2+346n_f^4$

Sterile neutrino

[Huayang Song, Hao Sun, **J.H.Yu**, 2306.05999]

EFTs at Broken Phase

[Hao Sun, Ming-Lei Xiao, **J.H.Yu**, 2206.07722]

[Hao Sun, Ming-Lei Xiao, **J.H.Yu**, 2210.14939]

[Huayang Song, Hao Sun, **J.H.Yu**, 2305.16770]

[Huayang Song, Hao Sun, **J.H.Yu**, 2306.05999]

[Chuan-Qiang Song, Hao Sun, **J.H.Yu**, 2404.15047]

[Xuan-He Li, Hao Sun, Feng-Jie Tang, **J.H.Yu**, 2404.14152]

[Hao Sun, Yi-Ning Wang, **J.H.Yu**, in préparation]

EFTs at Broken Phase

Standard Model Effective Field Theory → (Matching / Running) → Low Energy Effective Field Theory

approximate custodial symmetry SU(2) x SU(2)

$$\Sigma \equiv (\Phi^c, \Phi) = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix} \to g_L \, \Sigma \, g_R^\dagger$$

$$\langle \Sigma \rangle = \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix} \neq 0$$

approximate chiral symmetry SU(3) x SU(3)

$$\mathbf{q}_L \to g_L \, \mathbf{q}_L \, , \quad \mathbf{q}_R \to g_R \, \mathbf{q}_R \, ,$$

$$\langle 0 | \left(\bar{\mathbf{q}}_L \mathbf{q}_R + \bar{\mathbf{q}}_R \mathbf{q}_L \right) | 0 \rangle \neq 0$$

Electroweak Chiral Lagrangian

$$\Phi \equiv \frac{1}{\sqrt{2}} \vec{\sigma} \cdot \vec{\varphi} = \begin{pmatrix} \frac{1}{\sqrt{2}} \varphi^0 & \varphi^+ \\ \varphi^- & -\frac{1}{\sqrt{2}} \varphi^0 \end{pmatrix}$$

SM fermions, W/Z and Goldstone

QCD Chiral Lagrangian

$$\Phi \equiv \frac{\vec{\lambda}}{\sqrt{2}} \vec{\phi} = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}} \eta \end{pmatrix}$$

meson and baryon

SM Fermion masses from Higgs VEV

Baryon masses around cutoff scale from Trace anomaly

Goldstone EFT and Power Counting

Construct generic EFT for Goldstone at IR broken phase

PHYSICAL REVIEW VOLUME 166, NUMBER 5 25 FEBRUARY 1968

Nonlinear Realizations of Chiral Symmetry*

STEVEN WEINBERG†

Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 25 September 1967)

Shift symmetry:

$$\pi \to \pi + \epsilon + \cdots$$

Goldstone mode is a fluctuation around the background in the direction of broken generator

Gapless mode

Weakly coupled at IR

Non-linear transform under G/H

No interaction at long-wave limit

$$\mathscr{L}_{\text{EFT}} = \mathscr{L}_{p^2} + \mathscr{L}_{p^3} + \mathscr{L}_{p^4} + \mathscr{L}_{p^5} + \mathscr{L}_{p^6} + \ldots$$

$$\frac{f^2}{4}\langle D_\mu \mathbf{U}^\dagger D^\mu \mathbf{U}\rangle$$

$$\Pi_{\hat{a}} \to \Pi_{\hat{a}}{}^{(g_{\mathcal{H}})} = \left(e^{i\,\alpha_a t_\pi{}^a}\right)_{\hat{a}}^{\ \hat{b}} \Pi_{\hat{b}}$$

$$\Pi_{\hat{a}} \to \Pi^{(g_{\mathcal{G}/\mathcal{H}})}{}_{\hat{a}} = \Pi_{\hat{a}} + \frac{f}{\sqrt{2}}\alpha_{\hat{a}} + \mathcal{O}\left(\alpha\frac{\Pi^2}{f} + \alpha\frac{\Pi^3}{f^2}\ldots\right)$$

$$U[\Pi] = e^{i\frac{\sqrt{2}}{f}\Pi_{\hat{a}}(x)\widehat{T}^{\hat{a}}}$$

$$g \cdot U[\Pi] = U[\Pi^{(g)}] \cdot h[\Pi;\, g] \qquad h[\Pi;\, g] = e^{i\,\zeta_a[\Pi;g]T^a}$$

Power counting: Derivative expansion

Coset Construction

[Callan, Coleman, Wess, Zumino, 1969]

CCWZ Chiral Lagrangian

Define the nonlinear Goldstone matrix

[Callan, Coleman, Wess, Zumino, 1969]

$$\Omega(\Pi) \equiv \exp\left[\frac{i}{2f}\Pi(x)\right] \to \Omega(\Pi^{(\mathfrak{g})}) = \mathfrak{g}\Omega(\Pi)\mathfrak{h}^{-1}(\Pi;\mathfrak{g})$$

$[T_{\hat{a}}, T_{\hat{b}}] \propto T_c$

CCWZ Coset

$$-i\Omega^\dagger\partial_\mu\Omega = d_\mu^{\hat{a}}T^{\hat{a}} + E_\mu^a T^a \equiv d_\mu + E_\mu$$

$$d_\mu \to \mathfrak{h}d_\mu\mathfrak{h}^{-1}, \quad E_\mu \to \mathfrak{h}E_\mu\mathfrak{h}^{-1} - ih\partial_\mu\mathfrak{h}^{-1}$$

$$\partial_\mu \to D_\mu = \partial_\mu + iA_\mu$$
$$A_\mu = A_\mu^{\hat{a}}T^{\hat{a}} + A_\mu^a T^a$$

Building block

$$d_\mu(\Pi), \quad E_{\mu\nu}(\Pi) \quad \nabla_\mu \equiv \partial_\mu + iE_\mu$$
$$f_{\mu\nu} = \Omega^\dagger F_{\mu\nu}\Omega = f_{\mu\nu}^{-\hat{a}}T^{\hat{a}} + f_{\mu\nu}^{+a}T^a$$

$$E_{\mu\nu} = -i[u_\mu, u_\nu] + f_{\mu\nu}^+$$
$$d_\mu = u_\mu$$

Symmetric Coset

$$\Omega \to \mathfrak{g}\Omega\mathfrak{h}^{-1}, \quad \Omega \to \mathfrak{h}\Omega\mathfrak{g}_{\mathcal{R}}^{-1}$$

$$U \equiv \Omega^2 \to \mathfrak{g}\,U\mathfrak{g}_{\mathcal{R}}^{-1}$$

$$D_\mu U \equiv \partial_\mu U + iA_\mu U - iUA_\mu^{(R)}$$
$$A_\mu^{(R)} \equiv A_\mu^a T^a - A_\mu^{\hat{a}}T^{\hat{a}}$$

Building block

$$u_\mu = i\Omega(D_\mu U)^\dagger\Omega \quad D_\mu$$
$$f_{\mu\nu}^\pm = \frac{1}{2}\left(f_{\mu\nu} \pm f_{\mu\nu}^{(R)}\right) = \Omega^\dagger F_{\mu\nu}\Omega \pm F_{\mu\nu}^{(R)}$$

$$\Omega(\Pi) \equiv \begin{bmatrix} u(\Pi) & 0 \\ 0 & u^\dagger(\Pi) \end{bmatrix} \qquad u \to \sqrt{\mathfrak{g}_L U \mathfrak{g}_R^\dagger} = \mathfrak{g}_L\, u\mathfrak{h}^{-1} = \mathfrak{h}^{-1}u\mathfrak{g}_R$$

$$U(\Pi) \equiv u^2(\Pi) \longrightarrow \mathfrak{g}_L\mathbf{U}(\Pi)\mathfrak{g}_R^\dagger$$

QCD Chiral Lag

$$u_\mu \to \mathfrak{h}\,u_\mu\,\mathfrak{h}^{-1}$$

$$u_\mu = \mathrm{i}\left[u^\dagger\left(\partial_\mu - \mathrm{i}r_\mu\right)u - u\left(\partial_\mu - \mathrm{i}l_\mu\right)u^\dagger\right]$$

$$\chi_\pm = u^\dagger\chi u^\dagger \pm u\chi^\dagger u\,,$$
$$f_{\mu\nu}^\pm = uf_{\mu\nu}^{\mathrm{L}}u^\dagger \pm u^\dagger f_{\mu\nu}^{\mathrm{R}}u\,,$$

EW Chiral Lag

$$\mathbf{V}_\mu \to \mathfrak{g}_L\,\mathbf{V}\,\mathfrak{g}_L^{-1}$$

$$\mathbf{V}_\mu(x) = i\mathbf{U}(x)D_\mu\mathbf{U}(x)^\dagger$$

$$\mathbf{T} = \mathbf{U}\mathcal{T}_R\mathbf{U}^\dagger \longrightarrow \mathfrak{g}_L\mathbf{T}\mathfrak{g}_L^\dagger \qquad \hat{W}_{\mu\nu} \longrightarrow \mathfrak{g}_L\hat{W}_{\mu\nu}\mathfrak{g}_L^\dagger$$

$$\mathbf{Y} = \mathbf{U}\mathcal{Y}_R\mathbf{U}^\dagger \longrightarrow \mathfrak{g}_L\mathbf{Y}\mathfrak{g}_L^\dagger \qquad \hat{B}_{\mu\nu} \longrightarrow \mathfrak{g}_R\hat{B}_{\mu\nu}\mathfrak{g}_R^\dagger$$

Massive On-shell Amplitudes

Little Group = U(1), helicity-spinor

$$p_{\alpha\dot\beta} \equiv \lambda_\alpha \tilde\lambda_{\dot\beta} \equiv |p\rangle_\alpha [p|_{\dot\beta}$$

$$\lambda \to e^{-i\frac{\theta}{2}}\lambda, \quad \tilde\lambda \to e^{i\frac{\theta}{2}}\tilde\lambda.$$

$$\epsilon^+_\mu = \frac{\langle\zeta|\sigma_\mu|\lambda]}{\sqrt{2}\langle\lambda\zeta\rangle}, \quad \epsilon^-_\mu = \frac{\langle\lambda|\sigma_\mu|\zeta]}{\sqrt{2}[\lambda\zeta]}$$

$$\lambda_\alpha = \sqrt{2E}\begin{pmatrix} c \\ s \end{pmatrix}, \quad \tilde\lambda_{\dot\alpha} = \sqrt{2E}\begin{pmatrix} c \\ s^* \end{pmatrix}$$

Little Group = SU(2), spin-spinor

$$\mathbf{p}_{\alpha\dot\beta} = \lambda^I_\alpha \tilde\lambda_{\dot\beta I} = \epsilon_{IJ}|p^I\rangle_\alpha [p^J|_{\dot\beta}$$

$$\lambda^{I\alpha} \to U^I{}_J \lambda^{J\alpha}, \quad \tilde\lambda^{I\dot\alpha} \to U^I{}_J \lambda^{J\alpha}$$

$$\varepsilon^{IJ}_\mu = \frac{\langle p^{(I}|\sigma_\mu|p^{J)}]}{\sqrt{2}m}$$

$$u^I(p) = \begin{pmatrix} |\,\boldsymbol{p}_I]^{\dot\alpha} \\ -|\boldsymbol{p}_I\rangle_\alpha \end{pmatrix} \qquad v(p) = \begin{pmatrix} |\,\boldsymbol{p}_I]^{\dot\alpha} \\ |\boldsymbol{p}_I\rangle_\alpha \end{pmatrix}$$

Spin-s = 2s symmetrized LG indices

$$\epsilon^{(I_1J_1}_{i\mu_1}\epsilon^{I_2J_2}_{i\mu_2}\dots\epsilon^{I_SJ_S)}_{i\mu_S} = \frac{1}{(\sqrt{2}M)^{2S}}\left\langle \mathbf{i}^{(I_1}|\sigma_{\mu_1}|\mathbf{i}^{J_1}\right]\left\langle \mathbf{i}^{I_2}|\sigma_{\mu_2}|\mathbf{i}^{J_2}\right]\dots\left\langle \mathbf{i}^{I_S}|\sigma^{\mu_S}|\mathbf{i}^{J_S)}\right]$$

Bolding $\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle \propto \langle 1^I 3^{K_1}\rangle\langle 2^J 3^{K_2}\rangle + \langle 1^I 3^{K_2}\rangle\langle 2^J 3^{K_1}\rangle$

[Arkani-Hamed, Huang, Huang, 2017]

Massive Amplitudes

Little Group = U(1), helicity-spinor

$$p_{\alpha\dot\beta} \equiv \lambda_\alpha \tilde\lambda_{\dot\beta} \equiv |p\rangle_\alpha [p|_{\dot\beta}$$

$$\lambda \to e^{-i\frac{\theta}{2}}\lambda, \quad \tilde\lambda \to e^{i\frac{\theta}{2}}\tilde\lambda.$$

$$\epsilon^+_\mu = \frac{\langle\zeta|\sigma_\mu|\lambda]}{\sqrt{2}\langle\lambda\zeta\rangle}, \quad \epsilon^-_\mu = \frac{\langle\lambda|\sigma_\mu|\zeta]}{\sqrt{2}[\lambda\zeta]}$$

$$\lambda_\alpha = \sqrt{2E}\begin{pmatrix} c \\ s \end{pmatrix}, \quad \tilde\lambda_{\dot\alpha} = \sqrt{2E}\begin{pmatrix} c \\ s^* \end{pmatrix}$$

$$\lambda^I_\alpha = \eta_\alpha \zeta^{+I} - \lambda_\alpha \zeta^{-I}$$
$$\tilde\lambda^I_{\dot\alpha} = \tilde\eta_{\dot\alpha}\zeta^{-I} + \tilde\lambda_{\dot\alpha}\zeta^{+I}$$

basis in 2-dim little group space

$$\zeta^+_I = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \zeta^-_I = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

2 null spinors in spinor space

$$\eta_\alpha = \sqrt{E-p}\begin{pmatrix} -s^* \\ c \end{pmatrix}, \quad \tilde\eta_{\dot\alpha} = \sqrt{E-p}\begin{pmatrix} -s \\ c \end{pmatrix}$$

$$\mathbf{p}_{\alpha\dot\alpha} = \lambda_\alpha\tilde\lambda_{\dot\alpha}\zeta^{-I}\zeta^+_I + \eta_\alpha\tilde\eta_{\dot\alpha}\zeta^{+I}\zeta^-_I$$
$$= \lambda_\alpha\tilde\lambda_{\dot\alpha} + \eta_\alpha\tilde\eta_{\dot\alpha}$$

Little Group = SU(2), spin-spinor

$$\mathbf{p}_{\alpha\dot\beta} = \lambda^I_\alpha \tilde\lambda_{\dot\beta I} = \epsilon_{IJ}\left|p^I\right\rangle_\alpha \left[p^J\right|_{\dot\beta}$$

$$\lambda^{I\alpha} \to U^I{}_J\lambda^{J\alpha}, \quad \tilde\lambda^{I\dot\alpha} \to U^I{}_J\lambda^{J\alpha}$$

$$\varepsilon^{IJ}_\mu = \frac{\langle p^{(I}|\sigma_\mu|p^{J)}]}{\sqrt{2}m}$$

$$u^I(p) = \begin{pmatrix} |\,\boldsymbol{p}_I]^{\dot\alpha} \\ -|\boldsymbol{p}_I\rangle_\alpha \end{pmatrix} \qquad v(p) = \begin{pmatrix} |\,\boldsymbol{p}_I]^{\dot\alpha} \\ |\boldsymbol{p}_I\rangle_\alpha \end{pmatrix}$$

Spin-s = 2s symmetrized LG indices

$$\epsilon^{(I_1J_1}_{i\mu_1}\epsilon^{I_2J_2}_{i\mu_2}\dots\epsilon^{I_SJ_S)}_{i\mu_S} = \frac{1}{(\sqrt{2}M)^{2S}}\left\langle \mathbf{i}^{(I_1}|\sigma_{\mu_1}|\mathbf{i}^{J_1}\right]\left\langle \mathbf{i}^{I_2}|\sigma_{\mu_2}|\mathbf{i}^{J_2}\right]\dots\left\langle \mathbf{i}^{I_S}|\sigma^{\mu_S}|\mathbf{i}^{J_S)}\right]$$

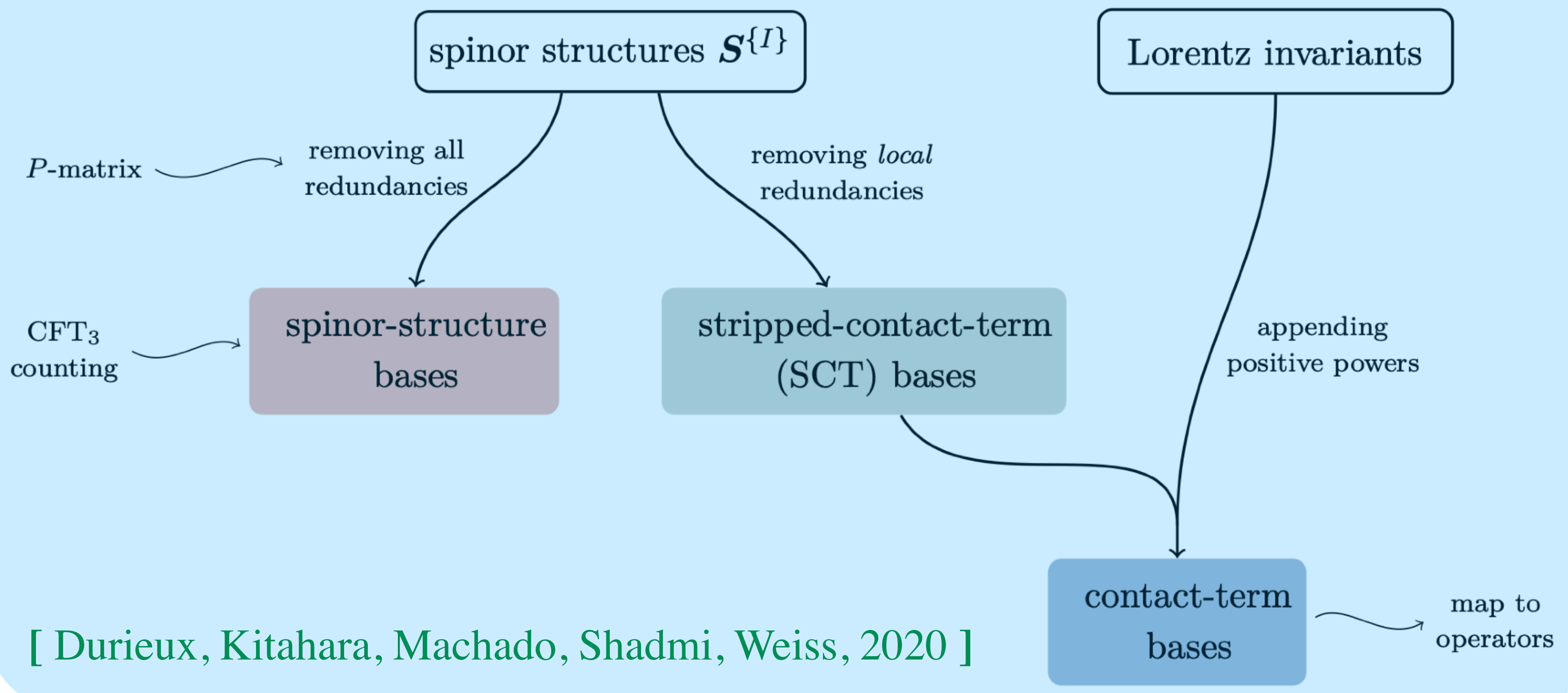
Bolding $\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle \propto \langle 1^I 3^{K_1}\rangle\langle 2^J 3^{K_2}\rangle + \langle 1^I 3^{K_2}\rangle\langle 2^J 3^{K_1}\rangle$

Unbolding

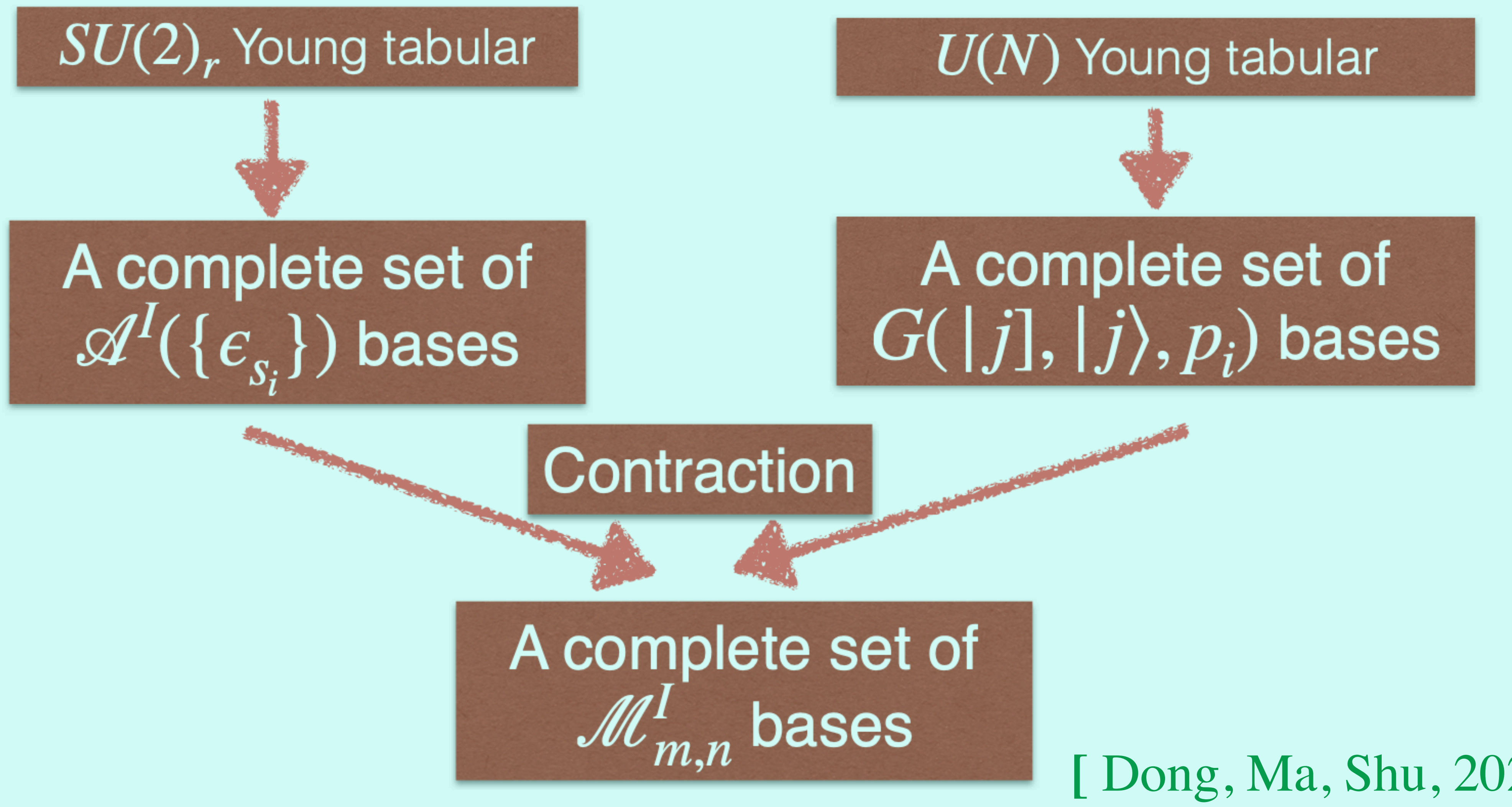
$$\langle\mathbf{12}\rangle^{\text{IJ}} \approx \begin{pmatrix} \left(1 - \frac{m_1^2}{8E_1^2} - \frac{m_2^2}{8E_2^2}\right)\langle 12\rangle & -\frac{m_2}{\sqrt{2E_2}}\langle 1\eta_2\rangle \\ -\frac{m_1}{\sqrt{2E_1}}\langle\eta_1 2\rangle & \frac{m_1m_2}{4E_1E_2}[12] \end{pmatrix}$$

[Arkani-Hamed, Huang, Huang, 2017]

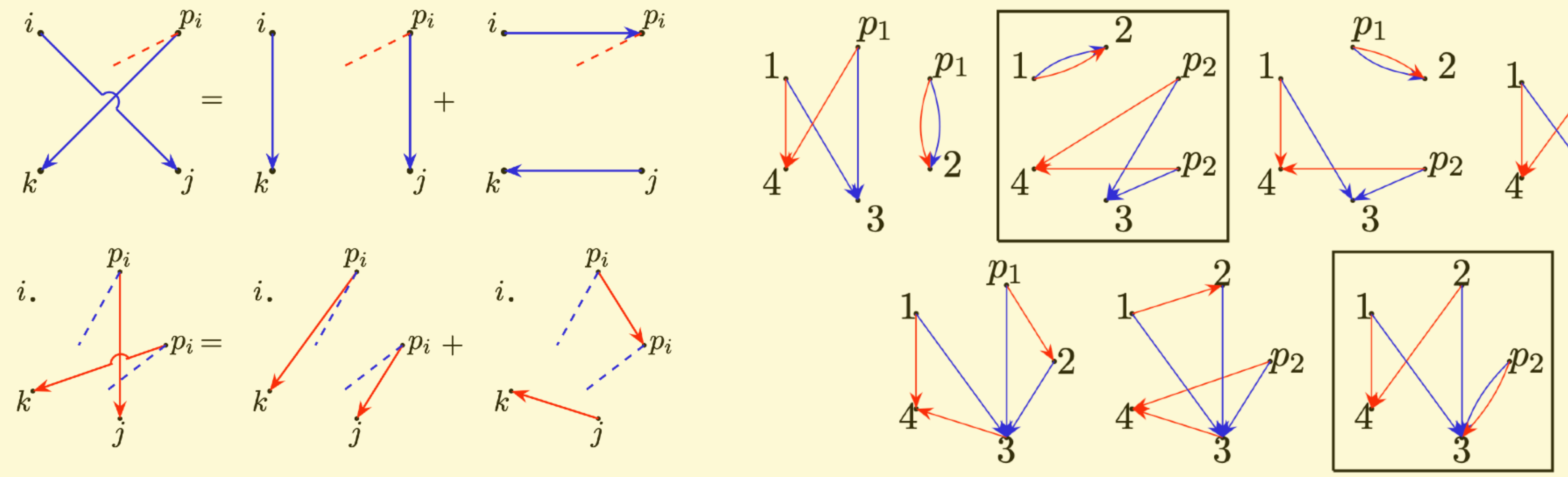
Massive Amplitude Basis



[Durieux, Kitahara, Machado, Shadmi, Weiss, 2020]



[Dong, Ma, Shu, 2021]



[de Augelis, 2022]

Massive-Massless Correspondence (using Goldstone)

$$h=0 \qquad \lambda^r\tilde\lambda^r \longrightarrow \left(\lambda^J\tilde\lambda_J\right)^r \qquad S=0,\ n=0$$

$$h=-\tfrac{1}{2} \qquad \lambda^{1+r}\tilde\lambda^r \longrightarrow \lambda^I\left(\lambda^J\tilde\lambda_J\right)^r \qquad S=\tfrac{1}{2},\ n=0 \qquad \Leftrightarrow D^n\psi_i$$

$$h=\tfrac{1}{2} \qquad \lambda^r\tilde\lambda^{1+r} \longrightarrow \tilde\lambda^I\left(\lambda^J\tilde\lambda_J\right)^r \qquad S=\tfrac{1}{2},\ n=1 \qquad \Leftrightarrow D^n\psi_i^\dagger$$

$c_1\frac{\langle 13\rangle^2}{\langle 12\rangle}+c_2\frac{[13]^2}{[12]}$ $\quad \bar t bW^+$

$\frac{m_1}{m_3}(c_1-c_2)\left(\langle 12\rangle-[12]\right)$ $\quad \bar t b\phi^+$

Spurious pole = mass singularity

Goldstone equivalence

$$c_1\frac{[\mathbf{13}]\langle\mathbf{23}\rangle}{m_3}+c_2\frac{\langle\mathbf{13}\rangle[\mathbf{23}]}{m_3}$$

Our treatment

Goldstone Boson on-shell amplitudes

Chiral symmetry (PCAC)

$$\alpha \to \beta \quad \xleftrightarrow{\text{At low energy}} \quad \alpha + n_1\pi \to \beta + n_2\pi$$

Goldberger-Trieman, Callan-Trieman, Adler-Weisberger, etc

Adler Zero condition [Adler, 1965]

$$T(\alpha + \phi(p), \beta) = -\frac{p_\mu}{F} R^\mu(p) \xrightarrow{p \to 0} 0$$

Amplitude (soft limit of external leg s)

$$\mathcal{A}(1, \ldots, N, s) \xrightarrow{p_s \to 0} \begin{cases} \left(S^{(0)}(s) + S^{(\text{sub})}(s)\right)\mathcal{A}(1, \ldots, N) \\ \mathcal{O}(p_s^\sigma) \qquad \text{for Goldstone Boson} \end{cases}$$

$$\{-1/2, -1/2, 1, 0, 0\} \qquad \begin{array}{|c|c|c|c|} \hline 1 & 1 & 1 & 4 \\ \hline 2 & 2 & 2 & 5 \\ \hline 4 & 5 \\ \cline{1-2} \end{array}, \quad \begin{array}{|c|c|c|c|} \hline 1 & 1 & 1 & 2 \\ \hline 2 & 2 & 5 & 5 \\ \hline 4 & 4 \\ \cline{1-2} \end{array}, \quad \begin{array}{|c|c|c|c|} \hline 1 & 1 & 1 & 2 \\ \hline 2 & 2 & 4 & 4 \\ \hline 5 & 5 \\ \cline{1-2} \end{array}, \quad \begin{array}{|c|c|c|c|} \hline 1 & 1 & 1 & 2 \\ \hline 2 & 2 & 4 & 5 \\ \hline 4 & 5 \\ \cline{1-2} \end{array},$$

Expand the soft-limit amplitude into the SSYT basis

Put constraints on the SSYT basis

$$\mathcal{B}_i^{(N)}(p_\pi \to 0) = \sum_{l=1}^{d_N} \mathcal{K}_{il} \mathcal{B}_l^{(N)}$$

$$\begin{array}{|c|c|c|c|} \hline 1 & 1 & 1 & 4 \\ \hline 2 & 2 & 2 & 5 \\ \hline 4 & 5 \\ \cline{1-2} \end{array}, \quad \begin{array}{|c|c|c|c|} \hline 1 & 1 & 1 & 2 \\ \hline 2 & 2 & 4 & 5 \\ \hline 4 & 5 \\ \cline{1-2} \end{array},$$

[Sun, Xiao, **Yu**, 2210.14939]
[Sun, Xiao, **Yu**, 2206.07722]
[Low, Shu, Xiao, Zheng, 2022]

custodial/chiral symmetry breaking: spurion

Spurion Technique

The SU(2) spurion is introduced to parametrize the custodial symmetry breaking

$$t_i \in \mathbf{2} \sim \square$$
$$\epsilon_{ij}t^j \in \bar{\mathbf{2}} \sim \square$$
$$t^I \tau_i^{Ik}\epsilon_{kj} \in \mathbf{3} \sim \square\square$$

$$\mathbf{T}^I \tau^{Ik}{}_i \epsilon_{kj} \in \boxed{i \mid j},$$

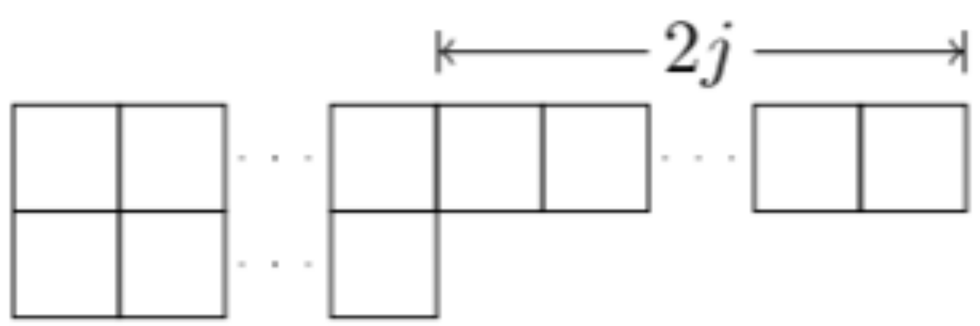
$$\mathbf{T}^{\{I_1} \ldots \mathbf{T}^{I_j\}} \in \text{spin } j$$

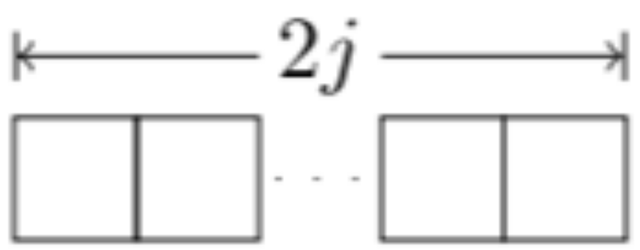
$$\begin{array}{ccccccccc} \mathbf{T}^I\mathbf{T}^J & = & \mathbf{T}^2\delta^{IJ} & + & \mathbf{T}^{[I}\mathbf{T}^{J]} & + & \mathbf{T}^{(I}\mathbf{T}^{J)}, \\ \mathbf{3}\otimes\mathbf{3} & = & \mathbf{1} & + & \mathbf{3} & + & \mathbf{5}. \end{array}$$

Littlewood-Richarson rules

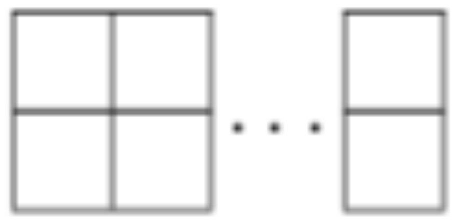
Symmetric highest weight

$$\epsilon^{IJK}\mathbf{T}^I\mathbf{T}^J A^K$$





Gauge Singlet

$SU(2) \sim$ 

[Sun, Xiao, **Yu**, 2206.07722]

External Sources

Equation of motion is allowed

[Ren, **Yu**, 2211.01420]

Operator

$$\mathcal{O}_N^{(d)} = (\epsilon^{\alpha_i\alpha_j})^{\otimes n}(\tilde{\epsilon}_{\dot{\alpha}_i\dot{\alpha}_j})^{\otimes \tilde{n}}\prod_{i=1}^{N}(D^{r_i-|h_i|}\Psi_{i,a_i})^{\dot{\alpha}_i^{r_i+h_i}}_{\alpha_i^{r_i-h_i}}$$

$$D^{r_i-1/2}\psi_i^{(\dagger)} \Leftrightarrow \lambda_i^{r_i\pm1/2}\tilde{\lambda}^{i,r_i\mp1/2}, \qquad D^{r_i-1}F_{\mathrm{L/R}\,i} \Leftrightarrow \lambda_i^{r_i\pm1}\tilde{\lambda}^{i,r_i\mp1},$$

$(\psi_1\psi_2)(D^\mu F_{R3}^{\nu\lambda})(D_\mu F_{R4\nu\lambda})$

↓

Spinor Tensor → **IBP relation**

$$|i_{\hat{d}_i}\rangle[i_{\hat{d}_i}| = -\sum_{j=1,j\neq i}^{N}|j_{\hat{d}_j+1}\rangle[j_{\hat{d}_j+1}|$$

$$(\epsilon^{\alpha_i\alpha_j})^{\otimes n}(\tilde{\epsilon}_{\dot{\alpha}_i\dot{\alpha}_j})^{\otimes\tilde{n}}\prod_{i=1}^{N}\lambda_i^{r_i-h_i}\tilde{\lambda}^{i,r_i+h_i}$$

↓

Schouten Identity

$$\langle i_{x_i}l_{x_l}\rangle\langle j_{x_j}k_{x_k}\rangle = -\langle i_{x_i}j_{x_j}\rangle\langle k_{x_k}l_{x_l}\rangle + \langle i_{x_i}k_{x_k}\rangle\langle j_{x_j}l_{x_l}\rangle$$

$$\lambda_{1,1}\lambda_{1,2}\tilde{\lambda}_{1,1}\tilde{\lambda}_{1,2},\quad \lambda_{2,1}\lambda_{2,2}\tilde{\lambda}_{2,1}\tilde{\lambda}_{2,2},\quad \lambda_{3,1}\lambda_{3,2}\tilde{\lambda}_{3,1}\tilde{\lambda}_{3,2},$$
$$\lambda_{1,1}\lambda_{3,1}\tilde{\lambda}_{1,1}\tilde{\lambda}_{3,1},\quad \lambda_{1,1}\lambda_{4,1}\tilde{\lambda}_{1,1}\tilde{\lambda}_{4,1},\quad \lambda_{2,1}\lambda_{3,1}\tilde{\lambda}_{2,1}\tilde{\lambda}_{3,1},$$
$$\lambda_{4,1}\lambda_{4,2}\tilde{\lambda}_{4,1}\tilde{\lambda}_{4,2},\quad \lambda_{1,1}\lambda_{2,1}\tilde{\lambda}_{1,1}\tilde{\lambda}_{2,1},$$
$$\lambda_{2,1}\lambda_{4,1}\tilde{\lambda}_{2,1}\tilde{\lambda}_{4,1},\quad \lambda_{3,1}\lambda_{4,1}\tilde{\lambda}_{3,1}\tilde{\lambda}_{4,1}.$$

↓

CDC

$$\{i_{d_i}|d_i\in\{1,2,\cdots,\hat{d}_i\}\}$$

↓

Symmetrize indices → **On-shell Amplitude**

$$D_{[\alpha\dot{\alpha}}D_{\beta]\dot{\beta}} = D_\mu D_\nu\sigma^\mu_{[\alpha\dot{\alpha}}\sigma^\nu_{\beta]\dot{\beta}} = -D^2\epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}} + \frac{i}{2}[D_\mu, D_\nu]\epsilon_{\alpha\beta}(\bar{\sigma}^{\mu\nu})_{\dot{\alpha}\dot{\beta}},$$
$$D_{[\alpha\dot{\alpha}}\psi_{\beta]} = D_\mu\sigma^\mu_{[\alpha\dot{\alpha}}\psi_{\beta]} = -\epsilon_{\alpha\beta}(D_\mu\sigma^\mu\psi)_{\dot{\alpha}},$$
$$D_{[\alpha\dot{\alpha}}F_{\mathrm{L}\beta]\gamma} = \frac{i}{2}D_\mu F_{\mathrm{L}\nu\rho}\sigma^\mu_{[\alpha\dot{\alpha}}\sigma^{\nu\rho}_{\beta]\gamma} = iD^\mu F_{\mathrm{L}\mu\nu}\epsilon_{\alpha\beta}\sigma^\nu_{\gamma\dot{\alpha}},$$

$$\lambda_{2,1}\lambda_{2,2}\tilde{\lambda}_{2,1}\tilde{\lambda}_{2,2},\quad \lambda_{3,1}\lambda_{3,2}\tilde{\lambda}_{3,1}\tilde{\lambda}_{3,2},\quad \lambda_{4,1}\lambda_{4,2}\tilde{\lambda}_{4,1}\tilde{\lambda}_{4,2},$$
$$\lambda_{2,1}\lambda_{3,1}\tilde{\lambda}_{2,1}\tilde{\lambda}_{3,1},\quad \lambda_{2,1}\lambda_{4,1}\tilde{\lambda}_{2,1}\tilde{\lambda}_{4,1},\quad \lambda_{3,1}\lambda_{4,1}\tilde{\lambda}_{3,1}\tilde{\lambda}_{4,1}.$$

Chiral Lagrangian for QCD and EW Theory

$$\mathscr{L}_{\mathrm{EFT}} = \mathscr{L}_{p^2} + \mathscr{L}_{p^3} + \mathscr{L}_{p^4} + \mathscr{L}_{p^5} + \mathscr{L}_{p^6} + \ldots$$

ChPT and Chiral EFT

LO Lagrangian

[Weinberg, 1979]

Pure mesonic

[Gasser, Leutwyler, 1984, 1985]
[Fearing, Scherer 1994]
[Bijnens, Colangelo, Ecker, 1999]
[Jiang, Ge, Wang, 2014]
[Bijnens, Hermansson, Wang, 2018]

nucleon-meson

[Li, Sun, Tang, **J.H.Yu**, 2404.14152]
[Krause, 1990]
[Ecker, 1994]
[Fettes, Meisner, Mojzis, Steininger, 2000]
[Oller, Verbeni, Prades, 2006]
[Frink, Meisner, 2006]
[Jiang, Chen, Liu, 2017]

nucleon-nucleon

[Song, Sun, **J.H.Yu**, 2405.15047]
[Weinberg 1990]
[van Kolck, Ordonez, 1992]
[Petschauer, Kaiser, 2013]
[Petschauer, Haidenbauer, Kaiser, Meisner, Weise, 2020]
[Sun, Wang, **Yu**, in préparation]

EW Chiral Lagrangian = HEFT

LO Lagrangian

[Weinberg, 1979]

NLO bosonic

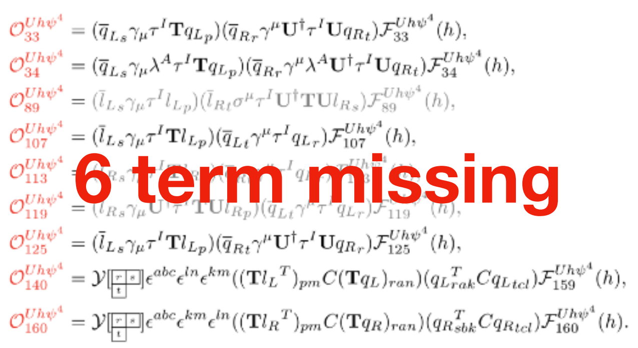
[Appelquist, Bernard, 1980]
[Longhitano, 1980, 1981]
[Feruglio, 1993]

NLO 2-fermion

[Buchalla, Cata, Krause, 2014]

NLO 4-fermion

[Buchalla, Cata, Krause, 2014]
[Pich, Rosell, Santos, Sanz-Cillero, 2015, 2018]
[Sun, Xiao, **Yu**, 2206.07722]



NNLO Basis

[Sun, Xiao, **Yu**, 2210.14939]

Axion and Dark Photon Effective Theories

Axion, ALP and Majoran EFT

Dark photon EFT

[Song, Sun, **J.H.Yu**, 2305.16770]

Goldstone for the PQ symmetry

$$\mathcal{A}_3[aVV] = C_{aVV}\frac{i}{f_a}\left(\langle\mathbf{12}\rangle^2 - [\mathbf{12}]^2\right) \qquad \mathcal{A}_3[af\bar{f}] = C_{af\bar{f}}\frac{im_f}{f_a}\left(\langle\mathbf{12}\rangle - [\mathbf{12}]\right)$$

$$\mathcal{L}_{\text{int}} = -\frac{g_{\phi\gamma}}{4}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{g_{\phi N}}{2m_N}\partial_\mu\phi(\bar{N}\gamma^\mu\gamma_5 N) + \frac{g_{\phi e}}{2m_e}\partial_\mu\phi(\bar{e}\gamma^\mu\gamma_5 e) - \frac{i}{2}g_d\phi\bar{N}\sigma_{\mu\nu}\gamma_5 N F^{\mu\nu}$$

Class	Type	Real	F	Axion	Majoron
$F_L^2\phi$	$B_L^2 s$	$sB_{L\mu\nu}B_L{}^{\mu\nu}$		✓	
	$W_L^2 s$	$sW_L{}^I{}_{\mu\nu}W_L{}^{I\mu\nu}$		✓	
	$G_L^2 s$	$sG_L{}^A{}_{\mu\nu}G_L{}^{A\mu\nu}$		✓	
$\psi^2\phi^2$	$e_c H^\dagger L s$	$sH^{\dagger i}(e_{cp}L_{ri})$		✓	
	$d_c H^\dagger Q s$	$sH^{\dagger i}(d_{cp}{}^a Q_{rai})$		✓	
	$HQu_c s$	$\epsilon^{ij}sH_j(Q_{pai}u_{cr}{}^a)$		✓	
$D^2\phi^4$	$D^2HH^\dagger s^2$	$(D_\mu s)(D^\mu s)H_i H^{\dagger i}$		✓	
$DF_L\phi\psi\bar{\psi}$	$DB_L u_c u_c^\dagger s$	$(D_\nu s)B_L{}^{\mu\nu}(u_{cp}{}^a\sigma_\mu u^\dagger_{cra})$		✓	
	$DB_L QQ^\dagger s$	$(D_\nu s)B_L{}^{\mu\nu}(Q_{pai}\sigma_\mu Q^\dagger{}_r{}^{ai})$		✓	
$Dd_c L^2 u_c^\dagger s$		$\epsilon^{ij}(D^\mu s)(d_{cp}{}^a L_{ri})(L_{sj}\sigma_\mu u^\dagger_{cta})$	$\mathcal{Y}[\boxed{r}\boxed{s}]$	✓	✓

Majoron, Goldstone for LNV, leading order at dim-8

Stueckelberg mechanism

$$\mathcal{L}_{\text{Stueck}} = -\frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{1}{2}m_X^2\left(X_\mu - \frac{1}{m_X}\partial_\mu\pi\right)\left(X^\mu - \frac{1}{m_X}\partial^\mu\pi\right) - \frac{1}{2}(\partial^\mu X_\mu + m_X\pi)(\partial^\nu X_\nu + m_X\pi)$$

[Song, Sun, **J.H.Yu**, 2306.05999]

Dark photon chiral Lagrangian

$$U(1)_L \times U(1)_R \to U(1)_V \qquad U \equiv e^{i\frac{\pi}{m_X}}$$

$$\mathcal{L}'_{\text{Stueck}} = -\frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{m_X^2}{2}(D_\mu U)^\dagger D^\mu U - \frac{1}{2\xi}\left(\partial^\mu X_\mu - i\xi m_X^2 \ln U\right)^2$$

$\xi = 1 \to$ Stueckelberg Lagrangian

$\xi = \infty \to$ Proca Lagrangian

Dark photon DM

Operators up to dim-8

Complete EFT Framework

Complete EFT construction from on-shell amplitudes at different scales

Jiang-Hao Yu (ITP-CAS)

UV Completion of EFT Operators

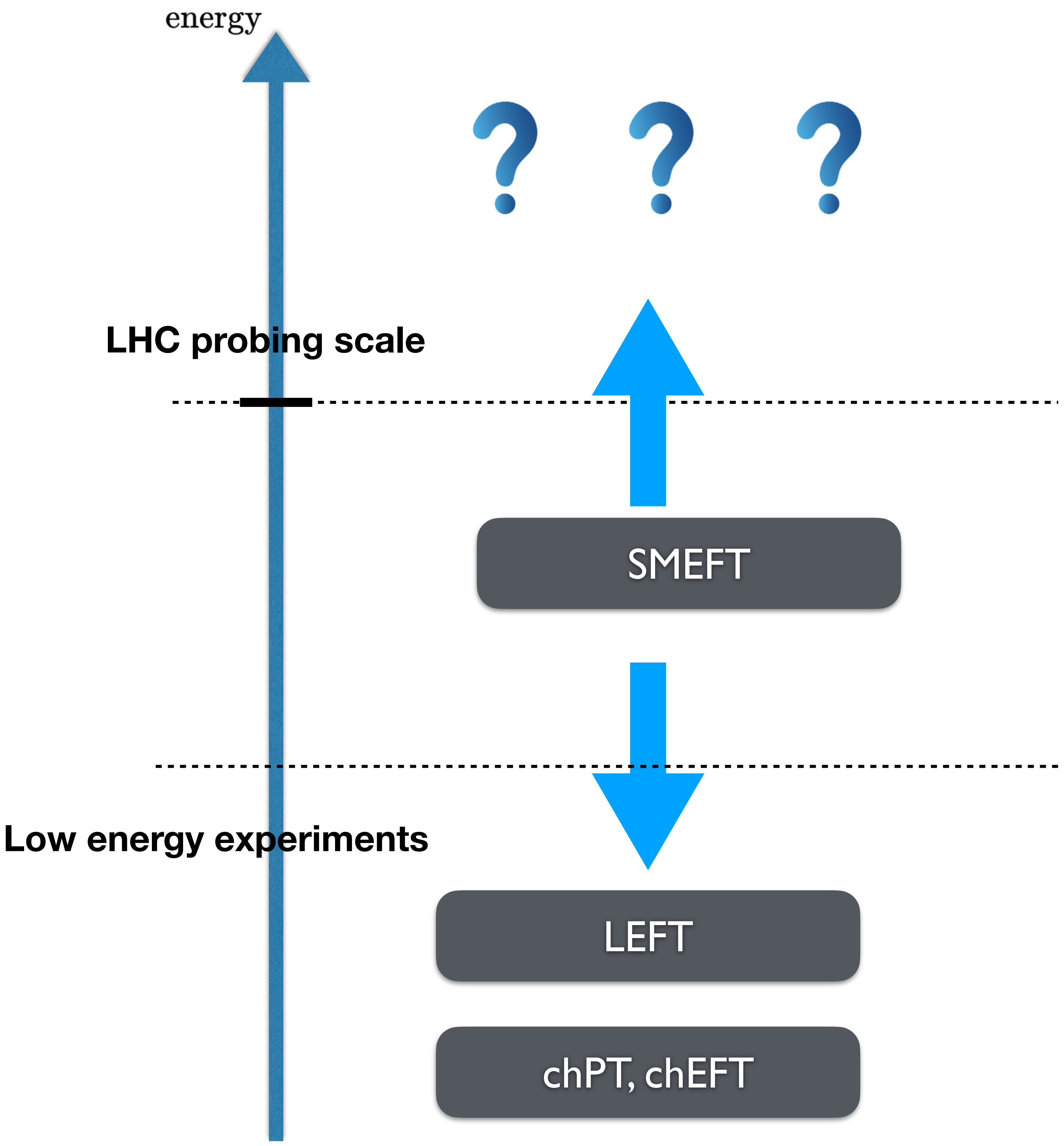
[Hao-Lin Li, Yu-Han Ni, Ming-Lei Xiao, **J.H.Yu**, 2204.03660]

[Xu-Xiang Li, Zhe Ren, **J.H.Yu,** 2307.10380]

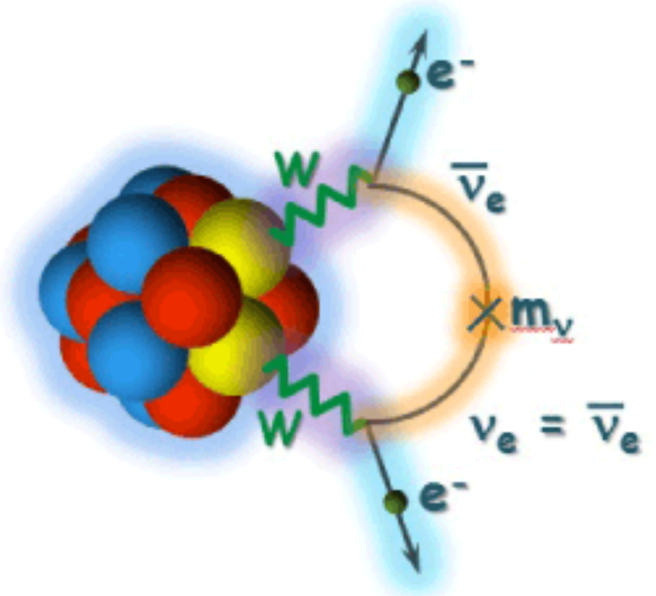
[Hao-Lin Li, Yu-Han Ni, Ming-Lei Xiao, **J.H.Yu**, 2309.15933]

EFT Inverse Problem

After writing down the effective operators, what is the next step?



$$\mathscr{L}_{\text{EFT}} = \mathscr{L}_{\mathscr{D}\leq 4} + \frac{\mathscr{L}_5}{\Lambda} + \frac{\mathscr{L}_6}{\Lambda^2} + \frac{\mathscr{L}_7}{\Lambda^3} + \frac{\mathscr{L}_8}{\Lambda^4} + \frac{\mathscr{L}_9}{\Lambda^5} + \ldots$$

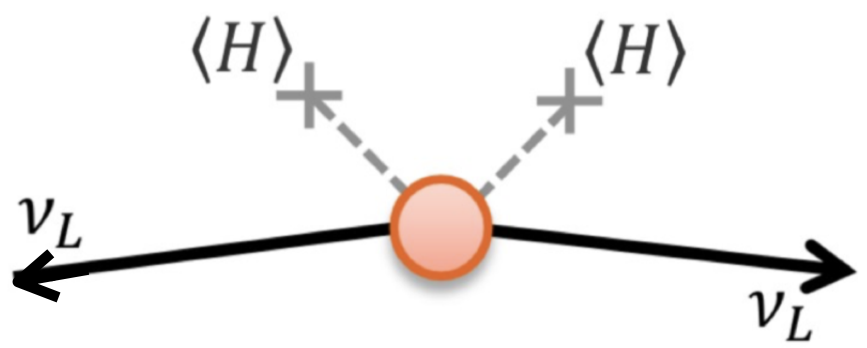


Jiang-Hao Yu (ITP-CAS)

Neutrino Masses

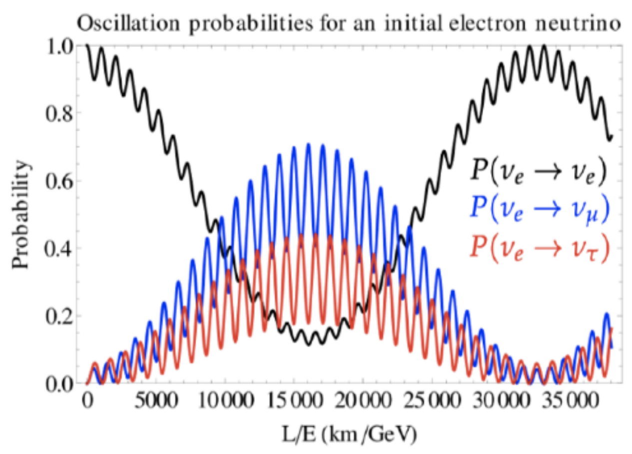
The existence of neutrino masses is the first evidence of new physics beyond standard model

?



SMEFT

$\frac{1}{2} m_\nu \, \overline{\nu_L^c} \nu_L$



LEFT

Seesaw tree-level UVs

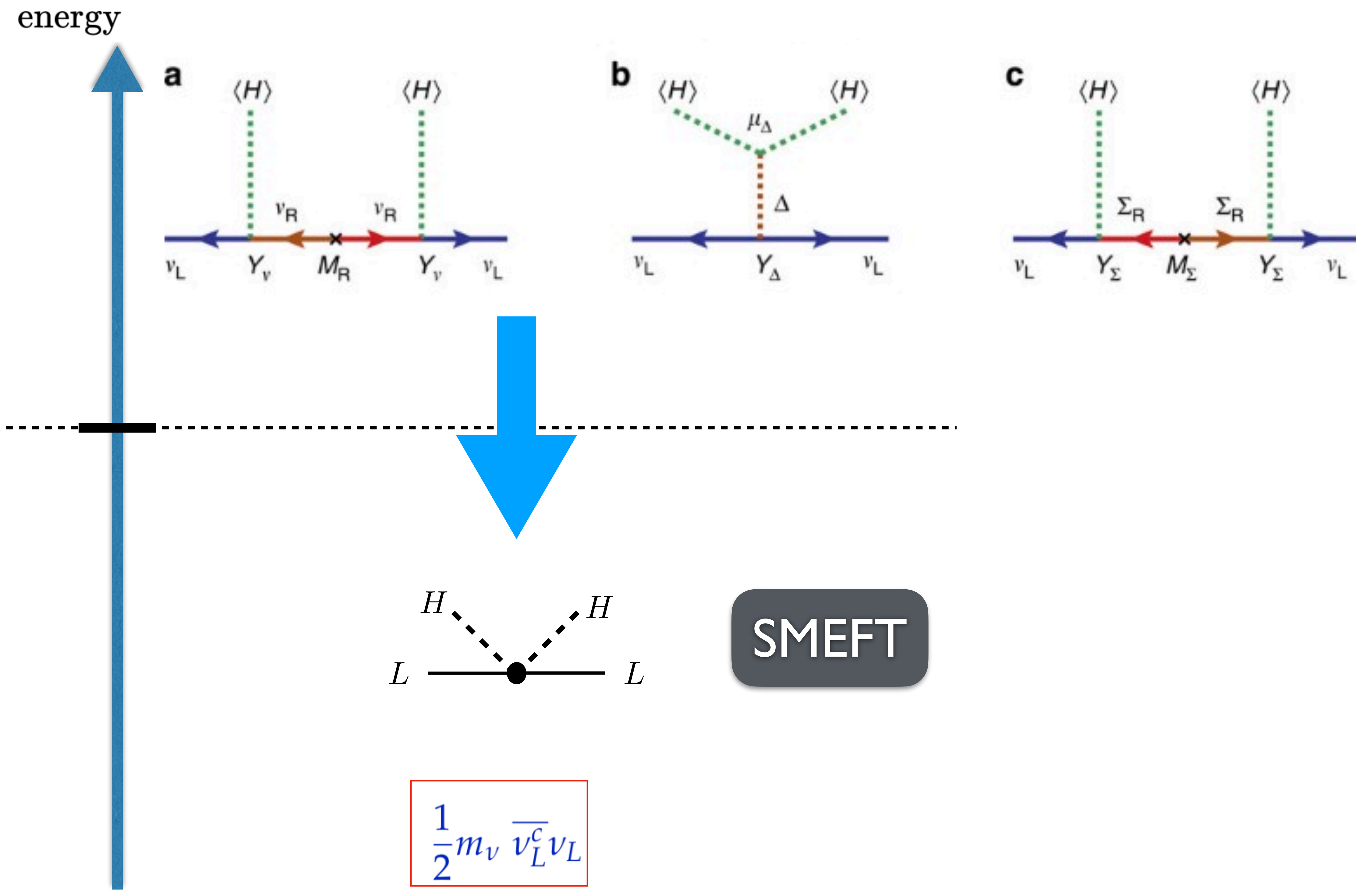
Top-down Approach

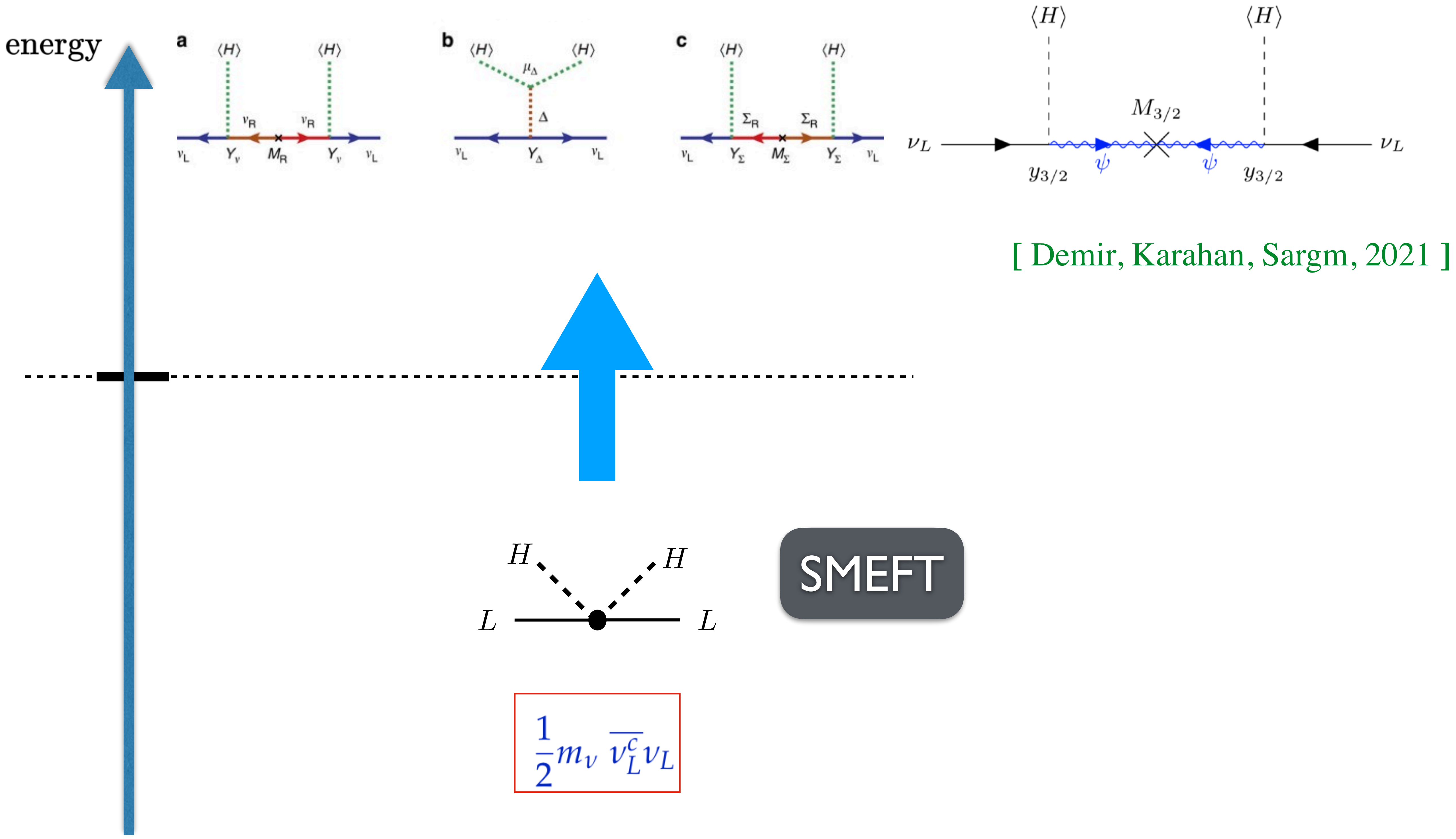
Bottom-up Approach

[Yanagida 1979, Gell-Mann, Ramond, Slansky1979, Mahapatra, Senjanovic, 1980]

[Schecheter, Walle, 1980, Cheng, Li 1980, Magg and Wetterich 1980]

[Foot, Lew, He, Joshi 1989]





Consider Angular momentum conservation of amplitudes

Conformal symmetry of amplitudes

Little group operator

$$\lambda \to t\lambda, \tilde{\lambda} \to t^{-1}\tilde{\lambda}$$
$$A(1^{h_1}\cdots n^{h_n}) \to \prod_i t_i^{-2h_i} A(1^{h_1}\cdots n^{h_n})$$

$$J^{i} = -\frac{1}{2}\left(\lambda_i \frac{\partial}{\partial \lambda_i} - \tilde{\lambda}_i \frac{\partial}{\partial \tilde{\lambda}_i}\right)$$
$$J^{i} M^{\{h_j\}}\left(\lambda_j, \tilde{\lambda}_j\right) = h_i M^{\{h_j\}}\left(\lambda_j, \tilde{\lambda}_j\right)$$

Pauli-Lubanski operator

$$[D, P_\mu] = -iP_\mu,$$
$$[D, K_\mu] = iK_\mu,$$
$$[K_\mu, P_\nu] = 2i(\eta_{\mu\nu} D + M_{\mu\nu}),$$
$$[M_{\mu\nu}, K_\rho] = i(\eta_{\mu\rho} K_\nu - \eta_{\nu\rho} K_\mu),$$

Spinor representation

$$P^{\alpha\dot{\alpha}} = \sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}}$$
$$K_{\alpha\dot{\alpha}} = -4 \sum \partial_{i\alpha} \bar{\partial}_{i\dot{\alpha}}$$
$$W_{\alpha\dot{\alpha}} = \frac{i}{2}\left(P_{\alpha\dot{\beta}} \bar{M}^{\dot{\beta}}{}_{\dot{\alpha}} - M_\alpha{}^\beta P_{\beta\dot{\alpha}}\right)$$

$$-iD = n + \frac{1}{2}\sum_i \left(\lambda_i^\alpha \partial_{i\alpha} + \tilde{\lambda}_i^{\dot{\alpha}} \bar{\partial}_{i\dot{\alpha}}\right),$$
$$-iM_{\alpha\beta} = \sum_i \lambda_{i\alpha} \partial_{i\beta} + \lambda_{i\beta} \partial_{i\alpha},$$
$$-i\bar{M}_{\dot{\alpha}\dot{\beta}} = \sum_i \tilde{\lambda}_{i\dot{\alpha}} \bar{\partial}_{i\dot{\beta}} + \tilde{\lambda}_{i\dot{\beta}} \bar{\partial}_{i\dot{\alpha}}.$$

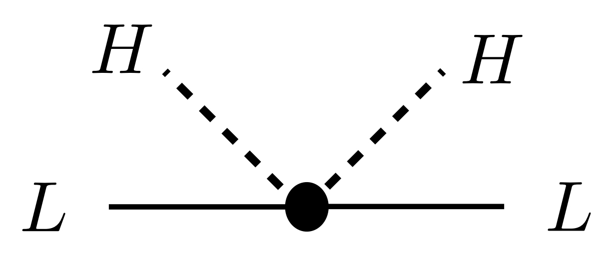
Little group J	Special conformal K	Pauli-Lubanski W	Dilatation D
3-pt/4-pt	Y-basis	J-basis	Anomalous dim

New framework for EFT programs!

Pauli-Lubanski Casimir

Weinberg operator as on-shell amplitude

$$\mathcal{O}^S = (HL)(HL) \qquad \mathcal{B}^y = \langle 12 \rangle$$



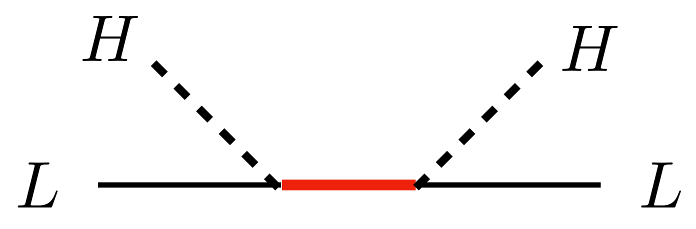
$$\langle p_L, h_L; p_H, h_H | p'_L, h'_L; p'_H, h'_H \rangle$$

[Li, Ni, Xiao, **Yu**, 2204.03660]

Acting on the Pauli-Lubanski Casimir, obtain the eigenvalues on spin!

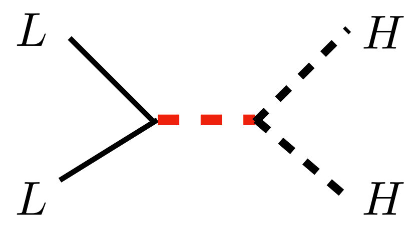
$$\mathbf{W}^2 \langle p_L, h_L; p_H, h_H | P, J, J_z \rangle = -P^2 J(J+1) \langle p_L, h_L; p_H, h_H | P, J, J_z \rangle \quad = -s \sum_J J(J+1) \mathcal{O}^{\mathrm{J}}$$

$$W^2_{\{1,3\}} \mathcal{B}^y = -\frac{3}{4} s_{13} \langle 12 \rangle$$



$$J = \frac{1}{2}$$

$$W^2_{\{1,2\}} \mathcal{B}^y = 0$$



$$J = 0$$

Acting on the SU(2) Casimir, obtain the eigenvalues on gauge!

$$\begin{array}{|c|c|}\hline i & j \\ \hline k & l \\ \hline \end{array} \qquad \begin{array}{|c|c|}\hline i & k \\ \hline j & l \\ \hline \end{array}$$

$$\mathcal{B}^R_1 = \epsilon^{ik}\epsilon^{jl}$$
$$\mathcal{B}^R_2 = \epsilon^{ij}\epsilon^{kl}$$

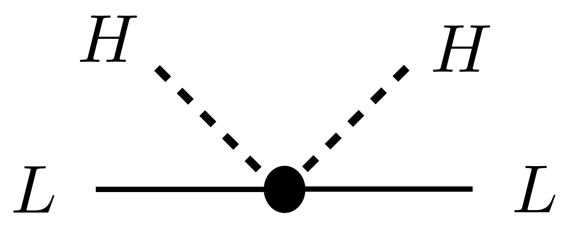
$$\mathbf{C}^2 \mathcal{B}^R = r(r+1) \mathcal{B}^R$$

$$\mathcal{B}^{\mathbf{R}} = \begin{cases} \epsilon^{ik}\epsilon^{jl} & \mathbf{R} = 1 \\ \epsilon^{ik}\epsilon^{jl} - 2\epsilon^{ij}\epsilon^{kl} & \mathbf{R} = 3 \end{cases}$$

Only 3 Types of Seesaw at Dim-5

Generalized partial wave analysis for Poincare/Gauge Casimir

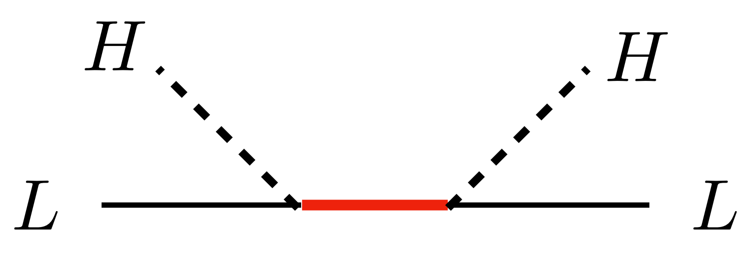
[Li, Ni, Xiao, **Yu**, 2204.03660]



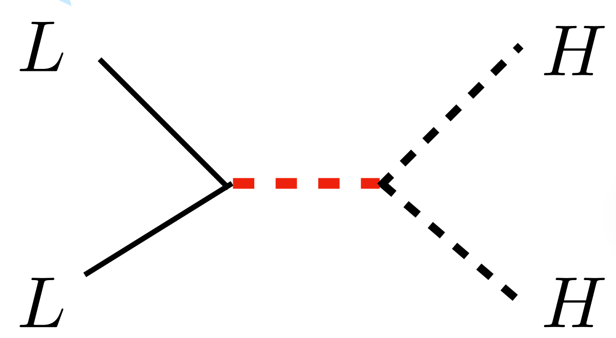
$$\mathbf{W}^2\mathcal{B}^J = -sJ(J+1)\mathcal{B}^J$$

$W^2_{\{1,3\}}\mathcal{B}^y = -\frac{3}{4}s_{13}\langle 12\rangle$ $LH \to LH$ channel

$LL \to HH$ channel $W^2_{\{1,2\}}\mathcal{B}^y = 0$



$J = \frac{1}{2}$



$J = 0$

Type-I and III: SU(2) **single** and **triplet**

Type-II: SU(2) **triplet**, or singlet (excluded by repeated field)

j-basis	Model
$\mathcal{O}^{(1/2,\mathbf{1})}_{HL\to HL} = \mathcal{O}^S + \mathcal{O}^A$	type I
$\mathcal{O}^{(1/2,\mathbf{3})}_{HL\to HL} = \mathcal{O}^S - 3\mathcal{O}^A$	type III

$\mathcal{O}^S = (HL)(HL), \quad \mathcal{O}^A = (HH)(LL)$

j-basis	Model
$\mathcal{O}^{(0,\mathbf{1})}_{HH\to LL} = \mathcal{O}^A$	N/A
$\mathcal{O}^{(0,\mathbf{3})}_{HH\to LL} = \mathcal{O}^S$	type II

Dim-7 Operators

$$\mathcal{B}^y_{\psi^2\phi^2 D^2} = \begin{pmatrix} s_{34}\langle 12\rangle \\ [34]\langle 13\rangle\langle 24\rangle \end{pmatrix} \qquad W^2_{\{1,3\}}\mathcal{B}^y = s_{24}\begin{pmatrix} -\frac{15}{4} & 2 \\ 0 & -\frac{3}{4} \end{pmatrix}\mathcal{B}^y \qquad \Rightarrow \mathcal{B}^j = \begin{cases} 3s_{34}\langle 12\rangle + 2[34]\langle 13\rangle\langle 24\rangle & J = \frac{3}{2} \\ \langle 13\rangle\langle 24\rangle & J = \frac{1}{2} \end{cases}$$

Complete Dim-6 Tree UVs

[Xu-Xiang Li, Zhe Ren, **J.H.Yu,** 2307.10380]

19 Scalars

Notation	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
Name	$\mathcal{S}$	$\mathcal{S}_1$	$\mathcal{S}_2$	φ	Ξ	Ξ_1	Θ_1	Θ_3
Irrep	$(\mathbf{1},\mathbf{1})_0$	$(\mathbf{1},\mathbf{1})_1$	$(\mathbf{1},\mathbf{1})_2$	$(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$(\mathbf{1},\mathbf{3})_0$	$(\mathbf{1},\mathbf{3})_1$	$(\mathbf{1},\mathbf{4})_{\frac{1}{2}}$	$(\mathbf{1},\mathbf{4})_{\frac{3}{2}}$
Notation	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}		
Name	ω_4	ω_1	ω_2	Π_1	Π_7	ζ		
Irrep	$(\mathbf{3},\mathbf{1})_{-\frac{4}{3}}$	$(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$	$(\mathbf{3},\mathbf{1})_{\frac{2}{3}}$	$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$	$(\mathbf{3},\mathbf{2})_{\frac{7}{6}}$	$(\mathbf{3},\mathbf{3})_{-\frac{1}{3}}$		
Notation	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}			
Name	Ω_2	Ω_1	Ω_4	Υ_1	Φ			
Irrep	$(\mathbf{6},\mathbf{1})_{-\frac{2}{3}}$	$(\mathbf{6},\mathbf{1})_{\frac{1}{3}}$	$(\mathbf{6},\mathbf{1})_{\frac{4}{3}}$	$(\mathbf{6},\mathbf{3})_{\frac{1}{3}}$	$(\mathbf{8},\mathbf{2})_{\frac{1}{2}}$			

14 Fermions

Notation	F_1	F_2	F_3	F_4	F_5	F_6	F_7
Name	N	E^c	Δ_1^c	Δ_3^c	Σ	Σ_1^c	
Irrep	$(\mathbf{1},\mathbf{1})_0$	$(\mathbf{1},\mathbf{1})_1$	$(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$(\mathbf{1},\mathbf{2})_{\frac{3}{2}}$	$(\mathbf{1},\mathbf{3})_0$	$(\mathbf{1},\mathbf{3})_1$	$(\mathbf{1},\mathbf{4})_{\frac{1}{2}}$
Notation	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}
Name	D	U	Q_5	Q_1	Q_7	T_1	T_2
Irrep	$(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$	$(\mathbf{3},\mathbf{1})_{\frac{2}{3}}$	$(\mathbf{3},\mathbf{2})_{-\frac{5}{6}}$	$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$	$(\mathbf{3},\mathbf{2})_{\frac{7}{6}}$	$(\mathbf{3},\mathbf{3})_{-\frac{1}{3}}$	$(\mathbf{3},\mathbf{3})_{\frac{2}{3}}$

14 Vectors

Notation	V_1	V_2	V_3	V_4	V_5	V_6	V_7
Name	$\mathcal{B}$	$\mathcal{B}_1$	$\mathcal{L}_3^\dagger$	$\mathcal{W}$	$\mathcal{U}_2$	$\mathcal{U}_5$	$\mathcal{Q}_5$
Irrep	$(\mathbf{1},\mathbf{1})_0$	$(\mathbf{1},\mathbf{1})_1$	$(\mathbf{1},\mathbf{2})_{\frac{3}{2}}$	$(\mathbf{1},\mathbf{3})_0$	$(\mathbf{3},\mathbf{1})_{\frac{2}{3}}$	$(\mathbf{3},\mathbf{1})_{\frac{5}{3}}$	$(\mathbf{3},\mathbf{2})_{-\frac{5}{6}}$
Notation	V_8	V_9	V_{10}	V_{11}	V_{12}	V_{13}	V_{14}
Name	$\mathcal{Q}_1$	$\mathcal{X}$	$\mathcal{Y}_1^\dagger$	$\mathcal{Y}_5^\dagger$	$\mathcal{G}$	$\mathcal{G}_1$	$\mathcal{H}$
Irrep	$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$	$(\mathbf{3},\mathbf{3})_{\frac{2}{3}}$	$(\mathbf{6},\mathbf{2})_{-\frac{1}{6}}$	$(\mathbf{6},\mathbf{2})_{\frac{5}{6}}$	$(\mathbf{8},\mathbf{1})_0$	$(\mathbf{8},\mathbf{1})_1$	$(\mathbf{8},\mathbf{3})_0$

Propose new way of searching new physics

LHC search these 47 resonances!

which covers all dim-6 NP scenarios

Vector $\mathcal{L}_1(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$ and vector $\mathcal{W}_1(\mathbf{1},\mathbf{3})_1$ $\quad \phi_1\phi_2 D_\mu V^\mu$

[de Blas, Vriado, Perez-Victoria, Santiago, 2017]

UV-IR Dictionary

[Xu-Xiang Li, Zhe Ren, **J.H.Yu,** 2307.10380]

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p \gamma^\mu d_r)$

$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p \gamma_\mu l_r)(\bar l_s \gamma^\mu l_t)$	Q_{ee}	$(\bar e_p \gamma_\mu e_r)(\bar e_s \gamma^\mu e_t)$	Q_{le}	$(\bar l_p \gamma_\mu l_r)(\bar e_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar q_s \gamma^\mu q_t)$	Q_{uu}	$(\bar u_p \gamma_\mu u_r)(\bar u_s \gamma^\mu u_t)$	Q_{lu}	$(\bar l_p \gamma_\mu l_r)(\bar u_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p \gamma_\mu \tau^I q_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar d_p \gamma_\mu d_r)(\bar d_s \gamma^\mu d_t)$	Q_{ld}	$(\bar l_p \gamma_\mu l_r)(\bar d_s \gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p \gamma_\mu l_r)(\bar q_s \gamma^\mu q_t)$	Q_{eu}	$(\bar e_p \gamma_\mu e_r)(\bar u_s \gamma^\mu u_t)$	Q_{qe}	$(\bar q_p \gamma_\mu q_r)(\bar e_s \gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p \gamma_\mu \tau^I l_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar e_p \gamma_\mu e_r)(\bar d_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar u_s \gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p \gamma_\mu u_r)(\bar d_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar u_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p \gamma_\mu T^A u_r)(\bar d_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar d_s \gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar d_s \gamma^\mu T^A d_t)$

$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\varepsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\varepsilon_{jk}(\bar q_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\varepsilon_{jk}(\bar q_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar q_s^k \sigma^{\mu\nu} u_t)$		

If excess, then find UV

	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}
$\mathcal{O}_{eH}$		✓	✓	✓	✓	✓								
$\mathcal{O}_{uH}$									✓		✓	✓	✓	✓
$\mathcal{O}_{dH}$								✓		✓	✓		✓	✓
$\mathcal{O}_{Hl}^{(1)}$	✓	✓			✓	✓								
$\mathcal{O}_{Hl}^{(3)}$	✓	✓			✓	✓								
$\mathcal{O}_{He}$			✓	✓										
$\mathcal{O}_{Hq}^{(1)}$								✓	✓				✓	✓
$\mathcal{O}_{Hq}^{(3)}$								✓	✓				✓	✓
$\mathcal{O}_{Hu}$											✓	✓		
$\mathcal{O}_{Hd}$										✓	✓			
$\mathcal{O}_{Hud}$											✓			

UV-IR Dictionary

[Xu-Xiang Li, Zhe Ren, **J.H.Yu,** 2307.10380]

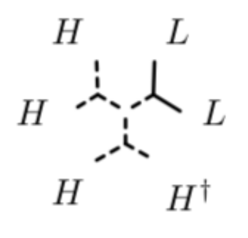
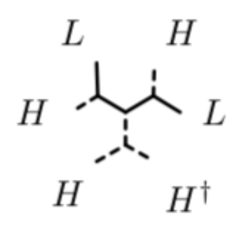
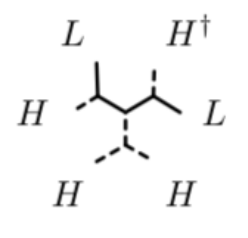
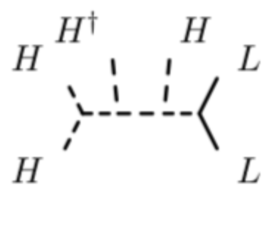
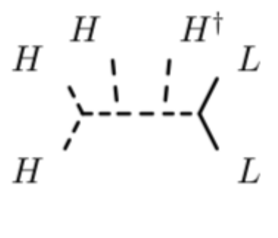
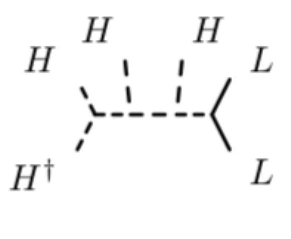
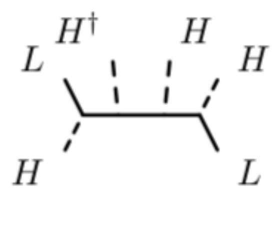
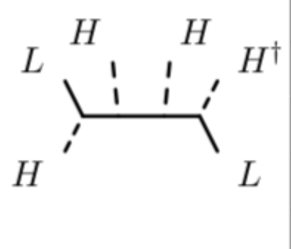
	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}
$\mathcal{O}_H$	✓			✓	✓	✓	✓	✓											
$\mathcal{O}_{H\square}$	✓				✓	✓													
$\mathcal{O}_{HD}$					✓	✓													
$\mathcal{O}_{eH}$	*			✓	✓	✓													
$\mathcal{O}_{uH}$	*			✓	✓	✓													
$\mathcal{O}_{dH}$	*			✓	✓	✓													
$\mathcal{O}_{ll}$		✓				✓													
$\mathcal{O}_{qq}^{(1)}$										✓				✓		✓		✓	
$\mathcal{O}_{qq}^{(3)}$										✓				✓		✓		✓	
$\mathcal{O}_{lq}^{(1)}$										✓				✓					
$\mathcal{O}_{lq}^{(3)}$										✓				✓					
$\mathcal{O}_{ee}$			✓																
$\mathcal{O}_{uu}$									✓								✓		
$\mathcal{O}_{dd}$											✓				✓				
$\mathcal{O}_{eu}$										✓									
$\mathcal{O}_{ed}$									✓										
$\mathcal{O}_{ud}^{(1)}$										✓						✓			
$\mathcal{O}_{ud}^{(8)}$										✓						✓			
$\mathcal{O}_{le}$				✓															
$\mathcal{O}_{lu}$													✓						
$\mathcal{O}_{ld}$												✓							
$\mathcal{O}_{qe}$													✓						
$\mathcal{O}_{qu}^{(1)}$				✓															✓
$\mathcal{O}_{qu}^{(8)}$				✓															✓
$\mathcal{O}_{qd}^{(1)}$				✓															✓
$\mathcal{O}_{qd}^{(8)}$				✓															✓
$\mathcal{O}_{ledq}$				✓															
$\mathcal{O}_{quqd}^{(1)}$				✓						✓						✓			
$\mathcal{O}_{quqd}^{(8)}$										✓						✓			✓
$\mathcal{O}_{lequ}^{(1)}$				✓						✓			✓						
$\mathcal{O}_{lequ}^{(3)}$										✓			✓						
$\mathcal{O}_{duq}$										✓									
$\mathcal{O}_{qqu}$										✓									
$\mathcal{O}_{qqq}$										✓				✓					
$\mathcal{O}_{duu}$									✓	✓									

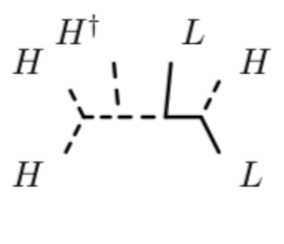
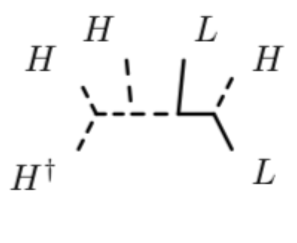
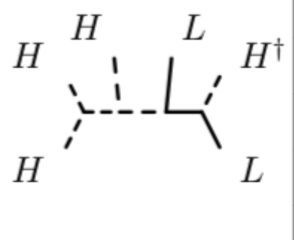
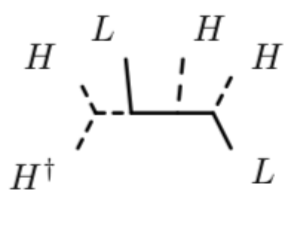
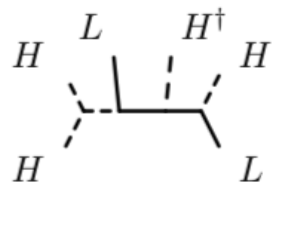
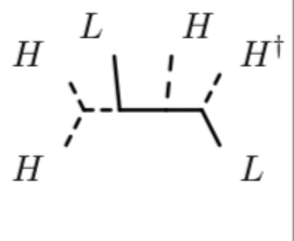
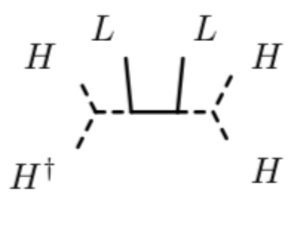
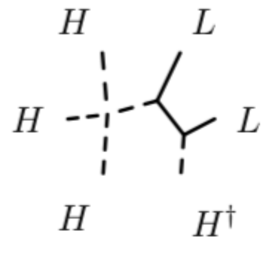
	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	V_{11}	V_{12}	V_{13}	V_{14}
$\mathcal{O}_H$		✓		✓										
$\mathcal{O}_{H\square}$	✓	✓		✓										
$\mathcal{O}_{HD}$	✓	✓		✓										
$\mathcal{O}_{eH}$	✓	✓		✓										
$\mathcal{O}_{uH}$	✓	✓		✓										
$\mathcal{O}_{dH}$	✓	✓		✓										
$\mathcal{O}_{Hl}^{(1)}$	✓													
$\mathcal{O}_{Hl}^{(3)}$				✓										
$\mathcal{O}_{He}$	✓													
$\mathcal{O}_{Hq}^{(1)}$	✓													
$\mathcal{O}_{Hq}^{(3)}$				✓										
$\mathcal{O}_{Hu}$	✓													
$\mathcal{O}_{Hd}$	✓													
$\mathcal{O}_{Hud}$		✓												
$\mathcal{O}_{ll}$	✓			✓										
$\mathcal{O}_{qq}^{(1)}$	✓											✓		✓
$\mathcal{O}_{qq}^{(3)}$				✓								✓		✓
$\mathcal{O}_{lq}^{(1)}$	✓				✓				✓					
$\mathcal{O}_{lq}^{(3)}$				✓	✓				✓					
$\mathcal{O}_{ee}$	✓													
$\mathcal{O}_{uu}$	✓											✓		
$\mathcal{O}_{dd}$	✓											✓		
$\mathcal{O}_{eu}$	✓					✓								
$\mathcal{O}_{ed}$	✓				✓									
$\mathcal{O}_{ud}^{(1)}$	✓	✓											✓	
$\mathcal{O}_{ud}^{(8)}$		✓										✓	✓	
$\mathcal{O}_{le}$	✓		✓											
$\mathcal{O}_{lu}$	✓							✓						
$\mathcal{O}_{ld}$	✓						✓							
$\mathcal{O}_{qe}$	✓						✓							
$\mathcal{O}_{qu}^{(1)}$	✓						✓				✓			
$\mathcal{O}_{qu}^{(8)}$							✓				✓	✓		
$\mathcal{O}_{qd}^{(1)}$	✓							✓		✓				
$\mathcal{O}_{qd}^{(8)}$								✓		✓		✓		
$\mathcal{O}_{ledq}$					✓		✓							
$\mathcal{O}_{duq}$							✓	✓						
$\mathcal{O}_{qqu}$							✓							

Dim-7 Tree-level Seesaw

59 UV tree-level models, 19 topologies, one genuine dim-7 seesaw

[Li, Ni, Xiao, **Yu**, 2204.03660]

Topology	j-basis	Quantum numbers $\{J, \mathbf{R}, Y\}$
	$\mathcal{O}_{\{12\|34\|56\},1} = 2\mathcal{O}^p_1 - 4\mathcal{O}^p_2,$	$\{0,3,-1\},\{0,3,1\},\{0,3,0\}$
	$\mathcal{O}_{\{12\|34\|56\},2} = 2\mathcal{O}^p_2 + 4\mathcal{O}^p_3,$	$\{0,3,-1\},\{0,1,1\},\{0,3,0\}$
	$\mathcal{O}_{\{12\|34\|56\},3} = 12\mathcal{O}^p_4,$	$\{0,1,-1\},\{0,3,1\},\{0,3,0\}$
	$\mathcal{O}_{\{12\|34\|56\},4} = 4\mathcal{O}^p_1 + 4\mathcal{O}^p_2,$	$\{0,3,-1\},\{0,3,1\},\{0,1,0\}$
	$\mathcal{O}_{\{12\|34\|56\},5} = 2\mathcal{O}^p_4 + 4\mathcal{O}^p_5.$	$\{0,1,-1\},\{0,1,1\},\{0,1,0\}$
	$\mathcal{O}_{\{13\|24\|56\},1} = -\mathcal{O}^p_2 - 2\mathcal{O}^p_3 + 3\mathcal{O}^p_4,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},3,0\},\{0,3,0\}$
	$\mathcal{O}_{\{13\|24\|56\},2} = -\mathcal{O}^p_1 + 3\mathcal{O}^p_2 + 2\mathcal{O}^p_3 + 3\mathcal{O}^p_4,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},1,0\},\{0,3,0\}$
	$\mathcal{O}_{\{13\|24\|56\},3} = -\mathcal{O}^p_1 + \mathcal{O}^p_2 - 2\mathcal{O}^p_3 - 3\mathcal{O}^p_4,$	$\{\frac{1}{2},1,0\},\{\frac{1}{2},3,0\},\{0,3,0\}$
	$\mathcal{O}_{\{13\|24\|56\},4} = -\mathcal{O}^p_1 - \mathcal{O}^p_2 + 3\mathcal{O}^p_4 + 6\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},3,0\},\{0,1,0\}$
	$\mathcal{O}_{\{13\|24\|56\},5} = \mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_4 + 2\mathcal{O}^p_5.$	$\{\frac{1}{2},1,0\},\{\frac{1}{2},1,0\},\{0,1,0\}$
	$\mathcal{O}_{\{16\|23\|45\},1} = 2\mathcal{O}^p_1 - 2\mathcal{O}^p_2 - 2\mathcal{O}^p_3 + 6\mathcal{O}^p_4 + 6\mathcal{O}^p_5,$	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},3,0\},\{0,3,1\}$
	$\mathcal{O}_{\{16\|23\|45\},2} = -2\mathcal{O}^p_1 - \mathcal{O}^p_2 - \mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5,$	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},1,0\},\{0,3,1\}$
	$\mathcal{O}_{\{16\|23\|45\},3} = 3\mathcal{O}^p_2 + 3\mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5,$	$\{\frac{1}{2},1,-1\},\{\frac{1}{2},3,0\},\{0,3,1\}$
	$\mathcal{O}_{\{16\|23\|45\},4} = \mathcal{O}^p_2 - \mathcal{O}^p_3 - 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5,$	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},3,0\},\{0,1,1\}$
	$\mathcal{O}_{\{16\|23\|45\},5} = \mathcal{O}^p_2 - \mathcal{O}^p_3 + \mathcal{O}^p_4 - \mathcal{O}^p_5.$	$\{\frac{1}{2},1,-1\},\{\frac{1}{2},1,0\},\{0,1,1\}$
	$\mathcal{O}_{\{12\|125\|34\},1} = \mathcal{O}^p_1 + 4\mathcal{O}^p_2,$	$\{0,3,-1\},\{0,4,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{12\|125\|34\},2} = -8\mathcal{O}^p_1 + 4\mathcal{O}^p_2,$	$\{0,3,-1\},\{0,2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{12\|125\|34\},3} = -12\mathcal{O}^p_4,$	$\{0,1,-1\},\{0,2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{12\|125\|34\},4} = -2\mathcal{O}^p_2 - 4\mathcal{O}^p_3,$	$\{0,3,-1\},\{0,2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{12\|125\|34\},5} = -2\mathcal{O}^p_4 - 4\mathcal{O}^p_5.$	$\{0,1,-1\},\{0,2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{12\|126\|34\},1} = 3\mathcal{O}^p_1,$	$\{0,3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{12\|126\|34\},2} = 12\mathcal{O}^p_2,$	$\{0,3,-1\},\{0,2,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{12\|126\|34\},3} = -12\mathcal{O}^p_4,$	$\{0,1,-1\},\{0,2,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{12\|126\|34\},4} = -2\mathcal{O}^p_2 - 4\mathcal{O}^p_3,$	$\{0,3,-1\},\{0,2,-\frac{3}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{12\|126\|34\},5} = -2\mathcal{O}^p_4 - 4\mathcal{O}^p_5.$	$\{0,1,-1\},\{0,2,-\frac{3}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{12\|124\|36\},1} = -\mathcal{O}^p_1 - 4\mathcal{O}^p_3,$	$\{0,3,-1\},\{0,4,-\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{12\|124\|36\},2} = 2\mathcal{O}^p_1 + 6\mathcal{O}^p_2 + 2\mathcal{O}^p_3,$	$\{0,3,-1\},\{0,2,-\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{12\|124\|36\},3} = -6\mathcal{O}^p_4 - 6\mathcal{O}^p_5,$	$\{0,1,-1\},\{0,2,-\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{12\|124\|36\},4} = -2\mathcal{O}^p_1 + 2\mathcal{O}^p_2 + 2\mathcal{O}^p_3,$	$\{0,3,-1\},\{0,2,-\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{12\|124\|36\},5} = -2\mathcal{O}^p_4 + 2\mathcal{O}^p_5.$	$\{0,1,-1\},\{0,2,-\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{13\|135\|24\},1} = \mathcal{O}^p_1 - 2\mathcal{O}^p_3 - 6\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},4,\frac{1}{2}\},\{\frac{1}{2},3,0\}$
	$\mathcal{O}_{\{13\|135\|24\},2} = -\mathcal{O}^p_1 - 3\mathcal{O}^p_2 - 4\mathcal{O}^p_3 + 9\mathcal{O}^p_4 + 6\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{\frac{1}{2},3,0\}$
	$\mathcal{O}_{\{13\|135\|24\},3} = \mathcal{O}^p_1 - \mathcal{O}^p_2 + 2\mathcal{O}^p_3 + 3\mathcal{O}^p_4,$	$\{\frac{1}{2},1,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{\frac{1}{2},3,0\}$
	$\mathcal{O}_{\{13\|135\|24\},4} = \mathcal{O}^p_1 - 3\mathcal{O}^p_2 - 2\mathcal{O}^p_3 - 3\mathcal{O}^p_4,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{\frac{1}{2},1,0\}$
	$\mathcal{O}_{\{13\|135\|24\},5} = \mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_4 + 2\mathcal{O}^p_5.$	$\{\frac{1}{2},1,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{\frac{1}{2},1,0\}$
	$\mathcal{O}_{\{16\|146\|23\},1} = -2\mathcal{O}^p_1 + 4\mathcal{O}^p_3 - 12\mathcal{O}^p_5,$	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},4,-\frac{1}{2}\},\{\frac{1}{2},3,0\}$
	$\mathcal{O}_{\{16\|146\|23\},2} = 2\mathcal{O}^p_1 - 3\mathcal{O}^p_2 - \mathcal{O}^p_3 + 9\mathcal{O}^p_4 + 3\mathcal{O}^p_5,$	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{\frac{1}{2},3,0\}$
	$\mathcal{O}_{\{16\|146\|23\},3} = 3\mathcal{O}^p_2 + 3\mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5,$	$\{\frac{1}{2},1,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{\frac{1}{2},3,0\}$
	$\mathcal{O}_{\{16\|146\|23\},4} = 2\mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_3 - 3\mathcal{O}^p_4 - 3\mathcal{O}^p_5,$	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{\frac{1}{2},1,0\}$
	$\mathcal{O}_{\{16\|146\|23\},5} = -\mathcal{O}^p_2 + \mathcal{O}^p_3 - \mathcal{O}^p_4 + \mathcal{O}^p_5,$	$\{\frac{1}{2},1,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{\frac{1}{2},1,0\}$

	$\mathcal{O}_{\{13\|123\|45\},1} = \mathcal{O}^p_1 - 4\mathcal{O}^p_2 - 4\mathcal{O}^p_3,$	$\{\frac{1}{2},3,0\},\{0,4,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{13\|123\|45\},2} = 2\mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_3 - 9\mathcal{O}^p_4 - 9\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{0,2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{13\|123\|45\},3} = 2\mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5,$	$\{\frac{1}{2},1,0\},\{0,2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{13\|123\|45\},4} = \mathcal{O}^p_2 - \mathcal{O}^p_3 + 3\mathcal{O}^p_4 - 3\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{0,2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{13\|123\|45\},5} = -\mathcal{O}^p_2 + \mathcal{O}^p_3 + \mathcal{O}^p_4 - \mathcal{O}^p_5.$	$\{\frac{1}{2},1,0\},\{0,2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{13\|123\|46\},1} = \mathcal{O}^p_1 - 4\mathcal{O}^p_2 - 4\mathcal{O}^p_3,$	$\{\frac{1}{2},3,0\},\{0,4,-\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{13\|123\|46\},2} = \mathcal{O}^p_1 + 2\mathcal{O}^p_2 - \mathcal{O}^p_3 - 9\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{0,2,-\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{13\|123\|46\},3} = -\mathcal{O}^p_1 - 2\mathcal{O}^p_2 + \mathcal{O}^p_3 - 3\mathcal{O}^p_5,$	$\{\frac{1}{2},1,0\},\{0,2,-\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{13\|123\|46\},4} = -\mathcal{O}^p_1 - \mathcal{O}^p_3 + 6\mathcal{O}^p_4 + 3\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{0,4,-\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{13\|123\|46\},5} = -\mathcal{O}^p_1 - \mathcal{O}^p_3 - 2\mathcal{O}^p_4 - \mathcal{O}^p_5.$	$\{\frac{1}{2},1,0\},\{0,2,-\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{16\|126\|34\},1} = 6\mathcal{O}^p_1,$	$\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{16\|126\|34\},2} = -3\mathcal{O}^p_2 + 9\mathcal{O}^p_4,$	$\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{16\|126\|34\},3} = -3\mathcal{O}^p_2 - 3\mathcal{O}^p_4,$	$\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{16\|126\|34\},4} = -\mathcal{O}^p_2 - 2\mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 6\mathcal{O}^p_5,$	$\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{16\|126\|34\},5} = \mathcal{O}^p_2 + 2\mathcal{O}^p_3 + \mathcal{O}^p_4 + 2\mathcal{O}^p_5.$	$\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{23\|235\|46\},1} = \mathcal{O}^p_1 - 2\mathcal{O}^p_3 + 6\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},4,\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{23\|235\|46\},2} = \mathcal{O}^p_1 - 6\mathcal{O}^p_2 - 5\mathcal{O}^p_3 - 3\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{23\|235\|46\},3} = \mathcal{O}^p_1 + 2\mathcal{O}^p_2 - \mathcal{O}^p_3 - 3\mathcal{O}^p_5,$	$\{\frac{1}{2},1,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{23\|235\|46\},4} = -\mathcal{O}^p_1 - \mathcal{O}^p_3 - 6\mathcal{O}^p_4 - 3\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{23\|235\|46\},5} = -\mathcal{O}^p_1 - \mathcal{O}^p_3 + 2\mathcal{O}^p_4 + \mathcal{O}^p_5.$	$\{\frac{1}{2},1,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{13\|136\|45\},1} = \mathcal{O}^p_1 + 2\mathcal{O}^p_2 + 2\mathcal{O}^p_3 - 6\mathcal{O}^p_4 - 6\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},4,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{13\|136\|45\},2} = 2\mathcal{O}^p_1 - 5\mathcal{O}^p_2 - 5\mathcal{O}^p_3 - 3\mathcal{O}^p_4 - 3\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{13\|136\|45\},3} = 2\mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5,$	$\{\frac{1}{2},1,0\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{13\|136\|45\},4} = \mathcal{O}^p_2 - \mathcal{O}^p_3 + 3\mathcal{O}^p_4 - 3\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{13\|136\|45\},5} = -\mathcal{O}^p_2 + \mathcal{O}^p_3 + \mathcal{O}^p_4 - \mathcal{O}^p_5.$	$\{\frac{1}{2},1,0\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{16\|156\|34\},1} = 2\mathcal{O}^p_1 - 4\mathcal{O}^p_2 + 12\mathcal{O}^p_4,$	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},4,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{16\|156\|34\},2} = 4\mathcal{O}^p_1 + \mathcal{O}^p_2 - 3\mathcal{O}^p_4,$	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{16\|156\|34\},3} = -3\mathcal{O}^p_2 - 3\mathcal{O}^p_4,$	$\{\frac{1}{2},1,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{16\|156\|34\},4} = -\mathcal{O}^p_2 - 2\mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 6\mathcal{O}^p_5,$	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{16\|156\|34\},5} = \mathcal{O}^p_2 + 2\mathcal{O}^p_3 + \mathcal{O}^p_4 + 2\mathcal{O}^p_5.$	$\{\frac{1}{2},1,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{34\|134\|56\},1} = -\mathcal{O}^p_1 + 2\mathcal{O}^p_2 + 6\mathcal{O}^p_4,$	$\{0,3,1\},\{\frac{1}{2},4,\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{34\|134\|56\},2} = 4\mathcal{O}^p_1 - 8\mathcal{O}^p_2 + 12\mathcal{O}^p_4,$	$\{0,3,1\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{34\|134\|56\},3} = 2\mathcal{O}^p_2 + 4\mathcal{O}^p_3,$	$\{0,1,1\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{34\|134\|56\},4} = -4\mathcal{O}^p_1 - 4\mathcal{O}^p_2,$	$\{0,3,1\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{34\|134\|56\},5} = 2\mathcal{O}^p_4 + 4\mathcal{O}^p_5.$	$\{0,1,1\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{16\|126\},1} = 6\mathcal{O}^p_1,$	$\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\}$
	$\mathcal{O}_{\{16\|126\},2,3} = \begin{pmatrix} \mathcal{O}^p_2 - \mathcal{O}^p_3 - 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5 \\ -2\mathcal{O}^p_2 - \mathcal{O}^p_3 + 6\mathcal{O}^p_4 + 3\mathcal{O}^p_5 \end{pmatrix},$	$\{\frac{1}{2},3,-1\},\{0,2,-\frac{3}{2}\}$
	$\mathcal{O}_{\{16\|126\},4,5} = \begin{pmatrix} -\mathcal{O}^p_2 - 2\mathcal{O}^p_3 - \mathcal{O}^p_4 - 2\mathcal{O}^p_5 \\ -2\mathcal{O}^p_2 - \mathcal{O}^p_3 - 2\mathcal{O}^p_4 - \mathcal{O}^p_5 \end{pmatrix}.$	$\{\frac{1}{2},1,-1\},\{0,2,-\frac{3}{2}\}$

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[Bonnet, Hernandez, Ota, Winter, 2009]

Complete dim-7 Tree UVs

[Li, Ni, Xiao, **Yu**, 2204.03660]

$(\mathrm{SU}(3)_c, \mathrm{SU}(2)_2, \mathrm{U}(1)_y)$	Scalar
$S1\ (\mathbf{1},\mathbf{1},0)$	$H^3H^\dagger L^2[(S6),(F5),(F1),(S4,S6),$ $(S4,F5),(S4,F1),(F3,F5),(F1,F3),(S6,F3)]$
$S2\ (\mathbf{1},\mathbf{1},1)$	$e_{\mathbb{C}}HL^3[(S4),(F4),(F1)]$ $d_{\mathbb{C}}HL^2Q[(S4),(F10),(F9)]$ $HL^2Q^\dagger u_{\mathbb{C}}^\dagger[(S4),(F8),(F12)]$ $De_{\mathbb{C}}H^{\dagger 3}L^\dagger[(F1),(F3),(V3)]$
$S4\ \left(\mathbf{1},\mathbf{2},\frac{1}{2}\right)$	$e_{\mathbb{C}}HL^3[(S6),(S2),(F5),(F1)]$ $d_{\mathbb{C}}HL^2Q[(S6),(S2),(F5),(F1)]$ $HL^2Q^\dagger u_{\mathbb{C}}^\dagger[(S6),(S2),(F5),(F1)]$ $H^3H^\dagger L^2[(S6),(F5),(F1),(S5,S6),(S1,S6),$ $(S6,F5),(S6,F1),(S5,F5),(S5,F1),(S1,F5),(S1,F1)]$
$S5\ (\mathbf{1},\mathbf{3},0)$	$H^3H^\dagger L^2[(S6),(F1,F5),(S6,S7),(S4,S6),(S7,F5),$ $(S4,F5),(S4,F1),(F5,F7),(F3,F5),(F1,F3),(S6,F7),(S6,F3)]$
$S6\ (\mathbf{1},\mathbf{3},1)$	$D^2H^2L^2$ $e_{\mathbb{C}}HL^3[(S4),(F4),(F5)]$ $d_{\mathbb{C}}HL^2Q[(S4),(F10),(F14)]$ $HL^2Q^\dagger u_{\mathbb{C}}^\dagger[(S4),(F13),(F12)]$ $De_{\mathbb{C}}H^{\dagger 3}L^\dagger[(F5),(F3),(V3)]$ $H^2L^2W_L[(F7)]$ $H^3H^\dagger L^2[(S4),(S8),(S7),(S5),(S1),(F5,F6),(F1,F6),$ $(S5,S7),(S4,S5),(S1,S4),(S7,F5),(S4,F5),(S4,F1),(F5,F7),(F3,F5),$ $(F1,F3),(S8,F6),(F6,F7),(F3,F6),(S5,F7),(S5,F3),(S1,F3)]$ $H^2L^2W_L$ $B_LH^2L^2$ $e_{\mathbb{C}}HL^3$ $HL^2Q^\dagger u_{\mathbb{C}}^\dagger$ $d_{\mathbb{C}}HL^2Q$ $De_{\mathbb{C}}^\dagger H^3L$
$S7\ \left(\mathbf{1},\mathbf{4},\frac{1}{2}\right)$	$H^3H^\dagger L^2[(S6),(F5),(S5,S6),(S6,F5),(S5,F5)]$
$S8\ \left(\mathbf{1},\mathbf{4},\frac{3}{2}\right)$	$H^3H^\dagger L^2[(S6),(F6),(S6,F6)]$
$S10\ \left(\mathbf{3},\mathbf{1},-\frac{1}{3}\right)$	$d_{\mathbb{C}}{}^2HLu_{\mathbb{C}}[(S12),(F10),(F1)]$ $d_{\mathbb{C}}HL^2Q[(S12),(F10),(F1)]$ $d_{\mathbb{C}}e_{\mathbb{C}}^\dagger HLu_{\mathbb{C}}^\dagger[(S12),(F10),(F1)]$ $d_{\mathbb{C}}HLQ^{\dagger 2}[(S12),(F10),(F1)]$
$S11\ \left(\mathbf{3},\mathbf{1},\frac{2}{3}\right)$	$d_{\mathbb{C}}{}^3H^\dagger L[(S12),(F11),(F2)]$ $d_{\mathbb{C}}{}^2HLu_{\mathbb{C}}[(F11),(S13),(F1)]$ $d_{\mathbb{C}}{}^2e_{\mathbb{C}}^\dagger HQ^\dagger[(S13),(F3),(F8)]$
$S12\ \left(\mathbf{3},\mathbf{2},\frac{1}{6}\right)$	$d_{\mathbb{C}}{}^3H^\dagger L[(S11),(F11)]$ $d_{\mathbb{C}}{}^2HLu_{\mathbb{C}}[(F11),(S10),(F10)]$ $d_{\mathbb{C}}HL^2Q[(S10),(S14),(F5),(F1),(F14),(F9)]$ $d_{\mathbb{C}}e_{\mathbb{C}}^\dagger HLu_{\mathbb{C}}^\dagger[(S10),(F3),(F12)]$
$S13\ \left(\mathbf{3},\mathbf{2},\frac{7}{6}\right)$	$d_{\mathbb{C}}{}^2HLu_{\mathbb{C}}[(S11),(F10)]$ $d_{\mathbb{C}}{}^2e_{\mathbb{C}}^\dagger HQ^\dagger[(S11),(F10)]$
$S14\ \left(\mathbf{3},\mathbf{3},-\frac{1}{3}\right)$	$d_{\mathbb{C}}HL^2Q[(S12),(F10),(F5)]$ $d_{\mathbb{C}}HLQ^{\dagger 2}[(S12),(F10),(F5)]$

$(\mathrm{SU}(3)_c, \mathrm{SU}(2)_2, \mathrm{U}(1)_y)$	Fermion
$F1\ (\mathbf{1},\mathbf{1},0)$	$D^2H^2L^2$ $e_{\mathbb{C}}HL^3[(S4),(S2)]$ $d_{\mathbb{C}}HL^2Q[(S4),(S10),(S12)]$ $HL^2Q^\dagger u_{\mathbb{C}}^\dagger[(S4),(V5),(V8)]$ $De_{\mathbb{C}}H^{\dagger 3}L^\dagger[(F3),(V2)]$ $d_{\mathbb{C}}{}^2HLu_{\mathbb{C}}[(S11),(S10)]$ $d_{\mathbb{C}}e_{\mathbb{C}}^\dagger HLu_{\mathbb{C}}^\dagger[(S10),(V5)]$ $d_{\mathbb{C}}HLQ^{\dagger 2}[(S10),(V8)]$ $H^2L^2W_L[(F5)]$ $H^3H^\dagger L^2[(S4),(S5,F5),(S1),(S6,F6),(F3,F5),(F3),(F3,F6),$ $(S4,S6),(S6,F3),(S4,S5),(S1,S4),(S5,F3),(S1,F3)]$ $H^2L^2W_L$ $B_LH^2L^2$ $e_{\mathbb{C}}HL^3$ $HL^2Q^\dagger u_{\mathbb{C}}^\dagger$ $d_{\mathbb{C}}HL^2Q$ $De_{\mathbb{C}}^\dagger H^3L$
$F2\ (\mathbf{1},\mathbf{1},1)$	$d_{\mathbb{C}}{}^3H^\dagger L[(S11)]$
$F3\ \left(\mathbf{1},\mathbf{2},\frac{1}{2}\right)$	$De_{\mathbb{C}}H^{\dagger 3}L^\dagger[(F5),(F1),(S6),(V2)]$ $d_{\mathbb{C}}e_{\mathbb{C}}^\dagger HLu_{\mathbb{C}}^\dagger[(S12),(V8)]$ $d_{\mathbb{C}}{}^2e_{\mathbb{C}}^\dagger HQ^\dagger[(V8),(S11)]$ $H^3H^\dagger L^2[(F5),(F1,F5),(F1),(F5,F6),(F1,F6),(S6,F5),$ $(S6,F1),(S5,F5),(S5,F1),(S1,F5),(S1,F1),(S6,F6),(S5,S6),(S1,S6)]$
$F4\ \left(\mathbf{1},\mathbf{2},\frac{3}{2}\right)$	$e_{\mathbb{C}}HL^3[(S6),(S2)]$
$F5\ (\mathbf{1},\mathbf{3},0)$	$e_{\mathbb{C}}HL^3[(S4),(S6)]$ $d_{\mathbb{C}}HL^2Q[(S4),(S12),(S14)]$ $HL^2Q^\dagger u_{\mathbb{C}}^\dagger[(S4),(V9),(V8)]$ $D^2H^2L^2$ $De_{\mathbb{C}}H^{\dagger 3}L^\dagger[(S6),(F3),(V5)]$ $d_{\mathbb{C}}HLQ^{\dagger 2}[(S14),(V8)]$ $H^2L^2W_L[(F7),(F1)]$ $H^3H^\dagger L^2[(S4),(S7),(S5,F1),(S1),(S6,F6),(F7),(F3),(F1,F3),$ $(F6,F7),(F3,F6),(S6,S7),(S4,S6),(S6,F7),(S6,F3),(S5,S7),$ $(S4,S5),(S1,S4),(S5,F7),(S5,F3),(S1,F3)]$ $H^2L^2W_L$ $B_LH^2L^2$ $e_{\mathbb{C}}HL^3$ $HL^2Q^\dagger u_{\mathbb{C}}^\dagger$ $d_{\mathbb{C}}HL^2Q$ $De_{\mathbb{C}}^\dagger H^3L$
$F6\ (\mathbf{1},\mathbf{3},1)$	$H^3H^\dagger L^2[(S8),(S6,F5),(S6,F1),(F5,F7),(F3,F5),(F1,F3),(S6,S8),(S6,F7),(S6,F3)]$
$F7\ \left(\mathbf{1},\mathbf{4},\frac{1}{2}\right)$	$H^2L^2W_L[(F5),(S6)]$ $H^3H^\dagger L^2[(F5),(S6,F5),(F5,F6),(S6,F6),(S5,F5),(S5,S6)]$
$F8\ \left(\mathbf{3},\mathbf{1},-\frac{1}{3}\right)$	$HL^2Q^\dagger u_{\mathbb{C}}^\dagger[(S2),(V8)]$ $d_{\mathbb{C}}HLQ^{\dagger 2}[(V8),(S12),(V5)]$ $d_{\mathbb{C}}{}^2e_{\mathbb{C}}^\dagger HQ^\dagger[(V5),(S11)]$
$F9\ \left(\mathbf{3},\mathbf{1},\frac{2}{3}\right)$	$d_{\mathbb{C}}HL^2Q[(S12),(S2)]$
$F10\ \left(\mathbf{3},\mathbf{2},-\frac{5}{6}\right)$	$d_{\mathbb{C}}{}^2HLu_{\mathbb{C}}[(S12),(S10),(S13)]$ $d_{\mathbb{C}}HL^2Q[(S10),(S6),(S2),(S14)]$ $d_{\mathbb{C}}e_{\mathbb{C}}^\dagger HLu_{\mathbb{C}}^\dagger[(S10),(V3),(V8)]$ $d_{\mathbb{C}}HLQ^{\dagger 2}[(S10),(S14),(V9),(V5)]$
$F11\ \left(\mathbf{3},\mathbf{2},\frac{1}{6}\right)$	$d_{\mathbb{C}}{}^3H^\dagger L[(S11),(S12)]$ $d_{\mathbb{C}}{}^2HLu_{\mathbb{C}}[(S11),(S12)]$
$F12\ \left(\mathbf{3},\mathbf{2},\frac{7}{6}\right)$	$HL^2Q^\dagger u_{\mathbb{C}}^\dagger[(S6),(S2),(V9),(V5)]$ $d_{\mathbb{C}}e_{\mathbb{C}}^\dagger HLu_{\mathbb{C}}^\dagger[(V5),(S12),(V3)]$
$F13\ \left(\mathbf{3},\mathbf{3},-\frac{1}{3}\right)$	$HL^2Q^\dagger u_{\mathbb{C}}^\dagger[(S6),(V8)]$ $d_{\mathbb{C}}HLQ^{\dagger 2}[(V8),(S12),(V9)]$
$F14\ \left(\mathbf{3},\mathbf{3},\frac{2}{3}\right)$	$d_{\mathbb{C}}HL^2Q[(S12),(S6)]$

Complete Dim-8 Tree UVs

[Li, Ni, Xiao, **Yu**, 2309.15933]
[Yang, Ren, **Yu**, 2312.04663]

Type: $D^4H^2H^{\dagger 2}$

$$\mathcal{O}_1^f = \frac{1}{4}\mathcal{Y}[\square\square_H]\mathcal{Y}[\square\square_{H^\dagger}]H_iH_j(D_\mu D_\nu H^{\dagger i})(D^\mu D^\nu H^{\dagger j}),$$
$$\mathcal{O}_2^f = \frac{1}{4}\mathcal{Y}[\square\square_H]\mathcal{Y}[\square\square_{H^\dagger}]H^{\dagger i}H_i(D_\mu D_\nu H_j)(D^\mu D^\nu H^{\dagger j}),$$
$$\mathcal{O}_3^f = \frac{1}{4}\mathcal{Y}[\square\square_H]\mathcal{Y}[\square\square_{H^\dagger}]H_i(D_\mu H_j)(D_\nu H^{\dagger i})(D^\mu D^\nu H^{\dagger j}).$$

group: (Spin, $SU(3)_c, SU(2)_w, U(1)_y$)		
	$\{H_1, H_2\}, \{H^\dagger{}_3, H^\dagger{}_4\}$	
*	$(2,1,3,1)$	$-8\mathcal{O}_1^f - 48\mathcal{O}_2^f - 48\mathcal{O}_3^f$
	$(0,1,3,1)$	$8\mathcal{O}_1^f$
	$(1,1,1,1)$	$8\mathcal{O}_1^f + 16\mathcal{O}_3^f$
	$\{H_1, H^\dagger{}_3\}, \{H_2, H^\dagger{}_4\}$	
*	$(2,1,3,0)$	$16\mathcal{O}_1^f - 4\mathcal{O}_2^f + 56\mathcal{O}_3^f$
	$(1,1,3,0)$	$8\mathcal{O}_1^f - 4\mathcal{O}_2^f + 8\mathcal{O}_3^f$
	$(0,1,3,0)$	$8\mathcal{O}_1^f + 4\mathcal{O}_2^f + 16\mathcal{O}_3^f$
*	$(2,1,1,0)$	$-24\mathcal{O}_1^f - 4\mathcal{O}_2^f - 24\mathcal{O}_3^f$
	$(1,1,1,0)$	$-4\mathcal{O}_2^f - 8\mathcal{O}_3^f$
	$(0,1,1,0)$	$4\mathcal{O}_2^f$

In the forward limit, a twice-subtracted dispersion relation

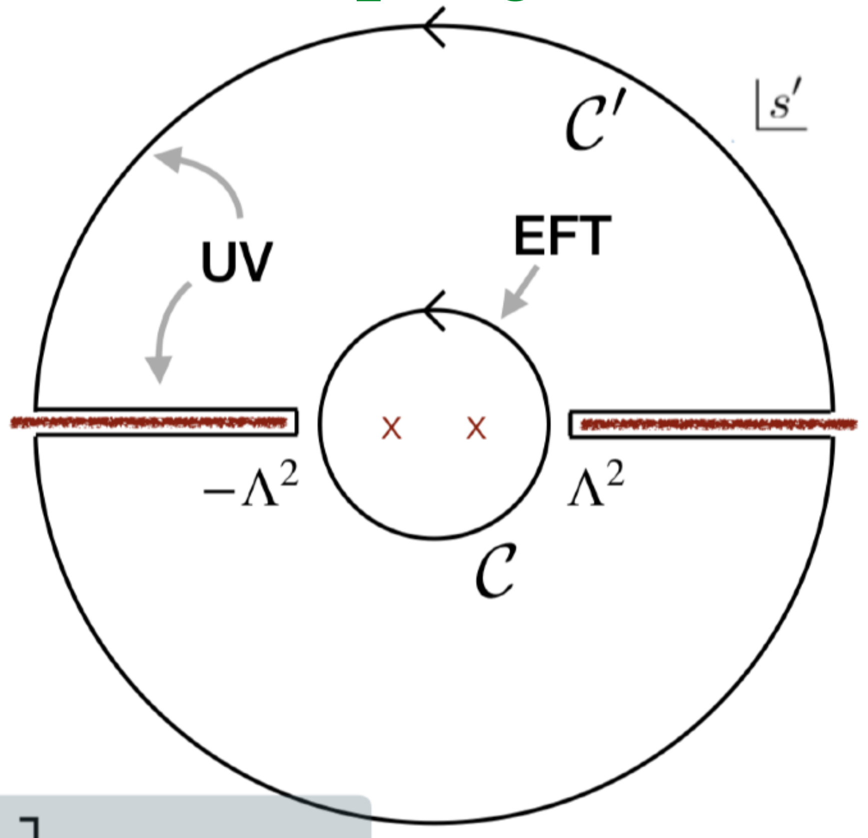
$$\mathcal{M}^{ijkl} = \frac{1}{2\pi}\int_{(\epsilon\Lambda)^2}^{\infty}\frac{ds}{s^3}\sum_X [\mathbf{M}_{ij\to X}\mathbf{M}^*_{kl\to X} + (j \leftrightarrow l)]$$

Particle	Spin	Charge/irrep	Interaction	ER	$\vec{c}$	$\vec{c}^{(6)}$
$\mathcal{B}_1$	1	1_1	$g\mathcal{B}_1^{\mu\dagger}(H^T\epsilon \overleftrightarrow{D}_\mu H) + h.c.$	✓	$8(1,0,-1)$	$2(-1,2)$
Ξ_1	0	3_1	$gM\Xi_1^{I\dagger}(H^T\epsilon\tau^I H) + h.c.$	✗	$8(0,1,0)$	$2(1,2)$
$\mathcal{S}$	0	$1_0(S)$	$gM\mathcal{S}(H^\dagger H)$	✓	$2(0,0,1)$	$-\frac{1}{2}(1,0)$
$\mathcal{B}$	1	$1_0(A)$	$g\mathcal{B}^\mu(H^\dagger \overleftrightarrow{D}_\mu H)$	✓	$2(-1,1,0)$	$-\frac{1}{2}(1,4)$
Ξ_0	0	$3_0(S)$	$gM\Xi_0^I(H^\dagger\tau^I H)$	✗	$2(2,0,-1)$	$\frac{1}{2}(1,-4)$
$\mathcal{W}$	1	$3_0(A)$	$g\mathcal{W}^{\mu I}(H^\dagger\tau^I \overleftrightarrow{D}_\mu H)$	✗	$2(1,1,-2)$	$-\frac{3}{2}(1,0)$

Analyticity in complex s plane (fixed t)

$$A(s,t) = \frac{1}{2\pi i}\oint_{\mathcal{C}} ds' \frac{A(s',t)}{s'-s}$$

Cauchy's integral formula



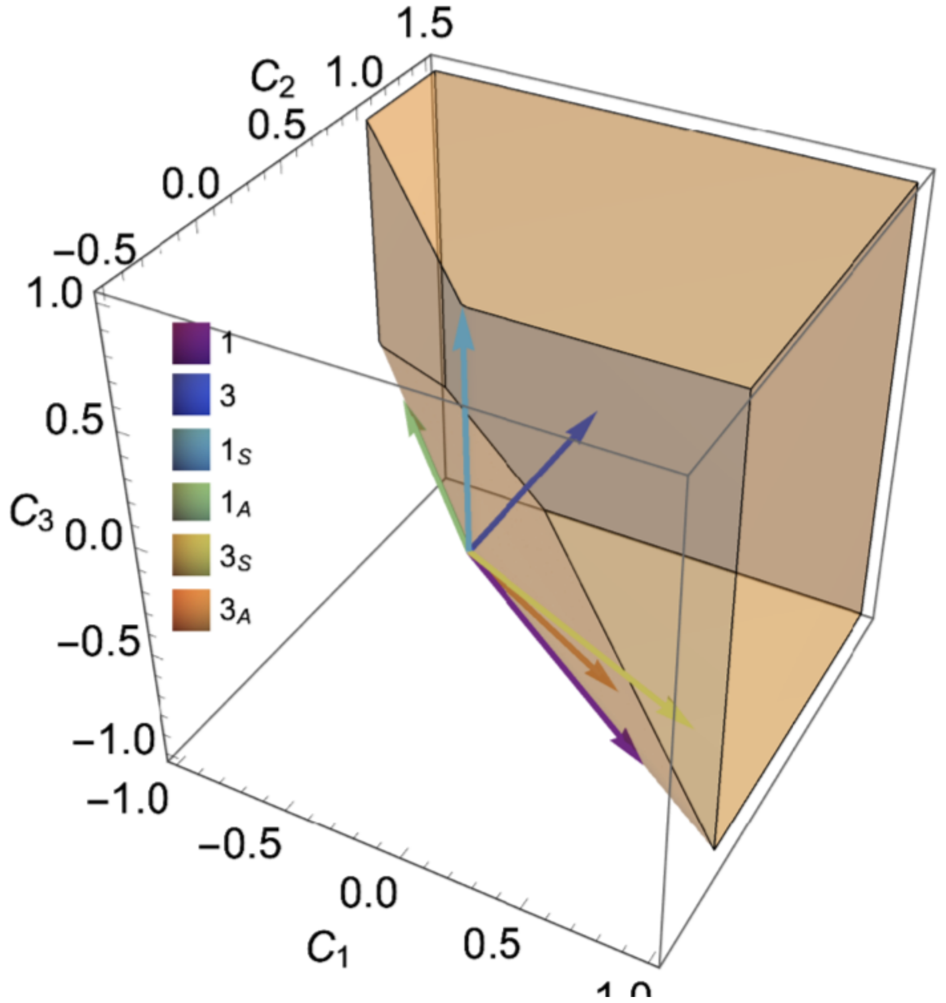
Fixed t dispersion relation

$$A(s,t) \sim \int_{\Lambda^2}^{\infty}\frac{d\mu}{\pi\mu^2}\left[\frac{s^2}{\mu-s} + \frac{u^2}{\mu-u}\right]\mathrm{Im}\,A(\mu,t)$$

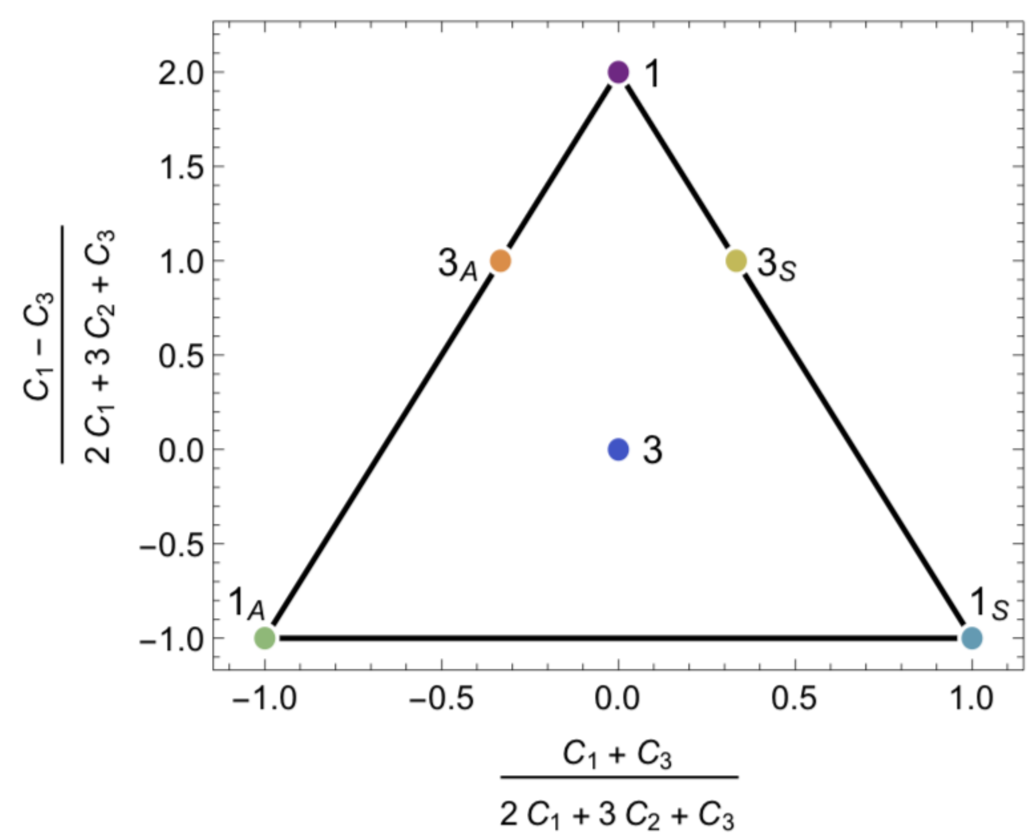
$\mu > \Lambda^2$

EFT amplitude — **IR ~ UV connection** — **UV full amplitude**

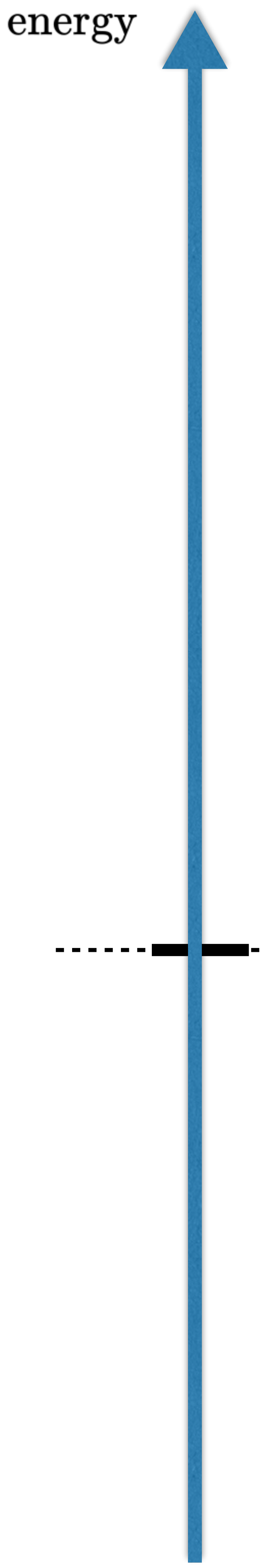
$$\mathrm{Disc}A_{ij\to kl}(s) = A_{ij\to kl}(s) - A_{kl\to ij}(s)^* = i\sum_X M_{ij\to X}(s)M_{kl\to X}(s)^*$$



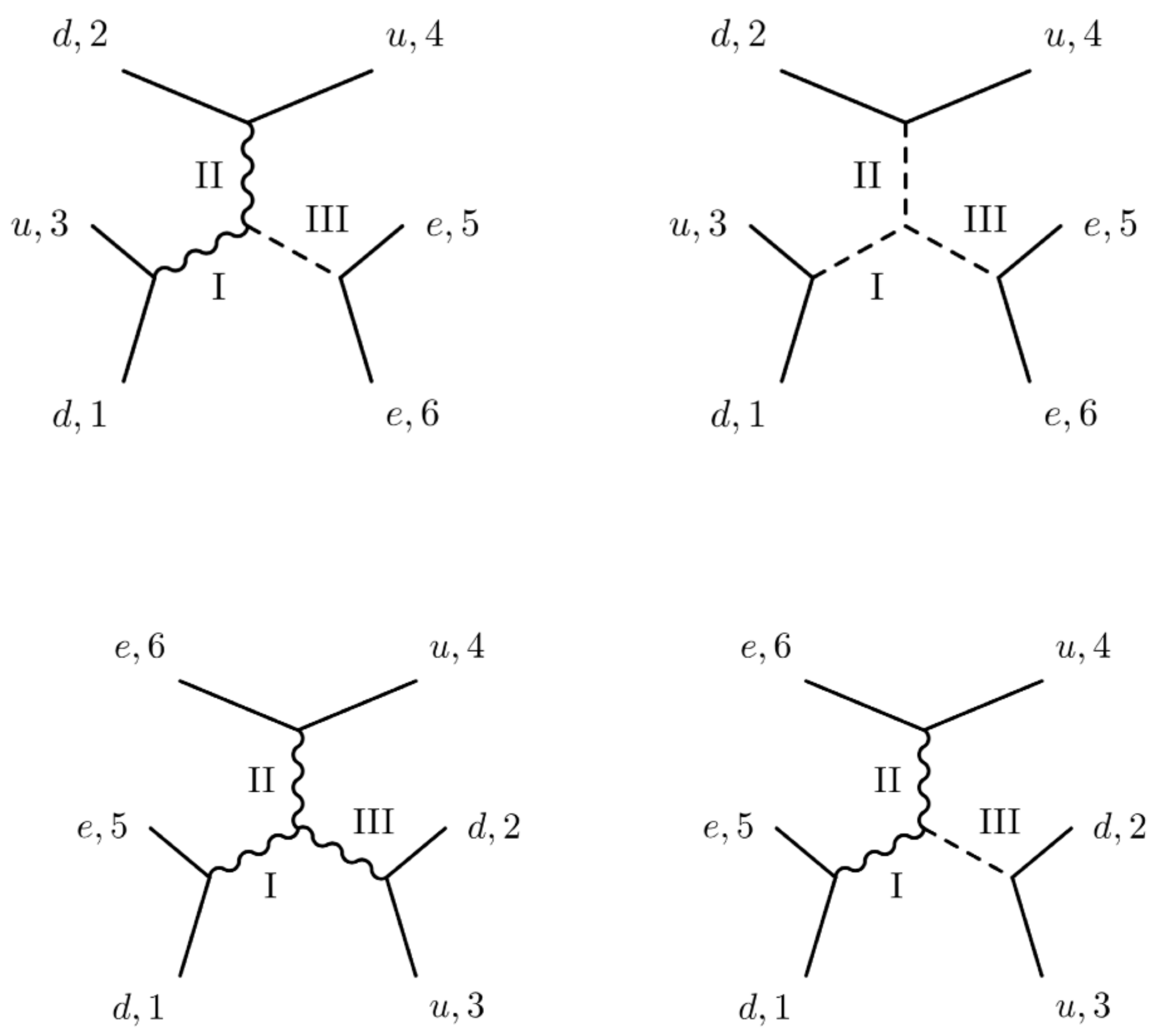
[Cen Zhang, S-Y Zhou]



Complete Dim-9 UV for 0vbb



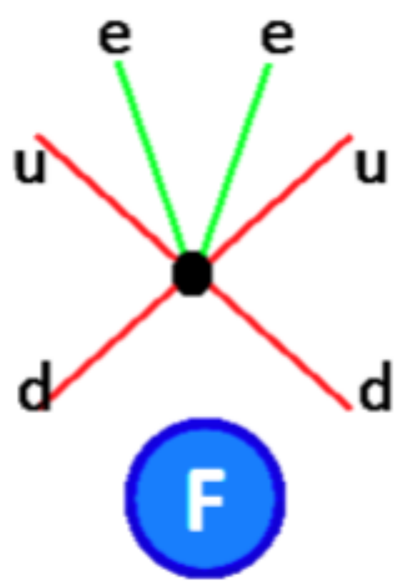
[Li, Ni, Xiao, **Yu**, in preparation]



$(\mathbf{r}_i, J_i)$	$(1,1,0)$	$(0,0,0)$
$(\mathbf{8}_2, \mathbf{8}_2, \mathbf{1}_3)$	$\frac{4}{3}\mathcal{O}_1 + 2\mathcal{O}_2$	$-\frac{2}{3}\mathcal{O}_2$
$(\mathbf{8}_2, \mathbf{8}_2, \mathbf{1}_1)$	$-2\mathcal{O}_2$	$\frac{4}{3}\mathcal{O}_1 + \frac{2}{3}\mathcal{O}_2$
$(\mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_3)$	$-4\mathcal{O}_1 - 6\mathcal{O}_2$	$2\mathcal{O}_2$
$(\mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_1)$	$6\mathcal{O}_2$	$-4\mathcal{O}_1 - 2\mathcal{O}_2$

$(\mathbf{r}_i, J_i)$	$(1,1,1)$	$(1,1,0)$
$(\mathbf{3}_3, \mathbf{8}_2, \bar{\mathbf{3}}_2)$	$-\frac{8}{3}\mathcal{O}_1$	$\frac{4}{3}\mathcal{O}_1 + \frac{8}{3}\mathcal{O}_2$
$(\mathbf{3}_1, \mathbf{8}_2, \bar{\mathbf{3}}_2)$	$-\frac{8}{3}\mathcal{O}_1$	0
$(\mathbf{3}_3, \mathbf{1}_2, \bar{\mathbf{3}}_2)$	$-\frac{8}{3}\mathcal{O}_1$	$\frac{4}{3}\mathcal{O}_1 + \frac{8}{3}\mathcal{O}_2$
$(\mathbf{3}_1, \mathbf{1}_2, \bar{\mathbf{3}}_2)$	$-\frac{8}{3}\mathcal{O}_1$	0

[Li, **Yu**, Zhao, 2311.15422]

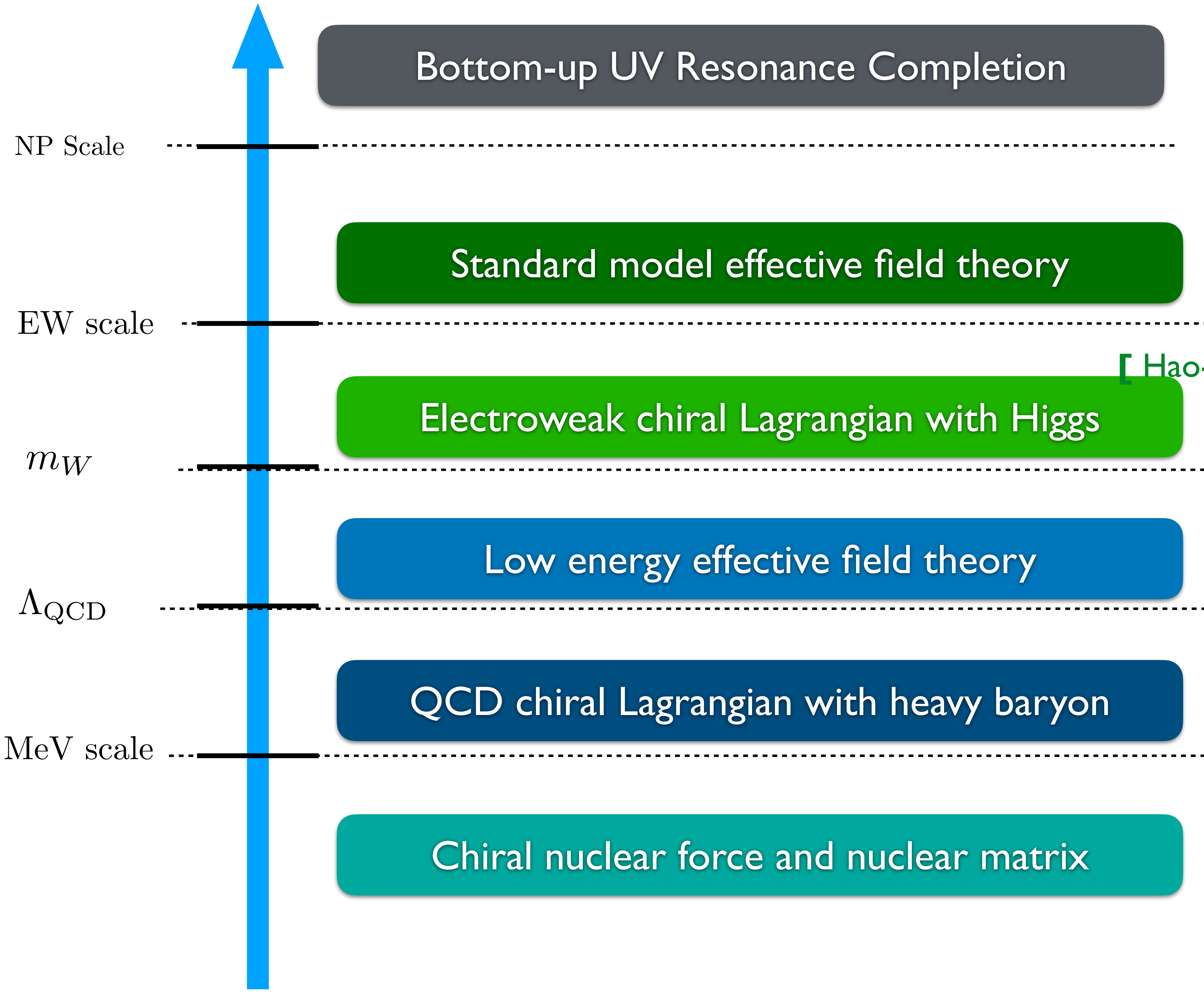


$$\mathcal{O}_1 = \frac{1}{4}(L^{\dagger}{}_u{}^j L^{\dagger}{}_v{}^i)(Q_{pai}Q_{rbj})(u_{cs}{}^b u_{ct}{}^a)$$

$$\mathcal{O}_2 = -\frac{1}{4}(L^{\dagger}{}_u{}^j L^{\dagger}{}_v{}^i)(Q_{pai}u_{cs}{}^b)(Q_{rbj}uc_t{}^a).$$

Summary

- EFT provides most general parametrization of new physics at different scales



RGE

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Thanks for your attention!