



Wess-Zumino-Witten Interactions of Axions

Jia Liu (刘佳)
Peking University

ArXiv: 2406.11948

Yang Bai, Ting-Kuo Chen, Jia Liu, Xiaolin Ma

International Workshop on New Opportunities for Particle Physics 2024

2024-07-20 @ IHEP

Outlines

- Axion effective and consistent ChPT Lagrangian
- WZW in QCD and counter-terms
- Full WZW interactions for axions
- Phenomenology at BESIII and STCF
- Summary

The QCD axion and the Strong CP problem

$$\mathcal{L} \supset -\frac{\theta g_s^2}{32\pi^2} G\tilde{G} - (\bar{u}_L M_u u_R + \bar{d}_L M_d d_R + \text{h.c.})$$

- The CKM matrix from $M_{u,d}$
 - CP violating phase $\theta_{\text{CP}} \sim 1.2$ radian
- QCD induced CP violating phase, $\bar{\theta}$

$$\bar{\theta} = \theta + \arg [\det [M_u M_d]]$$

- $\bar{\theta}$ is invariant under quark chiral rotation
- According to neutron EDM experiment

$$d_{\text{EDM}}^n \sim \theta \times 10^{-16} \text{ e cm}$$

$$d_{\text{exp}}^n < 10^{-26} \text{ e cm}$$


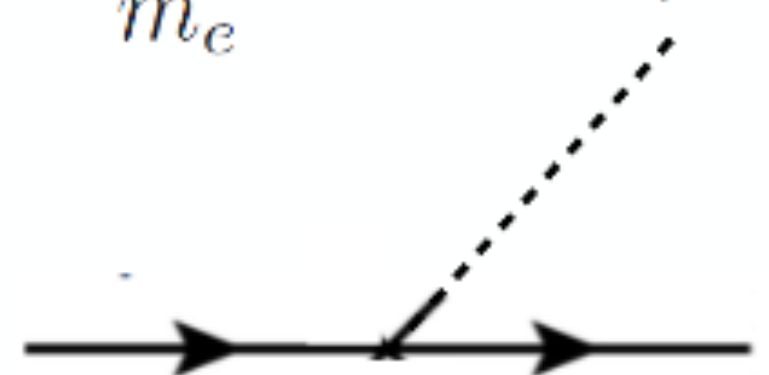
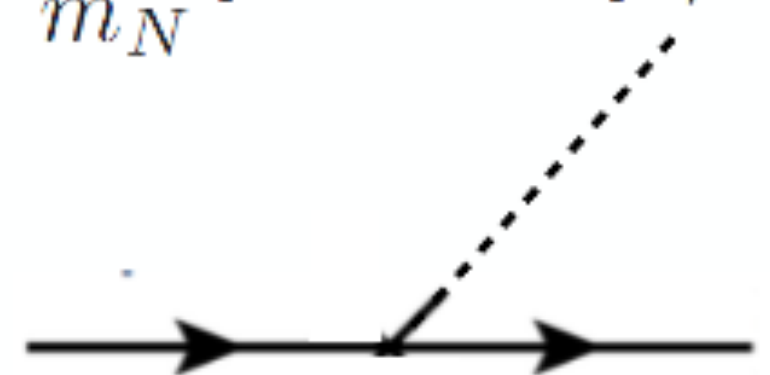
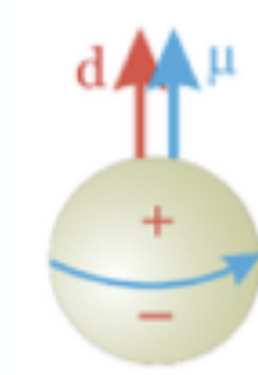
$$\bar{\theta} \lesssim 1.3 \times 10^{-10} \text{ radian}$$

The axion effective Lagrangian at quark-level

- Axion can couple to SM gauge bosons and fermions

$$\mathcal{L}_{\text{ALP}} = g_{ag} \frac{a}{f_a} G\tilde{G} + g_{a\gamma} \frac{a}{f_a} F\tilde{F} + g_{af} \frac{\partial_\mu a}{2f_a} \bar{f}\gamma^\mu\gamma_5 f$$

- Detection of axion through various couplings

photon coupling	electron coupling	nucleon coupling	CP Neutron electric dipole
$-\frac{g_{a\gamma}}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} a$ 	$\frac{g_{ae}}{m_e} [\bar{e}\gamma^\mu\gamma^5 e] \partial_\mu a$ 	$\frac{g_{aN}}{m_N} [\bar{N}\gamma^\mu\gamma^5 N] \partial_\mu a$ 	$\propto \frac{1}{m_n} [F_{\mu\nu} \bar{n}\sigma^{\mu\nu}\gamma_5 n] \frac{A}{f_A}$ <p style="font-size: 2em; font-weight: bold; margin: 0;">X</p> 

The axion effective Lagrangian at quark-level

- A more detailed effective Lagrangian

$$\mathcal{L}_{\text{eff},0} = \bar{q}_0(iD_\mu\gamma^\mu - \mathbf{m}_{q,0})q_0 + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 \\ + g_{ag,0}\frac{a}{f_a}G\tilde{G} + g_{a\gamma,0}\frac{a}{f_a}F\tilde{F} + \frac{\partial_\mu a}{f_a}(\bar{q}_L\mathbf{k}_{L,0}\gamma^\mu q_L + \bar{q}_R\mathbf{k}_{R,0}\gamma^\mu q_R + \dots)$$

Bauer et al, PRL 127 (2021), 081803

- Quark mass $\mathbf{m}_{q,0}$ diagonal and real
- Coupling to both left/right fermions $\mathbf{k}_{L,0}$ and $\mathbf{k}_{R,0}$

The axion-dependent chiral rotation

- Use an axion-dependent chiral rotation to eliminate $aG\tilde{G}$ term

$$q_0(x) = \exp \left[-i(\delta_{q,0} + \kappa_{q,0}\gamma_5)c_{gg} \frac{a(x)}{f_a} \right] q(x) \quad \text{Bauer et al, PRL 127 (2021), 081803}$$

$$\text{Tr}(\kappa_{q,0}) = 1$$

- New effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \bar{q}(iD_\mu\gamma^\mu - \mathbf{m}_q(a))q + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2$$

$$+ g_{a\gamma} \frac{a}{f_a} F\tilde{F} + \frac{\partial_\mu a}{f_a} (\bar{q}_L \mathbf{k}_L(a)\gamma^\mu q_L + \bar{q}_R \mathbf{k}_R(a)\gamma^\mu q_R + \dots)$$

The axion-dependent chiral rotation

- Define the chiral rotations (2-flavor for simplicity)

$$\begin{aligned}\boldsymbol{\theta}_L &\equiv \boldsymbol{\delta}_{q,0} - \boldsymbol{\kappa}_{q,0} & U_L &\equiv \exp \left[-i\boldsymbol{\theta}_L a / f_a \right] \\ \boldsymbol{\theta}_R &\equiv \boldsymbol{\delta}_{q,0} + \boldsymbol{\kappa}_{q,0} & U_R &\equiv \exp \left[-i\boldsymbol{\theta}_R a / f_a \right]\end{aligned}$$

- The relations between parameters

$$\mathbf{m}_q(a) = U_L^\dagger \mathbf{m}_0 U_R \rightarrow \begin{pmatrix} m_{u,0} e^{-2i\kappa_{u,0} c_{gg}} & 0 \\ 0 & m_{d,0} e^{-2i\kappa_{d,0} c_{gg}} \end{pmatrix}$$

$$\mathbf{k}_L(a) = U_L^\dagger [\mathbf{k}_{L,0} + c_{gg} \boldsymbol{\theta}_{L,0}] U_L \rightarrow \mathbf{k}_{L,0} + c_{gg} \boldsymbol{\theta}_{L,0}$$

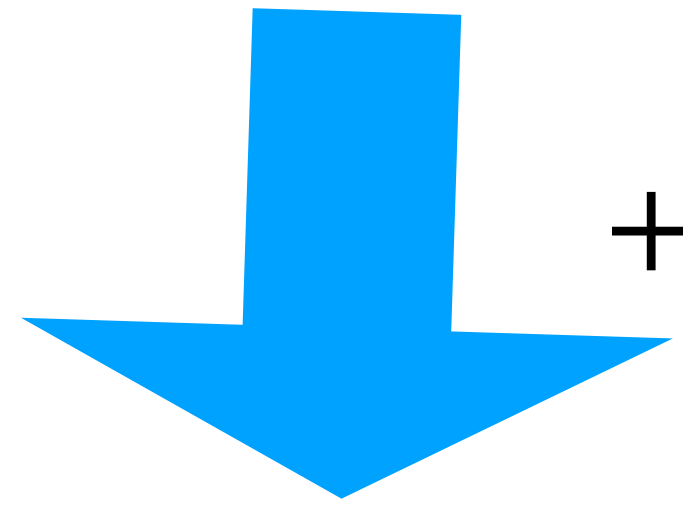
$$\mathbf{k}_R(a) = U_R^\dagger [\mathbf{k}_{R,0} + c_{gg} \boldsymbol{\theta}_{R,0}] U_R \rightarrow \mathbf{k}_{R,0} + c_{gg} \boldsymbol{\theta}_{R,0}$$

Anomalous axion contribution

$$g_{a\gamma} = g_{a\gamma_0} - 2N_c c_{gg} \text{Tr} \left[\mathbf{Q}^2 \boldsymbol{\kappa}_{q,0} \right]$$

The consistent ChPT axion Lagrangian

$$\mathcal{L}_{\text{eff}} = \bar{q}(iD_\mu\gamma^\mu - \mathbf{m}_q(a))q + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2$$



$$+ g_{a\gamma} \frac{a}{f_a} F\tilde{F} + \frac{\partial_\mu a}{f_a} (\bar{q}_L \mathbf{k}_L(a) \gamma^\mu q_L + \bar{q}_R \mathbf{k}_R(a) \gamma^\mu q_R + \dots)$$

- ChPT Lagrangian matching

$$U = \exp[(\sqrt{2}i/f_\pi)\pi^a \tau^a]$$

$$\mathcal{L}_{\chi\text{PT}} = \frac{f_\pi^2}{8} [(D^\mu U)(D_\mu U)^\dagger] + \frac{f_\pi^2}{4} B_0 \text{Tr} [\mathbf{m}_q(a) U^\dagger + h.c.] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 + g_{a\gamma} \frac{a}{f_a} F\tilde{F}$$

- The axion derivative coupling

Bauer et al, PRL 127 (2021), 081803

$$D^\mu U \rightarrow D^\mu U - i \frac{\partial^\mu a}{f_a} (\mathbf{k}_L U - U \mathbf{k}_R)$$

The importance of consistency

- The physical results should be independent of auxiliary parameters

$$q_0(x) = \exp \left[-i(\delta_{q,0} + \kappa_{q,0}\gamma_5)c_{gg} \frac{a(x)}{f_a} \right] q(x)$$

- The most important channel $\text{BR}(K \rightarrow \pi a)$ is off by a factor of 37

H. Georgi, D. B. Kaplan and L. Randall, Phys. Lett. B 169, 73-78 (1986)

- Model-independent expression for $K \rightarrow \pi a$ and $\pi^- \rightarrow e^- \bar{\nu}_e a$ have been obtained for all axion couplings

Bauer et al, PRL 127 (2021), 081803

Outlines

- Axion effective and consistent ChPT Lagrangian
- **WZW in QCD and counter-terms**
- Full WZW interactions for axions
- Phenomenology at BESIII and STCF
- Summary

Wess-Zumino-Witten Interactions in QCD

- Describing anomalies in QCD
- Ensuring gauge invariance and completing chiral L
- Low-energy dynamics of mesons
e.g. multiple mesons and photons interactions, $\pi_0 \rightarrow \gamma\gamma$

$$\Gamma_{WZW}(U, \mathcal{A}_L, \mathcal{A}_R) = \Gamma_0(U) + \mathcal{C} \int \text{Tr} \left\{ (\mathcal{A}_L \alpha^3 + \mathcal{A}_R \beta^3) - \frac{i}{2} [(\mathcal{A}_L \alpha)^2 - (\mathcal{A}_R \beta)^2] \right. \\ \left. + i(\mathcal{A}_L U \mathcal{A}_R U^\dagger \alpha^2 - \mathcal{A}_R U^\dagger \mathcal{A}_L U \beta^2) + i(d\mathcal{A}_R dU^\dagger \mathcal{A}_L U - d\mathcal{A}_L dU \mathcal{A}_R U^\dagger) \right. \\ \left. + i[(d\mathcal{A}_L \mathcal{A}_L + \mathcal{A}_L d\mathcal{A}_L) \alpha + (d\mathcal{A}_R \mathcal{A}_R + \mathcal{A}_R d\mathcal{A}_R) \beta] + (\mathcal{A}_L^3 \alpha + \mathcal{A}_R^3 \beta) \right. \\ \left. - (d\mathcal{A}_L \mathcal{A}_L + \mathcal{A}_L d\mathcal{A}_L) U \mathcal{A}_R U^\dagger + (d\mathcal{A}_R \mathcal{A}_R + \mathcal{A}_R d\mathcal{A}_R) U^\dagger \mathcal{A}_L U \right. \\ \left. + (\mathcal{A}_L U \mathcal{A}_R U^\dagger \mathcal{A}_L \alpha + \mathcal{A}_R U^\dagger \mathcal{A}_L U \mathcal{A}_R \beta) + i \left[\mathcal{A}_L^3 U \mathcal{A}_R U^\dagger - \mathcal{A}_R^3 U^\dagger \mathcal{A}_L U - \frac{1}{2} (U \mathcal{A}_R U^\dagger \mathcal{A}_L)^2 \right] \right\}.$$

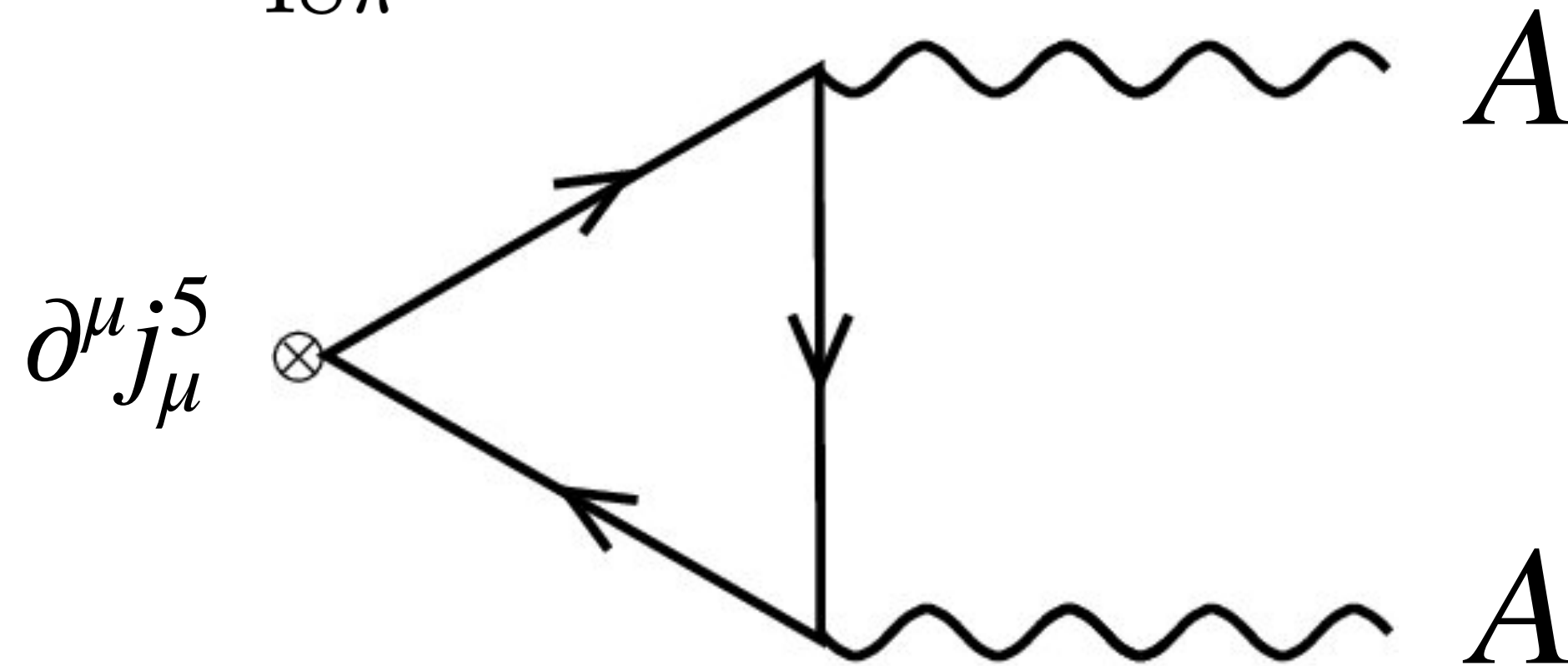
$$\alpha = dU U^\dagger \\ \beta = U^\dagger dU$$

$$\Gamma_0(U) = -\frac{i\mathcal{C}}{5} \int_{M^5} \text{Tr} (\alpha^5) = \frac{iN_c}{240\pi^2} \int d^5x \epsilon^{ABCDE} \text{Tr} (\alpha_A \alpha_B \alpha_C \alpha_D \alpha_E),$$

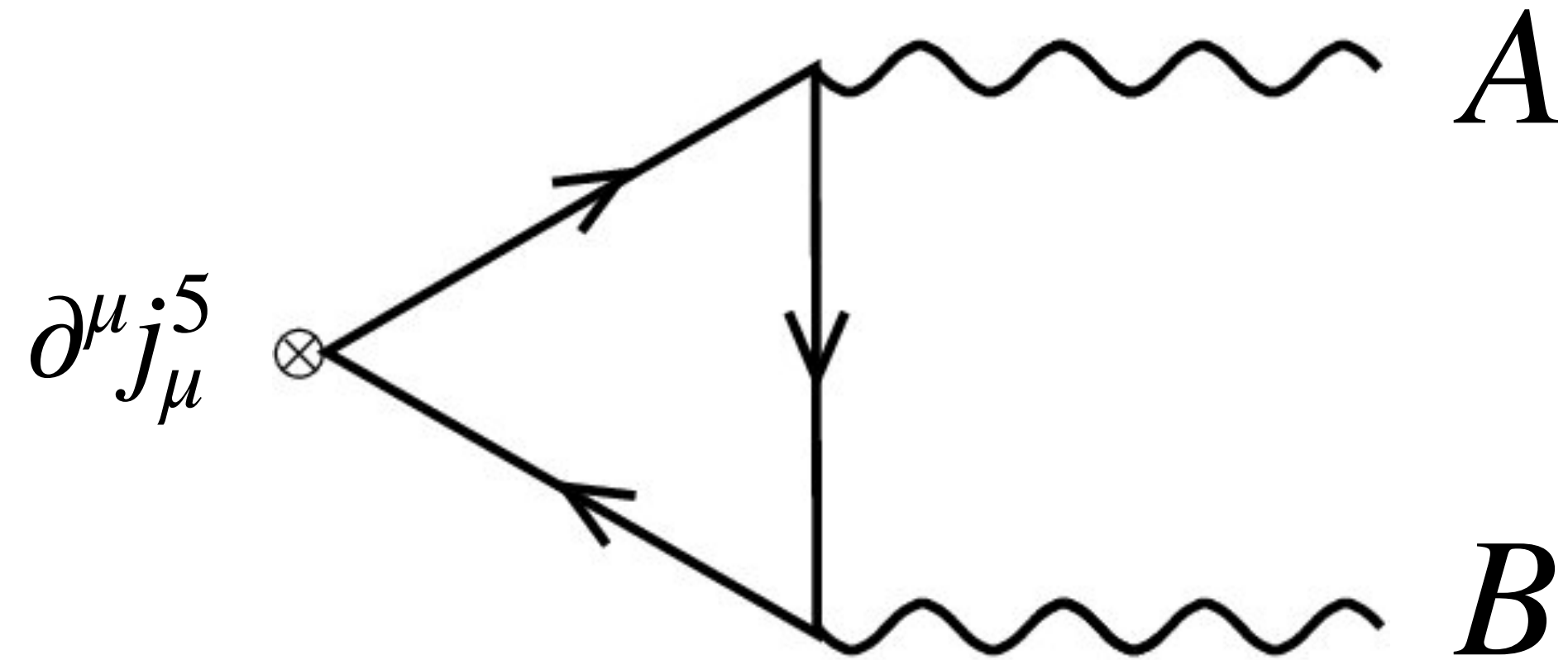
Global currents and background vector fields

- Background fields can couple to currents of $\mathcal{L}_{\chi\text{PT}}$
 - Baryon currents $U(1)_B$ in neutron star, ω meson
 - Z boson vector in neutrino dense environment
- SM gauge invariance needs counter terms

$$\partial^\mu j_\mu^5 = \frac{1}{48\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$$



$$\delta \partial^\mu j_\mu^5 \propto \epsilon_{\mu\nu\rho\sigma} A^{\mu\nu} B^{\rho\sigma}$$



WZW counter terms for global symmetry

J. A. Harvey, C. T. Hill, and R. J. Hill,
PRL 99 (2007) 261601,
PRD 77(2008) 085017

- Generic WZW interactions with counter terms
- Vector fields in 1-form: $\mathcal{A}_{L/R} \equiv \mathbb{A}_{L/R} + \mathbb{B}_{L/R}$
Similar to Hidden Local Symmetry

$$\mathcal{L}_{\text{WZW}}^{\text{full}}(U, \mathcal{A}_{L/R}) = \mathcal{L}_{\text{WZW}}(U, \mathcal{A}_L, \mathcal{A}_R) + \mathcal{L}_c(\mathbb{A}_{L/R}, \mathbb{B}_{L/R})$$

- Counter terms ensures SM invariance

$$\Gamma_c = -2\mathcal{C} \int \text{Tr} \left[(\mathbb{A}_L d\mathbb{A}_L + d\mathbb{A}_L \mathbb{A}_L) \mathbb{B}_L + \frac{1}{2} \mathbb{A}_L (\mathbb{B}_L d\mathbb{B}_L + d\mathbb{B}_L \mathbb{B}_L) - \frac{3}{2} i \mathbb{A}_L^3 \mathbb{B}_L - \frac{3}{4} i \mathbb{A}_L \mathbb{B}_L \mathbb{A}_L \mathbb{B}_L - \frac{i}{2} \mathbb{A}_L \mathbb{B}_L^3 \right] - (L \leftrightarrow R)$$

- Suitable for chiral gauge fields and background fields

Outlines

- Axion effective and consistent ChPT Lagrangian
- WZW in QCD and counter-terms
- **Full WZW interactions for axions**
- Phenomenology at BESIII and STCF
- Summary

Axion treatment as a fictitious background field

$$\mathcal{L}_{\text{eff}} = \bar{q}(iD_{\mu}\gamma^{\mu} - \mathbf{m}_q(a))q + \frac{1}{2}(\partial_{\mu}a)(\partial^{\mu}a) - \frac{m_{a,0}^2}{2}a^2$$

Yang Bai, Ting-Kuo Chen, JL, Xiaolin Ma
2406.11948

$$+ g_{a\gamma} \frac{a}{f_a} F\tilde{F} + \frac{\partial_{\mu}a}{f_a} (\bar{q}_L \mathbf{k}_L(a) \gamma^{\mu} q_L + \bar{q}_R \mathbf{k}_R(a) \gamma^{\mu} q_R + \dots)$$

- $D_{\mu} = \partial_{\mu} - ig(A_L P_L + A_R P_R)$

- Hints from quark-level L: $D_{\mu} \rightarrow D_{\mu} + i \frac{\partial_{\mu}a}{f_a} (\mathbf{k}_L P_L + \mathbf{k}_R P_R)$

- Hints from ChPT L: $D^{\mu}U \rightarrow D^{\mu}U - i \frac{\partial^{\mu}a}{f_a} (\mathbf{k}_L U - U \mathbf{k}_R)$

$$\mathcal{L}_{\chi\text{PT}} = \frac{f_{\pi}^2}{8} \left[(D^{\mu}U)(D_{\mu}U)^{\dagger} \right] + \frac{f_{\pi}^2}{4} B_0 \text{Tr} \left[\mathbf{m}_q(a) U^{\dagger} + h.c. \right] + \frac{1}{2}(\partial_{\mu}a)(\partial^{\mu}a) - \frac{m_{a,0}^2}{2}a^2 + g_{a\gamma} \frac{a}{f_a} F\tilde{F}$$

Axion treatment as a fictitious background field

- Vector fields in 1-form: $\mathcal{A}_{L/R} \equiv \mathbb{A}_{L/R} + \mathbb{B}_{L/R}$
Similar to Hidden Local Symmetry

- Axion 1-form field can be added into background fields:

$$\mathbb{B}_{L/R} \rightarrow \mathbb{B}_{L/R} + \mathbf{k}_{L/R,0} \frac{da}{f_a}$$

- 2-flavor ChPT with SM gauge bosons and background fields

$$\mathbb{A}_L = \frac{e}{s_w} W^a \frac{\boldsymbol{\tau}^a}{2} + \frac{e}{c_w} W^0 \mathbf{Y}_Q, \quad \mathbb{A}_R = \frac{e}{c_w} W^0 \mathbf{Y}_q$$

$$\mathbb{B}_V \equiv \mathbb{B}_L + \mathbb{B}_R = g \begin{pmatrix} \rho_0 & \sqrt{2}\rho^+ \\ \sqrt{2}\rho^- & -\rho_0 \end{pmatrix} + g' \begin{pmatrix} \omega & \\ & \omega \end{pmatrix} + (\mathbf{k}_{L,0} + \mathbf{k}_{R,0}) \frac{da}{f}$$

$$\mathbb{B}_A \equiv \mathbb{B}_L - \mathbb{B}_R = g \begin{pmatrix} a_1 & \sqrt{2}a^+ \\ \sqrt{2}a^- & -a_1 \end{pmatrix} + g' \begin{pmatrix} f_1 & \\ & f_1 \end{pmatrix} + (\mathbf{k}_{L,0} - \mathbf{k}_{R,0}) \frac{da}{f}$$

The consistent axion Lagrangian at low energy

- ChPT:

$$\mathcal{L}_{\chi\text{PT}} = \frac{f_\pi^2}{8} \text{Tr} [(D^\mu U)(D_\mu U)^\dagger] + \frac{f_\pi^2}{4} B_0 \text{Tr} [\mathbf{m}_q(a)U^\dagger + h.c.] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 + \frac{a}{f} \sum_{\mathcal{A}_{1,2}} c_{\mathcal{A}_1\mathcal{A}_2} F_{\mathcal{A}_1\mu\nu} \tilde{F}_{\mathcal{A}_2}^{\mu\nu}$$

- Full WZW: $\mathcal{L}_{\text{WZW}}^{\text{full}}(U, \mathcal{A}_{L/R}) = \mathcal{L}_{\text{WZW}}(U, \mathcal{A}_L, \mathcal{A}_R) + \mathcal{L}_c(\mathbb{A}_{L/R}, \mathbb{B}_{L/R})$

- Full \mathcal{L} : $\mathcal{L}_{\text{axion}}^{\text{full}} \equiv \left[\mathcal{L}_{\chi\text{PT}} + \mathcal{L}_{\text{WZW}}^{\text{full}} \right] \left(U, \mathbf{m}_q(a), \mathcal{A}_{L/R} + \mathbf{k}_{L/R}(a)da/f_a \right)$

Matching between \mathcal{L}_{eff} and $\mathcal{L}_{\text{axion}}^{\text{full}}$

$$\mathcal{L}_{\text{eff},0}(q_0, \mathbf{m}_{q,0}, \mathbf{k}_{L,0}, \mathbf{k}_{R,0})$$

$$\downarrow q_0 = \exp\left(-i c_{gg} \boldsymbol{\kappa}_{q,0} \gamma_5 \frac{a}{f}\right) q$$

$$\mathcal{L}_{\text{eff}}(q, \mathbf{m}_q, \mathbf{k}_L, \mathbf{k}_R) \xrightarrow{\hspace{10em}} \mathcal{L}_{\text{eff}}(q', \mathbf{m}'_q, \mathbf{k}'_L, \mathbf{k}'_R) + \delta\mathcal{L}_a^{\text{ano}}$$

$$q' = \exp\left[i\left(\delta_q + \boldsymbol{\kappa}_q \gamma_5\right) \frac{a}{f}\right] q$$

matching

matching

$$\begin{aligned} \mathcal{L}_{\text{axion}}^{\text{full}} &\equiv \mathcal{L}_{\chi\text{PT}}(U, \mathbf{m}_q, \mathcal{A}_{L/R} + \mathbf{k}_{L/R} da) + \mathcal{L}_{\text{WZW}}^{\text{full}}(U, \mathbf{m}_q, \mathcal{A}_{L/R} + \mathbf{k}_{L/R} da) \\ &\xrightarrow{U' = U_L^\dagger U U_R} \mathcal{L}_{\chi\text{PT}}(U', \mathbf{m}'_q, \mathcal{A}_{L/R} + \mathbf{k}'_{L/R} da) + \mathcal{L}_{\text{WZW}}^{\text{full}}(U', \mathbf{m}'_q, \mathcal{A}_{L/R} + \mathbf{k}'_{L/R} da) + \delta\mathcal{L}_{\text{WZW}}^{\text{ano}} \end{aligned}$$

Effective Lagrangian for axions

- Initial effective Lagrangian:

$$\mathcal{L}_{\text{eff},0} = \mathcal{L}_{\text{SM}} + \bar{q}_0(i\not{D} - \mathbf{m}_{q,0})q_0 + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 + c_{gg} \frac{\alpha_s}{4\pi} \frac{a}{f} G_{\mu\nu} \tilde{G}^{\mu\nu} + \frac{a}{f} \sum_{\mathcal{A}_{1,2}} c_{\mathcal{A}_1 \mathcal{A}_2}^0 F_{\mathcal{A}_1 \mu\nu} \tilde{F}_{\mathcal{A}_2}^{\mu\nu} + \mathcal{L}_c$$

- Eliminating aGG term:

$$q_0(x) = \exp \left[-i(\boldsymbol{\delta}_{q,0} + \boldsymbol{\kappa}_{q,0} \gamma_5) c_{gg} \frac{a(x)}{f} \right] q(x), \quad \text{with } \text{Tr}(\boldsymbol{\kappa}_{q,0}) = 1$$

- New effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \bar{q} i\not{D} q - [\bar{q}_L \mathbf{m}_q(a) q_R + h.c.] + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 + \frac{a}{f} \sum_{\mathcal{A}_{1,2}} c_{\mathcal{A}_1 \mathcal{A}_2} F_{\mathcal{A}_1 \mu\nu} \tilde{F}_{\mathcal{A}_2}^{\mu\nu} + \mathcal{L}_c$$

Auxiliary chiral rotation for effective Lagrangian

- Chiral rotation without regenerating aGG term $\text{Tr}(\boldsymbol{\kappa}_q) = 0$

$$q' = \exp \left[i \left(\boldsymbol{\delta}_q + \boldsymbol{\kappa}_q \gamma_5 \right) a/f \right] q$$

- Left/right rotation matrices

$$\boldsymbol{\theta}_{L/R} \equiv \boldsymbol{\delta}_q \mp \boldsymbol{\kappa}_q \quad U_{L/R} \equiv \exp \left[-i \boldsymbol{\theta}_{L/R} a/f \right]$$

- Mass and coupling shifts

$$\mathbf{m}'_q = U_L^\dagger \mathbf{m}_q U_R, \quad \mathbf{k}'_{L/R} = U_{L/R}^\dagger (\mathbf{k}_{L/R} + \boldsymbol{\theta}_{L/R}) U_{L/R} = \mathbf{k}_{L/R} + \boldsymbol{\theta}_{L/R}$$

- Chiral basis change for effective Lagrangian

$$\mathcal{L}_{\text{eff}}(q, \mathbf{m}_q, \mathbf{k}_L, \mathbf{k}_R) \rightarrow \mathcal{L}_{\text{eff}}(q', \mathbf{m}'_q, \mathbf{k}'_L, \mathbf{k}'_R) + \delta \mathcal{L}_a^{\text{ano}}$$

The axion anomalous interactions

$$\delta \mathcal{L}_a^{\text{ano}} = -\delta [\mathcal{L}_{\text{WZW}} + \mathcal{L}_c](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R)$$

- The exact expressions

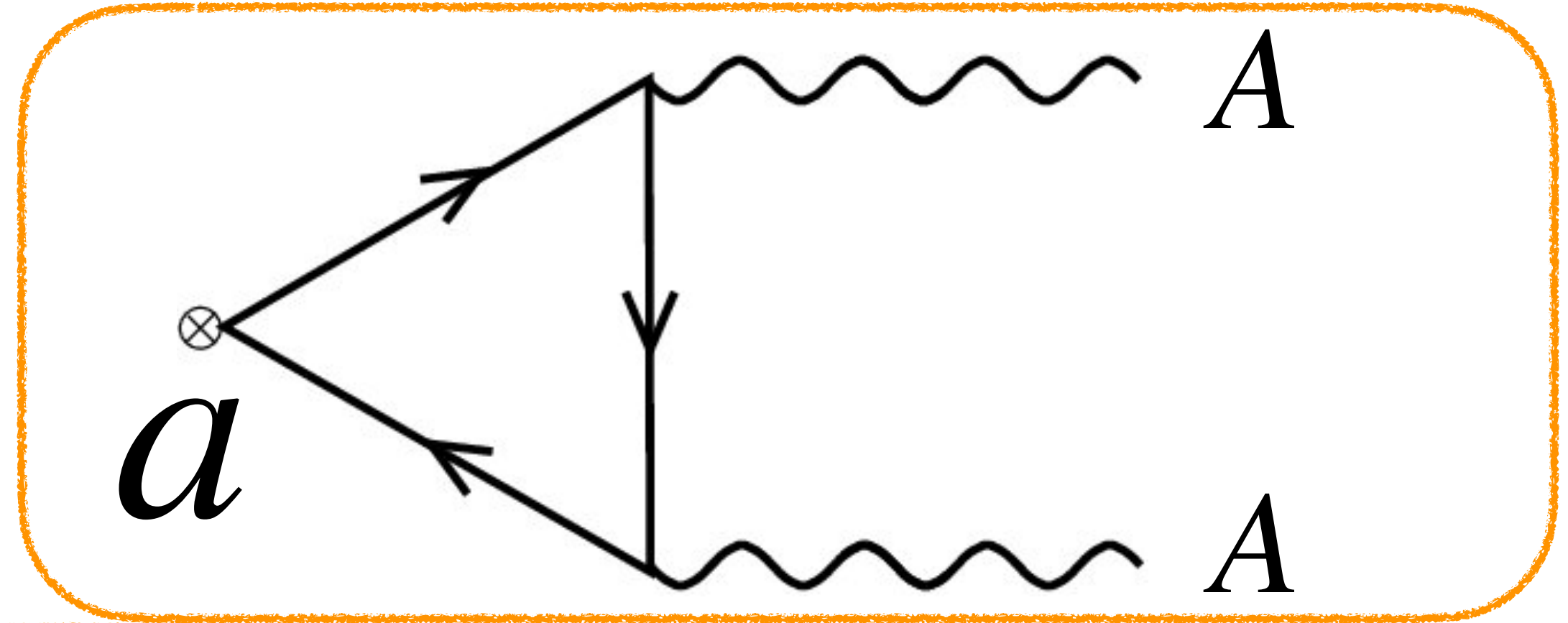
$$\delta [\Gamma_{\text{WZW}} + \Gamma_c](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R) = -2\mathcal{C} \frac{a}{f} \int \text{Tr} \left\{ \boldsymbol{\theta}_L \left[3(d\mathbb{A}_L - i\mathbb{A}_L^2)^2 + 3(d\mathbb{A}_L - i\mathbb{A}_L^2)(D\mathbb{B}_L) + D\mathbb{B}_L D\mathbb{B}_L - \frac{i}{2} D(\mathbb{B}_L^3) \right. \right. \\ \left. \left. + i\mathbb{B}_L(d\mathbb{A}_L - i\mathbb{A}_L^2)\mathbb{B}_L - i(d\mathbb{A}_L - i\mathbb{A}_L^2)\mathbb{B}_L^2 \right] \right\} - (L \leftrightarrow R),$$

- Covariant derivative $D\mathbb{B}_{L,R} = d\mathbb{B}_{L,R} - i\mathbb{A}_{L,R}\mathbb{B}_{L,R} - i\mathbb{B}_{L,R}\mathbb{A}_{L,R}$
- Covariant field strength: $F = d\mathbb{A}_L - i\mathbb{A}_L^2$

The axion anomalous interactions

$$\delta \mathcal{L}_a^{\text{ano}} = -\delta [\mathcal{L}_{\text{WZW}} + \mathcal{L}_c](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R)$$

- The exact expressions



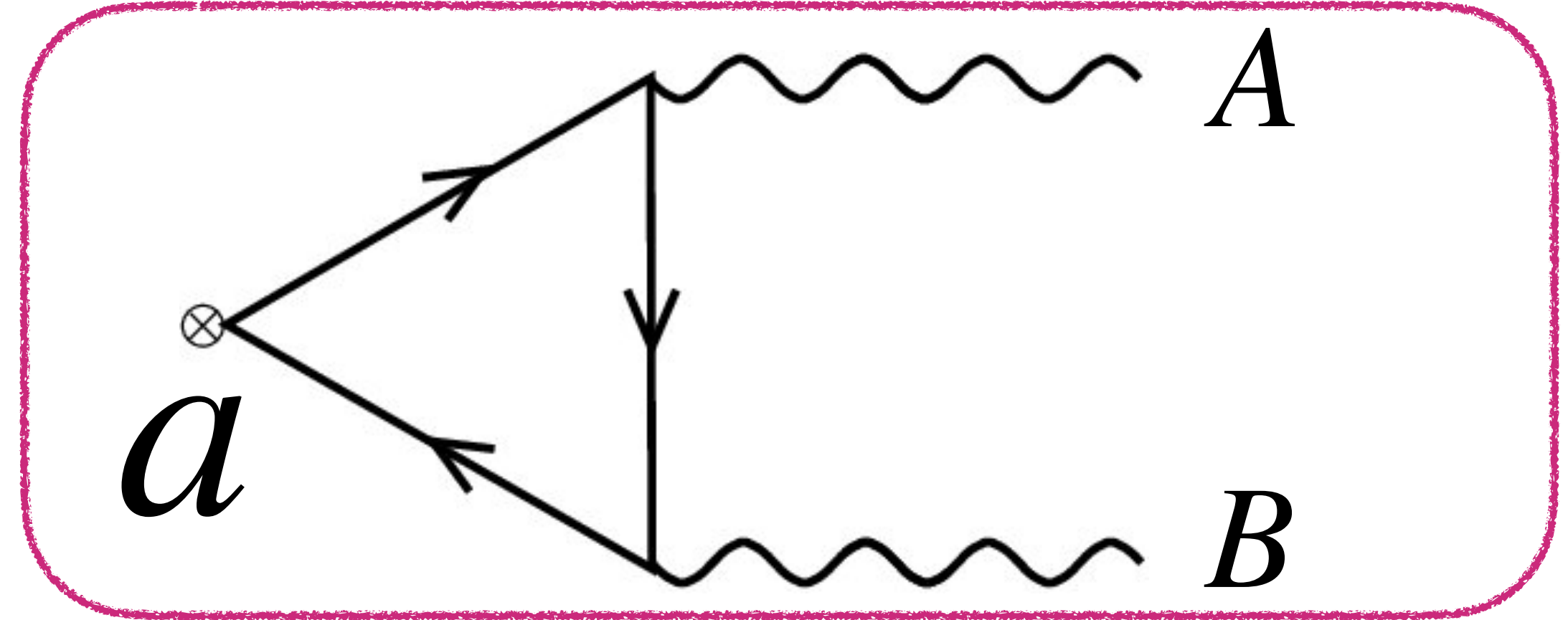
$$\delta [\Gamma_{\text{WZW}} + \Gamma_c](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R) = -2\mathcal{C} \frac{a}{f} \int \text{Tr} \left\{ \boldsymbol{\theta}_L \left[3(d\mathbb{A}_L - i\mathbb{A}_L^2)^2 + 3(d\mathbb{A}_L - i\mathbb{A}_L^2)(D\mathbb{B}_L) + D\mathbb{B}_L D\mathbb{B}_L - \frac{i}{2} D(\mathbb{B}_L^3) \right. \right. \\ \left. \left. + i\mathbb{B}_L(d\mathbb{A}_L - i\mathbb{A}_L^2)\mathbb{B}_L - i(d\mathbb{A}_L - i\mathbb{A}_L^2)\mathbb{B}_L^2 \right] \right\} - (L \leftrightarrow R),$$

- Covariant derivative $D\mathbb{B}_{L,R} = d\mathbb{B}_{L,R} - i\mathbb{A}_{L,R}\mathbb{B}_{L,R} - i\mathbb{B}_{L,R}\mathbb{A}_{L,R}$
- Covariant field strength: $F = d\mathbb{A}_L - i\mathbb{A}_L^2$

The axion anomalous interactions

$$\delta \mathcal{L}_a^{\text{ano}} = -\delta [\mathcal{L}_{\text{WZW}} + \mathcal{L}_c](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R)$$

- The exact expressions



$$\delta [\Gamma_{\text{WZW}} + \Gamma_c](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R) = -2\mathcal{C} \frac{a}{f} \int \text{Tr} \left\{ \boldsymbol{\theta}_L \left[3(d\mathbb{A}_L - i\mathbb{A}_L^2)^2 + 3(d\mathbb{A}_L - i\mathbb{A}_L^2)(D\mathbb{B}_L) + D\mathbb{B}_L D\mathbb{B}_L - \frac{i}{2} D(\mathbb{B}_L^3) \right. \right. \\ \left. \left. + i\mathbb{B}_L(d\mathbb{A}_L - i\mathbb{A}_L^2)\mathbb{B}_L - i(d\mathbb{A}_L - i\mathbb{A}_L^2)\mathbb{B}_L^2 \right] \right\} - (L \leftrightarrow R),$$

- Covariant derivative $D\mathbb{B}_{L,R} = d\mathbb{B}_{L,R} - i\mathbb{A}_{L,R}\mathbb{B}_{L,R} - i\mathbb{B}_{L,R}\mathbb{A}_{L,R}$
- Covariant field strength: $F = d\mathbb{A}_L - i\mathbb{A}_L^2$

Effective and Chiral Lagrangian matching

$$\mathcal{L}_{\text{eff}}(q, \mathbf{m}_q, \mathbf{k}_L, \mathbf{k}_R) \rightarrow \mathcal{L}_{\chi\text{PT}}(q, \mathbf{m}_q, \mathbf{k}_L, \mathbf{k}_R)$$

- The correspondence

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \bar{q} i \not{D} q - [\bar{q}_L \mathbf{m}_q(a) q_R + h.c.] + \frac{1}{2} (\partial_\mu a) (\partial^\mu a) - \frac{m_{a,0}^2}{2} a^2 - \frac{a}{f} \sum_{\mathcal{A}_{1,2}} c_{\mathcal{A}_1 \mathcal{A}_2} F_{\mathcal{A}_1 \mu\nu} \widetilde{F}_{\mathcal{A}_2}^{\mu\nu} + \mathcal{L}_c$$

$$\mathcal{L}_{\chi\text{PT}} = \frac{f_\pi^2}{8} \text{Tr} [(D^\mu U)(D_\mu U)^\dagger] + \frac{f_\pi^2}{4} B_0 \text{Tr} [\mathbf{m}_q(a) U^\dagger + h.c.] + \frac{1}{2} (\partial_\mu a) (\partial^\mu a) - \frac{m_{a,0}^2}{2} a^2 + \frac{a}{f} \sum_{\mathcal{A}_{1,2}} c_{\mathcal{A}_1 \mathcal{A}_2} F_{\mathcal{A}_1 \mu\nu} \widetilde{F}_{\mathcal{A}_2}^{\mu\nu}$$

- The anomalous matching condition between UV and IR

$$\mathcal{L}_{\chi\text{PT}}^{\text{ano}} \equiv \frac{a}{f_a} \sum_{\mathcal{A}_{1,2}} c_{\mathcal{A}_1 \mathcal{A}_2} F_{\mathcal{A}_1 \mu\nu} \widetilde{F}_{\mathcal{A}_2}^{\mu\nu}$$

Effective and Chiral Lagrangian matching

$$\mathcal{L}_{\text{eff}}(q, \mathbf{m}_q, \mathbf{k}_L, \mathbf{k}_R) \rightarrow \mathcal{L}_{\text{axion}}^{\text{full}} = \mathcal{L}_{\chi\text{PT}}(q, \mathbf{m}_q, \mathbf{k}_L, \mathbf{k}_R) + \mathcal{L}_{\text{WZW}}^{\text{full}}(U, \mathcal{A}_{L/R})$$

- The correspondence

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \bar{q} i \not{D} q - [\bar{q}_L \mathbf{m}_q(a) q_R + h.c.] + \frac{1}{2} (\partial_\mu a) (\partial^\mu a) - \frac{m_{a,0}^2}{2} a^2 + \frac{a}{f} \sum_{\mathcal{A}_{1,2}} c_{\mathcal{A}_1 \mathcal{A}_2} F_{\mathcal{A}_1 \mu\nu} \tilde{F}_{\mathcal{A}_2}^{\mu\nu} + \mathcal{L}_c$$

$$\mathcal{L}_{\chi\text{PT}} = \frac{f_\pi^2}{8} \text{Tr} [(D^\mu U)(D_\mu U)^\dagger] + \frac{f_\pi^2}{4} B_0 \text{Tr} [\mathbf{m}_q(a) U^\dagger + h.c.] + \frac{1}{2} (\partial_\mu a) (\partial^\mu a) - \frac{m_{a,0}^2}{2} a^2 + \frac{a}{f} \sum_{\mathcal{A}_{1,2}} c_{\mathcal{A}_1 \mathcal{A}_2} F_{\mathcal{A}_1 \mu\nu} \tilde{F}_{\mathcal{A}_2}^{\mu\nu}$$

- WZW term and counter terms

$$\mathcal{L}_{\text{WZW}}^{\text{full}}(U, \mathcal{A}_{L/R}) = \mathcal{L}_{\text{WZW}}(U, \mathcal{A}_L, \mathcal{A}_R) + \mathcal{L}_c(\mathbb{A}_{L/R}, \mathbb{B}_{L/R})$$

Consistent matching between \mathcal{L}_{eff} and $\mathcal{L}_{\text{axion}}^{\text{full}}$

$$\mathcal{L}_{\text{eff},0}(q_0, \mathbf{m}_{q,0}, \mathbf{k}_{L,0}, \mathbf{k}_{R,0})$$

$$\downarrow q_0 = \exp\left(-i c_{gg} \boldsymbol{\kappa}_{q,0} \gamma_5 \frac{a}{f}\right) q$$

$$\delta \mathcal{L}_a^{\text{ano}} = -\delta [\mathcal{L}_{\text{WZW}} + \mathcal{L}_c](\boldsymbol{\theta}_L, \boldsymbol{\theta}_R) = \delta \mathcal{L}_{\text{WZW}}^{\text{ano}}$$

$$\mathcal{L}_{\text{eff}}(q, \mathbf{m}_q, \mathbf{k}_L, \mathbf{k}_R)$$

$$\xrightarrow{q' = \exp\left[i\left(\delta_q + \boldsymbol{\kappa}_q \gamma_5\right) \frac{a}{f}\right] q}$$

$$\mathcal{L}_{\text{eff}}(q', \mathbf{m}'_q, \mathbf{k}'_L, \mathbf{k}'_R) + \delta \mathcal{L}_a^{\text{ano}}$$

matching

matching

$$\mathcal{L}_{\text{axion}}^{\text{full}} \equiv \mathcal{L}_{\chi\text{PT}}(U, \mathbf{m}_q, \mathcal{A}_{L/R} + \mathbf{k}_{L/R} da) \xrightarrow{U' = U_L^\dagger U U_R}$$

$$+ \mathcal{L}_{\text{WZW}}^{\text{full}}(U, \mathbf{m}_q, \mathcal{A}_{L/R} + \mathbf{k}_{L/R} da)$$

$$\mathcal{L}_{\chi\text{PT}}(U', \mathbf{m}'_q, \mathcal{A}_{L/R} + \mathbf{k}'_{L/R} da)$$

$$+ \mathcal{L}_{\text{WZW}}^{\text{full}}(U', \mathbf{m}'_q, \mathcal{A}_{L/R} + \mathbf{k}'_{L/R} da)$$

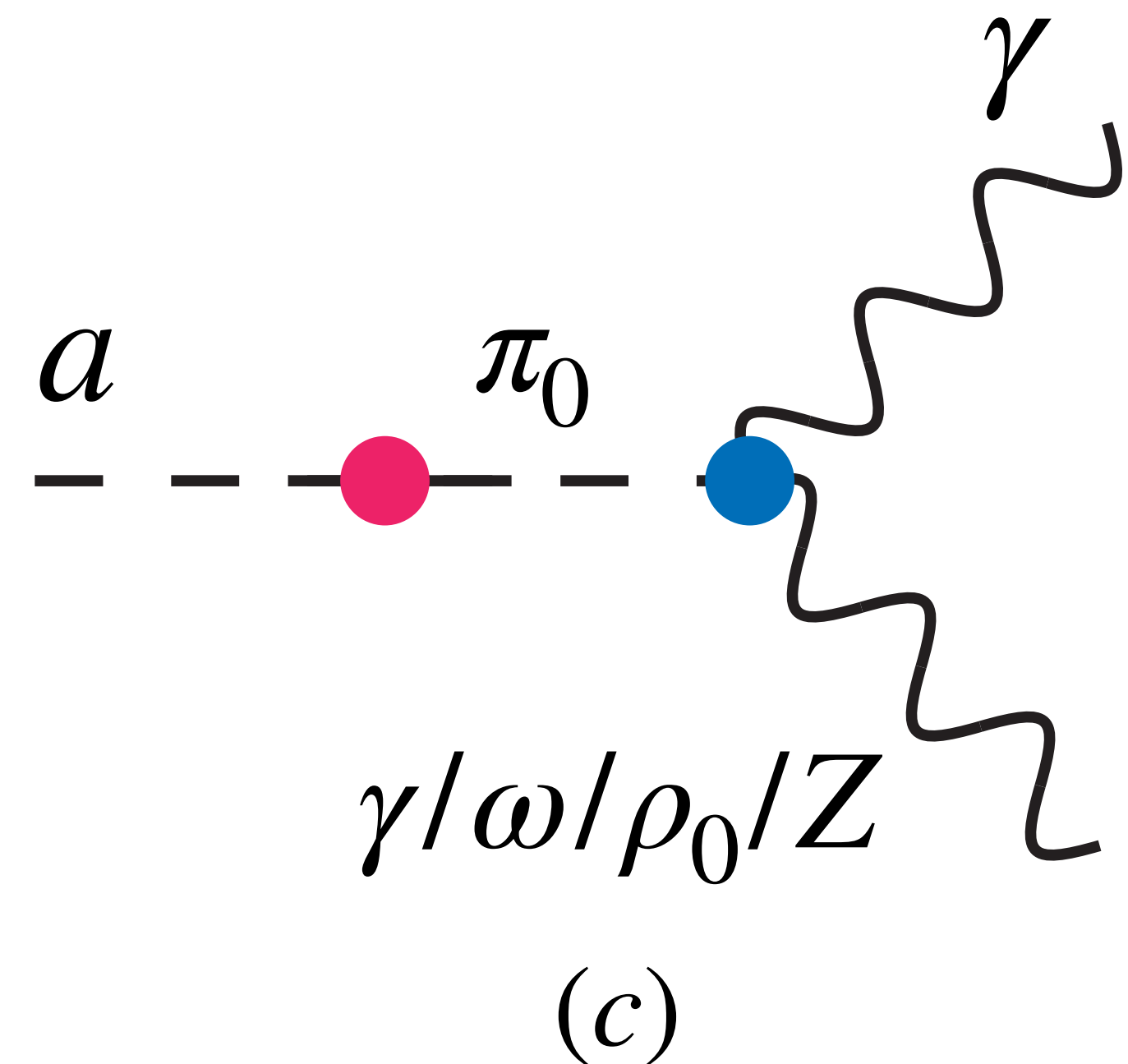
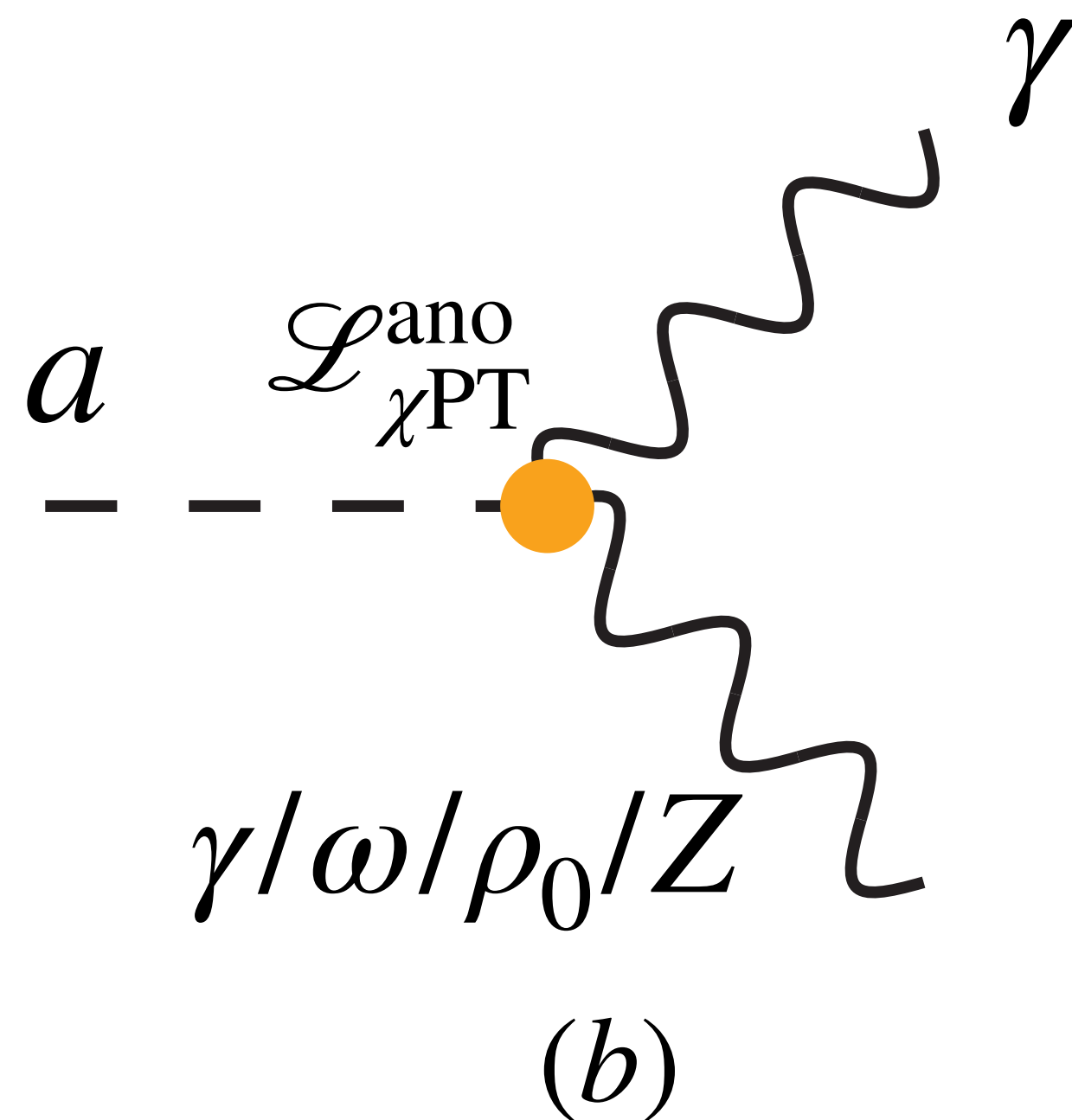
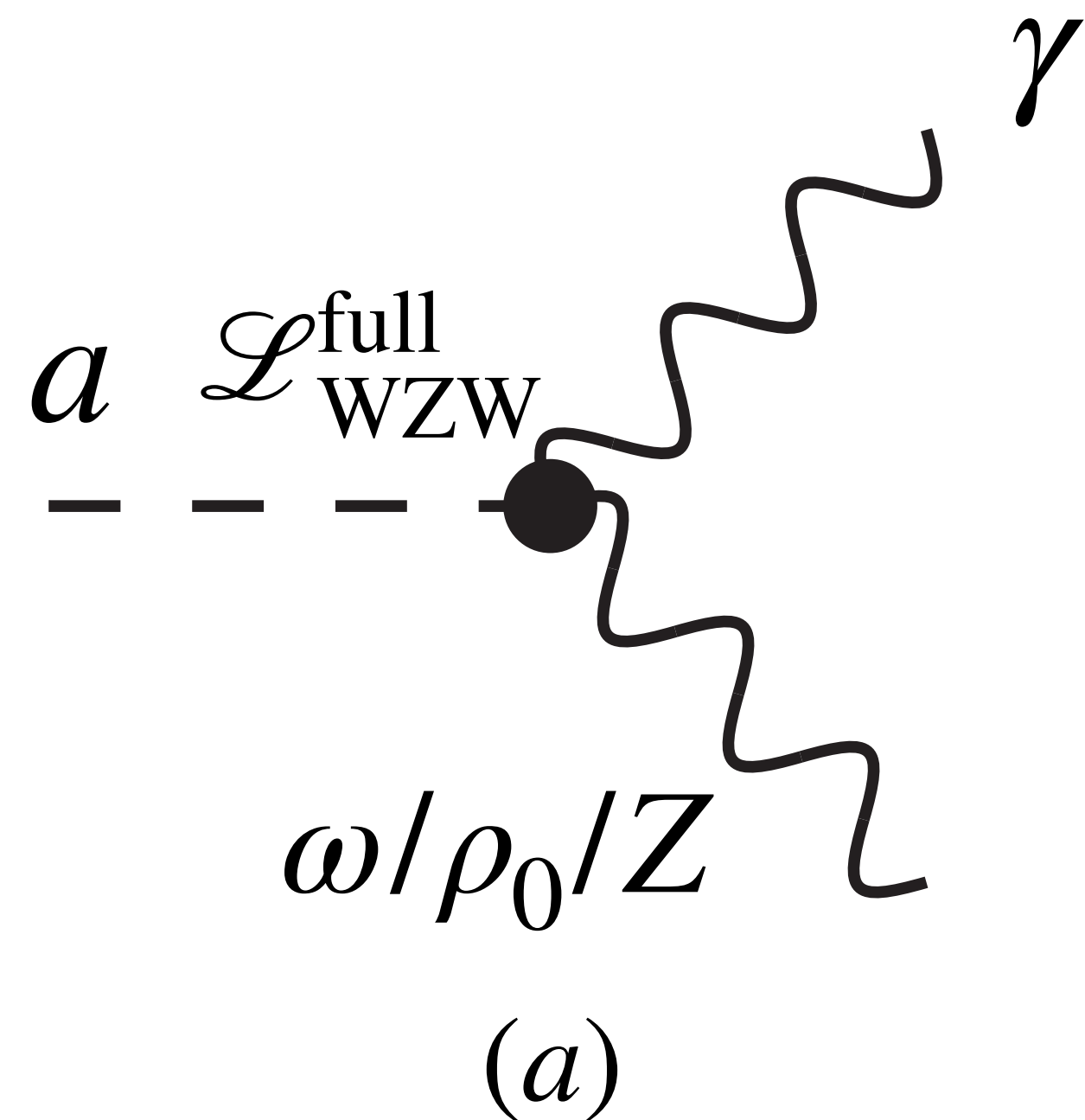
$$+ \delta \mathcal{L}_{\text{WZW}}^{\text{ano}}$$

Outlines

- Axion effective and consistent ChPT Lagrangian
- WZW in QCD and counter-terms
- Full WZW interactions for axions
- Phenomenology at BESIII and STCF
- Summary

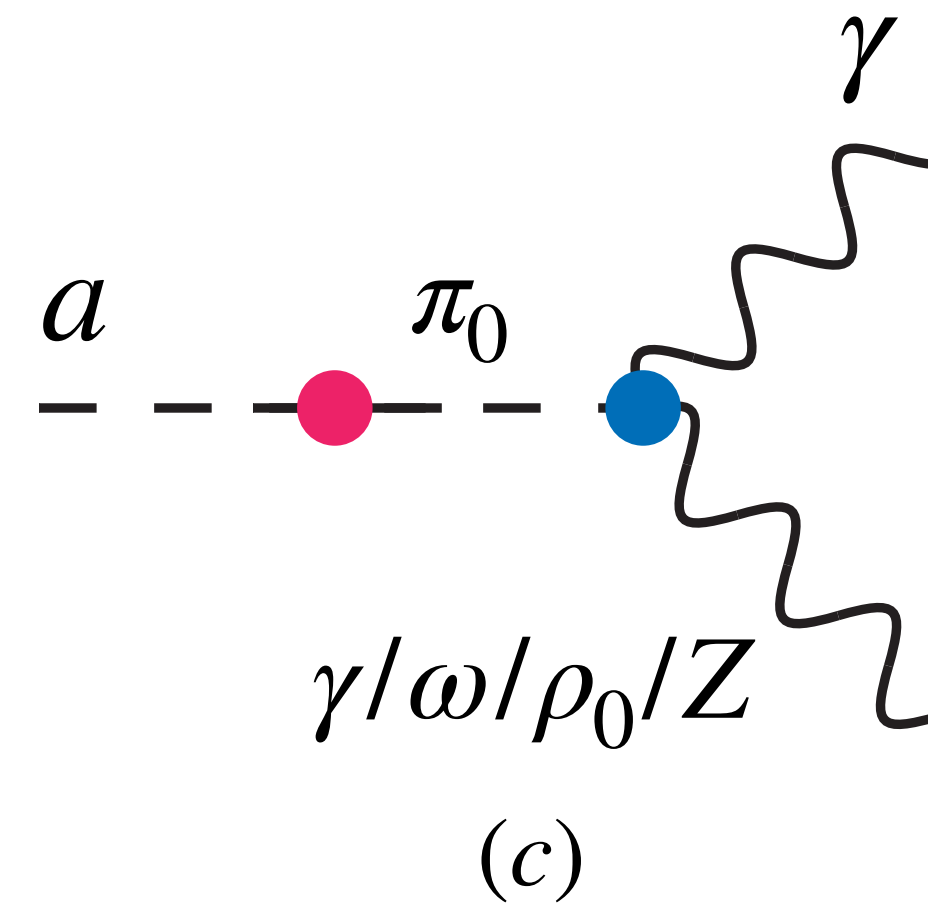
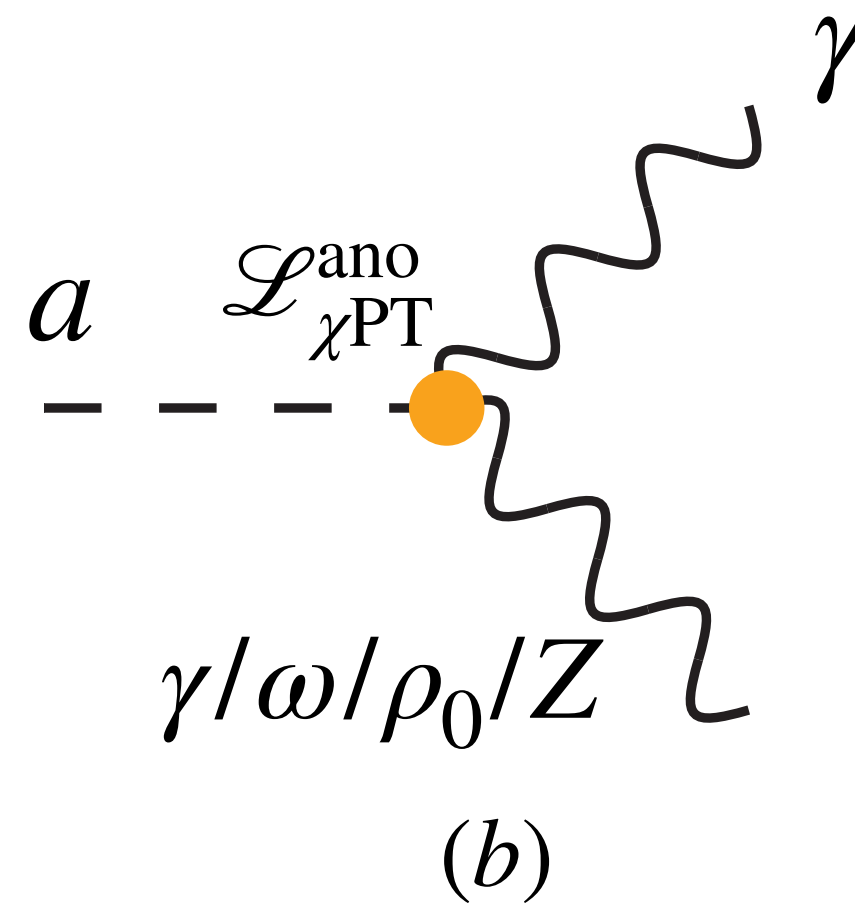
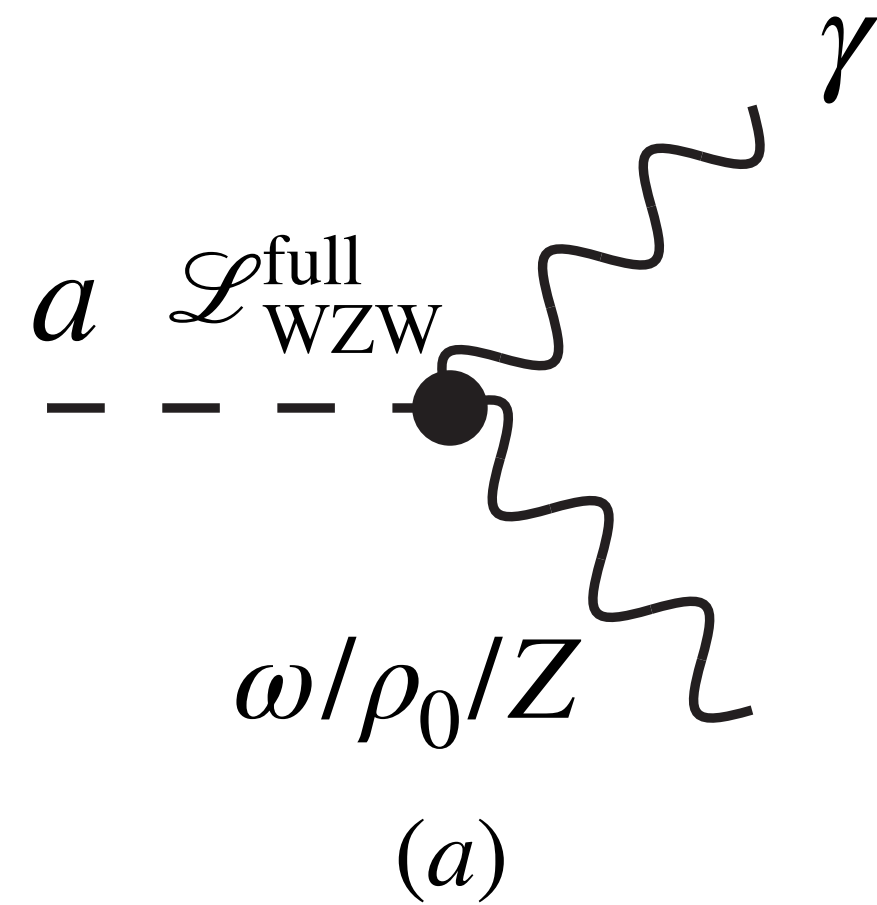
Consistent physical amplitudes

- A consistent Lagrangian will give physical amplitudes independent of auxiliary rotations
- Full WZW interactions are important for a-A-B amplitudes



Consistent physical amplitudes for $a - \gamma - \gamma$

- Auxiliary rotations are cancelled



$$ad\gamma d\gamma: c_{\text{WZW}} \equiv 0 \quad ad\gamma d\gamma: c_{\text{ano}} \equiv -\frac{e^2 N_c}{48\pi^2 f} 12(Q_u^2 \kappa_u + Q_d^2 \kappa_d) \quad \pi_0 d\gamma d\gamma: c_{\pi_0} \equiv \frac{e^2 N_c}{48\pi^2 f_\pi} 6\sqrt{2}(Q_d^2 - Q_u^2)$$

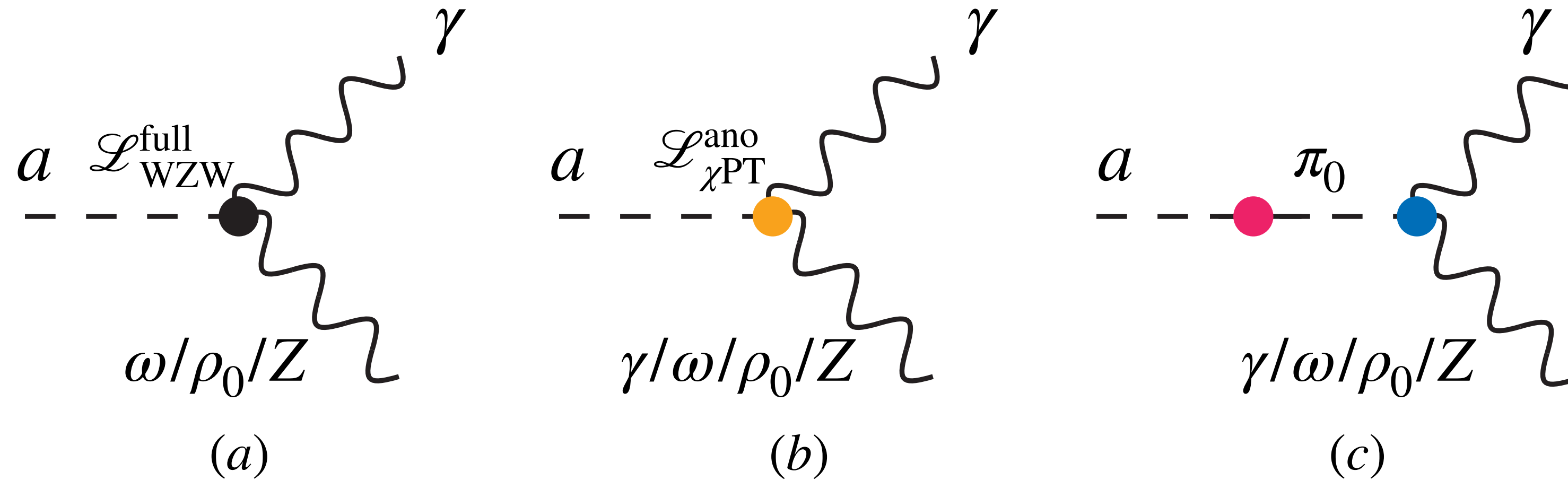
$$\mathcal{M}(a \rightarrow \gamma\gamma)(\text{auxiliary}) = CF \times (c_{\text{ano}} + \theta'_{a-\pi_0} c_{\pi_0} + c_{\text{WZW}})$$

$$= CF \times e^2 \left\{ \frac{-N_c}{48\pi^2 f_a} 12(Q_u^2 \kappa_u + Q_d^2 \kappa_d) + i \frac{f_\pi}{\sqrt{2}f} \left[(\kappa_u - \kappa_d) p_a^2 - 2 \frac{m_u \kappa_u - m_d \kappa_d}{m_u + m_d} m_\pi^2 \right] \frac{i}{p_a^2 - m_\pi^2} \times \frac{N_c}{48\pi^2 f_\pi} 6\sqrt{2}(Q_d^2 - Q_u^2) \right\}$$

$$\begin{aligned} \kappa_u + \kappa_d &= 0 \\ p_a^2 &= m_a^2 \end{aligned} \rightarrow 0$$

Consistent physical amplitudes for $a - \gamma - \omega$

- Auxiliary rotations are cancelled



$$ad\omega d\gamma: c_{\text{wzw}} = \frac{-eg'N_c}{48\pi^2 f} 6(Q_u\kappa_u + Q_d\kappa_d)$$

$$ad\omega d\gamma: c_{\text{ano}} = \frac{-eg'N_c}{48\pi^2 f} 6(Q_u\kappa_u + Q_d\kappa_d)$$

$$\pi_0 d\omega d\gamma: c_{\pi_0} = \frac{eg'N_c}{48\pi^2 f_\pi} 6\sqrt{2}(Q_d - Q_u)$$

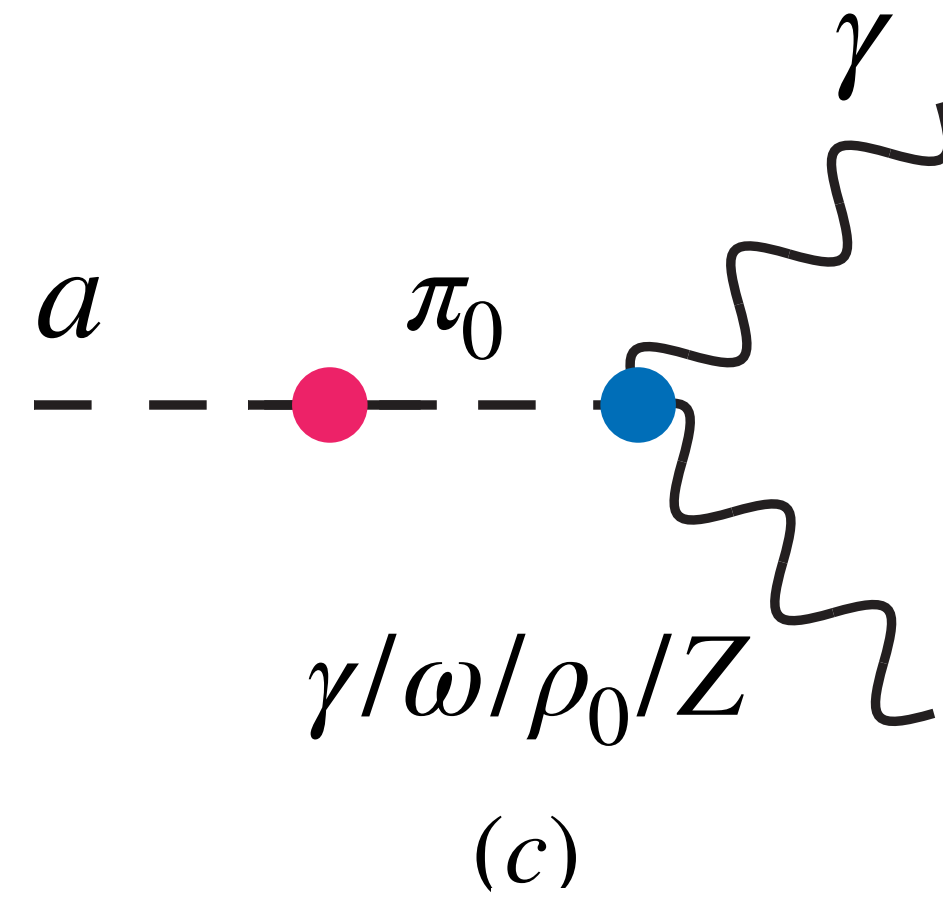
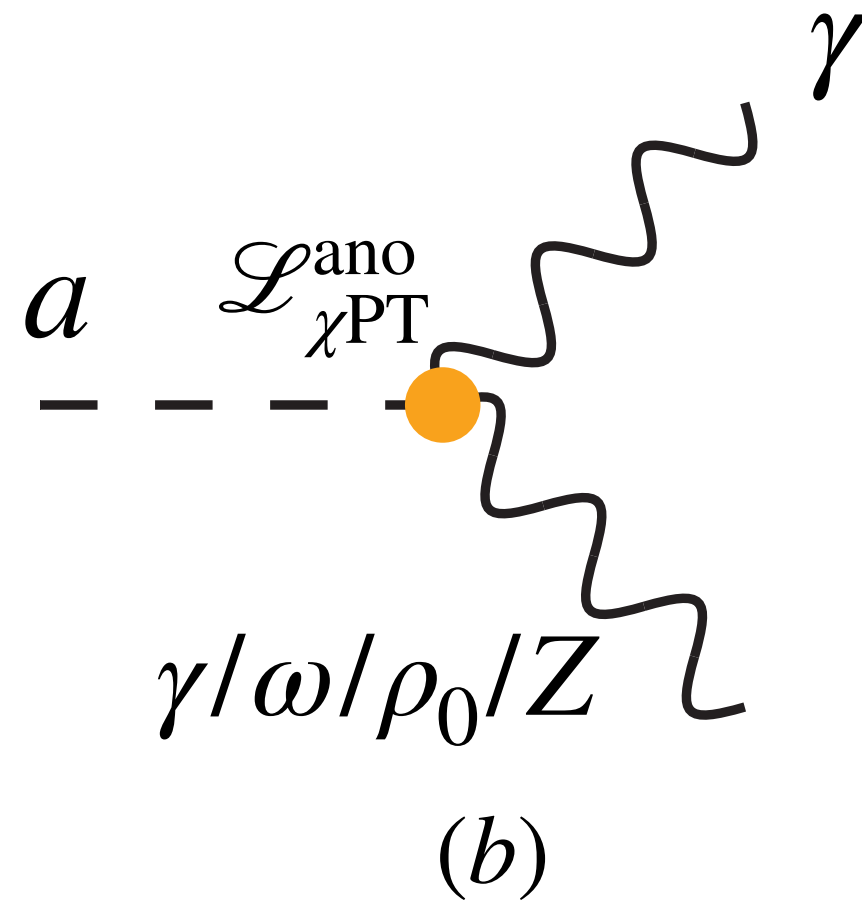
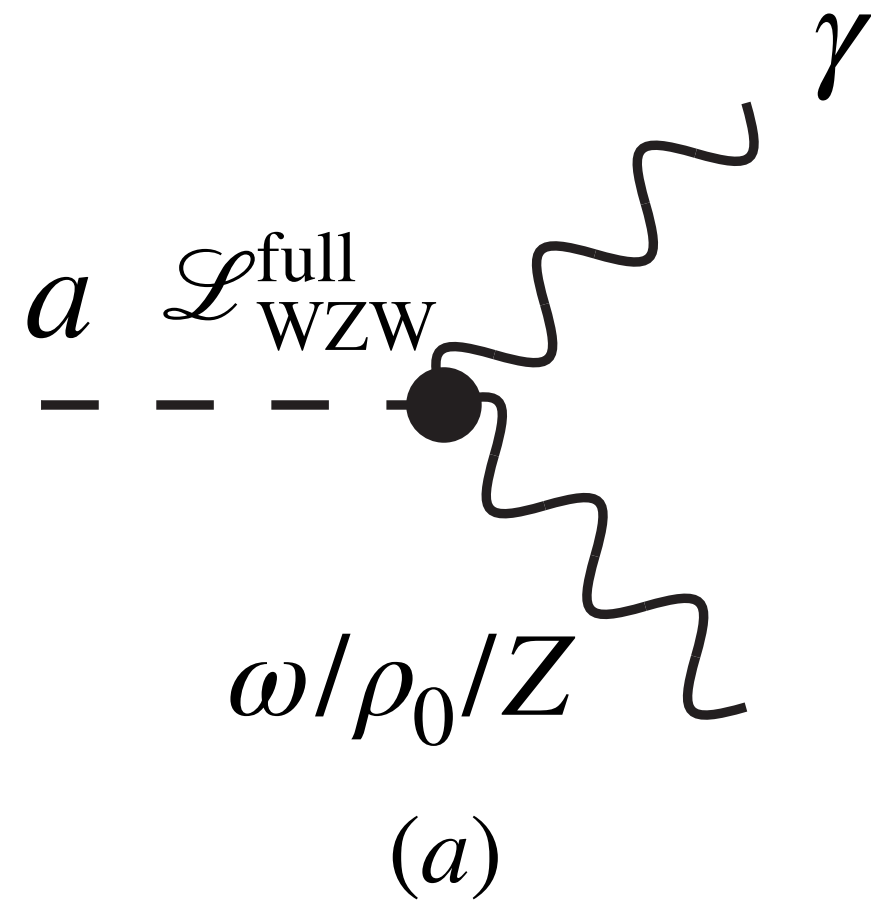
$$\mathcal{M}(a \rightarrow \omega\gamma) \text{(auxiliary)} = CF \times (c_{\text{ano}} + \theta'_{a-\pi_0} c_{\pi_0} + c_{\text{wzw}})$$

$$= CF \times eg' \left[\frac{-N_c}{48\pi^2 f} 12(Q_u\kappa_u + Q_d\kappa_d) + i \frac{f_\pi}{\sqrt{2}f} ((\kappa_u - \kappa_d)p_a^2 - 2 \frac{m_u\kappa_u - m_d\kappa_d}{m_u + m_d} m_\pi^2) \frac{i}{p_a^2 - m_\pi^2} \times \frac{N_c}{48\pi^2 f_\pi} 6\sqrt{2}(Q_d - Q_u) \right]$$

$$\rightarrow 0$$

Consistent physical amplitudes for $a - \gamma - Z$

- Auxiliary rotations are cancelled $ad\gamma dZ: c_{\text{ano}} = \frac{N_c}{48\pi^2 f} \frac{2e^2}{3c_w s_w} [3\delta_d + 6\delta_u - 3\kappa_d - 6\kappa_u + 4s_w^2(\kappa_d + 4\kappa_u)]$



$$ad\gamma dZ: c_{\text{wzw}} = \frac{-2e^2 N_c}{48\pi^2 f s_w c_w} (\delta_d + 2\delta_u)$$

$$\pi_0 d\gamma dZ: c_{\pi_0} = \frac{-e^2 N_c}{48\pi^2 f_\pi s_w c_w} \sqrt{2} (c_w^2 - 3s_w^2)$$

$$\mathcal{M}(a \rightarrow Z^* \gamma) (\text{auxiliary}) = CF \times (c_{\text{ano}} + \theta'_{a-\pi_0} c_{\pi_0} + c_{\text{wzw}})$$

$$= CF \times \left[c_{\text{wzw}} + c_{\text{ano}} + i \frac{f_\pi}{\sqrt{2}f} \left((\kappa_u - \kappa_d) p_a^2 - 2 \frac{m_u \kappa_u - m_d \kappa_d}{m_u + m_d} m_\pi^2 \right) \frac{i}{p_a^2 - m_\pi^2} \times c_{\pi_0} \right]$$

$$\rightarrow 0$$

Consistent amplitudes for three point vertex

$$c_{\gamma\gamma}^{\text{eff}} = c_{\gamma\gamma}^0 + \frac{e^2 c_{gg}}{16\pi^2 f} \left(-\frac{10}{3} - 2 \frac{m_u - m_d}{m_u + m_d} \frac{m_\pi^2}{m_a^2 - m_\pi^2} \right) - \frac{e^2}{16\pi^2 f} \frac{m_a^2}{m_\pi^2 - m_a^2} (c_u - c_d)$$

$$c_{\omega\gamma}^{\text{eff}} = eg' \left\{ \frac{-c_{gg}}{8\pi^2 f} - \frac{3}{8\pi^2 f} \left[\frac{m_a^2}{m_\pi^2 - m_a^2} \left(\frac{c_u - c_d}{2} \right) + c_{gg} \frac{m_u - m_d}{m_u + m_d} \frac{m_\pi^2}{m_a^2 - m_\pi^2} \right] + \frac{1}{16\pi^2 f} (c_d + c_Q - 2c_u) \right\}$$

$$c_{\rho\gamma}^{\text{eff}} = eg \left\{ \frac{-3c_{gg}}{8\pi^2 f} - \frac{1}{8\pi^2 f} \left[\frac{m_a^2}{m_\pi^2 - m_a^2} \left(\frac{c_u - c_d}{2} \right) + c_{gg} \frac{m_u - m_d}{m_u + m_d} \frac{m_\pi^2}{m_a^2 - m_\pi^2} \right] + \frac{1}{16\pi^2 f} (3c_Q - 2c_u - c_d) \right\}$$

$$c_{\gamma Z}^{\text{eff}} = c_{\gamma Z}^0 + \frac{N_c c_{gg}}{48\pi^2 f} \frac{e^2}{s_w c_w} (-9 + 20s_w^2) - c_{\pi_0} \frac{f_\pi}{\sqrt{2}f} \left(\frac{m_a^2}{m_\pi^2 - m_a^2} \frac{c_d - c_u}{2} - c_{gg} \frac{m_u - m_d}{m_u + m_d} \frac{m_\pi^2}{m_a^2 - m_\pi^2} \right) - \frac{N_c}{48\pi^2 f} \frac{2e^2}{s_{2w}} (c_d + 2c_u + 3c_Q)$$

- Vertex $\omega \rightarrow \gamma a$ benefit from large $g' \approx 5.7 \gg e$

$$\mathbf{k}_{L,0} = \{c_Q, c_Q\} \quad \mathbf{k}_{R,0} = \{c_u, c_d\}$$

Phenomenology at BESIII and STCF

- New channel $e^+e^- \rightarrow \gamma^*(J/\Psi) \rightarrow \omega a$

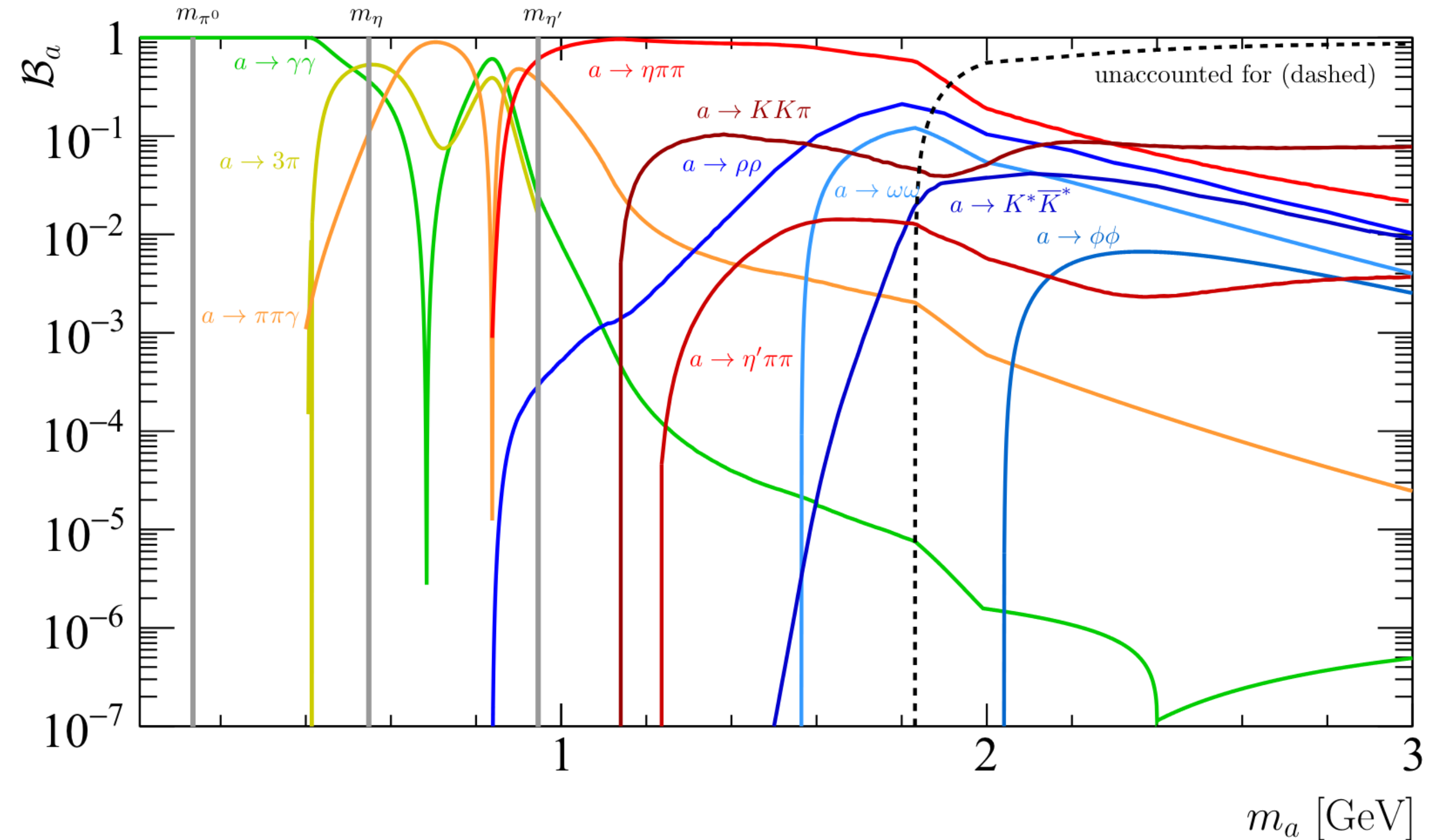
$$c_{\omega\gamma}^{\text{eff}}(q^2) = -\frac{eg'c_{gg}}{8\pi^2f} \frac{m_\omega^2}{m_\omega^2 - q^2 - i\sqrt{q^2}\Gamma_\omega} - \frac{3eg'c_{gg}}{8\pi^2f} \frac{m_u - m_d}{m_u + m_d} \frac{m_\pi^2}{m_a^2 - m_\pi^2} \sum_{i=0}^3 \frac{A_i M_i^2 e^{i\phi_i}}{M_i^2 - q^2 - i\sqrt{q^2}\Gamma_i(\sqrt{q^2})}$$

- The model satisfies partial Vector Meson Dominance, therefore we can use form factor for $\gamma^* \rightarrow \omega \rightarrow a$
- The differential cross-section

$$\frac{d\sigma(e^+e^- \rightarrow \omega a)}{d\cos\theta} = \frac{\alpha |c_{\omega\gamma}^{\text{eff}}(q^2)|^2 [m_a^4 + (m_\omega^2 - s)^2 - 2m_a^2(m_\omega^2 + s)]}{64f^2s^2} (1 + \cos\theta^2)$$

The decay of axion

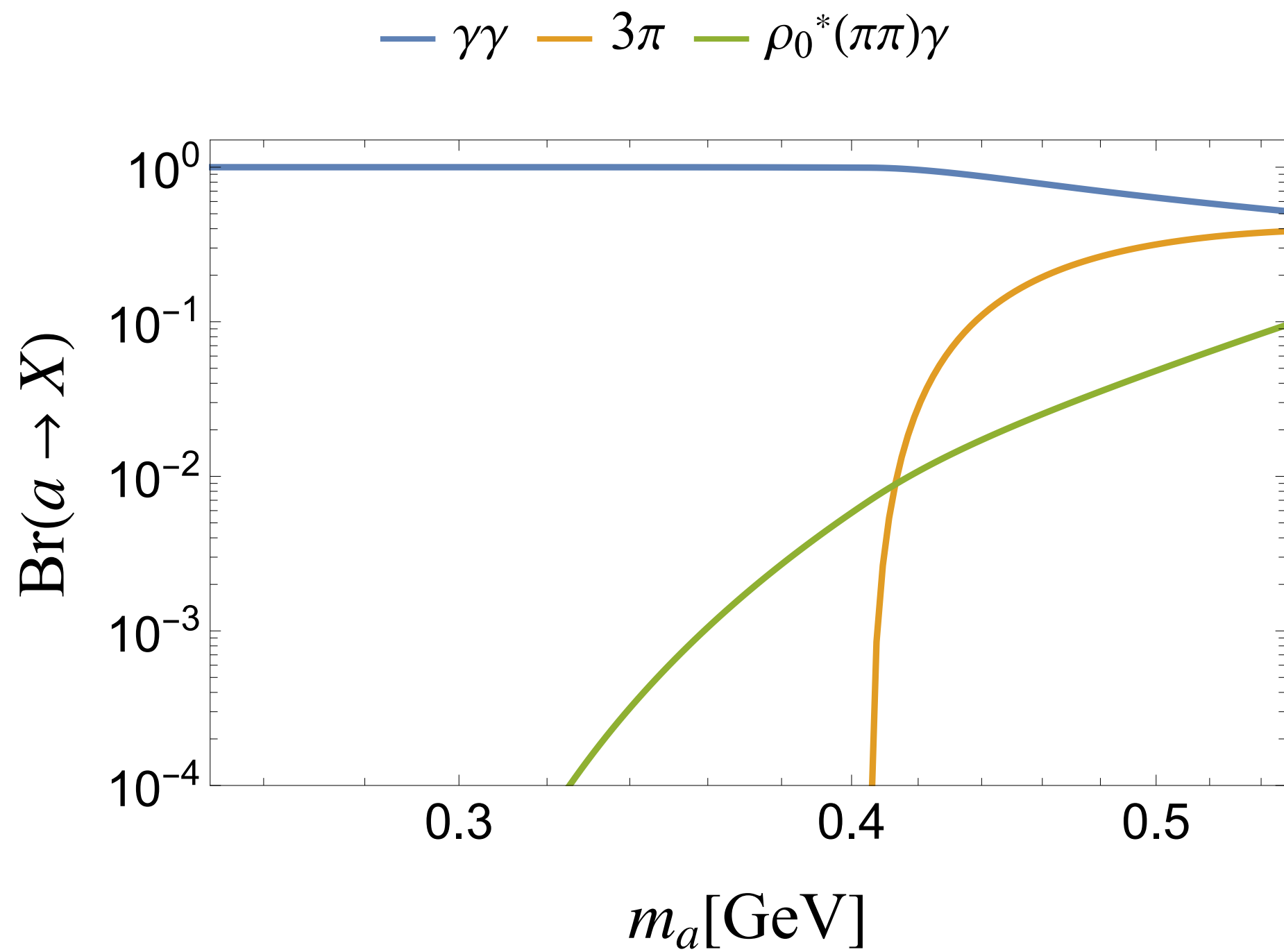
- Previous work (PRL 123 (2019) 031803) use Hidden Local Symmetry to describe pseudo scalar meson + vector meson interactions
- Assume axion mixes with π, η, η'
- Use data driven method to obtain form factor



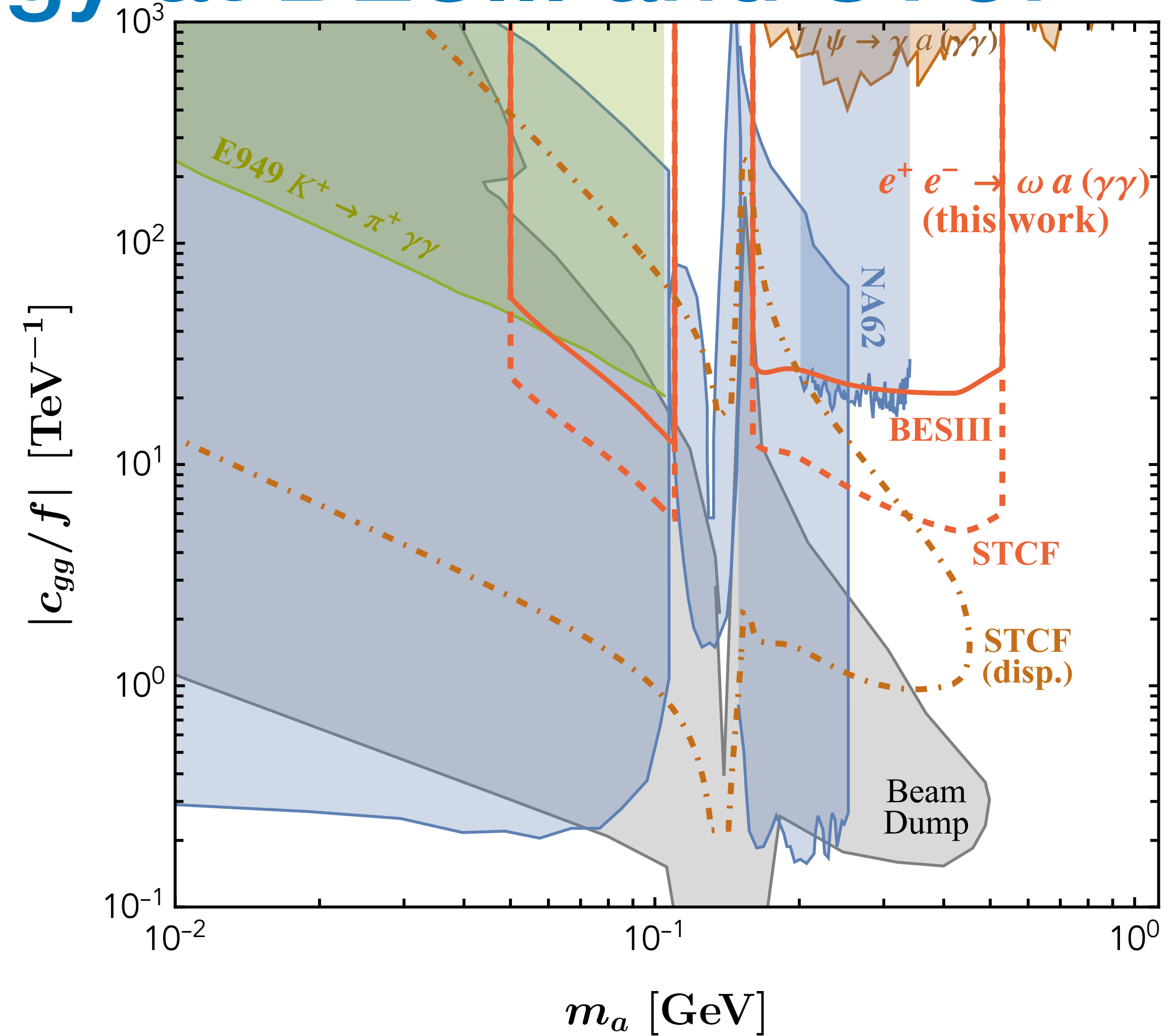
- Lacks first chiral rotation contribution from $\mathcal{L}_{\chi\text{PT}}^{\text{ano}}$

- Lacks full WZW contribution from $\mathcal{L}_{\text{WZW}}^{\text{full}}$

Light axion phenomenology at BESIII and STCF



- Production $e^+e^- \rightarrow \gamma^*(J/\Psi) \rightarrow \omega a$
- Prompt decay: $a \rightarrow \gamma\gamma$
- Displaced decay of a

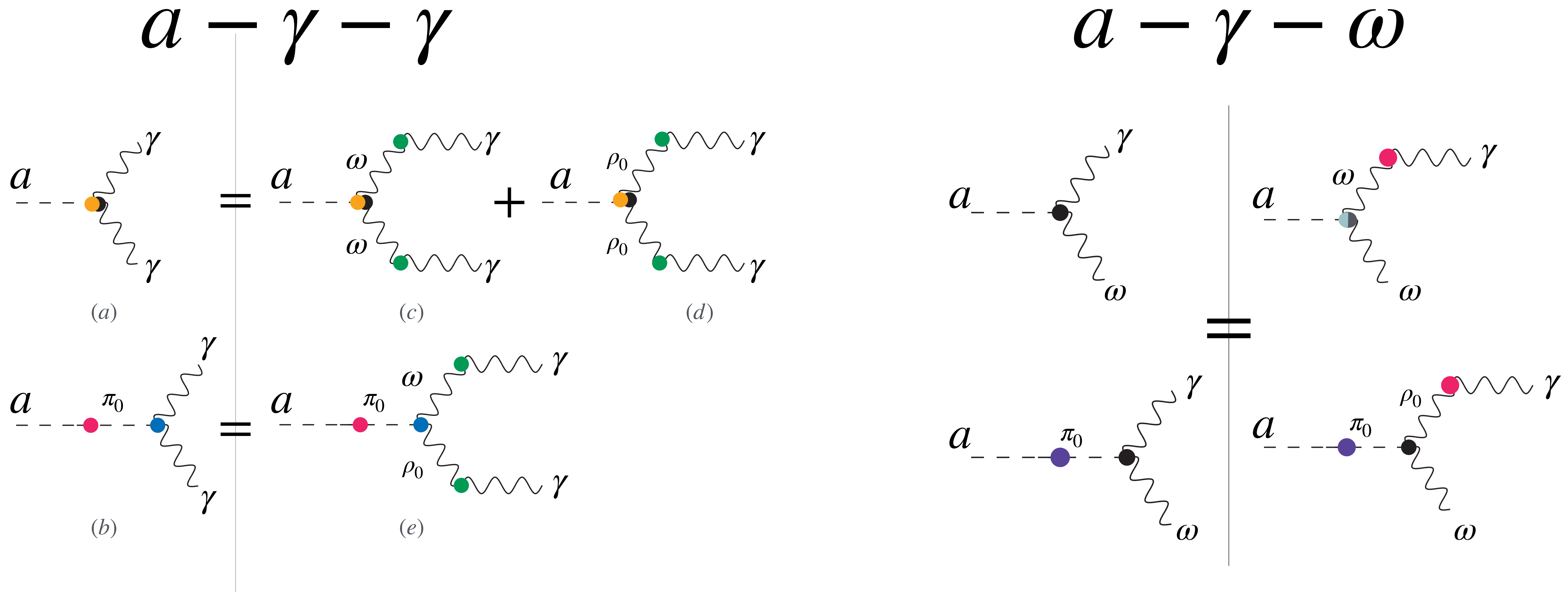


$$\frac{\text{BR}(J/\psi \rightarrow \omega a)}{\text{BR}(J/\psi \rightarrow ee)} = \frac{m_{J/\psi}^2}{32\pi\alpha} \left| c_{\omega\gamma}^{\text{eff}}(q^2 = m_{J/\psi}^2) \right|^2 \left[\left(1 - \frac{(m_a + m_\omega)^2}{m_{J/\psi}^2} \right) \left(1 - \frac{(m_a - m_\omega)^2}{m_{J/\psi}^2} \right) \right]^{\frac{3}{2}}$$

Summary

- A full chiral axion Lagrangian for axion-pseudo-vector meson
 - Wess-Zumino-Witten counter term is necessary for gauge invariance
 - $\mathbb{B}_{L/R} \rightarrow \mathbb{B}_{L/R} + \mathbf{k}_{L/R} da/f_a$
 - UV-IR anomaly matching is necessary
- Consistent physical amplitudes without auxiliary rotation parameters
- New search channel involving $\omega \rightarrow \gamma a$ vertex at BESIII & STCF
- Future plan: extending to three light quarks scheme

Backup: Vector Meson Dominance



Backup: axion related WZW interactions

- Convention $\int d^4x \epsilon_{\mu\nu\rho\sigma} A^\mu B^\nu \partial^\rho C^\sigma \equiv \int ABdC$

$$\begin{aligned}
 \Gamma_{XdYda} = & \frac{\mathcal{C}}{f} \int da \left\{ \frac{2e^2}{s_{2w}} (k_d + 2k_u + 3k_Q) \gamma dZ + eg(k_d + 2k_u + 3k_Q) \gamma da_1 - eg'(k_d - k_Q - 2k_u) \gamma df_1 \right. \\
 & + eg(k_d - 3k_Q + 2k_u) \gamma d\rho_0 - eg'(k_d + k_Q - 2k_u) \gamma d\omega + \frac{2e^2}{s_{2w}^2} \left[(k_d + 4k_Q + k_u) - 2s_w^2(k_d + 3k_Q + 2k_u) \right] ZdZ \\
 & + \frac{eg}{s_{2w}} \left[(k_d + 4k_Q + k_u) - 2s_w^2(k_d + 3k_Q + 2k_u) \right] Zda_1 - \frac{eg'}{s_{2w}} \left[k_d - k_u + s_w^2(-2k_d + 2k_Q + 4k_u) \right] Zdf_1 \\
 & - \frac{eg}{s_{2w}} \left[-3k_d - 3k_u + 2s_w^2(k_d - 3k_Q + 2k_u) \right] Zd\rho_0 - \frac{eg'}{s_{2w}} \left[3k_d - 3k_u - 2s_w^2(k_d + k_Q - 2k_u) \right] Zd\omega \\
 & + g^2(k_d + 2k_Q + k_u) a_1 d\rho_0 + gg'(k_u - k_d) a_1 d\omega + gg'(k_u - k_d) f_1 d\rho_0 + g^2(k_d + 2k_Q + k_u) f_1 d\omega \\
 & + g^2(k_d - 2k_Q + k_u) \rho_0 d\rho_0 + 2gg'(k_u - k_d) \rho_0 d\omega + g^2(k_d - 2k_Q + k_u) \omega d\omega + \frac{3eg}{2s_w} (k_u + k_d) W^\pm d\rho^\mp \\
 & \left. + \frac{eg}{2s_w} (k_d + 4k_Q + k_u) a^\mp dW^\pm + g^2(k_d + 2k_Q + k_u) a^\mp d\rho^\pm + \frac{e^2}{s_w^2} (k_d + 4k_Q + k_u) W^- dW^+ \right\}
 \end{aligned}$$