



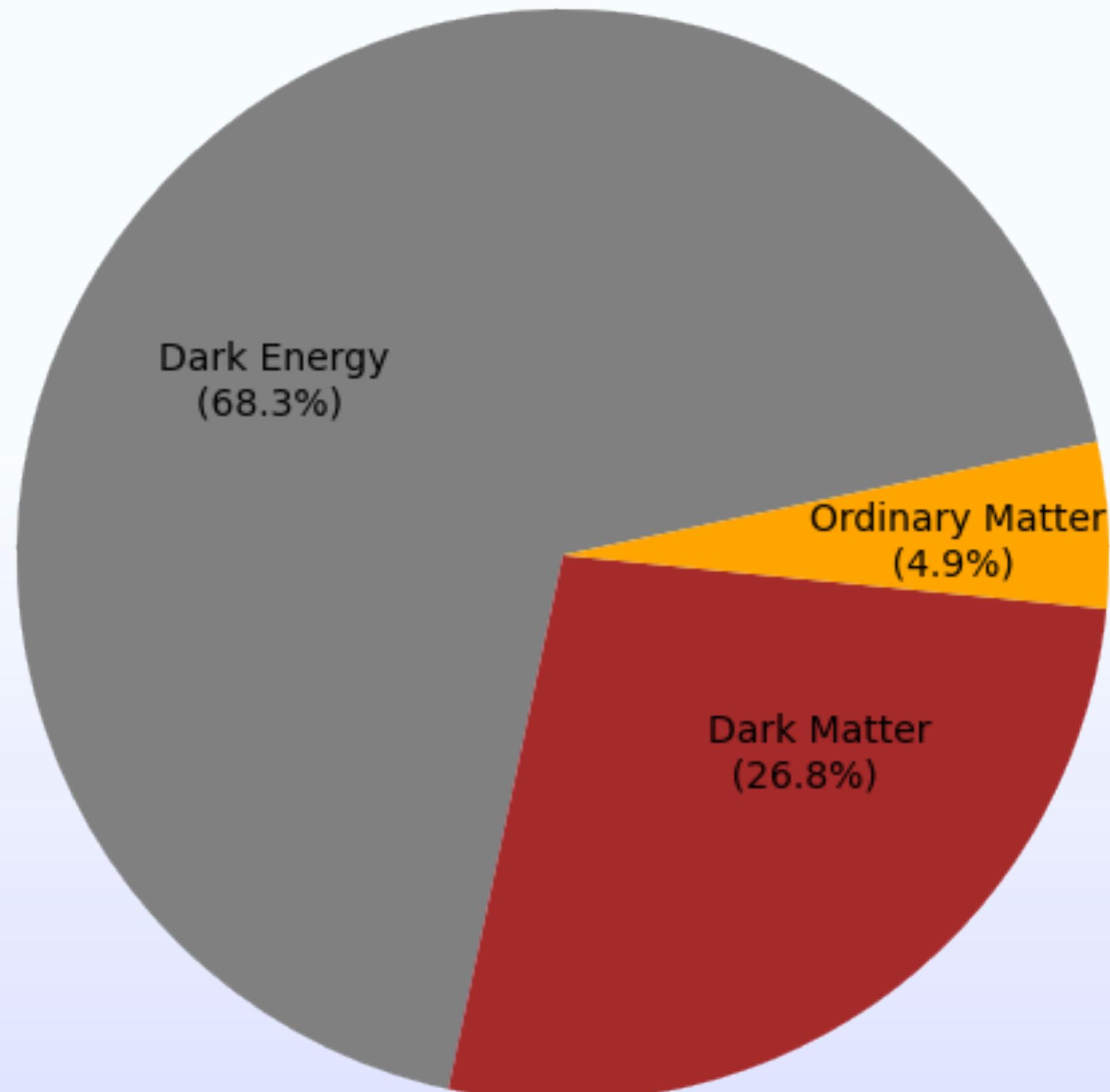
Investigating the general dark matter-bound-electron interactions in effective field theories

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arXiv: 2405. 04855, arXiv: 2406.10912

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Evidence of the DM



- Galaxy rotation curves
- Gravitational lensing
- Bullet Cluster
- CMB power spectrum
- Structure formation
- N-body simulation
- ...

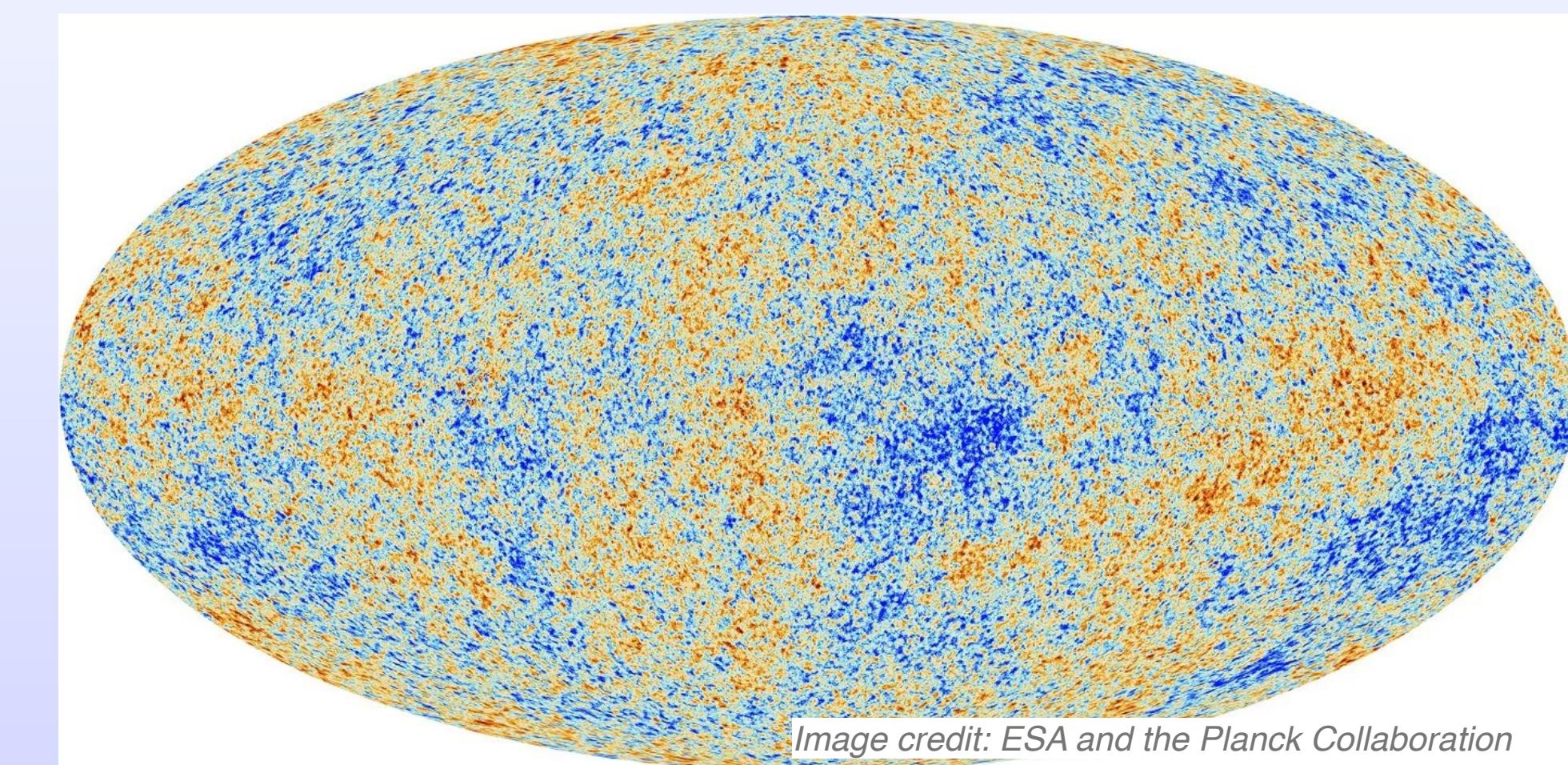
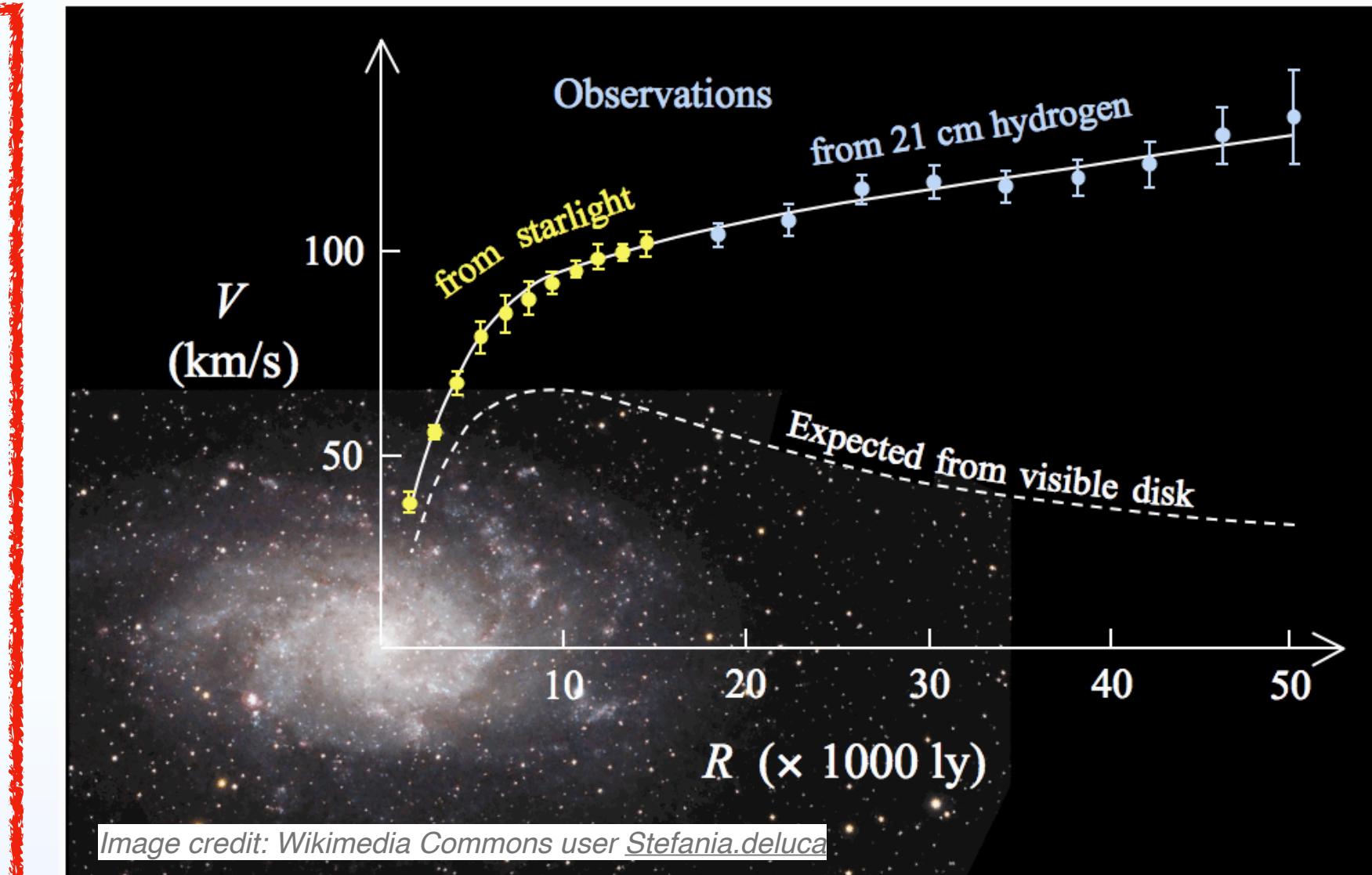
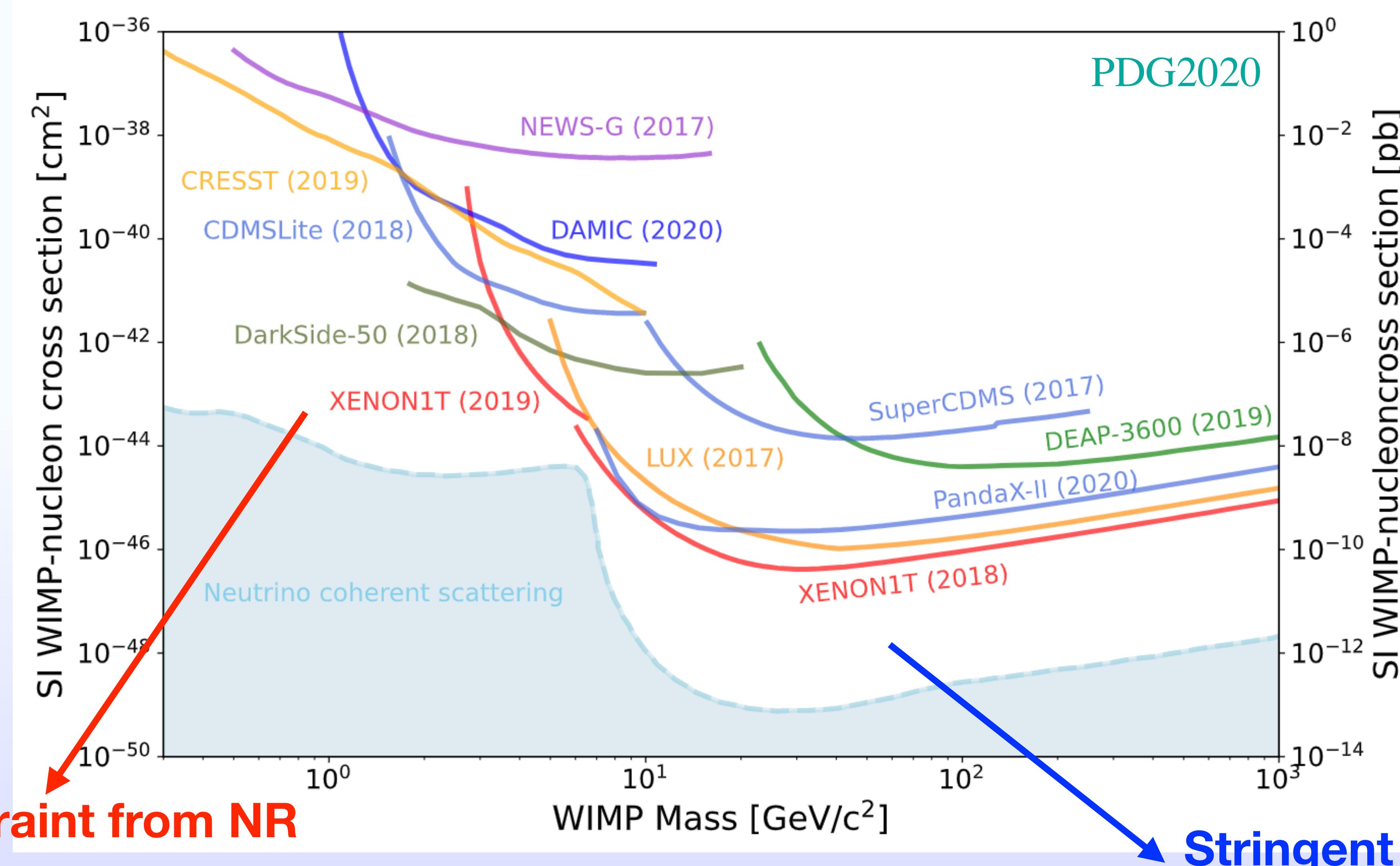


Image credit: ESA and the Planck Collaboration

Current constraints on DM-nucleon interaction



But can be probed through electron recoil via DM-electron interaction

Outline

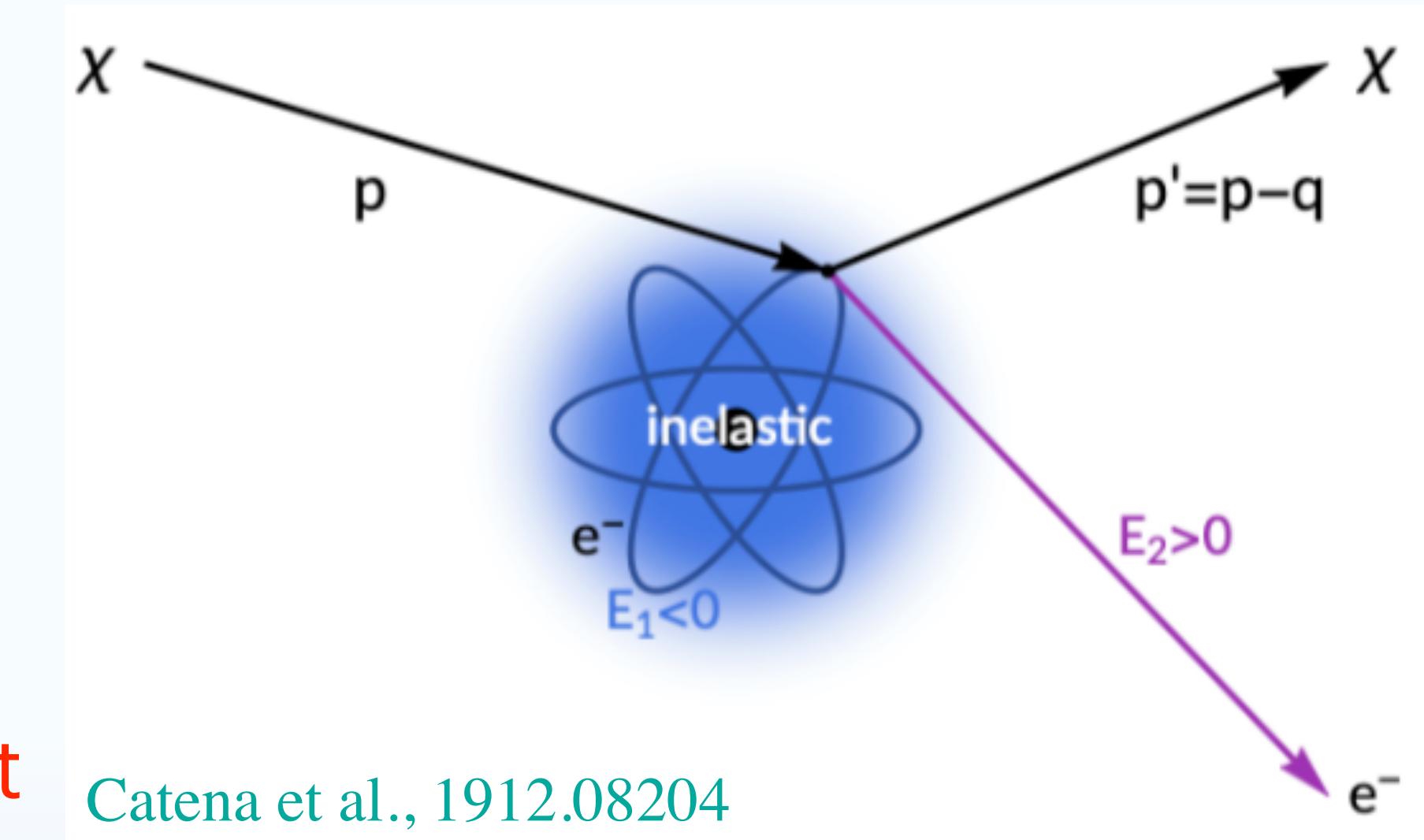
- General DM-atom scattering formalism
- DM-electron interactions in the EFTs
- Constraints on non-relativistic and relativistic interactions
- Summary

DM-atom scattering

$$\text{DM} + (n, \ell, m) \equiv |1\rangle \rightarrow \text{DM} + (k', \ell', m') \equiv |2\rangle$$

$$\frac{d\mathcal{R}_{\text{ion}}^{n\ell}}{d \ln E_e} = \frac{n_x}{128\pi m_x^2 m_e^2} \int dq \frac{\mathbf{q}}{\nu} f_x(\mathbf{v}) \Theta(\nu - \nu_{\min}) \overline{|\mathcal{M}_{\text{ion}}^{n\ell}|^2}$$

Momentum transfer Velocity distribution Key input



Summing over all final atomic states

$$\overline{|\mathcal{M}_{\text{ion}}^{n\ell}|^2} = \frac{4V k'^3}{(2\pi)^3} \sum_{m=-\ell}^{\ell} \sum_{\ell'=0}^{\infty} \sum_{m'=-\ell'}^{\ell'} |\mathcal{M}_{1 \rightarrow 2}|^2$$

Bound electron-DM amplitude

Free electron-DM amplitude

$$\mathcal{M}_{1 \rightarrow 2} = \int \frac{d^3 k}{(2\pi)^3} \tilde{\psi}_2^*(\mathbf{k} + \mathbf{q}) \mathcal{M}(\mathbf{q}, \mathbf{v}_{\text{el}}^\perp) \tilde{\psi}_1(\mathbf{k})$$

Atomic wave-functions

A. Dedes I. Giomataris, K. Suxho, J.D. Vergados, 0907.0758

J. Kopp, V. Niro, T. Schwetz, J. Zupan, 0907.3159

R. Essig, J. Mardon, T. Volansky, 1108.5383,

R. Essig, A. Manalaysay, J. Mardon, P. Sorensen, T. Volansky, 1206.2644

The usual case

– Dark photon model, SI and SD interactions, etc

$$\mathcal{M}(\mathbf{q}, \mathbf{v}_{\text{el}}^\perp) \rightarrow \mathcal{M}(\mathbf{q})$$



$$\mathcal{M}_{1 \rightarrow 2} = \mathcal{M}(q = \alpha m_e) F_{\text{DM}}(q) f_{1 \rightarrow 2}(\mathbf{q})$$



$$\frac{d\mathcal{R}_{\text{ion}}^{n\ell}}{d \ln E_e} = \frac{n_\chi \bar{\sigma}_e}{8\mu_{\chi e}} \times \int q dq \eta(v_{\min}) |f_{\text{ion}}^{n\ell}(q)|^2 |F_{\text{DM}}(q)|^2$$

A. Dedes I. Giomataris, K. Suxho, J.D. Vergados, 0907.0758

J. Kopp, V. Niro, T. Schwetz, J. Zupan, 0907.3159

R. Essig, J. Mardon, T. Volansky, 1108.5383,

R. Essig, A. Manalaysay, J. Mardon, P. Sorensen, T. Volansky, 1206.2644

$$F_{\text{DM}}(q) = \mathcal{M}(q)/\mathcal{M}(q = \alpha m_e)$$

$$f_{1 \rightarrow 2}(\mathbf{q}) = \int \frac{d^3 k}{(2\pi)^3} \tilde{\psi}_2^*(\mathbf{k} + \mathbf{q}) \tilde{\psi}_1(\mathbf{k}),$$

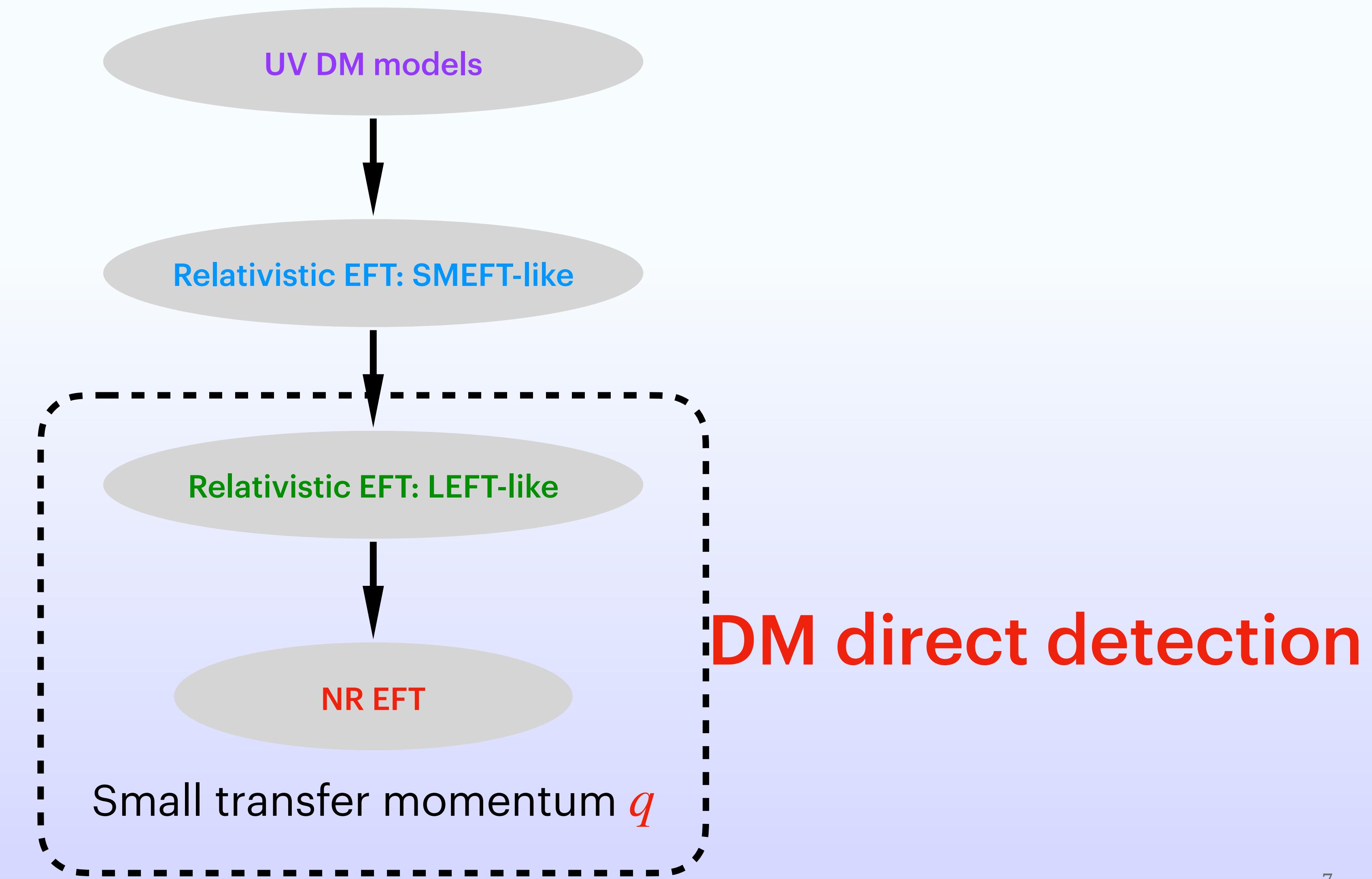
$$|f_{\text{ion}}^{n\ell}(q)|^2 \propto \sum_{l'm'm} |f_{1 \rightarrow 2}(\mathbf{q})|^2$$

$$\eta(v_{\min}) \equiv \int d^3 \vec{v} f_\chi(\vec{v}) \frac{1}{v} \Theta(v - v_{\min})$$

- Only one atomic response function (or K -factor) is used to describe the atom effect
- The velocity dependence can be integrated out

How about the general case ?

To be as general as possible, we focus on the EFT approach



NR operators

- Small momentum transfer q
- Rotational and Galilean invariance:

$$\{\mathbb{1}_e, \mathbf{S}_e\} \otimes \{\mathbb{1}_x, \mathbf{S}_x, \tilde{\mathcal{S}}_x\} \otimes \{iq, \mathbf{v}_{\text{el}}^\perp\}$$
- Works well for the DM-nucleus scattering
- Relativistic correction could be important for the DM-electron scattering
- J. Fan, M. Reece, L.-T. Wang, 1008.1591
- R. Catena, K. Fridell, M. B. Kraus, 1907.02910

NR operators	Power counting	DM type		
		scalar	fermion	vector
$\mathcal{O}_1 = \mathbb{1}_x \mathbb{1}_e$	1	✓	✓	✓
$\mathcal{O}_3 = \mathbb{1}_x \left(\frac{iq}{m_e} \times \mathbf{v}_{\text{el}}^\perp \right) \cdot \mathbf{S}_e$	qv	✓	✓	✓
$\mathcal{O}_4 = \mathbf{S}_x \cdot \mathbf{S}_e$	1	—	✓	✓
$\mathcal{O}_5 = \mathbf{S}_x \cdot \left(\frac{iq}{m_e} \times \mathbf{v}_{\text{el}}^\perp \right) \mathbb{1}_e$	qv	—	✓	✓
$\mathcal{O}_6 = \left(\mathbf{S}_x \cdot \frac{q}{m_e} \right) \left(\frac{q}{m_e} \cdot \mathbf{S}_e \right)$	q^2	—	✓	✓
$\mathcal{O}_7 = \mathbb{1}_x \mathbf{v}_{\text{el}}^\perp \cdot \mathbf{S}_e$	v	✓	✓	✓
$\mathcal{O}_8 = \mathbf{S}_x \cdot \mathbf{v}_{\text{el}}^\perp \mathbb{1}_e$	v	—	✓	✓
$\mathcal{O}_9 = -\mathbf{S}_x \cdot \left(\frac{iq}{m_e} \times \mathbf{S}_e \right)$	q	—	✓	✓
$\mathcal{O}_{10} = \mathbb{1}_x \frac{iq}{m_e} \cdot \mathbf{S}_e$	q	✓	✓	✓
$\mathcal{O}_{11} = \mathbf{S}_x \cdot \frac{iq}{m_e} \mathbb{1}_e$	q	—	✓	✓
$\mathcal{O}_{12} = -\mathbf{S}_x \cdot (\mathbf{v}_{\text{el}}^\perp \times \mathbf{S}_e)$	v	—	✓	✓
$\mathcal{O}_{13} = (\mathbf{S}_x \cdot \mathbf{v}_{\text{el}}^\perp) \left(\frac{iq}{m_e} \cdot \mathbf{S}_e \right)$	qv	—	✓	✓
$\mathcal{O}_{14} = (\mathbf{S}_x \cdot \frac{iq}{m_e})(\mathbf{v}_{\text{el}}^\perp \cdot \mathbf{S}_e)$	qv	—	✓	✓
$\mathcal{O}_{15} = \mathbf{S}_x \cdot \frac{q}{m_e} \left[\frac{q}{m_e} \cdot (\mathbf{v}_{\text{el}}^\perp \times \mathbf{S}_e) \right]$	q^2v	—	✓	✓
$\mathcal{O}_{17} = \frac{iq}{m_e} \cdot \tilde{\mathcal{S}}_x \cdot \mathbf{v}_{\text{el}}^\perp \mathbb{1}_e$	qv	—	—	✓
$\mathcal{O}_{18} = \frac{iq}{m_e} \cdot \tilde{\mathcal{S}}_x \cdot \mathbf{S}_e$	q	—	—	✓
$\mathcal{O}_{19} = \frac{q}{m_e} \cdot \tilde{\mathcal{S}}_x \cdot \frac{q}{m_e} \mathbb{1}_e$	q^2	—	—	✓
$\mathcal{O}_{20} = -\frac{q}{m_e} \cdot \tilde{\mathcal{S}}_x \cdot \left(\frac{q}{m_e} \times \mathbf{S}_e \right)$	q^2	—	—	✓
$\mathcal{O}_{21} = \mathbf{v}_{\text{el}}^\perp \cdot \tilde{\mathcal{S}}_x \cdot \mathbf{S}_e$	v	—	—	✓
$\mathcal{O}_{22} = \left(\frac{iq}{m_e} \times \mathbf{v}_{\text{el}}^\perp \right) \cdot \tilde{\mathcal{S}}_x \cdot \mathbf{S}_e + \mathbf{v}_{\text{el}}^\perp \cdot \tilde{\mathcal{S}}_x \cdot \left(\frac{iq}{m_e} \times \mathbf{S}_e \right)$	qv	—	—	✓
$\mathcal{O}_{23} = -\frac{iq}{m_e} \cdot \tilde{\mathcal{S}}_x \cdot (\mathbf{v}_{\text{el}}^\perp \times \mathbf{S}_e)$	qv	—	—	✓
$\mathcal{O}_{24} = \frac{q}{m_e} \cdot \tilde{\mathcal{S}}_x \cdot \left(\frac{q}{m_e} \times \mathbf{v}_{\text{el}}^\perp \right)$	q^2v	—	—	✓
$\mathcal{O}_{25} = \left(\frac{q}{m_e} \cdot \tilde{\mathcal{S}}_x \cdot \mathbf{v}_{\text{el}}^\perp \right) \left(\frac{q}{m_e} \cdot \mathbf{S}_e \right)$	q^2v	—	—	✓
$\mathcal{O}_{26} = \left(\frac{q}{m_e} \cdot \tilde{\mathcal{S}}_x \cdot \frac{q}{m_e} \right) (\mathbf{v}_{\text{el}}^\perp \cdot \mathbf{S}_e)$	q^2v	—	—	✓

Calculation of matrix element squared— method 1

Free electron case

$$\mathcal{M}(\mathbf{q}, \mathbf{v}_{\text{el}}^\perp) = \mathcal{M}(\mathbf{q}, \mathbf{v}_0^\perp) + \mathbf{k} \cdot \nabla_{\mathbf{k}} \mathcal{M}(\mathbf{q}, \mathbf{v}_{\text{el}}^\perp)$$

Catena et al., Phys. Rev. Res. 2, 033195 (2020) (110+ citations)

$$\begin{aligned}\mathbf{v}_{\text{el}}^\perp &= \mathbf{v} - \mathbf{q}/(2\mu_{xe}) - \mathbf{k}/m_e \\ \mathbf{v}_0^\perp &\equiv \mathbf{v} - \mathbf{q}/(2\mu_{xe})\end{aligned}$$

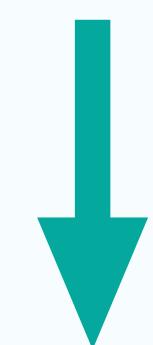
Bound electron case

$$\mathcal{M}_{1 \rightarrow 2} = f_{1 \rightarrow 2}(\mathbf{q}) \mathcal{M}(\mathbf{q}, \mathbf{v}_0^\perp) + \mathbf{f}_{1 \rightarrow 2}(\mathbf{q}) \cdot \nabla_{\mathbf{k}} \mathcal{M}(\mathbf{q}, \mathbf{v}_{\text{el}}^\perp)$$

$$f_{1 \rightarrow 2}(\mathbf{q}) = \int \frac{d^3k}{(2\pi)^3} \tilde{\psi}_2^*(\mathbf{k} + \mathbf{q}) \tilde{\psi}_1(\mathbf{k})$$

$$\mathbf{f}_{1 \rightarrow 2}(\mathbf{q}) = \int \frac{d^3k}{(2\pi)^3} \tilde{\psi}_2^*(\mathbf{k} + \mathbf{q}) \frac{\mathbf{k}}{m_e} \tilde{\psi}_1(\mathbf{k})$$

A new vector form factor



$$\overline{\left| \mathcal{M}_{\text{ion}}^{nl} \right|^2} = \sum_{i=1}^4 R_i^{nl}(q, v) W_i^{nl}(k', q)$$

DM response functions

1 usual $|f_{\text{ion}}^{nl}(q)|^2$ + 3 new atomic response functions

9

- A crucial minus sign is missed from p -space to x -space

$$\mathbf{f}_{1 \rightarrow 2}(\mathbf{q}) = \int \frac{d^3k}{(2\pi)^3} \tilde{\psi}_2^*(\mathbf{k} + \mathbf{q}) \frac{\mathbf{k}}{m_e} \tilde{\psi}_1(\mathbf{k}) \quad \rightarrow \quad \mathbf{f}_{1 \rightarrow 2}(\mathbf{q}) = \int d^3\mathbf{r} \psi_2^*(\mathbf{r}) e^{i\mathbf{q} \cdot \mathbf{r}} \frac{-i\nabla}{m_e} \psi_1(\mathbf{r})$$



Will affect all operators containing \mathbf{v}_{el}^\perp and lead to almost an order of magnitude difference for the rate

- The DM response functions R_s also contain atomic information and the power counting is not obvious

Calculation of matrix element squared— our method

Free electron case

$$\mathcal{M}(\mathbf{q}, \mathbf{v}_{\text{el}}^\perp) = \mathcal{M}(\mathbf{q}, 0) + \mathbf{v}_{\text{el}}^\perp \cdot \nabla_{\mathbf{v}_{\text{el}}^\perp} \mathcal{M}(\mathbf{q}, \mathbf{v}_{\text{el}}^\perp)$$

arXiv: 2405. 04855

Bound electron case

$$\mathcal{M}_{1 \rightarrow 2} = f_S(\mathbf{q}) \mathcal{M}_S + \mathbf{f}_V(\mathbf{q}) \cdot \mathbf{M}_V$$

$$f_S(\mathbf{q}) \equiv f_{1 \rightarrow 2}(\mathbf{q})$$

$$\mathbf{f}_V(\mathbf{q}) \equiv \mathbf{v}_0^\perp f_{1 \rightarrow 2}(\mathbf{q}) - \mathbf{f}_{1 \rightarrow 2}(\mathbf{q})$$

$$\overline{|\mathcal{M}_{1 \rightarrow 2}|^2} = a_0 |f_S|^2 + a_1 |\mathbf{f}_V|^2 + \frac{a_2}{x_e} \left| \frac{\mathbf{q}}{m_e} \cdot \mathbf{f}_V \right|^2 + i a_3 \frac{\mathbf{q}}{m_e} \cdot (\mathbf{f}_V \times \mathbf{f}_V^*) + 2 \Im \left[a_4 f_S \mathbf{f}_V^* \cdot \frac{\mathbf{q}}{m_e} \right]$$

$$\overline{|\mathcal{M}_{\text{ion}}^{n\ell}|^2} \equiv \frac{4Vk^3}{(2\pi)^3} \sum_{m=-\ell}^{\ell} \sum_{\ell'=0}^{\infty} \sum_{m'=-\ell'}^{\ell'} \overline{|\mathcal{M}_{1 \rightarrow 2}|^2} = a_0 \widetilde{W}_0 + a_1 \widetilde{W}_1 + a_2 \widetilde{W}_2$$

Due to the property of atomic wave functions

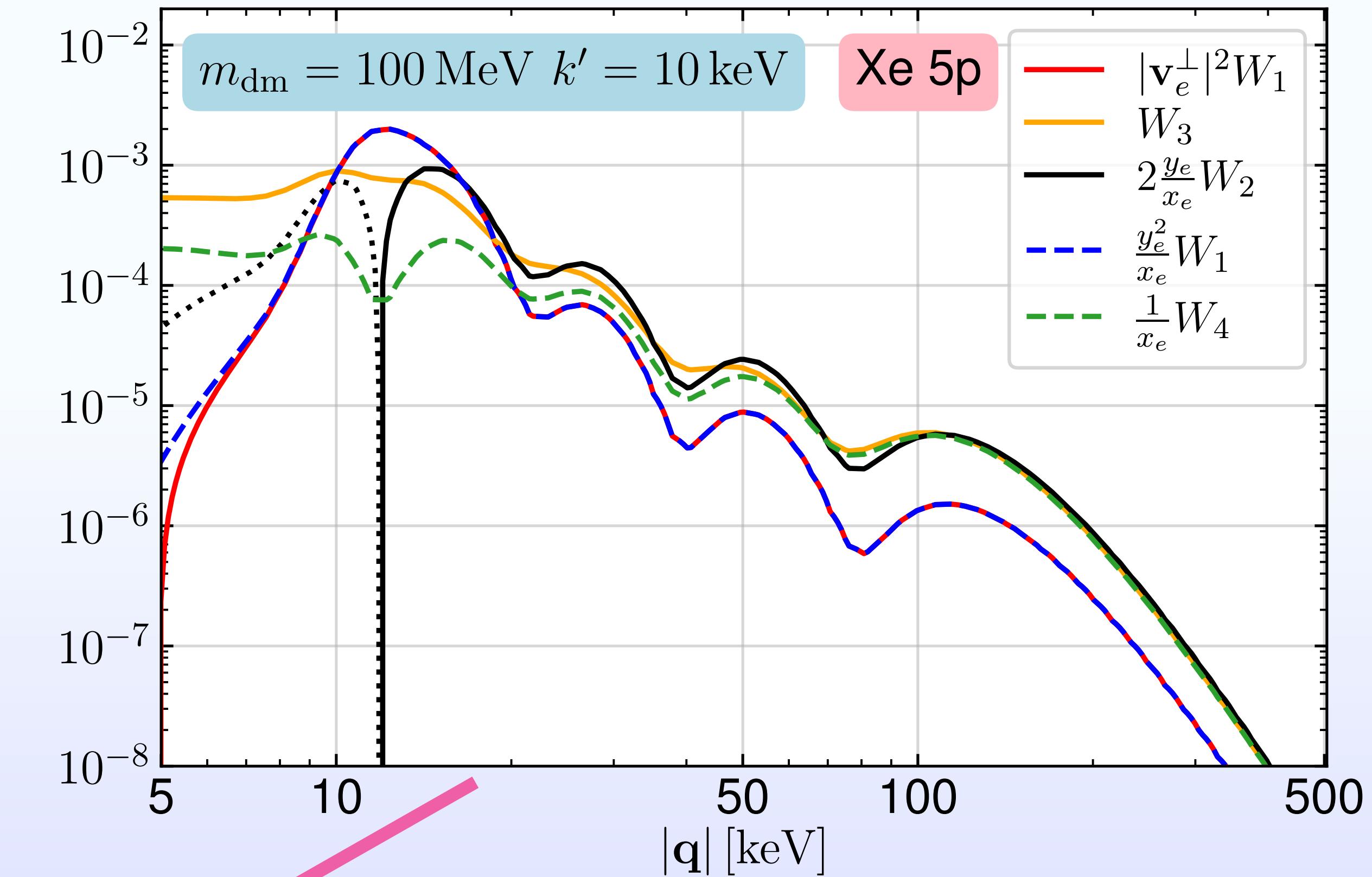
The merit of our approach

- Correctly incorporate the minus sign
- Only three atomic response functions

$$\widetilde{W}_0 = W_1$$

$$\widetilde{W}_1 = |\mathbf{v}_0^\perp|^2 W_1 - 2 \frac{m_e \mathbf{q} \cdot \mathbf{v}_0^\perp}{\mathbf{q}^2} W_2 + W_3$$

$$\widetilde{W}_2 = \frac{(\mathbf{q} \cdot \mathbf{v}_0^\perp)^2}{\mathbf{q}^2} W_1 - 2 \frac{m_e \mathbf{q} \cdot \mathbf{v}_0^\perp}{\mathbf{q}^2} W_2 + \frac{m_e^2}{\mathbf{q}^2} W_4$$



The minus sign leads to a strong cancellation among W_2 and $W_{3,4}$

DM response functions

$$\left| \mathcal{M}_{\text{ion}}^{n\ell} \right|^2 = a_0 \widetilde{W}_0 + a_1 \widetilde{W}_1 + a_2 \widetilde{W}_2$$

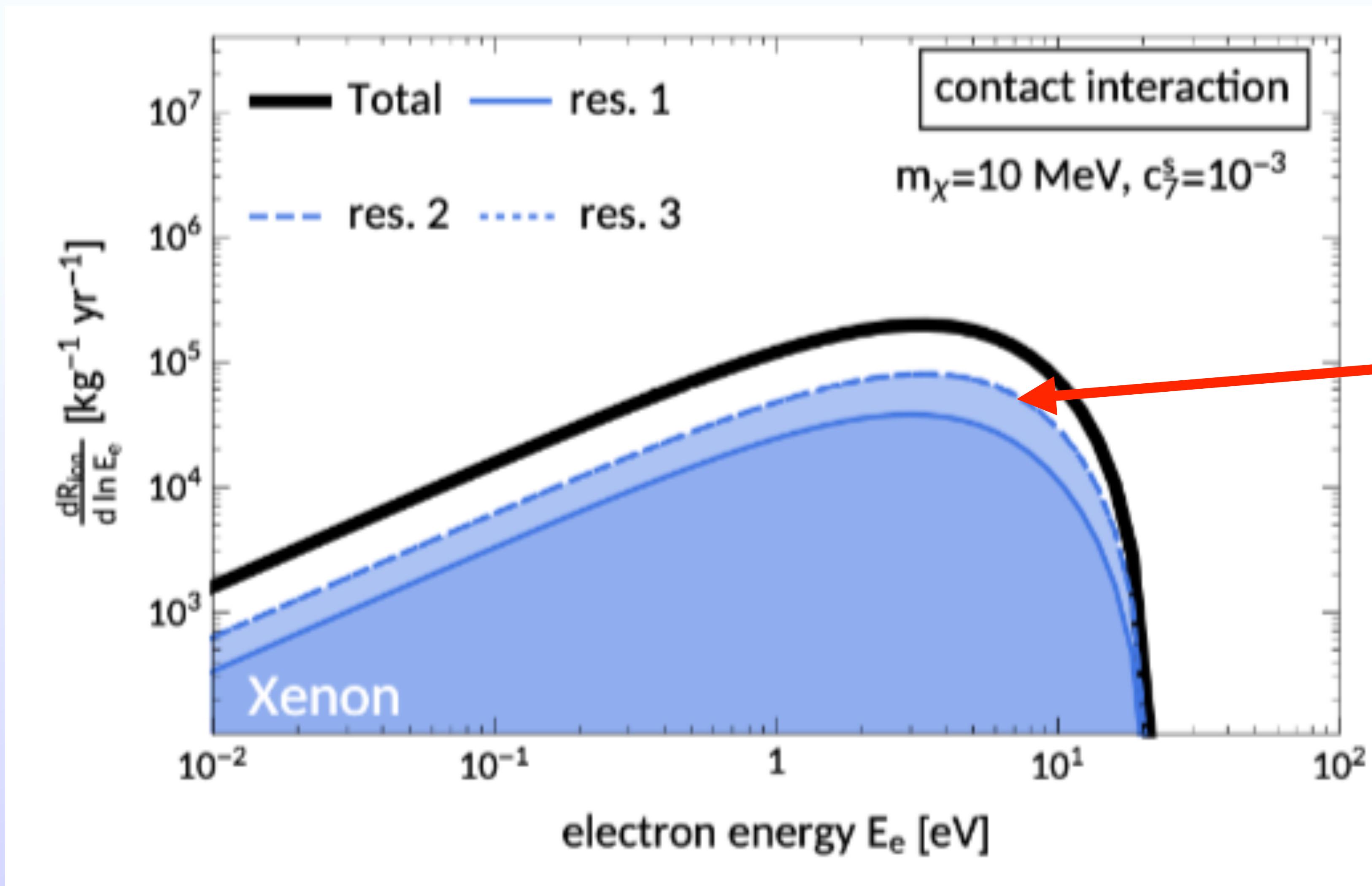
- a_0 and $a_{1,2}$ involve different NR operators
- Clear power counting for q
- Do not contain any atomic properties

Type	DM response functions
Scalar DM	$a_0 = c_1 ^2 + \frac{1}{4} c_{10} ^2 \mathbf{x}_e$ $a_1 = \frac{1}{4} c_7 ^2 + \frac{1}{4} c_3 ^2 \mathbf{x}_e$ $a_2 = -\frac{1}{4} c_3 ^2 \mathbf{x}_e$ <p style="color: green; margin-left: 20px;">• Usual SI and SD interactions</p> <p style="background-color: green; color: black; padding: 5px; margin-left: 20px;">Only $\widetilde{W}_1(\widetilde{W}_0)$ is enough</p>
Fermion DM	$a_0 = c_1 ^2 + \frac{3}{16} c_4 ^2 + (\frac{1}{8} c_9 ^2 + \frac{1}{4} c_{10} ^2 + \frac{1}{4} c_{11} ^2 + \frac{1}{8}\Re[c_4c_6^*]) \mathbf{x}_e + \frac{1}{16} c_6 ^2 \mathbf{x}_e^2$ $a_1 = \frac{1}{4} c_7 ^2 + \frac{1}{4} c_8 ^2 + \frac{1}{8} c_{12} ^2 + (\frac{1}{4} c_3 ^2 + \frac{1}{4} c_5 ^2 + \frac{1}{16} c_{13} ^2 + \frac{1}{16} c_{14} ^2 - \frac{1}{8}\Re[c_{12}c_{15}^*]) \mathbf{x}_e$ $+ \frac{1}{16} c_{15} ^2 \mathbf{x}_e^2$ $a_2 = -(\frac{1}{4} c_3 ^2 + \frac{1}{4} c_5 ^2 - \frac{1}{8}\Re[c_{12}c_{15}^*] - \frac{1}{8}\Re[c_{13}c_{14}^*]) \mathbf{x}_e - \frac{1}{16} c_{15} ^2 \mathbf{x}_e^2$
Vector DM	$a_0 = c_1 ^2 + \frac{1}{2} c_4 ^2 + (\frac{1}{3} c_9 ^2 + \frac{1}{4} c_{10} ^2 + \frac{2}{3} c_{11} ^2 + \frac{5}{36} c_{18} ^2 + \frac{1}{3}\Re[c_4c_6^*]) \mathbf{x}_e$ $+ (\frac{1}{6} c_6 ^2 + \frac{2}{9} c_{19} ^2 + \frac{1}{12} c_{20} ^2) \mathbf{x}_e^2$ $a_1 = \frac{1}{4} c_7 ^2 + \frac{2}{3} c_8 ^2 + \frac{1}{3} c_{12} ^2 + \frac{5}{36} c_{21} ^2 + (\frac{1}{4} c_3 ^2 + \frac{2}{3} c_5 ^2 + \frac{1}{6} c_{13} ^2 + \frac{1}{6} c_{14} ^2 + \frac{1}{6} c_{17} ^2$ $+ \frac{3}{8} c_{22} ^2 + \frac{7}{72} c_{23} ^2 - \frac{1}{3}\Re[c_{12}c_{15}^*] + \frac{1}{12}\Re[c_{21}c_{25}^*] - \frac{1}{18}\Re[c_{21}c_{26}^*] + \frac{1}{12}\Re[c_{22}c_{23}^*]) \mathbf{x}_e$ $+ (\frac{1}{6} c_{15} ^2 + \frac{1}{6} c_{24} ^2 + \frac{1}{24} c_{25} ^2 + \frac{1}{18} c_{26} ^2) \mathbf{x}_e^2$ $a_2 = -(\frac{1}{4} c_3 ^2 + \frac{2}{3} c_5 ^2 - \frac{1}{18} c_{17} ^2 + \frac{7}{24} c_{22} ^2 + \frac{1}{72} c_{23} ^2 - \frac{1}{3}\Re[c_{12}c_{15}^*] - \frac{1}{3}\Re[c_{13}c_{14}^*]$ $- \frac{1}{36}\Re[c_{21}c_{25}^*] - \frac{1}{6}\Re[c_{21}c_{26}^*] + \frac{1}{4}\Re[c_{22}c_{23}^*]) \mathbf{x}_e$ $- (\frac{1}{6} c_{15} ^2 + \frac{1}{6} c_{24} ^2 - \frac{1}{72} c_{25} ^2 - \frac{1}{9}\Re[c_{25}c_{26}^*]) \mathbf{x}_e^2$ <p style="color: cyan; margin-left: 20px;">$x_e = \frac{\mathbf{q}^2}{m_e^2}$</p>

Example: contributions from different response functions for \mathcal{O}_7

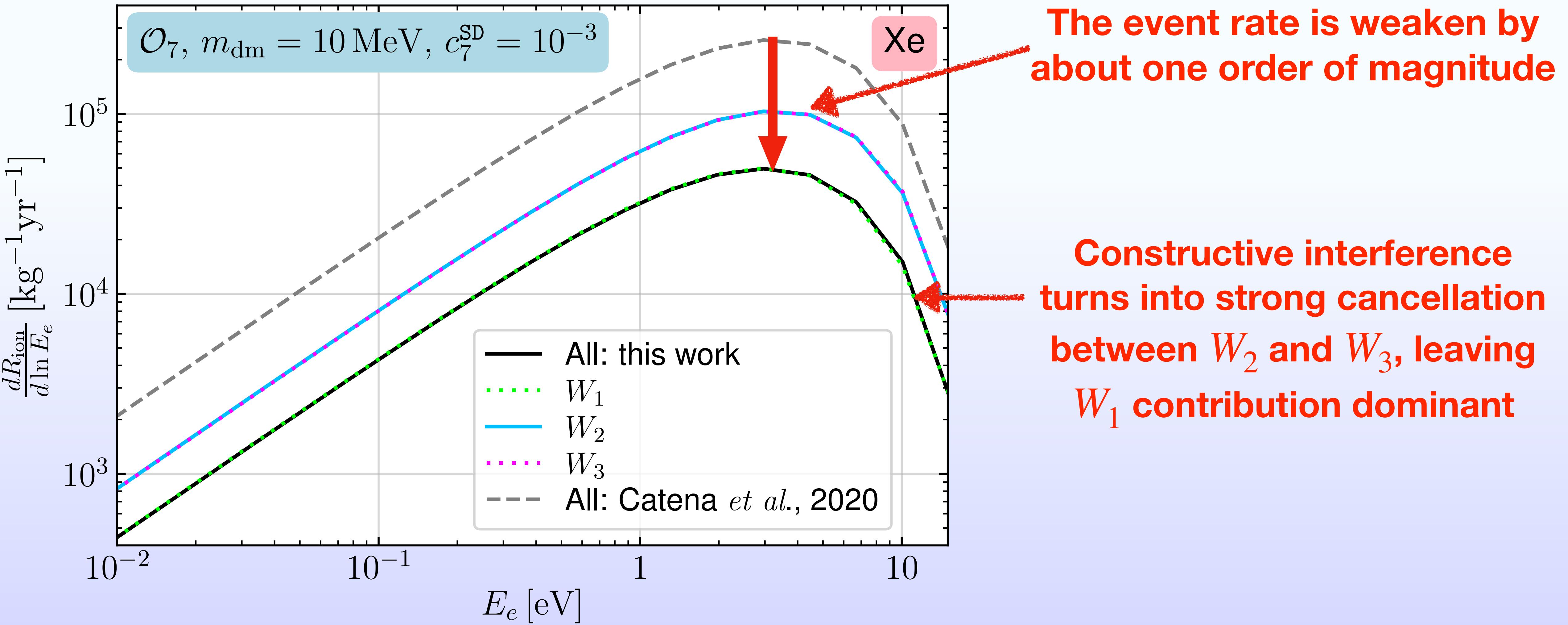
Catena et al., 1912.08204

$$\mathcal{O}_7 = \mathbf{1}_x \mathbf{v}_{\text{el}}^\perp \cdot \mathbf{S}_e$$



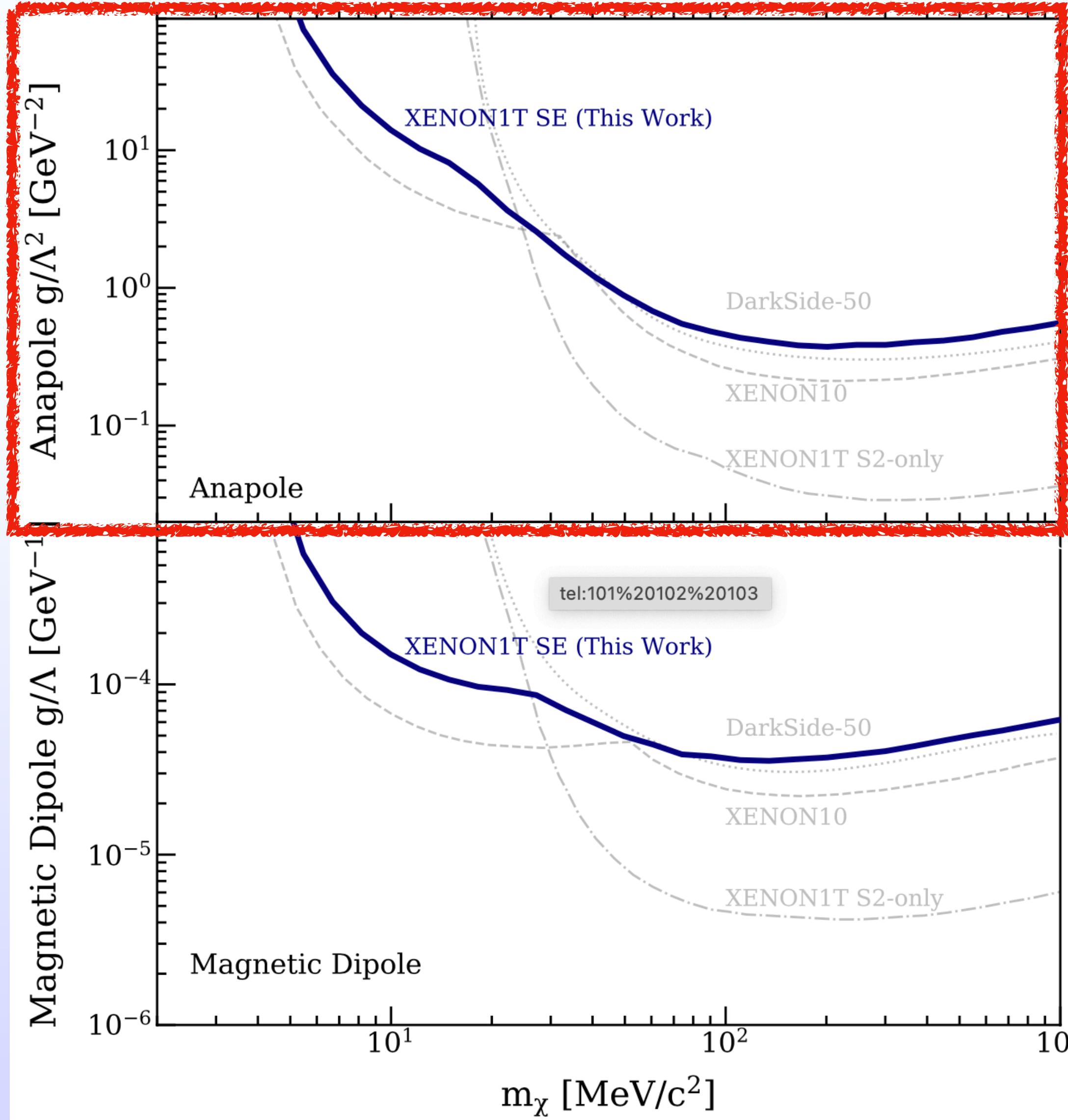
Contributions from W_2 and W_3 dominate

Revisited event rate from \mathcal{O}_7



XENON1T SE constraints on EM form factors of DM

XENON Collaboration, 2112.12116

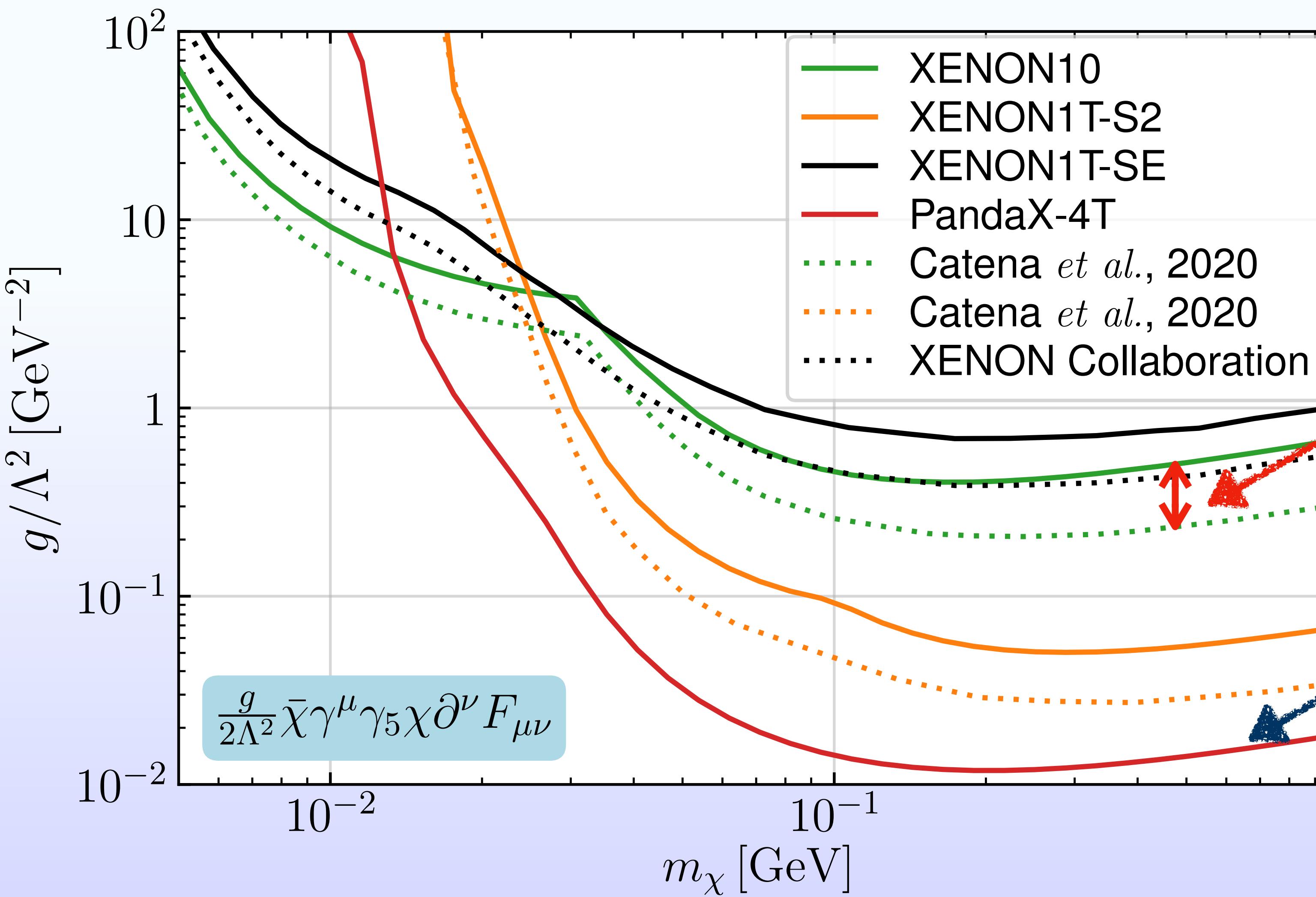


$$a_\chi (\bar{\chi} \gamma^\mu \gamma_5 \chi) \partial^\nu F_{\mu\nu} \rightarrow 8a_\chi e m_\chi m_e (\mathcal{O}_8 - \mathcal{O}_9)$$

The constraint on the anapole operator greatly affected by the new atomic response functions

Revisited constraints on the anapole operator

$$a_\chi (\bar{\chi} \gamma^\mu \gamma_5 \chi) \partial^\nu F_{\mu\nu} \rightarrow 8a_\chi e m_\chi m_e (\mathcal{O}_8 - \mathcal{O}_9)$$



The constraints are weakened by a factor of 2 if the sign of W_2 is corrected.

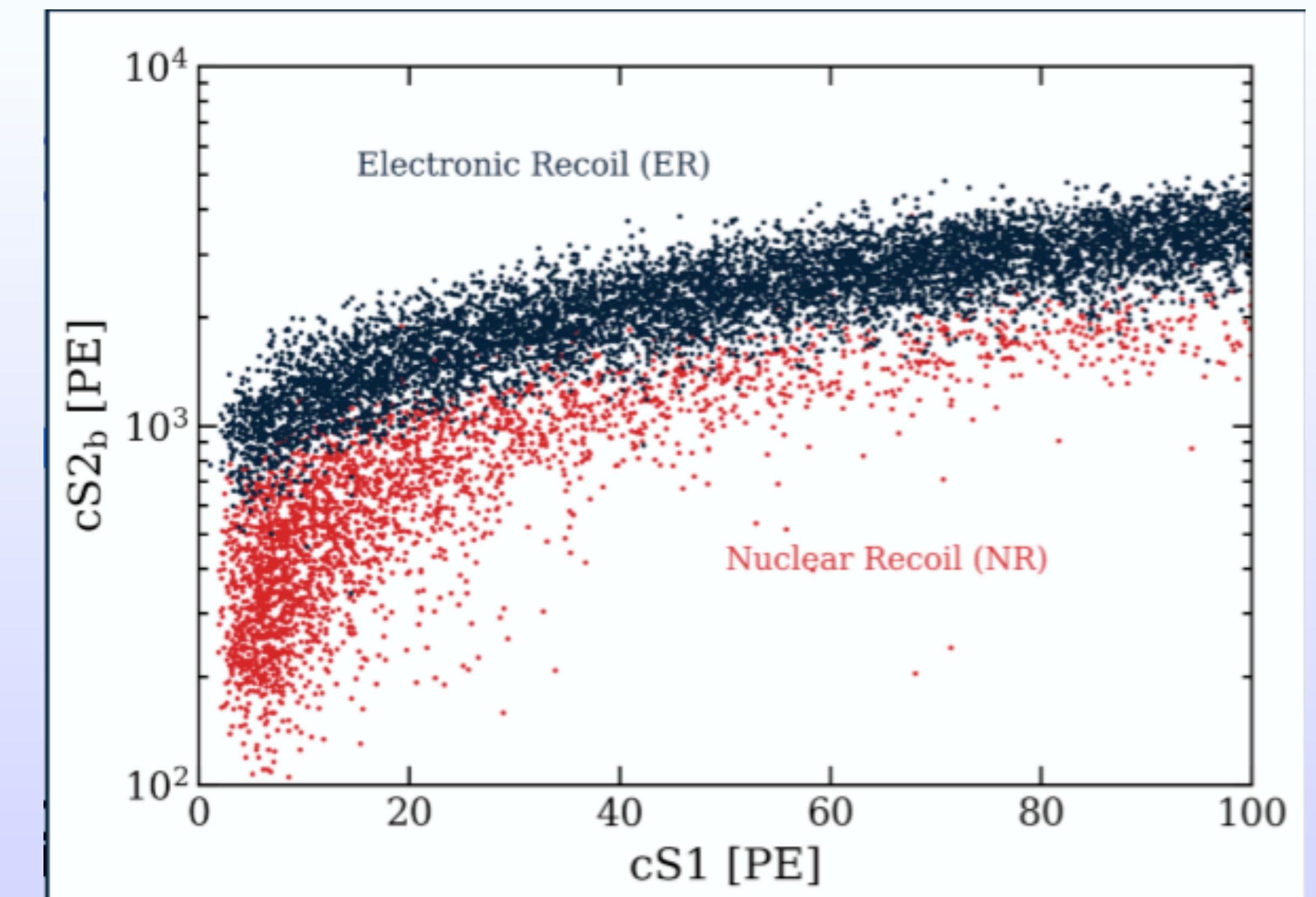
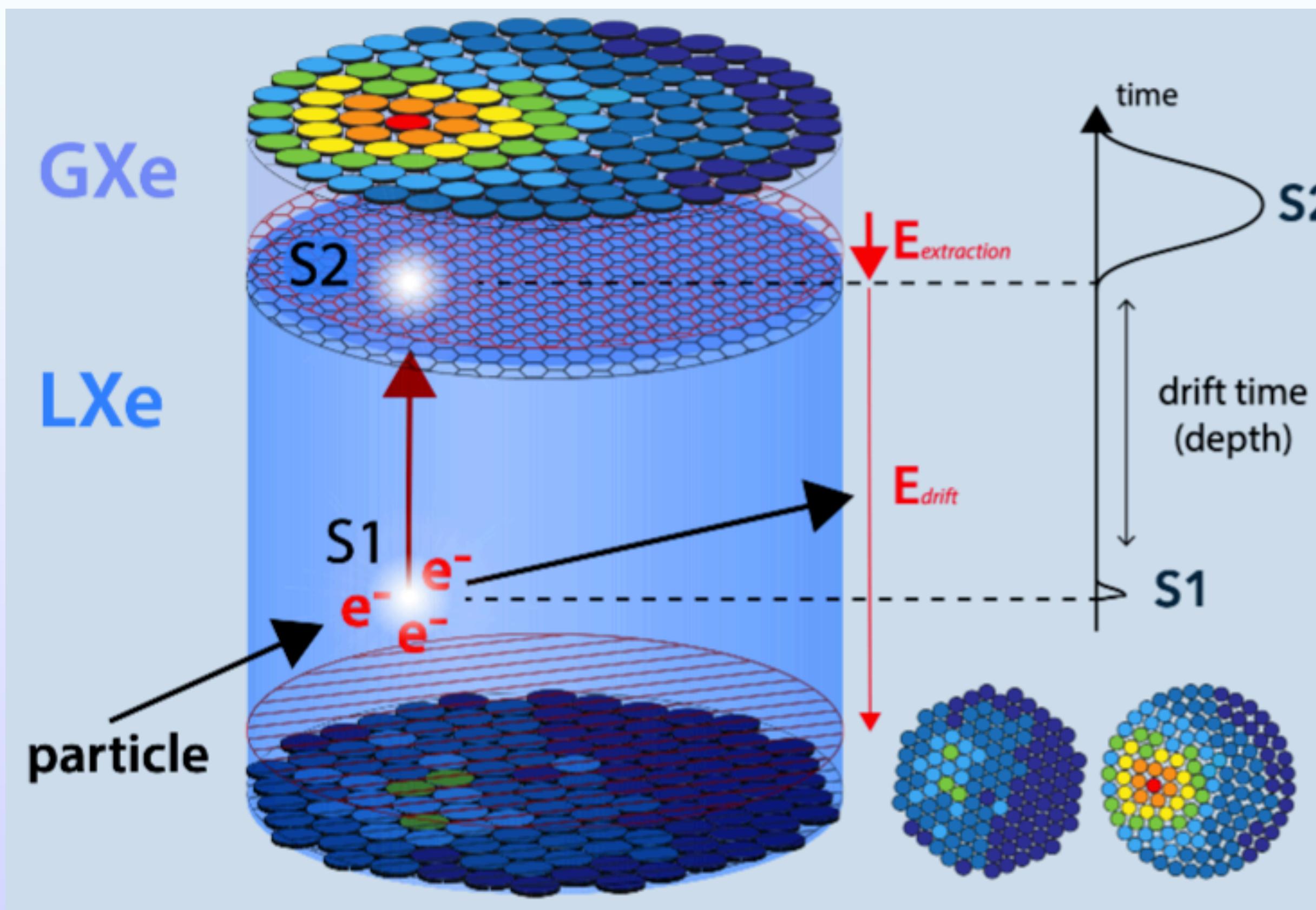
PandaX-4T set the most stringent constraint when $m_\chi \gtrsim 20$ MeV

Constraints on non-relativistic interactions

$$\frac{dN_s}{dN_{\text{PE}}} = \epsilon \omega \frac{1}{m_T} \sum_{n \ell} \sum_{n_e=1}^{\infty} \frac{d\mathcal{R}_{\text{ion}}^{n\ell}}{dn_e} P(N_{\text{PE}}|n_e)$$

XENON10 collaboration, 1104.3088
 XENON collaboration, 1907.11485]
 PandaX collaboration , 2212.10067

Using S2-only data from xenon experiments, including XENON10, XENON1T, and PandaX-4T



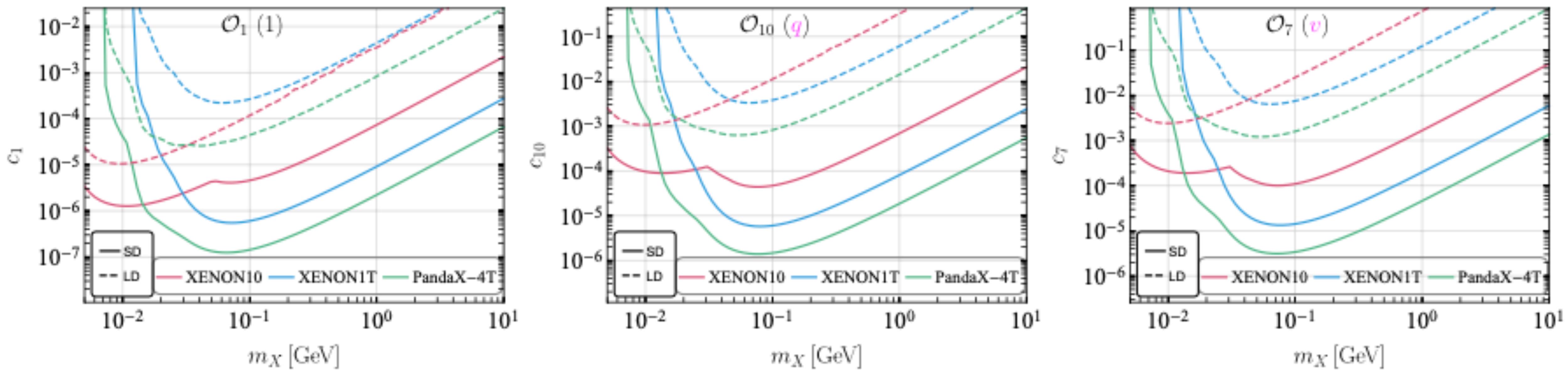
12 independent constraints

Class	Chosen one	Scalar DM (c_i^s)	Fermion DM (c_i^f)	Vector DM (c_i^v)	RFs
a_0	$x_e^0 : c_1^v$	$c_1^s = c_1^v$	$c_{1,4}^f = \left(1, \sqrt{\frac{16}{3}}\right) c_1^v$	$c_4^v = \sqrt{2} c_1^v$	\widetilde{W}_0
	$x_e^1 : c_{10}^v$	$c_{10}^s = c_{10}^v$	$c_{9,10,11}^f = (\sqrt{2}, 1, 1)c_{10}^v$	$c_{9,11,18}^v = \left(\sqrt{\frac{3}{4}}, \sqrt{\frac{3}{8}}, \sqrt{\frac{9}{5}}\right) c_{10}^v$	$\frac{1}{4}\widetilde{W}_0$
	$x_e^2 : c_6^v$	—	$c_6^f = \sqrt{\frac{8}{3}} c_6^v$	$c_{19,20}^v = \left(\sqrt{\frac{3}{4}}, \sqrt{2}\right) c_6^v$	$\frac{1}{6}\widetilde{W}_0$
$a_{1,2}$	$x_e^0 : c_7^v$	$c_7^s = c_7^v$	$c_{7,8,12}^f = (1, 1, \sqrt{2})c_7^v$	$c_{8,12,21}^v = \left(\sqrt{\frac{3}{8}}, \sqrt{\frac{3}{4}}, \sqrt{\frac{9}{5}}\right) c_7^v$	$\frac{1}{4}\widetilde{W}_1$
	$x_e^1 : c_3^v$	$c_3^s = c_3^v$	$c_{3,5}^f = (1, 1)c_3^v$	$c_5^v = \sqrt{\frac{3}{8}} c_3^v$	$\frac{1}{4}(\widetilde{W}_1 - \widetilde{W}_2)$
	$x_e^1 : c_{13}^v$	—	$c_{13,14}^f = \left(\sqrt{\frac{8}{3}}, \sqrt{\frac{8}{3}}\right) c_{13}^v$	$c_{14}^v = c_{13}^v$	$\frac{1}{6}\widetilde{W}_1$
	$x_e^2 : c_{15}^v$	—	$c_{15}^f = \sqrt{\frac{8}{3}} c_{15}^v$	$c_{24}^v = c_{15}^v$	$\frac{1}{6}(\widetilde{W}_1 - \widetilde{W}_2)$
	$x_e^1 : c_{17}^v$	—	—	✓	$\frac{1}{18}(3\widetilde{W}_1 + \widetilde{W}_2)$
	$x_e^1 : c_{22}^v$	—	—	✓	$\frac{1}{24}(9\widetilde{W}_1 - 7\widetilde{W}_2)$
	$x_e^1 : c_{23}^v$	—	—	✓	$\frac{1}{72}(7\widetilde{W}_1 - \widetilde{W}_2)$
	$x_e^2 : c_{25}^v$	—	—	✓	$\frac{1}{72}(3\widetilde{W}_1 + \widetilde{W}_2)$
	$x_e^2 : c_{26}^v$	—	—	✓	$\frac{1}{18}\widetilde{W}_1$

Velocity-independent

Velocity-dependent

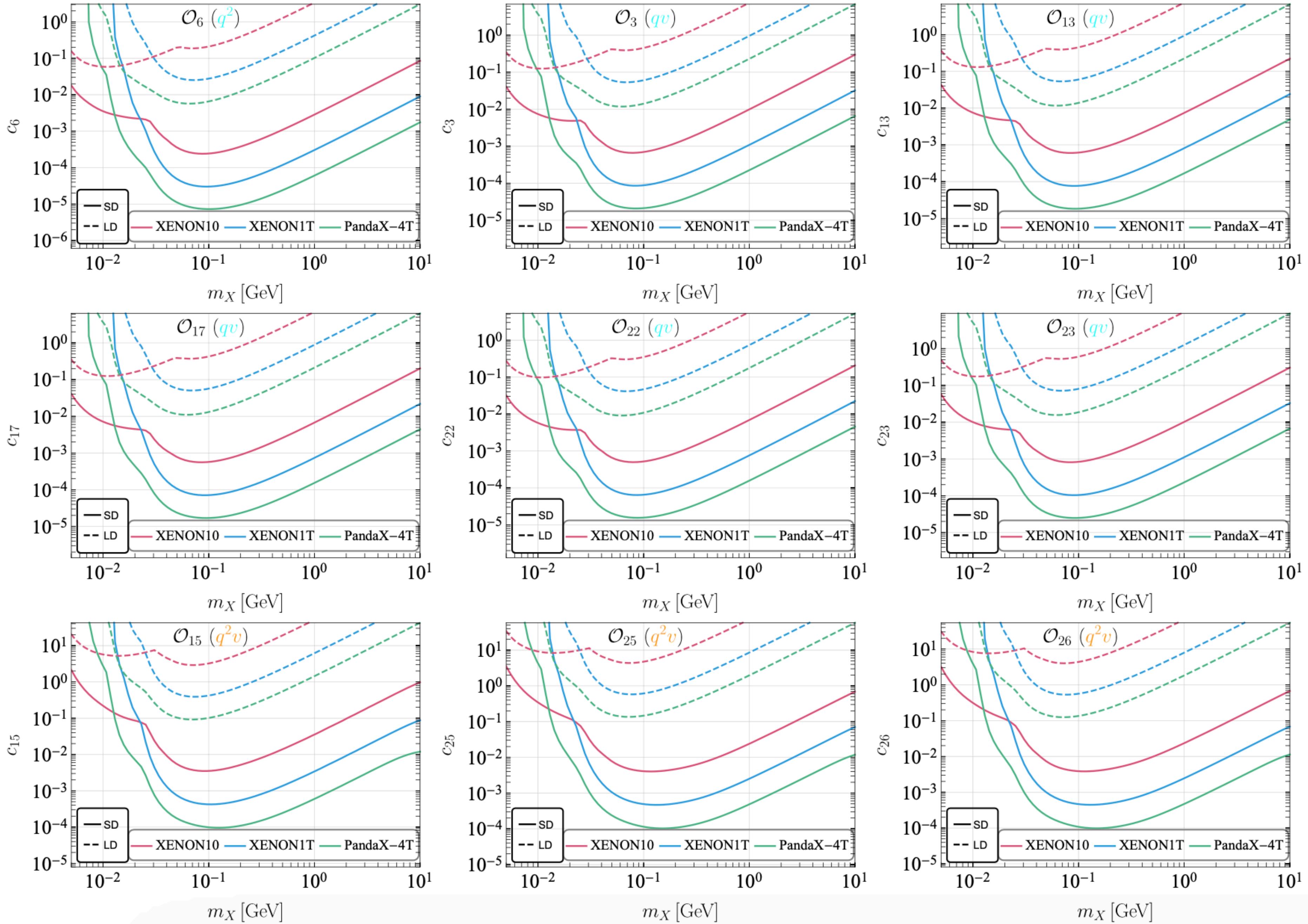
Constraints on the NR operators



$$\text{SD} \rightarrow \text{LD} : \quad c_i \rightarrow c_i (\alpha_{\text{em}} m_e)^2 / q^2$$

- The constraints follows consistently with the power counting of q and/or v
- The suppression from v is comparable with that from q
- $q/v \rightarrow \mathcal{O}(10^{-1})$

PandaX-4T sets the most stringent constraints when $m_X \gtrsim 20$ MeV



Constraints on the relativistic operators

Scalar and fermion DM cases

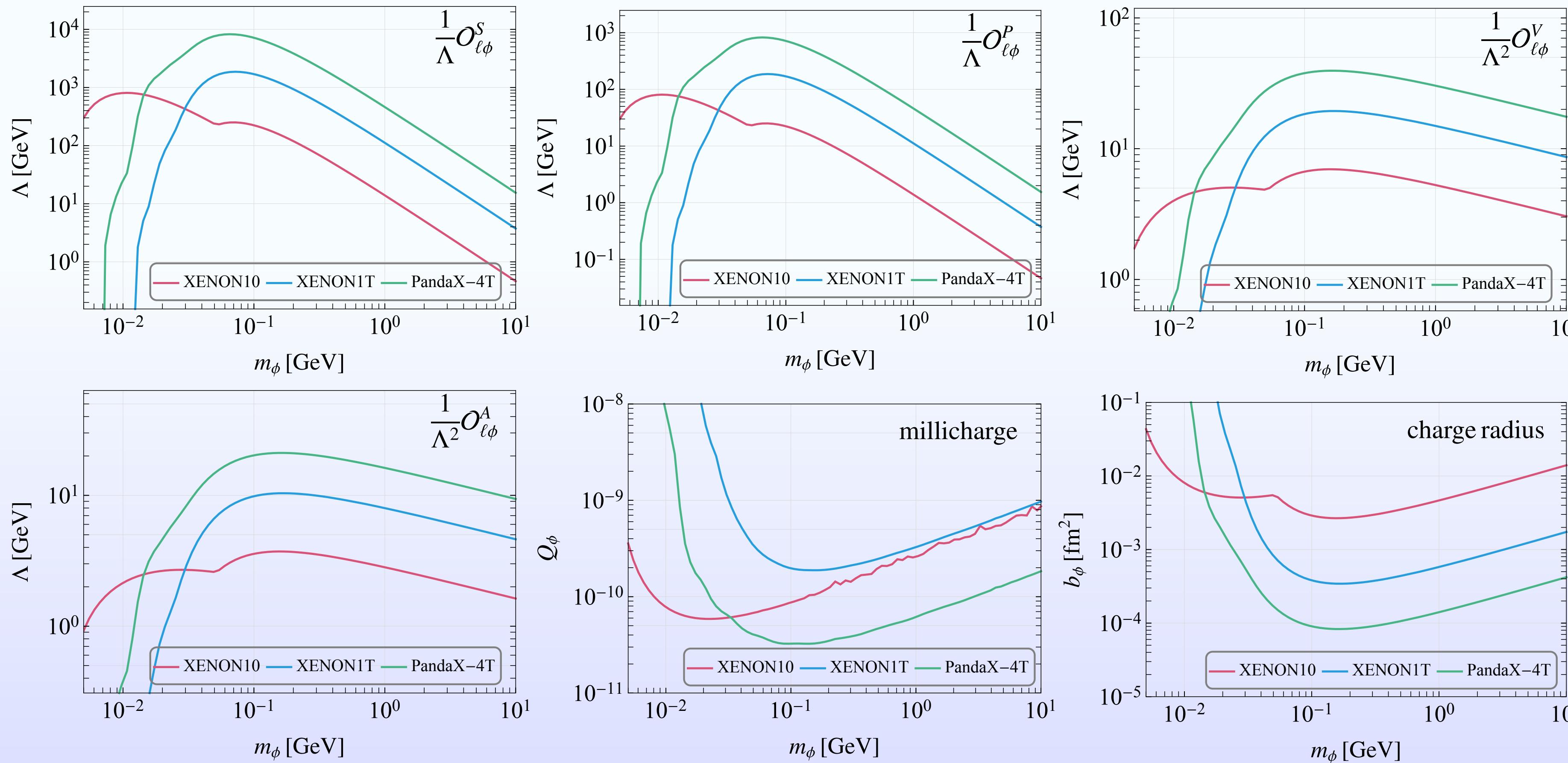
Dim	Relativistic operators	NR reduction
Scalar case		
dim-5	$\mathcal{O}_{\ell\phi}^S = (\bar{\ell}\ell)(\phi^\dagger\phi)$	$2m_e \mathcal{O}_1$
	$\mathcal{O}_{\ell\phi}^P = (\bar{\ell}i\gamma_5\ell)(\phi^\dagger\phi)$	$-2m_e \mathcal{O}_{10}$
dim-6	$\mathcal{O}_{\ell\phi}^V = (\bar{\ell}\gamma^\mu\ell)(\phi^\dagger i\overleftrightarrow{\partial}_\mu\phi) (\times)$	$4m_e m_\phi \mathcal{O}_1$
	$\mathcal{O}_{\ell\phi}^A = (\bar{\ell}\gamma^\mu\gamma_5\ell)(\phi^\dagger i\overleftrightarrow{\partial}_\mu\phi) (\times)$	$-8m_e m_\phi \mathcal{O}_7$
	$\mathcal{L}_\phi^Q = (\partial_\mu - iQ_\phi e A_\mu)\phi ^2 (\times)$	$-4Q_\phi e^2 \frac{m_e m_\phi}{q^2} \mathcal{O}_1$
	$\mathcal{L}_\phi^{\text{cr}} = b_\phi (\phi^\dagger i\overleftrightarrow{\partial}^\mu\phi) \partial^\nu F_{\mu\nu} (\times)$	$4b_\phi e m_e m_\phi \mathcal{O}_1$
Fermion case		
dim-6	$\mathcal{O}_{\ell\chi_1}^S = (\bar{\ell}\ell)(\bar{\chi}\chi)$	$4m_e m_\chi \mathcal{O}_1$
	$\mathcal{O}_{\ell\chi_2}^S = (\bar{\ell}\ell)(\bar{\chi}i\gamma_5\chi)$	$4m_e^2 \mathcal{O}_{11}$
	$\mathcal{O}_{\ell\chi_1}^P = (\bar{\ell}i\gamma_5\ell)(\bar{\chi}\chi)$	$-4m_e m_\chi \mathcal{O}_{10}$
	$\mathcal{O}_{\ell\chi_2}^P = (\bar{\ell}i\gamma_5\ell)(\bar{\chi}i\gamma_5\chi)$	$4m_e^2 \mathcal{O}_6$
	$\mathcal{O}_{\ell\chi_1}^V = (\bar{\ell}\gamma^\mu\ell)(\bar{\chi}\gamma_\mu\chi) (\times)$	$4m_e m_\chi \mathcal{O}_1$
	$\mathcal{O}_{\ell\chi_2}^V = (\bar{\ell}\gamma^\mu\ell)(\bar{\chi}\gamma_\mu\gamma_5\chi)$	$8m_e m_\chi (\mathcal{O}_8 - \mathcal{O}_9)$
	$\mathcal{O}_{\ell\chi_1}^A = (\bar{\ell}\gamma^\mu\gamma_5\ell)(\bar{\chi}\gamma_\mu\chi) (\times)$	$-8m_e (m_\chi \mathcal{O}_7 + m_e \mathcal{O}_9)$
	$\mathcal{O}_{\ell\chi_2}^A = (\bar{\ell}\gamma^\mu\gamma_5\ell)(\bar{\chi}\gamma_\mu\gamma_5\chi)$	$-16m_e m_\chi \mathcal{O}_4$
	$\mathcal{O}_{\ell\chi_1}^T = (\bar{\ell}\sigma^{\mu\nu}\ell)(\bar{\chi}\sigma_{\mu\nu}\chi) (\times)$	$32m_e m_\chi \mathcal{O}_4$
	$\mathcal{O}_{\ell\chi_2}^T = (\bar{\ell}\sigma^{\mu\nu}\ell)(\bar{\chi}i\sigma_{\mu\nu}\gamma_5\chi) (\times)$	$8m_e (m_e \mathcal{O}_{10} - m_\chi \mathcal{O}_{11} - 4m_\chi \mathcal{O}_{12})$
	$\mathcal{L}_\chi^Q = \bar{\chi}i\gamma^\mu(\partial_\mu - iQ_\chi e A_\mu)\chi (\times)$	$-4Q_\chi e^2 \frac{m_e m_\chi}{q^2} \mathcal{O}_1$
	$\mathcal{L}_\chi^{\text{mdm}} = \mu_\chi (\bar{\chi}\sigma^{\mu\nu}\chi) F_{\mu\nu} (\times)$	$4\mu_\chi e \left(m_e \mathcal{O}_1 + 4m_\chi \mathcal{O}_4 + \frac{4m_e^2 m_\chi}{q^2} (\mathcal{O}_5 - \mathcal{O}_6) \right)$
	$\mathcal{L}_\chi^{\text{edm}} = d_\chi (\bar{\chi}i\sigma^{\mu\nu}\gamma_5\chi) F_{\mu\nu} (\times)$	$d_\chi e \frac{16m_e^2 m_\chi}{q^2} \mathcal{O}_{11}$
	$\mathcal{L}_\chi^{\text{cr}} = b_\chi (\bar{\chi}\gamma^\mu\chi) \partial^\nu F_{\mu\nu} (\times)$	$4b_\chi e m_e m_\chi \mathcal{O}_1$
	$\mathcal{L}_\chi^{\text{anap.}} = a_\chi (\bar{\chi}\gamma^\mu\gamma_5\chi) \partial^\nu F_{\mu\nu}$	$8a_\chi e m_e m_\chi (\mathcal{O}_8 - \mathcal{O}_9)$

$$u^s(\mathbf{p}) = \frac{1}{\sqrt{4m}} \begin{pmatrix} (2m - \mathbf{p} \cdot \boldsymbol{\sigma}) \xi^s \\ (2m + \mathbf{p} \cdot \boldsymbol{\sigma}) \xi^s \end{pmatrix} + \mathcal{O}(\mathbf{p}^2)$$

$$\xi^{s'\dagger} \xi^s \rightarrow \mathbb{1}, \quad \xi^{s'\dagger} \frac{\boldsymbol{\sigma}}{2} \xi^s \rightarrow \mathbf{S},$$

→ interference effect

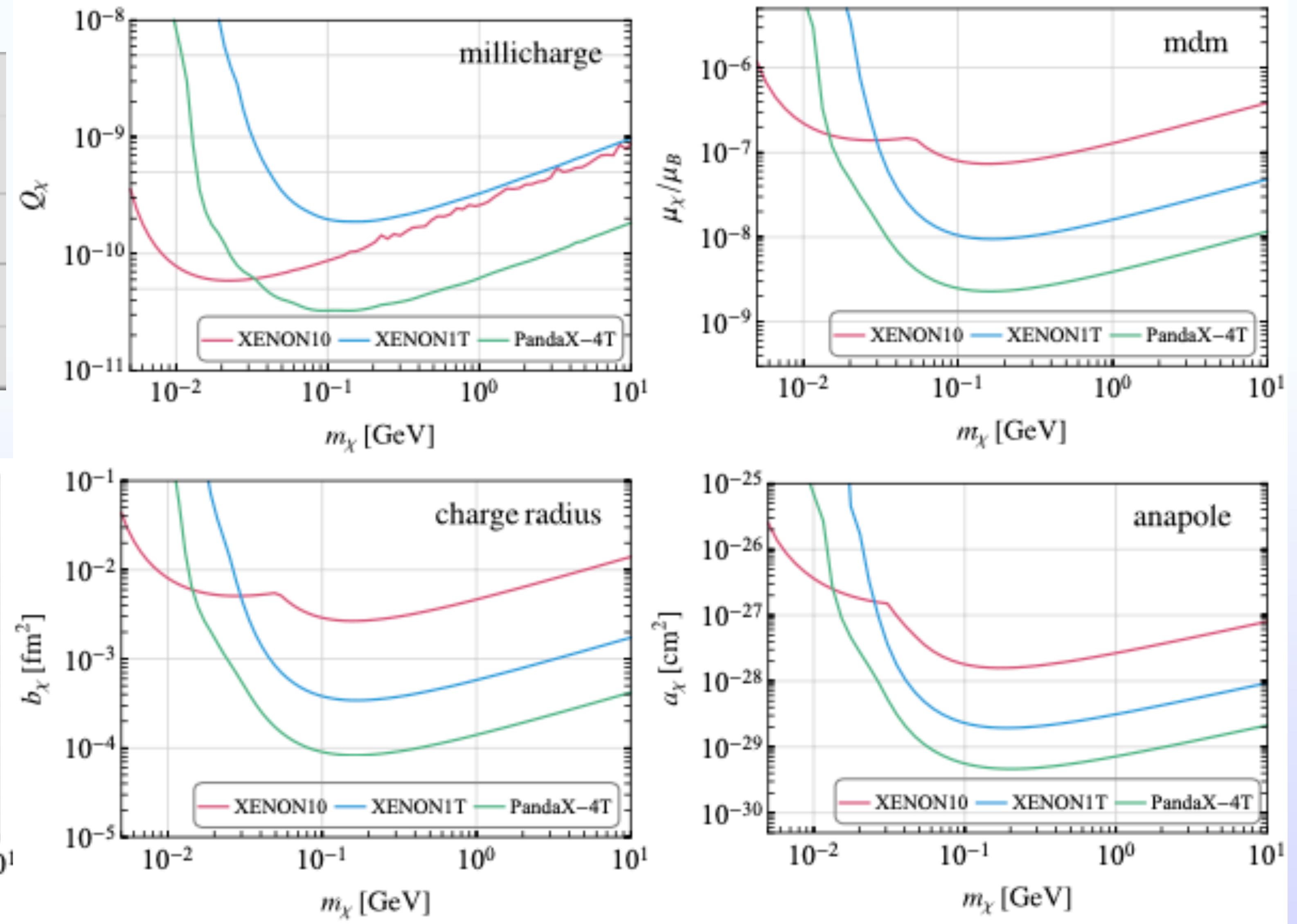
Constraints on the scalar DM case



$$\begin{aligned}
 \mathcal{O}_{\ell\phi}^S &= (\bar{\ell}\ell)(\phi^\dagger\phi) \\
 \mathcal{O}_{\ell\phi}^P &= (\bar{\ell}i\gamma_5\ell)(\phi^\dagger\phi) \\
 \mathcal{O}_{\ell\phi}^V &= (\bar{\ell}\gamma^\mu\ell)(\phi^\dagger i\overleftrightarrow{\partial}_\mu\phi) \quad (\times) \\
 \mathcal{O}_{\ell\phi}^A &= (\bar{\ell}\gamma^\mu\gamma_5\ell)(\phi^\dagger i\partial_\mu\phi) \quad (\times) \\
 \mathcal{L}_\phi^Q &= |(\partial_\mu - iQ_\phi e A_\mu)\phi|^2 \quad (\times) \\
 \mathcal{L}_\phi^{\text{cr}} &= b_\phi(\phi^\dagger i\overleftrightarrow{\partial}^\mu\phi)\partial^\nu F_{\mu\nu} \quad (\times)
 \end{aligned}$$

Constraints on the fermion DM EM property

$$\begin{aligned}\mathcal{L}_\chi^Q &= \bar{\chi} i \gamma^\mu (\partial_\mu - i Q_\chi e A_\mu) \chi \quad (\times) \\ \mathcal{L}_\chi^{\text{mdm}} &= \mu_\chi (\bar{\chi} \sigma^{\mu\nu} \chi) F_{\mu\nu} \quad (\times) \\ \mathcal{L}_\chi^{\text{edm}} &= d_\chi (\bar{\chi} i \sigma^{\mu\nu} \gamma_5 \chi) F_{\mu\nu} \quad (\times) \\ \mathcal{L}_\chi^{\text{cr}} &= b_\chi (\bar{\chi} \gamma^\mu \chi) \partial^\nu F_{\mu\nu} \quad (\times) \\ \mathcal{L}_\chi^{\text{anap.}} &= a_\chi (\bar{\chi} \gamma^\mu \gamma_5 \chi) \partial^\nu F_{\mu\nu}\end{aligned}$$



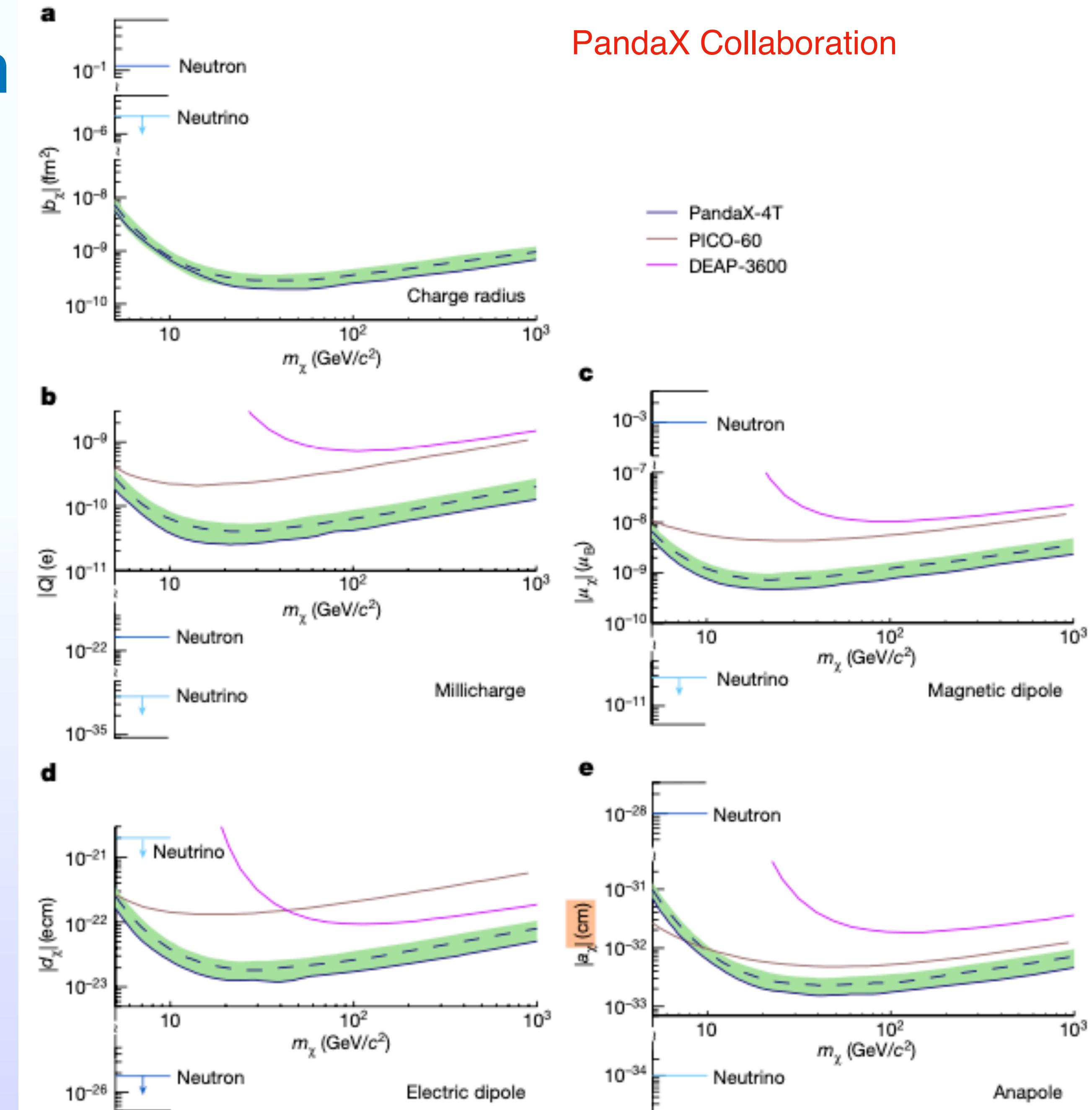
Comparison with the NR data

- Complementarity

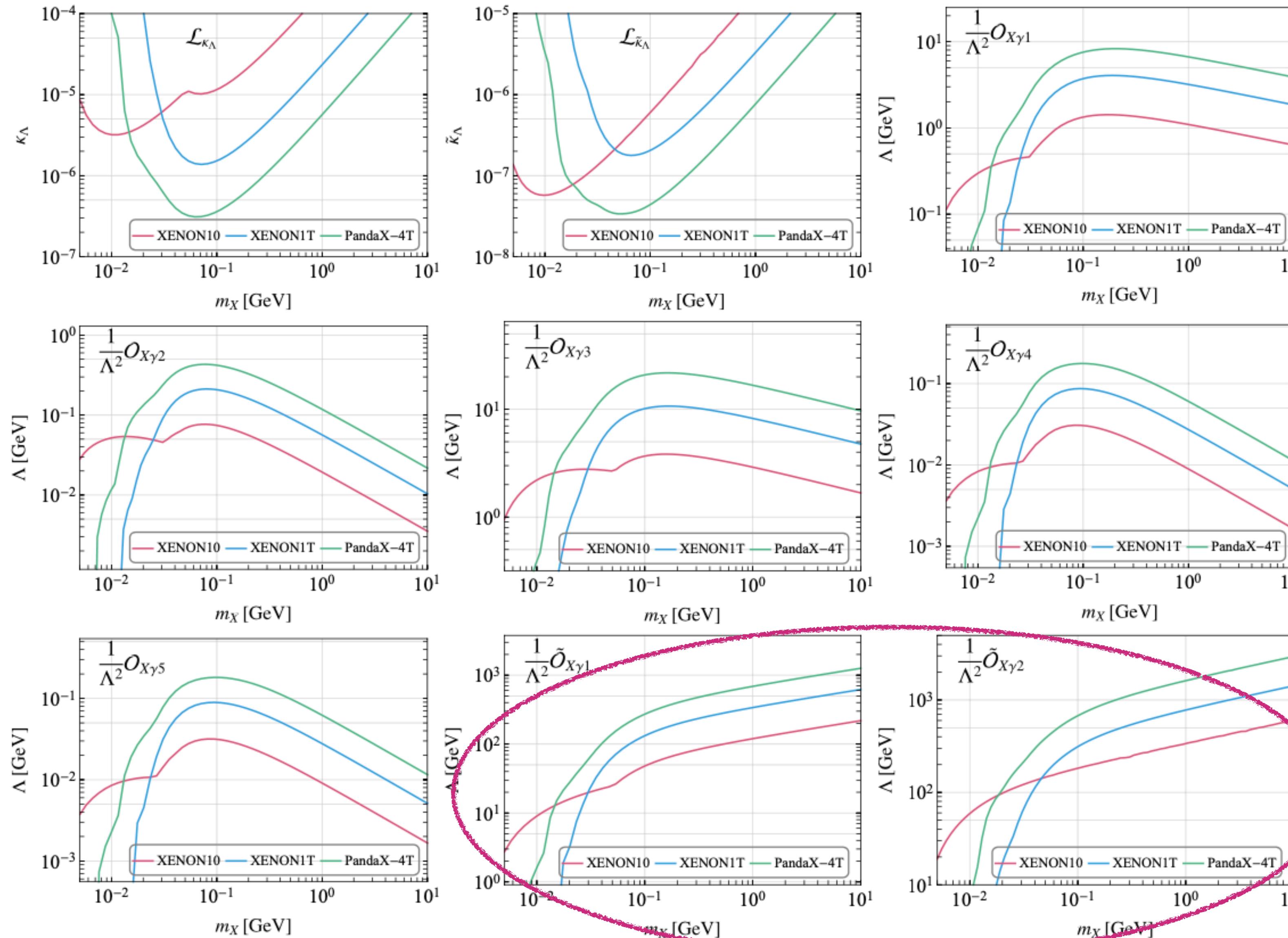
NR is sensitive for m_χ above a few GeVs

ER is sensitive to m_χ below 10 GeV

- Generally, the NR constraints are stronger than those of ER.
- For millicharge and mdm, the ER constraint is comparable to that of NR



Constraints on the vector DM case



Some examples involving photon field

Different behavior due to different parametrization and DM mass dependence

Become stronger as m_X increase

Summary

- We find a crucial minus sign was missed for W_2 in 1912.08204, which has significant phenomenological consequences on some specific DM scenarios.
- A more compact amplitude squared is provided for the general DM-electron interactions for three DM scenarios.
- A matching dictionary between the relativistic and NR operators is given.
- The constraints from the xenon target experiments were studied, and we find the PandaX-4T set the most stringent constraints on the effective operators when $m_{\text{DM}} \gtrsim 20$ MeV.

Thanks for your attention!