



Degenerate Oscillation in Neutron Star

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Fu, **SFG**, Guo & Wang [arXiv:2405.08591]



上海交通大学
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July 21, 2024
New Opportunities

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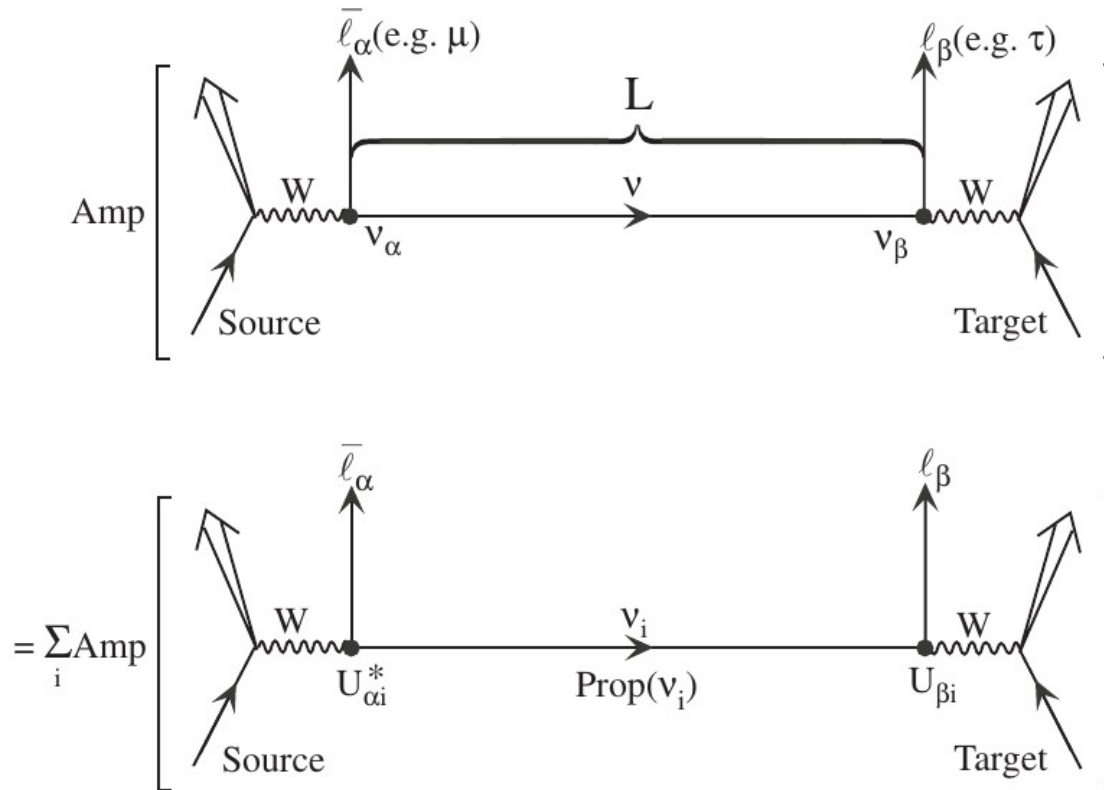
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- **Degenerate Oscillation**
- **Neutron-Antineutron Oscillation in NS**
- **Neutron Star Cooling & GUT**

Neutrino Oscillation



[Kayser, <https://arxiv.org/abs/hep-ph/0506165>]

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i \rightarrow \boxed{\sum_i U_{\alpha i} e^{i(E_i t - \vec{P}_i \cdot \vec{x})} \nu_i} = \boxed{\sum_i U_{\alpha i} P_i U_{\beta i}^\dagger \nu_\beta} \equiv \sum_\beta A_{\alpha\beta} \nu_\beta$$

Neutrino oscillations in matter

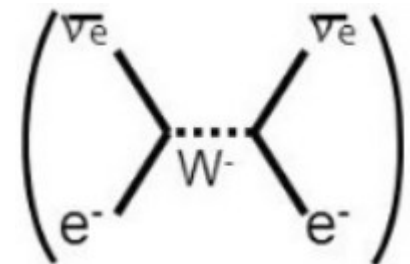
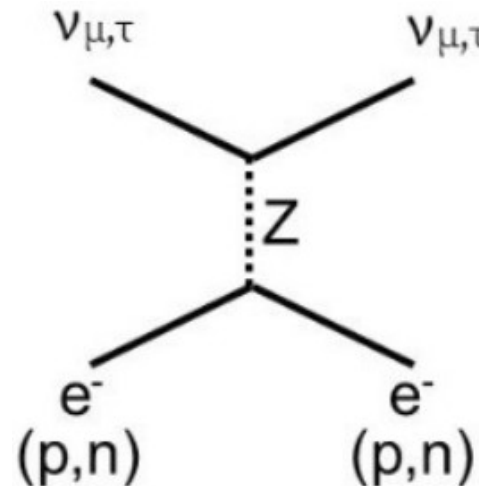
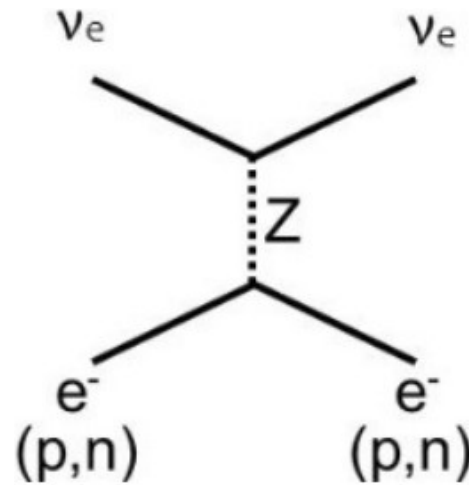
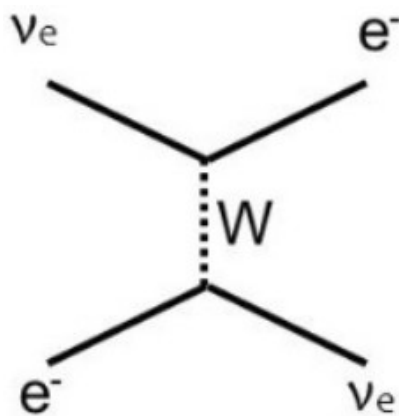
L. Wolfenstein

Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213

(Received 6 October 1977; revised manuscript received 5 December 1977)

The effect of coherent forward scattering must be taken into account when considering the oscillations of neutrinos traveling through matter. In particular, for the case of massless neutrinos for which vacuum oscillations cannot occur, oscillations can occur in matter if the neutral current has an off-diagonal piece connecting different neutrino types. Applications discussed are solar neutrinos and a proposed experiment involving transmission of neutrinos through 1000 km of rock.

$$\mathcal{H} = \frac{MM^\dagger}{2E_\nu} \pm \mathbf{V}$$



$$i \frac{d}{dx} \psi = H \psi$$

$$H = \frac{\Delta m^2}{2E} B + \lambda L + H_{\nu\nu}$$

$$B = U \left(\frac{1}{2} \text{diag}[-1, 1] \right) U^\dagger = \frac{1}{2} \begin{bmatrix} -\cos 2\theta_\nu & \sin 2\theta_\nu \\ \sin 2\theta_\nu & \cos 2\theta_\nu \end{bmatrix}$$

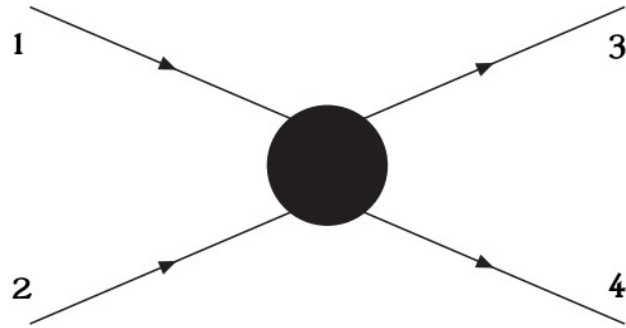
$$= \frac{\Delta m^2}{2E} B + \sqrt{2} G_F n_e L + \sqrt{2} G_F \int d^3 \mathbf{p}' (1 - \hat{\mathbf{p}} \cdot \hat{\mathbf{p}}') (\rho_{\mathbf{p}'} - \bar{\rho}_{\mathbf{p}'})$$

$$[\rho_{\mathbf{p}'}(t, \mathbf{x})]_{\alpha\beta} = \sum_{\nu'} n_{\nu', \mathbf{p}'}(t, \mathbf{x}) \langle \nu_\alpha | \psi_{\nu', \mathbf{p}'}(t, \mathbf{x}) \rangle \langle \psi_{\nu', \mathbf{p}'}(t, \mathbf{x}) | \nu_\beta \rangle,$$

$$[\bar{\rho}_{\mathbf{p}'}(t, \mathbf{x})]_{\beta\alpha} = \sum_{\bar{\nu}'} n_{\bar{\nu}', \mathbf{p}'}(t, \mathbf{x}) \langle \bar{\nu}_\alpha | \psi_{\bar{\nu}', \mathbf{p}'}(t, \mathbf{x}) \rangle \langle \psi_{\bar{\nu}', \mathbf{p}'}(t, \mathbf{x}) | \bar{\nu}_\beta \rangle,$$

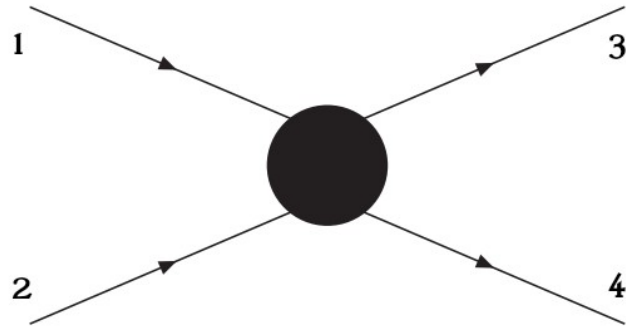
Duan, Fuller & Qian [arXiv:1001.2799]

Boltzmann Equation:
$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{x}} \frac{d\mathbf{x}}{dt} + \frac{\partial f}{\partial \mathbf{p}} \frac{d\mathbf{p}}{dt} = \mathbb{C}[f]$$



$$\mathbb{C}[f] = \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \int \frac{d^3 p_3}{(2\pi)^3 2E_3} \int \frac{d^3 p_4}{(2\pi)^3 2E_4} (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) |\mathcal{M}|^2 \{f_1 f_2 [1 \pm f_3][1 \pm f_4] - f_3 f_4 [1 \pm f_1][1 \pm f_2]\}$$

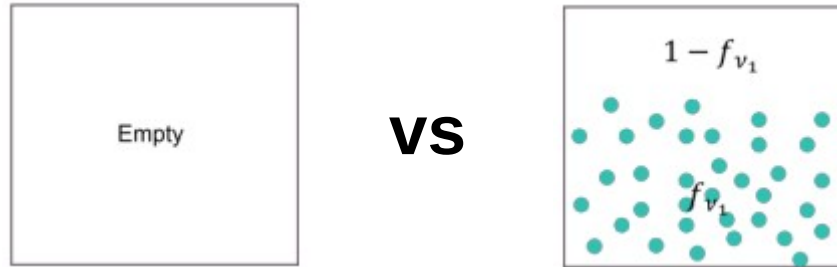
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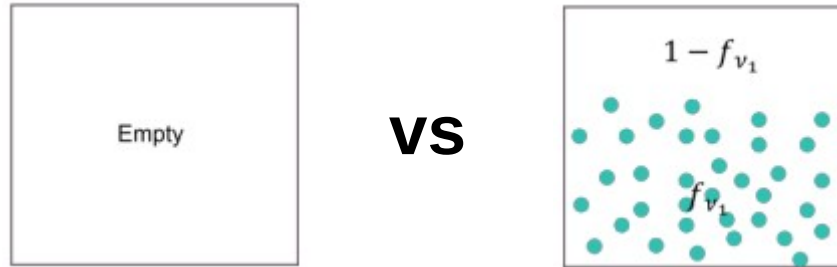
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Degeneracy is described by phase space factors: $1 \pm f$

Description with 2nd Quantization

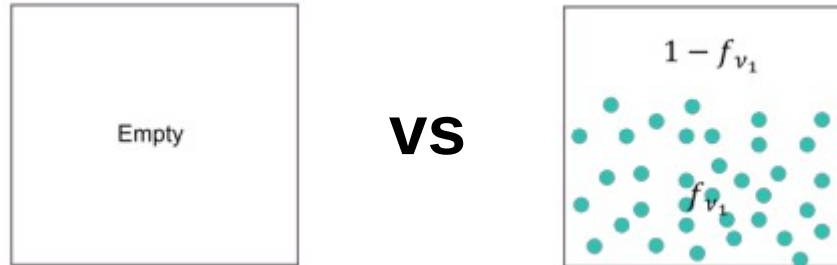


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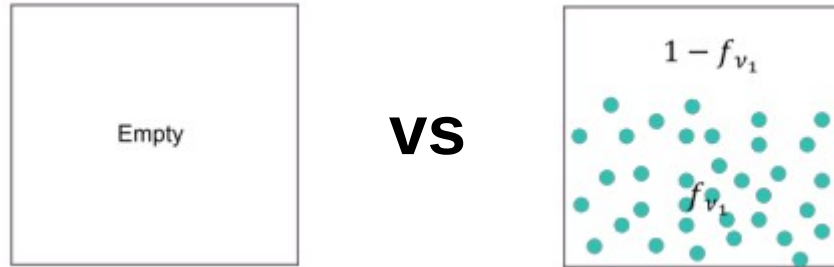
$$|N\rangle \equiv \frac{(a^\dagger)^N}{\sqrt{N!}} |0\rangle$$

Description with 2nd Quantization



$$|N\rangle \equiv \frac{(a^\dagger)^N}{\sqrt{N!}} |0\rangle$$
$$a^\dagger |N\rangle = \sqrt{N+1} |N+1\rangle$$
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Description with 2nd Quantization



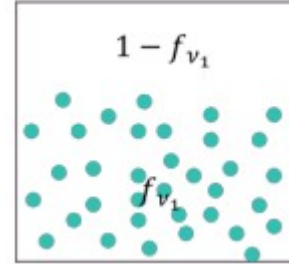
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Description with 2nd Quantization



VS



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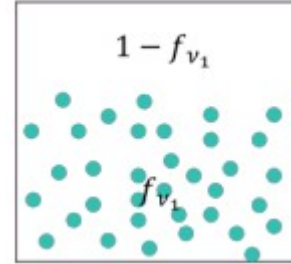
$$a^\dagger a |N\rangle = N |N\rangle$$

$$a a^\dagger |N\rangle = (1 \pm N) |N\rangle$$

Description with 2nd Quantization



VS



$|\Omega\rangle$

$f(\mathbf{x}, \mathbf{p})$

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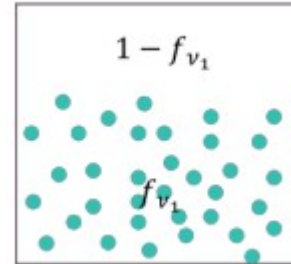
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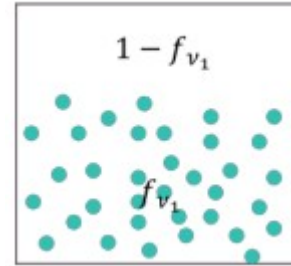
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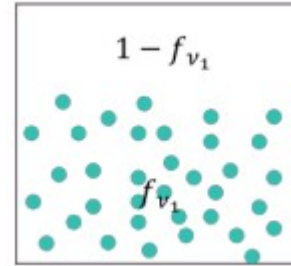
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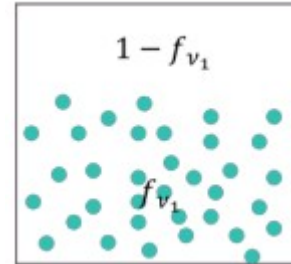
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$$a_{\mathbf{p}}^\dagger \quad a_{\mathbf{p}}$$

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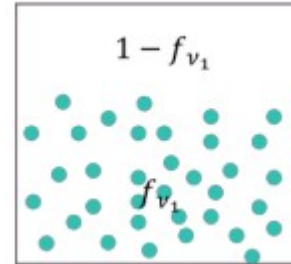
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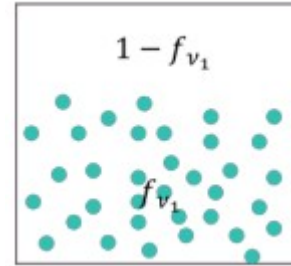
$$a_{\mathbf{p}}^\dagger \quad a_{\mathbf{p}} \quad \hat{n}_{\mathbf{p}} = a_{\mathbf{p}}^\dagger a_{\mathbf{p}}$$

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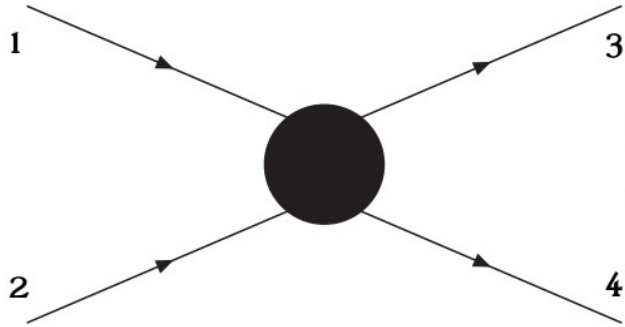
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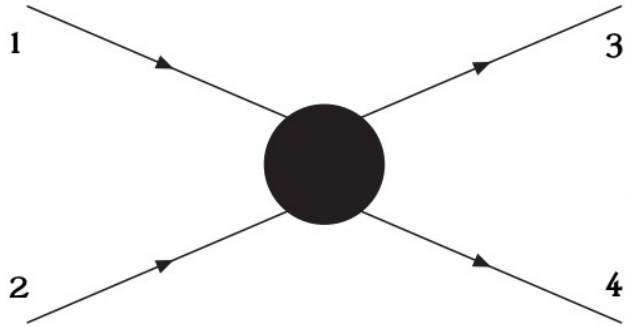
$$a_{\mathbf{p}} a_{\mathbf{p}}^\dagger |\Omega\rangle = \int d^3 \mathbf{x} [1 \pm f(\mathbf{x}, \mathbf{p})] |\Omega\rangle$$

Degeneracy with External State



$$|\mathcal{M}|^2 \{f_1 f_2 [1 \pm f_3][1 \pm f_4] - f_3 f_4 [1 \pm f_1][1 \pm f_2]\}$$

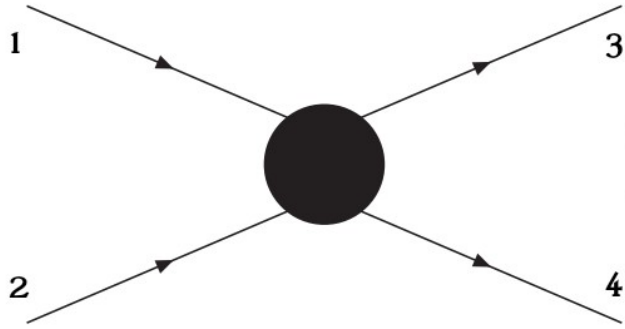
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$$\mathcal{M} \sim \langle \Omega + f | \cdots a | i + \Omega \rangle$$

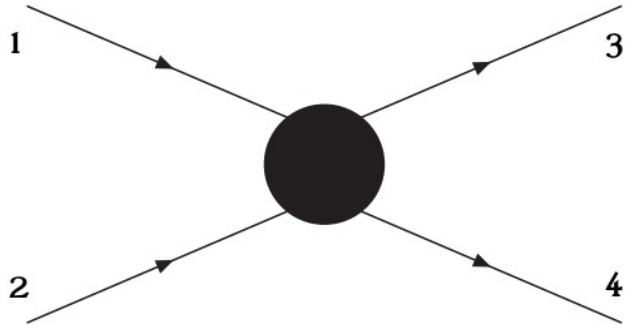


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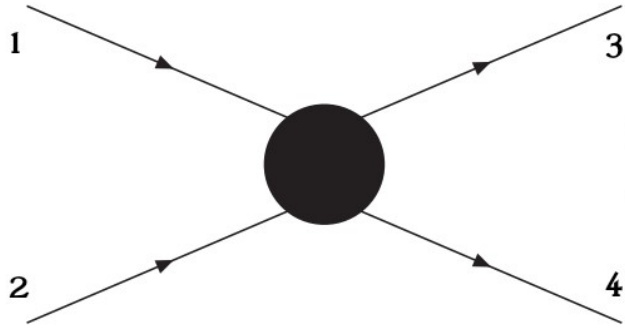
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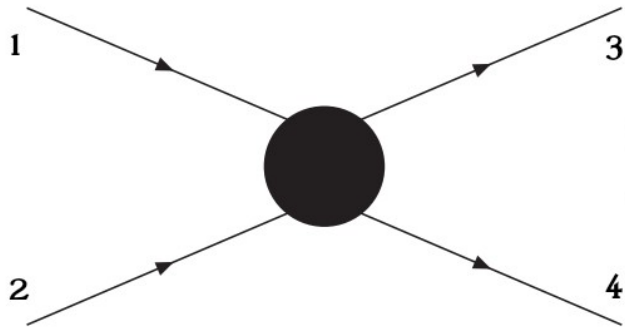
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↓
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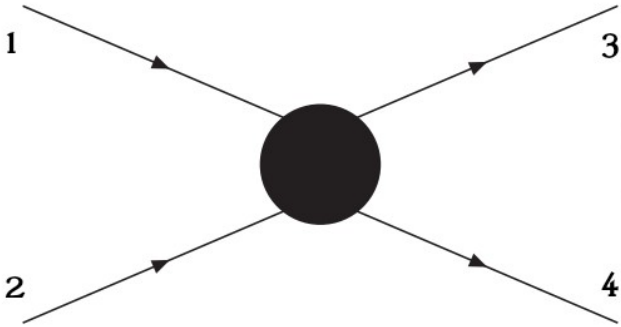
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↓
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Final State:

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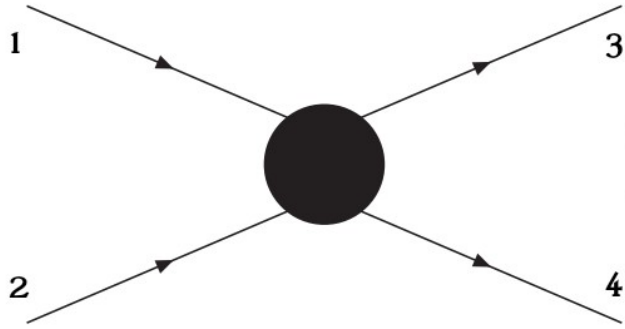
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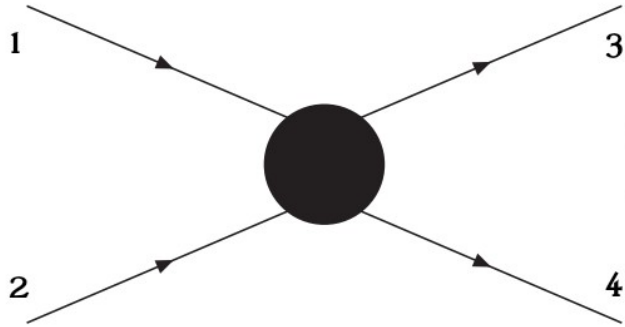
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↓
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↓
 $1 \pm f$

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SFG, Chui-Fan Kong, Pedro Pasquini [2310.04077]

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Fermion oscillation needs to involve 3 parts:

$$\mathcal{M}_{\beta\alpha} \equiv \mathcal{M}_d \left[\sum_i U_{\beta i} U_{\alpha i}^* e^{ip_i \cdot (x-y)} \langle \Omega | a_{p_i} a_{p_i}^{\dagger} | \Omega \rangle \right] \mathcal{M}_p$$

Spinors u & v combined into M_p & M_d

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↓
 $1 - f_i$

Pauli blocking factor already appears in amplitude!

Degenerate Oscillation in QFT

$$\mathcal{M}_{\beta\alpha} = \mathcal{M}_d \left[\sum_i U_{\beta i} U_{\alpha i}^* e^{ip_i \cdot (x-y)} (1 - f_i) \right] \mathcal{M}_p$$

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Fraction of events in β flavor

$$P_{\alpha\beta}(x - y) \equiv N_{\alpha \rightarrow \beta}(x - y) / N_\alpha(x = y)$$

- **Degenerate Oscillation**
- **Neutron-Antineutron Oscillation in NS**
- **Neutron Star Cooling & GUT**

GUT & Baryon Number Violation

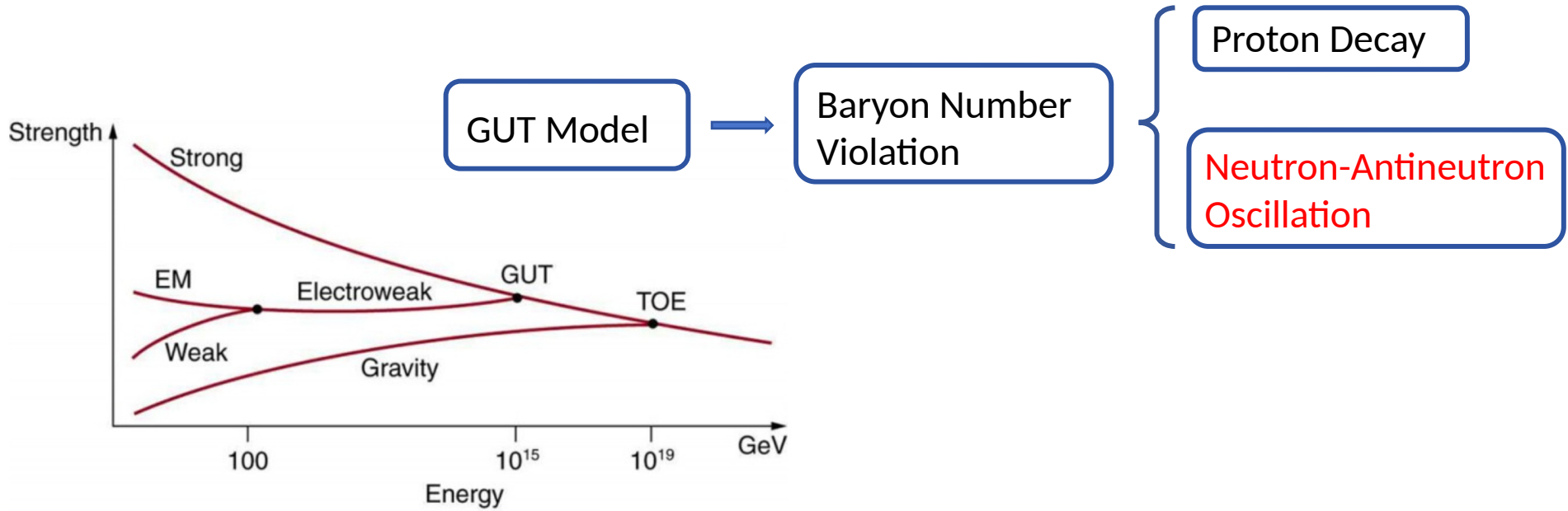


Table 1

GUT model	Is $N - \bar{N}$ observable?	Implications
(NON SUSY)		
$SU(5)$	No	$\Delta(B - L) = 0$
$SU(2)_L \times SU(2)_R \times SU(4)_c$	Yes	$M_c \simeq 10^5$ GeV
Minimal $SO(10)$	No	
E_6	No	
(SUSY GUT)		
$[SU(3)]^3$	Yes	Induced breaking of R-parity
$SO(10)$	No	

Table Caption: This table summarizes the observability of neutron-anti-neutron oscillation in various GUT models.

Being neutral, neutron can have **Majorana mass term**:

$$H \approx \begin{pmatrix} H_{11} & \delta m \\ \delta m & H_{22} \end{pmatrix}$$

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Note that only neutron is in thermal equilibrium!

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Neutron Degeneracy in NS

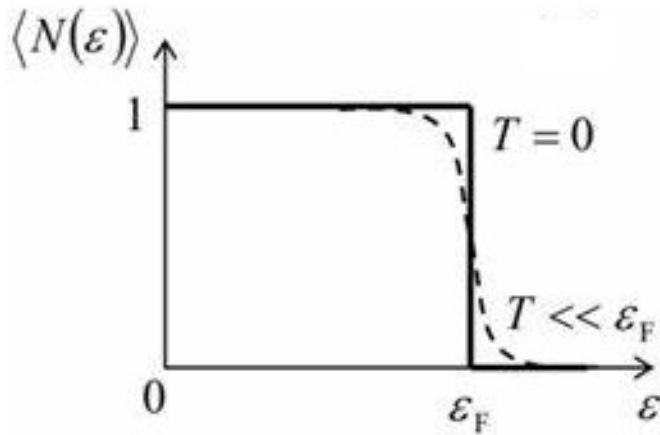
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$$k_F = (3\pi^2 n_n)^{1/3}$$



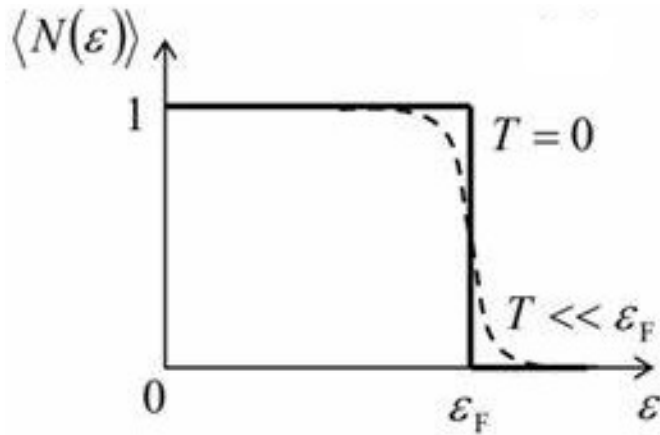
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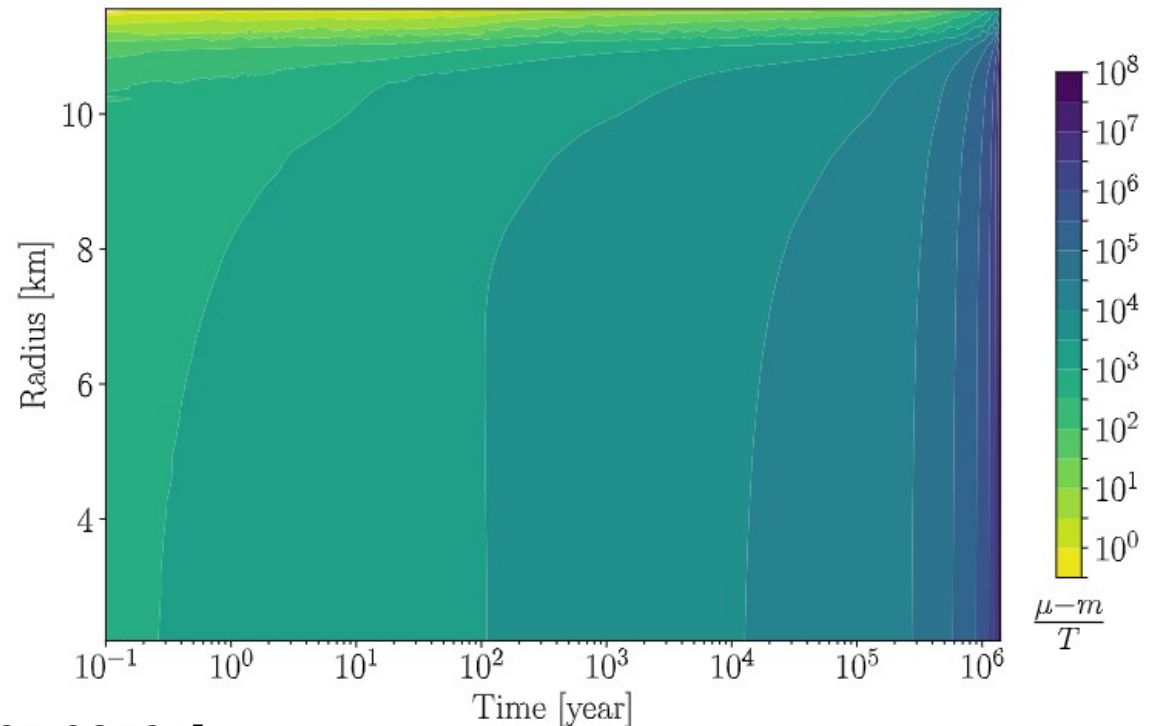
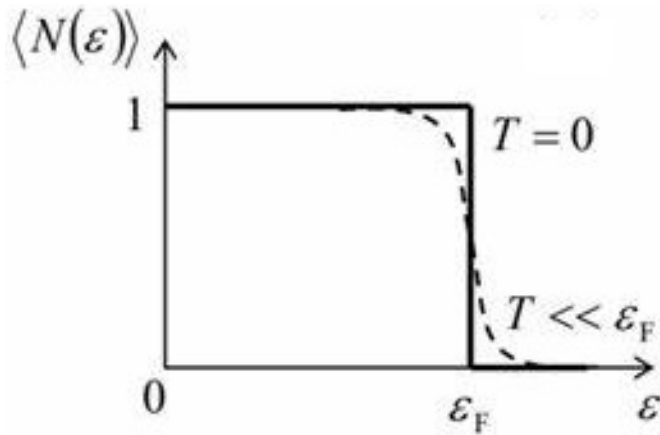


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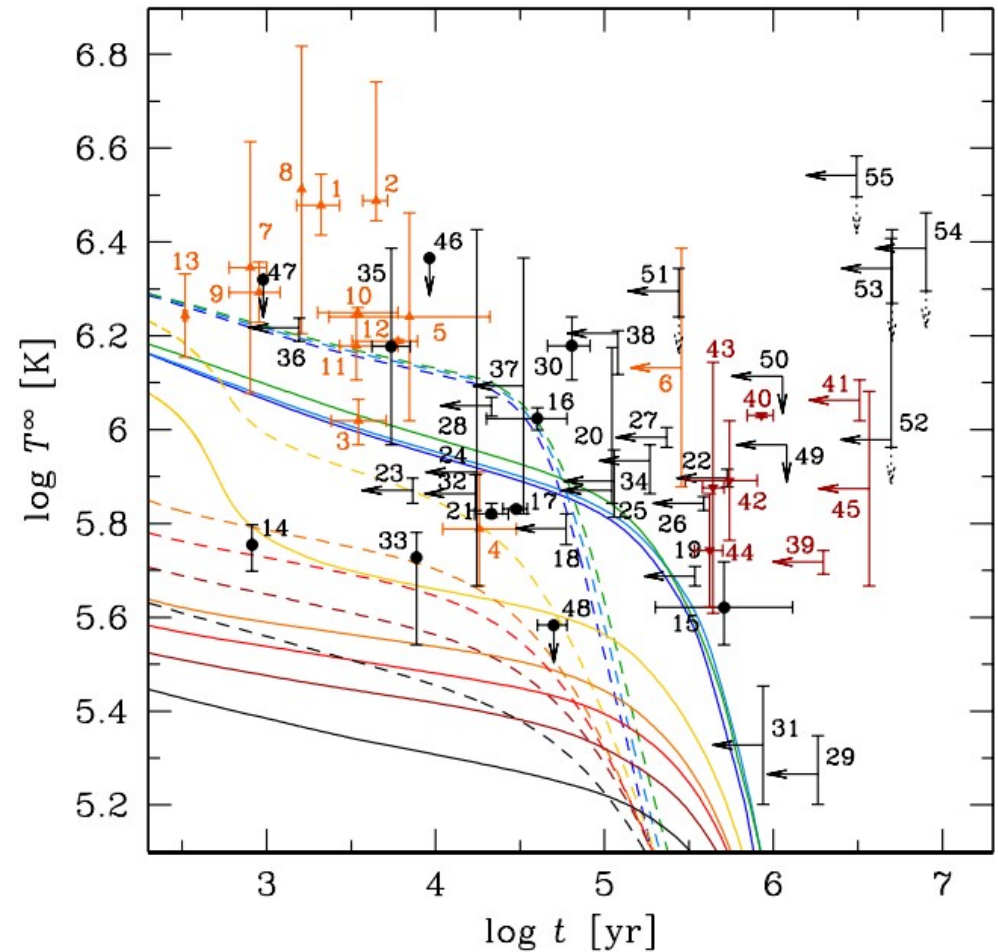
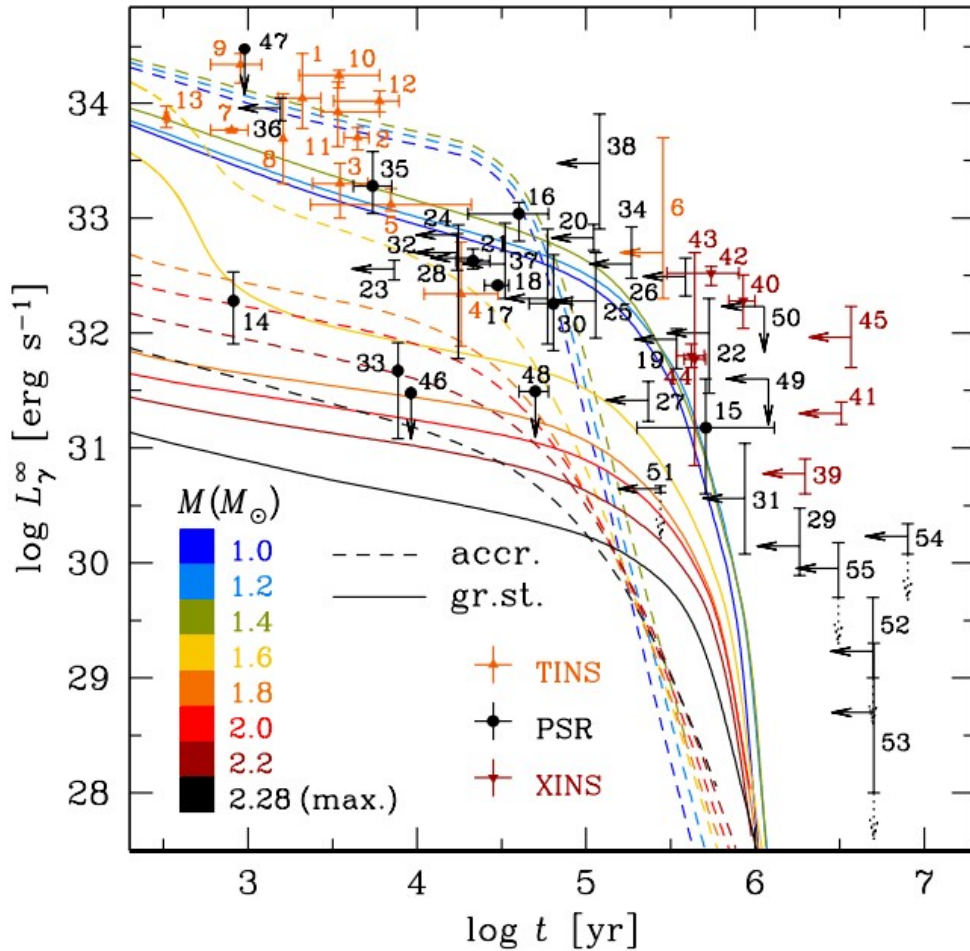
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NS lifetime > 10⁶ yrs

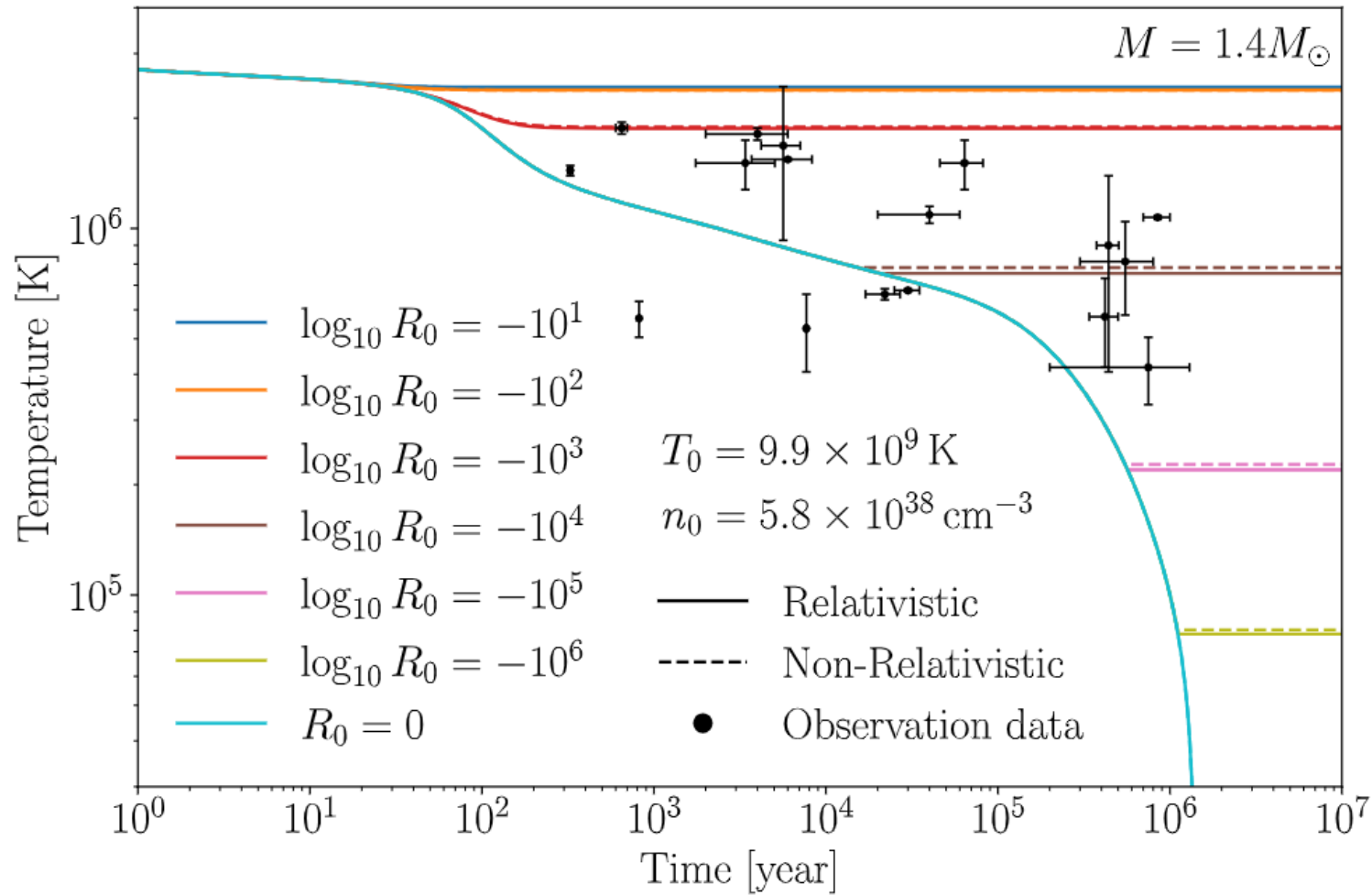


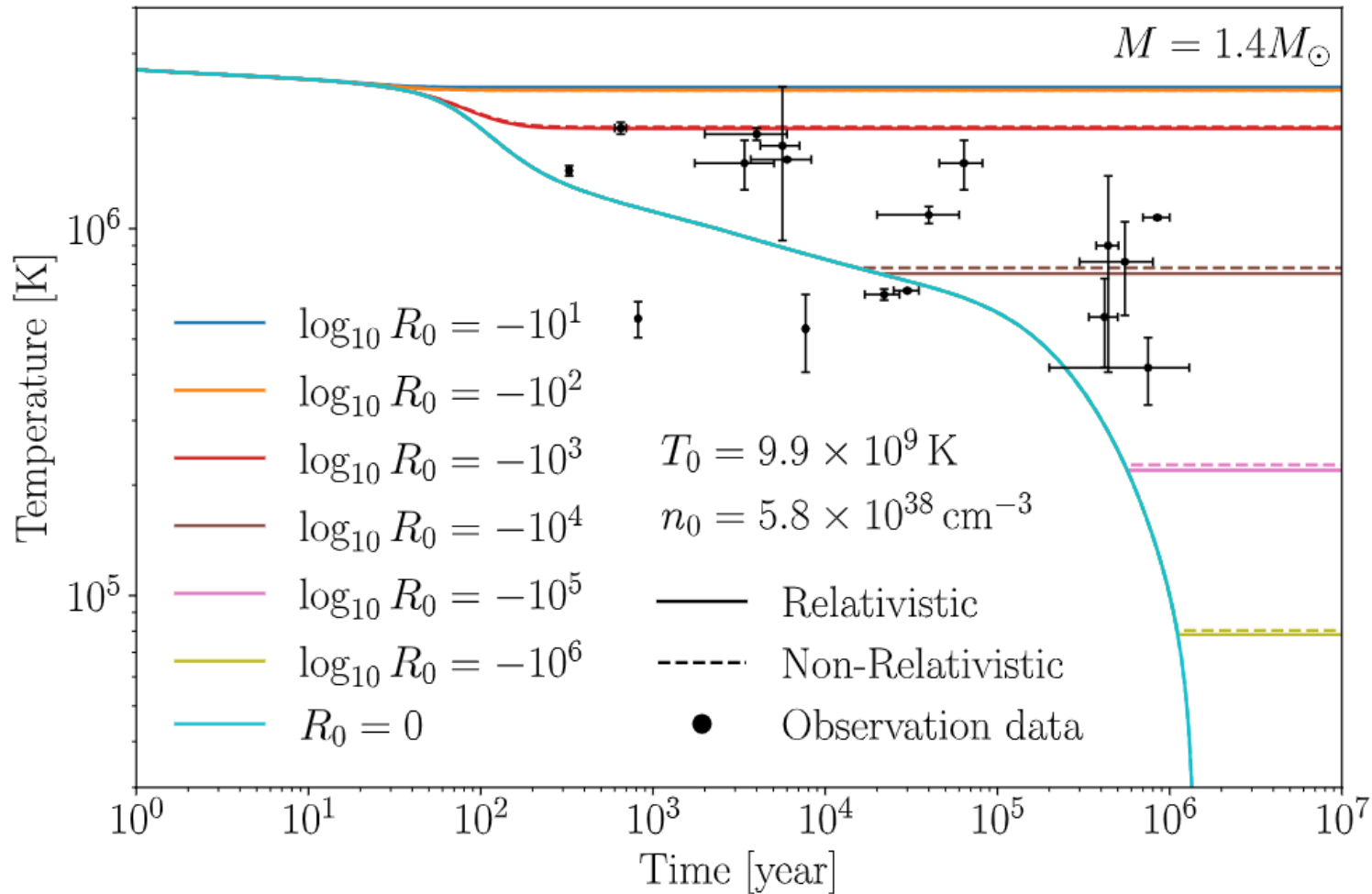
$$R \approx \frac{s^2}{[1 - f_1(\mathbf{p})]^2} \lesssim 10^{-37}$$

NS Cooling Data

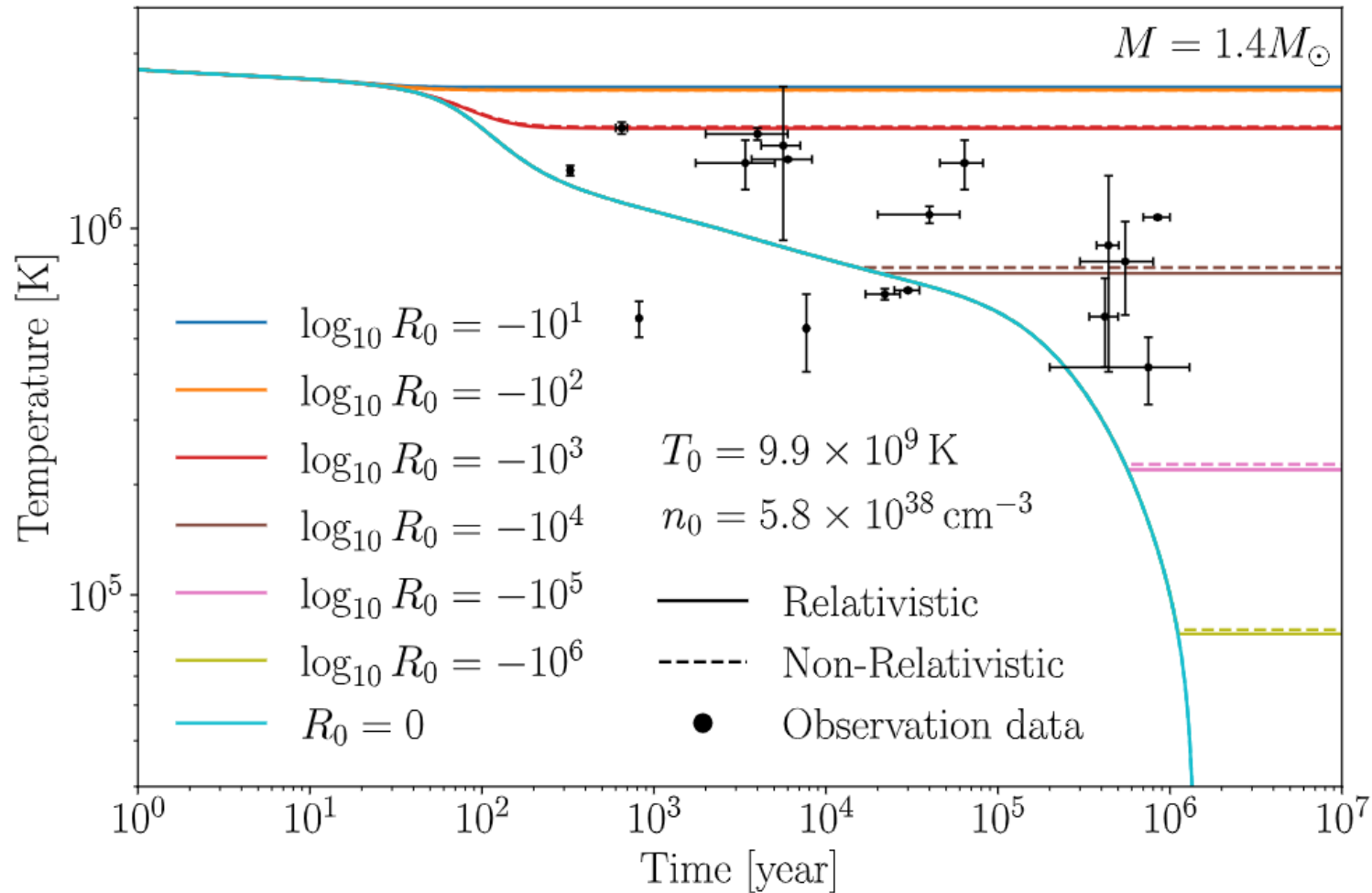


Potekhin, Zyuzin, Yakovlev, Beznogov & Shibano
MNRAS 496, 5052-5071 (2020) [arXiv:2006.15004]





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Cooling power

$$4\pi R^2 \sigma T^4 \sim 10^{21} \text{ W}$$

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$$\delta m \sim \frac{\Lambda_{\text{QCD}}^6}{M_X^5} \quad \Lambda_{\text{QCD}} \sim 180 \text{ MeV}$$

$$H \approx \begin{pmatrix} H_{11} & \delta m \\ \delta m & H_{22} \end{pmatrix} \quad s \sim \frac{\delta m}{\Delta H}$$

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which is far beyond the Planck scale!

- **Degenerate Oscillation**

1. Consistent picture of degeneracy in external & intermediate state

- **Neutron-Antineutron Oscillation in NS**

1. Concrete realization with n - \bar{n} oscillation
2. Standing fraction of antineutron

- **Neutron Star Cooling & GUT**

1. Degeneracy enhancement
2. Very strong constraint



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Thank You

Superfluidity in NS

