The Right-Handed Slepton Bulk Region for Dark Matter in Generalized No-scale $\mathcal{F}$-$SU(5)$ with Effective Super-Natural Supersymmetry

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- G. Du, T. Li, D. V. Nanopoulos and S. Raza, Phys. Rev. D **92**, no. 2, 025038 (2015) [arXiv:1502.06893 [hep-ph]].
- R. Ding, T. Li, F. Staub and B. Zhu, Phys. Rev. D **93**, no.9, 095028 (2016) [arXiv:1510.01328 [hep-ph]].
- T. Li, S. Raza and X. C. Wang, Phys. Rev. D **93**, no.11, 115014 (2016) [arXiv:1510.06851 [hep-ph]].
- X. Yin, J. A. Maxin, D. V. Nanopoulos and T. Li, Phys. Rev. D **109**, no.11, 115031 (2024) [arXiv:2310.03622 [hep-ph]].
- I. Khan, W. Ahmed, T. Li and S. Raza, Phys. Rev. D **109**, no.7, 075051 (2024) [arXiv:2312.07863 [hep-ph]].

Outline

Outline

The Standard Model

- The SM has been confirmed.
- However, it cannot be the final theory for particle physis.

Motivation for New Physics beyong the Standard Model

- The convincing evidence

  Dark energy; dark matter; neutrino masses and mixing; baryon asymmetry; inflation; ...

- Fine-tuning problems

  cosmological constant problem; gauge hierarchy problem; strong CP problem; SM fermion masses and mixings; ...

- Aesthetic problems

  Interaction and fermion unification; gauge coupling unification; charge quantization; too many parameters; ...

- The electroweak vacuum stability problem

  The stability problem can be easily solved in the new physics models.

- New Physics beyond the SM!
- Question: why do we still believe in supersymmetry?

Motivations for Supersymmetry

- Supersymmetric Albgebra is a generalization of the Poincare Algebra; The most general symmetry of the S-matrix; The local supersymmetry includes gravity naturally, *i.e.*, supergravity, etc.
- Supersymmetry provides a natural solution to the gauge hiearchy problem.
- Supersymmetry partially solves the cosmological constant problem: $M_{\rm Pl} \to M_{SUSY}$.
- Supersymmetry is a bridge between the promising low energy phenomenology and the high-energy fundamental physics such as the Grand Unified Theory and String Theory.

The Supersymmetric Standard Models (SSMs)

- Solving the gauge hierarchy problem
- Gauge coupling unification
- Radiatively electroweak symmetry breaking
- Natural dark matter candidates
- Electroweak baryogenesis
- Electroweak precision: R parity
- μ problem in the MSSM: $\mu H_d H_u$.

The Grand Unified Theories: $SU(5)$ and $SO(10)$

- Unification of the gauge interactions, and unifications of the SM fermions
- Charge quantization
- Gauge coupling unification in the MSSM, and Yukawa unification
- Radiative electroweak symmetry breaking due to the large top quark Yukawa coupling
- Weak mixing angle at weak scale M_Z
- Neutrino masses and mixings by seesaw mechanism
- Prediction: dim-6 proton decay via heavy gauge boson exchange.

Problems

- Gauge symmetry breaking
- Doublet-triplet splitting problem
- Proton decay problem
- Fermion mass problem: $m_e/m_\mu = m_d/m_s$

String Models

- Calabi-Yau compactification of heterotic string theory
- Orbifold compactification of heterotic string theory
- D-brane models on Type II orientifolds
- Free fermionic string model builing
- $\mathcal{F}$-Theory Model Building

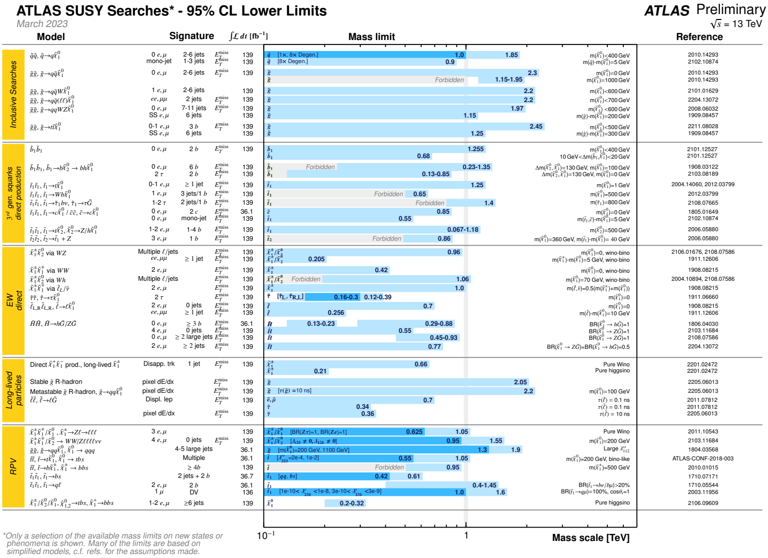
Supersymmetry is a bridge between the promising low energy phenomenology and high-energy fundamental physics.

Particle Physics Paradigm

String Theory $\rightarrow$ String Models $\rightarrow$ GUTs $\rightarrow$ SSMs $\rightarrow$ SM

The LHC Supersymmetry Search Contraints:

- The gluino mass low bound is around 2.3 TeV in the CMSSM/mSUGRA
- The first two-generation squark mass low bounds are around 1.85 TeV.
- The stop/sbottom mass low bounds are around 1.3 TeV. **The SSMs are fine-tuned!!!**


ATLAS SUSY Searches* - 95% CL Lower Limits
March 2023
ATLAS Preliminary
$\sqrt{s}$ = 13 TeV
Model
Signature
$\int \mathcal{L}\, dt$ [fb^{-1}]
Mass limit
Reference
Inclusive Searches
3rd gen. squarks direct production
EW direct
Long-lived particles
RPV
Forbidden
10^{-1}
1
Mass scale [TeV]
*Only a selection of the available mass limits on new states or phenomena is shown. Many of the limits are based on simplified models, c.f. refs. for the assumptions made.

Interesting Questions in Supersymmetry Phenomenology

- Can we solve the supersymmetry electroweak fine-tuning problem natually?
- Do we still have the bulk region for dark matter?
- Can we probe this entire bulk region at the future experiments?

Outline

High Eneryg Fine-Tuning Definition

- Fine-tuning Definition [1]: the quantitative measure $\Delta_{\rm FT}^{\rm EENZ-BG}$ for fine-tuning is the maximum of the logarithmic derivative of M_Z with respect to all the fundamental parameters a_i at the GUT scale

$$\Delta_{\rm FT}^{\rm EENZ-BG} = {\rm Max}\{\Delta_i^{\rm GUT}\}~, \quad \Delta_i^{\rm GUT} = \left| \frac{\partial {\rm ln}(M_Z)}{\partial {\rm ln}(a_i^{\rm GUT})} \right| ~.$$

[1] J. R. Ellis, K. Enqvist, D. V. Nanopoulos and F. Zwirner, Mod. Phys. Lett. A **1**, 57 (1986); R. Barbieri and G. F. Giudice, Nucl. Phys. B **306**, 63 (1988).

Question: Super-Natural Supersymmetry

Can we propose a supersymmetry scenario whose the EENZ-BG fine-tuning measure is automatically 1 or order 1 ($\mathcal{O}(1)$)?

Fundamental physics principles: simplicity and naturalness.

Super-Natural Supersymmetry [2]

- Fine-Tuning Definition:

$$\Delta_{\rm FT} ~=~ {\rm Max}\{\Delta_i^{\rm GUT}\}~,~~~\Delta_i^{\rm GUT} ~=~ \left|\frac{\partial {\rm ln}(M_Z)}{\partial {\rm ln}(a_i^{\rm GUT})}\right|~.$$

- Natural Solution:

$$M_Z^n ~=~ f_n\left(\frac{M_Z}{M_*}\right)~M_*^n~.$$

$$\frac{\partial {\rm ln}(M_Z^n)}{\partial {\rm ln}(M_*^n)} ~\simeq~ \frac{M_*^n}{M_Z^n}\frac{\partial M_Z^n}{\partial M_*^n} ~\simeq~ \frac{1}{f_n}f_n \simeq \mathcal{O}(1)~.$$

- For no-scale supergravity and M-theory on S^1/Z_2, we have $M_* = M_{1/2}$ and $M_* = M_{3/2}$, respectively.

[2] T. Leggett, T. Li, J. A. Maxin, D. V. Nanopoulos and J. W. Walker, arXiv:1403.3099 [hep-ph]; Phys. Lett. B **740**, 66 (2015) [arXiv:1408.4459 [hep-ph]]; G. Du, T. Li, D. V. Nanopoulos and S. Raza, Phys. Rev. D **92**, no. 2, 025038 (2015) [arXiv:1502.06893 [hep-ph]]; T. Li, S. Raza and X. C. Wang, Phys. Rev. D **93**, no. 11, 115014 (2016) [arXiv:1510.06851 [hep-ph]]

The Super-Natural Supersymmetry and Its Generalizations

- The Kähler potential and superpotential can be calculated in principle or at least inspired from a fundamental theory such as string theory with suitable compactifications. In other words, one cannot add arbitrary high-dimensional terms in the Kähler potential and superpotential.
- There is one and only one chiral superfield or modulus whose F-term breaks supersymmetry. And all the supersymmetry breaking soft terms are obtained from the above Kähler potential and superpotential.
- All the other mass parameters, if there exist such as the μ term in the MSSM, must arise from supersymmetry breaking.

Effectively Super-Natural Supersymmetry [3]

A supersymmetry breaking scenario is natural if all the fundamental parameters that have large EENZ-BG fine-tuning measures are correlated.

[3] R. Ding, T. Li, F. Staub and B. Zhu, Phys. Rev. D **93**, no.9, 095028 (2016) [arXiv:1510.01328 [hep-ph]].

Outline

SUSY Dark Matter Scenarios

- **Bulk region**

  The sfermions (supersymmetric partners of the SM fermions) are light.

- **The Z/Higgs funnels or Z/Higgs resonances**

  The LSP neutralino mass is about half of the masses of the Z boson, SM Higgs, CP-even Higgs H_0, or CP-odd Higgs A_0.

- **Coannihilations**

  The sfermion masses are close to the LSP neutralino.

- **Mixing scenario or well-tempered scenario**

  The LSP neutralino has enough Wino or Higgsino component to significantly increase the annihilation cross section.

SUSY Dark Matter Scenarios

- The bulk region may be the most natural.
- Because the LHC SUSY searches have given strong constraints on the SUSY parameter space, the possible bulk region is the light right-handed sleptons and LSP neutralino annihilations.
- This is an interesting scenario for the SUSY searches at the $\mathrm{FCC_{ee}}$ and CEPC.
- Question: Is it possible to have such kind of viable bulk region for dark matter in a natural SUSY scenario?

Outline

$\mathcal{F}$-$SU(5)$ Models

- The gauge group $SU(5) \times U(1)_X$ can be embedded into $SO(10)$ model.
- Generator $U(1)_{Y'}$ in $SU(5)$

$$T_{\rm U(1)_{Y'}} = {\rm diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right) ~.$$

- Hypercharge

$$Q_Y = \frac{1}{5}\left(Q_X - Q_{Y'}\right) ~.$$

- SM fermions

$$F_i = (\mathbf{10}, \mathbf{1}),~ \bar{f}_i = (\bar{\mathbf{5}}, -\mathbf{3}),~ \bar{l}_i = (\mathbf{1}, \mathbf{5})~,$$

$$F_i = (Q_i, D_i^c, N_i^c),~ \overline{f}_i = (U_i^c, L_i),~ \overline{l}_i = E_i^c~.$$

- Higgs particles:

$$H = (\mathbf{10}, \mathbf{1}),~ \overline{H} = (\overline{\mathbf{10}}, -\mathbf{1}),~ h = (\mathbf{5}, -\mathbf{2}),~ \overline{h} = (\bar{\mathbf{5}}, \mathbf{2})~,$$

$$H = (Q_H, D_H^c, N_H^c)~,~ \overline{H} = (\overline{Q}_{\overline{H}}, \overline{D}^c_{\overline{H}}, \overline{N}^c_{\overline{H}})~,$$

$$h = (D_h, D_h, D_h, H_d)~,~ \overline{h} = (\overline{D}_{\overline{h}}, \overline{D}_{\overline{h}}, \overline{D}_{\overline{h}}, H_u)~.$$

- Flip

$$U \leftrightarrow D~,~ N \leftrightarrow E~,~ H_d \leftrightarrow H_u~.$$

Symmetry breaking:

- Superpotential

$$W_{\rm GUT} = \lambda_1 HHh + \lambda_2 \overline{HHh} + \Phi(\overline{H}H - M_{\rm H}^2) ~.$$

- There is only one F-flat and D-flat direction along the N_H^c and $\overline{N_H^c}$ directions: $< N_H^c > = < \overline{N_H^c} > = M_{\rm H}$.
- The doublet-triplet splitting due to the missing partner mechanism
- No dimension-5 proton decay problem.

Flipped $SU(5) \times U(1)_X$ Models: [6]

- Doublet-triplet splitting via missing partner mechanism [4].
- No dimension-five proton decay problem.
- Little hierarchy problem in string models: $M_{\rm String} \sim 20 \times M_{\rm GUT}$

$$M_{\rm String} = g_{\rm String} \times 5.27 \times 10^{17} ~{\rm GeV} ~.$$

- Testable flipped $SU(5) \times U(1)_X$ models: TeV-scale vector-like particles [5].

[4] I. Antoniadis, J. R. Ellis, J. S. Hagelin and D. V. Nanopoulos, Phys. Lett. B **194**, 231 (1987).

[5] J. Jiang, T. Li and D. V. Nanopoulos, Nucl. Phys. B **772**, 49 (2007).

[6] S. M. Barr, Phys. Lett. B **112**, 219 (1982); J. P. Derendinger, J. E. Kim and D. V. Nanopoulos, Phys. Lett. B **139**, 170 (1984).

Flipped $SU(5) \times U(1)_X$ Models:

- Four-dimensional free-fermionic string construction [7].
- F-theory model building [8].
- Heterotic string constructions: Calabi-Yau [9]; Orbifold [10].
- Orbifold GUTs [11].

[7] J. L. Lopez, D. V. Nanopoulos and K. j. Yuan, Nucl. Phys. B **399**, 654 (1993).

[8] C. Beasley, J. J. Heckman and C. Vafa, JHEP **0901**, 059 (2009); J. Jiang, T. Li, D. V. Nanopoulos and D. Xie, Phys. Lett. B **677**, 322 (2009); Nucl. Phys. B **830**, 195 (2010).

[9] A. E. Faraggi, R. S. Garavuso and J. M. Isidro, Nucl. Phys. B **641**, 111 (2002)

[10] J. E. Kim and B. Kyae, Nucl. Phys. B **770**, 47 (2007).

[11] S. M. Barr and I. Dorsner, Phys. Rev. D **66**, 065013 (2002).

$\mathcal{F}$-$SU(5)$ Models

- To achieve the string scale gauge coupling unification, we introduce sets of vector-like particles in complete $SU(5) \times U(1)_X$ multiplets, whose contributions to the one-loop beta functions of the $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ gauge symmetries, Δb_1, Δb_2 and Δb_3 respectively, satisfy $\Delta b_1 < \Delta b_2 = \Delta b_3$.
- To avoid the Landau pole problem for the gauge couplings, we can only introduce the following two sets of vector-like particles around the TeV scale, which could be observed at the LHC

$$Z1 : XF = (\mathbf{10}, \mathbf{1}) \equiv (XQ,\ XD^c,\ XN^c)\ ,\ \overline{XF} = (\overline{\mathbf{10}}, -\mathbf{1})\ ;$$
$$Z2 : XF\ ,\ \overline{XF}\ ,\ Xl = (\mathbf{1}, -\mathbf{5})\ ,\ \overline{Xl} = (\mathbf{1}, \mathbf{5}) \equiv XE^c\ .$$

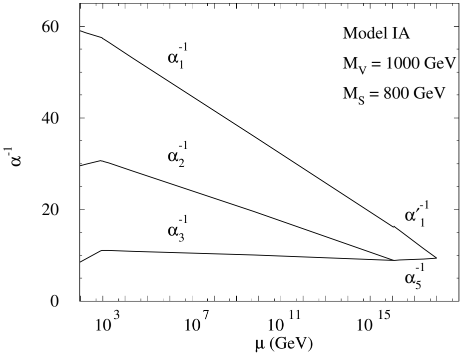


Figure: Gauge coupling unification in the Type IA model.

No-Scale Supergravity [12]:

- The vacuum energy vanishes automatically due to the suitable Kähler potential.
- At the minimum of the scalar potential, there are flat directions which leave the gravitino mass $M_{3/2}$ undertermined.
- The super-trace quantity $\mathrm{Str}\mathcal{M}^2$ is zero at the minimum.

$$K = -3\ln(T + \overline{T} - \sum_i \overline{\Phi}_i \Phi_i) \ .$$

[12] E. Cremmer, S. Ferrara, C. Kounnas and D. V. Nanopoulos, Phys. Lett. B **133**, 61 (1983); A. B. Lahanas and D. V. Nanopoulos, Phys. Rept. **145**, 1 (1987).

No-Scale Supergravity

No-scale supergravity can be realized in the compactification of the weakly coupled heterotic string theory [13] and the compactification of M-theory on S^1/Z_2 at the leading order [14].

[13] E. Witten, Phys. Lett. B **155**, 151 (1985).

[14] T. Li, J. L. Lopez and D. V. Nanopoulos, Phys. Rev. D **56**, 2602 (1997).

No-Scale Supergravity:

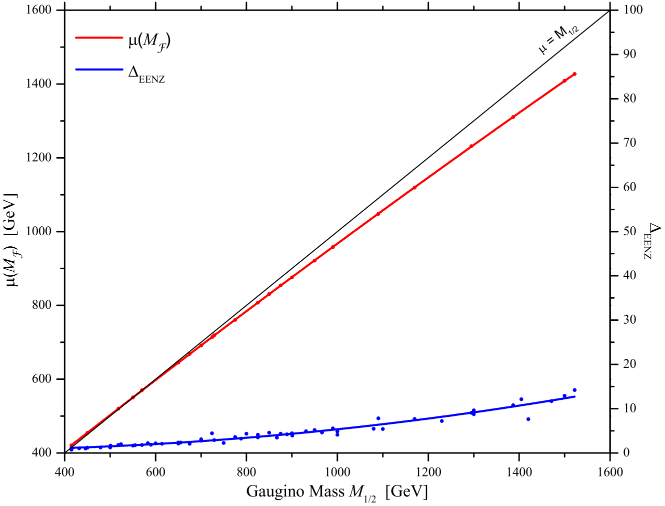
- mSUGRA/CMSSM: $M_{1/2}$, M_0, A, $\tan\beta$, $\mathrm{sign}(\mu)$.
- No-scale boundary condition: $M_{1/2} \neq 0$, $M_0 = A = B_\mu = 0$
- Natural solution to CP violation and FCNC problem.
- Disfavored by phenomenology

  The light stau is the LSP if we run the RGEs from the traditional GUT scale.
- No-scale $\mathcal{F}$-$SU(5)$

The Bino dominant neutralino is the LSP if we run the RGEs from the string scale.

Miracle of Vector-Like Particles

- String-scale gauge coupling unification.
- Lifting the lightest CP-even Higgs boson mass.
- The proton decay $p \to e^+\pi^0$ from the heavy gauge boson exchange is within the reach of the future Hyper-Kamiokande experiments for a majority of the most plausible parameter space.
- Explaining the muon anomalous magnetic moment easily due to the chirality flip if it is needed.
- Special sparticle spectra.


$\mu(M_{\mathcal{F}})$
Δ_{EENZ}
$\mu = M_{1/2}$
$\mu(M_{\mathcal{F}})$ [GeV]
Δ_{EENZ}
Gaugino Mass $M_{1/2}$ [GeV]
1600
1400
1200
1000
800
600
400
100
90
80
70
60
50
40
30
20
10
0
400
600
800
1000
1200
1400
1600

The No-Scale Supergravity and Its Generalization

- No-Scale Supergravity is only valid at tree level.
- There exist corrections to the gauge kinetic functions and Kähler potential at one loop.
- The Generalized No-Scale Supergravity: $M_{1/2} \neq 0$, $M_0 \sim \mathcal{O}(10^{-2})M_{1/2}$, $A \sim \mathcal{O}(10^{-2})M_{1/2}$, $B_\mu \sim \mathcal{O}(10^{-2})M_{1/2}$.

The Generalized No-Scale Supergravity for $\mathcal{F}$-$SU(5)$

- At the $SU(5) \times U(1)_X$ unification scale (string scale), we vary the $SU(5)$ gaugino mass M_5 from 1200 GeV to 5000 GeV, yielding a large gluino mass.
- To produce a light Bino, we vary the $U(1)_X$ gaugino mass M_{1X} from 100 GeV to 600 GeV.
- Note that No-Scale SUGRA is obtained at tree level and can be violated at one loop, so we assume the universal supersymmetry breaking soft mass M_0 and trilinear soft term A are smaller than about 1% of M_5. For simplicity, we take $A = 0$.
- We span $\tan\beta$ from 2 to 65, and the vector-like particle mass scale M_V from 1 TeV to 10 TeV.

The Bino Mass

$$\frac{M_1}{\alpha_1} \equiv \frac{24}{25}\frac{M_{1X}}{\alpha_{1X}} + \frac{1}{25}\frac{M_5}{\alpha_5} ~,$$

where α_i are the gauge couplings at their respective scales.

Effective Super-Natural SUSY

- A supersymmetry breaking scenario is natural if all the fundamental parameters that have large EENZ-BG fine-tuning measures are correlated.
- In our generalized No-Scale SUGRA, the fine-tuning measures for the SUSY breaking soft terms M_{1X}, M_0, and A_0 are all small, and only M_5 might have a large fine-tuning measure.
- Our generalized No-Scale SUGRA is approximately Super-Natural SUSY, and thus indeed natural.
  More specifically, it is only a small deviation from Super-Natural SUSY, and hence the simplest scenario for Effective Super-Natural SUSY.

Experimental Constraints

- Require neutralino LSP
- Constraints on the masses of the gluino and first/second generation squarks: $m_{\tilde{g}} \gtrsim 2.2$ TeV, $m_{\tilde{q}} \gtrsim 2.0$ TeV.
- Rare B-meson decay constraint $1.6 \times 10^{-9} \leq \text{BR}(B_s^0 \to \mu^+\mu^-) \leq 4.2 \times 10^{-9}$ and branching ratio of rare b-quark decay of $2.99 \times 10^{-4} \leq \text{BR}(b \to s\gamma) \leq 3.87 \times 10^{-4}$.
- The Higgs boson mass range 122 GeV$\leq m_h \leq$ 128 GeV.
- Constraints on spin-independent DM-nuclei cross sections from the XENONnT and LUX-ZEPLIN experiments.
- Relic density of cold DM measured by the 5σ Planck 2018 of $0.114 \leq \Omega_{\rm DM} h^2 \leq 0.126$ where below this range is regarded as under-saturated and above is over-saturated.

Suppression of Coannihilation

With the absent of coannihilation, we have

$$\Omega_{\tilde{B}} h^2 = 1.3 \times 10^{-2} \left(\frac{m_{\tilde{e}_R}}{100\mathrm{GeV}}\right)^2 \frac{(1+r)^4}{r\,(1+r^2)} \left(1 + 0.07 \log \frac{\sqrt{r}100\mathrm{GeV}}{m_{\tilde{e}_R}}\right) ,$$

where $r \equiv M_1^2 / m_{\tilde{e}_R}^2$.

- MicrOMEGAs 2.1, with and without coannihilation
- $\mathcal{R}_\phi \equiv (m_\phi - m_{\tilde{\chi}_1^0})/m_{\tilde{\chi}_1^0}$ with $\phi = \tilde{e}_R$ or $\tilde{\tau}_1$.
- $\mathcal{R}_\phi = 5 - 8\%$, deviates from the coannihilation $\geq 50\%$.
- $\mathcal{R}_\phi = 10 - 12\%$, co-annihilation $20 - 30\%$, annihilation $70 - 80\%$.

Bulk Region Condition

- 99.9% Bino-like LSPs are selected to prohibit large annihilation cross sections induced by Higgsino or Wino components.
- $2m_{\tilde{\chi}_1^0} \ll m_{H^0}, m_{A^0}$ and $2m_{\tilde{\chi}_1^0} \gg m_h$ are enforced to avoid the "Higgs funnel".
- $\tilde{\tau}_1$ and $\tilde{e}_R$ are naturally light, so coannihilation processes are negligible when $\mathcal{R}_{\tilde{\tau}_1} \equiv \frac{m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0}}{m_{\tilde{\chi}_1^0}} \gtrsim 10\%$ and $\mathcal{R}_{\tilde{e}_R} \equiv \frac{m_{\tilde{e}_R} - m_{\tilde{\chi}_1^0}}{m_{\tilde{\chi}_1^0}} \gtrsim 10\%$.
- Require the SM Higgs resonance to vanish, which transpires when $|\mu|^2 \gg M_Z^2$, via the coupling $g_{h\tilde{\chi}_1^0\tilde{\chi}_1^0} \propto \frac{M_Z(2\mu\cos\beta + M_1)}{\mu^2 - M_1^2}$.

The Criteria for Right-Handed Slepton Bulk Region

- The ratio of the mass difference $\mathcal{R}_\phi \equiv (m_\phi - m_{\tilde{\chi}_1^0})/m_{\tilde{\chi}_1^0}$ is more important than the absolute mass difference, where ϕ is $\tilde{\tau}_1$ (light stau) or $\tilde{e}_R$ (light selectron).
- Comprehensive numerical studies that we present in this work show that $\mathcal{R}_\phi \gtrsim 10\%$ is a conservative criterion to formulate the bulk region, *i.e.*, the observed dark matter density is obtained via traditional annihilations, not from coannihilations or resonances, etc.

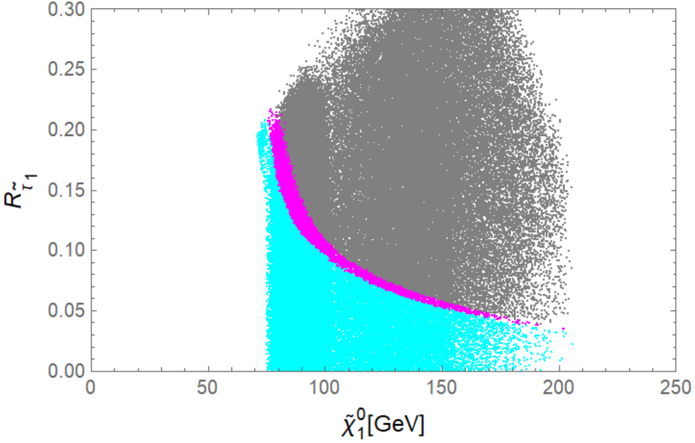


Figure: Bulk region in Generalized No-Scale $\mathcal{F}$-$SU(5)$. Cyan, magenta, gray points correspond to under-saturated, saturated, over-saturated DM relic density.

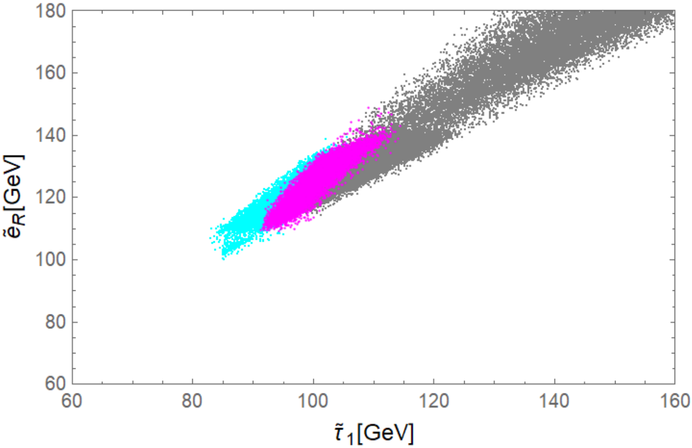


Figure: Light right-handed slepton masses in the Generalized No-Scale $\mathcal{F}$-$SU(5)$ bulk region. Cyan, magenta, gray points correspond to under-saturated, saturated, over-saturated DM relic density with $\mathcal{R}_{\tilde{\tau}_1} \gtrsim 10\%$.

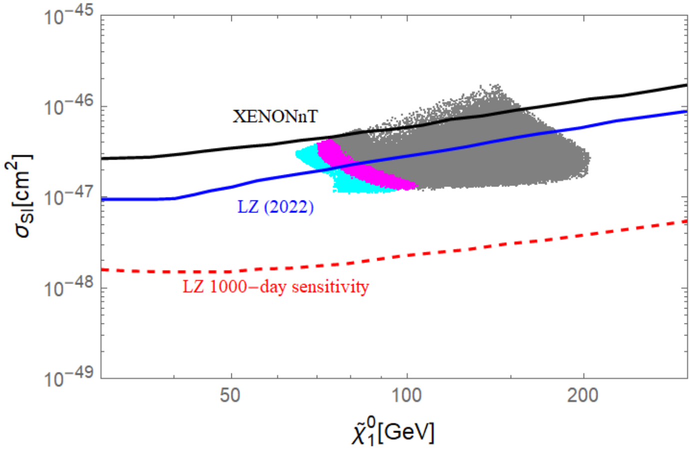


Figure: Generalized No-Scale $\mathcal{F}$-$SU(5)$ bulk region LSPs plot in reference to spin-independent DM-nuclei cross sections from XENONnT and LUX-ZEPLIN. Cyan, magenta, gray points correspond to under-saturated, saturated, over-saturated DM relic density with $\mathcal{R}_{\tilde{\tau}_1} \gtrsim 10\%$. We underscore the significance of the 1000-day LUX-ZEPLIN run that should fully probe the $\mathcal{F}$-$SU(5)$ bulk and about 50% of the pMSSM bulk (not shown).

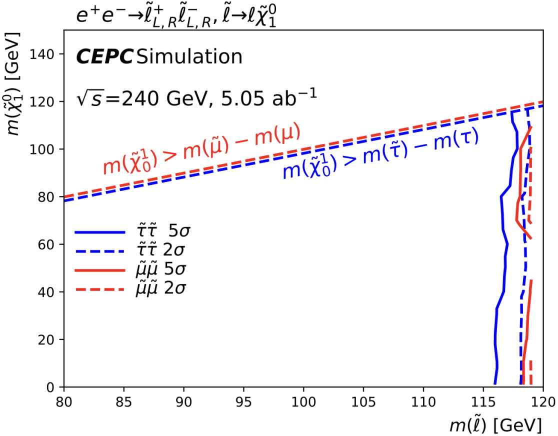


Figure: CEPC sensitivity.

The Light Right-Handed Stau Bulk Region in $\mathcal{F}$-$SU(5)$

- The LSP neutralino is Bino dominant.
- The mass hierarchy in $\mathcal{F}$-$SU(5)$ is $m_{\tilde{\chi}_1^0} < m_{\tilde{\tau}_1} < m_{\tilde{e}_R} = m_{\tilde{\mu}_R}$.
- If the Bino contributes all the DM abundance, the ratio $\mathcal{R}_{\tilde{\tau}_1} \gtrsim 10\%$ implies $m_{\tilde{\chi}_1^0} \leq 103.0$ GeV.
- The upper bounds on $\tilde{\tau}_1$ and $\tilde{e}_R$ are around 115 GeV and 150 GeV, respectively.

The Light Right-Handed Stau Bulk Region in $\mathcal{F}$-$SU(5)$

- The entire viable parameter space can be probed by the the 1000-day LUX-ZEPLIN experiment within the next a few years.
- The proton lifetime via dimension-six proton decay is around $3 - 4 \times 10^{34}$ years, so it is within reach of the future Hyper-Kamiokande experiment.
- The entire viable parameter space can be probed by the Future Circular Collider (FCC-ee) at CERN, Circular Electron Positron Collider (CEPC)
- The pure supersymmetry contribution to the muon anomalous magnetic moment Δa_μ is small, around 5×10^{-11}, which is consistent with the BMW results. Δa_μ can be explained via vectot-like particles if it is needed.

M_5	3996	2591
M_{1X}	473	268
M_0	23.15	21.49
A_0	0	0
$\tan\beta$	3.04	2.53
M_V	9063	4603
m_h	125.43	123.06
m_A	7325	5395
$m_{\tilde{\chi}_1^0}$	161.3	92.3
$m_{\tilde{\tau}_1}$	169.6	103.4
$m_{\tilde{e}_R}$	216.1	130.1
$m_{\tilde{t}_1}$	4273	2747
$m_{\tilde{g}}$	4986	3259
$m_{\tilde{u}_R}$	6798	4606
$BR(B_S^0 \to \mu^+\mu^-) \times 10^{-9}$	3.03	3.05
$BR(b \to s\gamma) \times 10^{-4}$	3.61	3.61
$\sigma_{SI} \times 10^{-12}$pb	6.28	17.10
$\tau_p \times 10^{34}$yrs	5.01	3.95
$\mathcal{R}_{\tilde{\tau}_1}$	5%	12%
$\Omega_{\tilde{\chi}} h^2$	0.1256	0.118
$\Omega_{\tilde{\chi}} h^2$(No co-annihilation)	0.386	0.147
co-annihilation rate	$> 50\%$	$\sim 20\%$

Figure: Two benchmark points for Generalized No-Scale $\mathcal{F}$-$SU(5)$. All masses are in GeV..

The pMSSM

The methodology just discussed is extended to include the much less constrained generic pMSSM. The pMSSM contains 22 free parameters, and we input M_A and μ in lieu of $m^2_{H_u}$ and $m^2_{H_d}$. The scanning ranges of the pMSSM parameters are as follows:

$$20 \text{ GeV} \leq M_1 \leq 1000 \text{ GeV} \qquad 2 \leq tan\beta \leq 65$$
$$1000 \text{ GeV} \leq M_2 \leq 5000 \text{ GeV} \qquad 1000 \text{ GeV} \leq M_A, \mu \leq 6000 \text{ GeV}$$
$$1200 \text{ GeV} \leq M_3 \leq 5000 \text{ GeV} \qquad M_1 \leq m_{\tilde{e}_R}, m_{\tilde{\tau}_R} \leq 2M_1$$
$$2500 \text{ GeV} \leq m_{\tilde{q}}, m_{\tilde{Q}}, m_{\tilde{u}_R}, m_{\tilde{t}_R}, m_{\tilde{d}_R}, m_{\tilde{b}_R} \leq 5000 \text{ GeV}$$
$$700 \text{ GeV} \leq m_{\tilde{l}} \leq 2000 \text{ GeV} \qquad 1200 \text{ GeV} \leq m_{\tilde{L}} \leq 5000 \text{ GeV}$$
$$-5000 \text{ GeV} \leq A_u, A_d, A_e, A_t, A_b, A_\tau \leq 5000 \text{ GeV}$$

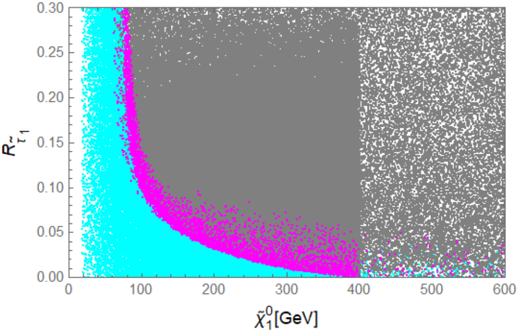


Figure: Bulk region in pMSSM. Cyan, magenta, gray points correspond to under-saturated, saturated, over-saturated DM relic density.

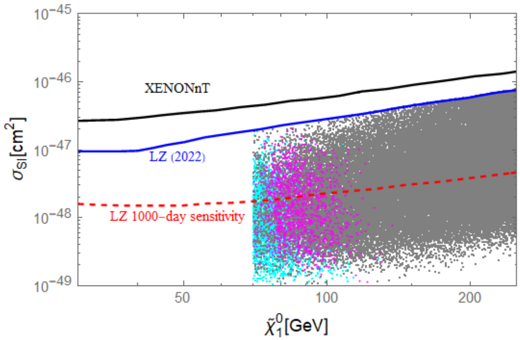


Figure: The pMSSM bulk region LSPs plot in reference to spin-independent DM-nuclei cross sections from XENONnT and LUX-ZEPLIN. Cyan, magenta, gray points correspond to under-saturated, saturated, over-saturated DM relic density with $\mathcal{R}_{\tilde{\tau}_1} \gtrsim 10\%$. The LUX-ZEPLIN 1000-day should fully probe about 50% of the pMSSM bulk.

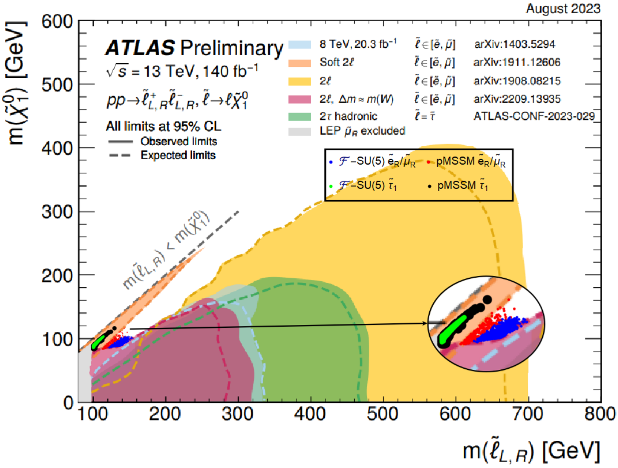


Figure: Generalized No-Scale $\mathcal{F}$-$SU(5)$ and pMSSM bulk regions superimposed over the August 2023 ATLAS Summary Plot [?] of SUSY searches for electroweak production of sleptons.

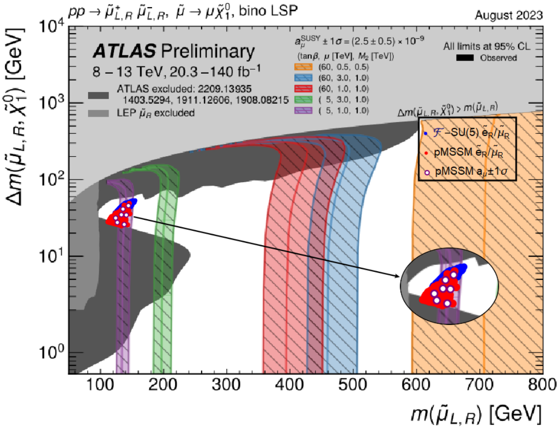


Figure: Generalized No-Scale $\mathcal{F}$-$SU(5)$ and pMSSM bulk regions superimposed over the August 2023 ATLAS Summary Plot of SUSY searches for electroweak production of smuons, plot here in terms of $\Delta m = (\tilde{\mu}_{L,R}, \tilde{\chi}^0_1)$ for a Bino LSP, emphasizing consistency of the bulk with recent muon anomalous magnetic moment measurements . The inset is a zoom of the bulk.

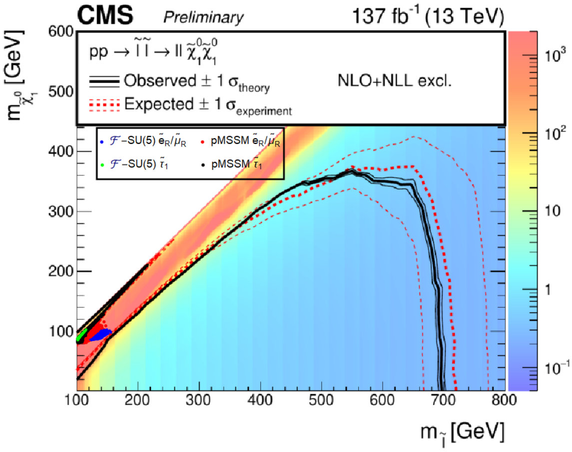


Figure: Generalized No-Scale $\mathcal{F}$-$SU(5)$ and pMSSM bulk regions consistent with the CMS SUSY searches for the sleptons.

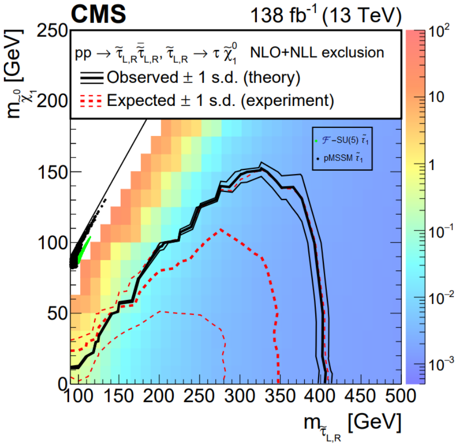


Figure: Generalized No-Scale $\mathcal{F}$-$SU(5)$ and pMSSM bulk regions consistent with the CMS SUSY searches for the light stau.

The Light Right-Handed Stau Bulk Region in the pMSSM

- All pMSSM points with an $\tilde{e}_R$ NLSP are excluded by the ATLAS soft lepton SUSY search.
- Therefore, like Generalized No-Scale $\mathcal{F}$-$SU(5)$, the only viable pMSSM region in the bulk is for the case $m_{\tilde{\chi}_1^0} < m_{\tilde{\tau}_1} < m_{\tilde{e}_R} = m_{\tilde{\mu}_R}$.
- Numerical findings disclose the ratio $\mathcal{R}_{\tilde{\tau}_1} \gtrsim 10\%$ implies $m_{\tilde{\chi}_1^0} \leq 117.7$ GeV. The upper bound on the light stau mass is about 129.3 GeV.
- The bulk alone can explain recent muon anomalous magnetic moment measurements.

Outline

Summary

- We propose Generalized No-Scale Supergravity, the simplest scenario for Effective Super-Natural Supersymmetry, which solve the supersymmetry electroweak fine-tuning problem naturally.
- A light right-handed slepton bulk region is realized in the Generalized No-Scale $\mathcal{F}$-$SU(5)$ and the pMSSM.
- The bulk region may be beyond the LHC reach, though can be probed at the 1000-day LUX-ZEPLIN, Future Circular Collider (FCC-ee) at CERN, Circular Electron Positron Collider (CEPC), and Hyper-Kamiokande experiments.

Thank You Very Much for Your Attention!