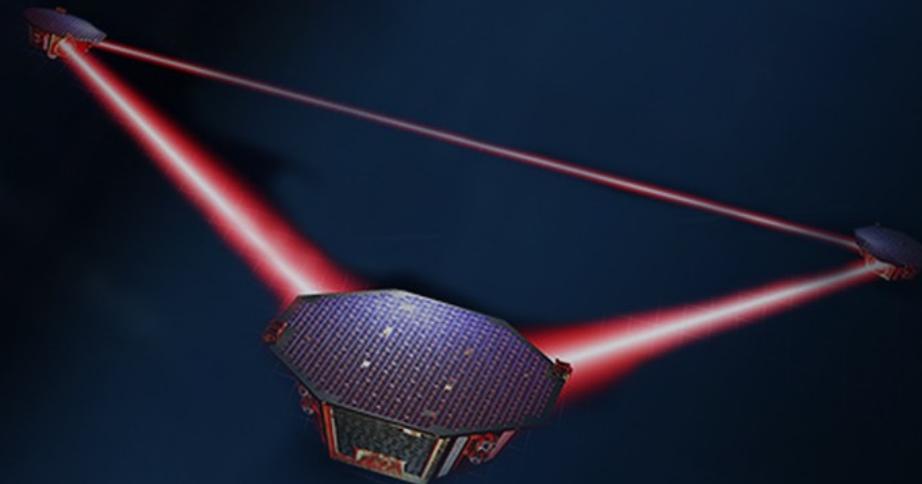


# International Workshop on New Opportunities for Particle Physics 2024



中国科学院高能物理研究所  
*Institute of High Energy Physics, Chinese Academy of Sciences*

## Grand unified theories in era of neutrino precision measurements and gravitational wave observations



Ye-Ling Zhou (HIAS-UCAS)

2024-07-21



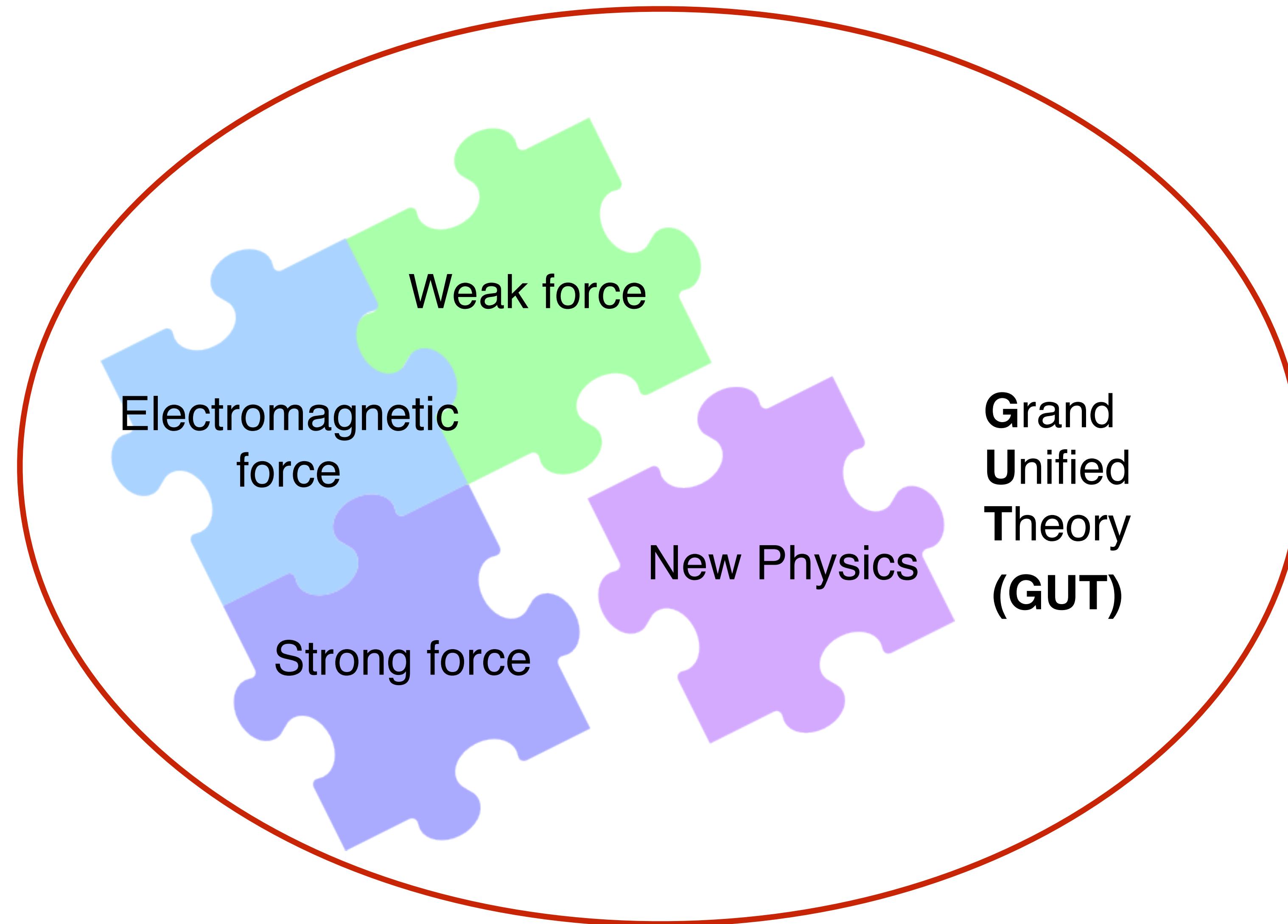
國科大杭州高學研究院  
Hangzhou Institute for Advanced Study, UCAS

基础物理与数学科学学院  
School of Fundamental Physics and Mathematical Sciences



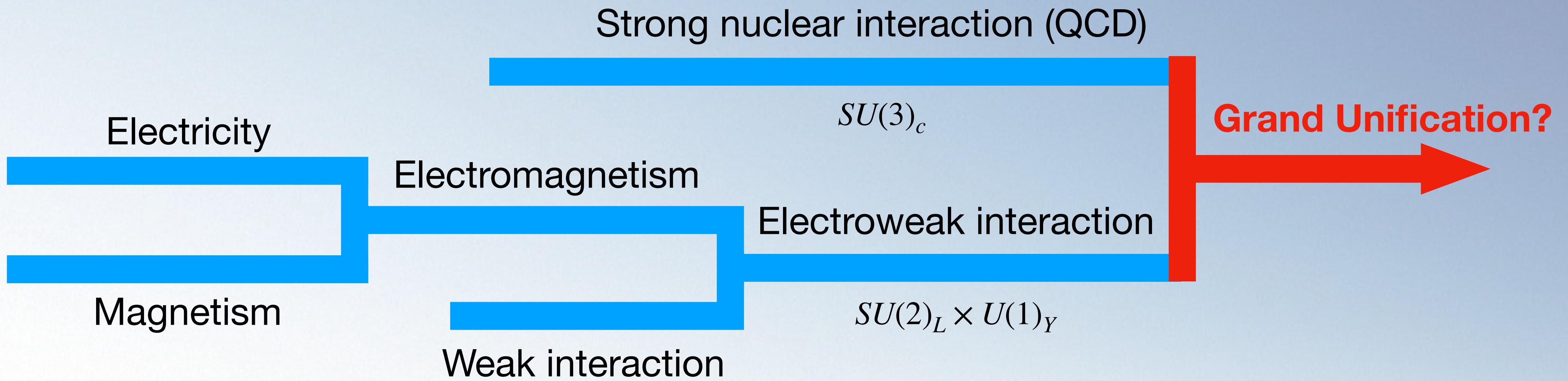
# Framework of GUTs

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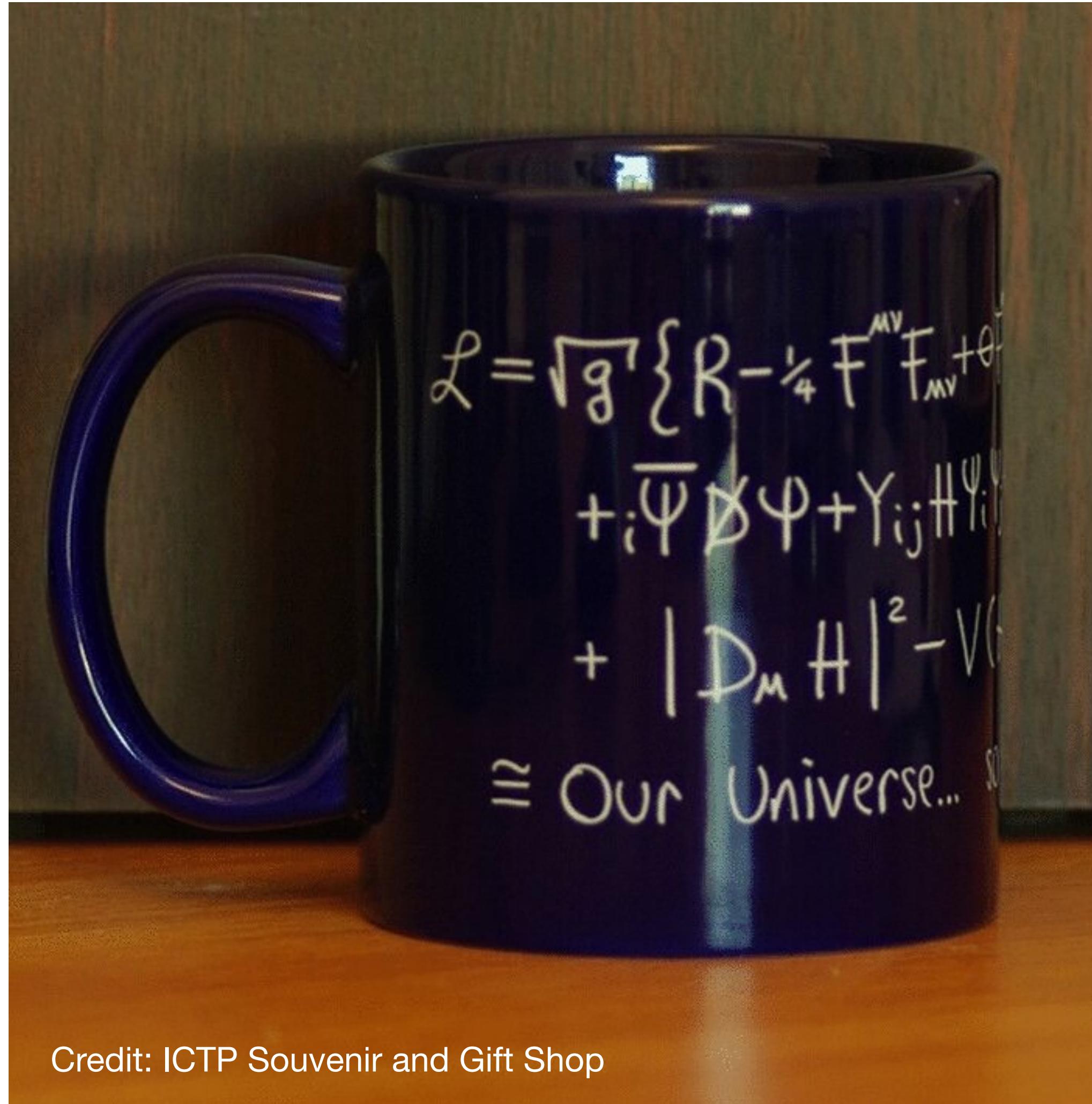


Gravity... not included

# Road to unifications



# What we know so far...



## The Standard Model of Particle Physics

- Gauge theories in

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

Strong force      Weak & EM force

- Particle content

$\nu_\tau$	$\tau$	$t$	$b$	$\gamma$	$W^\pm$	$H$
------------	--------	-----	-----	----------	---------	-----

$\nu_\mu$	$\mu$	$c$	$s$	$g$	$Z^0$
-----------	-------	-----	-----	-----	-------

$\nu_e$	$e$	$u$	$d$
---------	-----	-----	-----

- Yukawa couplings

- Higgs mechanism

And neutrinos have masses...

# How to get a GUT?

- Unification of symmetries

$$G_{\text{GUT}} \supset G_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\begin{array}{c} \downarrow \\ g_3 \end{array} = \begin{array}{c} \downarrow \\ g_2 \end{array} = \begin{array}{c} \downarrow \\ g_1 \end{array}$$

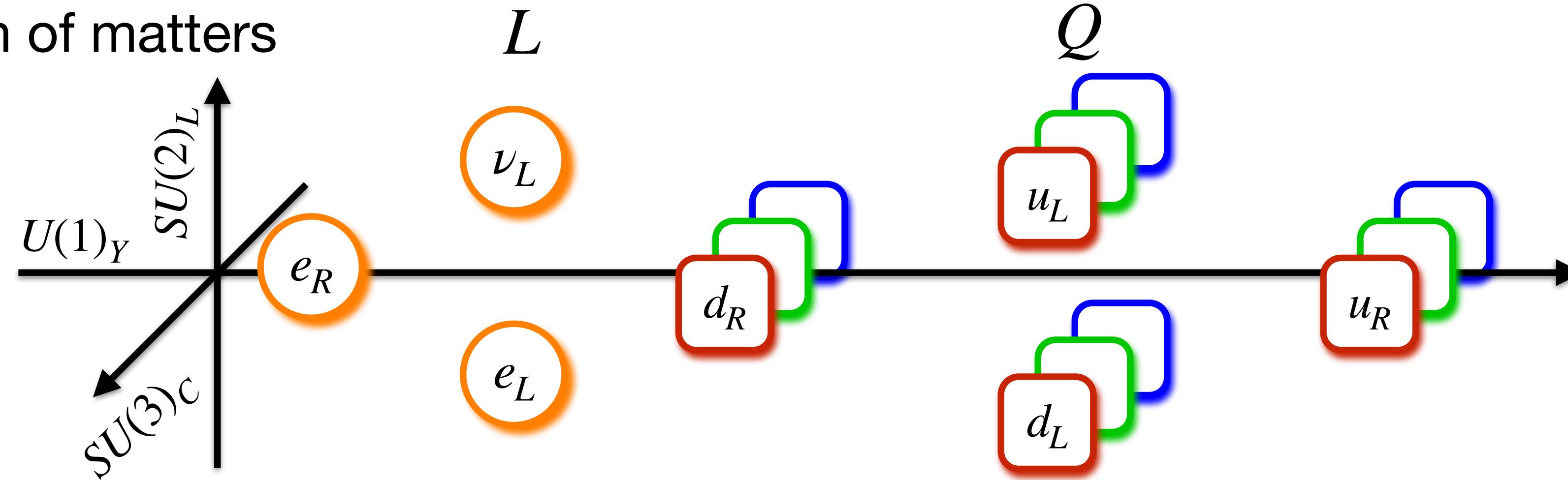
EW

(up to loop correction factors, and if  $G_{\text{GUT}}$  is a simple Lie group)

- Unification of couplings

The scale where three gauge couplings are unified, denoted as  $M_{\text{GUT}}$  in this talk

- Unification of matters



Weak hypercharge:  $Y = -1$     $Y = -\frac{1}{2}$     $Y = -\frac{1}{3}$

$Y = \frac{1}{6}$     $Y = \frac{2}{3}$

# The beginning of GUTs

Pati-Salam (1973, 1974)  $SU(4)_c \times SU(2)_L \times SU(2)_R := G_{422}$

PHYSICAL REVIEW D VOLUME 8, NUMBER 4 15 AUGUST 1973

**Unified Lepton-Hadron Symmetry and a Gauge Theory of the Basic Interactions\***

Jogesh C. Pati†  
*Department of Physics and Astronomy, University of Maryland, College Park, Maryland*

Abdus Salam  
*International Centre for Theoretical Physics, Miramare, Trieste, Italy  
 and Imperial College, London*  
 (Received 5 February 1973)

An attempt is made to unify the fundamental hadrons and leptons into a common irreducible representation  $F$  of the same symmetry group  $G$  and to generate a gauge theory of strong, electromagnetic, and weak interactions. Based on certain constraints from the hadronic side, it is proposed that the group  $G$  is  $SU(4') \times SU(4'')$ , which contains a Han-Nambu-type  $SU(3') \times SU(3'')$  group for the hadronic symmetry, and that the representation  $F$  is  $(4, 4^*)$ . There exist four possible choices for the lepton number  $L$  and accordingly four possible assignments of the hadrons and leptons within the  $(4, 4^*)$ . Two of these require

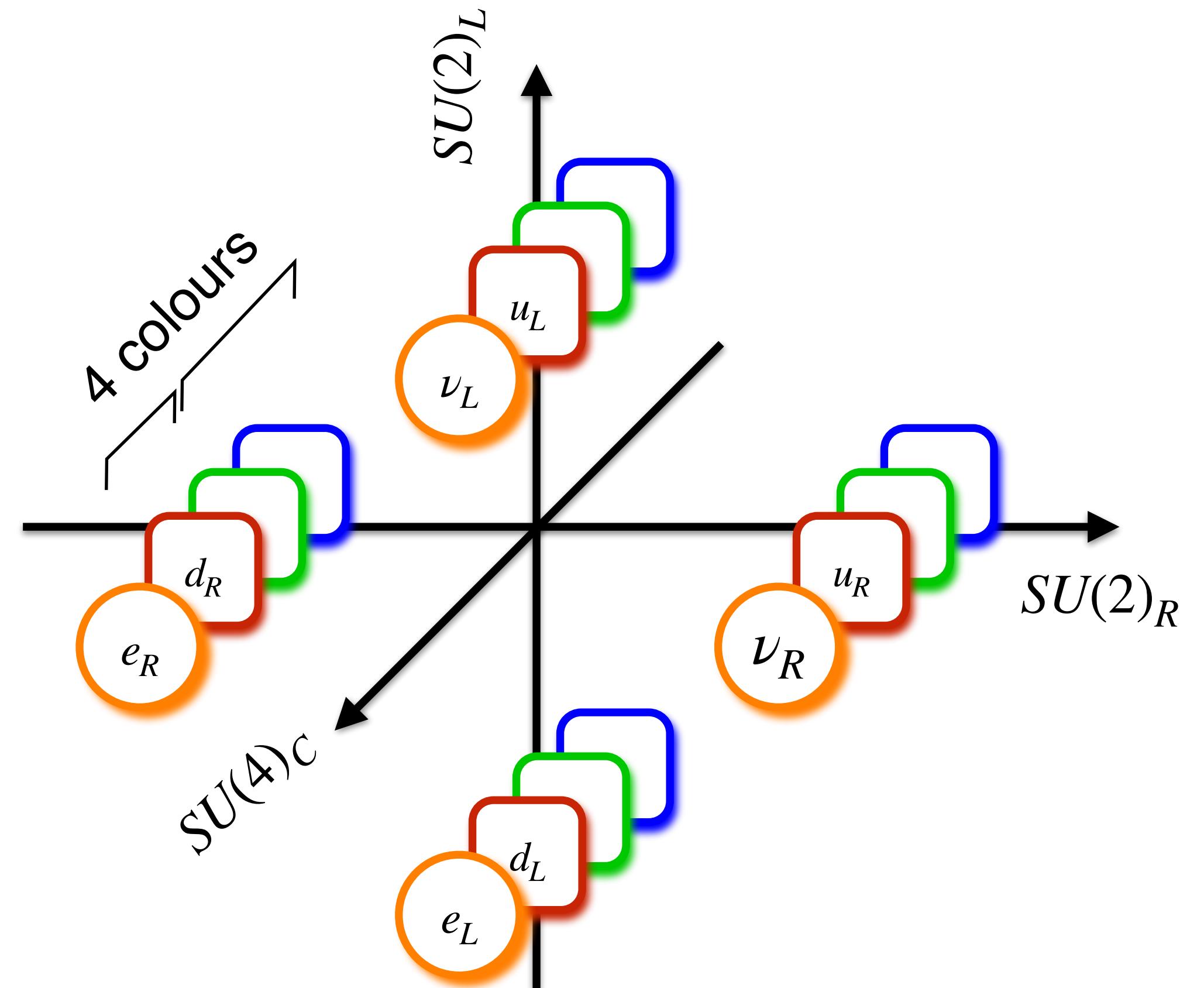
PHYSICAL REVIEW D VOLUME 10, NUMBER 1 1 JULY 1974

**Lepton number as the fourth “color”**

Jogesh C. Pati\*  
*Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742*

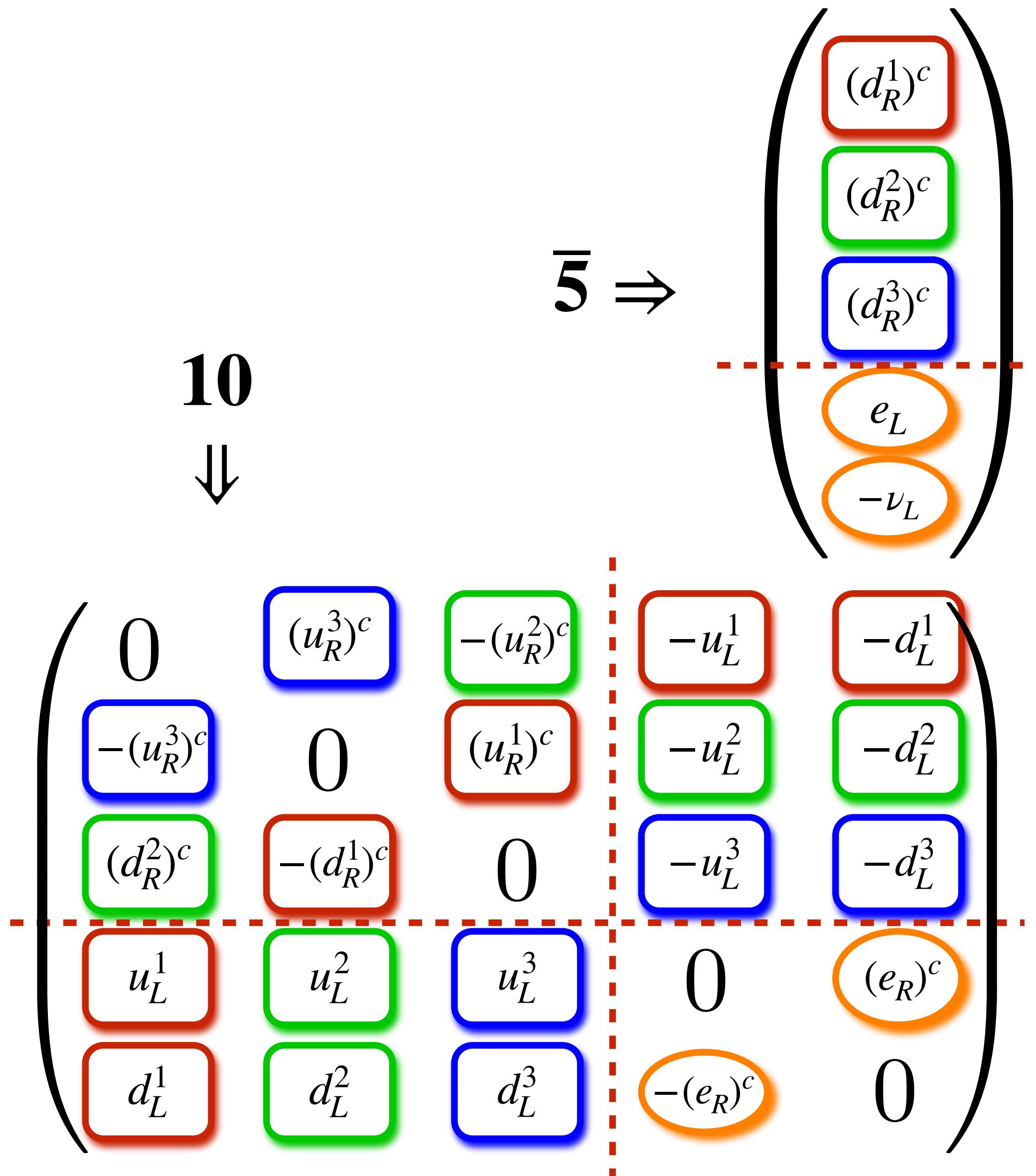
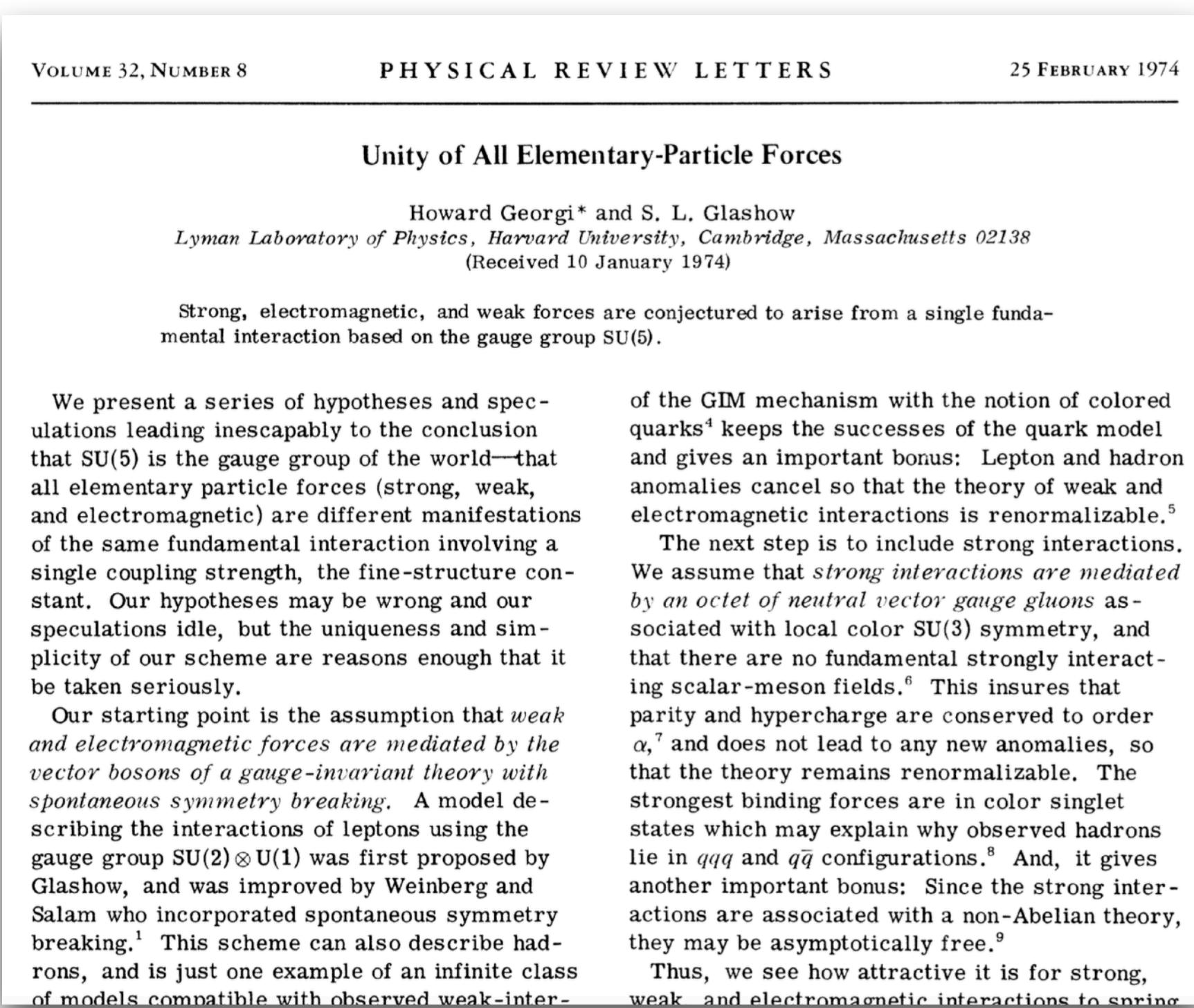
Abdus Salam  
*International Centre for Theoretical Physics, Trieste, Italy  
 and Imperial College, London, England*  
 (Received 25 February 1974)

Universal strong, weak, and electromagnetic interactions of leptons and hadrons are generated by gauging a non-Abelian renormalizable anomaly-free subgroup of the fundamental symmetry structure  $SU(4)_L \times SU(4)_R \times SU(4')$ , which unites three quartets of “colored” baryonic quarks and the quartet of known leptons into 16-folds of chiral fermionic multiplets, with lepton number treated as the fourth “color” quantum number. Experimental consequences of this scheme are discussed. These include (1) the emergence and effects of exotic gauge mesons carrying both baryonic as well as leptonic quantum numbers, particularly in semileptonic processes, (2) the manifestation of anomalous strong interactions among leptonic and semi-leptonic processes at high energies, (3) the independent possibility of baryon-lepton number violation in quark and proton decays, and (4) the occurrence of  $(V+A)$  weak-current effects.



# The beginning of GUTs

Georgi-Glashow (1974),  $SU(5)$  GUT



And Higgses 5, 45, 24.

# The beginning of GUTs

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- $SO(10)$  GUTs      Fritzsch, Minkowski (1975)

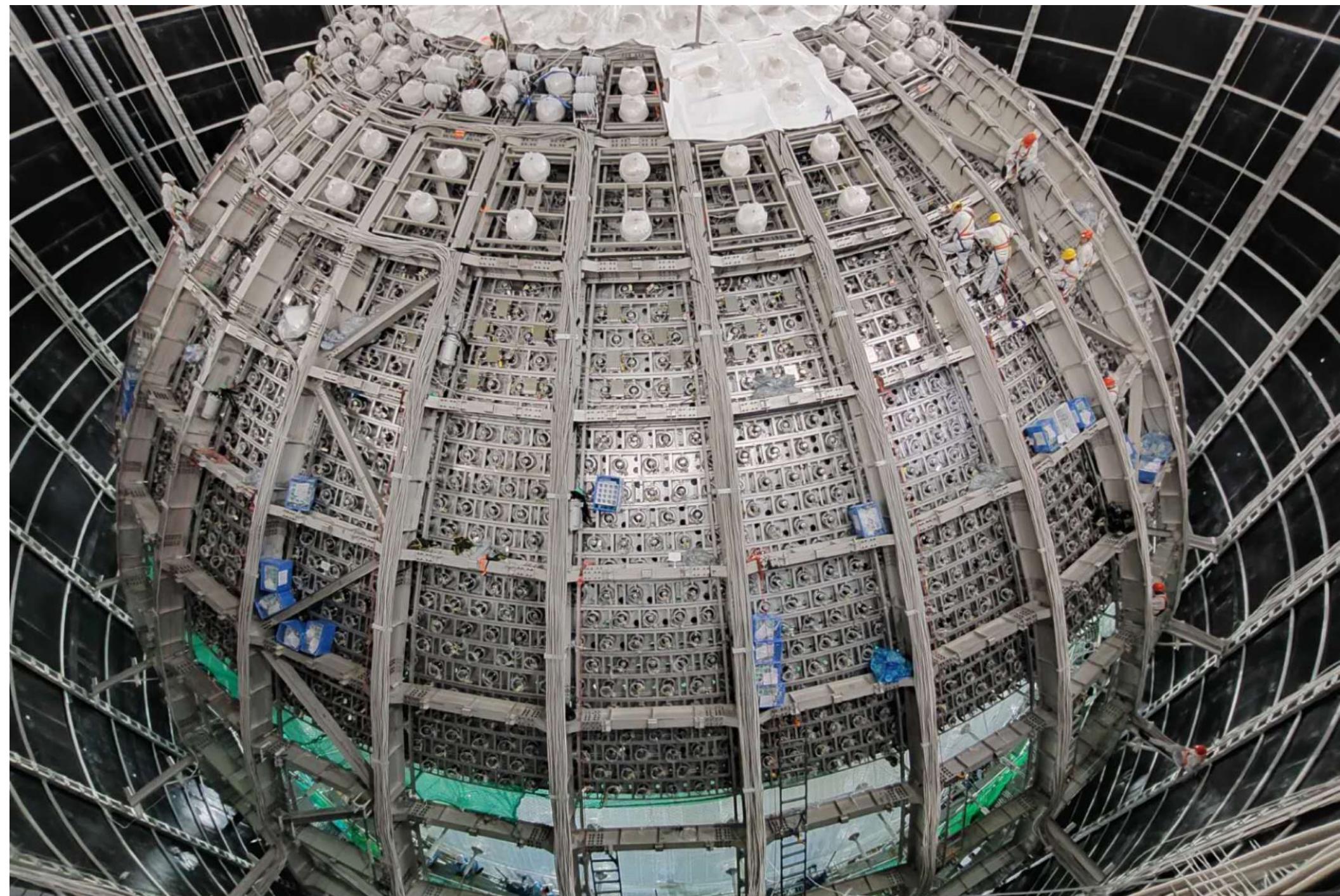
$$\mathbf{16} = \bar{\mathbf{5}} + \mathbf{10} + \mathbf{1} = (\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\mathbf{4}, \mathbf{1}, \mathbf{2})$$
$$SO(10) \quad SU(5) \quad SU(4)_c \times SU(2)_L \times SU(2)_R$$

Contains  $SU(5)$  and Pati-Salam, and more ...

- Not minimal but realistic  $SU(5)$       e.g., with extra  $\mathbf{15}_H$ ,  $\mathbf{24}_F$
- $SU(5) \times U(1)_{B-L} := G_{51}$        $\bar{\mathbf{5}} + \mathbf{10} + \mathbf{1}$ ,  $\nu_R \sim \mathbf{1}$
- Flipped  $SU(5) \times U(1)_\chi := G_{51}^{\text{flip}}$        $u \leftrightarrow d$ ,  $\nu \leftrightarrow e$        $\Rightarrow$  See Tianjun's talk

Rujula, Georgi, Glashow (1980); Barr,(1982); Derendinger, Kim,  
Nanopoulos (1984); Antoniadis, Ellis, Hagelin, Nanopoulos (1989)

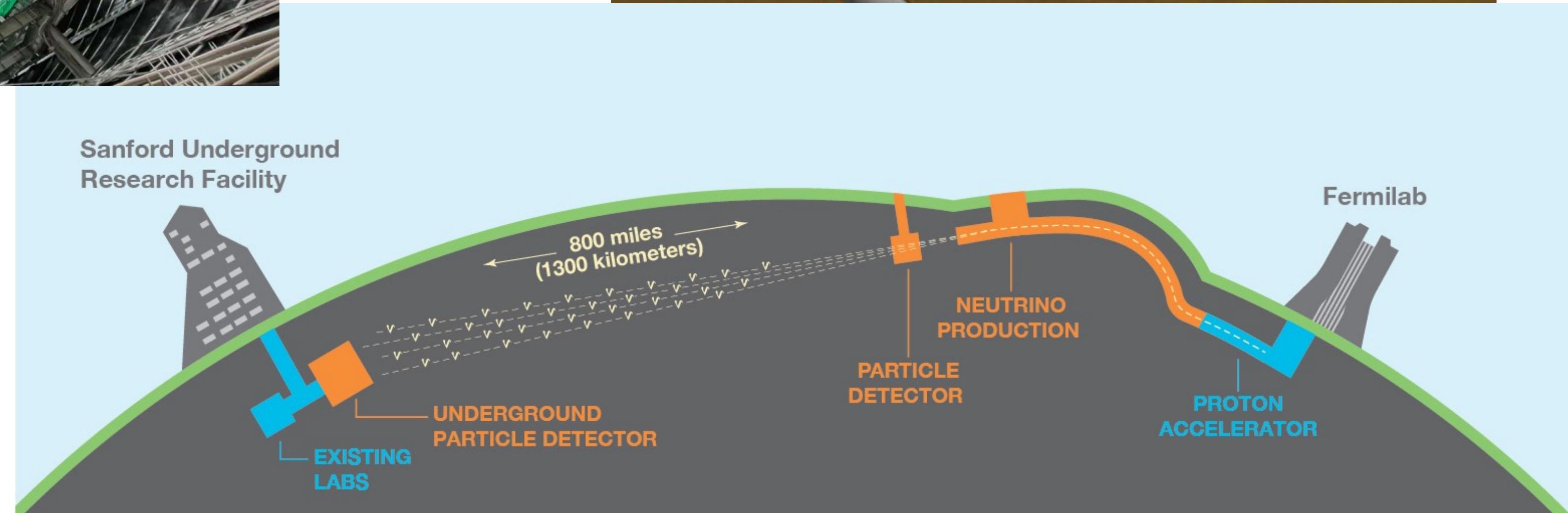
# Upcoming large-scale neutrino experiments



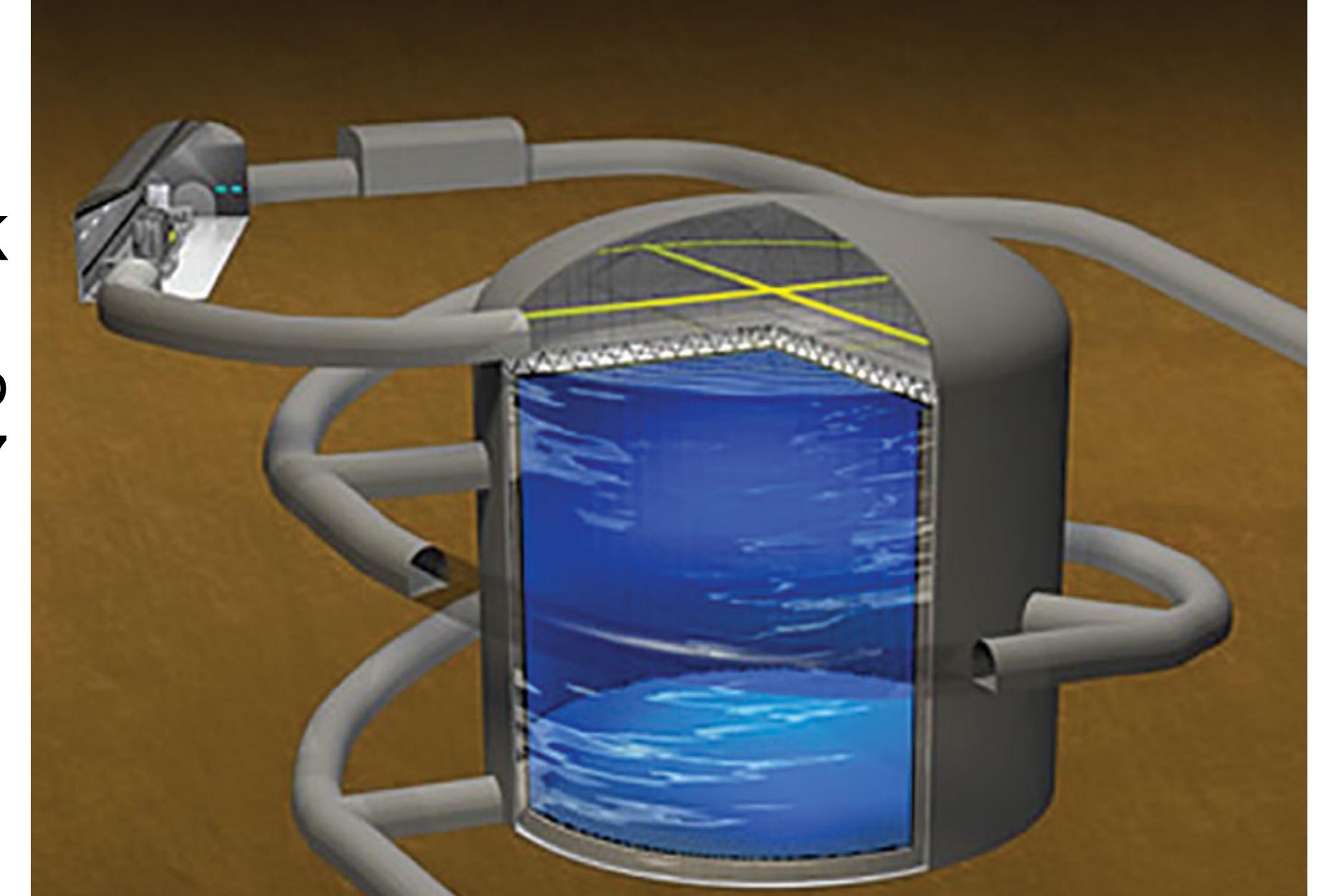
JUNO, about to run very soon

Neutrino precision measurements is coming!

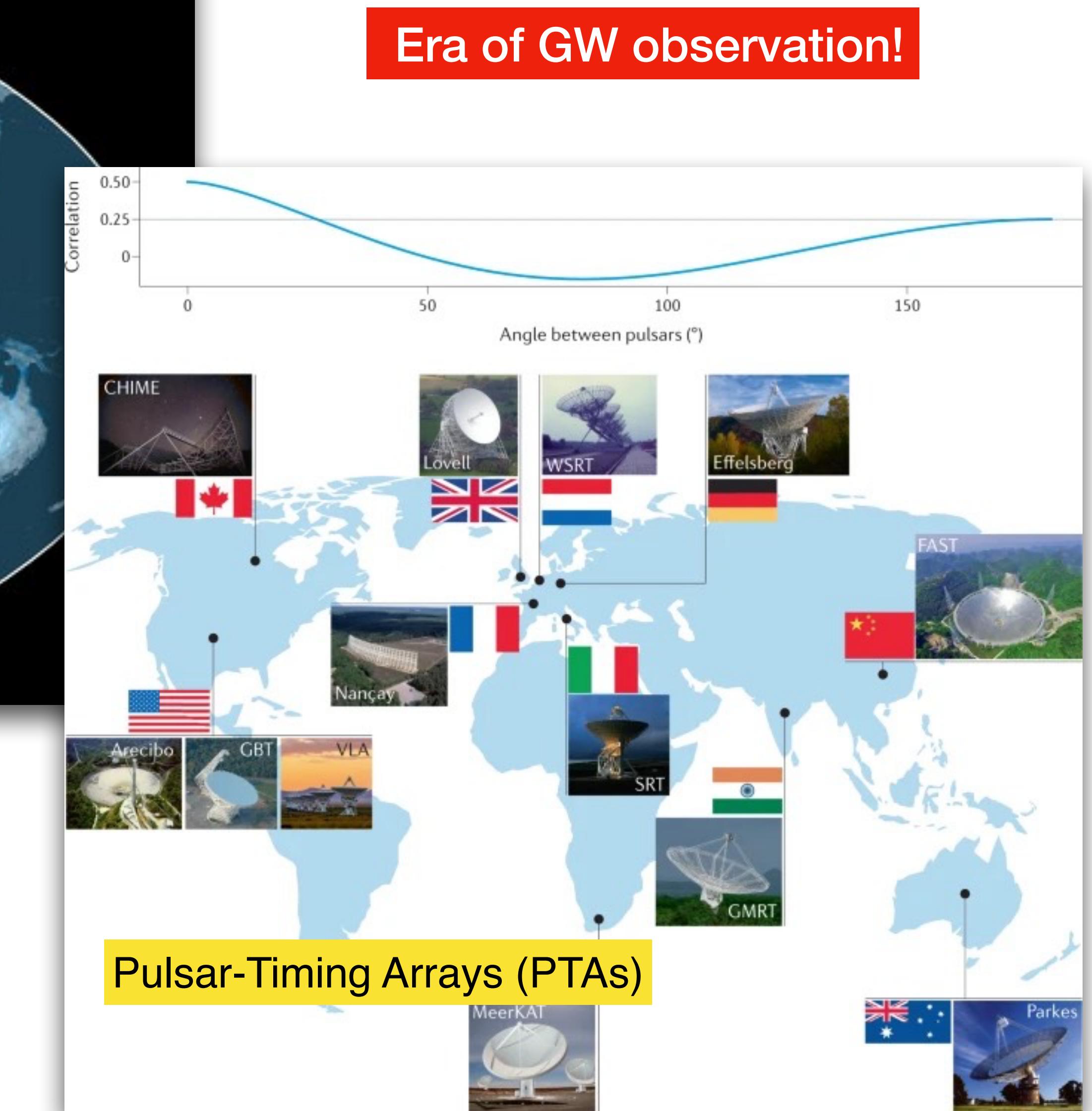
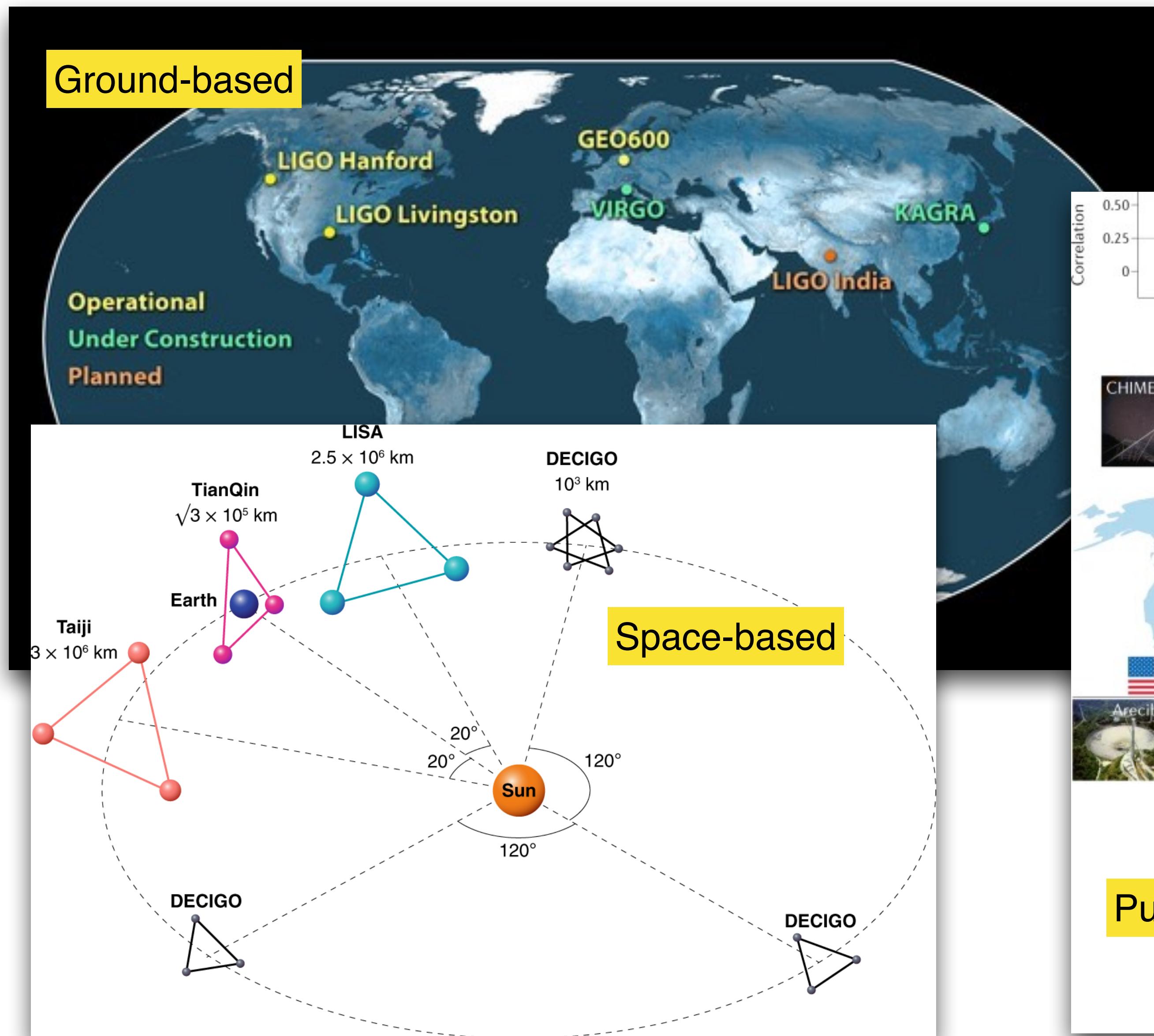
DUNE  
supposed to run in 2030?



Hyper-K  
expected to run in 2027

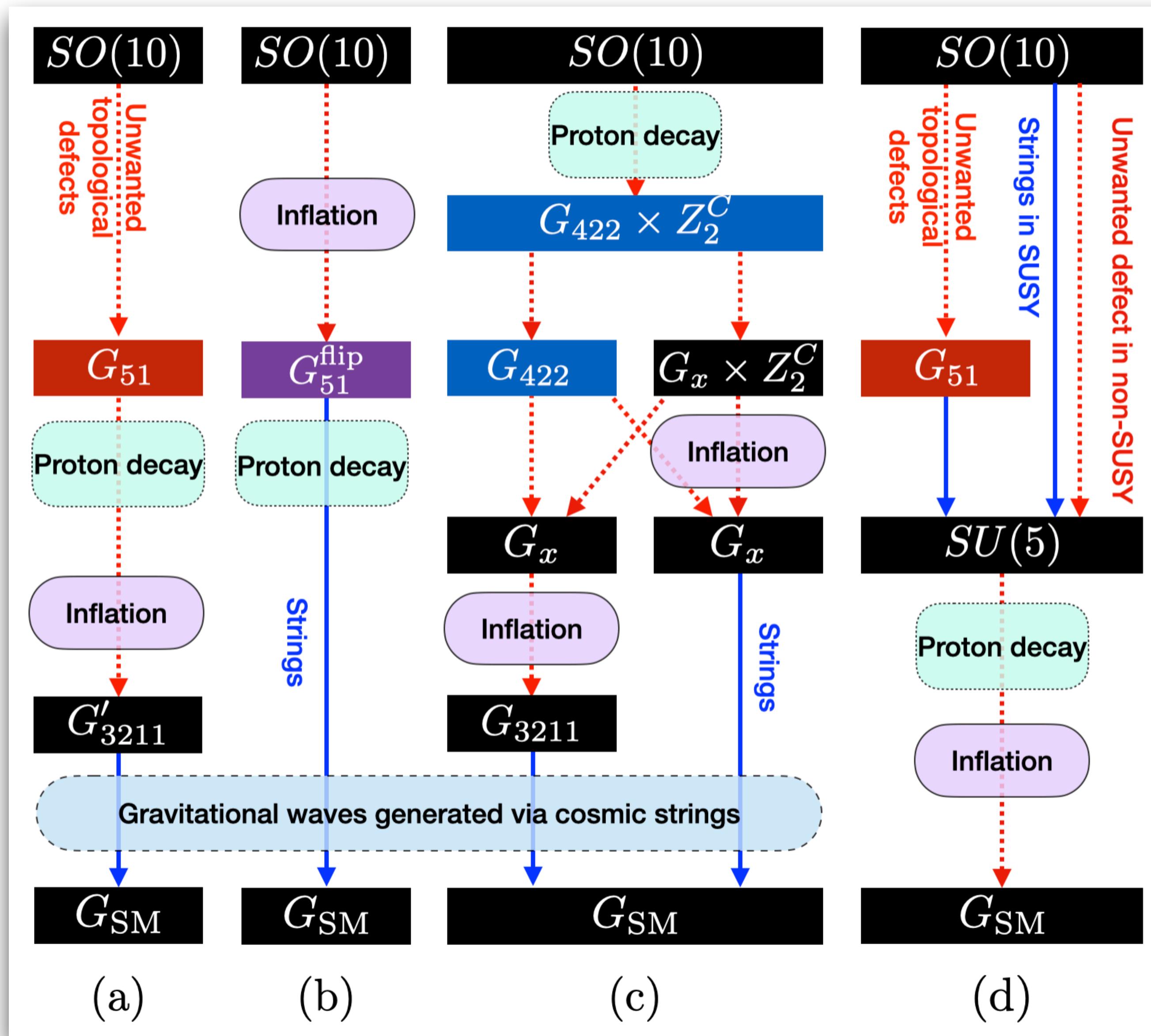


# Undergoing and upcoming GW measurements



# GUT phenos

## Fermion masses and mixing



**Unwanted topological defects:  
monopoles and domain walls**

In any breaking chains, inflation has to be introduced to inflate unwanted defects

$$G_{422} = SU(4)_C \times SU(2)_L \times SU(2)_R$$

$$G_{51} = SU(5) \times U(1)_X$$

$$G_{51}^{\text{flip}} = SU(5)_{\text{flip}} \times U(1)_{\text{flip}}$$

$$Z_2^C: \quad \psi_L \leftrightarrow \psi_R^c$$

$$G_x = G_{421} \text{ or } G_{3221}$$

$$G_{3221} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$G_{421} = SU(4)_C \times SU(2)_L \times U(1)_Y$$

$$G_{3211} = SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$$

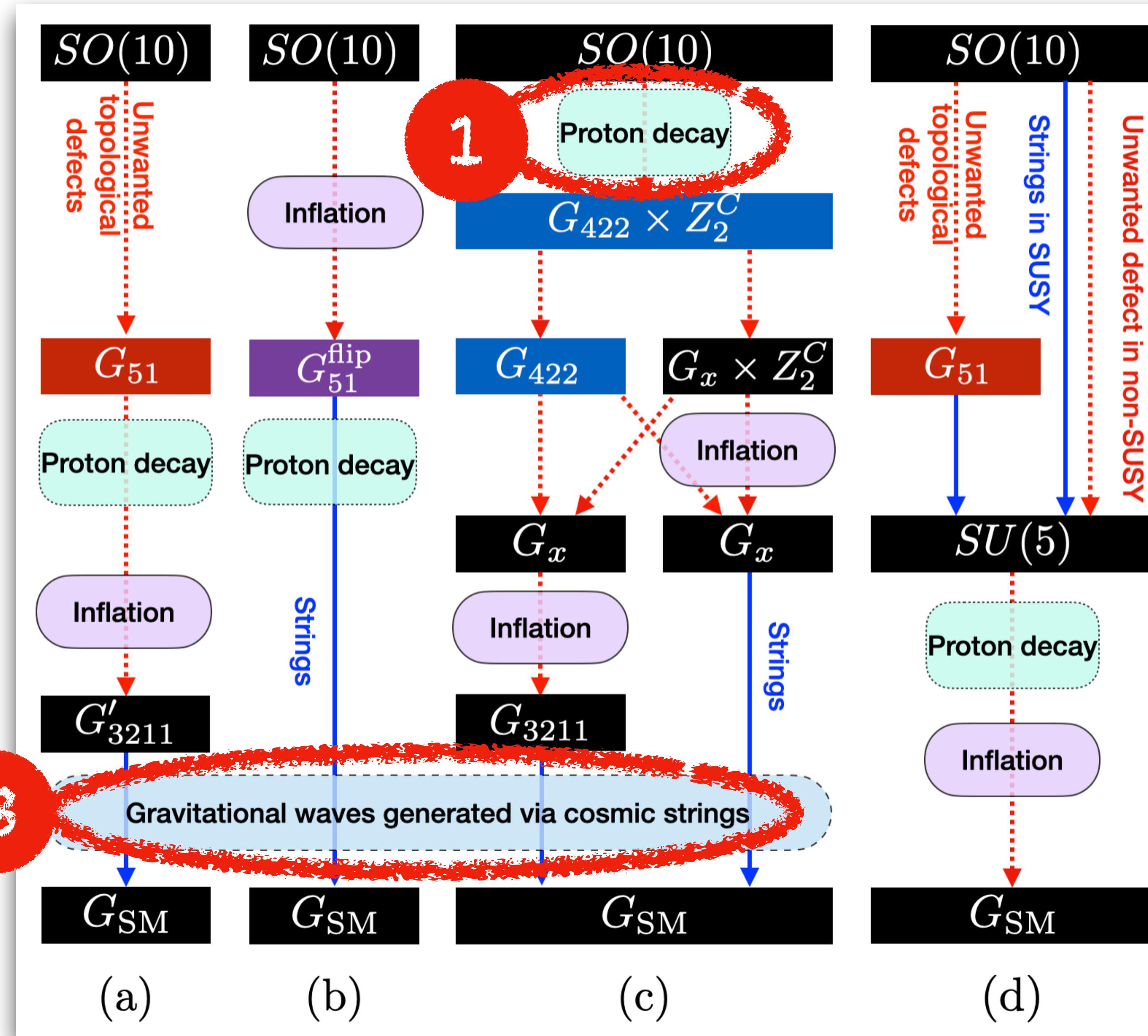
$$G'_{3211} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$$

$$G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

# GUT phenos

2

## Fermion masses and mixing



## Unwanted topological defects: monopoles and domain walls

In any breaking chains, inflation has to be introduced to inflate unwanted defects

$$G_{422} = SU(4)_C \times SU(2)_L \times SU(2)_R$$

$$G_{51} = SU(5) \times U(1)_X$$

$$G_{51}^{\text{flip}} = SU(5)_{\text{flip}} \times U(1)_{\text{flip}}$$

$$Z_2^C: \psi_L \leftrightarrow \psi_R^c$$

$$G_x = G_{421} \text{ or } G_{3221}$$

$$G_{3221} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$G_{421} = SU(4)_C \times SU(2)_L \times U(1)_Y$$

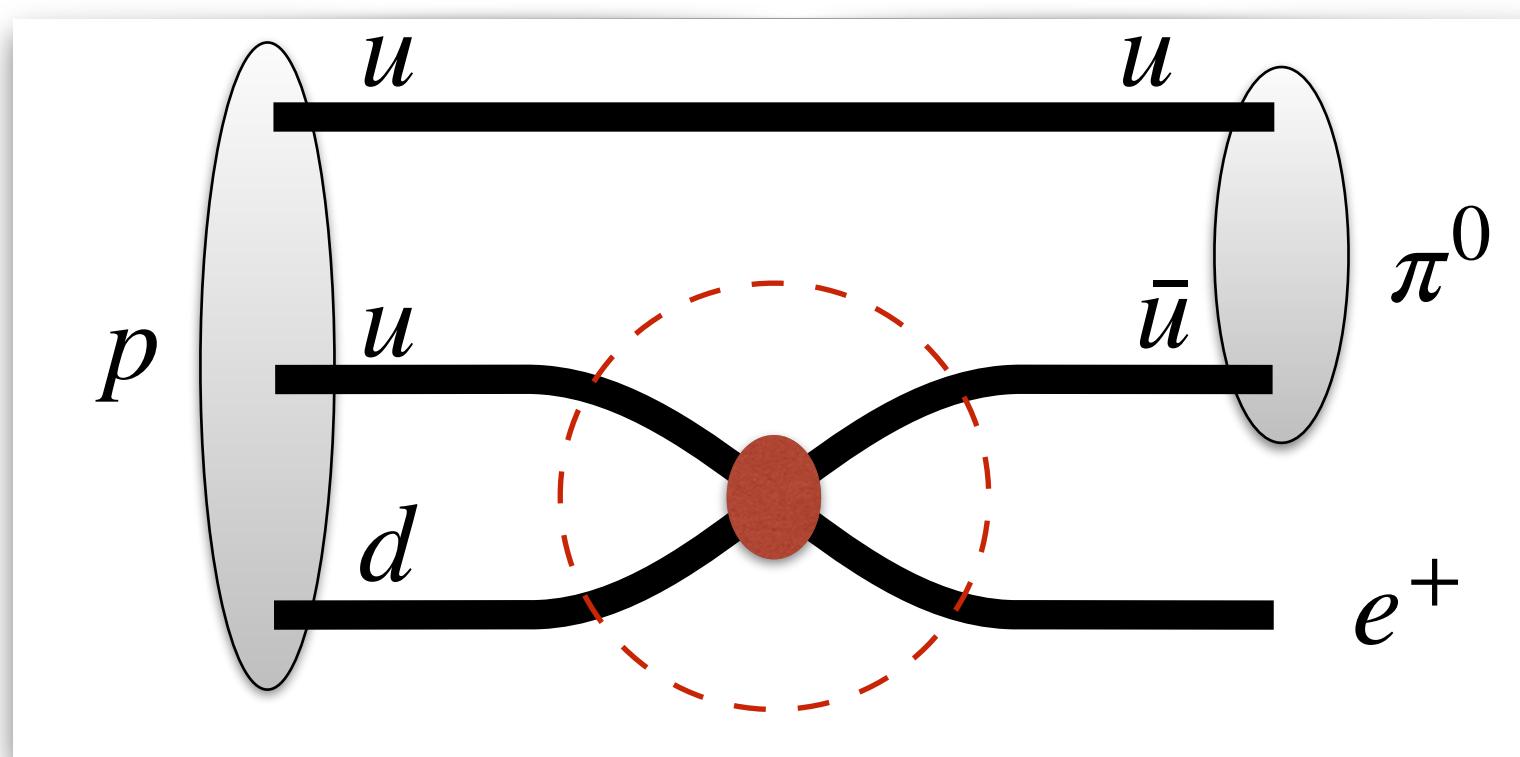
$$G_{3211} = SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$$

$$G'_{3211} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$$

$$G_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

# Proton decay

- GUTs unify baryons and leptons and naturally induce baryon-violating interactions, which appear as higher dimensional operators at low energy.
  - Dim-6 operators Dominated by gauge mediators

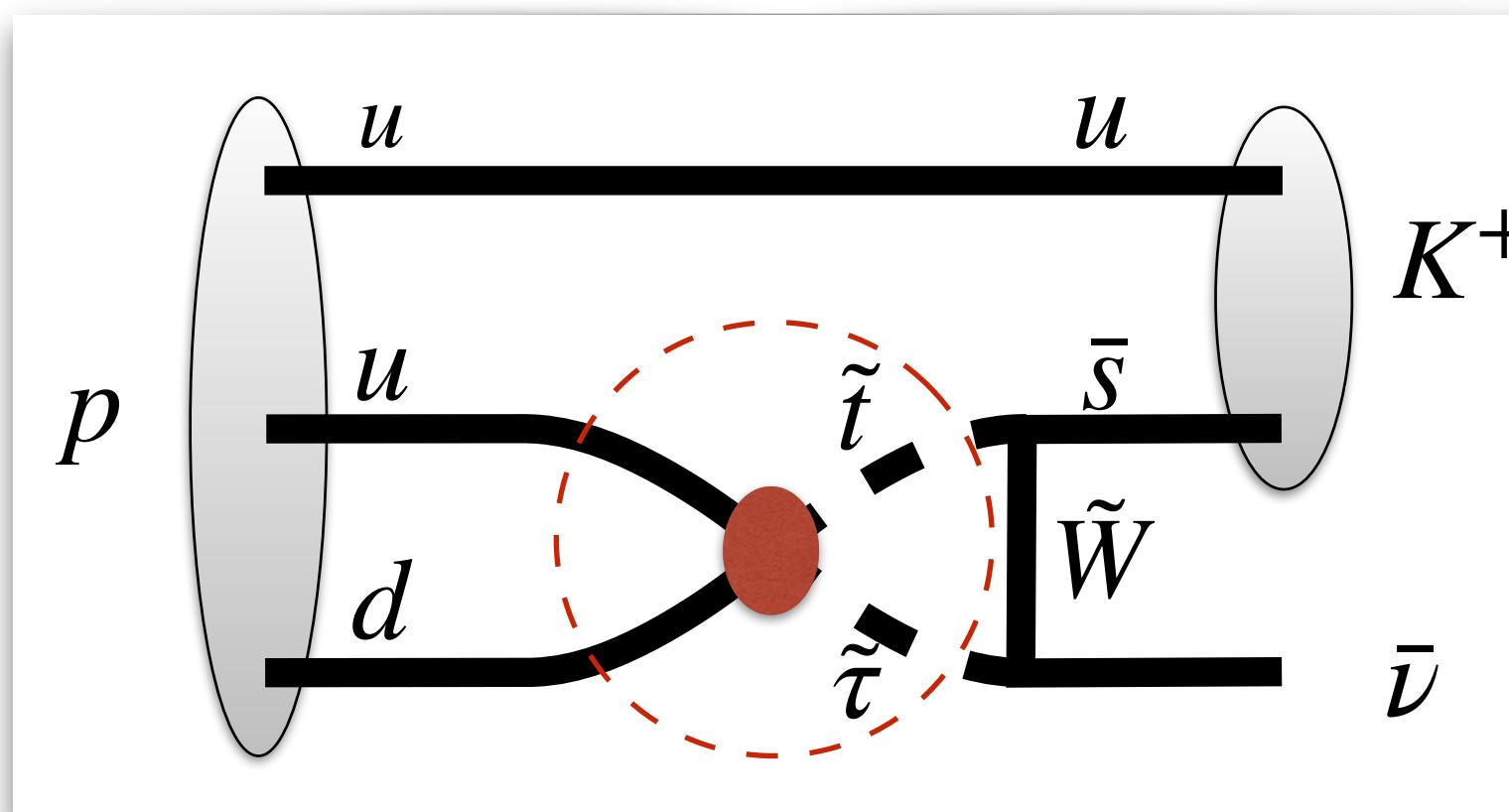


# Dominated by gauge mediators

$$p \rightarrow \pi^0 e^+$$

$$\tau_{\pi^0 e^+} \sim 10^{35} \text{ years} \times \left( \frac{M_{\text{GUT}}}{10^{16} \text{ GeV}} \right)^4$$

- # • Dim-5 operators      Induced by SUSY



# Induced by SUSY

$$p \rightarrow K^+ \bar{\nu}$$

$$\tau_{K^+\bar{\nu}} \propto (M_{\text{GUT}} \cdot M_{\text{SUSY}} \cdot Y_{\text{Yukawa}})^2$$

# Measuring proton decay in neutrino experiments



20 kton water / liquid scintillator

$$\simeq 2 \times 10^{10} \text{ g} \times (2 + 8)/18 \text{ mol/g} \times N_A / \text{mol}$$

$$\simeq 7 \times 10^{33} \text{ proton}$$

# Capability of JUNO

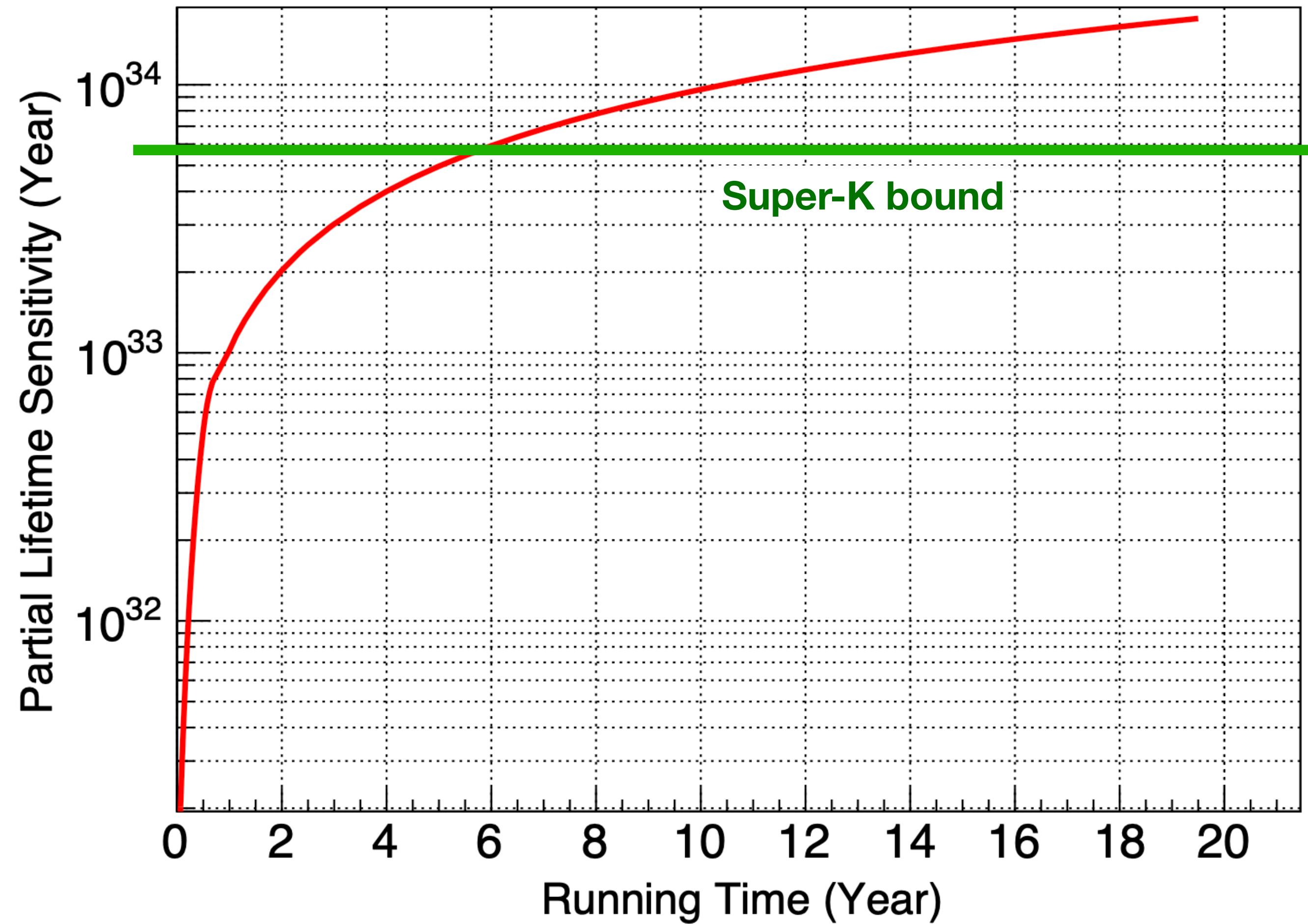
$$\tau/B(p \rightarrow \bar{\nu}K^+) = \frac{N_p T \epsilon}{n_{90}}$$

Efficiency  $\epsilon_{\text{JUNO}} = 36.9\%$

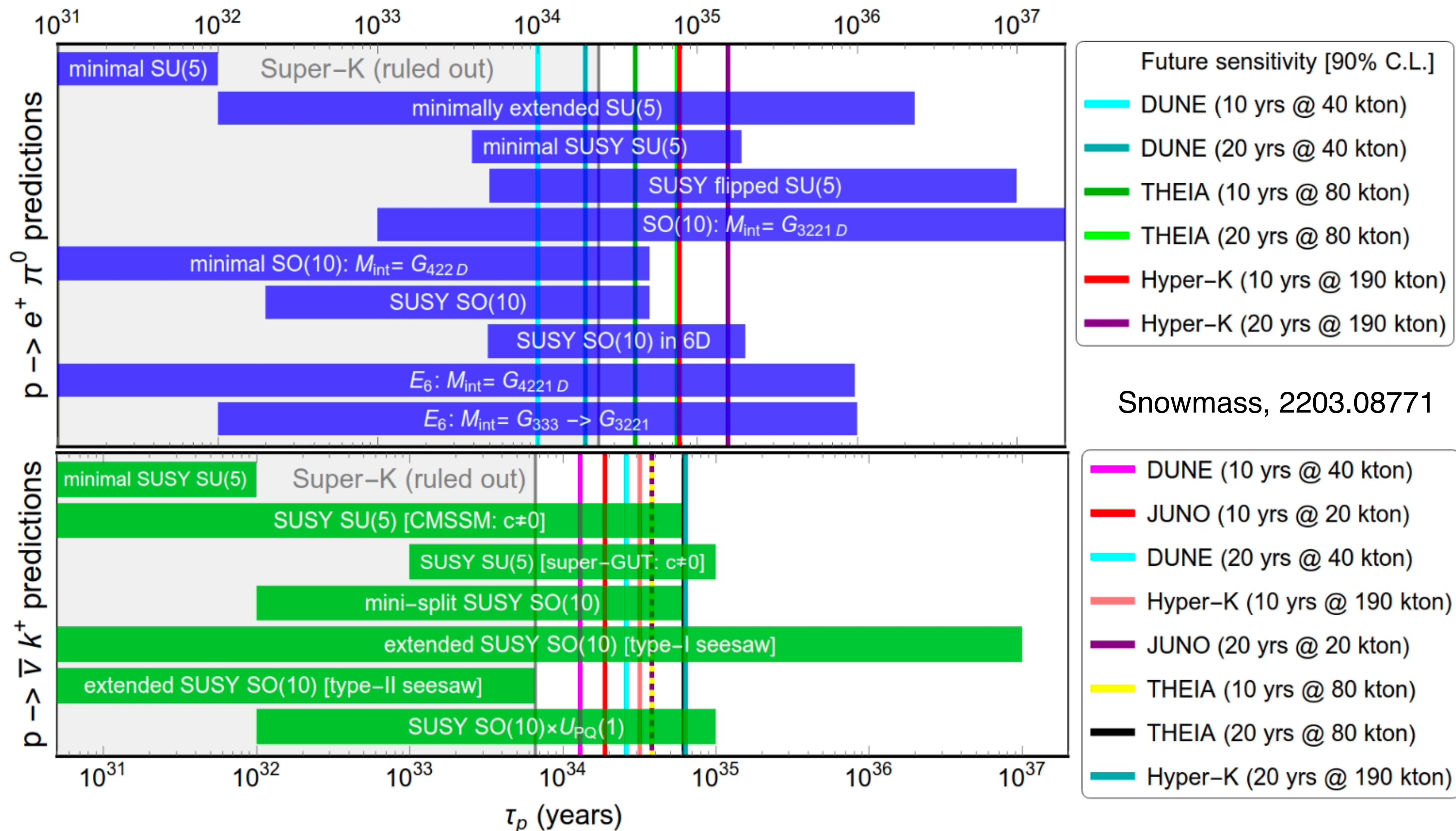
↑  
↓

Statistics (90%CL)

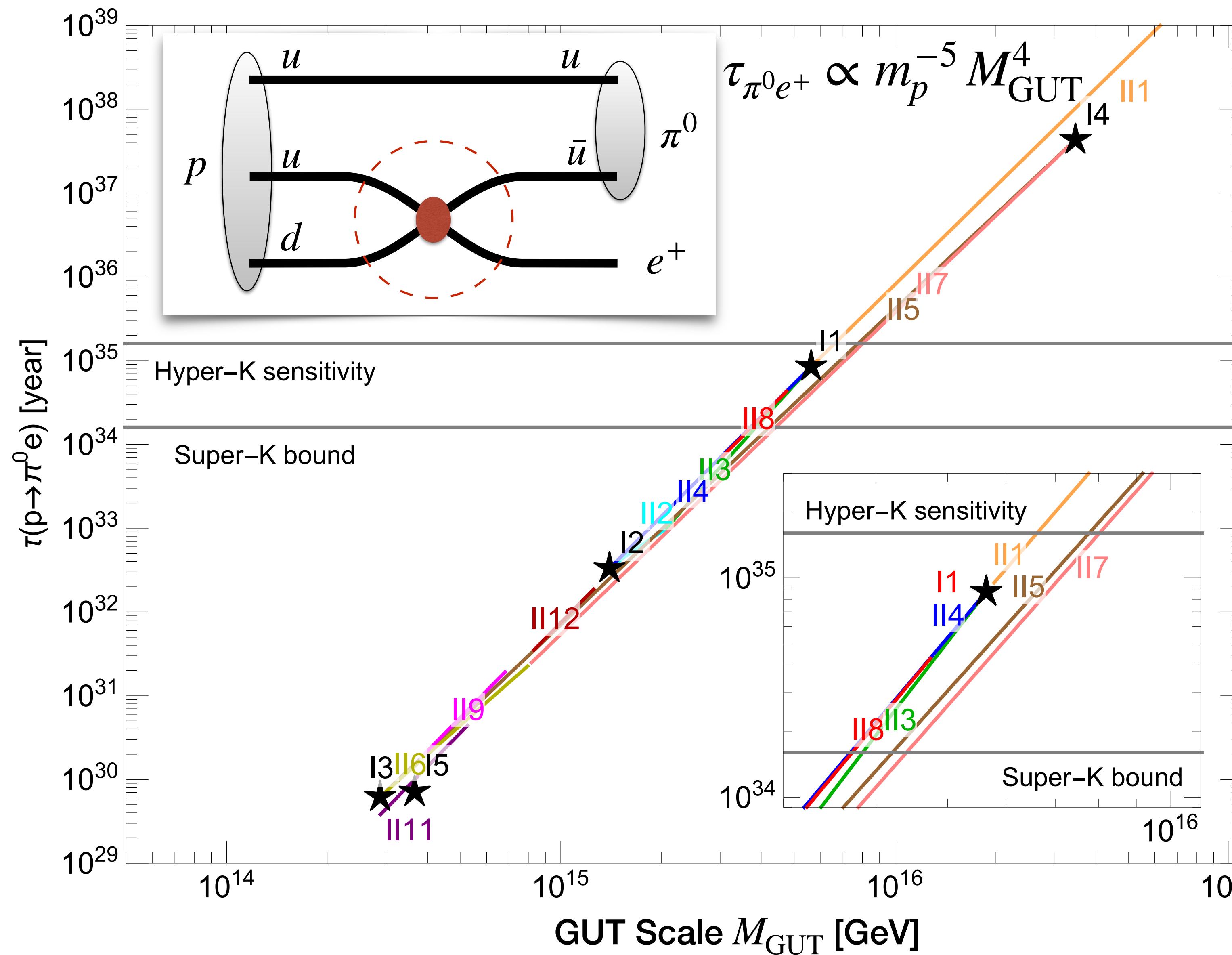
JUNO, 2212.08502



# Potential in upcoming neutrino experiments



# Proton decay in SO(10) GUTs

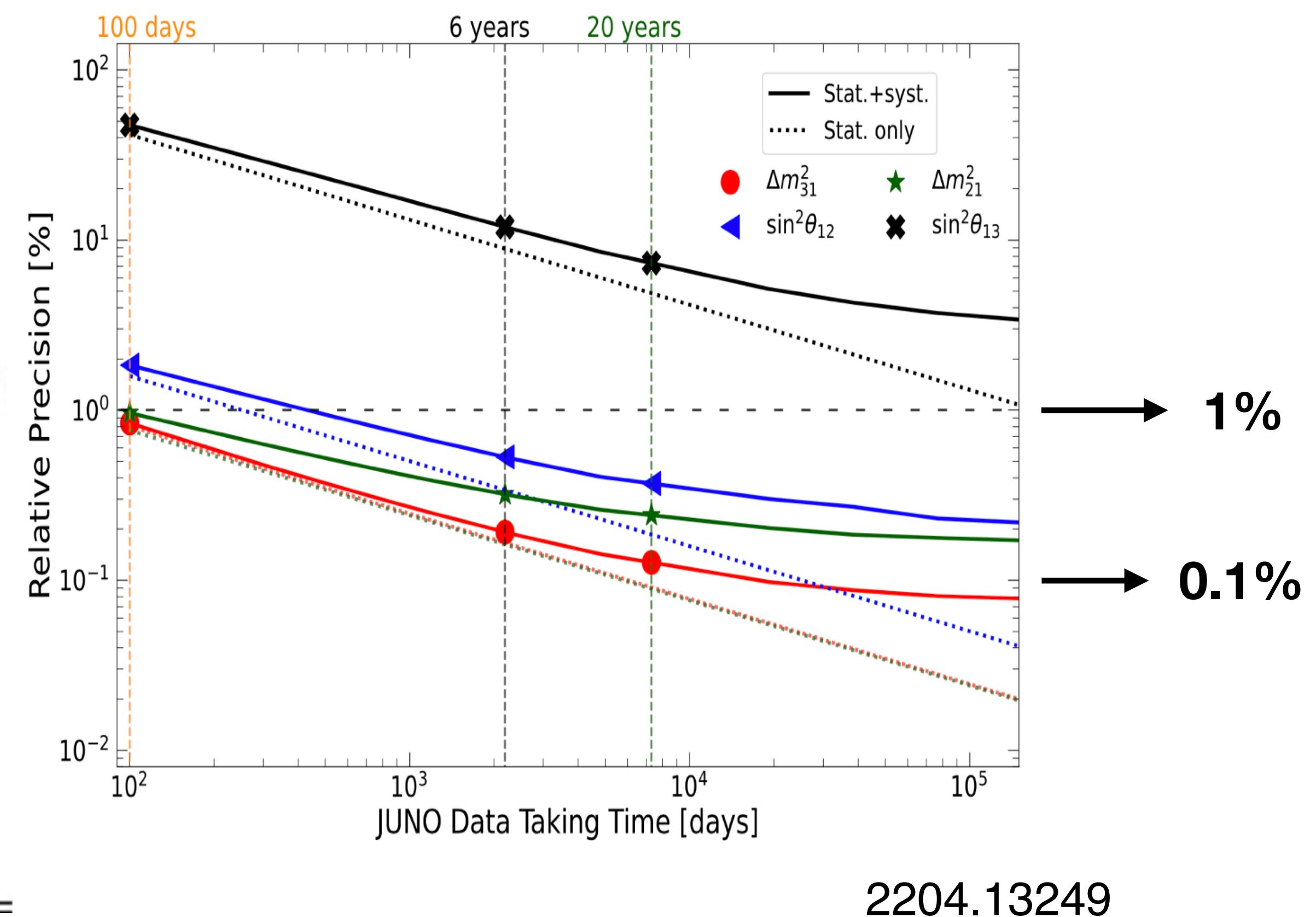


$SO(10)$	$\xrightarrow{\text{defect}} Higgs$	$G_2$	$\xrightarrow{\text{defect}} Higgs$	$G_1$	$\xrightarrow{\text{defect}} Higgs$	$G_{\text{SM}}$
II1:	$\frac{m}{210}$	$G_{422}$	$\frac{m}{45}$	$G_{3221}$	$\frac{s}{126}$	
II2:	$\frac{m,s}{54}$	$G_{422}^C$	$\frac{m}{210}$	$G_{3221}^C$	$\frac{s,w}{126}$	
II3:	$\frac{m,s}{54}$	$G_{422}^C$	$\frac{m,w}{45}$	$G_{3221}$	$\frac{s}{126}$	
II4:	$\frac{m,s}{210}$	$G_{3221}^C$	$\frac{w}{45}$	$G_{3221}$	$\frac{s}{126}$	
II5:	$\frac{m}{210}$	$G_{422}$	$\frac{m}{45}$	$G_{421}$	$\frac{s}{126}$	
II6:	$\frac{m,s}{54}$	$G_{422}^C$	$\frac{m}{45}$	$G_{421}$	$\frac{s}{126}$	
II7:	$\frac{m,s}{54}$	$G_{422}^C$	$\frac{w}{210}$	$G_{422}$	$\frac{m}{126,45}$	
II8:	$\frac{m}{45}$	$G_{3221}$	$\frac{m}{45}$	$G_{3211}$	$\frac{s}{126}$	
II9:	$\frac{m,s}{210}$	$G_{3221}^C$	$\frac{m,w}{45}$	$G_{3211}$	$\frac{s}{126}$	
II10:	$\frac{m}{210}$	$G_{422}$	$\frac{m}{210}$	$G_{3211}$	$\frac{s}{126}$	
II11:	$\frac{m,s}{54}$	$G_{422}^C$	$\frac{m,w}{210}$	$G_{3211}$	$\frac{s}{126}$	
II12:	$\frac{m}{45}$	$G_{421}$	$\frac{m}{45}$	$G_{3211}$	$\frac{s}{126}$	

King, Pascoli, Turner, **YLZ**,  
2106.15634

# Fermion masses and mixing

		NuFIT 5.3 (2024)			
		Normal Ordering (best fit)		Inverted Ordering ( $\Delta\chi^2 = 2.3$ )	
		bfp $\pm 1\sigma$	3 $\sigma$ range	bfp $\pm 1\sigma$	3 $\sigma$ range
without SK atmospheric data	$\sin^2 \theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \rightarrow 0.344$	$0.307^{+0.012}_{-0.011}$	$0.275 \rightarrow 0.344$
	$\theta_{12}/^\circ$	$33.66^{+0.73}_{-0.70}$	$31.60 \rightarrow 35.94$	$33.67^{+0.73}_{-0.71}$	$31.61 \rightarrow 35.94$
	$\sin^2 \theta_{23}$	$0.572^{+0.018}_{-0.023}$	$0.407 \rightarrow 0.620$	$0.578^{+0.016}_{-0.021}$	$0.412 \rightarrow 0.623$
	$\theta_{23}/^\circ$	$49.1^{+1.0}_{-1.3}$	$39.6 \rightarrow 51.9$	$49.5^{+0.9}_{-1.2}$	$39.9 \rightarrow 52.1$
	$\sin^2 \theta_{13}$	$0.02203^{+0.00056}_{-0.00058}$	$0.02029 \rightarrow 0.02391$	$0.02219^{+0.00059}_{-0.00057}$	$0.02047 \rightarrow 0.02396$
	$\theta_{13}/^\circ$	$8.54^{+0.11}_{-0.11}$	$8.19 \rightarrow 8.89$	$8.57^{+0.11}_{-0.11}$	$8.23 \rightarrow 8.90$
	$\delta_{\text{CP}}/^\circ$	$197^{+41}_{-25}$	$108 \rightarrow 404$	$286^{+27}_{-32}$	$192 \rightarrow 360$
	$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.81 \rightarrow 8.03$	$7.41^{+0.21}_{-0.20}$	$6.81 \rightarrow 8.03$
	$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.511^{+0.027}_{-0.027}$	$+2.428 \rightarrow +2.597$	$-2.498^{+0.032}_{-0.024}$	$-2.581 \rightarrow -2.409$



# Fermion masses and mixing in SO(10)

- Matter fields  $\{Q = (u_L, u_R), u_R, d_R, L = (\nu_L, e_L), e_R, \nu_R\}$  are arranged in a single **16** of SO(10)

$$\mathbf{16} \times \mathbf{16} = \mathbf{10} + \mathbf{126} + \mathbf{120}$$

- Three Higgs fields  $\mathbf{10}_H, \overline{\mathbf{126}}_H, \mathbf{120}_H$  can be introduced

And the SM Higgs should be the lightest one after mixing

- Gauge-invariant Yukawa interactions

$$\mathcal{L}_Y = Y_{\mathbf{10}}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10}_H + Y_{\overline{\mathbf{126}}}^* \mathbf{16} \cdot \mathbf{16} \cdot \overline{\mathbf{126}}_H + Y_{\mathbf{120}}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{120}_H + \text{h.c.}$$

- Correlations of Yukawa matrices of quarks and leptons Dutta, Mimura, Mohapatra, 0412105

$$Y_d = r_1(h + f + i h')$$

$$Y_u = h + r_2 f + i r_3 h'$$

$$M_{\nu_R} = m_{\nu_R} f$$

$$Y_e = r_1(h - 3f + i c_e h')$$

$$Y_\nu = h - 3r_2 f + i c_\nu h'$$

Yukawa/mass matrices in SO(10)

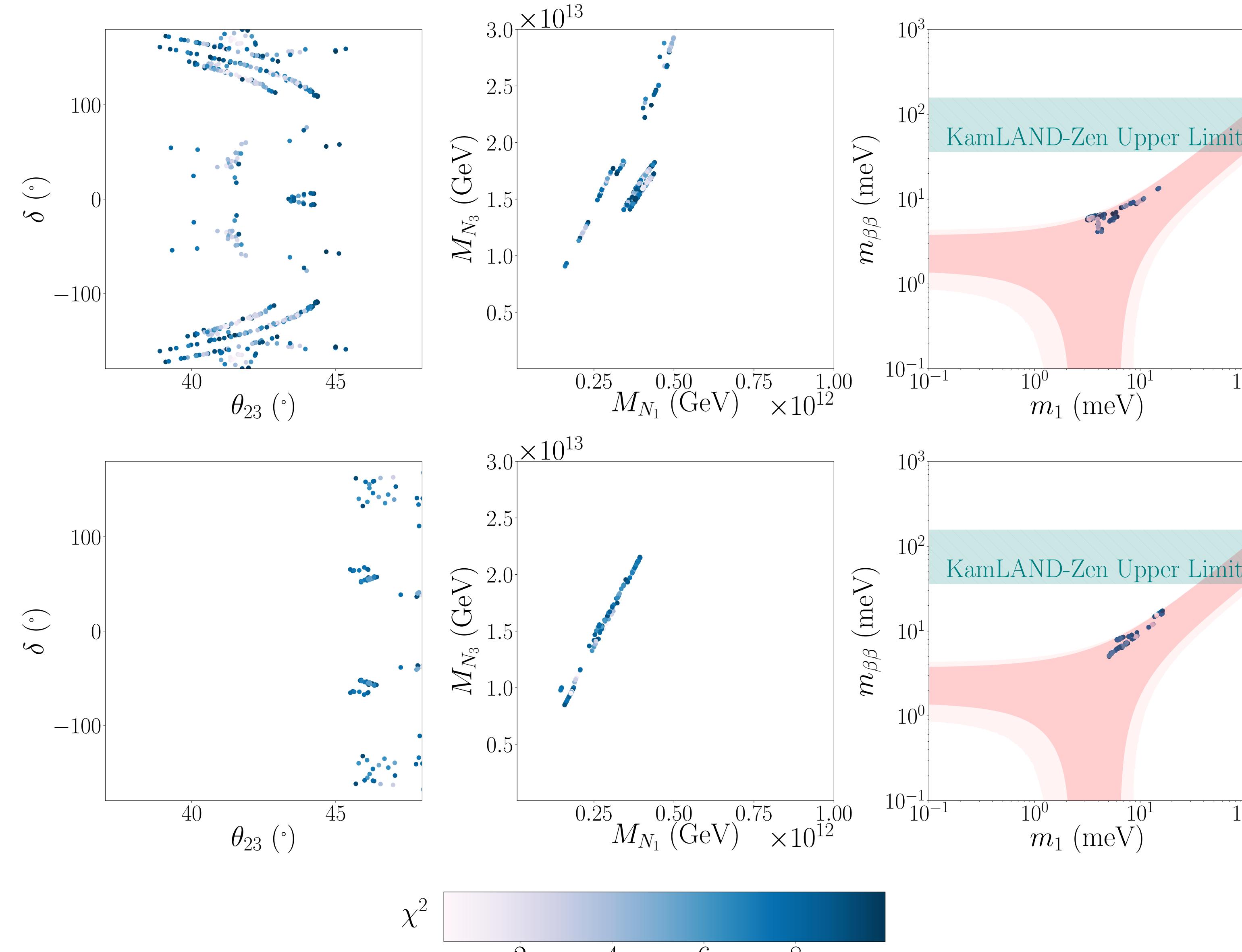
$3 \times 3$  matrices

$h \propto Y_{\mathbf{10}}$

$f \propto Y_{\overline{\mathbf{126}}}$

$h' \propto Y_{\mathbf{120}}$

# Fermion masses and mixing in SO(10)



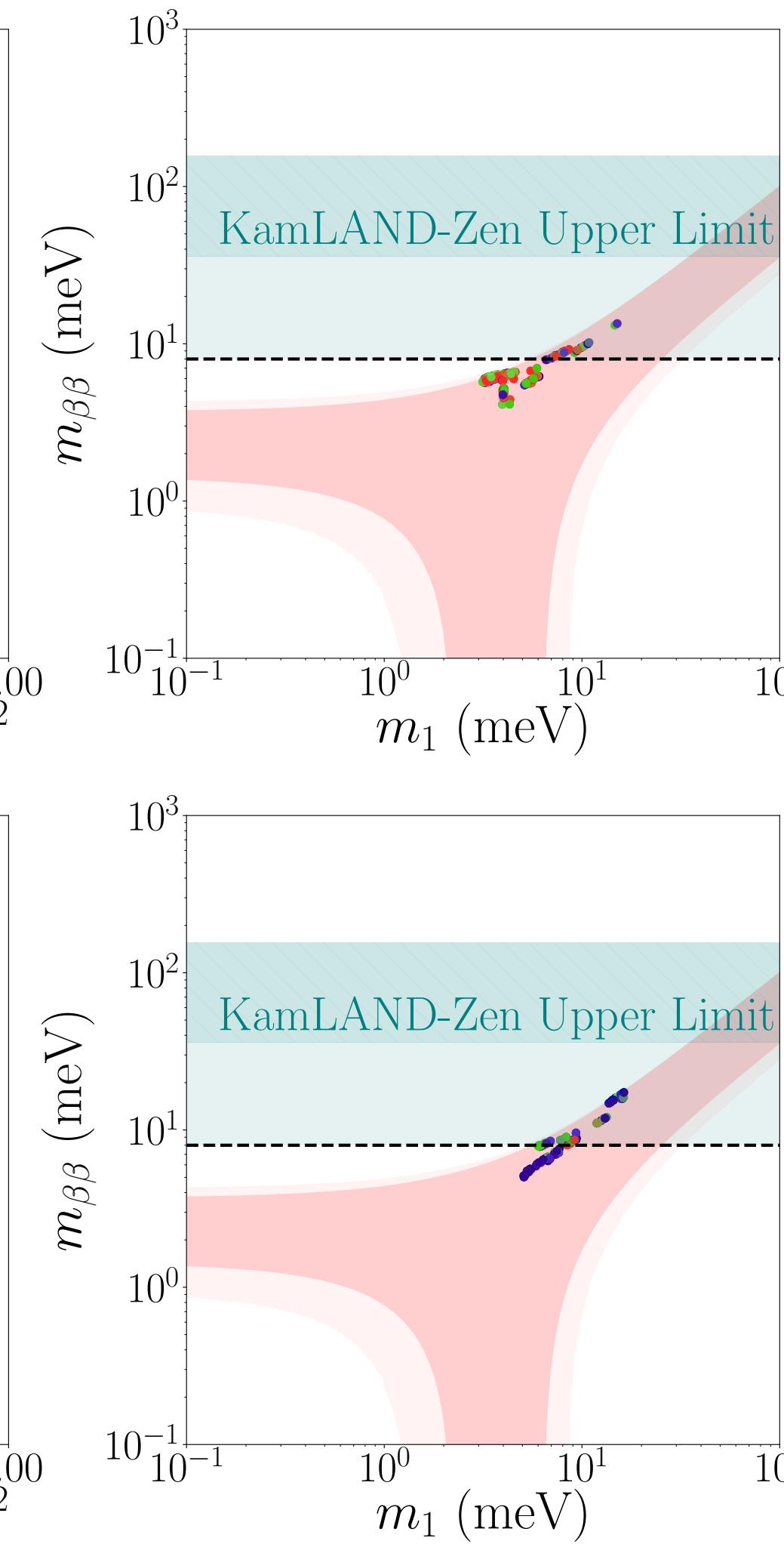
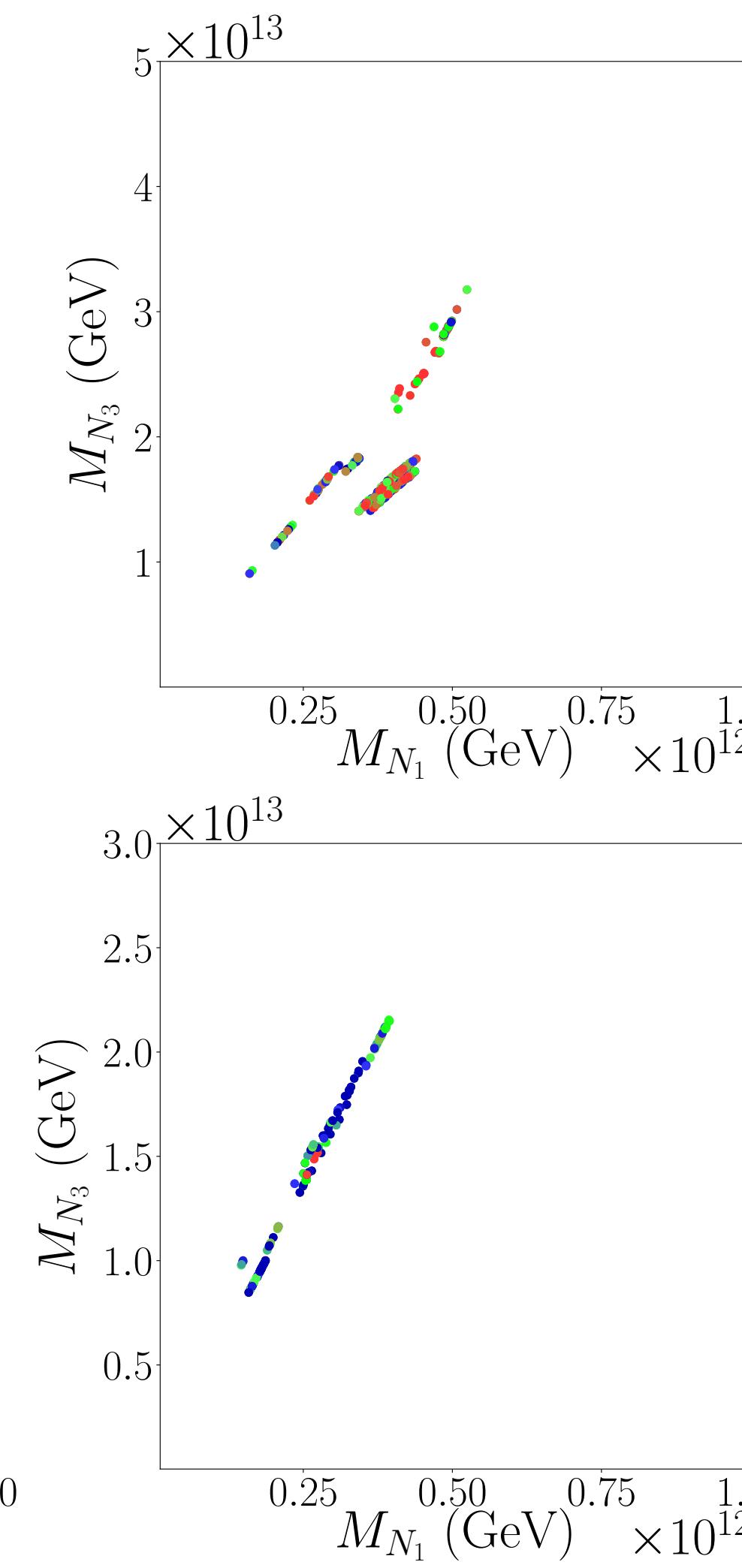
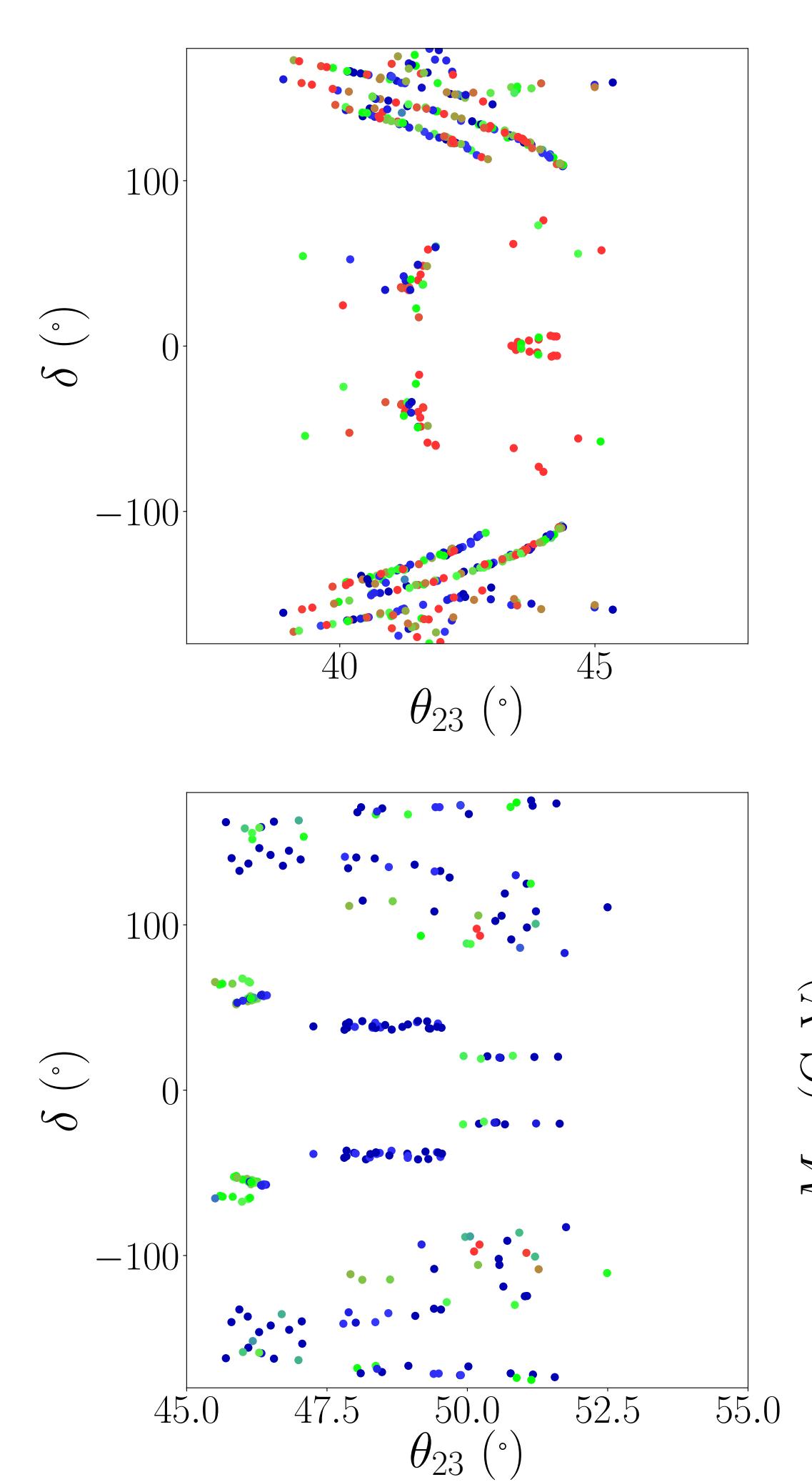
**Take-away message:**

$M_1 \sim (2, 5) \times 10^{11} \text{ GeV}$   
 $M_3 \sim (1, 3) \times 10^{13} \text{ GeV}$   
 $m_{\beta\beta} \sim 10^{-2} \text{ eV}$

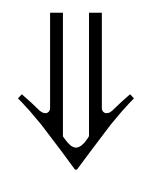
And normal ordering preferred

Fu, King, Marsili, Pascoli,  
Turner, **YLZ**, 2209.00021

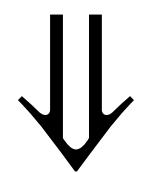
# Thermal leptogenesis in SO(10)



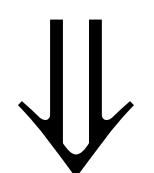
Data of quark masses,  
CKM mixing, lepton  
masses, PMNS mixing



Heavy neutrino masses  
and Dirac v Yukawa  
couplings



CP violation in heavy  
neutrino decay

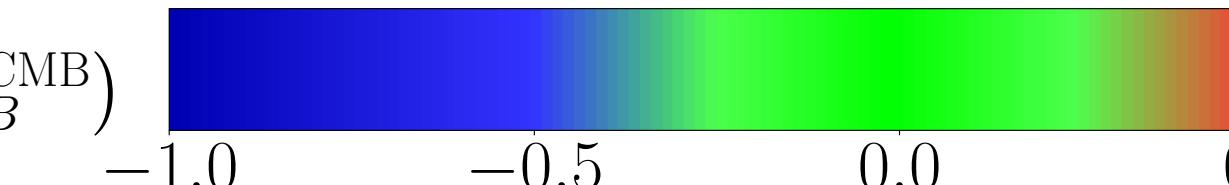


**Thermal leptogenesis**

Fu, King, Marsili, Pascoli,  
Turner, **YLZ**, 2209.00021

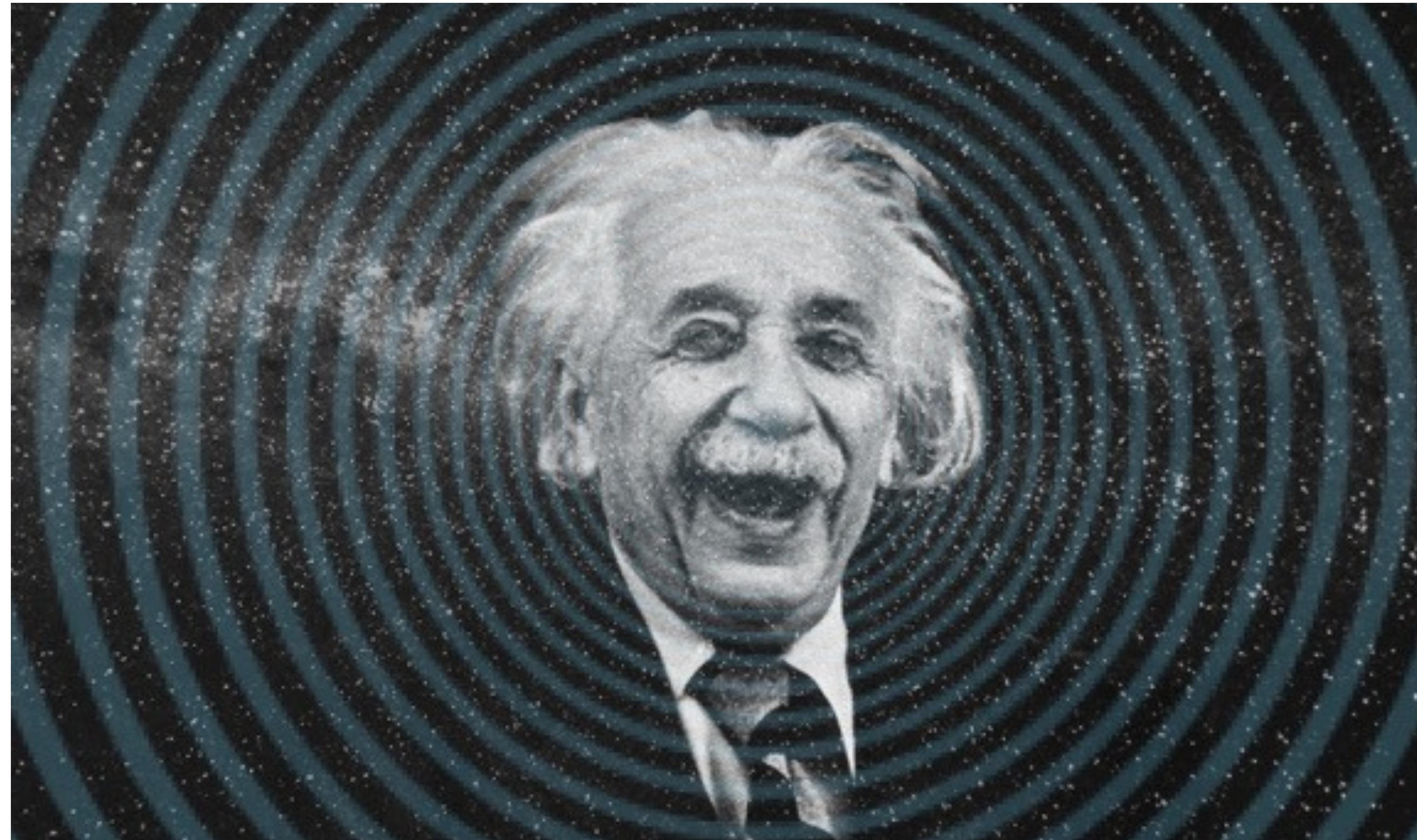
$$\chi^2 < 10 \text{ & }$$

$$\log_{10}(\eta_B/\eta_B^{\text{CMB}})$$



# Gravitational waves as a probe to GUTs

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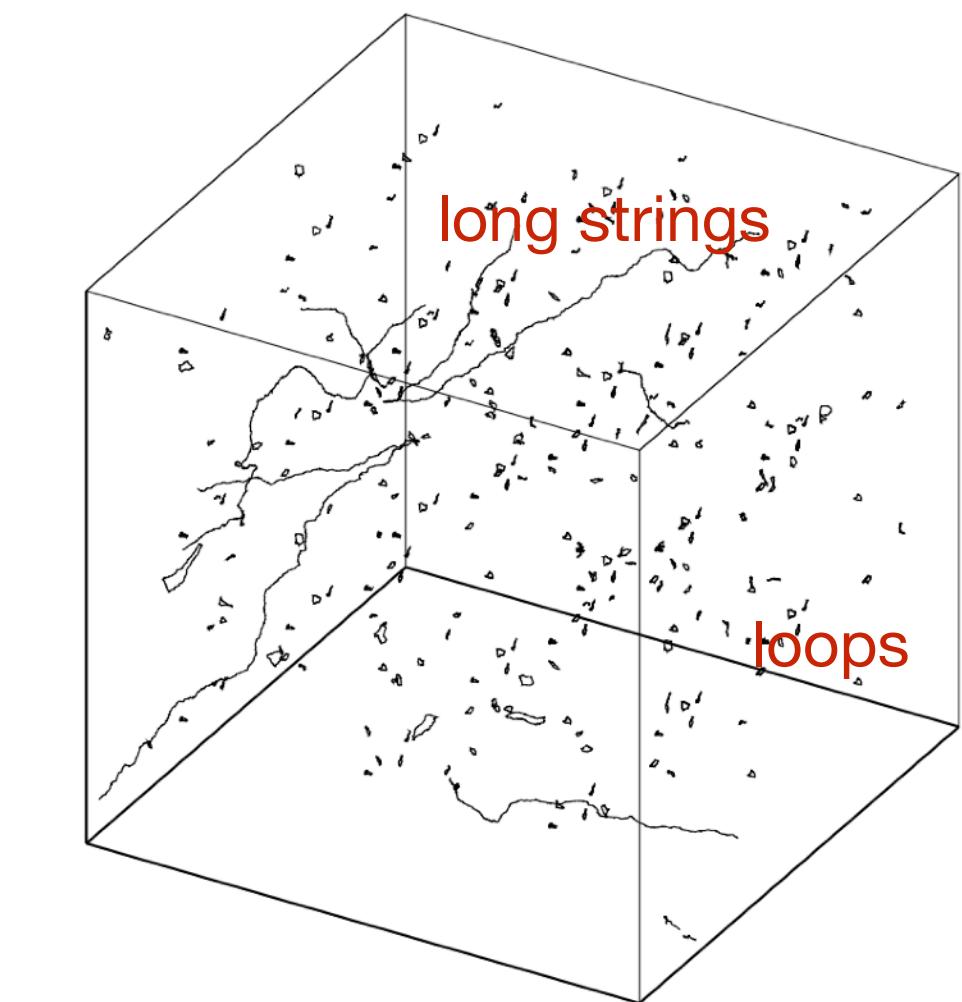


# Mechanisms of gravitational wave (GW) genesis in GUTs

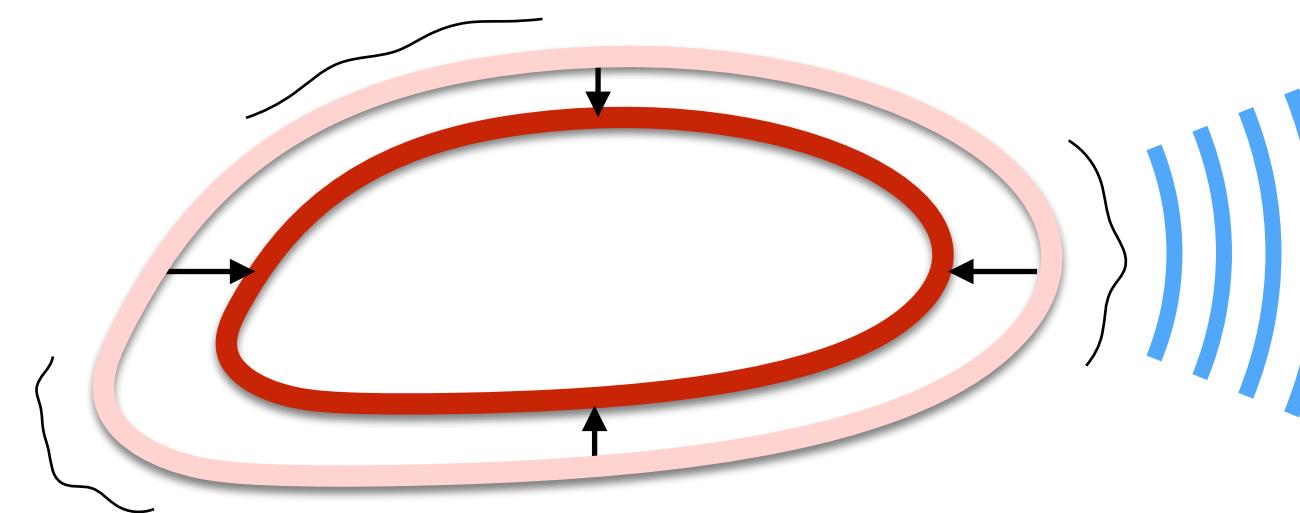
## GW via cosmic strings

- Most GUTs include a  $U(1)_{B-L}$  symmetry.
- Spontaneous breaking of this  $U(1)$  generates cosmic strings.
- Strings intersect and intercommute to form loops and cusps
- Loops oscillates via gravitational radiation

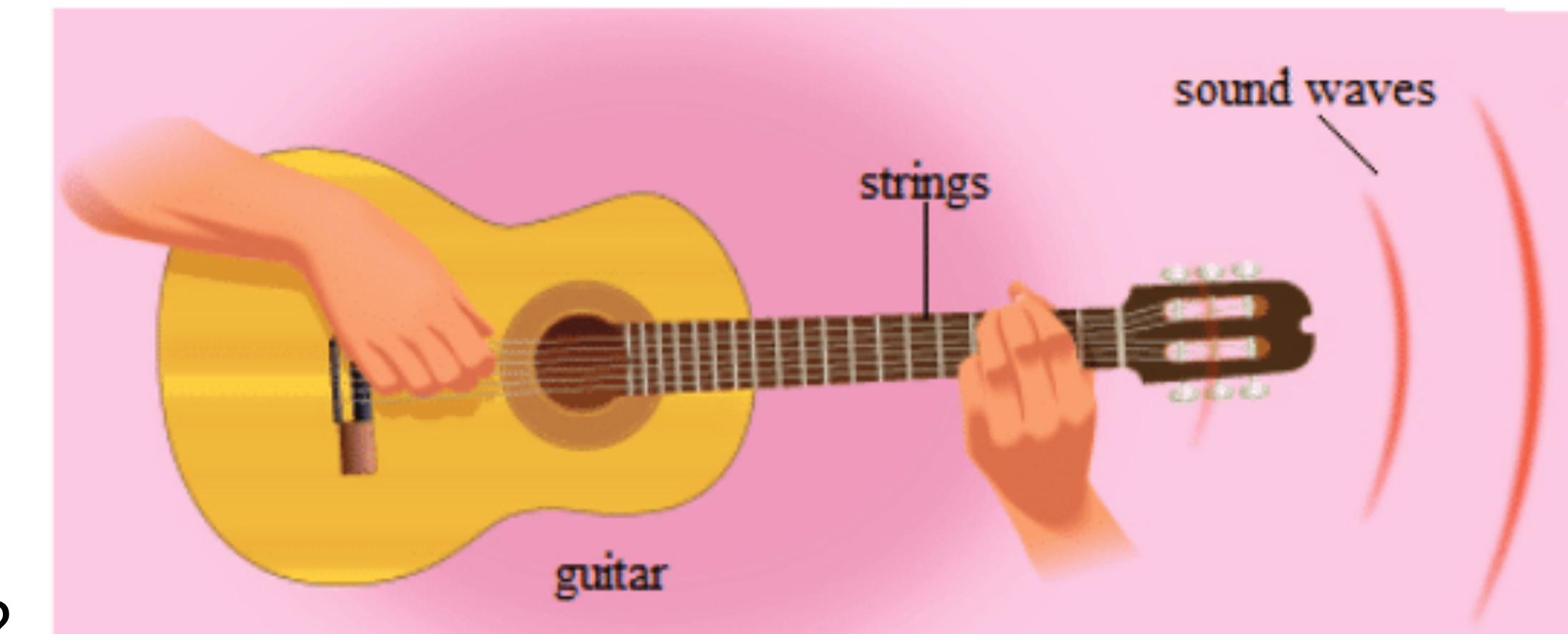
$$\pi_1(U(1)) = \mathbb{Z}$$



Vanchurin, Olum, Vilenkin, 0511159

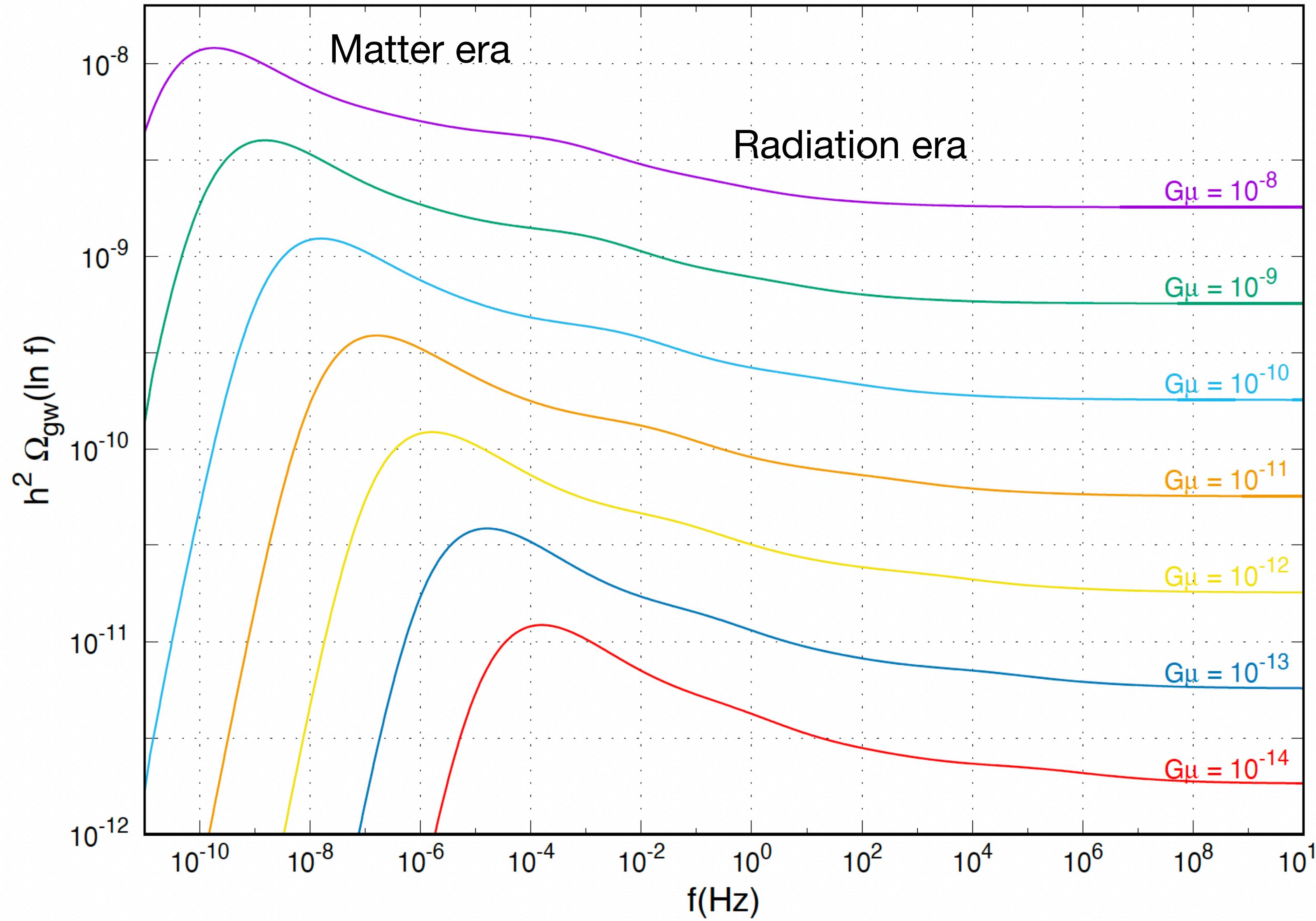


Another mechanism: GW via GUT phase transition?



— — require technique developments to measure high-frequency GW, see e.g., 2011.12414, 2310.06607

# Typical GW spectrum from string loops



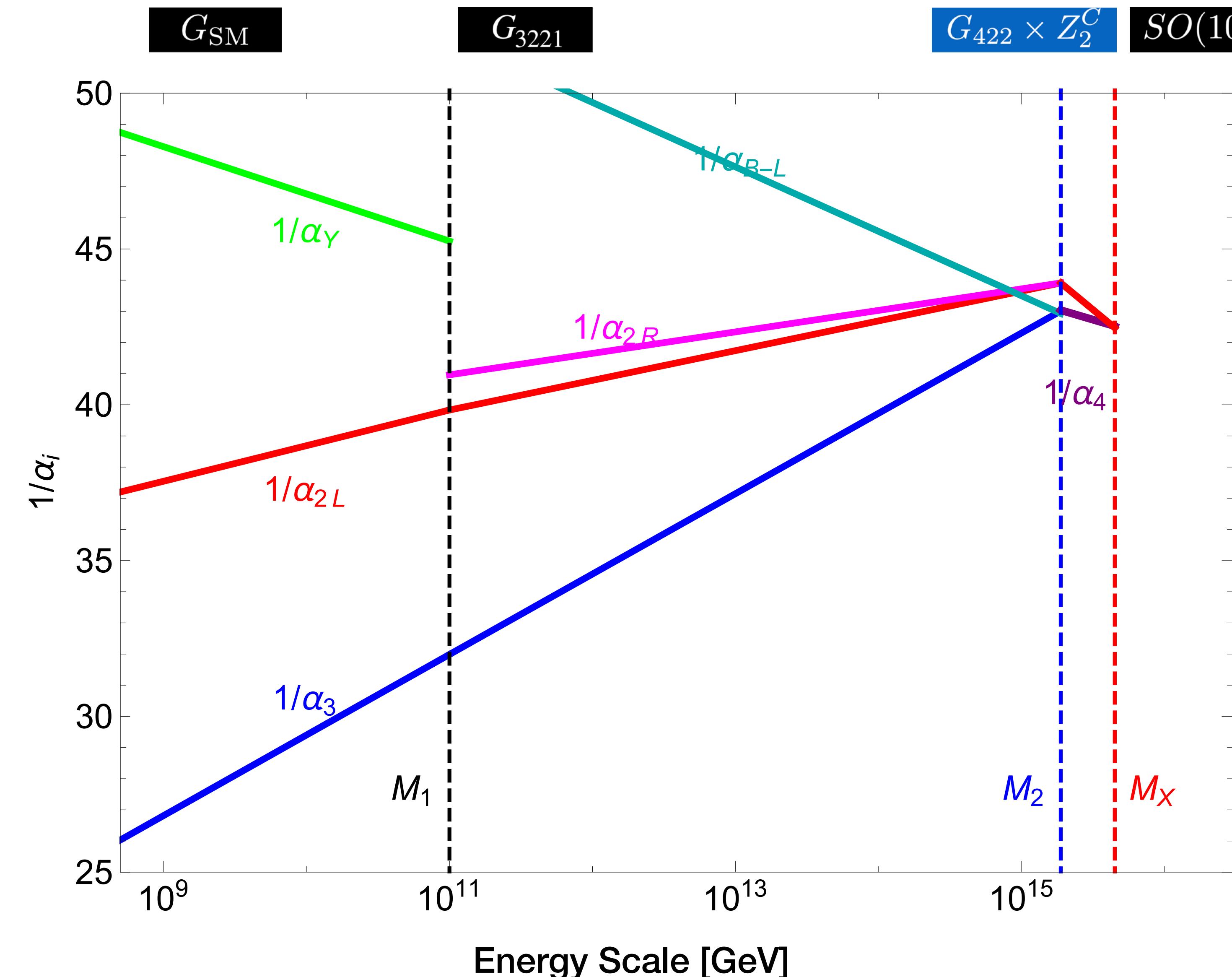
In the Nambu-Goto case,  
the spectrum depends  
on one single parameter  
 $\mu$  (string tension)

$$\Omega_{\text{GW}} h^2 \sim 5 \times 10^{-5} \sqrt{G\mu}$$
$$\propto \frac{M_{B-L}}{M_{\text{Planck}}} \quad \text{in GUTs}$$

$$G = M_{\text{Planck}}^{-2}$$

Blanco-Pillado, Olum  
1709.02693

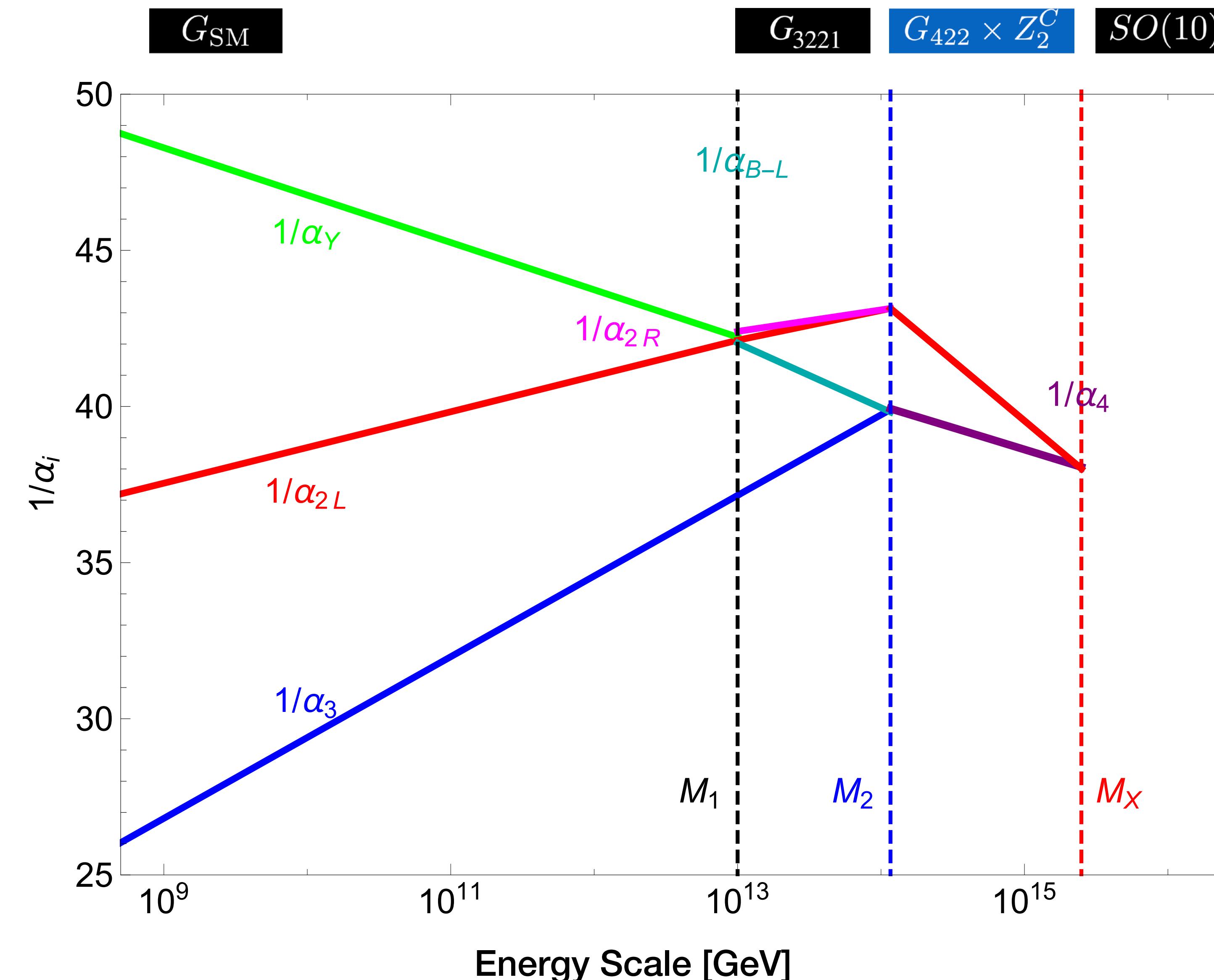
# Gauge unification correlates GUT scale with intermediate scales



$M_i$  refers to  
the gauge  
boson mass of  
the  $i$ -th gauge  
symmetry  $G_i$

No new particle  
introduced if not  
necessary

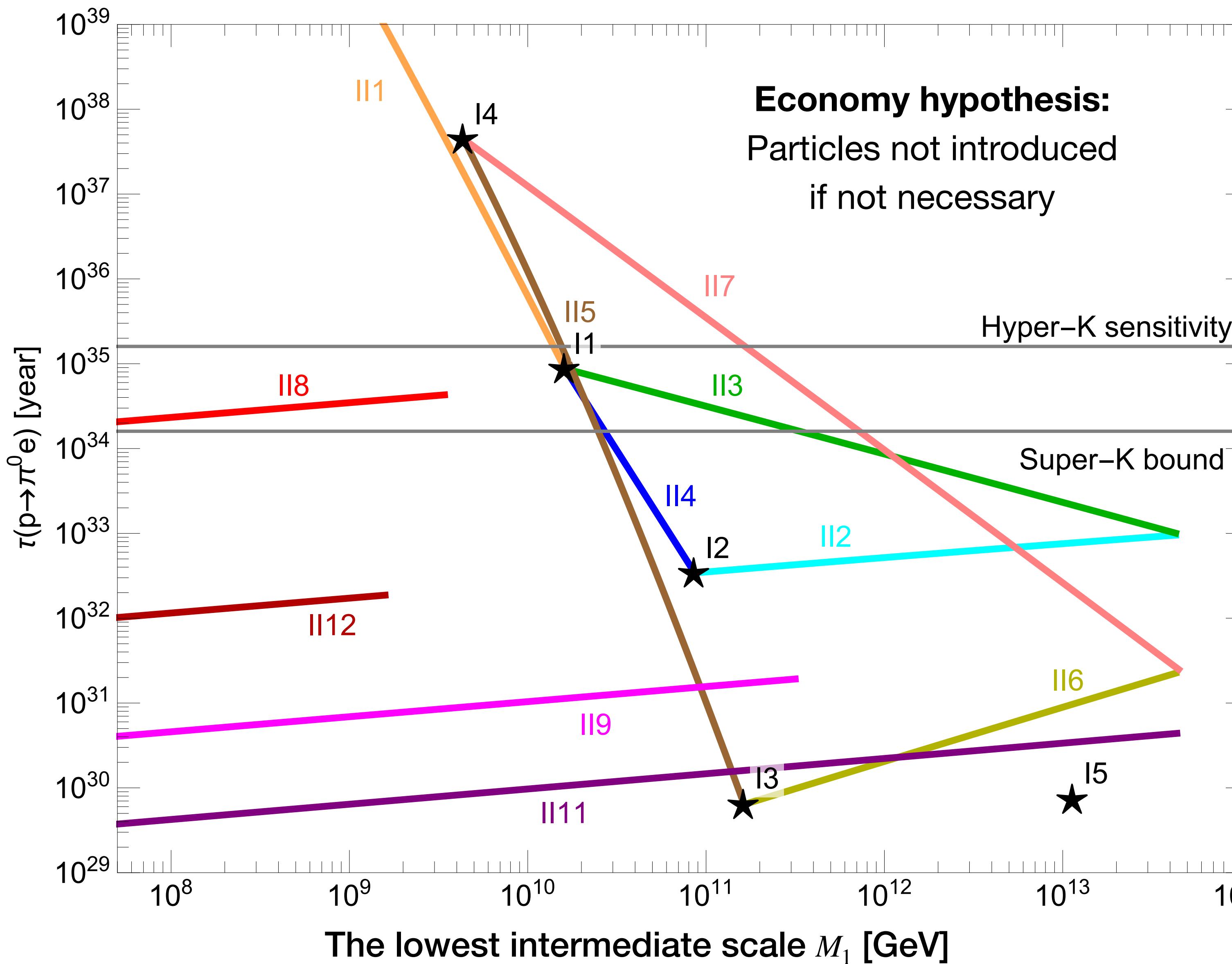
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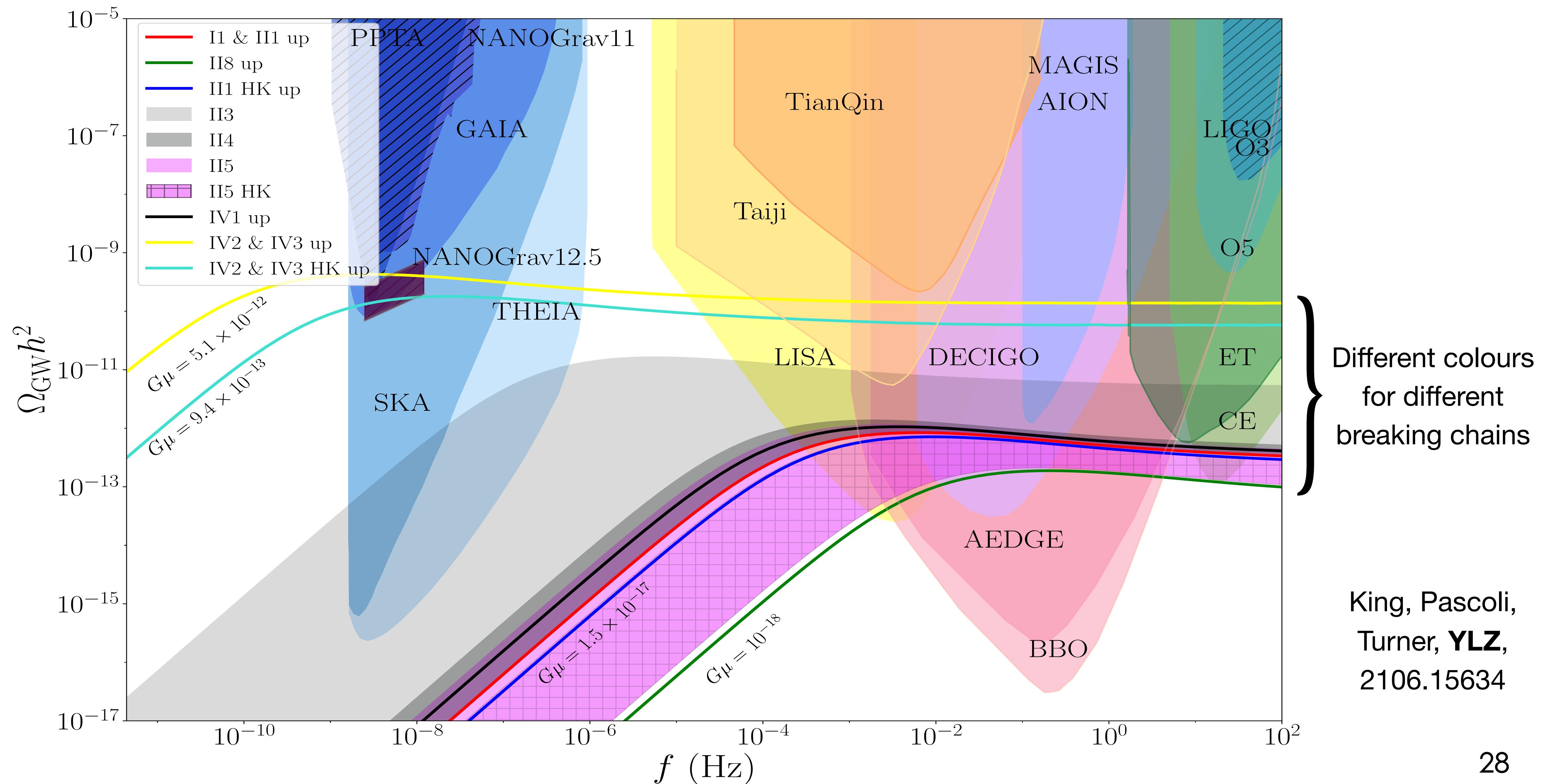
# Constraints on B-L scale in SO(10)



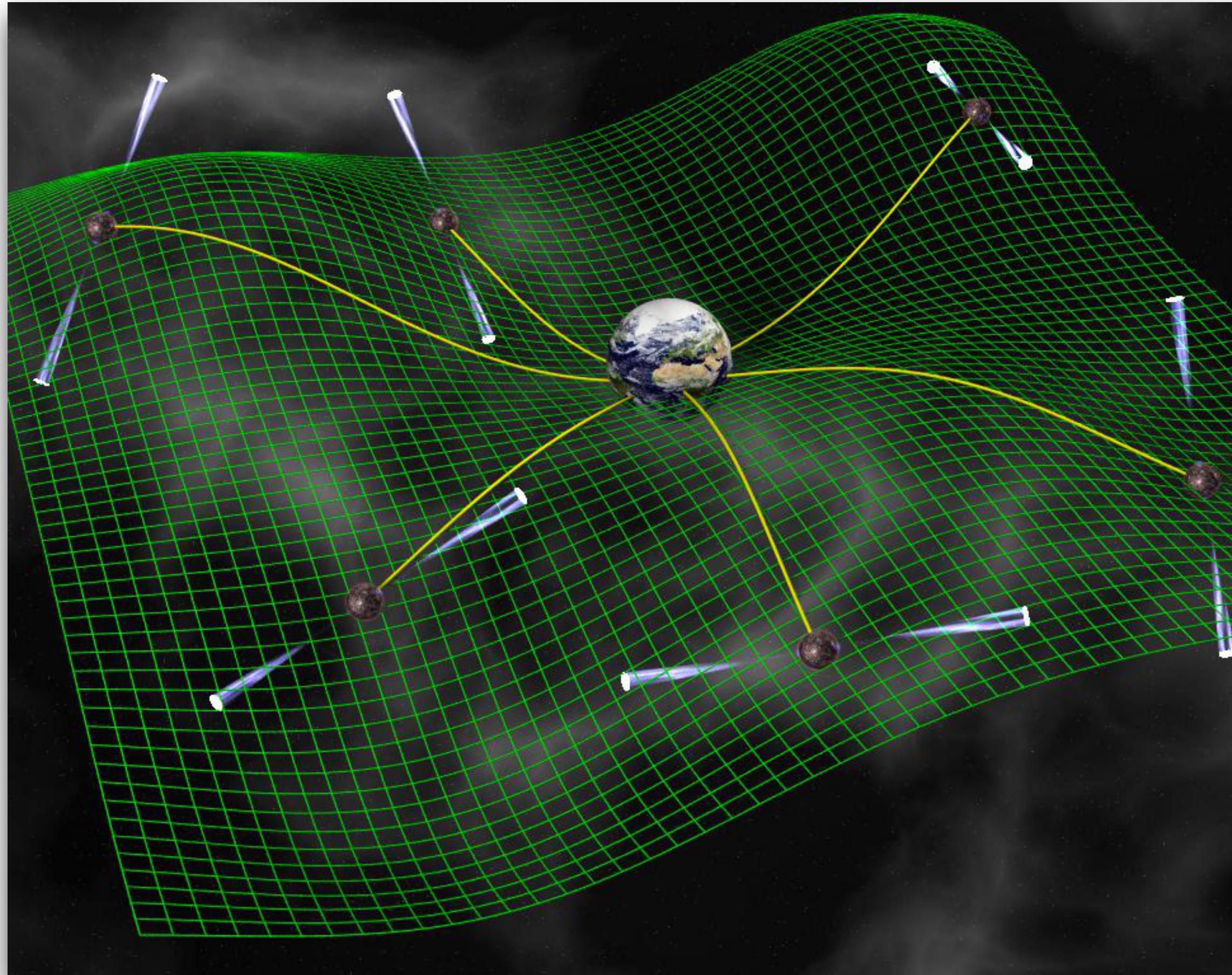
$SO(10)$	$\xrightarrow[\text{defect}]{\text{Higgs}} G_2$	$\xrightarrow[\text{defect}]{\text{Higgs}} G_1$	$\xrightarrow[\text{defect}]{\text{Higgs}} G_{\text{SM}}$
II1:	$\xrightarrow[m]{210} G_{422}$	$\xrightarrow[m]{45} G_{3221}$	$\xrightarrow[s]{126} \overline{126}$
II2:	$\xrightarrow[m,s]{54} G_{422}^C$	$\xrightarrow[m]{210} G_{3221}^C$	$\xrightarrow[s,w]{126} \overline{126}$
II3:	$\xrightarrow[m,s]{54} G_{422}^C$	$\xrightarrow[m,w]{45} G_{3221}$	$\xrightarrow[s]{126} \overline{126}$
II4:	$\xrightarrow[m,s]{210} G_{3221}^C$	$\xrightarrow[w]{45} G_{3221}$	$\xrightarrow[s]{126} \overline{126}$
II5:	$\xrightarrow[m]{210} G_{422}$	$\xrightarrow[m]{45} G_{421}$	$\xrightarrow[s]{126} \overline{126}$
II6:	$\xrightarrow[m,s]{54} G_{422}^C$	$\xrightarrow[m]{45} G_{421}$	$\xrightarrow[s]{126} \overline{126}$
II7:	$\xrightarrow[m,s]{54} G_{422}^C$	$\xrightarrow[w]{210} G_{422}$	$\xrightarrow[m]{126,45} \overline{126,45}$
II8:	$\xrightarrow[m]{45} G_{3221}$	$\xrightarrow[m]{45} G_{3211}$	$\xrightarrow[s]{126} \overline{126}$
II9:	$\xrightarrow[m,s]{210} G_{3221}^C$	$\xrightarrow[m,w]{45} G_{3211}$	$\xrightarrow[s]{126} \overline{126}$
II10:	$\xrightarrow[m]{210} G_{422}$	$\xrightarrow[m]{210} G_{3211}$	$\xrightarrow[s]{126} \overline{126}$
II11:	$\xrightarrow[m,s]{54} G_{422}^C$	$\xrightarrow[m,w]{210} G_{3211}$	$\xrightarrow[s]{126} \overline{126}$
II12:	$\xrightarrow[m]{45} G_{421}$	$\xrightarrow[m]{45} G_{3211}$	$\xrightarrow[s]{126} \overline{126}$

King, Pascoli, Turner, **YLZ**,  
2106.15634

# Predictions for GW spectrum in SO(10) GUTs



# Influence of recent PTA measurements



On 28 Jun 2023

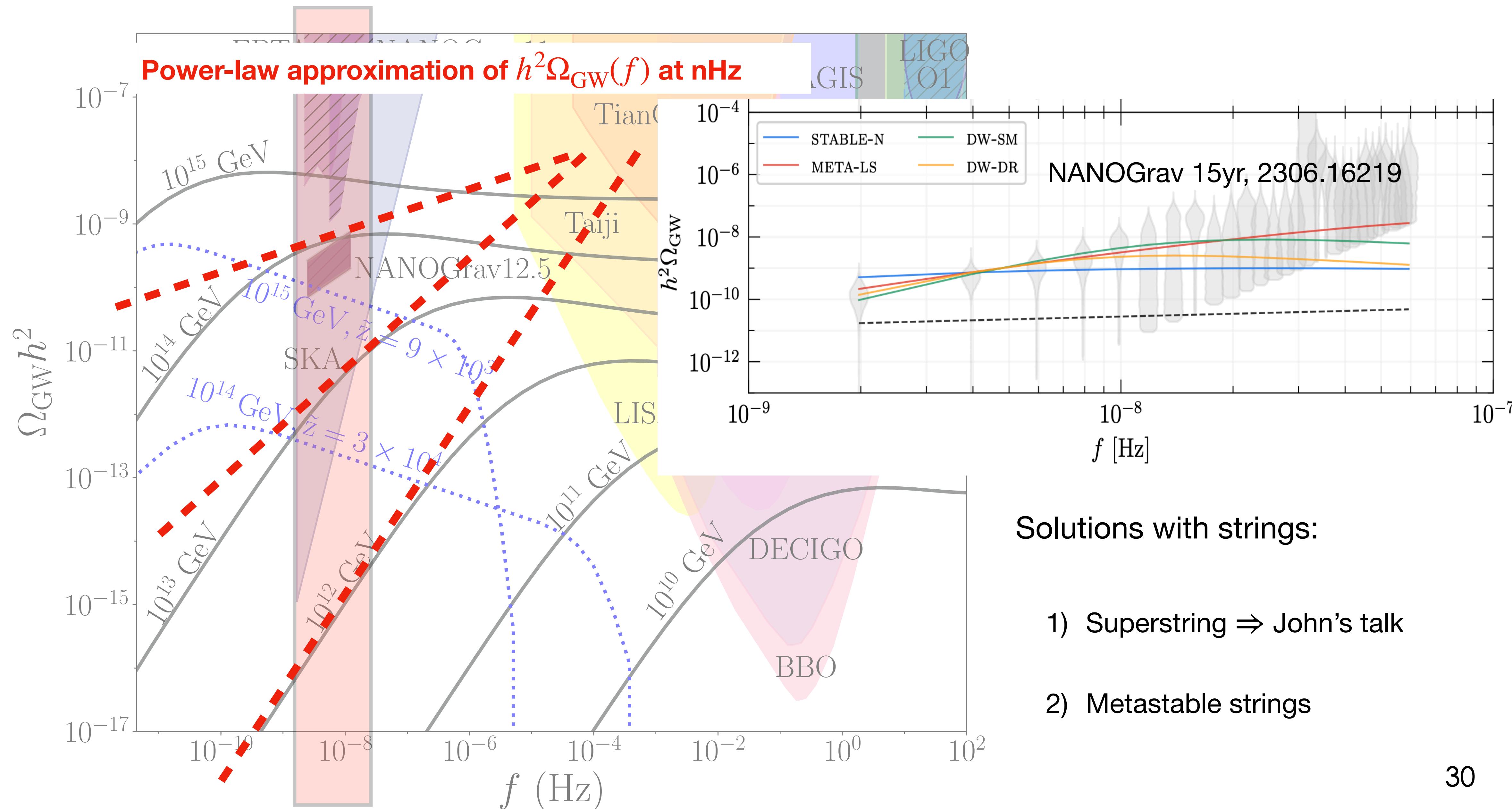
**NANOGrav 15yr,**  
2306.16213

**European Pulsar Timing Array III,**  
2306.16214

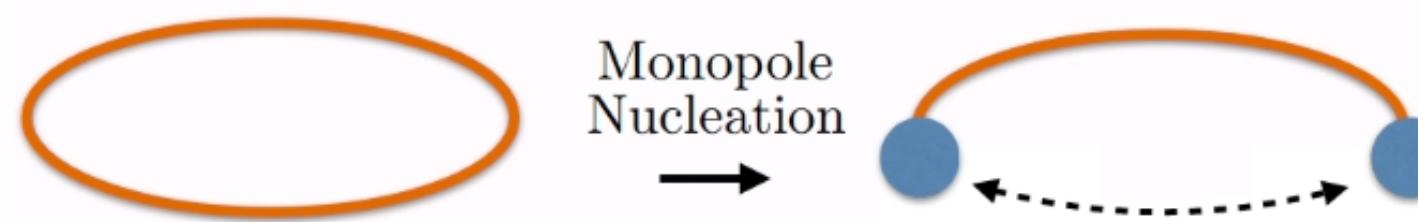
**Parkes Pulsar Timing Array III,**  
2306.16215

**Chinese Pulsar Timing Array,**  
2306.16216

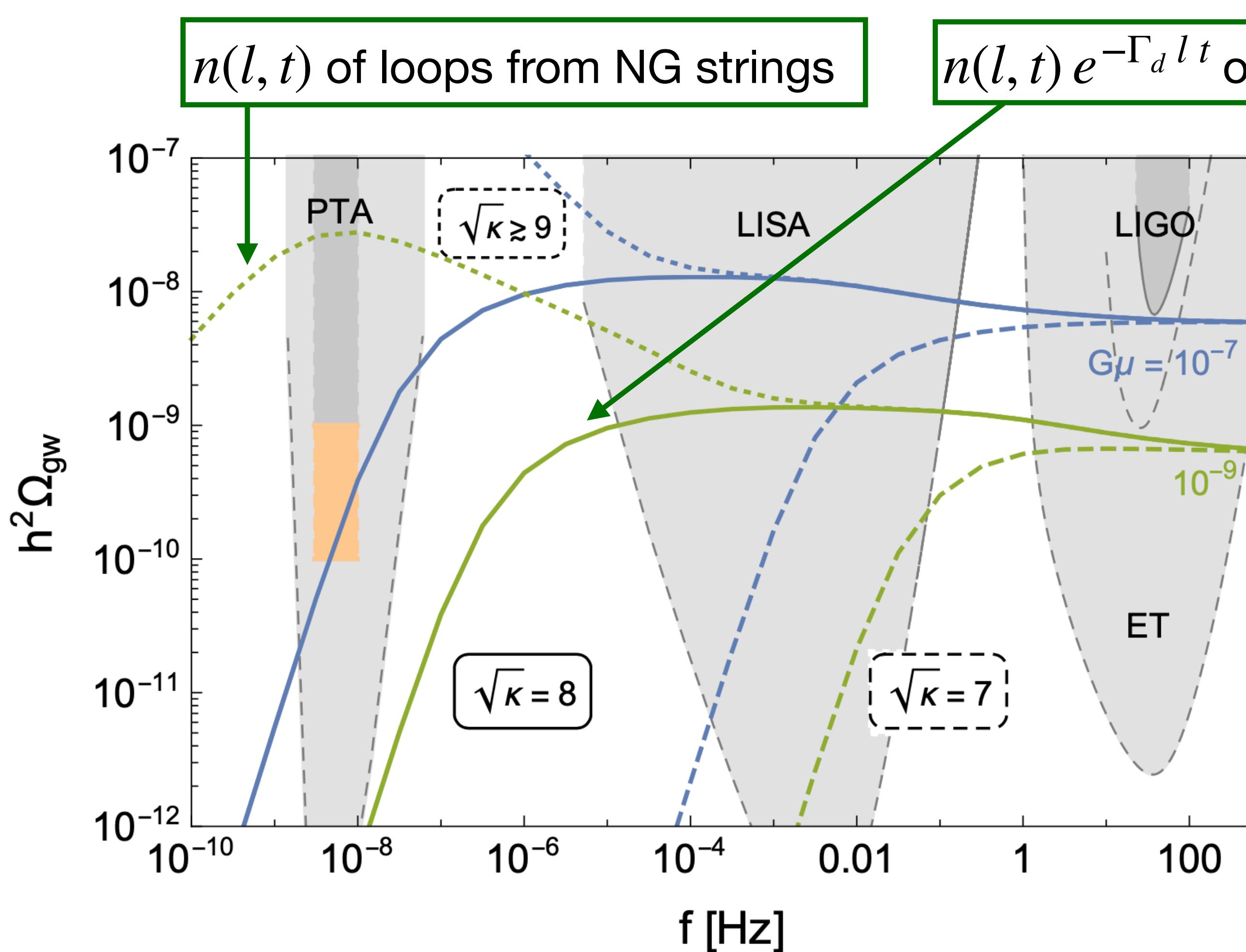
# Tensions between NANOGrav and GWs via Nambu-Goto strings



# GWs via metastable strings



Vilenkin, NPB(1982); Preskill, Vilenkin, 9209210;  
Leblond, Shlaer, Siemens, 0903.4686; Monin, Voloshin,  
0808.1693; Buchmuller, Domcke, Schmitz, 2107.04578



$$\sqrt{\kappa} = \frac{m_{\text{monopole}}}{\sqrt{\mu_{\text{string}}}} \sim \alpha_{\text{GUT}}^{-1/2} \frac{M_{\text{GUT}}}{M_{B-L}}$$

$$\sqrt{\kappa} \simeq (8, 9) \Rightarrow M_{\text{GUT}} \sim M_{B-L}$$

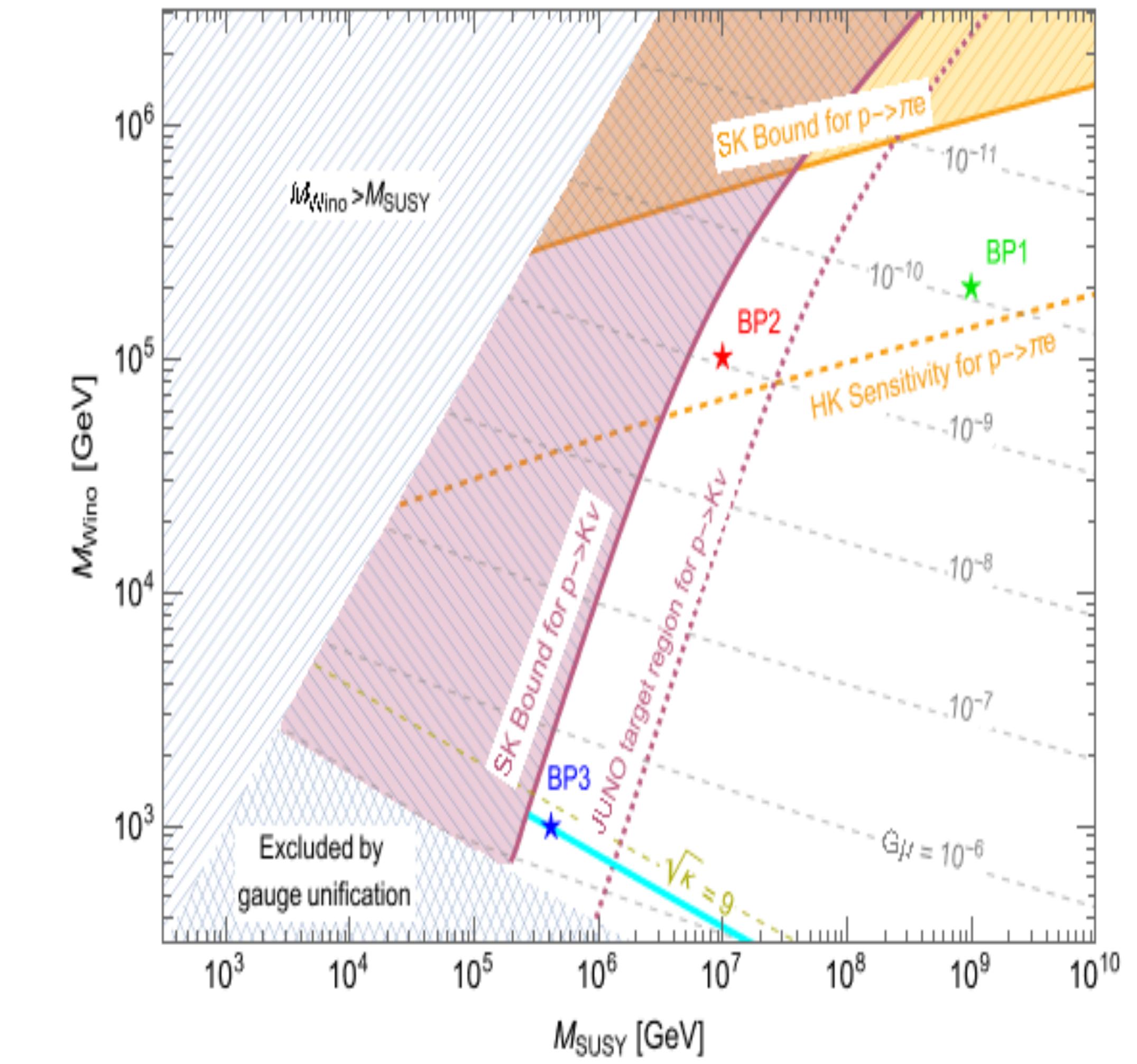
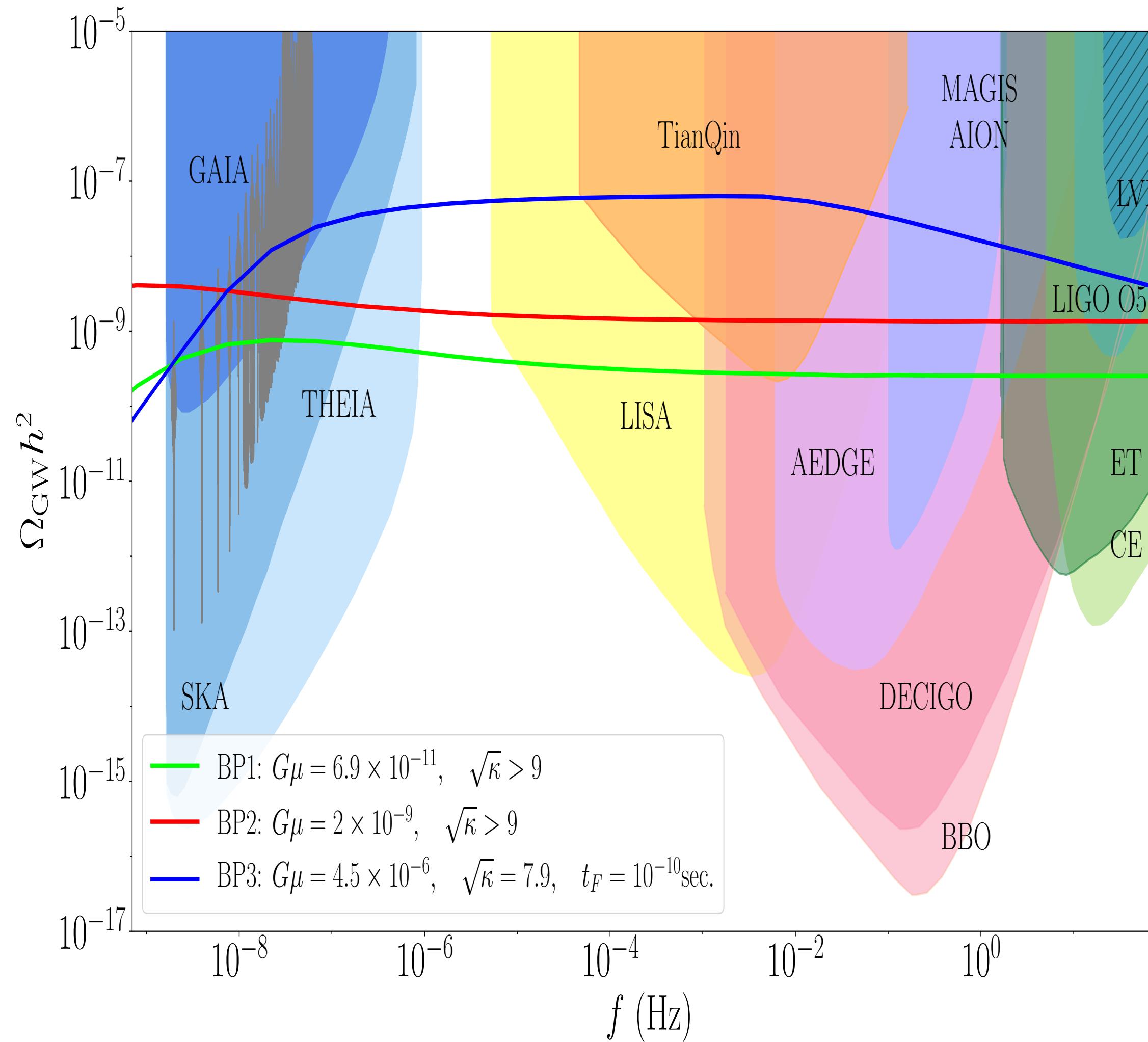
Buchmuller, Domcke, Schmitz, 2307.04691

A GUT inflation separates the GUT breaking and B-L breaking in the time scale is required.

Antusch, Hinze, Saad, Steiner, 2307.04595

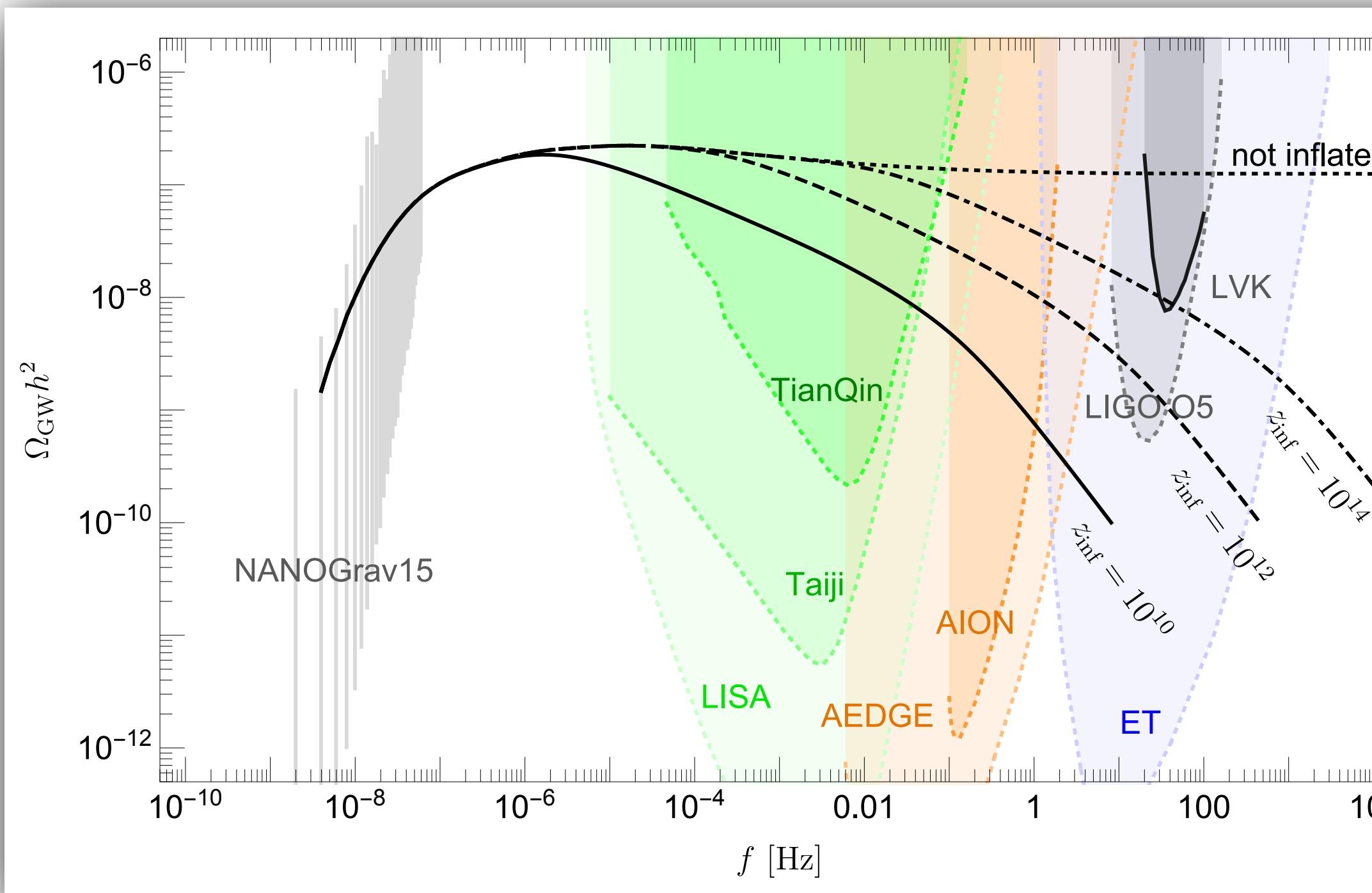
See also Lazarides, Maji, Moursy, Shafi, 2308.07094

# PTA-favoured GUTs: SUSY SO(10)

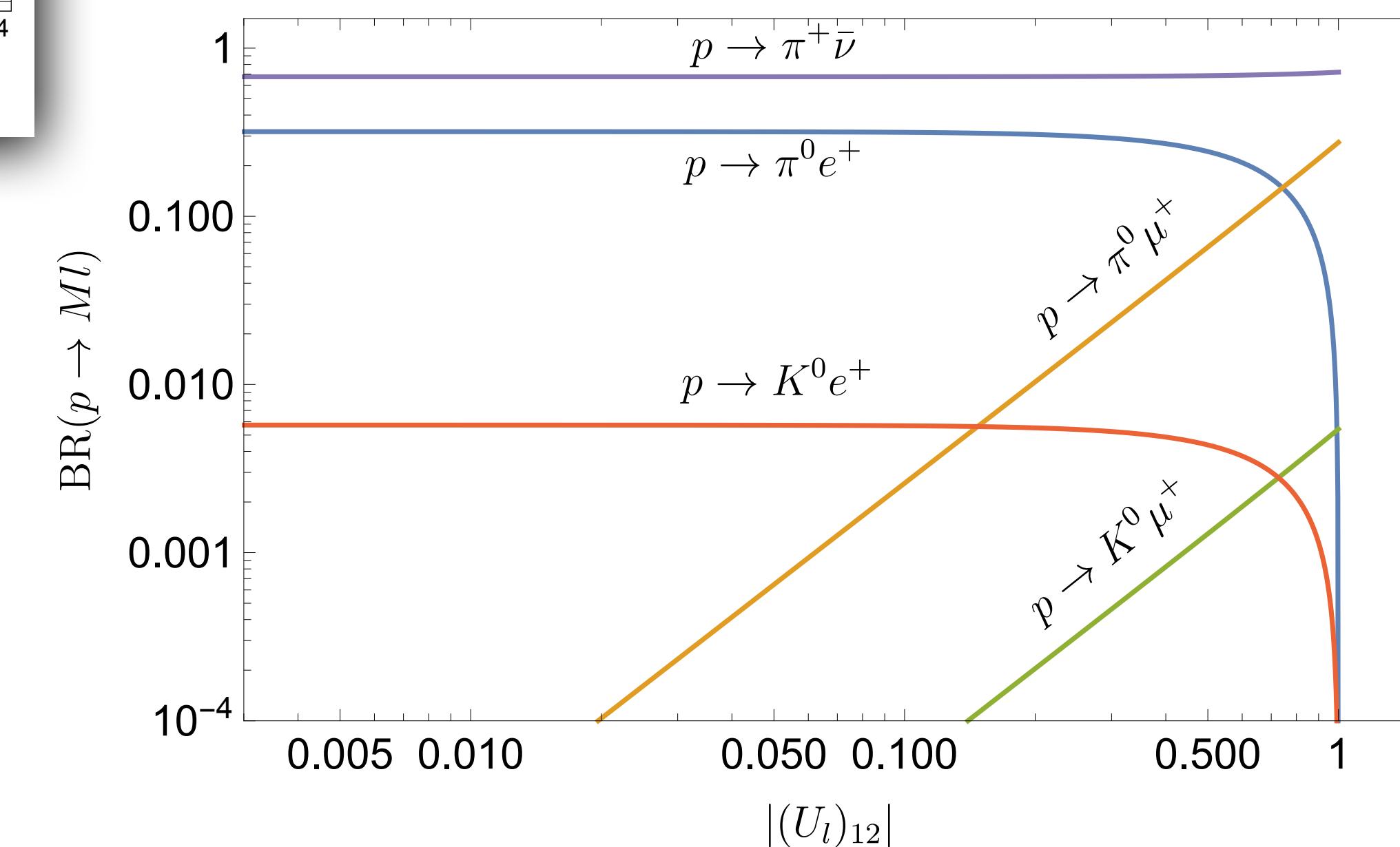
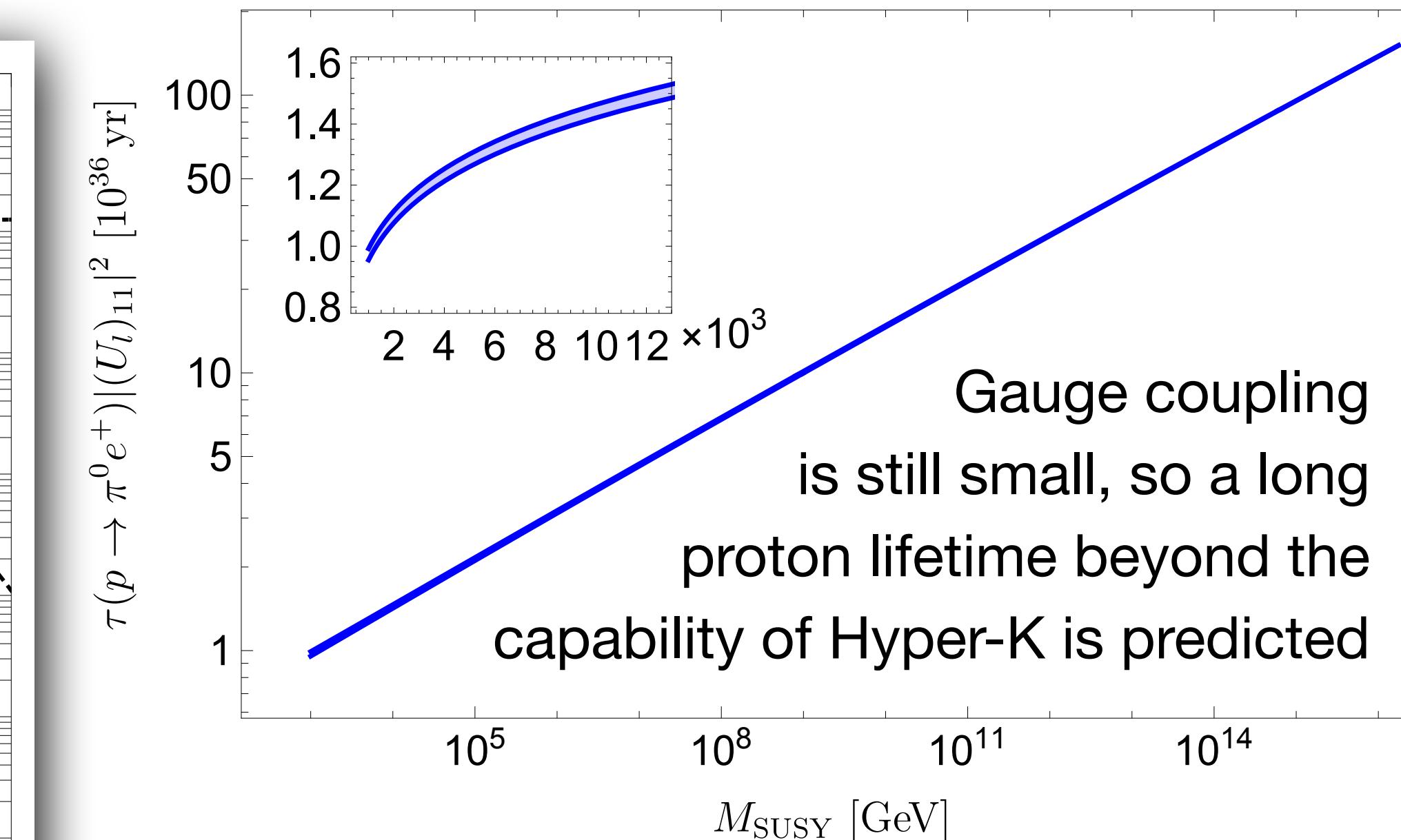


# PTA-favoured GUTs: flipped SU(5)

$u \leftrightarrow d, \nu \leftrightarrow e$



King, Leontaris, YLZ,  
2311.11857



# Leptogenesis in flipped SU(5)

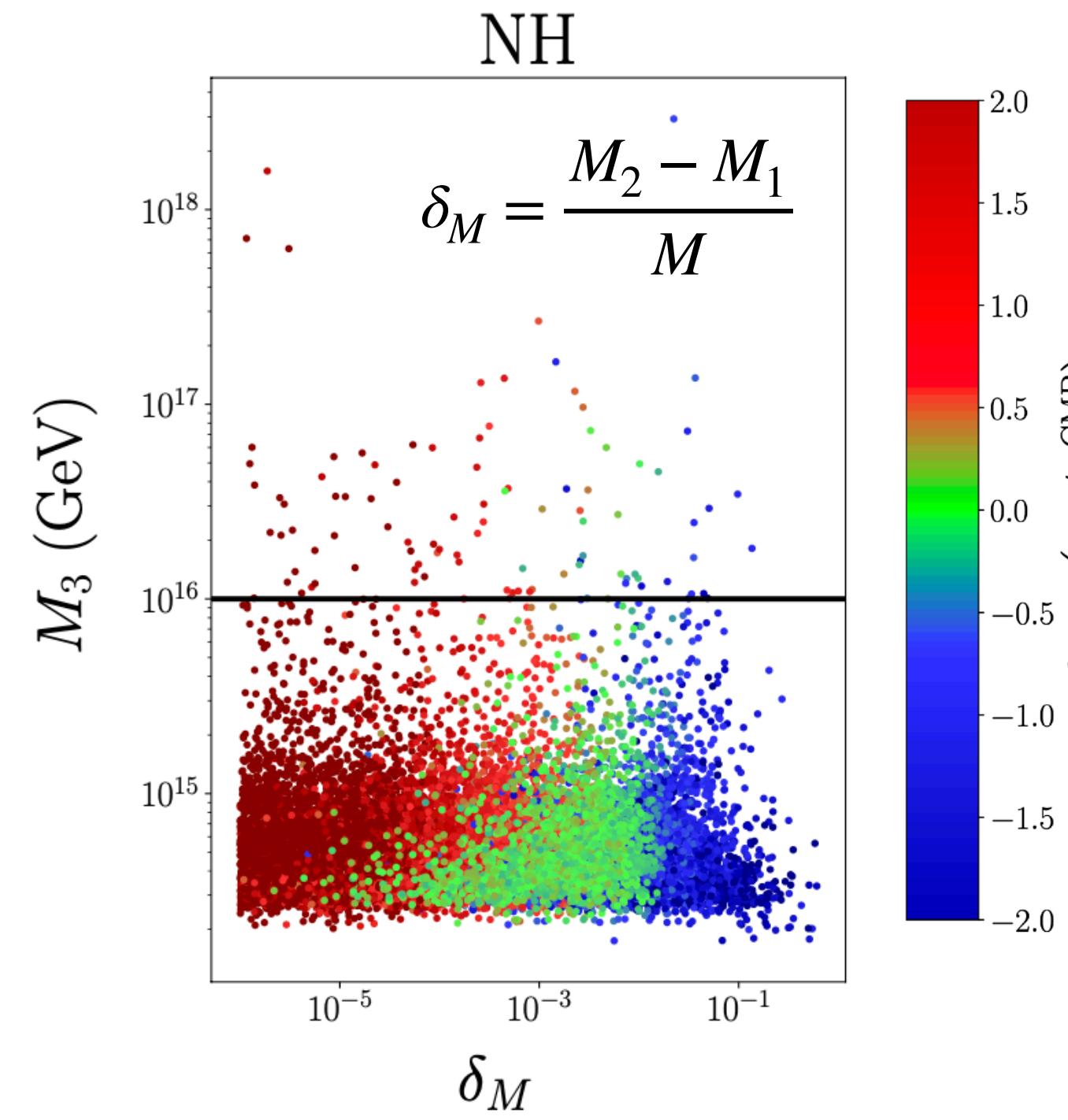
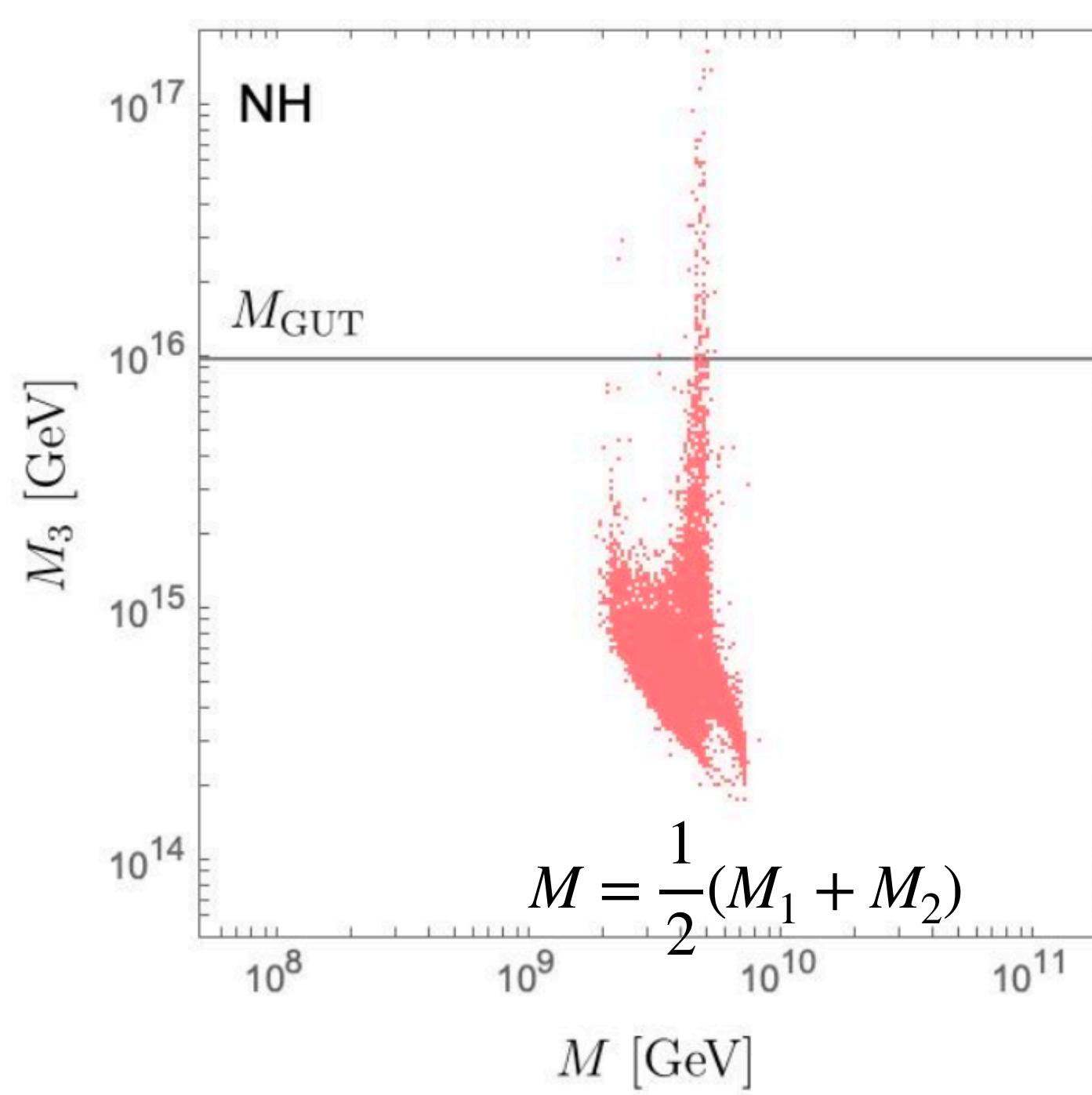
King, Leontaris, Marsili, **YLZ**, 2407.02701

- Flipped SU(5) usually predicts very hierarchical RHN masses,

$$M_u = M_D, \quad M_R = M_D^T M_\nu^{-1} M_D \quad \Rightarrow \quad M_1 : M_2 : M_3 \simeq m_u^2 : m_c^2 : m_t^2$$

The lightest RHN neutrino is too light and cannot be used to achieve leptogenesis

- But it is not always true, with special flavour texture assumed,  $M_1 \sim M_2 \ll M_3$  achieved



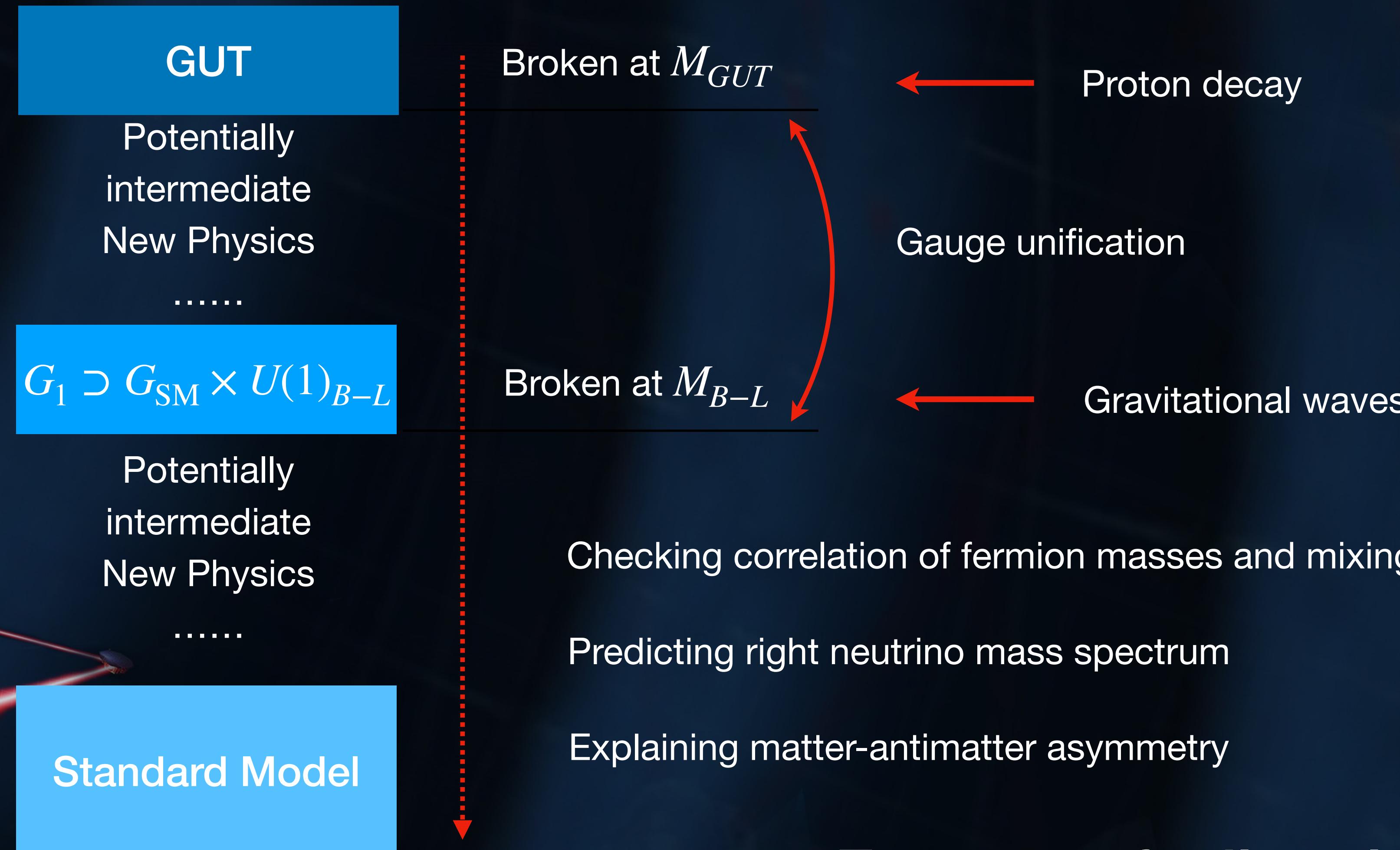
$$a = \left( \frac{y_t}{y_c} \sin \theta_{13}^R \right)^2, \quad b = \left( \frac{y_t}{y_c} \sin \theta_{23}^R \right)^2$$

$$m_1 \approx \frac{m_c^2 y_t^2 \kappa}{M y_c^2 a' y_c^2}, \quad \lesssim 10^{-7} \text{ eV}$$

$$m_{2,3} \approx \frac{m_c^2}{2M} \left[ 1 + a' + b \mp \sqrt{(1 + a' + b)^2 - 4a'} \right]$$

Both Normal Hierarchy (NH) and Inverted Hierarchy (IH) are predicted. Here we show NH for illustration

# Summary



***Thank you for listening***