Nuclear effects on longitudinal-transverse structure function ratio

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26th International Symposium on Spin Physics,
Qingdao, Shandong, China, September 22–26, 2025
https://indico.ihep.ac.cn/e/spin2025

Ref. S. Kumano, arXiv:2506.18305

September 23, 2025

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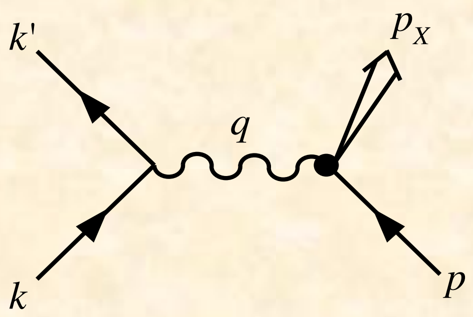
Introduction to longitudinal-transverse structure function ratio *R*

$$R = \frac{F_L}{F_T} = \frac{(1 + Q^2/\nu^2)F_2 - 2xF_1}{2xF_1}$$

Cross section for charged-lepton nucleon scattering



$$d\sigma = \frac{1}{4\sqrt{(k\cdot p)^2 - m^2 M_N^2}} \overline{\sum_{pol}} \sum_X (2\pi)^4 \delta^4(k+p-k'-p_X)\, |M|^2 \frac{d^3k'}{(2\pi)^3 2E'}$$

$$M = e\,\bar{u}(k',\lambda')\gamma_\mu u(k,\lambda)\, \frac{g^{\mu\nu}}{q^2} \langle X | e J_\nu^{em}(0) | p, \lambda_N \rangle$$

$$L^{\mu\nu} = \overline{\sum_{\lambda,\,\lambda'}} \left[\bar{u}(k',\lambda')\gamma^\mu u(k,\lambda)\right]^* \left[\bar{u}(k',\lambda')\gamma^\nu u(k,\lambda)\right]$$
$$= 2\left[k^\mu k'^\nu + k'^\mu k^\nu - (k\cdot k' - m^2) g^{\mu\nu}\right]$$

$$W_{\mu\nu} = \frac{1}{4\pi M_N} \overline{\sum_{\lambda_N}} \sum_X (2\pi)^4 \delta^4(k+p-k'-p_X) \langle p,\lambda_N | J_\mu^{em}(0) | X\rangle \langle X | J_\nu^{em}(0) | p,\lambda_N\rangle$$
$$= \frac{1}{4\pi M_N} \overline{\sum_{\lambda_N}} \int d^4\xi\, e^{iq\cdot\xi} \langle p,\lambda_N | \left[J_\mu^{em}(\xi), J_\nu^{em}(0)\right] | p,\lambda_N\rangle$$

By the current conservation, $W_{\mu\nu}$ is expressed by two functions W_1 and W_2 as

$$W_{\mu\nu} = -W_1\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + W_2 \frac{1}{M_N^2}\left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right)$$

Combining $L^{\mu\nu}$ and $W_{\mu\nu}$, we have the cross section

$$\frac{d\sigma}{d\Omega\, dE} = \frac{\alpha^2}{4E^2 \sin^4(\theta/2)}\left[2W_1(\nu,Q^2)\sin^2\frac{\theta}{2} + W_2(\nu,Q^2)\cos^2\frac{\theta}{2}\right]$$

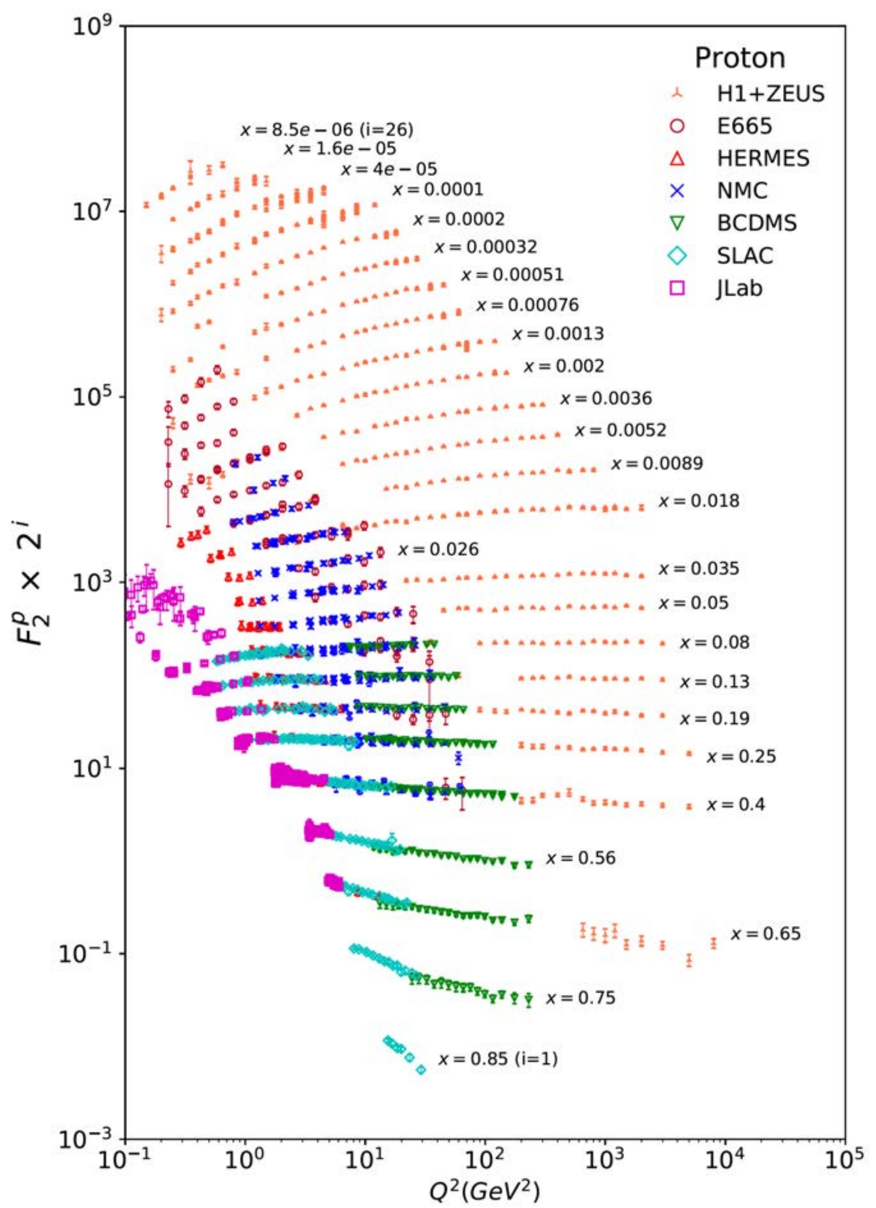
Instead of W_1 and W_2, we use F_1 and F_2 by defining

$$F_1 = M_N W_1, \quad F_2 = \nu W_2.$$

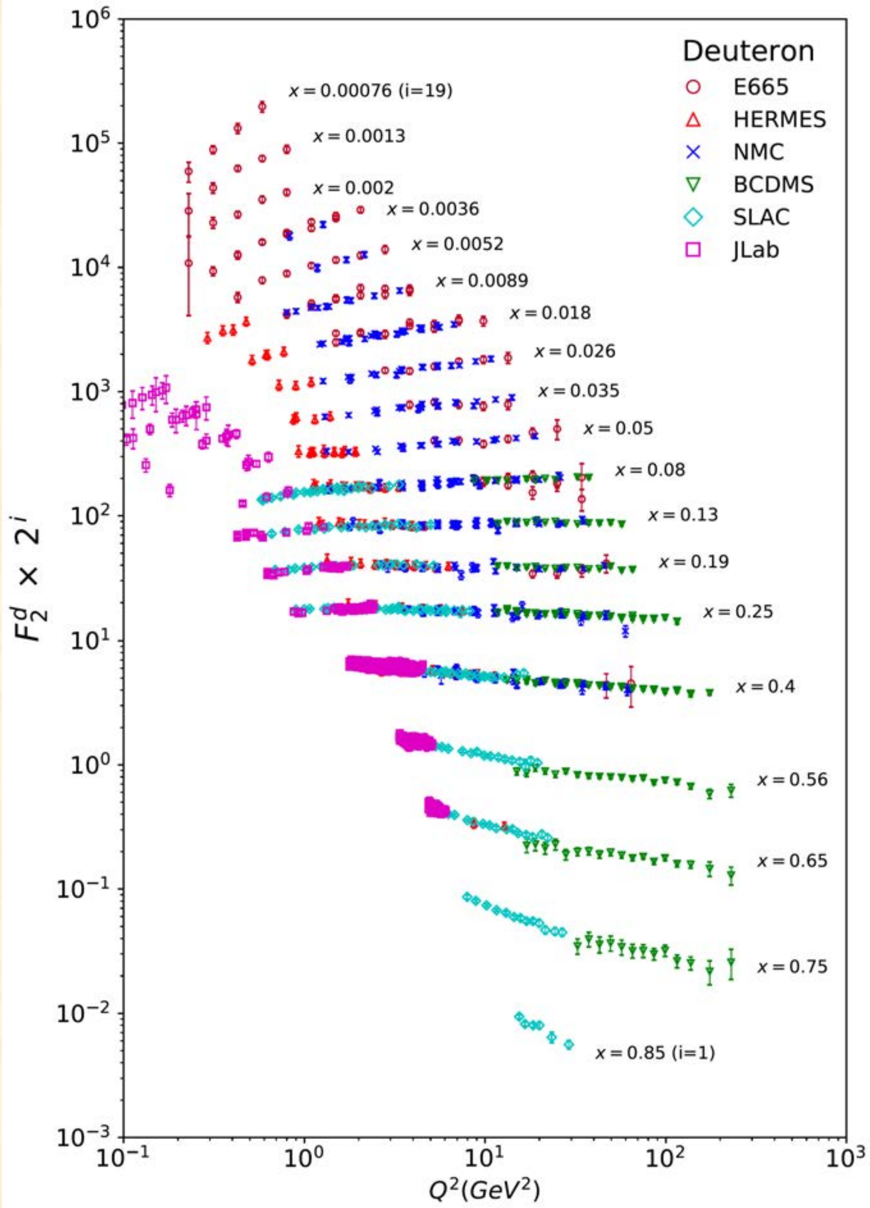
Structure function F_2

S. Navas *et al.* (Particle Data Group), Phys. Rev. D 110 (2024) 030001

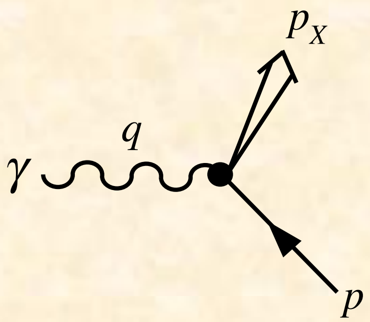
proton



deuteron



Longitudinal and transverse cross sections of $\gamma^*+N\to X$ and their relations to W_1 and W_2



$$\sigma_\lambda^{tot} = \frac{1}{2M_N\, 2K}\overline{\sum_{\lambda_N}}\sum_X (2\pi)^4\delta^4(q+p-p_X)\,|M|^2, \quad M=\varepsilon^\mu(\lambda,q)\,\langle X\,|e\,J_\mu^{em}(0)|\,p,\lambda_N\rangle$$

λ = helicity, photon polarization vector: $\varepsilon^\mu(\lambda,q)$

transverse: $\lambda=\pm1$: $\varepsilon_\pm = \mp\frac{1}{\sqrt{2}}\left(0,\,1,\,\pm i,\,0\right)$

longitudinal: $\lambda=0$: $\varepsilon_0 = \frac{1}{\sqrt{-q^2}}\left(\sqrt{\nu^2-q^2},\,0,\,0,\,\nu\right)$

For real photon: $K=\nu,\ \ W^2=(p+q)^2=M_N^2+2M_NK$

For virtual photon, the factor $(2K)$ is arbitrary

- Hand convention: $K=\frac{W^2-M_N^2}{2M_N}=\nu+\frac{q^2}{2M_N}$
- others: $K=\nu,\quad K=|\vec{q}|$

$$\sigma_\lambda^{tot} = \frac{e^2}{4M_NK}\overline{\sum_{\lambda_N}}\sum_X (2\pi)^4\delta^4(q+p-p_X)\langle p,\lambda_N\,|\,\varepsilon^{\mu*}J_\mu^{em}(0)|X\rangle\langle X\,|\,\varepsilon^\nu J_\nu^{em}(0)|\,p,\lambda_N\rangle = \frac{4\pi\alpha}{4M_NK}\varepsilon^{\mu*}\varepsilon^\nu 4\pi M_N W_{\mu\nu}$$

$$=\frac{4\pi^2\alpha}{K}\varepsilon^{\mu*}\varepsilon^\nu W_{\mu\nu}$$

$$\sigma_T \equiv \frac{\sigma_{+1}^{tot}+\sigma_{-1}^{tot}}{2} = \frac{4\pi^2\alpha}{K}W_1, \quad \sigma_L \equiv \sigma_0^{tot} = \frac{4\pi^2\alpha}{K}\left[(1-\nu^2/q^2)W_2 - W_1\right]$$

W_1 = transverse, W_2 = transverse + longitudinal

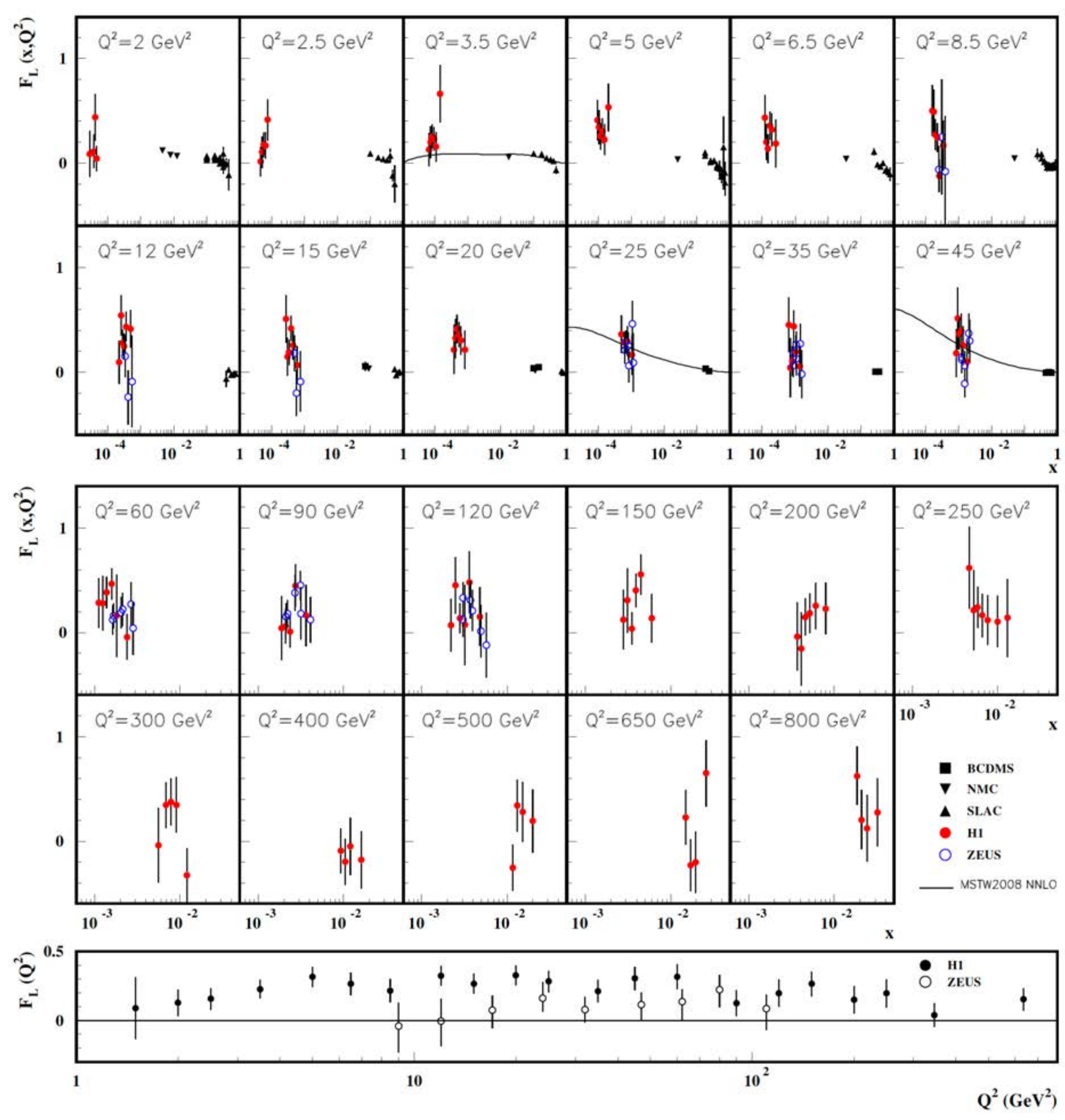
$(1-\nu^2/q^2)W_2 - W_1 \equiv W_L$ = longitudinal

Instead of W_1, W_2 and W_L, the functions F_1, F_2 and F_L are usually used for showing experimental results.

$$F_1 = M_N W_1, \quad F_2 = \nu W_2, \quad F_L = \frac{Q^2}{\nu}W_L = \left(1+\frac{Q^2}{\nu^2}\right)F_2 - 2xF_1$$

Structure function F_L

S. Navas *et al*. (Particle Data Group), Phys. Rev. D 110 (2024) 030001



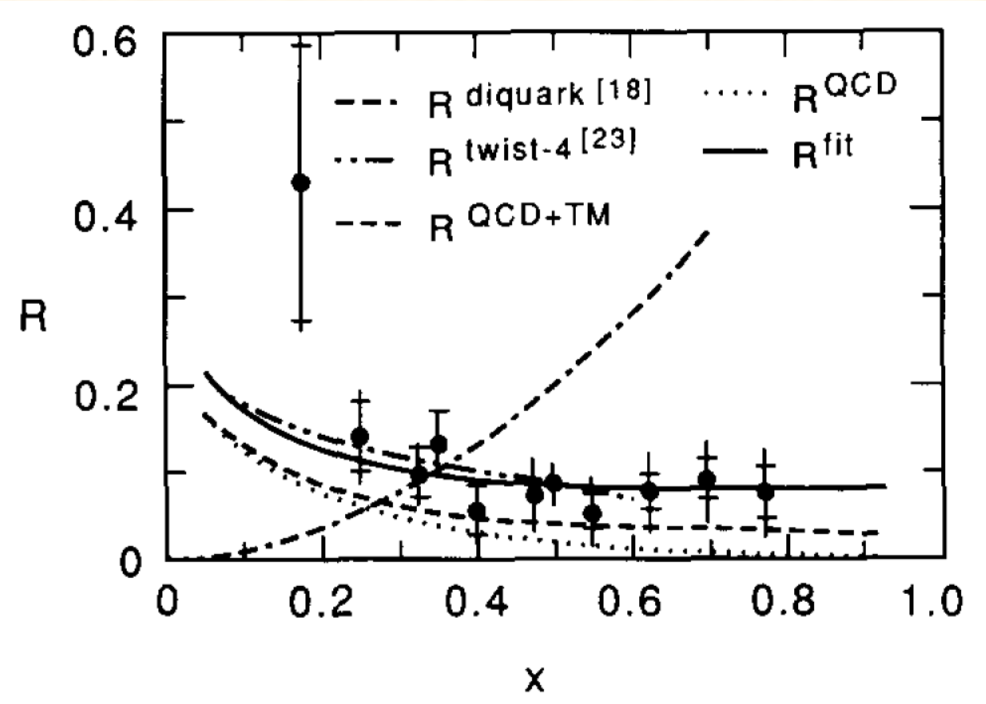
Longitudinal-transverse structure function ratio *R*

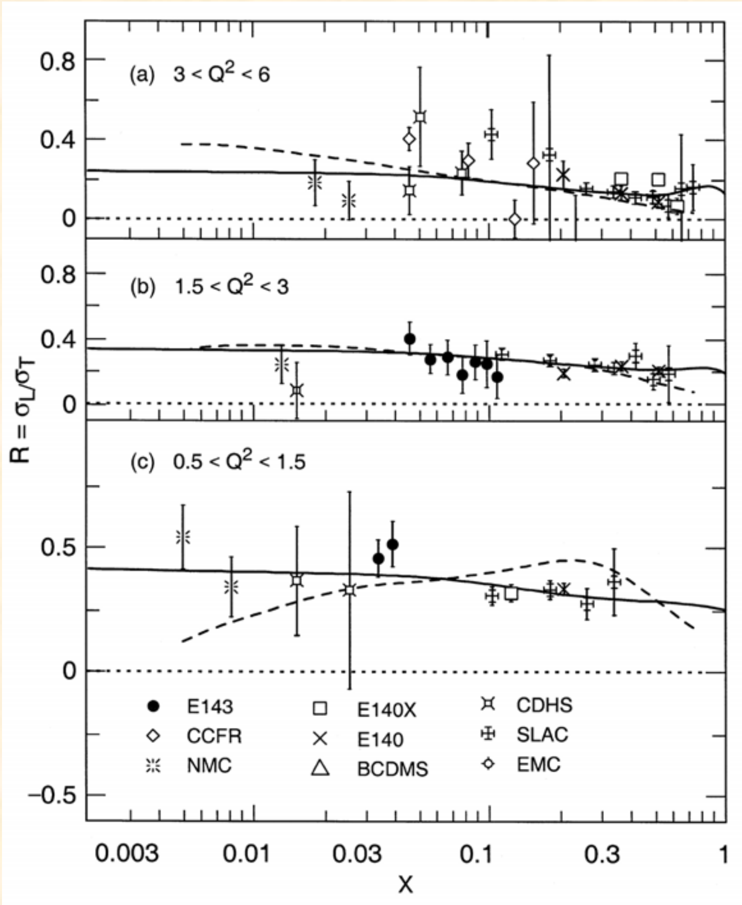
$$R = \frac{F_L}{F_T} = \frac{(1+Q^2/\nu^2)F_2 - 2xF_1}{2xF_1} = \frac{(1+4M_N^2x^2/Q^2)F_2 - 2xF_1}{2xF_1} \to 0 \quad \text{in the scaling limit}$$

Finite R = higher-twist and higher-order (of α_s) effects

SLAC parametrizations for *R*

R1990: L. W. Whitlow *et al*., PLB 250 (1990) 193; R1998: K. Abe *et al*., PLB 452 (1999) 194.

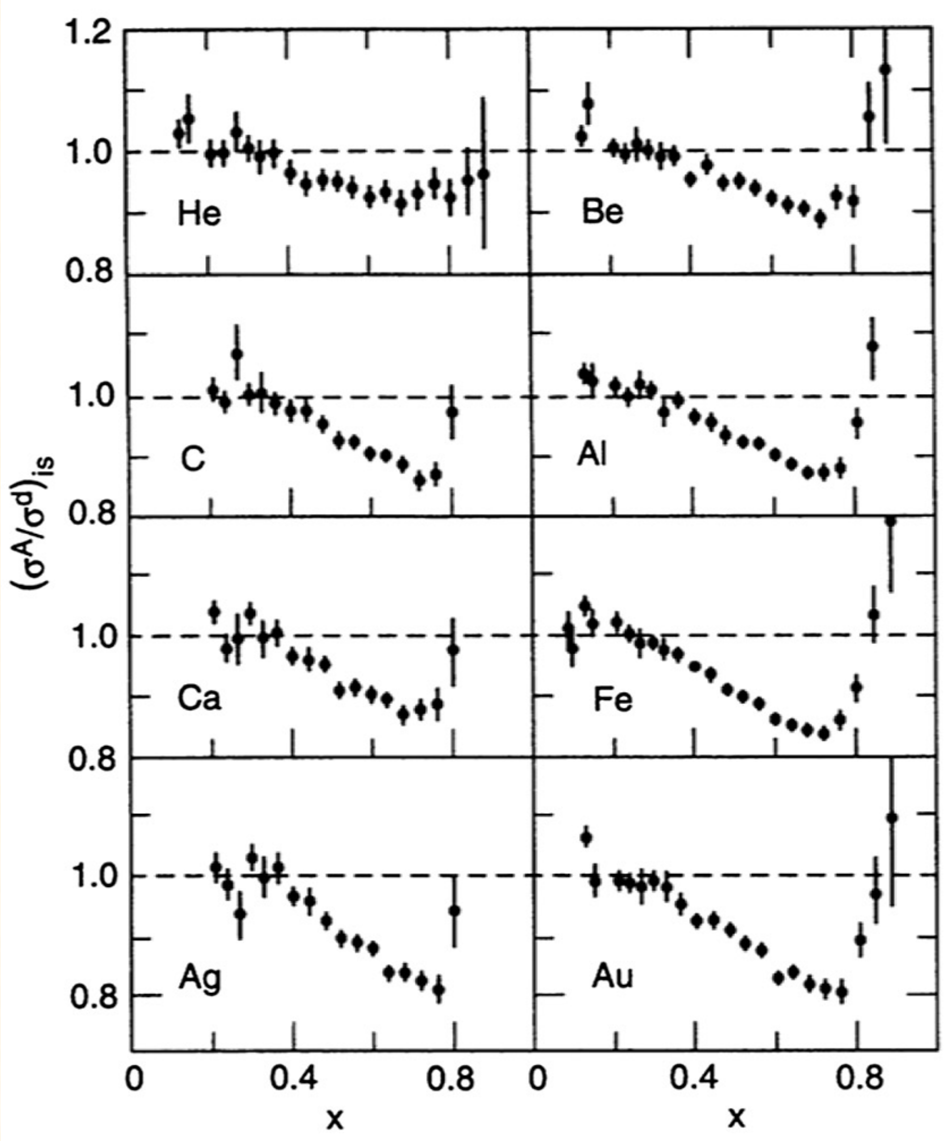


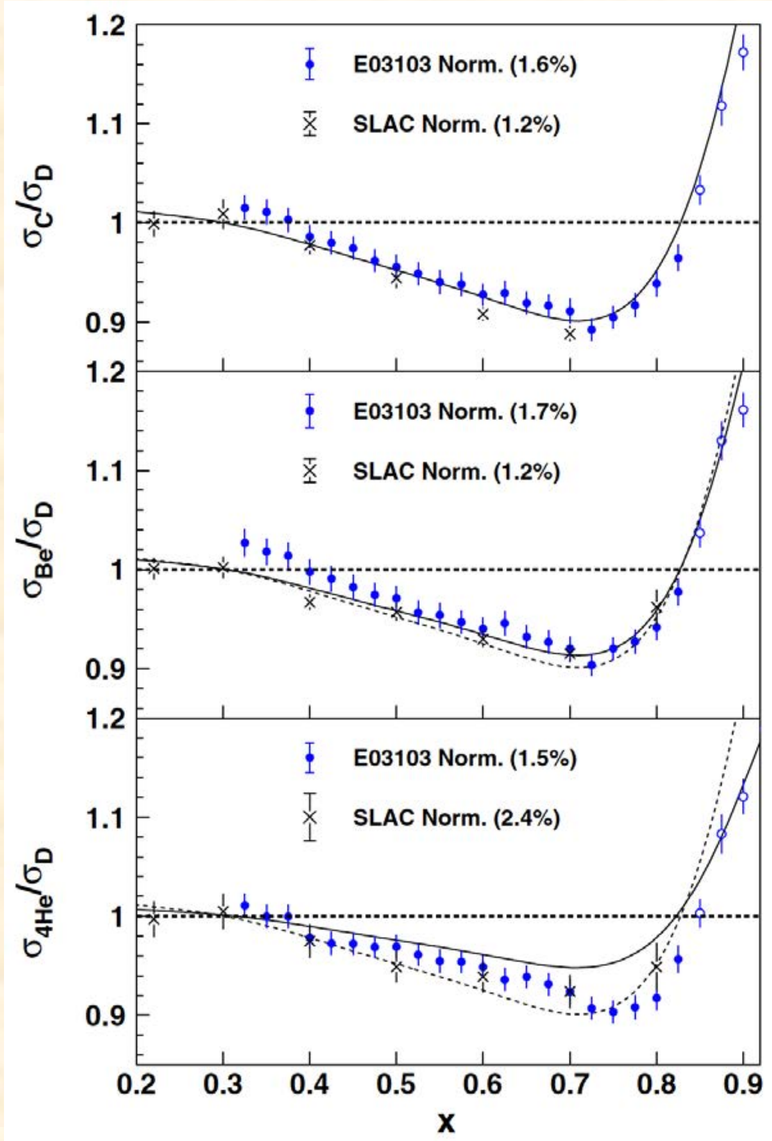


Nuclear modifications of structure functions F_2

Nuclear modifications at SLAC and JLab

EMC effect,
J. J. Aubert *et al*. (EMC), Phys. Lett. B123 (1983) 275.

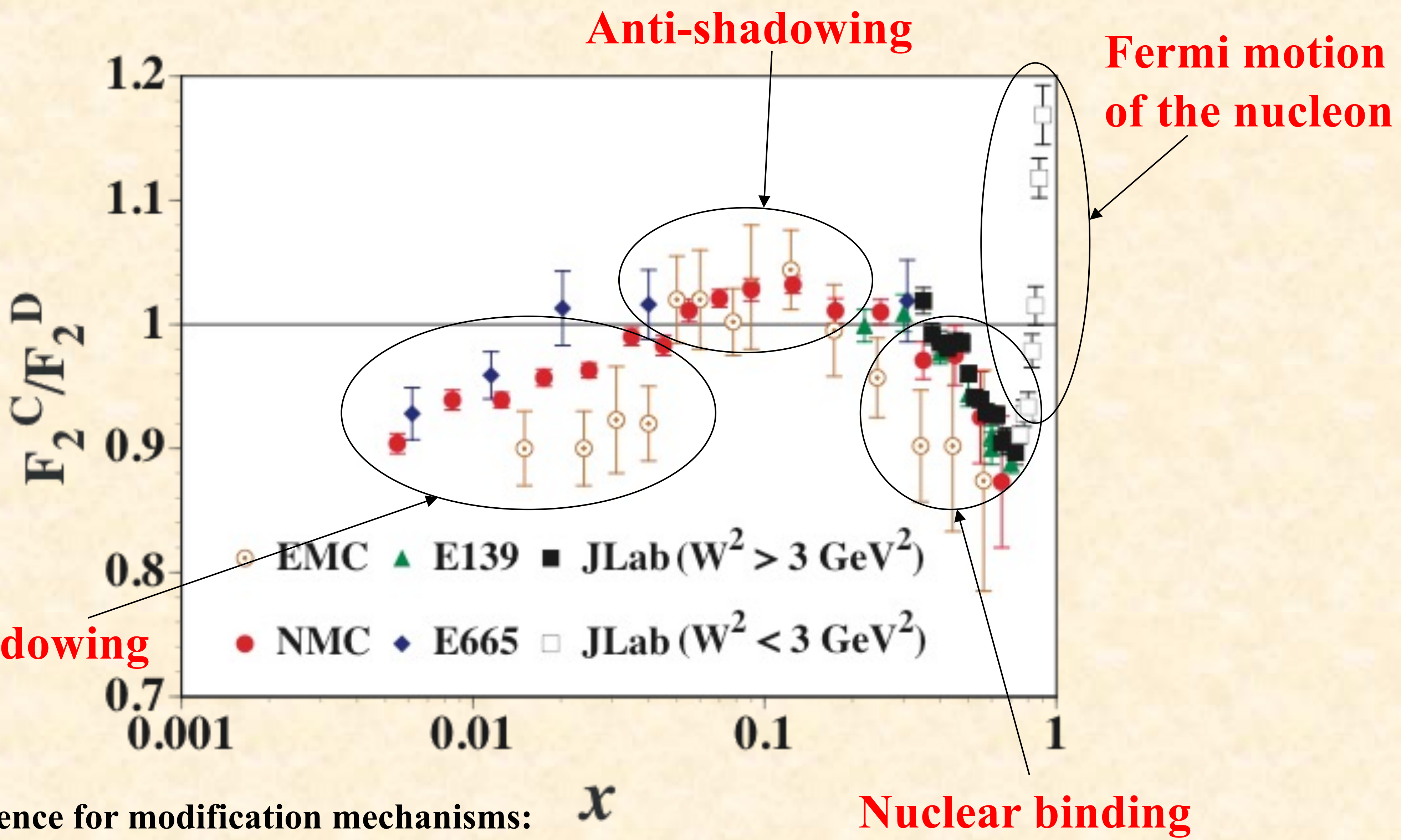




SLAC: J. Gomez *et al*., Phys. Rev. D 49 (1994) 4348.

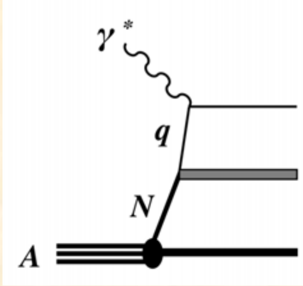
JLab: J. Seely *et al*., Phys. Rev. Lett. 103 (2009) 202301.

Nuclear modifications of structure function F_2



Reference for modification mechanisms:
D. F. Geesaman, K. Saito, A. W. Thomas,
Ann. Rev. Nucl. Part. Sci. 45 (1995) 337.

Binding and Fermi motion



Convolution: $W^A_{\mu\nu}(p_A, q) = \int d^4 p\, S(p)\, W^N_{\mu\nu}(p_N, q)$

$S(p)$ = Spectral function = nucleon momentum distribution in a nucleus

In a simple shell model: $S(p) = \sum_i |\phi_i(\vec{p})|^2 \delta(p_0 - M_N - \varepsilon_i)$ Separation energy: ε_i

Projecting out F_2: $F_2^A(x, Q^2) = \sum_i \int dz\, f_i(z)\, F_2^N(x/z, Q^2)$

$f_i(z) = \int d^3 p\, z\, \delta\left(z - \frac{p \cdot q}{M_N \nu}\right) |\phi_i(\vec{p})|^2$

lightcone momentum distribution for a nucleon i

$z = \frac{p \cdot q}{M_N \nu} = \frac{p^0 \nu - \vec{p} \cdot \vec{q}}{M_N \nu} = 1 - \frac{|\varepsilon_i|}{M_N} - \frac{\vec{p} \cdot \vec{q}}{M_N \nu} \approx 1.00 - 0.02 \pm 0.20$ for a medium-size nucleus

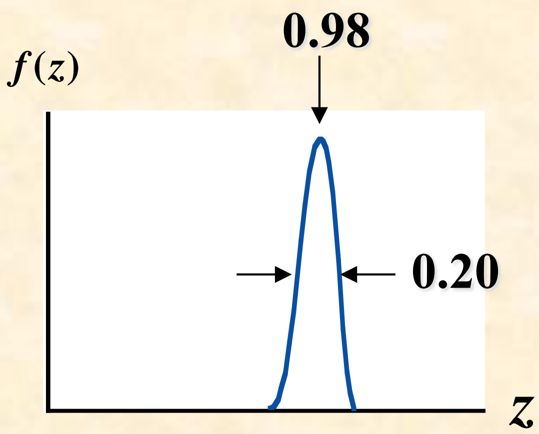
If $f_i(z)$ were $f_i(z) = \delta(z-1)$, there is no nuclear modification: $F_2^A(x, Q^2) = F_2^N(x, Q^2)$.

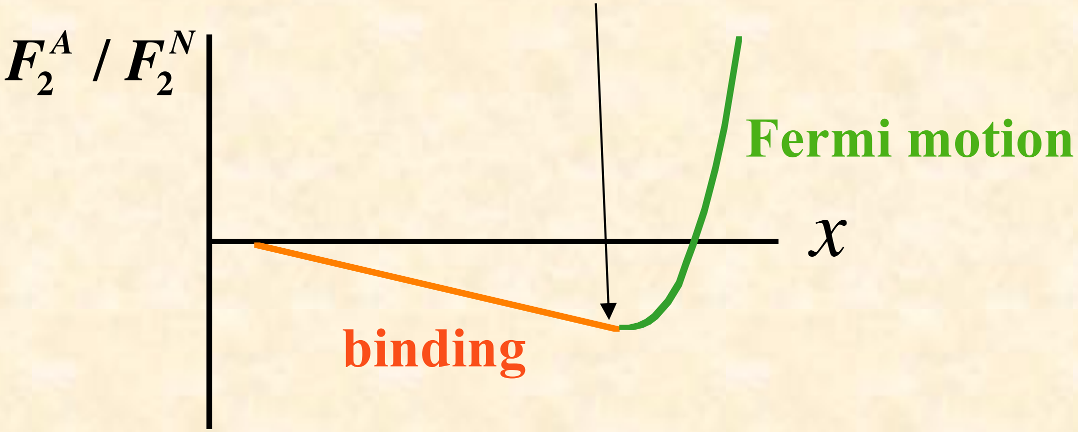
Because the peak shifts slightly (1→ 0.98), nuclear modification of F_2 is created.

$F_2^A(x, Q^2) \simeq F_2^N(x/0.98, Q^2)$

For $x = 0.60$, $x/0.98 = 0.61$

$\frac{F_2^N(x=0.61)}{F_2^N(x=0.60)} = \frac{0.021}{0.024} = 0.88$





Nuclear modifications of structure-function ratio *R*

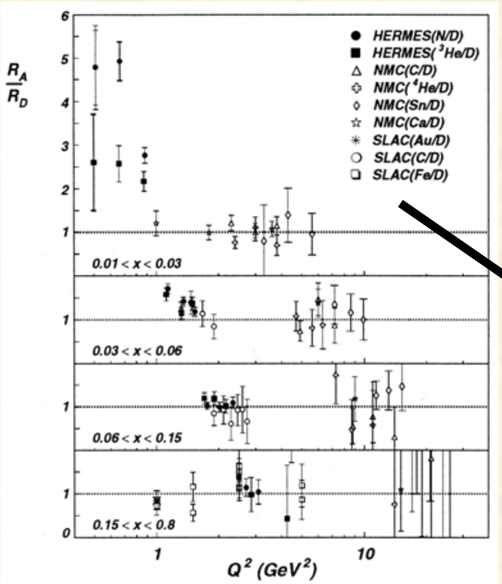
S. Kumano, arXiv:2506.18305

Experimental situation on nuclear modifications of *R*

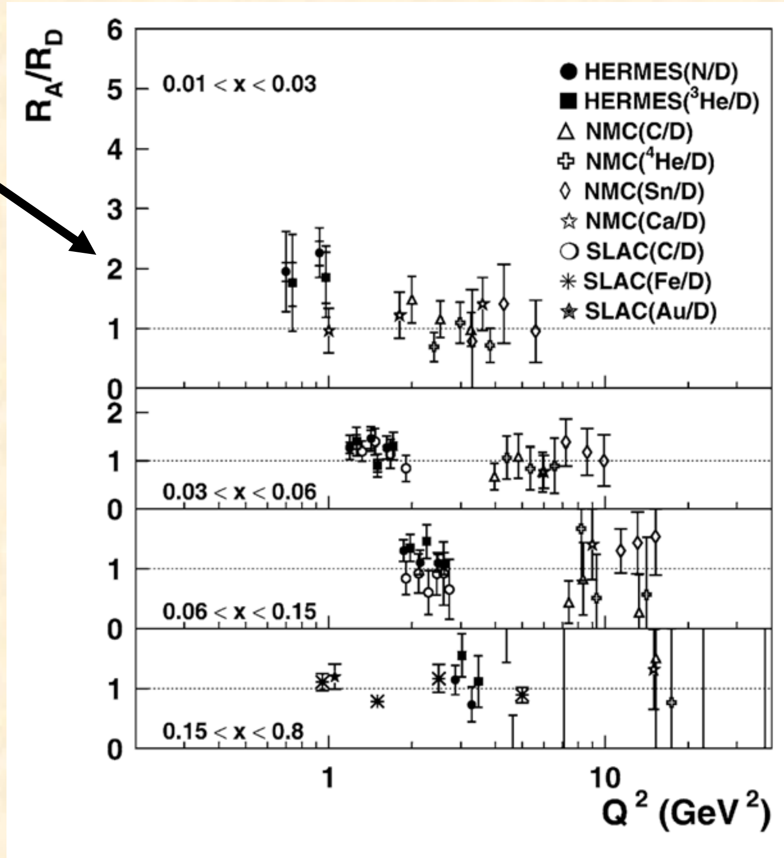
Ther is no nuclear modification in *R* within the experimental errors.

HERMES ^{14}N/D, ^{3}He/D
(2000, 2003)

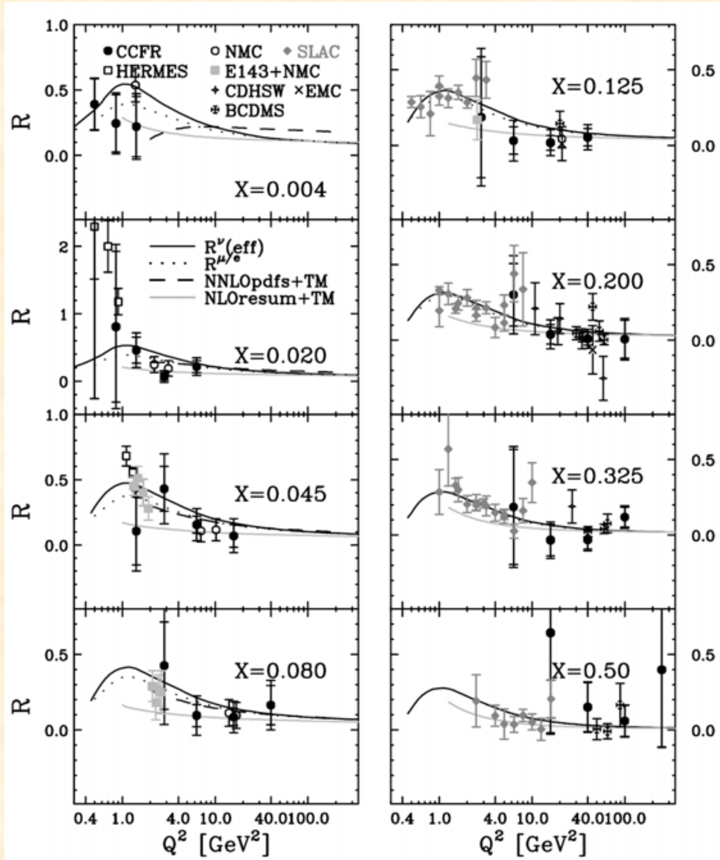
CCFR Fe
(2001)



K. Ackerstaff *et al.*,
PLB 475 (2000) 386.



Erratum, A. Airapetian et al., PLB 567 (2003) 339.

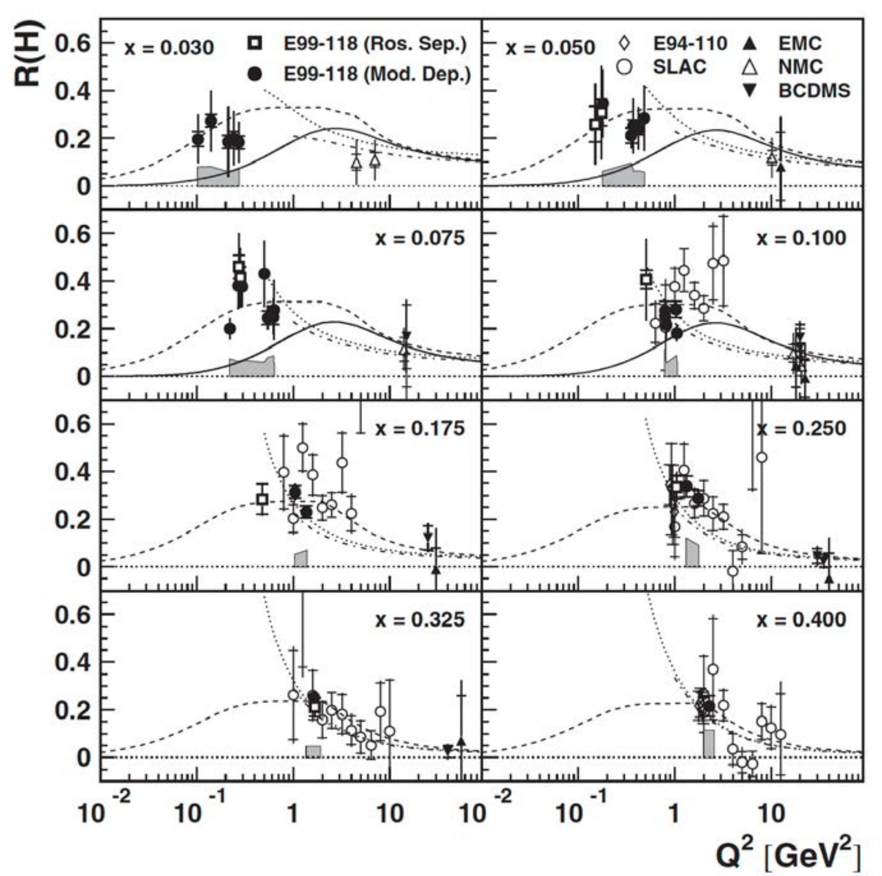


U. K. Yang *et al.*, PRL 87 (2001) 251802.

JLab measurements for the deuteron (2007)

V. Tvaskis et al., PRL 98 (2007) 142301

proton



deuteron − proton

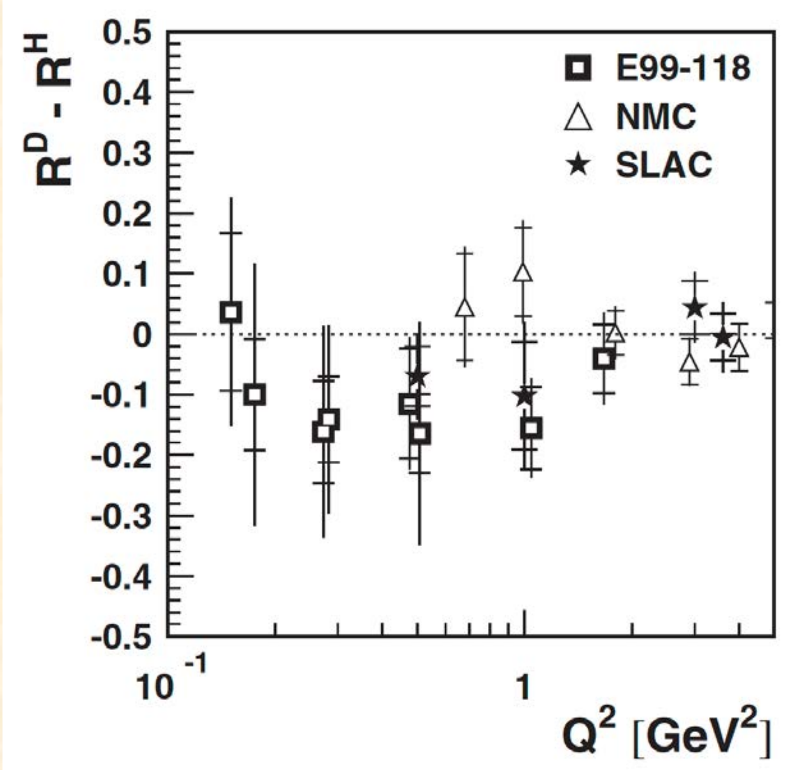
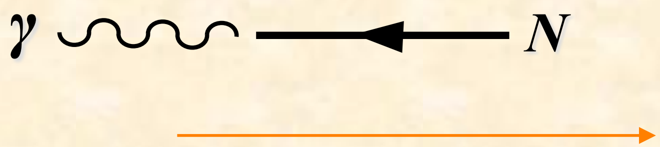


TABLE I. The values of R_H and $R^D - R^H$ calculated via the Rosenbluth separation. Note that the systematic error in the difference accounts for the correlation between the uncertainties in the hydrogen and deuterium data.

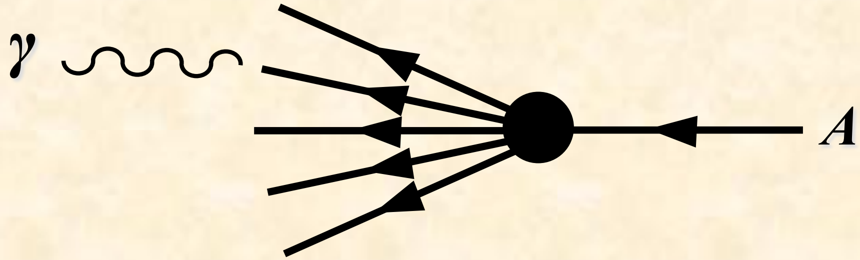
Q^2 (GeV^2)	x	R^H	Stat.	Syst.	$R^D - R^H$	Stat.	Syst.
0.150	0.041	0.259	0.074	0.153	0.036	0.131	0.136
0.175	0.050	0.307	0.056	0.188	−0.100	0.091	0.196
0.273	0.077	0.460	0.049	0.132	−0.162	0.084	0.153
0.283	0.081	0.414	0.045	0.117	−0.141	0.071	0.138
0.476	0.156	0.283	0.063	0.025	−0.115	0.091	0.061
0.508	0.091	0.406	0.038	0.168	−0.164	0.065	0.172
1.045	0.200	0.335	0.048	0.041	−0.155	0.068	0.046
1.670	0.320	0.211	0.038	0.021	−0.040	0.057	0.051

Nuclear modifications exist in *R* theoretically

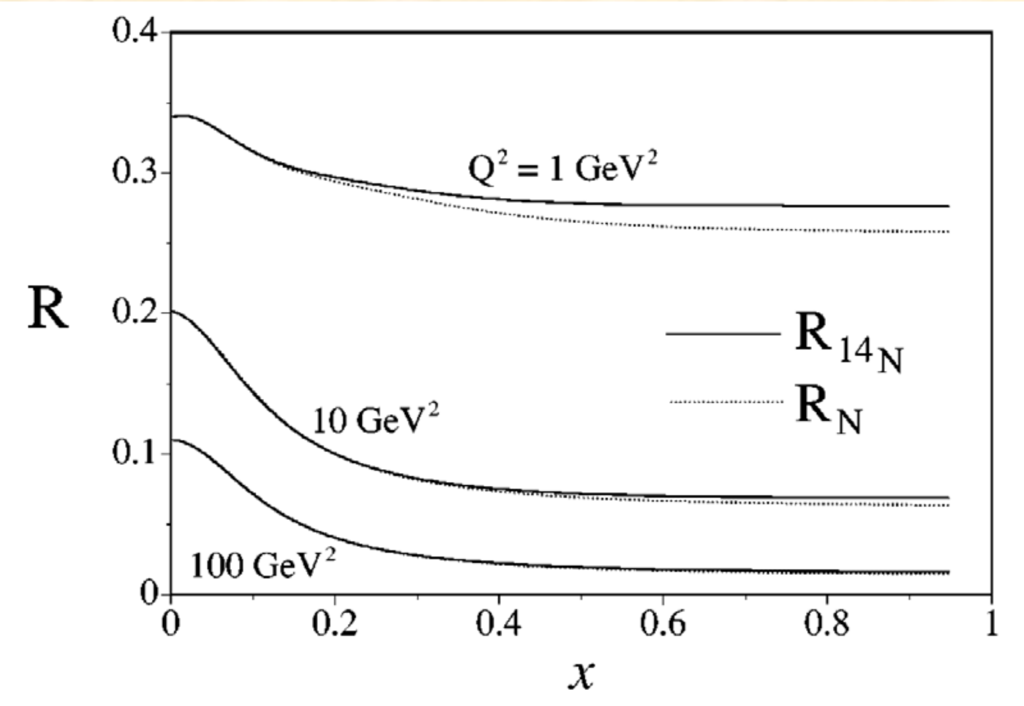
M. Ericson and S. Kumano
PRC 67 (2003) 022201

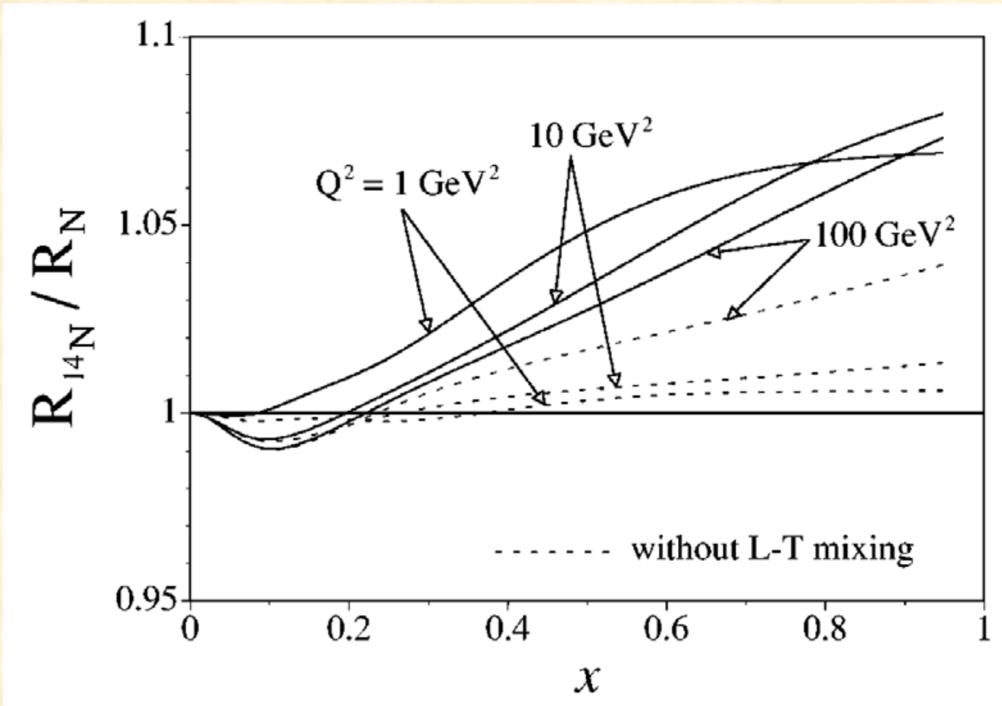


Longitudinal direction

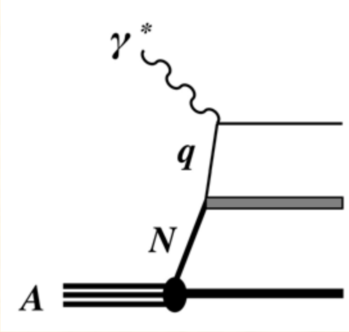


A nucleon in a nucleus moves any direction, which is not necessarily the longitudinal one.





Nuclear modifications of *R* in a nucleus



Convolution: $W^{A}_{\mu\nu}(p_A, q) = \int d^4p\, S(p)\, W^{N}_{\mu\nu}(p_N, q)$

$S(p)$ = Spectral function

= nucleon momentum distribution in a nucleus

In a simple shell model,

$$S(p) = \sum_i \left|\phi_i(\vec{p})\right|^2 \delta(p_0 - M_N - \varepsilon_i), \quad \varepsilon_i = \text{Separation energy}$$

Projection operators of F_1, F_2, and F_L from $W_{\mu\nu} = -F_1 \frac{1}{M_N}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + F_2 \frac{\tilde{p}_\mu \tilde{p}_\nu}{M_N^2 \nu}$

$$\tilde{p}_\mu \equiv p_\mu - \frac{p \cdot q}{q^2} q_\mu, \quad \tilde{p} \cdot q = 0$$

$$\hat{P}_1^{\mu\nu} = -\frac{M_N}{2}\left(g^{\mu\nu} - \frac{\tilde{p}^\mu \tilde{p}^\nu}{\tilde{p}^2}\right), \quad \hat{P}_2^{\mu\nu} = -\frac{M_N^2 \nu}{2\tilde{p}^2}\left(g^{\mu\nu} - \frac{3\tilde{p}^\mu \tilde{p}^\nu}{\tilde{p}^2}\right), \quad \hat{P}_L^{\mu\nu} = \frac{Q^2}{\nu}\frac{\tilde{p}^\mu \tilde{p}^\nu}{\tilde{p}^2}$$

$$\hat{P}_1^{\mu\nu} W_{\mu\nu} = F_1, \quad \hat{P}_2^{\mu\nu} W_{\mu\nu} = F_2, \quad \hat{P}_L^{\mu\nu} W_{\mu\nu} = F_L$$

Theoretical nuclear modifications of *R*

$$R_A(x_A,Q^2)=\frac{F_L^A(x_A,Q^2)}{2x_A F_1^A(x_A,Q^2)},\quad x_A=\frac{M_N}{M_A}x,\quad 0\le x_A\le 1,\quad 0\le x_A\le\frac{M_A}{M_N}\simeq A$$

$$\begin{pmatrix} F_2^A(x_A,Q^2) \\ 2x_A F_1^A(x_A,Q^2) \\ F_L^A(x_A,Q^2)\end{pmatrix}=\int_x^A dy\begin{pmatrix} f_{22}(y) & 0 & 0 \\ 0 & f_{11}(y) & f_{1L}(y) \\ 0 & f_{L1}(y) & f_{LL}(y)\end{pmatrix}\begin{pmatrix} F_2^N(x/y,Q^2) \\ \frac{2x}{y}F_1^N(x/y,Q^2) \\ F_L^N(x/y,Q^2)\end{pmatrix}$$

$$f_{22}(y)=\int_0^\infty dp_{N\perp}2\pi p_{N\perp}y\,\frac{M_N\nu}{|\vec q|}\left|\phi\left(|\vec p_N|\right)\right|^2\left[\frac{2x_N^2p_{N\perp}^2}{(1+Q^2/\nu^2)Q^2}+\left(1+\frac{2x_Np_{N\parallel}}{\sqrt{Q^2+\nu^2}}\right)^2\right]$$

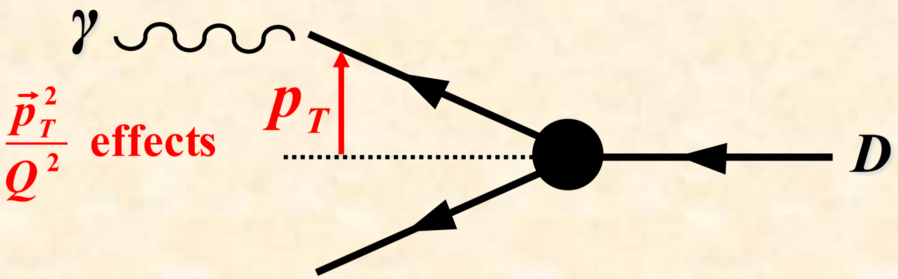
$$f_{11}(y)=f_{LL}(y)=\int_0^\infty dp_{N\perp}2\pi p_{N\perp}y\,\frac{M_N\nu}{|\vec q|}\left|\phi\left(|\vec p_N|\right)\right|^2\left(1+\frac{\vec p_{N\perp}^{\,2}}{\tilde p_N^{\,2}}\right)$$

$$f_{1L}(y)=f_{L1}(y)=\int_0^\infty dp_{N\perp}2\pi p_{N\perp}y\,\frac{M_N\nu}{|\vec q|}\left|\phi\left(|\vec p_N|\right)\right|^2\frac{\vec p_{N\perp}^{\,2}}{\tilde p_N^{\,2}}$$

- The transverse structure function $2xF_1^N(x,Q^2)$ and the longitudinal one $F_1^N(x,Q^2)$ mix with each other in a nucleus with the mixture coefficeint $\frac{\vec p_{N\perp}^{\,2}}{\tilde p_N^{\,2}}\sim\frac{\vec p_{N\perp}^{\,2}}{Q^2}$.
- It should be prominent at small Q^2.

Results for the deuteron

$$\begin{pmatrix} F_2^A(x_A,Q^2) \\ 2x_A F_1^A(x_A,Q^2) \\ F_L^A(x_A,Q^2) \end{pmatrix} = \int_x^A dy \begin{pmatrix} f_{22}(y) & 0 & 0 \\ 0 & f_{11}(y) & f_{1L}(y) \\ 0 & f_{L1}(y) & f_{LL}(y) \end{pmatrix} \begin{pmatrix} F_2^N(x/y,Q^2) \\ \frac{2x}{y} F_1^N(x/y,Q^2) \\ F_L^N(x/y,Q^2) \end{pmatrix}$$



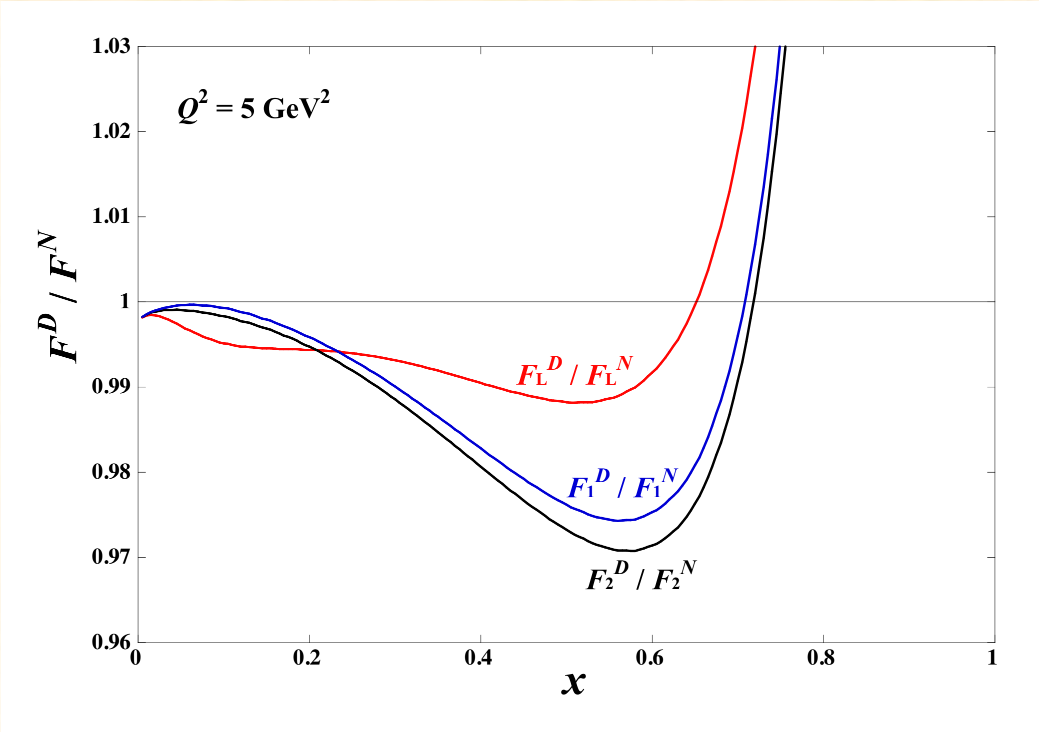
$$f_{22}(y) = \int_0^\infty dp_{N\perp} 2\pi p_{N\perp} y \frac{M_N \nu}{|\vec{q}|} |\phi(|\vec{p}_N|)|^2 \left[\frac{2x_N^2 p_{N\perp}^2}{(1+Q^2/\nu^2)Q^2} + \left(1 + \frac{2x_N p_{N\parallel}}{\sqrt{Q^2+\nu^2}}\right)^2 \right]$$

$$f_{11}(y) = f_{LL}(y) = \int_0^\infty dp_{N\perp} 2\pi p_{N\perp} y \frac{M_N \nu}{|\vec{q}|} |\phi(|\vec{p}_N|)|^2 \left(1 + \frac{\vec{p}_{N\perp}^{\,2}}{\vec{p}_N^{\,2}}\right)$$

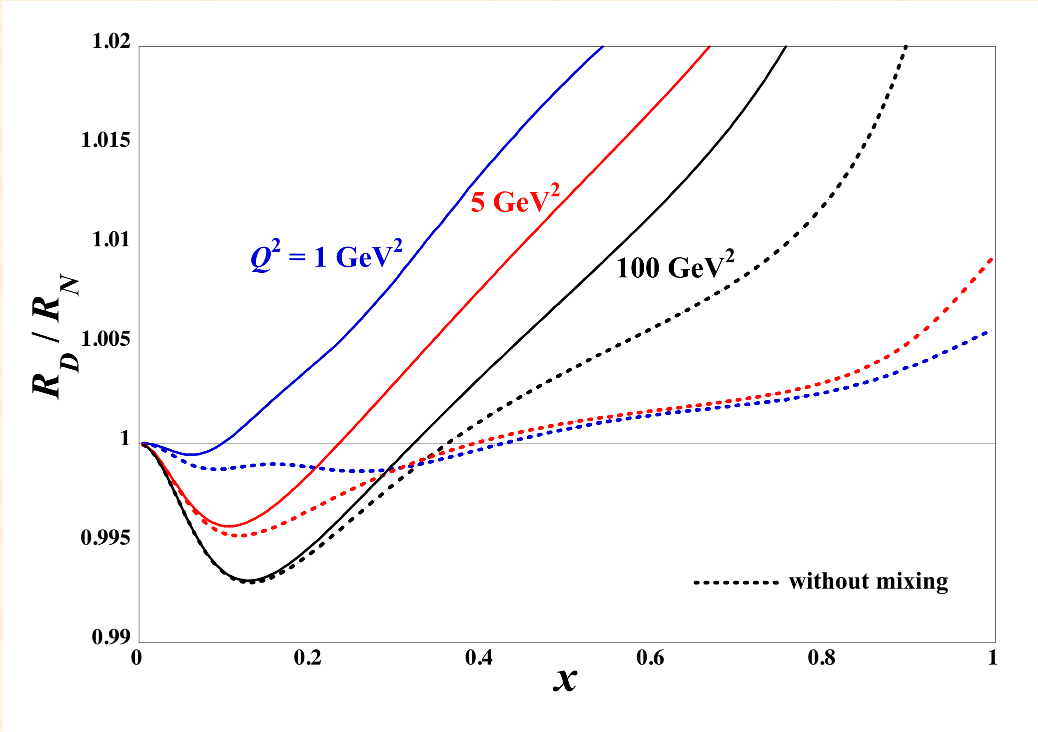
$$f_{1L}(y) = f_{L1}(y) = \int_0^\infty dp_{N\perp} 2\pi p_{N\perp} y \frac{M_N \nu}{|\vec{q}|} |\phi(|\vec{p}_N|)|^2 \frac{\vec{p}_{N\perp}^{\,2}}{\vec{p}_N^{\,2}}$$

Without mixing
(dotted curves)

Nuclear modifications of F_2, F_1, F_L



Nuclear modifications of R



* Modifications of F_L are different from the others.

* Modifications of R are large at small Q^2.

* However, modifications exist even at large Q^2 because x dependent functional forms are different between F_L and $2xF_1$.

We have been assuming nuclear modification does not exist in *R*.

For example, in the polarized PDF analysis

Experimental data are shown by the spin asymmetry

$$A_1(x,Q^2) = \frac{g_1(x,Q^2)}{F_1(x,Q^2)}, \quad F_1(x,Q^2) = \frac{F_2(x,Q^2)}{2x\left[1+R(x,Q^2)\right]}.$$

In global analyses for obtaining the longitudinally-polarized PDFs, the same function (x,Q^2) is uased for the deuteron and ^{3}He data.

One needs to be carful about the nuclear modifications of *R* as studied in this work, especially for large nuclei.

- **Nuclear modifications of *R* could be important for obtaining accurate PDFs.**
- **There could be interesting mechanisms for such nuclear modifications.**
- **Nuclear modifications are large as the mass number *A* increases.**

Experimental prospects
and
Summary

JLab experiment for *R* in 2026

Update of Experiment E12-06-104

Measurement of the Ratio $R = \sigma_L/\sigma_T$
in Semi-Inclusive Deep-Inelastic Scattering
July 01, 2010

R. Ent (spokesperson), H.C. Fenker, D. Gaskell,
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JLab experiment E12-06-104 for *R* will run in 2026, so it would be nice to study possible nuclear modifications.

In future, we expect to have EIC experiments.

Summary

$$R = \frac{F_L}{F_T} = \frac{(1 + Q^2/\nu^2)F_2 - 2xF_1}{2xF_1}$$

- We showed that finite nuclear modifications exist
  for the longitudinal-transverse structure function ratio *R* in the deuteron,
  mainly due to the transverse motion of a nucleon in the nucleus.

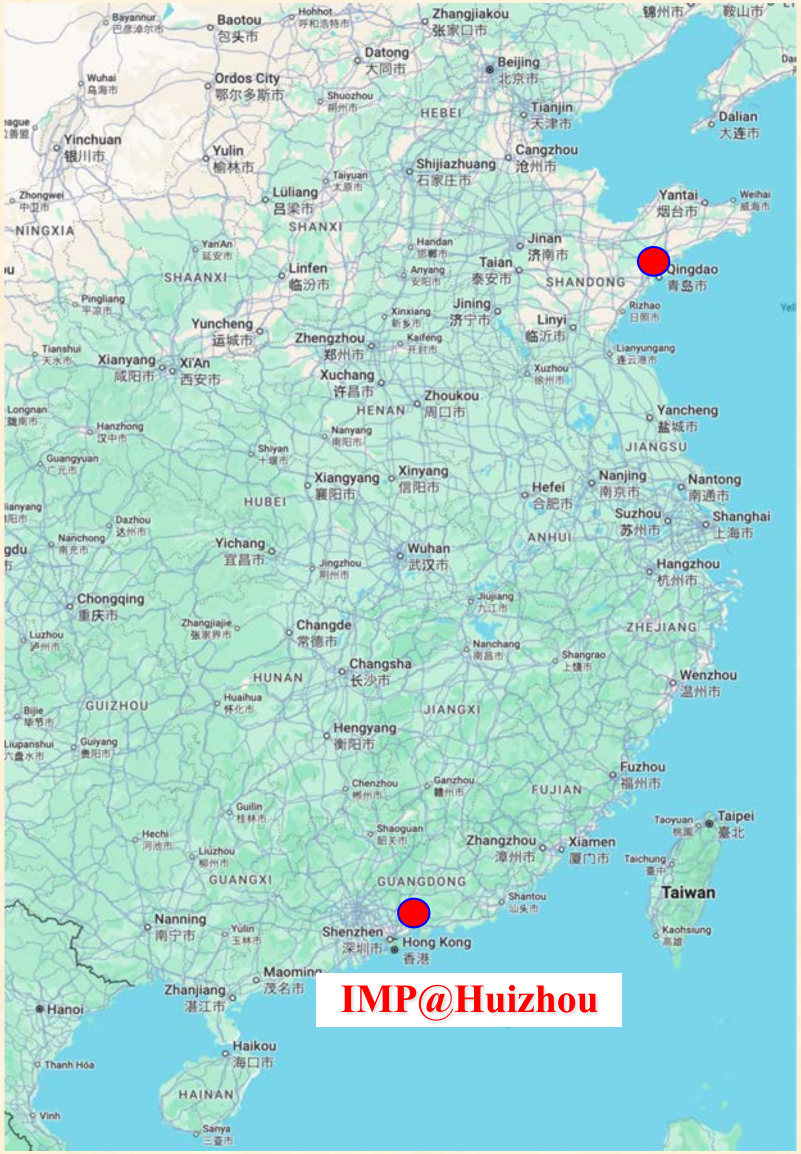
- Nuclear modifications of *R* could be important practically,
  for example, in order to determine accurate PDFs.

- There could be interesting mechanisms for such nuclear modifications.

- Modifications are large for large size nuclei.

- The JLab experiment on *R* will start next year 2026.
  We should aware that the nuclear modifications exist in *R*
  although there is no indication in the existing experimental data.

Postdoctoral research associate at IMP



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HIAF (High Intensity heavy ion Accelerator Facility)

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