Strong CP-Violation and Nucleon EDM on the Lattice

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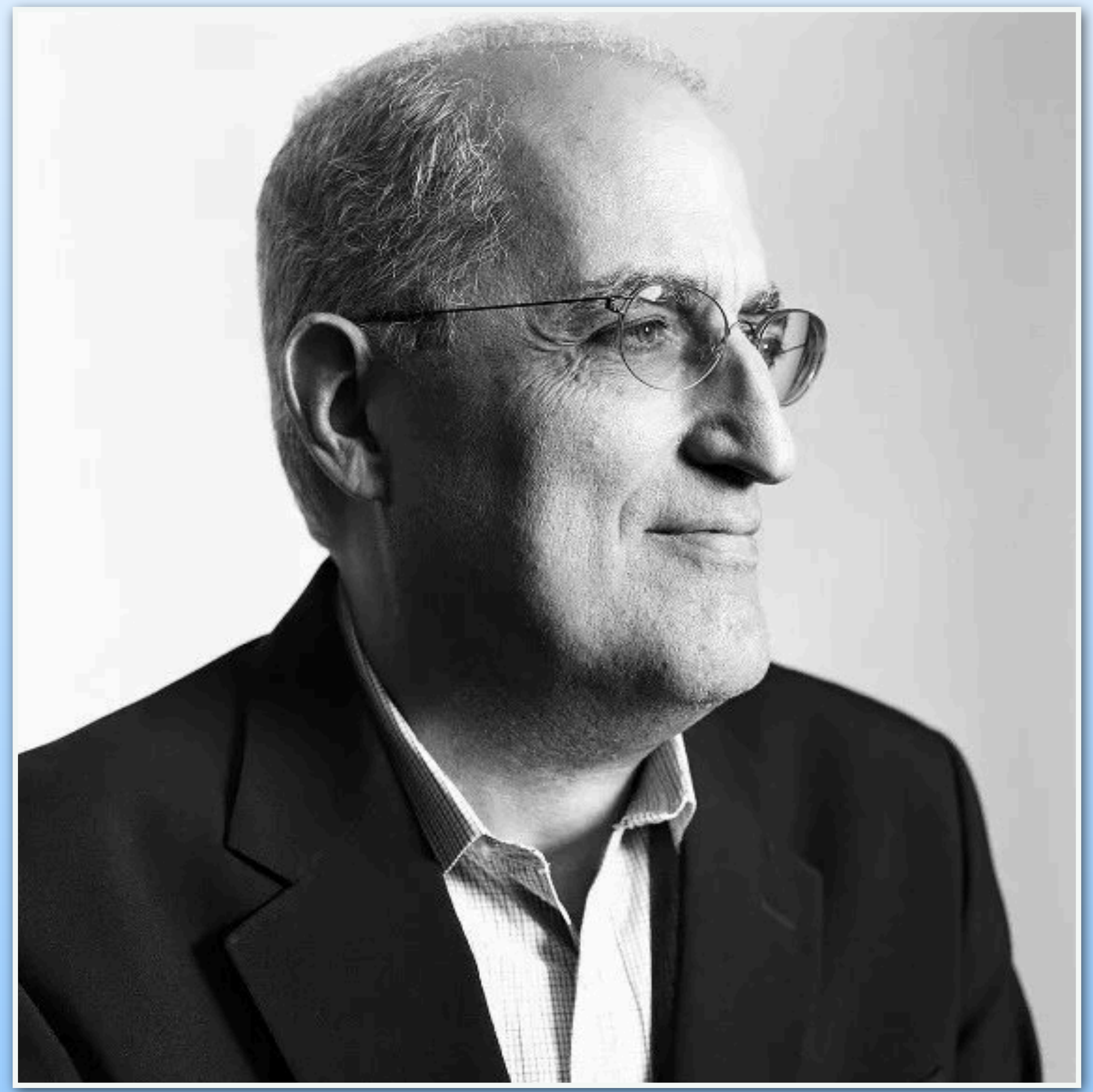
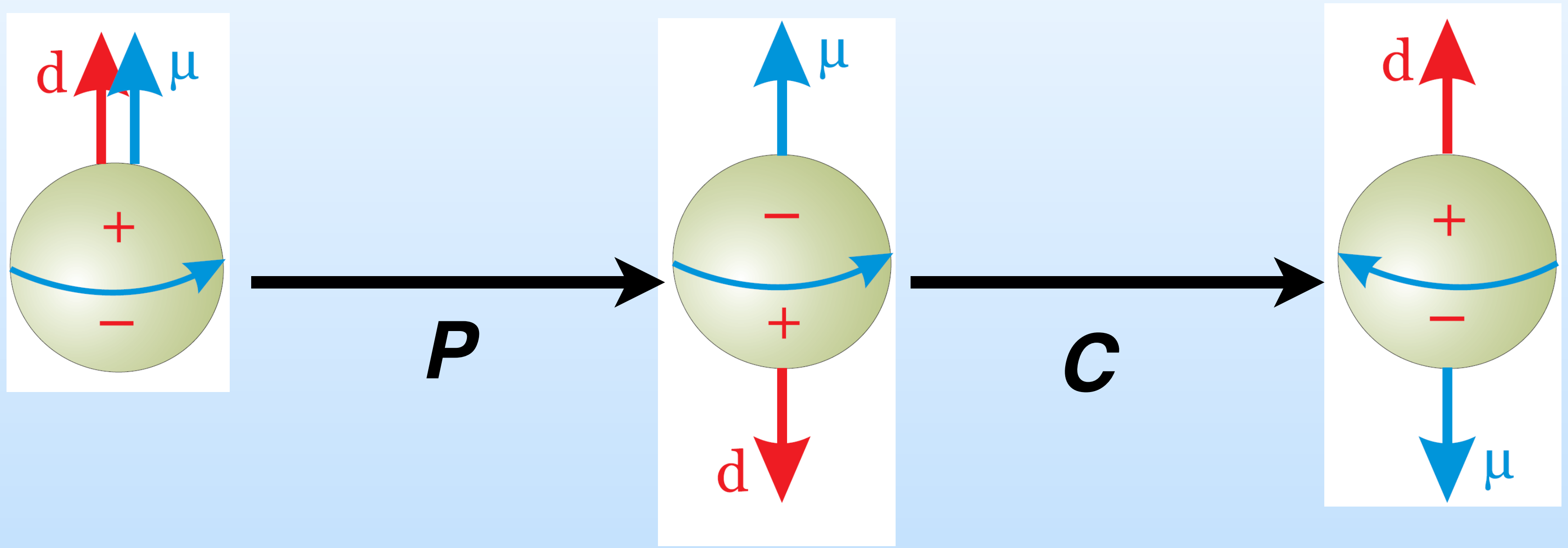
Nucleon EDM

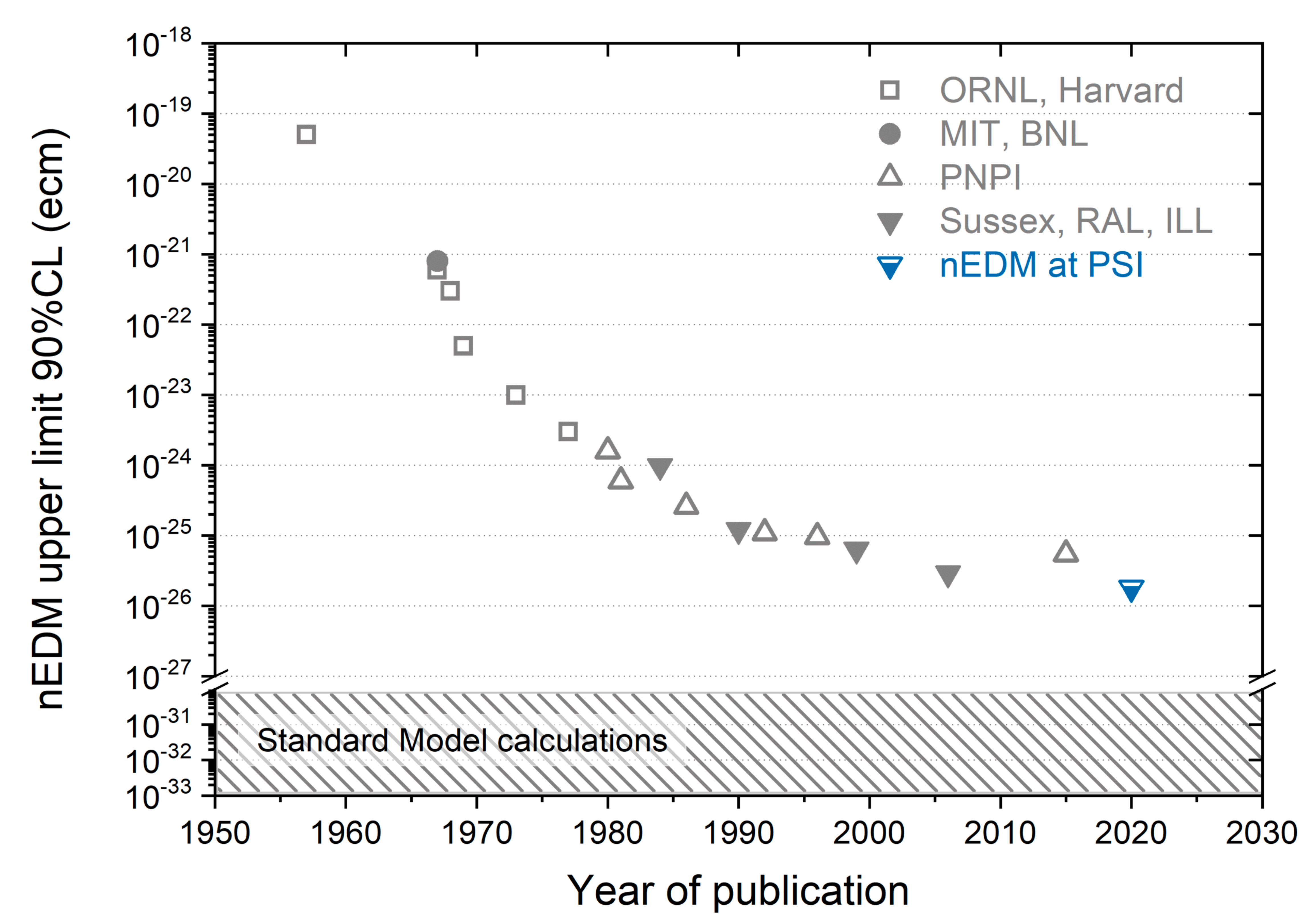
... It is also important to look for electron or neutron electric dipole moments, which may give a new physical signal, and under the energy scales we have already observed...



— Edward Witten: Where is the New Physics Lurking? (cern courier)

Nucleon EDM





- ✦ A non-zero intrinsic electric dipole moment (EDM) of a fundamental particle violates the CP (T) symmetry

- ✦ Nucleon EDM (NEDM) is a sensitive probe of BSM/ Strong CPV: the contribution to the nEDM from the weak CPV phase ~10^{-31} e·cm, 10^{-5} of the current experimental limit: 0.0(1.1)(0.2)×10-26 e·cm

C. Abel et al., PRL124:081803 (2020)

- ✦ Experiments are aiming at improving the limit down to 10^{-28} e·cm in the next ~10 years (?)

- ✦ Still a long way to trek, room for the strong CPV and/or BSM physics

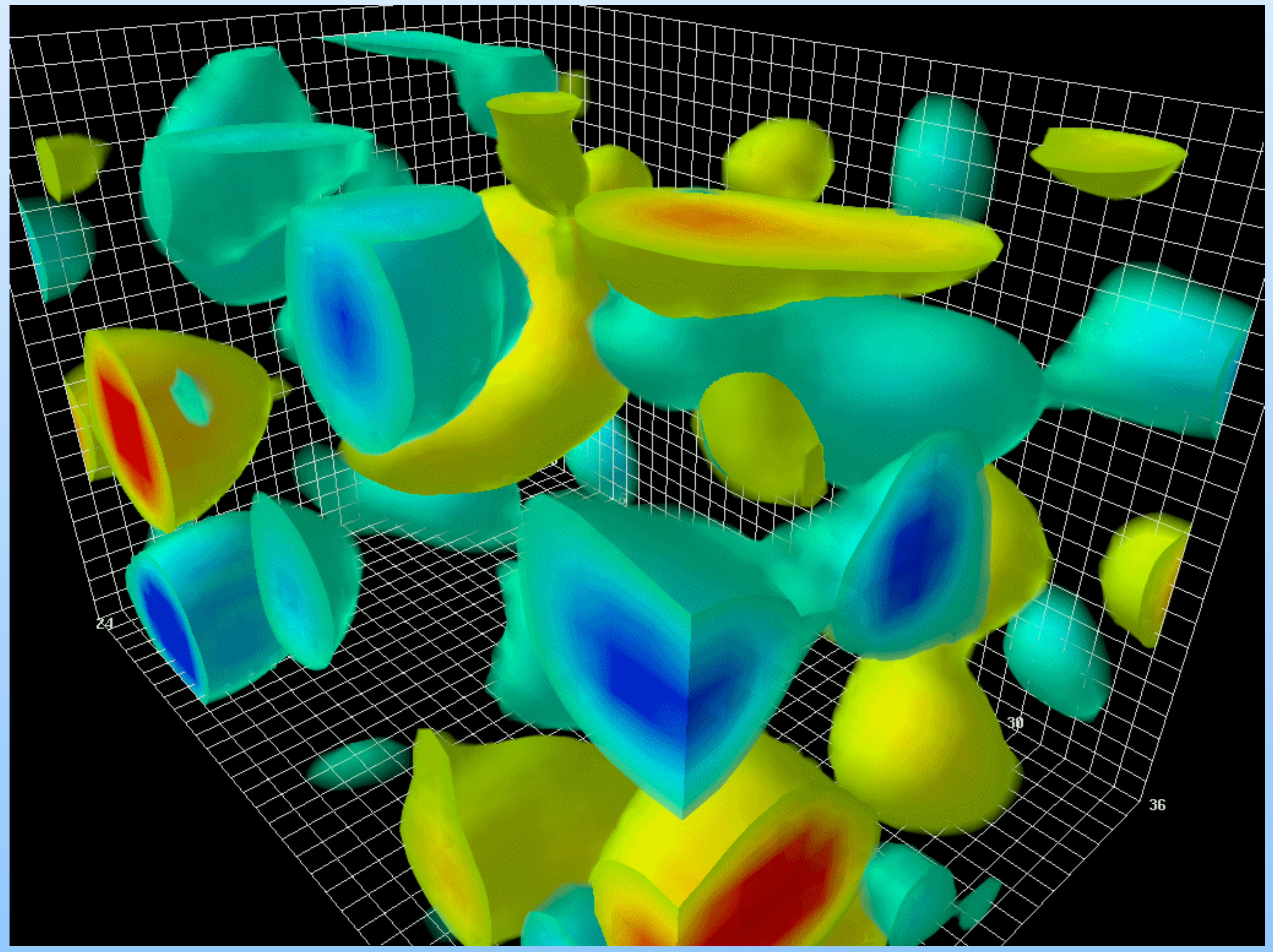
Strong CP-Violation

$$\mathscr{L}^E + \mathscr{L}^E_\theta = \bar{\psi}\left(\not{D}^E + m\right)\psi + \frac{1}{2}\mathrm{Tr}\left[F^E_{\mu\nu}F^E_{\mu\nu}\right] - i\bar{\theta}\frac{g^2}{16\pi^2}\mathrm{Tr}\left[F^E_{\mu\nu}\tilde{F}^E_{\mu\nu}\right]$$

- ✦ Additional term (the θ-term) can be introduced into the QCD Lagrangian
- ✦ Related to the phase of quark mass matrix by chiral rotations
- ✦ Contributes as a total divergence (surface term) to the QCD action
  - not affecting the classical equations of motion
  - not participating in perturbative expansions
  - influencing the QCD vacuum structure, non-perturbative observable CP-violating effects
- ✦ No direct experimental evidence of strong CP violation: $\bar{\theta}$ is either extremely small or strictly zero
  - naturalness issue known as the "fine-tuning problem" or the strong CP problem

Topological Charges

$$\mathcal{L}_\theta^E = -i\bar{\theta}\frac{g^2}{16\pi^2}\mathrm{Tr}\left[F_{\mu\nu}^E \tilde{F}_{\mu\nu}^E\right] \equiv -i\bar{\theta}q(x) \qquad Q = \int d^4x q(x)$$

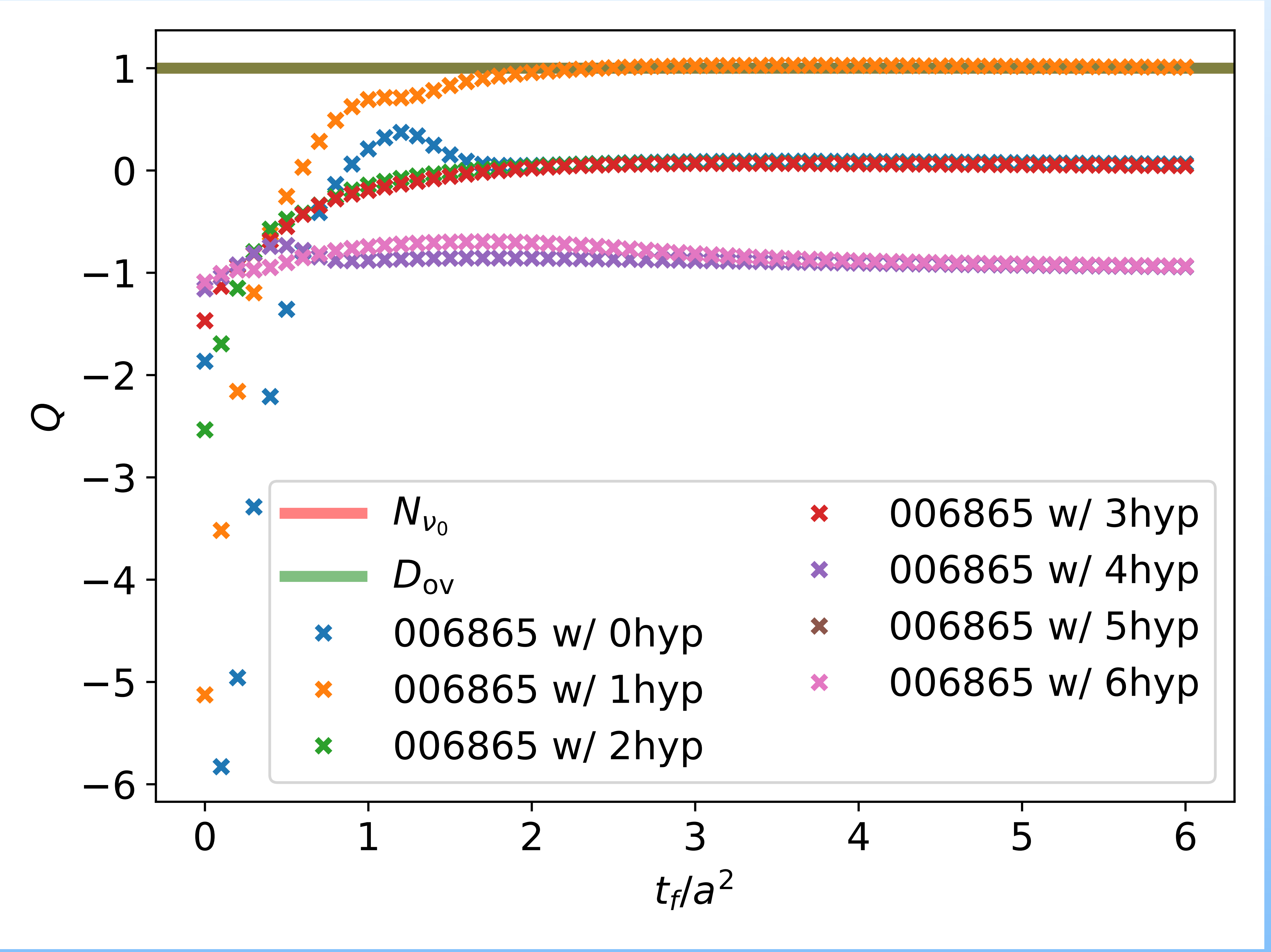


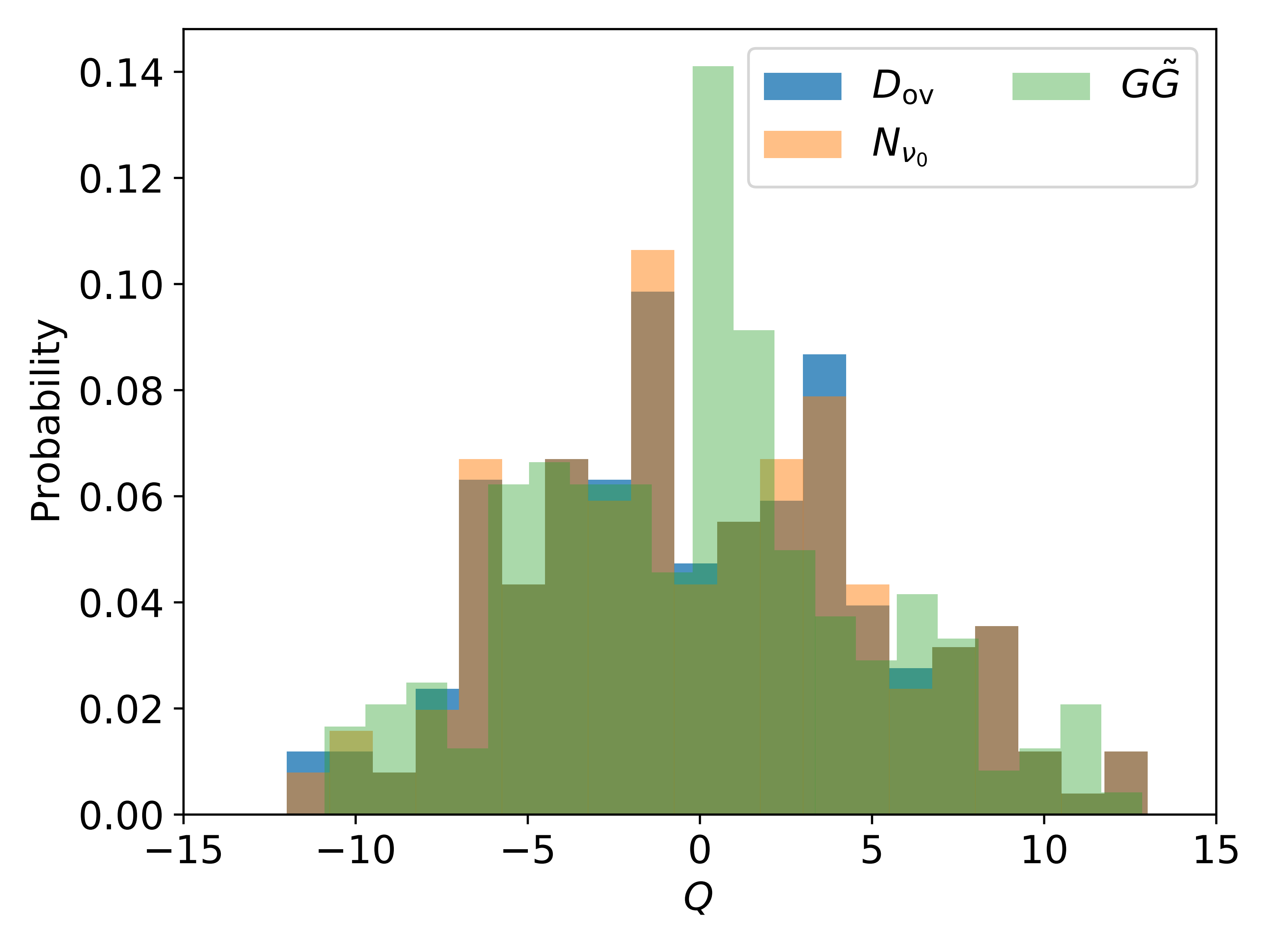
Leinweber/VisualQCD

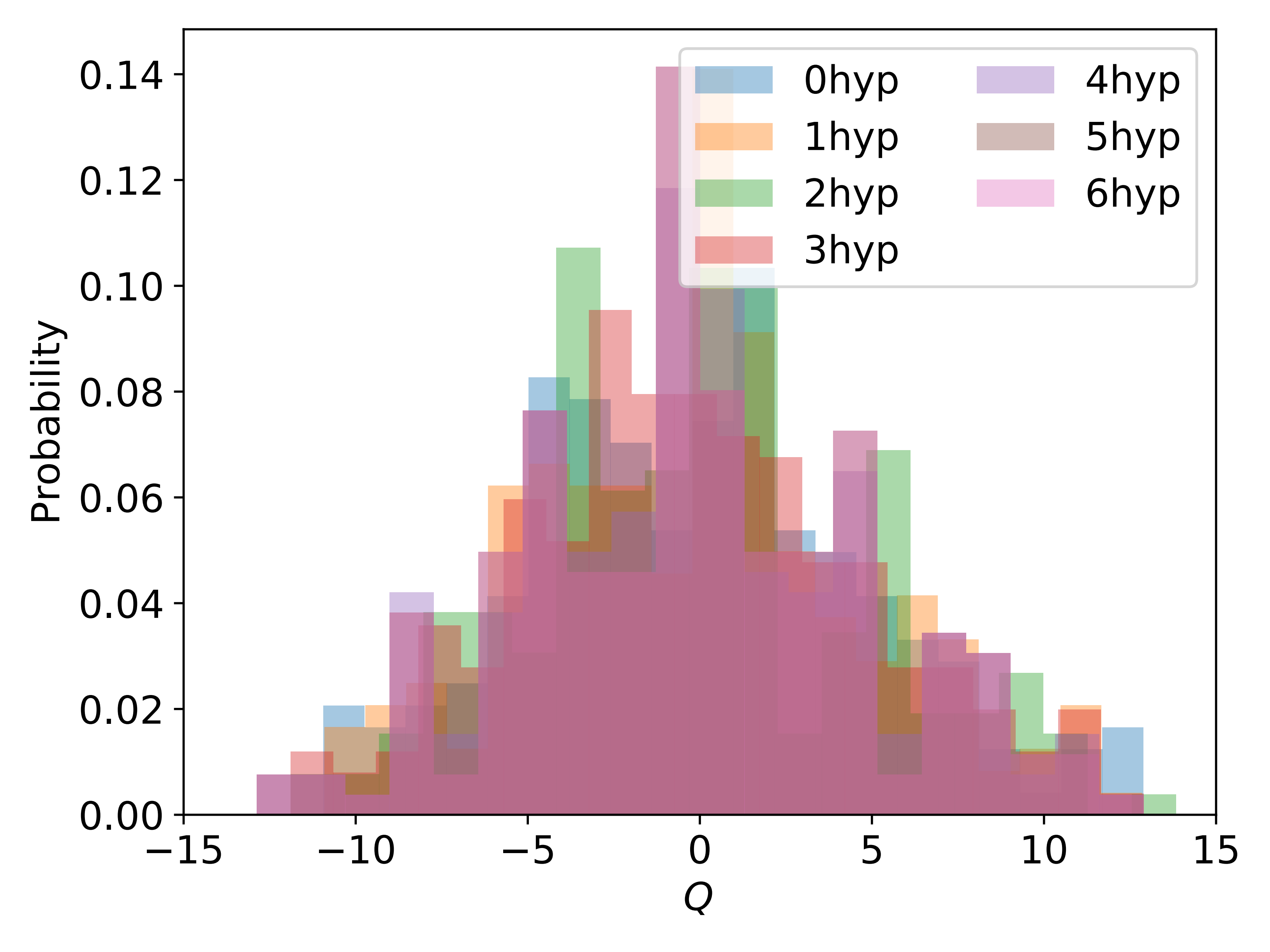
- ✦ Q is the topological charge of the gauge field
- ✦ Q should be integer

$$Q = n_- - n_+ \qquad q(x) = \frac{1}{2}\mathrm{Tr}\left[\gamma_5 D\right] = -\mathrm{Tr}\left[\gamma_5\left(1-\frac{D}{2}\right)\right]$$

Topological Charges on the Lattice



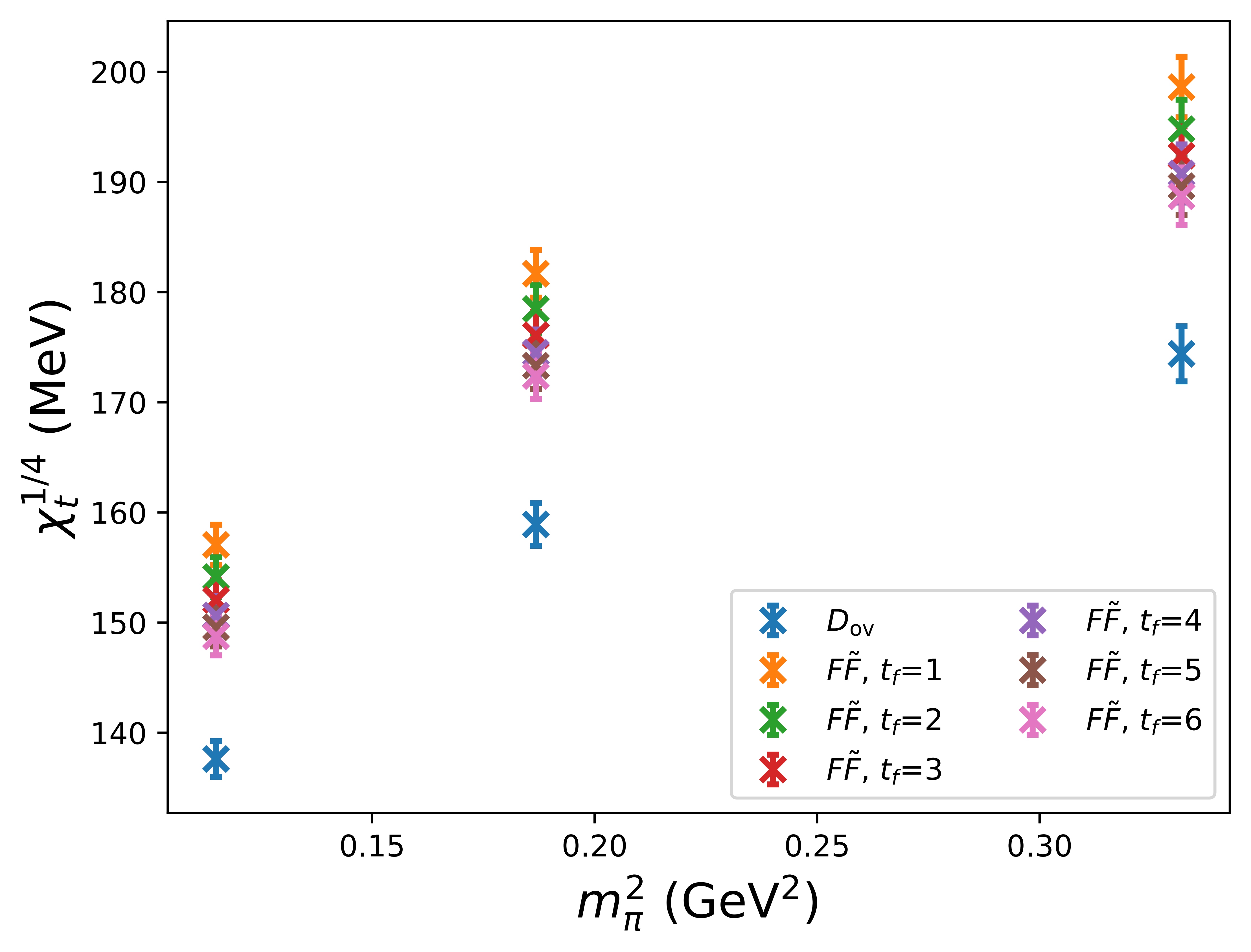




- ✦ The topological charges of individual configurations with different definitions (smearings) can be different, while the distributions are similar

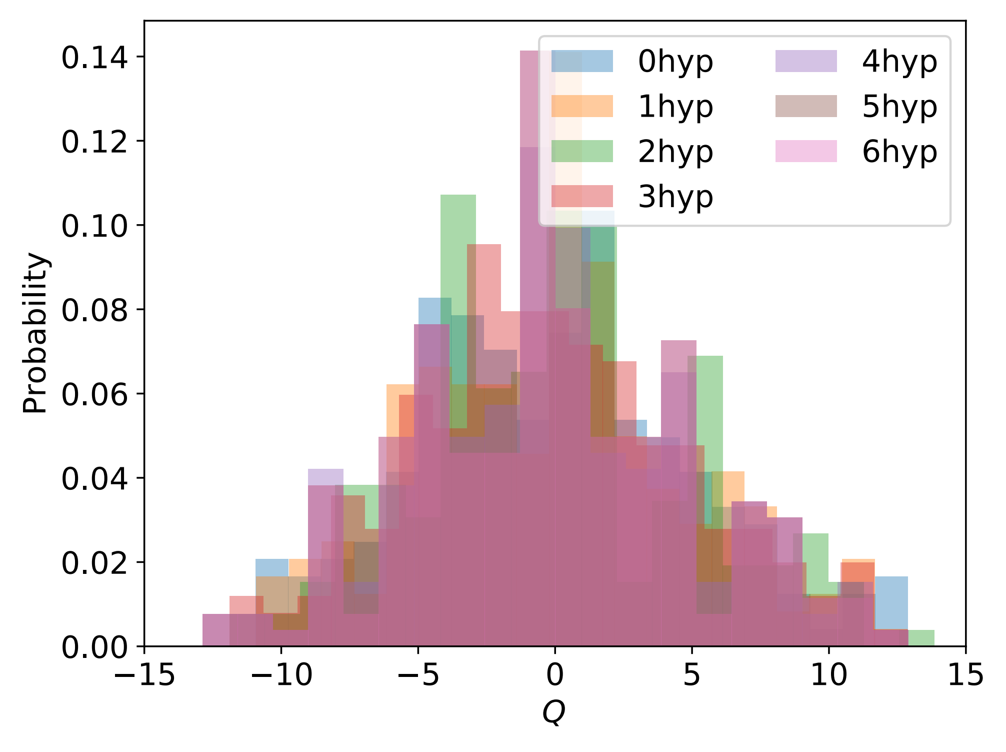
Topological Charges on the Lattice

$$\chi_t = \frac{1}{V}\langle Q^2 \rangle$$



✦ For physical observables such as the topological susceptibility, the $D_{\rm ov}$ definition has smaller discrete effects than the $G\tilde{G}$ definition with Wilson flow

Strong CP-Violation on the Lattice



$$\langle T[\hat{O}_1\hat{O}_2,\ldots]\rangle = \frac{1}{Z}\int \mathscr{D}\psi\mathscr{D}\bar{\psi}\mathscr{D}A_\mu O_1(\psi,\bar{\psi},A_\mu)O_2(\psi,\bar{\psi},A_\mu)\ldots e^{i\mathcal{S}_{QCD}}$$

$$\langle T[\hat{O}_1\hat{O}_2,\ldots]\rangle^E = \frac{1}{Z^E}\int \mathscr{D}\psi\mathscr{D}\bar{\psi}\mathscr{D}A_\mu O_1(\psi,\bar{\psi},A_\mu)O_2(\psi,\bar{\psi},A_\mu)\ldots e^{-\mathcal{S}^E_{QCD}}$$

$$\int \mathscr{D}\psi = \prod_x \int d\psi(x) \qquad \int \mathscr{D}\psi = \prod_{n\in L} \int d\psi(n)$$

$$\langle T[\hat{O}_1\hat{O}_2,\ldots]\rangle^E_\theta \sim \int \mathscr{D}\ldots O_1O_2\ldots e^{-\mathcal{S}^E_{QCD}+i\bar{\theta}Q} \sim \int \mathscr{D}\ldots O_1O_2\ldots(1+i\bar{\theta}Q+\ldots)e^{-\mathcal{S}^E_{QCD}}$$

$$\langle T[\hat{O}_1\hat{O}_2,\ldots]\rangle^E_\theta \sim \langle T[\hat{O}_1\hat{O}_2,\ldots]\rangle^E + i\bar{\theta}\langle T[\hat{O}_1\hat{O}_2,\ldots]Q\rangle^E$$

- ✦ Sign problem and the "signal-to-noise" problem are essentially the same one
- ✦ Operators or vacuum (action)? different points of views

Nucleon EDM from Lattice QCD

			m_π [MeV]	m_N [GeV]	F_2	α	$\tilde{F}_3$	F_3
PRD93:074503 (2016)	[10]	n	373	1.216(4)	−1.50(16)[a]	−0.217(18)	−0.555(74)	0.094(74)
PRD72:014504 (2005)	[5]	n	530	1.334(8)	−0.560(40)	−0.247(17)[b]	−0.325(68)	−0.048(68)
		p	530	1.334(8)	0.399(37)	−0.247(17)[b]	0.284(81)	0.087(81)
PRD73:054509 (2006)	[6]	n	690	1.575(9)	−1.715(46)	−0.070(20)	−1.39(1.52)	−1.15(1.52)
		n	605	1.470(9)	−1.698(68)	−0.160(20)	0.60(2.98)	1.14(2.98)
PRL115:062001 (2015)	[8]	n	465	1.246(7)	−1.491(22)[c]	−0.079(27)[d]	−0.375(48)	−0.130(76)[d]
		n	360	1.138(13)	−1.473(37)[c]	−0.092(14)[d]	−0.248(29)	0.020(58)[d]

Watershed: *Abramczyk et al., PRD96:014501 (**2017**)* $F_3 = \tilde{F}_3 + 2\alpha^1 F_2$

		Neutron $\bar{\Theta}$ e · fm	Proton $\bar{\Theta}$ e · fm
	This Work	$d_n = -0.003(7)(20)$	$d_p = 0.024(10)(30)$
	This Work with $N\pi$	$d_n = -0.028(18)(54)$	$d_p = 0.068(25)(120)$
PRD103:054501 (2021)	ETMC [66]	$\|d_n\| = 0.0009(24)$	–
PRC103:015202 (2021)	Dragos et al. [44]	$d_n = -0.00152(71)$	$d_p = 0.0011(10)$
arXiv:1901.05455	Syritsyn et al. [67]	$d_n \approx 0.001$	–

T. Bhattacharya et. at., PRD103:114507 (2021)

Detailed Formulas

$$u^\theta = e^{i\alpha^1\theta\gamma_5}u$$

$$\bar{u}^\theta = \bar{u}e^{i\alpha^1\theta\gamma_5}$$

$$u^\theta(p)\bar{u}^\theta(p) = \frac{-i\not{p} + me^{i2\alpha^1\gamma_5\theta}}{2m}$$

$$\langle N'|\gamma_\mu|N\rangle^\theta = \bar{u}^\theta(p')\left[\gamma_\mu F_1(q^2) - \sigma_{\mu\nu}q_\nu\frac{F_2(q^2)}{2m} - \sigma_{\mu\nu}q_\nu\gamma_5\frac{F_3'(q^2)}{2m}\right]u^\theta(p)$$

$$u^\theta(p) \to u^\theta(\tilde{p}) = e^{i\alpha_1\theta\gamma_5}\gamma_4 u(p) = (1 + i\alpha_1\theta\gamma_5)\gamma_4 u$$

$$i\theta F_3 = 2i\alpha^1\theta F_2 + i\theta F_3' = i\theta(2\alpha^1 F_2 + F_3')$$

Detailed Formulas

$$d_{n/p} = \frac{F_{3,n/p}(q^2 \to 0)}{2m}\theta$$

$$\langle N' | \gamma_\mu | N \rangle^\theta = M_\mu^{(3)} + i\theta M_\mu^{(3)Q}$$

$$\begin{aligned} M^{(2)} &= \langle N(p) | N(p) \rangle, \\ M_\mu^{(3)} &= \langle N(p_f) | V_\mu | N(p_i) \rangle, \\ M^{(2)Q} &= \langle N(p) | Q_t | N(p) \rangle, \\ M_\mu^{(3)Q} &= \langle N(p_f) | Q_t V_\mu | N(p_i) \rangle, \end{aligned}$$

$$F_3(q^2) = \frac{2m}{E_f + m} \left\{ \frac{2E_f}{q_i} \frac{\mathrm{Tr}[\Gamma_i M_4^{(3)Q}]}{\mathrm{Tr}[\Gamma_e M^{(2)}]} - \alpha^1 G_E(q^2) \right\},$$

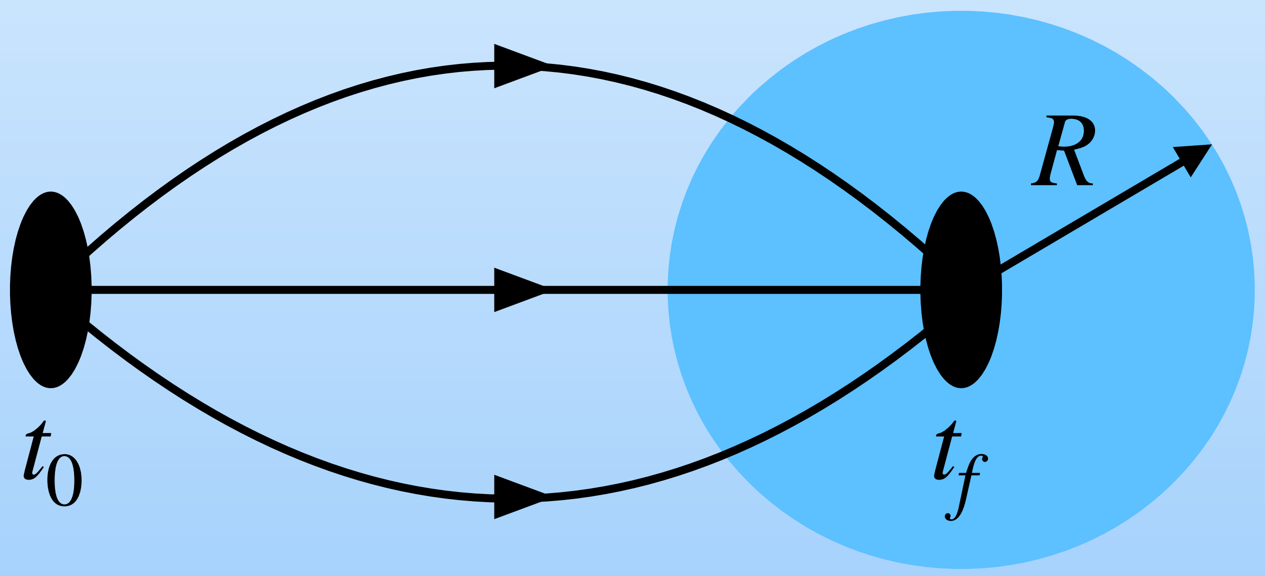
$$G_E(q^2) = \frac{2E_f}{E_f + m} \frac{\mathrm{Tr}[\Gamma_e M_4^{(3)}]}{\mathrm{Tr}[\Gamma_e M^{(2)}]}, \quad \alpha^1 = \frac{\mathrm{Tr}[\gamma_5 M^{(2)Q}]}{2\mathrm{Tr}[\Gamma_e M^{(2)}]},$$

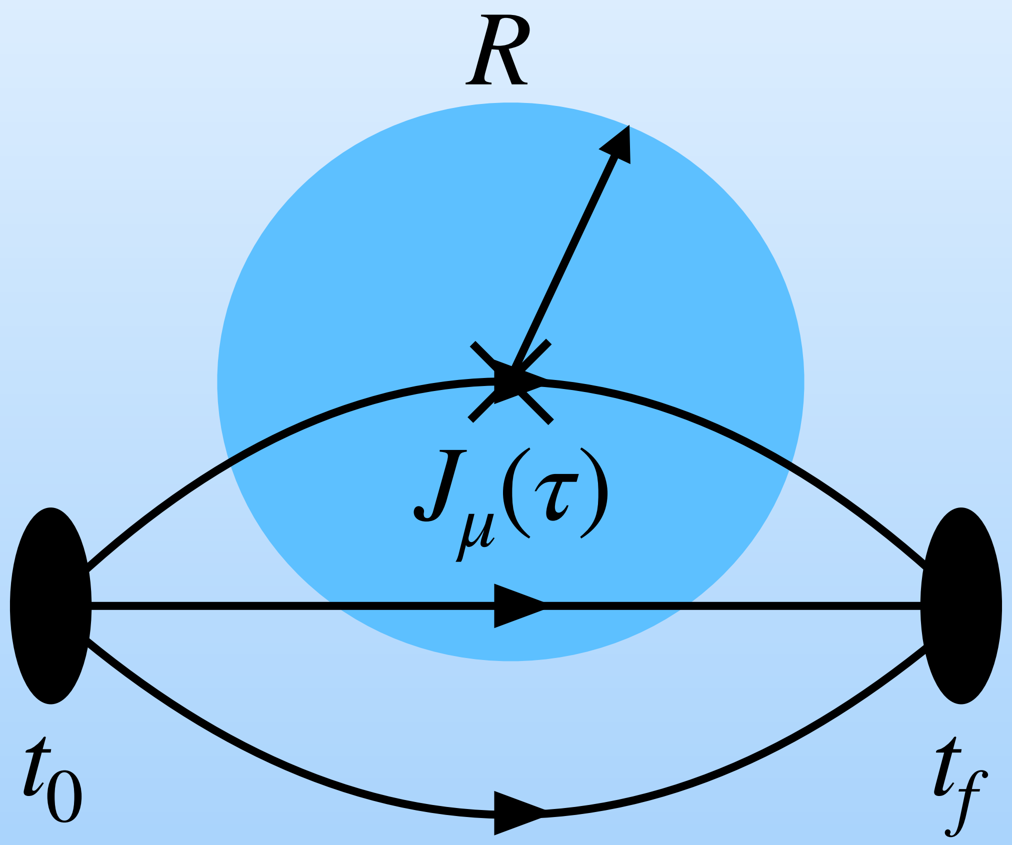
$$\begin{aligned} G^{(2)Q}(t_f) &\sim \sum_{\vec{x}} \left\langle Q\, \chi(x) \bar{\chi}(t_0, \mathcal{G}) \right\rangle \\ &\sim M^{(2)Q} + \mathcal{O}(e^{-\delta m t_f}) \end{aligned}$$

$$\begin{aligned} G^{(3)Q} &\sim \sum_{\vec{x}\vec{y}} e^{-i\vec{q}(\vec{x}-\vec{y})} \left\langle \chi(x)\ Q\ J_\mu(y) \bar{\chi}(t_0, \mathcal{G}) \right\rangle \\ &\sim M^{(3)Q} + \mathcal{O}\left(e^{-\delta m(t_c - t_0)}, e^{-\delta E(\vec{q})(t_f - t_c)}\right), \end{aligned}$$

CDER

✦ cluster decomposition error reduction (CDER)





$$G^{(2)Q}(t_f) \sim \sum_{\vec{x}} \langle Q\, \chi(x)\bar{\chi}(t_0, \mathcal{G}) \rangle$$
$$\sim \sum_{\vec{x}} \left\langle \sum_{r}^{|r|<R} q_t(x+r)\chi(x)\bar{\chi}(t_0, \mathcal{G}) \right\rangle$$
$$\sim M^{(2)Q} + \mathcal{O}(e^{-\delta m t_f}, e^{-m_\eta R})$$

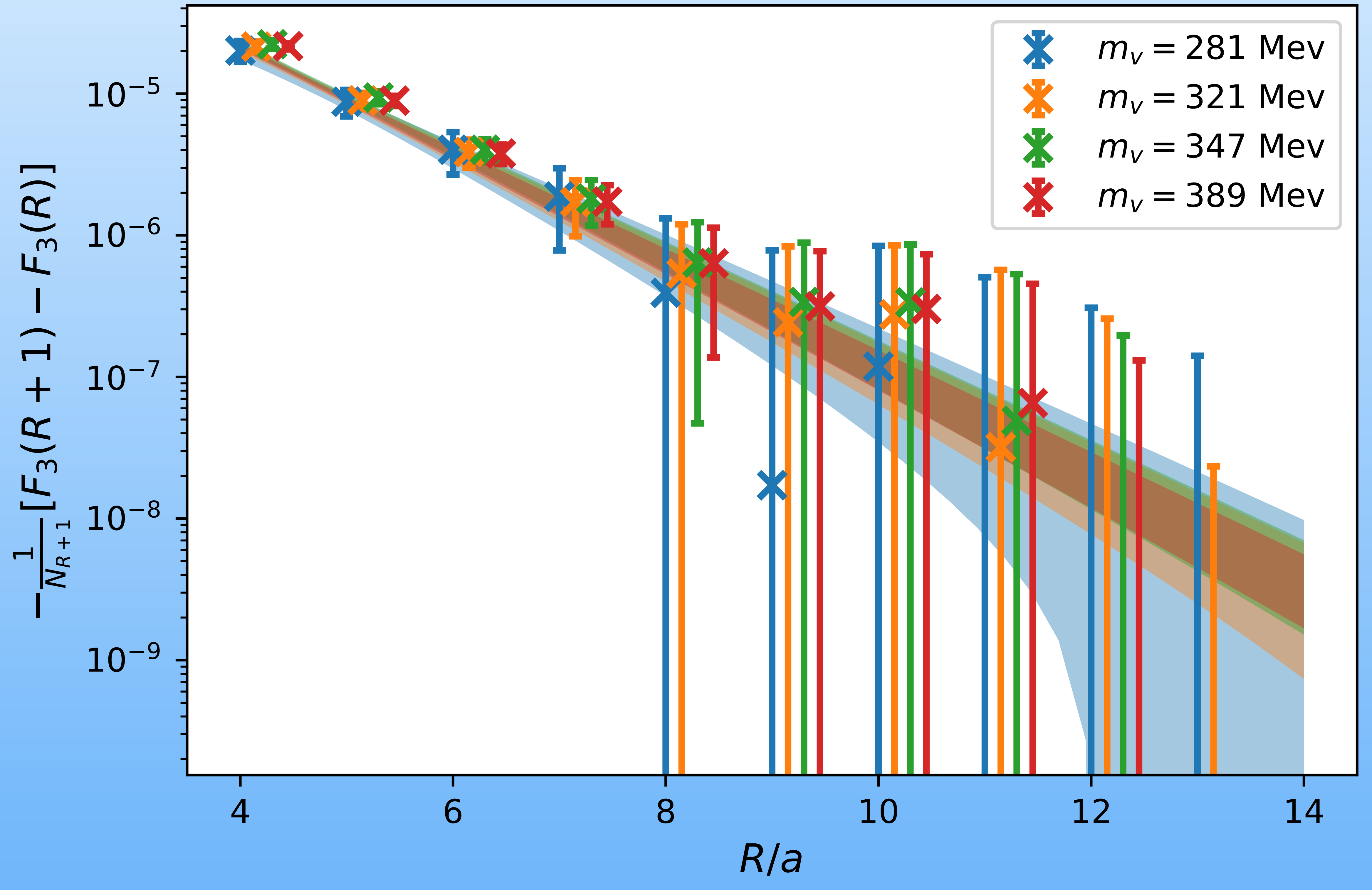
$$G^{(3)Q} \sim \sum_{\vec{x}\vec{y}} e^{-i\vec{q}(\vec{x}-\vec{y})} \left\langle \chi(x)\; Q\; J_\mu(y)\bar{\chi}(t_0, \mathcal{G}) \right\rangle$$
$$\sim \sum_{\vec{x}\vec{y}} e^{-i\vec{q}(\vec{x}-\vec{y})} \left\langle \chi(x) \sum_{r}^{|r|<R} q_t(y+r) J_\mu(y)\bar{\chi}(t_0, \mathcal{G}) \right\rangle$$
$$\sim M^{(3)Q} + \mathcal{O}\left(e^{-\delta m(t_c-t_0)}, e^{-\delta E(\vec{q})(t_f-t_c)}, e^{-m_\eta R}\right)$$

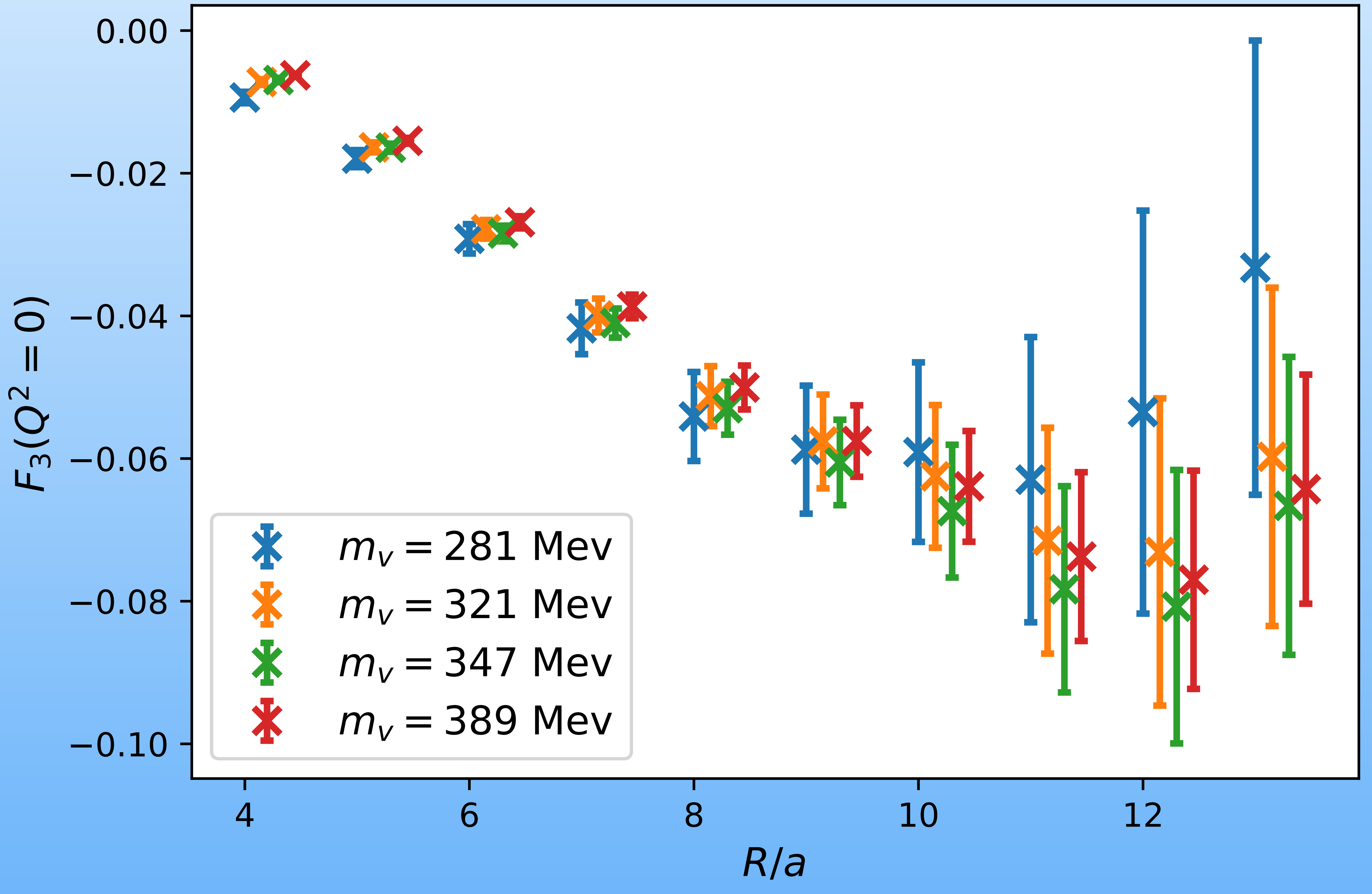
*Liu, **Liang** and Yang, PRD97:034507 (2018)*

CDER

$$f'(r) \sim \left\langle \chi(x) q_t(y+r) J_\mu(y) \bar{\chi}(t_0, \mathcal{G}) \right\rangle$$

$$f(R) \sim \left\langle \chi(x) \sum_{r}^{|r|<R} q_t(y+r) J_\mu(y) \bar{\chi}(t_0, \mathcal{G}) \right\rangle$$





Liang *and et. al., PRD108 094512 (2023)*

Chiral Fermions

$$\partial_\mu A_\mu^0 = \sum_{f=u,d,s} 2m_f P_f - 2iN_f q$$

For overlap fermions, the anomalous Ward identity (AMI) has been proven (with chiral axial vector current) and numerically checked at finite lattice spacings

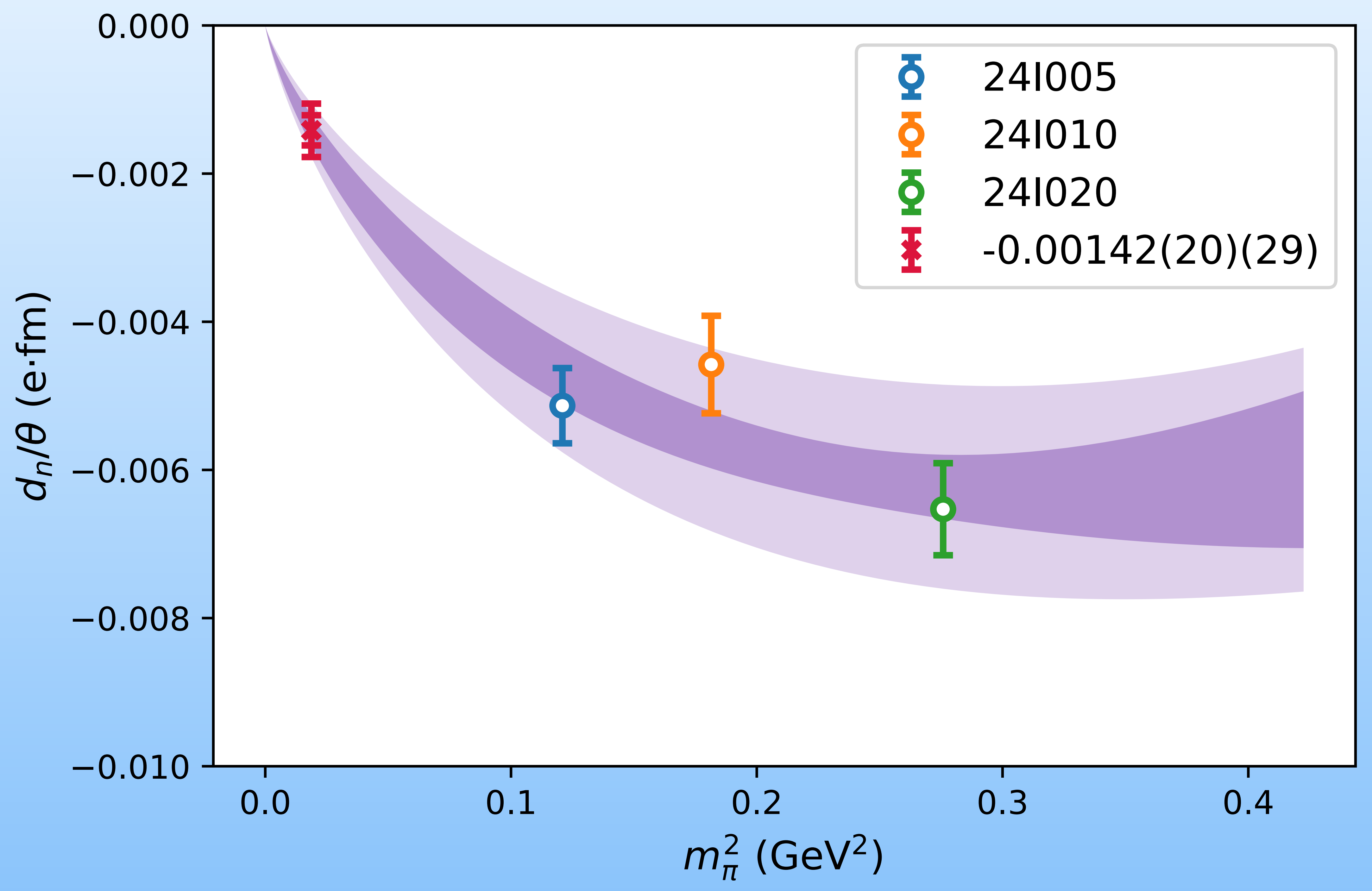
P. Hasenfratz, et. al., NPB643:280 (2002)

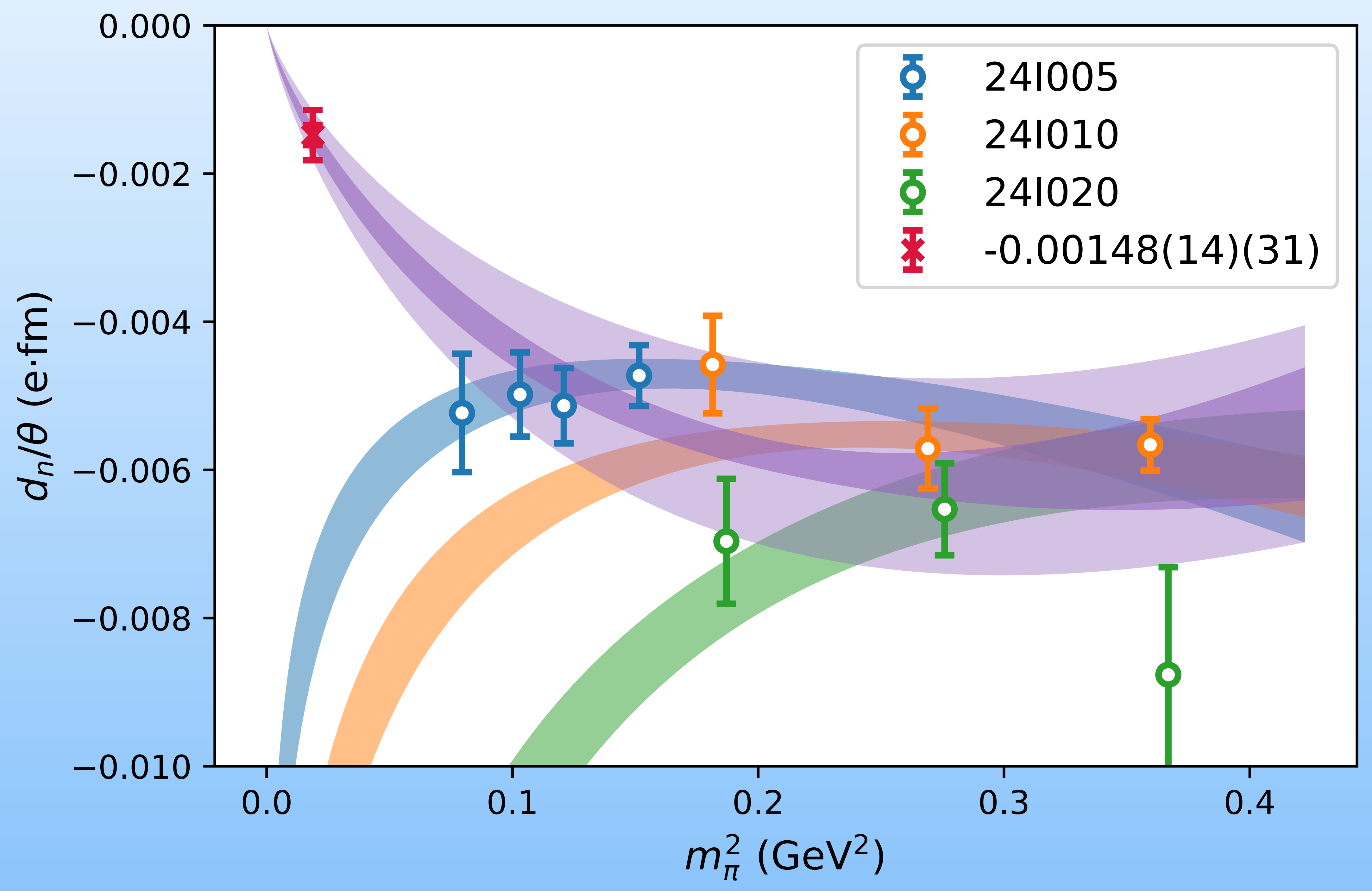
J. Liang *et. al., PRD98:074505 (2018)*

With the AWI respected, one has $d_n \to 0$ when $m_q \to 0$ even at finite lattice spacings, reliable chiral limit

D. Guadagnoli, et. al., JHEP 0304, 019 (2003)

Nucleon EDM from Lattice QCD



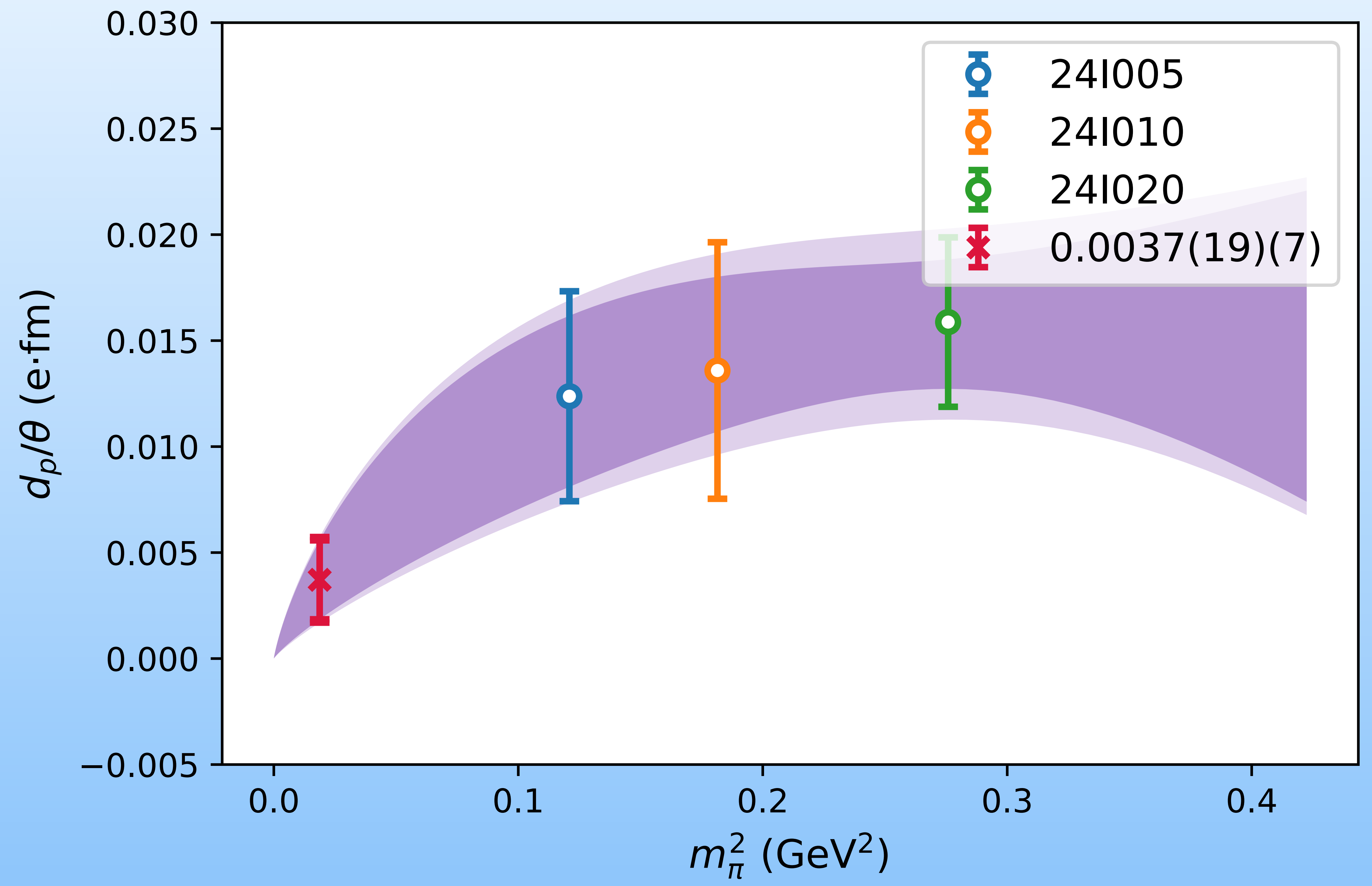


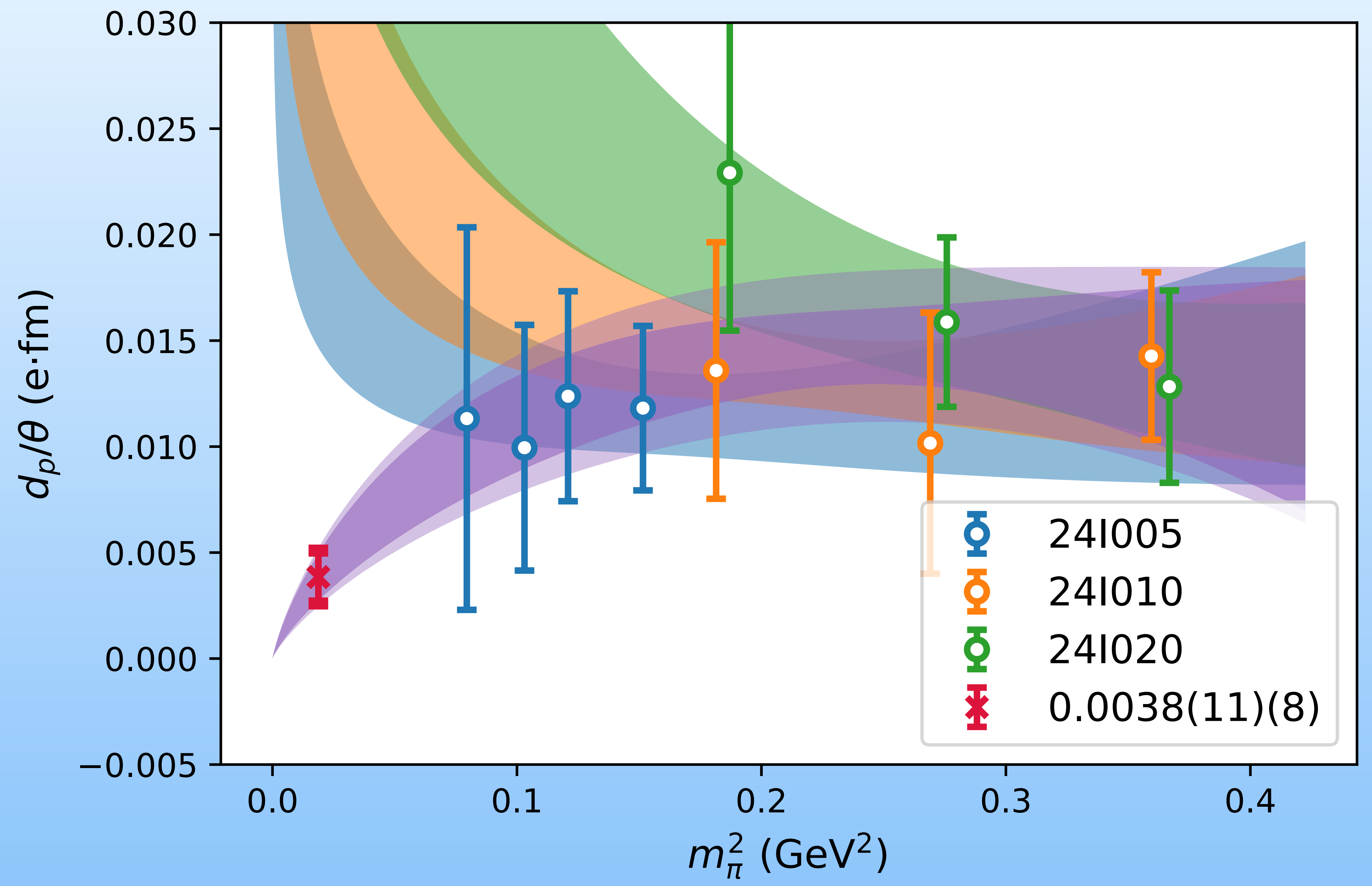
***Liang** and et. al., PRD108 094512 (2023)*

$$d_n^{(PQ)} = \frac{e\,\bar{\theta}\,m_s}{4\pi^2 f^2}\left[F_\pi \log\left(\frac{m_\pi^2}{\mu^2}\right) + F_J \log\left(\frac{m_J^2}{\mu^2}\right)\right] + \bar{\theta}\,\frac{e}{\Lambda_\chi^2}\left[\frac{m_s}{2}\,c(\mu) + \underline{d\,(m_s - m_v)} + f q_{jl}\,\underline{(m_s - m_v)}\right]$$

D.O'Connell and M. J. Savage, PLB633:319 (2006)

Nucleon EDM from Lattice QCD





Liang *and et. al., PRD108 094512 (2023)*

$$d_n^{(PQ)} = \frac{e\,\bar{\theta}\,m_s}{4\pi^2 f^2}\left[F_\pi \log\left(\frac{m_\pi^2}{\mu^2}\right) + F_J \log\left(\frac{m_J^2}{\mu^2}\right)\right] + \bar{\theta}\,\frac{e}{\Lambda_\chi^2}\left[\frac{m_s}{2}\,c(\mu) + \underline{d\,(m_s - m_v)} + f q_{jl}\,\underline{(m_s - m_v)}\right]$$

D.O'Connell and M. J. Savage, PLB633:319 (2006)

CPV Pion-Nucleon Coupling

$$D_n = g_{\pi \mathrm{NN}} \bar{g}_{\pi \mathrm{NN}} \ln \left(M_N / m_\pi \right) / 4\pi^2 M_N$$

Crewther, Di Vecchia, Veneziano, Witten, PLB 88:123 (1979)

$$d_0^\theta = \bar{d}_0^\theta - \frac{e g_A \bar{g}_0}{16\pi F_\pi} \frac{3 m_\pi}{M_N},$$

$$d_1^\theta = \bar{d}_1^\theta(\mu) + \delta_1(\mu) + \frac{e g_A \bar{g}_0}{4\pi^2 F_\pi} \left[\ln \frac{m_\pi^2}{m_N^2} - \frac{5\pi m_\pi}{4 m_N} \right]$$

A. Shindler, EPJA57: 128 (2021)

- ✦ In effective theories, nucleon EDM is determined by the CPV pion-nucleon coupling
- ✦ The CPV nucleon-nucleon interactions, mainly the one-pion-exchange contributions (CPV pion-nucleon vertices) according to the chiral power counting, often dominate nuclear and atomic EDMs compared to the EDMs of constituent nucleons
- ✦ No direct first-principles calculations

J. de Vries et al., PLB766:254 (2017)

J. de Vries et al., PRC92:045201 (2015)

CPV Pion-Nucleon Coupling

- To leading order the coupling is related to the octet baryon masses

$$\bar{g}_{\pi \mathrm{NN}} = -\theta \frac{(M_{\Xi} - M_N) m_u m_d}{F_\pi (m_u + m_d)(2m_s - m_u - m_d)} \qquad |\bar{g}_{\pi \mathrm{NN}}| \approx 0.038\,|\theta|$$

Crewther, Di Vecchia, Veneziano, Witten, PLB 88:123 (1979)

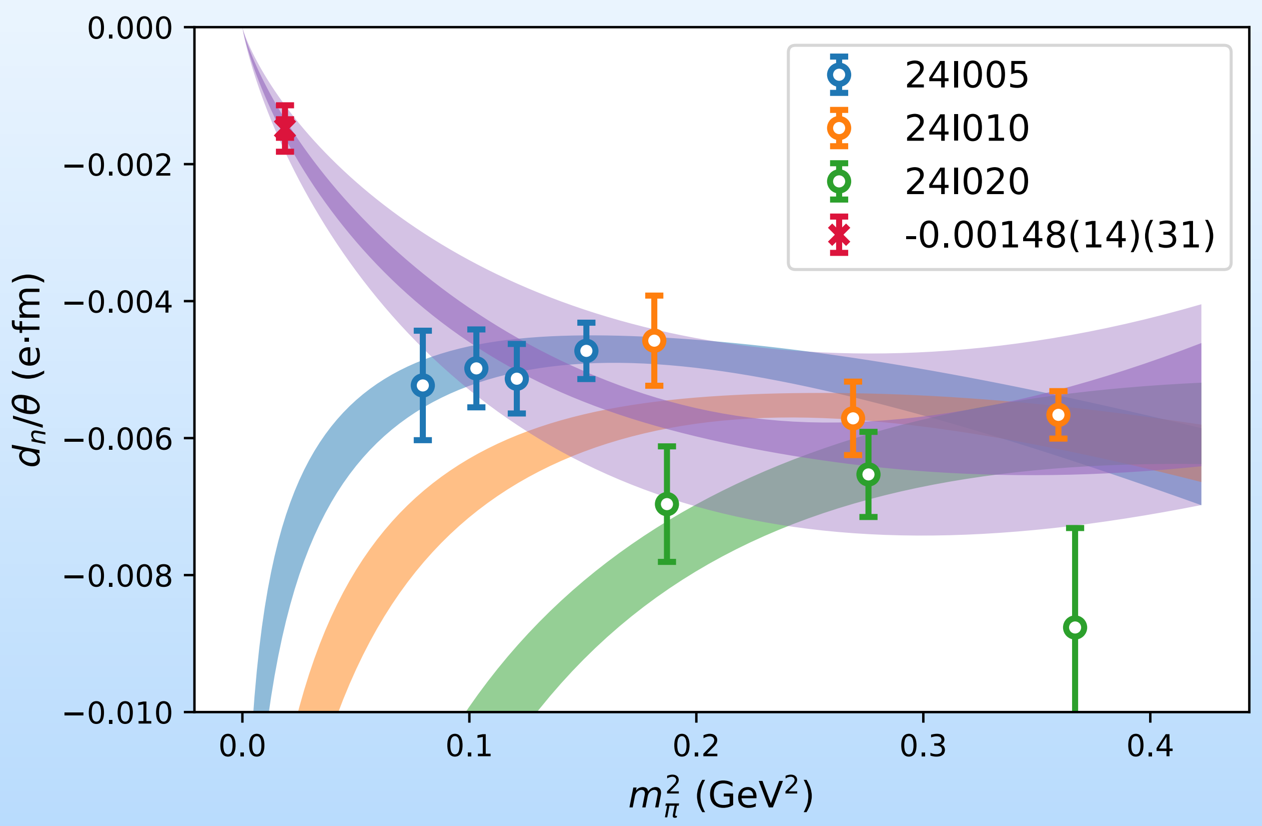
- A more recent work shows that this coupling can be achieved using the proton-neutron mass difference, which receives no SU(3)-flavor breaking corrections up to NNLO in chiral expansion

$$\bar{g}_0 = \delta m_N \frac{m_* \bar{\theta}}{\bar{m}\varepsilon} \qquad m_* = \frac{m_u m_d m_s}{m_s(m_u + m_d) + m_u m_d} = \frac{\bar{m}(1-\varepsilon^2)}{2 + \frac{\bar{m}}{m_s}(1-\varepsilon^2)} \qquad \frac{\bar{g}_0}{2F_\pi} = (15.5 \pm 2.0 \pm 1.6) \times 10^{-3} \bar{\theta}$$

$$2\bar{m} = m_u + m_d \qquad \varepsilon = (m_d - m_u)/(m_d + m_u)$$

J. de Vries et al., PRC92:045201 (2015)

A Rough Check



$$d_n^{(PQ)} \sim \frac{e\,\bar{\theta}\,m_s}{4\pi^2 f^2}\left[F_\pi \log\left(\frac{m_\pi^2}{\mu^2}\right) + F_J \log\left(\frac{m_J^2}{\mu^2}\right)\right] + \ldots$$

$$d_n \sim A \times m_{\text{sea}} \log\frac{m_\pi^2}{\Lambda^2} + \ldots \sim A' \times m_{\pi,\text{sea}}^2 \log\frac{m_\pi^2}{\Lambda^2} + \ldots$$

$$d_n \sim \frac{g_A \bar{g}_0}{4\pi^2 f} \log\frac{m_\pi^2}{\Lambda^2} + \ldots$$

$$\bar{g}_0(m_\pi \sim 0.140) = \sim 0.0061$$

Detailed Formulas of Direct Calculation

$$G^{(3)Q}(t_f,\tau) = \sum_{\vec{x}\vec{y}} e^{-i\vec{p}\cdot\vec{x}} e^{i\vec{q}\cdot\vec{y}} \left\langle \chi(x)\ Q\ \cancel{J_\mu(y)}\ P(y)\ \bar{\chi}(0) \right\rangle$$

$$R_3(\Gamma_e,\gamma_5) = \frac{\mathrm{Tr}\left[\Gamma_e G^{(3)Q}(\gamma_5)\right]}{\mathrm{Tr}\left[\Gamma_e G^{(2)}\left(\vec{p}=0\right)\right]}$$

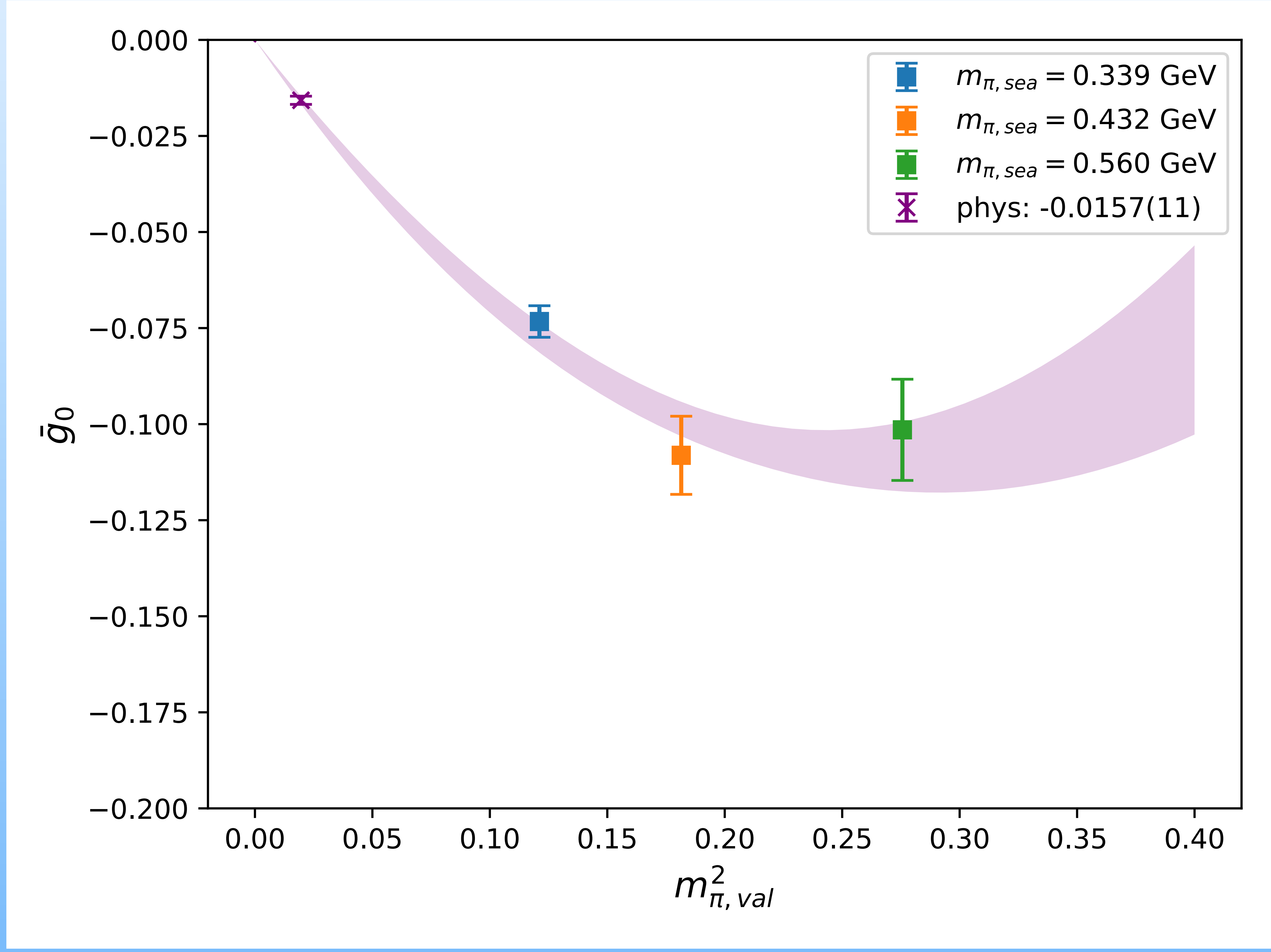
$$= \frac{\mathrm{Tr}\left[\frac{1+\gamma_4}{2}\left[\alpha^1\gamma_5 G_p \gamma_5 \frac{1+\gamma_4}{2} + \frac{1+\gamma_4}{2} G_p \gamma_5 \alpha^1 \gamma_5 + \frac{1+\gamma_4}{2}\tilde{G}'_P \frac{1+\gamma_4}{2}\right]\right]}{2}$$

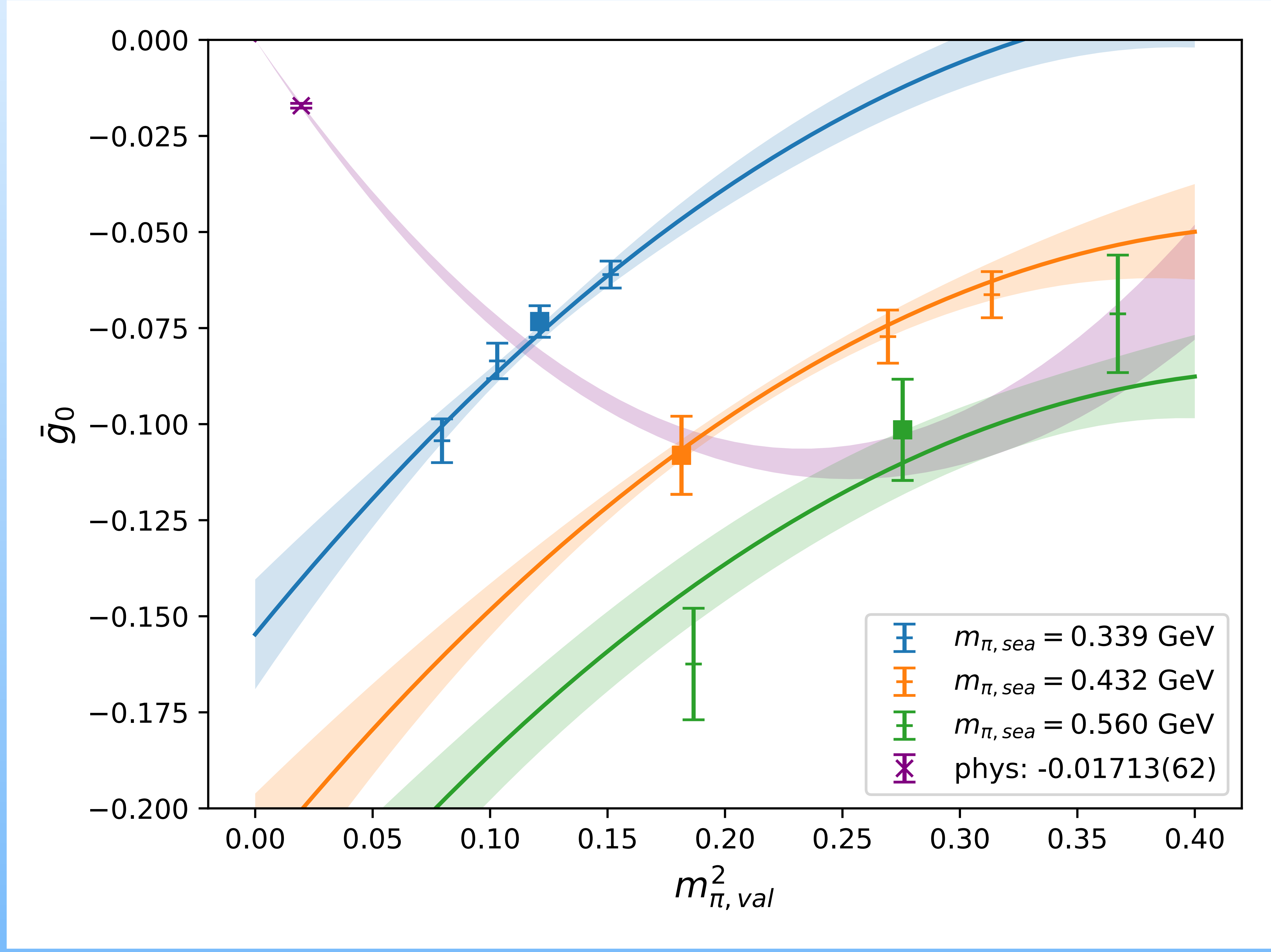
$$= 2\alpha^1 G_p + \tilde{G}'_p$$

$$2i\alpha_1\theta G_p + i\theta\tilde{G}'_P = i\theta\left(2\alpha_1 G_P + \tilde{G}'_P\right) \equiv i\theta\tilde{G}_P$$

$$m_q\tilde{G}_P = \frac{m_\pi^2 f_\pi}{m_\pi^2 - q^2}\bar{g}_{\pi NN}$$

$\bar{g}_{\pi NN}$ from Direct Calculation (preliminary)





Liang *and et. al., in preparation*

Summary and Take-Home

- ✦ Strong CPV and nucleon EDM can be studied on the lattice with decent precision
- ✦ Many other related physical topics are also of great interest and can be studied as well

Thank you for your attention!