Nucleon Structure from Basis Light-Front Quantization : Status and Prospects

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100 YEARS SPIN 2025

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A Century of Spin

BLFQ Collaboration

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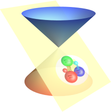


Qingdao, China, September 23, 2025

Overview

Introduction

Basis Light-Front Quantization (BLFQ)

Properties of proton

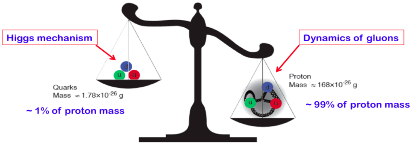
Conclusions

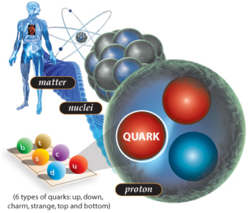
(Satvir Kaur, 9/23/25, 2:25 PM : Hidden-Color Effects in Deuteron Structure)

(Xingbo Zhao, 9/24/25, 8:30 AM : Proton Spin Structure from a Light-Front Hamiltonian Approach)

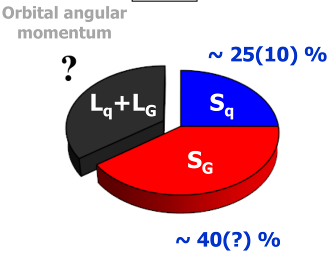
Fundamental Properties: Mass, Spin and Structure

Mass

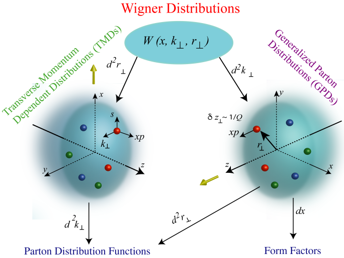




Spin



Structure

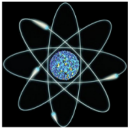


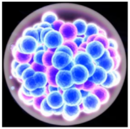
Ideal facilities : EIC & EicC

Nonperturbative Approaches

Lagrangian formalism	**Hamiltonian formalism**
Euclidean space-time	Minkowski space-time
correlators: $\langle \mathcal{O}(x_1, \cdots, x_n) \rangle$	wave functions: $\lvert \psi_h \rangle$
$\langle \mathcal{O} \rangle = \int \mathcal{D}_\psi \mathcal{O} \exp\Big(- S_E[\psi] \Big)$	$H \lvert \psi_h \rangle = E_h \lvert \psi_h \rangle$
—	$i \frac{\partial}{\partial t} \lvert \psi_h(t) \rangle = H \lvert \psi_h(t) \rangle$
Lattice QCD, Dyson-Schwinger, FRG	DLCQ, BLFQ, Tamm-Dancoff, RGPEP

Schrödinger Equation $H \lvert \psi \rangle = E \lvert \psi \rangle$



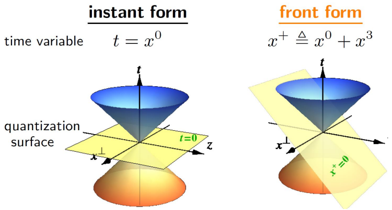


nonrelativistic	nonrelativistic	relativistic
few-body	many-body	many-body
quant, AMO	nucl, quant chem	hadron, QFT

[1] Adopted from Yang Li

Basis Light-Front Quantization (BLFQ)

A computational framework for solving relativistic many-body bound state problems in quantum field theories



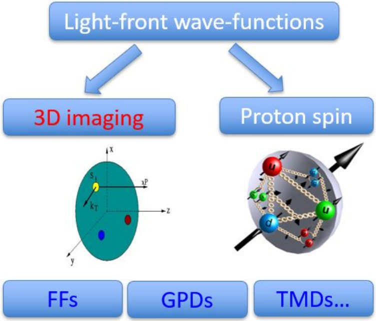
$$P^- P^+ |\Psi\rangle = M^2 |\Psi\rangle$$

- $P^\pm \equiv P^0 \pm P^3$: light-front momentum (Hamiltonian)
- First-principle / effective Hamiltonian as input
- Access to mass (M) and LFWFs

- Evaluate observables

$$O \sim \langle \Psi | \hat{O} | \Psi \rangle$$

GOAL



[1] Vary, Honkanen, Li, Maris, Brodsky, Harindranath, *et. al.*, Phys. Rev. C 81, 035205 (2010).

- Fock expansion of baryonic bound states:

$$|\text{Proton}\rangle = \psi_{(3q)}|qqq\rangle + \psi_{(3q+1g)}|qqqg\rangle + \psi_{(3q+q\bar{q})}|qqqq\bar{q}\rangle + \dots ,$$

Solution proposed by BLFQ

Discrete basis and their direct product	**Truncation**
2D HO $\phi_{nm}(p^{\perp})$ in the transverse plane	$\sum_i (2n_i + \lvert m_i\rvert + 1) \leq N_{\max}$
Plane-wave in the longitudinal direction	$\sum_i k_i = K, \quad x_i = \frac{k_i}{K}$
Light-front helicity state for spin d.o.f.	$\sum_i (m_i + \lambda_i) = M_J$
$\alpha_i = (k_i, n_i, m_i, \lambda_i)$ $\lvert\alpha\rangle = \otimes_i \lvert\alpha_i\rangle$	Fock sector truncation

$$\Lambda_{\text{IR}} \sim \frac{b}{\sqrt{N_{\max}}} \quad \text{and} \quad \Lambda_{\text{UV}} \sim b\sqrt{N_{\max}}, \quad (b: \text{HO energy scale})$$

Large $N_{\max}$ and K $\rightarrow$ High UV cutoff & low IR cutoff

- Exact factorization between center-of-mass motion and intrinsic motion

[1] Vary, Honkanen, Li, Maris, Brodsky, Harindranath, *et. al.*, Phys. Rev. C 81, 035205 (2010).

BLFQ Road-map toward First Principles

Basis Light-Front Quantization (BLFQ)

2020

$|qqq\rangle$ → K.E. + Confinement + One Gluon Exchange interaction

- EMFFs, PDFs & Axial FFs →
  - PRD **102**, 016008 (2020)
  - PRD **104**, 094036 (2021)
- GPDs (Twist-2, -3) →
  - PRD **105**, 094018 (2022)
  - PRD **109**, 014015 (2024)
  - PRD **109**, 034031 (2024)
  - PLB **855**, 138809 (2024)
- TMDs →
  - PLB **833**, 137360 (2022)
  - PRD **109**, 014015 (2024)
- GFFs →
  - PRD **110**, 056027 (2024)
- DPDs →
  - PRD **112**, 014018 (2025)

2023

$|qqq\rangle + |qqqg\rangle$ → K.E. + Confinement in $|qqq\rangle$ + **QCD interactions (Instantaneous gluon interaction + quark-gluon vertex interaction)**

- PDFs & Spin Decomposition →
  - PRD **108**, 094002 (2023)
- GPDs (Twist-2, -3) →
  - PLB **847**, 138305 (2023)
  - PLB **860**, 139153 (2025)
  - PLB **866**, 139584 (2025)
- TMDs (Twist-2, -3) →
  - PLB **855**, 138831 (2024)
  - PLB **855**, 138829 (2024)
- GFFs & Mass Decomposition →
  - arXiv-hep: 2506.07554

2025

$|qqq\rangle + |qqqg\rangle + |qqqq\bar{q}\rangle$ → K.E. + **QCD interactions (Instantaneous gluon interaction + Instantaneous fermion interaction + quark-gluon vertex interaction)**

- EMFFs, PDFs & Spin Decomposition →
  - PLB **867**, 139599 (2025)

⋮

First Principles of LF QCD → K.E. + **All QCD interactions**

[0] BLFQ Collaboration

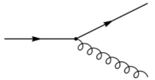
Proton with One Dynamical Gluon

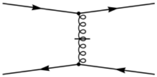
$$P^+P^-|\Psi\rangle = M^2|\Psi\rangle \qquad |\text{proton}\rangle = \psi_{uud}|uud\rangle + \psi_{uudg}|uudg\rangle$$

$$P^- = P^-_{\rm QCD} + P^-_C$$

QCD Interaction:

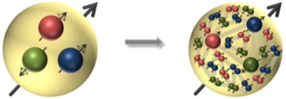
$$P^-_{\rm QCD} = \int dx^- d^2x^\perp \Big\{ \frac{1}{2}\bar{\psi}\gamma^+ \frac{m_0^2 + (i\partial^\perp)^2}{i\partial^+}\psi - \frac{1}{2}A_a^i\left[m_g^2 + (i\partial^\perp)^2\right]A_a^i + g_s\bar{\psi}\gamma_\mu T^a A_a^\mu \psi + \frac{1}{2}g_s^2\bar{\psi}\gamma^+ T^a\psi \frac{1}{(i\partial^+)^2}\bar{\psi}\gamma^+ T^a \psi \Big\},$$





Confinement only in leading Fock:

$$P_C^- P^+ = \frac{\kappa^4}{2}\sum_{i\neq j}\left\{ \vec{r}_{ij\perp}^{\,2} - \frac{\partial_{x_i}(x_i x_j \partial_{x_j})}{(m_i + m_j)^2}\right\}$$



Parameters:

Truncation: Nmax=9, K=16.5
HO parameters: b=0.7GeV, b_{inst}=3GeV

m_u	m_d	m_g	κ	m_f	g
0.31GeV	0.25GeV	0.50GeV	0.54GeV	1.80GeV	2.40

[1] S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, Phys.Rev.D 108 (2023) 094002.
[2] Brodsky, Teramond, Dosch and Erlich, Phys. Rep. 584, 1 (2015).
[3] Li, Maris, Zhao and Vary, Phys. Lett. B (2016); M. Burkardt, Phys. Rev. D 58, 096015 (1998).

Unpolarized PDFs

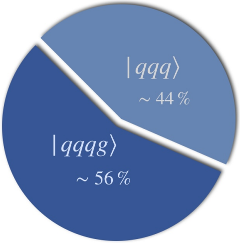
Fock expansion:

$$|\text{Proton}\rangle = a\,|uud\rangle + b\,|uudg\rangle + \ldots$$

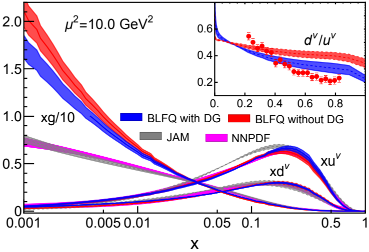
Light-front effective Hamiltonian :

$$H_{\text{eff}} = \sum_a \frac{\vec{p}^{\,2}_{\perp a} + m_a^2}{x_a} + H_{\text{confinement}} + H_{\text{vertex}} + H_{\text{inst}}$$

Fock Sector Decomposition



Model scale $\mu_0^2 = 0.24 \pm 0.01$ GeV^2

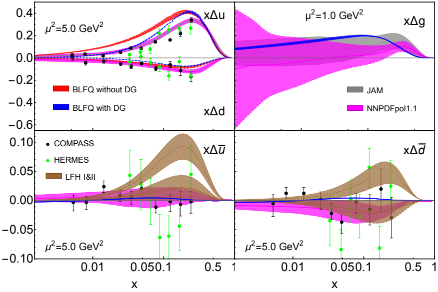


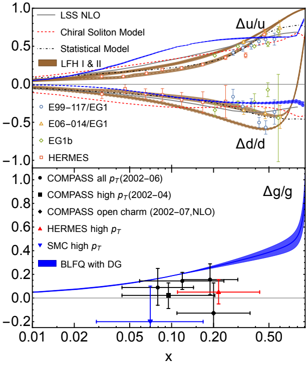
Diagonalizing H_{eff} ⇒ LF wavefunction ⇒ Initial PDFs ⇒ Scale evolution

- $\langle x \rangle_u = 0.261 \pm 0.005$, $\langle x \rangle_d = 0.109 \pm 0.005$ at 10 GeV^2.

[1] BLFQ Collaboration, Phys.Rev.D 108 (2023) 094002.

Helicity PDFs





- Quark spin: $\frac{1}{2}\Sigma_u = 0.438 \pm 0.004, \quad \frac{1}{2}\Delta\Sigma_d = -0.080 \pm 0.002.$
- Gluon spin: $\Delta G = 0.131 \pm 0.003$, (PHENIX: $\Delta G^{[0.02,0.3]} = 0.2 \pm 0.1$).

[1] BLFQ: PRD 108 (2023) 094002; LFH: PRL 124 (2020), 082003; PHENIX: PRL 103 (2009) 012003.

BLFQ Results for Spin Decomposition

Fock expansion:

$$|\,\text{Proton}\rangle = a\,|\,uud\rangle + b\,|\,uudg\rangle + \ldots$$

Quark and gluon helicities :

$$\Delta\Sigma_q = \int \mathrm{d}x\,\Delta q(x)$$

$$\Delta\Sigma_g = \int \mathrm{d}x\,\Delta G(x)$$

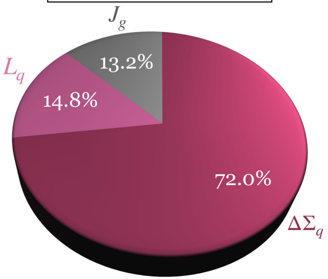
Total AM[2] :

$$J_i = \frac{1}{2}\int \mathrm{d}x\, x\,[H_i(x,0,0) + E_i(x,0,0)]$$

Kinetic OAM[2] :

$$L_q = \frac{1}{2}\int \mathrm{d}x\,[x\,\{H_q(x,0,0)+E_q(x,0,0)\}-\widetilde{H}_q(x,0,0)]$$

$$\boxed{\frac{1}{2}\Sigma_q + L_q + J_g = \frac{1}{2}}$$



Kinetic[1]

[1] S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, Phys.Rev.D 108, 094002 (2023).
[2] X. Ji, Phys.Rev.Lett. 78, 610 (1997).

BLFQ Results for Spin Decomposition

Fock expansion:

$$|\,\text{Proton}\rangle = a\,|\,uud\rangle + b\,|\,uudg\rangle + \ldots$$

Quark and gluon helicities :

$$\Delta\Sigma_q = \int \mathrm{d}x\,\Delta q(x)$$

$$\Delta\Sigma_g = \int \mathrm{d}x\,\Delta G(x)$$

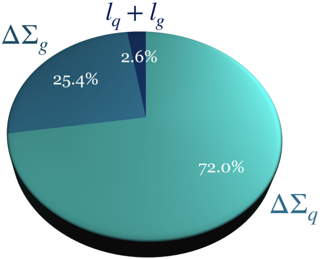
Total AM :

$$J_i = \frac{1}{2}\int \mathrm{d}x\,x\,[H_i(x,0,0) + E_i(x,0,0)]$$

Canonical OAM :

$$l_i^z = -\int \mathrm{d}x\,\mathrm{d}^2\vec{p}_\perp\,\frac{\vec{p}_\perp^{\,2}}{M^2}\,F_{1,4}^i(x,0,\vec{p}_\perp^{\,2},0,0)$$

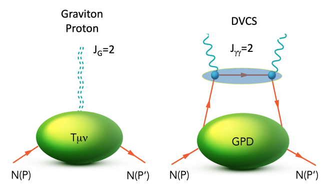
$$\boxed{\frac{1}{2}\Sigma_q + \Sigma_g + l_q^z + l_g^z = \frac{1}{2}}$$



Canonical[1]

[1] S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, Phys.Rev.D 108, 094002 (2023).
[2] Jaffe and Manohar, Nucl.Phys.B 337, 509–546 (1990).

Proton Gravitational Form Factors



- Parametrization of matrix element in terms of GFFs

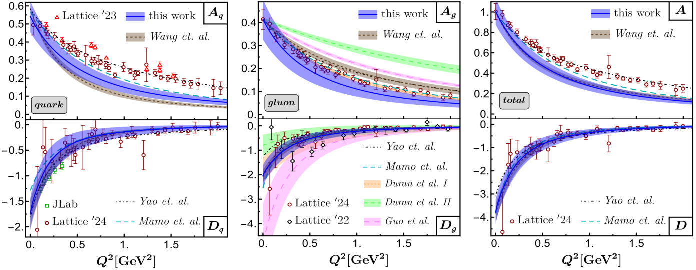
$$\langle P'|T_i^{\mu\nu}(0)|P\rangle = \bar{U}'\Big[-B_i(q^2)\frac{\bar{P}^\mu\bar{P}^\nu}{M} + (A_i(q^2)+B_i(q^2))\frac{1}{2}(\gamma^\mu\bar{P}^\nu+\gamma^\nu\bar{P}^\mu)$$
$$+C_i(q^2)\frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} + \bar{C}_i(q^2)Mg^{\mu\nu}\Big]U$$

- Momentum sum rule : $\sum_i A^i(0) = 1$
- Gravitomagnetic moment sum rule : $\sum_i B^i(0) = 0$
- Spin sum rule: $J^i = \frac{1}{2}\left[A^i(0) + B^i(0)\right]$
- $4C(q^2) = D(q^2)$ provides shear forces and the pressure distributions

[1] Burkert *et. al.*: Rev. Mod. Phys. 95, 041002 (2023)
[2] Ji, Phys. Rev. Lett. 78, 610 (1997)

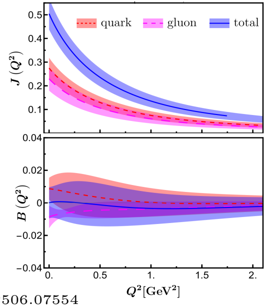
BLFQ Results for Proton GFFs

$$|\text{Nucleon}\rangle = \psi_{(3q)}|qqq\rangle + \psi_{(3q+1g)}|qqqg\rangle$$



- QCD EMT : $T^{\mu\nu} = T_q^{\mu\nu} + T_g^{\mu\nu}$, where $T_q^{\mu\nu} = i\overline{\psi}\gamma^{\{\mu}\mathcal{D}^{\nu\}}\psi$ and $T_g^{\mu\nu} = -F^{\mu\lambda a}F^{\nu}_{\lambda a} + \frac{1}{4}g^{\mu\nu}\left(F_{\lambda\sigma a}\right)^2$
- $A(Q^2)$ and $B(Q^2)$: T^{++} component
- $D(Q^2) = 4C(Q^2)$: T^{ij} components

$\sum_i A^i(0) = 1$ and $\sum_i B^i(0) = 0$



[1] S. Nair, CM, Siqi Xu, Xingbo Zhao, James P. Vary, arXiv: 2506.07554

Effective Hamiltonian with Dynamical Gluon and Sea Quarks

Fock expansion:

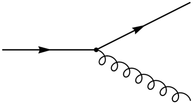
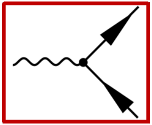
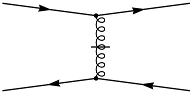
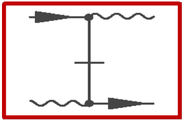
$$|\,\text{Proton}\rangle = a\,|\,uud\rangle + b\,|\,uudg\rangle + c_1\,|\,uudu\bar{u}\rangle + c_2\,|\,uudd\bar{d}\rangle + c_3\,|\,uuds\bar{s}\rangle + \ldots$$

Light-front QCD Hamiltonian :

$$H_{\text{LF}} = \sum_a \frac{\vec{p}^{\,2}_{\perp a} + m_a^2}{x_a} + \cancel{H_{\text{confinement}}} + H_{\text{vertex}} + H_{\text{inst}}$$

$$H_{\text{vertex}} + H_{\text{inst}} = g_s \bar{\psi}\gamma_\mu T^a A^\mu_a \psi + \frac{1}{2} g_s^2 \bar{\psi}\gamma^+ T^a \psi \frac{1}{(i\partial^+)^2} \bar{\psi}\gamma^+ T^a \psi$$

$$+ \frac{1}{2} g_s^2 \bar{\psi}\gamma^\mu A_\mu \frac{\gamma^+}{(i\partial^+)} A_\nu \gamma^\nu \psi$$

[1] Brodsky, Pauli, and Pinsky, Phys. Rep. 301, 299 (1998).
[2] Siqi Xu, Yiping Liu, CM, *et. al.*, PLB 867, 139599 (2025)

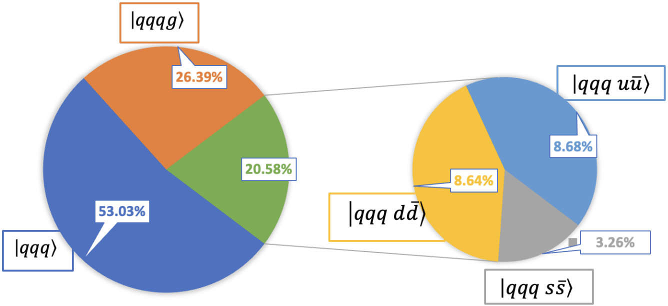
Fock Sector Decomposition

$$|P_{proton}\rangle \rightarrow |qqq\rangle + |qqqg\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle$$

Truncation parameter: $N_{\max} = 7$ and $K_{\max} = 16$

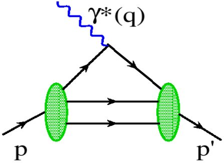
m_u	m_d	m_f	g	b	b_{inst}
0.99 GeV	0.94 GeV	5.9 GeV	3.0	0.6 GeV	2.7 GeV

In five quark Fock sector, we use current quark mass



[2] Siqi Xu, Yiping Liu, CM, *et. al.*, PLB 867, 139599 (2025)

Proton EM Form Factors



Sach's form factors

$$G_E(q^2) = F_1(q^2) - \frac{q^2}{4M^2} F_2(q^2),$$

$$G_M(q^2) = F_1(q^2) + F_2(q^2).$$

- EM current: $J^\mu = \bar{\psi}\gamma^\mu\psi$
- $\langle p'; \uparrow | J^+(0) | p; \uparrow (\downarrow) \rangle \sim F_{1(2)}(q^2)$
- Two FFs: $F_{1(2)}(q^2 = -Q^2)$

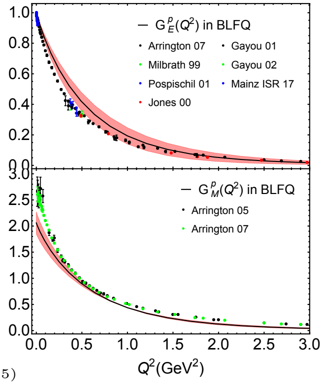
Proton Radii

$$\langle r_\mathrm{E}^2 \rangle = -6 \frac{\mathrm{d}G_\mathrm{E}(Q^2)}{\mathrm{d}Q^2}\bigg|_{Q^2=0},$$

$$\langle r_\mathrm{M}^2 \rangle = -\frac{6}{G_\mathrm{M}(0)} \frac{\mathrm{d}G_\mathrm{M}(Q^2)}{\mathrm{d}Q^2}\bigg|_{Q^2=0}.$$

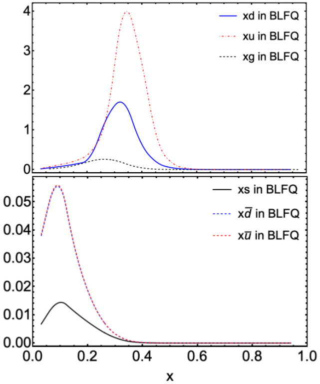
$$\sqrt{\langle r_E^2 \rangle} = 0.72 \pm 0.05 \, (0.840^{+0.003}_{-0.002}) \text{ fm}$$

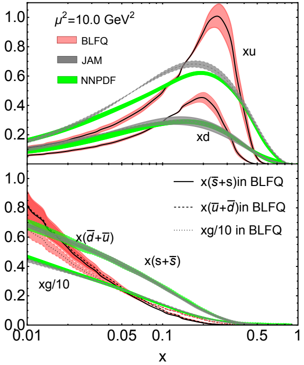
$$\sqrt{\langle r_M^2 \rangle} = 0.73 \pm 0.02 \, (0.849^{+0.003}_{-0.003}) \text{ fm}$$



[1] Siqi Xu, Yiping Liu, CM, *et. al.*, PLB 867, 139599 (2025)

Unpolarized PDFs



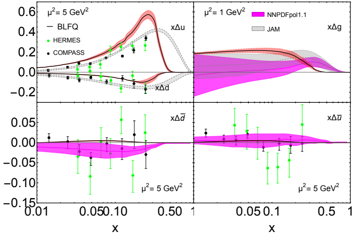


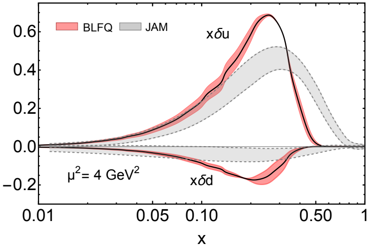
Diagonalizing H_{LFQCD} ⇒ LFWFs ⇒ Initial PDFs ⇒ Scale evolution

- Model scale $\mu_0^2 = 0.22 \pm 0.02$ GeV^2, $\langle x \rangle_{u+d} = 0.37 \pm 0.01$ 10 GeV^2.
- Longitudinal excitations is challenging, in absence of confining potential.

[1] Siqi Xu, Yiping Liu, CM, *et. al.*, PLB 867, 139599 (2025)

Helicity and Transversity PDFs

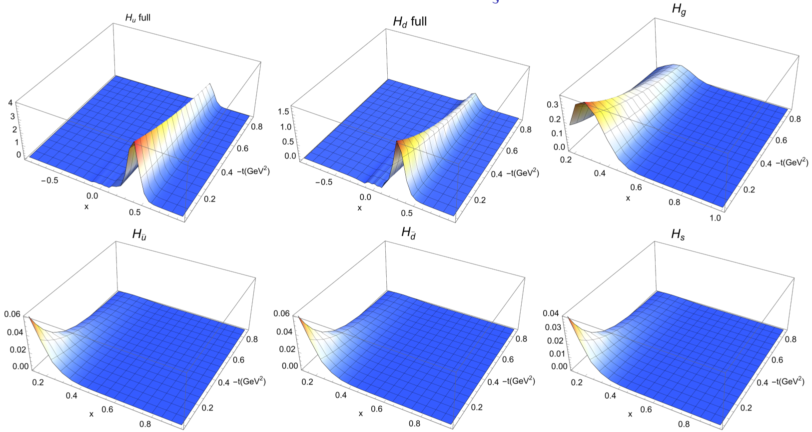




- Gluon spin $\Delta G = 0.29 \pm 0.03$[1] for $x_g \in [0.05, 0.2]$ at 10 GeV^2
- NNPDF[2] analysis: $\Delta G = 0.23(6)$; lattice QCD[3]: $\Delta G = 0.251(47)(16)$
- Tensor Charges: $\delta u = 0.81 \pm 0.08$, $\delta d = -0.22 \pm 0.01$[1]
- JAM[4] analysis: $\delta u = 0.71(2)$, $\delta d = -0.200(6)$; lattice QCD[5]: $\delta u = 0.784(28)$, $\delta d = -0.204(11)$.

[1] Siqi Xu, Yiping Liu, CM, *et. al.*, PLB 867, 139599 (2025)
[2] E. R. Nocera, *et. al.* (NNPDF), Nuclear Physics B 887, 276 (2014)
[3] Y.-B. Yang, *et. al.* (Lattice) Phys. Rev. Lett. 118, 102001 (2017)
[4] C. Cocuzza, *et. al.* (JAM), Phys. Rev. Lett. 132, 091901 (2024)
[5] R. Gupta, *et. al.* (Lattice), Phys. Rev. D 98, 091501 (2018)

GPDs at Skewness $\xi = 0.1$



- u and d GPDs from $-1 < x < 1$; exhibit similar distributions.
- At $\xi = 0.1$, DGLAP region dominates
- Gluon and sea quarks GPDs in DGLAP region

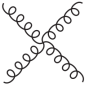
[1] Y. Liu, S. Xu, CM, *et. al.*, coming soon...

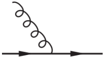
Towards Full BLFQ

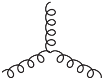
Fock expansion:

Xingbo Zhao: 9/24/25, 8:30 AM

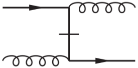
$$|\,\text{Proton}\rangle = |\,uud\rangle + |\,uudg\rangle + |\,uudq\bar{q}\rangle + |\,uudgg\rangle + |\,uudq\bar{q}g\rangle + |\,uudggg\rangle$$

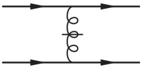


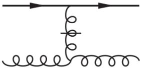


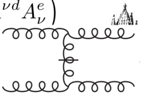


$$\begin{aligned}
\hat{P}^{-}_{\text{LFQCD}} = {} & \frac{1}{2}\int \mathrm{d}x^{-}\mathrm{d}^{2}x^{\perp}\;\; \overline{\psi}\gamma^{+}\frac{(i\partial^{\perp})^{2}+m^{2}}{i\partial^{+}}\psi + A^{ia}(i\partial^{\perp})^{2}A^{ia} \\
& + g_{s}\int \mathrm{d}x^{-}\mathrm{d}^{2}x^{\perp}\,\overline{\psi}\gamma_{\mu}A^{\mu a}T^{a}\psi \\
& + \frac{g_{s}^{2}}{2}\int \mathrm{d}x^{-}\mathrm{d}^{2}x^{\perp}\,\overline{\psi}\gamma_{\mu}A^{\mu a}T^{a}\frac{\gamma^{+}}{i\partial^{+}}\left(\gamma_{\nu}A^{\nu b}T^{b}\psi\right) \\
& + \frac{g_{s}^{2}}{2}\int \mathrm{d}x^{-}\mathrm{d}^{2}x^{\perp}\,\overline{\psi}\gamma^{+}T^{a}\psi\frac{1}{(i\partial^{+})^{2}}\left(\overline{\psi}\gamma^{+}T^{a}\psi\right) \\
& - g_{s}^{2}\int \mathrm{d}x^{-}\mathrm{d}^{2}x^{\perp}\; if^{abc}\;\overline{\psi}\gamma^{+}T^{c}\psi\frac{1}{(i\partial^{+})^{2}}\left(i\partial^{+}A^{\mu a}A^{b}_{\mu}\right) \\
& + g_{s}\int \mathrm{d}x^{-}\mathrm{d}^{2}x^{\perp}\; if^{abc}\; i\partial^{\mu}A^{\nu a}A^{b}_{\mu}A^{c}_{\nu} \\
& + \frac{g_{s}^{2}}{2}\int \mathrm{d}x^{-}\mathrm{d}^{2}x^{\perp}\; if^{abc}\; if^{ade}\; i\partial^{+}A^{\mu b}A^{c}_{\mu}\frac{1}{(i\partial^{+})^{2}}\left(i\partial^{+}A^{\nu d}A^{e}_{\nu}\right) \\
& - \frac{g_{s}^{2}}{4}\int \mathrm{d}x^{-}\mathrm{d}^{2}x^{\perp}\; if^{abc}\; if^{ade}A^{\mu b}A^{\nu c}A^{d}_{\mu}A^{e}_{\nu}.
\end{aligned}$$







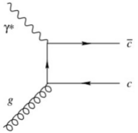


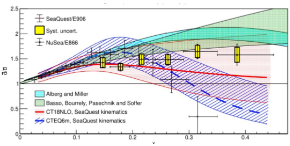
[1] Siqi Xu, *et. al.*, work in progress

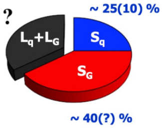
Outlook

Current status

- Full QCD interaction
  - Fock sector expansion $|qqq\ q\bar{q}\ g\rangle$ and $|qqq\ ggg\rangle$......
    - Intrinsic charm
    - Sea asymmetry
    - Origin of spin and mass
- Deutron calculation $|qqq\ qqq\rangle + |qqq\ qqq\ g\rangle$
  - EMC effect







Conclusions

- BLFQ: A non-perturbative approach based on light-front QCD Hamiltonian
- LF Hamiltonian ⇒ Wavefunctions ⇒ Observables
- $|qqq\rangle + |qqqg\rangle$ $(P^- = P^-_{\rm QCD} + P^-_{\rm C})$ ⇒ Provides good description of data/global fits and lattice QCD results for various observables
- $|qqq\rangle + |qqqg\rangle + |qqqq\bar{q}\rangle$, $(P^- = P^-_{\rm QCD})$ ⇒ Provides qualitative description of data/global fits for mass, spin, EMFFs, PDFs, axial and tensor charges ...

Outlook

- Expand Fock sectors Xingbo Zhao: 9/24/25, 8:30 AM
- Include four-gluon interactions in the Hamiltonian

Thank You